Experimental Test of Special Relativity from a High-γ Electron $g-2$ Measurement

P. S. Cooper, M. J. Alguard,[(a)] R. D. Ehrlich,[(b)] V. W. Hughes, H. Kobayakawa,[(c)] J. S. Ladish,[(d)] M. S. Lubell, N. Sasao,[(e)] K. P. Schüler, and P. A. Souder
J. W. Gibbs Laboratory, Yale University, New Haven, Connecticut 06520

and

D. H. Coward, R. H. Miller, C. Y. Prescott, D. J. Sherden, and C. K. Sinclair
Stanford Linear Accelerator Center, Stanford, California 94305

and

G. Baum and W. Raith
University of Bielefeld, Bielefeld, West Germany

and

K. Kondo
University of Tsukuba, Ibaraki, Japan
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We report a verification of the theory of special relativity at a value of $\gamma \approx 2.5\times10^4$ based upon a comparison of electron $g-2$ measurements at meV and GeV kinetic energies. Specially we obtain a measure of the equivalence between the quantities $\gamma \equiv (1-\beta^2)^{-1/2}$ and $\tilde{\gamma} \equiv (p/m_0)dp/dE$.

A recent publication[1] has pointed out that an experimental test of special relativity is provided by comparing the values of the electron g-factor anomaly a $[a \equiv \frac{1}{2}(g-2)]$ for electrons with different velocities or γ values. Special relativity predicts that the value of a should be independent of the electron velocity. Newman *et al.*,[1] refer to two measurements of a, one done with electrons of 1 meV kinetic energy ($\gamma - 1 = 10^{-9}$) and the other done with electrons of 100 keV kinetic energy ($\gamma = 1.2$). These two measured values agree. We point out here that another measurement of a has been done with electrons of about 12 GeV kinetic energy ($\gamma \sim 2.5\times10^4$), which is relevant to this test of special relativity.[2]

The high-γ $g-2$ measurement[3] was obtained as a by-product of the measurement of the polarization of the high-energy longitudinally polarized electron beam at the Stanford Linear Accelerator Center (SLAC). After acceleration to high energy the longitudinally polarized beam was deflected through the beam switchyard by an angle $\theta_c = 24.5°$ into the experimental area, with the spin precessing relative to the momentum by an angle

$$\theta_a = \gamma a \theta_c. \tag{1}$$

The longitudinal component of the beam polarization is then given by

$$P(E) = P_0 \cos(\pi E/E_0 + \varphi_0), \tag{2}$$

in which P_0 is the magnitude of the initial vector polarization, $\vec{P}_0$, of the electron beam before the magnetic deflection, φ_0 is projected angle of $\vec{P}_0$ with respect to the electron momentum in the plane of the bent trajectory, E is the electron energy, and E_0 is defined as

$$E_0 = \left(\frac{180°}{24.5°}\right)\frac{m_0 c^2}{a} \simeq 3.2 \text{ GeV}, \tag{3}$$

where m_0 is the electron rest mass.

The longitudinal polarization of the deflected beam was measured by Møller scattering[4] from a Supermendur target foil magnetized to saturation in a 90-G longitudinal magnetic field and inclined at 20° with respect to the beam direction in order to provide a large component of longitudinal polarization. Reversal of the 90-G field reversed the polarization of the target. The Møller-scattered electrons were observed by conventional particle-detection techniques with the SLAC 8-GeV/c spectrometer.[5]

The results of the Møller measurement are shown in Fig. 1 together with the fitted curve $P(E)$ given by Eq. (2) with P_0 and a as free parameters and φ_0 fixed at zero. The data points shown are taken from the earlier publication.[3] From the fit, the value $a = (1.1622 \pm 0.0200)\times10^{-3}$ is obtained, where the quoted 1.7% uncertainty is the linear contribution of counting statistics (0.7%) and possible systematic effects (1.0%). The systematic contributions are the estimated 0.3% uncertainty in the absolute momentum calibration of the beam switchyard magnet system,[6] and an uncertainty of 82 mrad in the value of φ_0, which

