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The analysis of Polyakov loop and spin correlators in finite volumes *

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We derive an analytic expression for point-to-point correlation functions of the Polyakov loop based on the transfer matrix formalism. The contributions from the eigenvalues of the transfer matrix including and beyond the mass gap are investigated both for the $2d$ Ising model and in finite temperature $SU(2)$ gauge theory. We find that the leading matrix element shows similar scaling properties in both models. Just above the critical point we obtain for $SU(2)$ a Debye screening mass $\mu_D/T \approx 4$, independent of the volume.

1. Introduction

The determination of the correlation length ξ and the screening mass μ_D from point-to-point correlation functions of the Polyakov loop is a non-trivial task, especially close to the critical point of lattice gauge theories. The difficulties are resulting on one hand from finite volume effects due to the nearby transition and on the other hand from the unknown parametrisation of the heavy quark potential in the non-perturbative regime.

In the transfer matrix (TM) formalism the levels of the transfer matrix provide an access to both ξ and μ_D without the introduction of an ansatz for the quark potential. The levels and matrix elements may be obtained easily from fits to measured plane-plane (or zero momentum) correlation functions since their TM form is known and simply exponential. In this paper we intend to derive the corresponding expression for point-to-point correlation functions. In the $2d$ Ising model we test the validity of our TM formula by comparison to the results obtained from plane-plane correlators. Simultaneously we are able to determine, where levels beyond the mass gap are of importance and what can be expected from such an analysis.

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2. Correlation functions in the transfer matrix formalism

Let us consider $d-$dimensional spatial lattices of size $N^{d-1}L$, where N denotes the number of points in each transverse direction and L that in one selected direction (the $z-$direction). The lattice spacing a is set to unity in the following. The partition funtion is then

$$Z \equiv \mathrm{Tr}\left(\mathbf{V}^L\right), \qquad (1)$$

and $\mathbf{V}$ is the transfer matrix in $z-$ direction. Its eigenstates $|\, n\rangle$ and eigenvalues λ_n ($n =$ 0, 1, 2, ...)

$$\mathbf{V}\,|n\rangle = \lambda_n \cdot |n\rangle\,; \quad \lambda_n \equiv e^{-\mu_n}; \qquad (2)$$

$$\mu_0 < \mu_1 < \mu_2 < \dots\,. \qquad (3)$$

are chosen to be orthonormal. In addition we normalize our partition function such that we have for the vacuum state

$$\lambda_0 = 1, \ \mu_0 = 0. \qquad (4)$$

Next we define zero momentum operators by

$$\tilde{\mathcal{O}}(z) = N^{-\frac{d-1}{2}} \cdot \sum_{\vec{x}_\perp} \mathcal{O}(\vec{x}_\perp, z)\,, \qquad (5)$$

where $\mathcal{O}(\vec{x}_\perp, z)$ is the Polyakov loop $\mathcal{P}(\vec{x}_\perp, z)$ for the $3+1$ dimensional $SU(2)$ gauge theory and the spin $\sigma_{x,z}$ for the $2d$ Ising model. The corresponding correlation functions are

$$\tilde{\Gamma}(z) = \langle \tilde{\mathcal{O}}(z) \cdot \tilde{\mathcal{O}}(0) \rangle, \qquad (6)$$

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resulting in

$$\tilde{\Gamma}(z) = \sum_{n<m} \frac{c_{mn}^2}{Z} e^{-\mu_n L} \left[e^{-\mu_{mn} z} + e^{-\mu_{mn}(L-z)} \right], \quad (7)$$

where

$$\mu_{mn} = \mu_m - \mu_n ; \; c_{mn} = \langle n \,|\, \tilde{\mathcal{O}}(0) \,|\, m \rangle \; , \quad (8)$$

are the level difference and the transition matrix element. Due to the symmetry properties of the eigenstates under tranformations, which change the sign of $\mathcal{O}$, $c_{nn} = 0$.

Below the critical point $\beta < \beta_c$ the lowest nonzero energy level μ_1 (the mass gap) defines the large distance behaviour of the correlator. We therefore define the correlation length at $\beta \lesssim \beta_c$ by

$$\xi_-(\beta) \equiv \mu_1^{-1} \sim |\beta - \beta_c|^{-\nu} \; . \quad (9)$$

At $\beta > \beta_c$ the mass gap $\mu_1 \approx 0$ and the large distance behaviour is given by the next level difference $\Delta\mu$, so that the Debye mass is

$$\mu_D = 2m_D \equiv \Delta\mu \; , \quad (10)$$

where m_D is the perturbative screening mass.

