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# The onset of deconfinement in $SU(2)$ lattice gauge theory

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**Abstract.** In $SU(2)$ lattice gauge theory, we study deviations from ideal gas behaviour near the deconfinement point. On lattices of size $N_\sigma^3 \times 4$, $N_\sigma = 8$, 12, 18 and 26, we calculate the quantity $\Delta \equiv (\varepsilon - 3P)/T^4$. It increases sharply just above $T_c$, peaks at $T/T_c = 1.15 \pm 0.05$ and then drops quickly. This form of behaviour is shown to be the consequence of a second order phase transition. Dynamically it could arise because just above $T_c$, the low momentum states of the system are remnant massive modes rather than deconfined massless gluons.

## 1 Introduction

The thermodynamics of $SU(2)$ gauge theory leads to a second order transition between a low temperature phase with gluonium constituents and a high temperature phase containing deconfined gluons. As a consequence of this transition, the energy density $\varepsilon$ of the system changes rapidly in a small interval around the critical temperature $T = T_c$; it grows from a value of the size expected for an ideal gas of gluonium states to a much larger value close to the limiting form

$$\varepsilon_{SB}/T^4 = \pi^2/5 \tag{1}$$

of an ideal gas of gluons with two spin and three colour degrees of freedom [1]. It was noted immediately that the weak temperature dependence of $\varepsilon$ soon after $T_c$ and its rapid convergence to values near $\varepsilon_{SB}$ do not imply the absence of interactions in the deconfined phase. The quantity

$$\Delta \equiv (\varepsilon - 3P)/T^4, \tag{2}$$

where $P$ denotes the pressure, vanishes for a massless ideal gas; for $SU(2)$ gauge theory, it was found to differ from zero over a considerable range of temperatures above $T_c$ [2]. Other lattice studies [3] have since then confirmed this. Once the temperature is above the transition region, $\Delta$ decreases, and this decrease has been compared to the predictions of perturbation theory and of the bag model [4], as well as to a description including remnant massive modes at low momenta [3]. The functional behaviour of $\Delta$ has also been parametrized in terms of an ideal gas form for $\varepsilon$, but with a pressure containing interaction effects linear in $T$ [6, 7]; very recently, it was interpreted in terms of differences between chromoelectric and chromomagnetic contributions [3]. From all these studies it has become increasingly clear that there are considerable thermodynamic interaction effects in the temperature range $1 \leq T/T_c \leq 2-3$. However, the detailed behaviour in the transition region and the origin of the deviations from the ideal gas form (including the question of finite lattice size effects) has remained open. The aim of the present study is therefore to first calculate the relevant thermodynamic quantities near $T_c$ in a high statistics lattice evaluation, for a number of different spatial lattice sizes; we then show that the form of $\Delta$ we obtain is a consequence of the critical behaviour of the theory. Finally, we shall see that such behaviour can be understood dynamically in terms of a rapid transition from massive to massless modes just above deconfinement [5, 8].

## 2 The lattice formulation

On an $N_\sigma^3 \times N_\tau$ lattice, the energy density $\varepsilon$ and the pressure $P$ are determined by

$$\varepsilon/T^4 = 12 N_\tau^4 \{g^{-2}(P_\sigma - P_\tau) + c_\sigma(P_0 - P_\sigma) + c_\tau(P_0 - P_\tau)\}, \tag{3}$$

$$P/T^4 = \varepsilon/(3T^4) - 4 N_\tau^4 \, a \frac{dg^{-2}}{da} (P_\tau + P_\sigma - 2P_0). \tag{4}$$

Here $P_\sigma$ and $P_\tau$ denote the space-space and space-temperature plaquette averages, and $P_0$ that for a symmetric ($N_\sigma^4$) lattice; $g$ is the coupling constant. Asymptotically, it is related to the lattice spacing $a$ by

$$a\Lambda_L = \exp\left\{-\frac{12\pi^2}{11g^2} - \frac{51}{121}\ln\frac{11g^2}{24\pi^2}\right\}. \qquad (5)$$

The derivatives $c_\sigma$ and $c_\tau$ have been evaluated in lowest order perturbation theory [9] giving

$$c_\sigma = c_0 + c_1, \qquad (6)$$

$$c_\tau = -c_0 + c_2, \qquad (7)$$

with $c_0 = 0.1100325$, $c_1 = 0.0040$, $c_2 = 0.04248$ for the $SU(2)$ case. Using (5), one has, again in lowest order,

$$a\frac{dg^{-2}}{da} = -2(c_1 + c_2); \qquad (8)$$

this means that the entropy density $s$,

$$s/T^3 = (\varepsilon + P)/T^4 = 16 N_\tau^4 g^{-2} \rightleftharpoons [1 - g^2(c_0 + 1/2 c_1 - 1/2 c_2)](P_\sigma - P_\tau) \qquad (9)$$

is determined completely in terms of the difference between space-space and space-time plaquettes. – From (3), (4), we get

$$\Delta = 12 N_\tau^4 a \frac{dg^{-2}}{da} [P_\tau + P_\sigma - 2P_0] \qquad (10)$$

for the deviation $\Delta$ from an ideal gas of massless gluons.