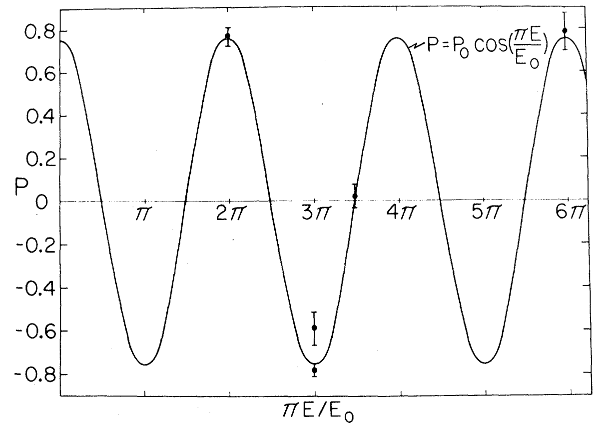

FIG. 1. The longitudinal component P of the electron beam polarization plotted as a function of E/E_0, the angle through which the spin precesses relative to the momentum during the 24.5° magnetic bend into the experimental area. The curve shown is the best fit to the data, with a and P_0 as free parameters.

results in a 0.7% uncertainty in a. The estimate of the uncertainty in φ_0 is obtained by considering the calculated upper bound to φ_0 for a single electron in the beam[7] as a 3-standard-deviation effect. Had data been taken at more than one zero crossing point, φ_0 and a could have been separately determined. However, with only one zero crossing the two parameters are highly correlated, which requires φ_0 to be estimated independently as we have done above. In this respect the measurement of a could be significantly improved by the addition of data taken at one or more of the five remaining zero-crossing points in the present SLAC energy range. Finally, the fitted value of P_0 is 0.755 ± 0.026, which agrees with both the theoretical expectations and experimental measurements of the polarization of the injected electrons.

We note that our value of a from the high-γ measurement agrees with the more precise values of a determined in the lower-γ measurements. In Table I, we summarize the lepton $g-2$ measurements for electrons and muons. For the high-γ electron $g-2$ measurement reported in this paper, we use the average value of γ over the range covered by the experiment; namely, $\gamma=1.27\times10^4$ to $\gamma=3.81\times10^4$. We note that this average value, $\gamma=2.5\times10^4$, is very close to the only zero-crossing point, $\gamma=2.22\times10^4$, at which we have obtained data. Since the measured value of $P(E)$ obtained at the crossing point most strongly influences our value of a, the choice of $\gamma=2.5\times10^4$ to characterize our measurement of a seems well justified.

TABLE I. Lepton $g-2$ measurements.

Lepton	Reference	γ	$a\times10^3$ [a]
e^-	8	$1+10^{-9}$	1.159 652 41(20)
	9	1.2	1.159 657 70(350)
	The present work	2.5×10^4	1.1622(200)
μ^-,μ^+	10	12	1.166 16(31) [b]
	11	29.2	1.165.922(9) [b]

[a] The errors quoted are 1-standard-deviation uncertainties in the last digits.
[b] Average μ^- and μ^+.

Any discussion of the sensitivity of these measurements as a test of special relativity requires, of course, some theoretical model for, or at least a parametrization of, a breakdown of special relativity. As a theoretical problem applied to the $g-2$ measurements, some breakdown of relativistic quantum field theory may be involved. This is a very profound problem which must involve some preferred frame of reference, perhaps determined from cosmological considerations. A systematic phenomenological viewpoint might involve an analysis of the accuracy with which the coefficients of the Lorentz transformation are tested. A recent theoretical model[12,13] for a breakdown of special relativity predicts effects proportional to γ^2.

In their parametrization, Newman *et al.* introduce $\gamma\equiv(P/m_0)dp/dE$ which they allow to be different from $\gamma=(1-\beta^2)^{-1/2}$. Hence the cyclotron, spin, and $g-2$ precession frequencies for motions perpendicular to a magnetic field $\vec{B}$ are given, respectively, by

$$\omega_c=eB/\tilde{\gamma}m_0c, \tag{4}$$

$$\omega_s=geB/2m_0c+(1-\gamma)\omega_c, \tag{5}$$

$$\omega_a=\omega_s-\omega_c=(\tfrac{1}{2}g-\gamma/\tilde{\gamma})eB/m_0c. \tag{6}$$

The term $(1-\gamma)\omega_c$ in Eq. (5) is the Thomas precession frequency and is regarded by Newman *et al.* as of kinematic origin and hence involves the usual γ term, whereas the term $\tilde{\gamma}$ in Eq. (4) is regarded as arising from electron dynamics and hence as possibly different. The $g-2$ experiments determine the quantity $\omega_a(eB/m_0c)^{-1}$ which in the conventional theory equals $\frac{1}{2}(g-2)=a$. Following the parametrization of Newman *et al.*, we

TABLE II. Summary of lepton $g-2$ relativity tests.