A similar formula as eq.7 may now be found for the point-to-point correlator

$$\Gamma(\vec{x}) = \langle \mathcal{O}(\vec{x}) \cdot \mathcal{O}(0) \rangle , \quad (11)$$

in the following way. The Fourier transforms of $\tilde{\Gamma}(z)$ and $\Gamma(\vec{x})$ are related by

$$\tilde{\Gamma}(p_z) \equiv \Gamma(\vec{p}_\perp = 0, p_z) \; . \quad (12)$$

This leads us to the ansatz

$$\Gamma(\vec{p}) = Z^{-1} \sum_{n<m} c_{mn}^2 e^{-\mu_n L} G(\vec{p}; \mu_{mn}) \; , \quad (13)$$

where

$$G(\vec{p}; \mu) = \frac{2\left(1 - e^{-\mu L}\right) \sinh \mu}{4 \sinh^2 \frac{\mu}{2} + \sum_{i=1}^{d} 4 \sin^2 \frac{p_i}{2}} \; , \quad (14)$$

and $G(\vec{p}_\perp = 0, p_z; \mu)$ is just the Fourier transform of $\left[e^{-\mu_{mn} z} + e^{-\mu_{mn}(L-z)} \right]$, i.e. we have added corresponding contributions for the missing momenta components in the denominator of eq.14. It is now straightforward to obtain $\Gamma(\vec{x})$ by another Fourier transformation of eq.13.

3. Results

We have used the $2d$ Ising model to test our ansatz, eq.13. To this end we have measured plane-plane and point-to-point correlators on $N = L = 30, 40, 50, 60$ lattices. At each point 500000 cluster updates were performed and measurements taken every 10th update. In the twodimensional model the levels μ_n are explicitly known [1,2]. We have carried out fits to both correlators with varying numbers of levels to obtain the matrix elements. Both formulae lead to the same results, when the maximal number of levels is taken into account which lead to non-negative c_{mn}^2; i.e. our ansatz is definitely confirmed. The final result is shown in Fig.1 for $N = L = 30$. We find that for $\beta < \beta_c$ only one term with $\mu_{10} = \mu_1$, the mass gap, contributes; near β_c up to three terms are essential and well above β_c, where $\mu_1 \approx 0$ only one more term is present.

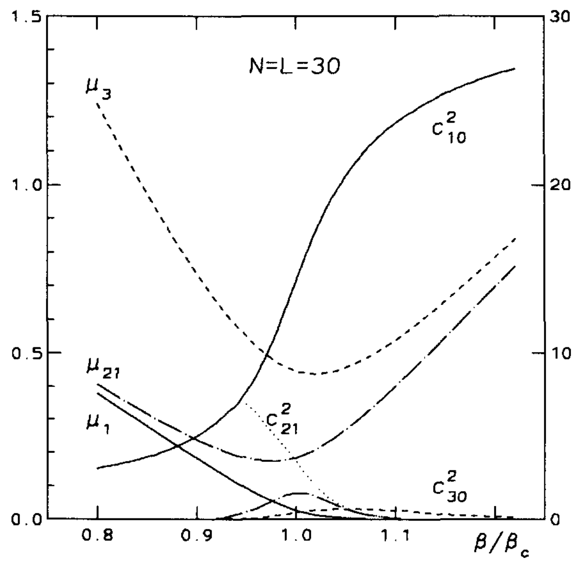

Figure 1. The lowest level differences μ_1, μ_3 and μ_{21} and the respective matrix elements c_{10}^2, c_{30}^2 and $c_{21}^2 e^{-\mu_1 L}$ vs. β/β_c in the $2d$ Ising model.

We now want to apply our TM formula for the point-to-point correlator to $SU(2)$ gauge theory. The Monte Carlo data [3] were computed on $N_\sigma^3 \times 4$ lattices, $N_\sigma = 12, 18, 26$ with $10^5 - 4 \cdot 10^5$ updates and measurements every 10th sweep.