## 3 Lattice results

Our calculations were performed on $N_\sigma^3 \times 4$ lattices, with $N_\sigma = 8$, 12 and 18; at a few temperature values, data were also taken with $N_\sigma = 26$. In general, we used 100000 sweeps per temperature point; very close to $T_c$, this was increased to 400000 sweeps per point. The evaluation was carried out with a full group heatbath vector program. For thermalization, the first 1000 iterations were discarded in data-taking.

With the same method, the symmetric plaquette values $P_0$ were calculated on a $16^4$ lattice in the range $2.1 \leqq 4/g^2 \leqq 2.95$ at 30 points with 14000–30000 sweeps. Part of these data are already published [10]. The results for $\varepsilon$ and $P$ are shown in Figs. 1–2; $\Delta$ is shown in Fig. 3. The data are compiled in Tables 1, 2 and 3. We note some finite size dependence near and just below $T_c$, in particular for $N_\sigma = 8$ and 12, and for $\varepsilon$ and hence $\Delta$. On the whole, however, the results appear to stabilize with growing $N_\sigma$. In Fig. 4, we show $\Delta$ at $T/\Lambda_L = 42.3$ $(T/T_c \simeq 1)$ for different $N_\sigma$; for $N_\sigma \geqq 18$, $\Delta$ seems to become quite constant. This is also seen when we compare the ratios of $\Delta$ for different $N_\sigma$ as a function of $4/g^2$. For $T \geqq T_c$, we can thus consider the results with $N_\sigma = 18$ as at least indicative of the infinite volume limit.

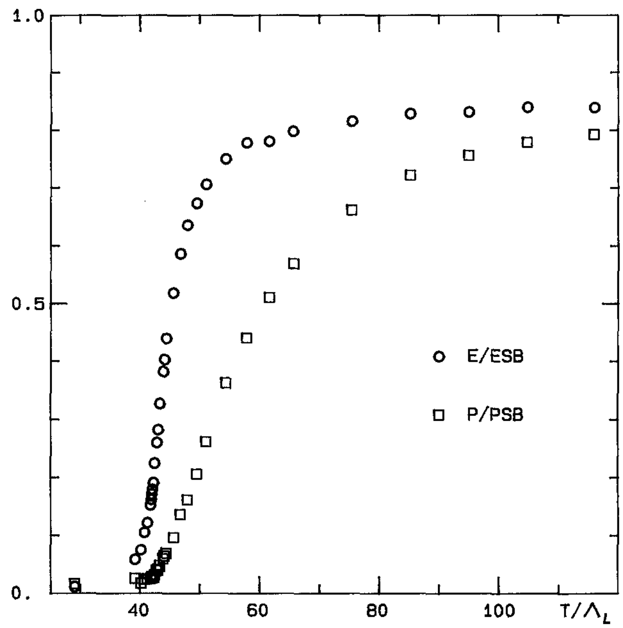


**Fig. 1.** Energy density and pressure, calculated on the $18^3 \times 4$ lattice, normalized to the respective finite lattice Stefan-Boltzmann values [12]

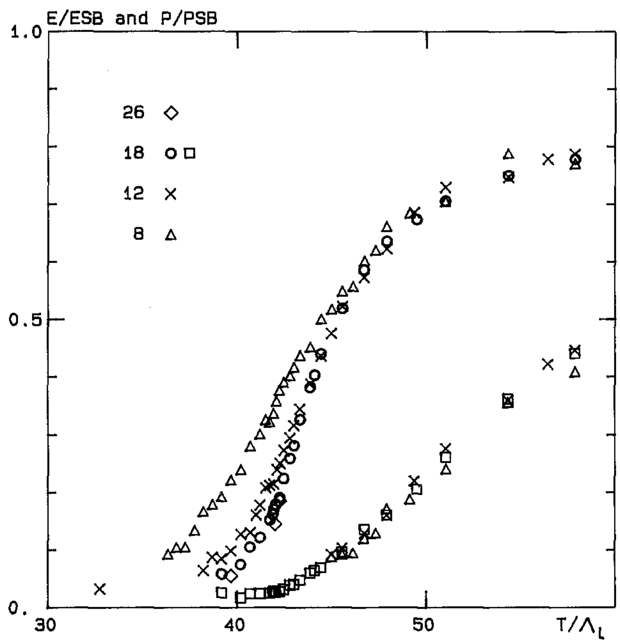


**Fig. 2.** Energy density and pressure, calculated on the $26^3 \times 4$, $18^3 \times 4$, $12^3 \times 4$ and $8^3 \times 4$ lattices and normalized as in Fig. 1; for the pressure only the points for $18^3 \times 4$ are shown below $T/\Lambda_L = 45$