Method	References	$\gamma^{(1)}$	$\gamma^{(2)}$	C_1
μ^-, μ^+ g factor	10 and 11	12	29.2	$(1.4 \pm 1.8)\times 10^{-8}$
e^- g factor	8 and 9	1	1.2	$(-2.6 \pm 1.8)\times 10^{-8}$
	8 and the present work	1	2.5×10^{4}[a]	$(-1.0 \pm 8.0)\times 10^{-10}$

[a]The g factor was measured over the γ interval $(1.3-3.8)\times 10^4$.

set

$$\omega_a(eB/m_0c)^{-1} = \tfrac{1}{2}g - \gamma/\tilde{\gamma} = a \qquad (7)$$

and regard the various $g-2$ measurements done at different electron velocities as determining $\gamma/\tilde{\gamma}$.

In the accompanying Letter by Combley *et al.*,[14] a more general phenomenological viewpoint of a breakdown of special relativity is taken and four distinct γ factors—γ_t, γ_E, γ_M, and γ_T—are introduced for the transformation of time, electromagnetic fields and mass, and for determining the Thomas precession. With certain assumptions about relations among these four γ factors, the parametrization of Combley *et al.* reduces to that of Newman *et al.*

We use the phenomenological model of Newman *et al.*, and, in addition, as do Combley *et al.*, and, in addition, as do Combley *et al.*,[14] assume a power-series expansion for $\gamma/\tilde{\gamma}$ of the form

$$\gamma/\tilde{\gamma} = 1 + C_1(\gamma - 1) + \dots, \qquad (8)$$

which preserves the nonrelativistic equivalence of γ and $\tilde{\gamma}$ in the limit $\gamma \to 1$. In order to define a figure of merit, we retain only the leading nonconstant term in Eq. (8). Then for each lepton, $g-2$ measurements at two values of γ suffice to determine C_1 according to

$$C_1 = (a^{(2)} - a^{(1)})/(\gamma^{(1)} - \gamma^{(2)}) \qquad (9)$$

for measurements $a^{(1)}$ and $a^{(2)}$ at $\gamma^{(1)}$ and $\gamma^{(2)}$, respectively. In Table II, we present the values of C_1 derived from various pairs of lepton $g-2$ measurements given in Table I. Implicit in this parametrization are the assumptions that any violation of special relativity vanishes as one approaches the nonrelativistic limit, and that g is a constant independent of γ. We note that although our measurement of a is relatively imprecise, our value of γ is comparatively very large. Thus our experiment provides a sensitive determination of the coefficients in a power-series expansion such as given by Eq. (8).

As can be seen from Table II, the limit on $|C_1|$ of $< 1.7\times 10^{-9}$ measurement is the most sensitive upper limit obtained to date. Within the framework of the relativity-breaking model expressed by Eq. (8), we have thus demonstrated the equivalence of γ and $\tilde{\gamma}$. Of course, a linear dependence on $\gamma - 1$ is but one possible choice. Indeed, Rédei,[12,13] in a discussion of the validity of special relativity at small distances and the existence of a universal length, suggests that for the lifetime of the muon a modification with a leading term quadratic in γ should be introduced. In the context of higher-order terms we wish to point out that the relative sensitivity of our measurement is enhanced by any higher-order dependence on $\gamma - 1$.

In conclusion, we emphasize that we have included in our discussion only those tests of special relativity which are directly comparable to ours. For reference to other tests see Newman *et al.*[1] and Bailey *et al.*[15]

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[(a)]Present address: Department of Electrical Engineering, Stanford University, Stanford, Calif. 94305.

[(b)]Present address: Department of Physics, Cornell University, Ithaca, N. Y. 14850.

[(c)]Present address: Nagoya University, Nagoya, Japan.

[(d)]Present address: Los Alamos Scientific Laboratory, Los Alamos, N. M. 87545.

[(e)]Present address: Kyoto University, Kyoto, Japan.

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