Here the level differences are unknown and have to be determined through the fit. In general we find a very similar behaviour as in the $2d$ Ising model. Fits with more than two levels are only possible on the largest lattice very close to the transition. Otherwise one either obtains negative squares of matrix elements or there is no minimum of χ^2. Taking into account more terms in eq.13 for the fits tends to decrease the result for the mass gap level. This is shown in Fig.2.

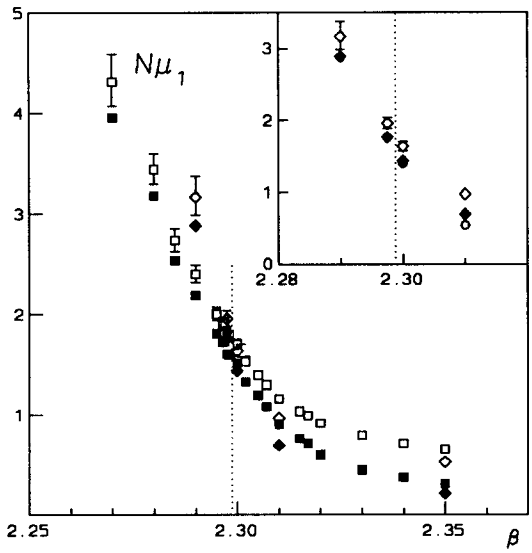

Figure 2. $N\mu_1$ resulting from one- and two-level fits (open and filled symbols) on $N = 18, 26$ (squares and diamonds) lattices for $SU(2)$. The inset shows for $N = 26$ also 3-level fits (circles). The dotted line indicates β_c.

It is interesting to look at the behaviour of the next to leading level (or level difference) μ_2. As can be seen from Fig.3, μ_2 drops from a higher value below β_c at the transition to a value near to one (in lattice units) and stays then relatively constant and moreover independent of the lattice sizes used here. This second level fixes the large distance behaviour above β_c of the correlation functions, since μ_1, as is evident from Fig.2, is essentially zero there and a third level does not contribute. Therefore we identify it with μ_D. Because we have $N_\tau = 4$ we are led to a ratio $\mu_D/T \approx 4$, slightly higher than found with conventional methods[4].

It can be shown [5], that for $N \to \infty$

$$c_{mn}^2 \sim N^0 \text{ ; for } \beta < \beta_c, \tag{15}$$

$$c_{10}^2 \sim N^{d-1}\text{; for } \beta > \beta_c, \tag{16}$$

and from finite size scaling theory [3] for $\beta \approx \beta_c$

$$c_{10}^2 = N^{\gamma/\nu - 1} f(xN^{1/\nu}). \tag{17}$$

These scaling properties are all well confirmed by both the $2d$ Ising model and the $SU(2)$ gauge theory results.

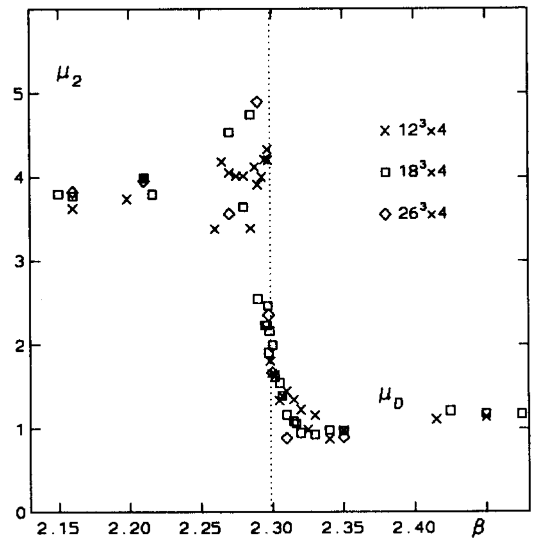

Figure 3. The second level μ_2 from two-level fits in $SU(2)$ gauge theory for $N = 12, 18, 26$ (crosses,squares,diamonds) vs. β .

REFERENCES

1. L. Onsager, Phys. Rev. **65** (1944) 117
2. T.D. Schultz, D.C. Mattis and E.H. Lieb, Rev. Mod. Phys. **36** (1964) 856
3. J. Engels and V.K. Mitrjushkin, Phys. Lett. B282 (1992) 415.
4. J. Engels, F. Karsch and H. Satz, Nucl. Phys.B315 (1989) 419.
5. J. Engels, V.K. Mitrjushkin and T. Neuhaus, Bielefeld Preprint (1993), to appear .