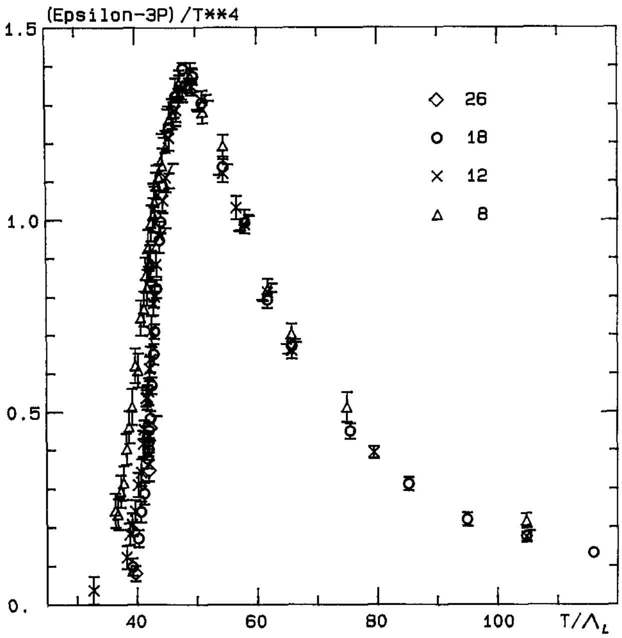


**Fig. 3.** The interaction measure $\Delta$ for different lattice sizes

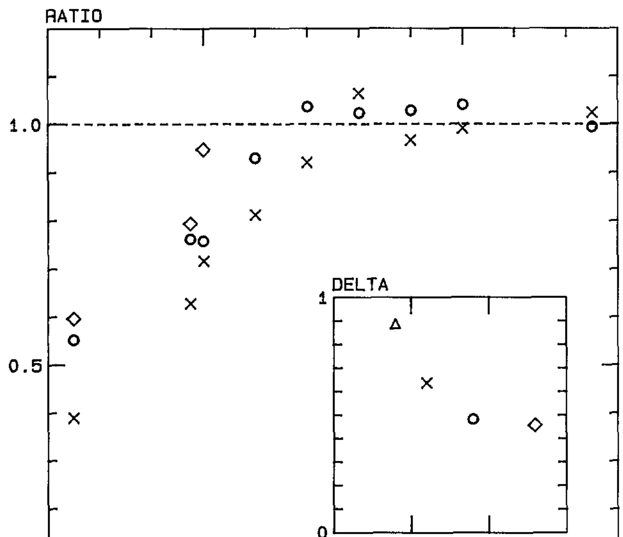


**Fig. 4.** The ratios of $\Delta$ for $N_\sigma=26$ to $N_\sigma=18$ (◇), 18 to 12 (○) and 12 to 8 (×) as function of $4/g^2$. In the inset, $\Delta$ is shown at the critical coupling $4/g^2=2.3$ for different $N_\sigma$

The most striking feature is the pronounced sharp peak in $\Delta$, which occurs at

$$T/T_c \simeq 1.15 \pm 0.05 \tag{11}$$

and thus is clearly above $T_c$. We shall now show that this peak is associated to the occurence of a second order phase transition, and then study some possible dynamical mechanisms underlying this transition.

**Table 1.** Numerical results from the $8^3 \times 4$ lattice

| $4/g^2$ | $T$ | $\varepsilon/T^4$ | $P/T^4$ | $s/T^3$ | $\Delta$ |
|---|---|---|---|---|---|
| 2.2400 | 36.37 | 0.2567 | 0.0041 | 0.2608 (458) | 0.2444 (443) |
| 2.2450 | 36.83 | 0.2892 | 0.0182 | 0.3073 (460) | 0.2346 (403) |
| 2.2500 | 37.29 | 0.2896 | −0.0014 | 0.2882 (458) | 0.2940 (414) |
| 2.2550 | 37.76 | 0.3715 | 0.0187 | 0.3902 (460) | 0.3153 (433) |
| 2.2600 | 38.24 | 0.4620 | 0.0196 | 0.4816 (459) | 0.4032 (404) |
| 2.2650 | 38.72 | 0.4958 | 0.0116 | 0.5074 (459) | 0.4608 (430) |
| 2.2700 | 39.21 | 0.5336 | 0.0068 | 0.5404 (459) | 0.5133 (469) |
| 2.2750 | 39.70 | 0.6136 | −0.0023 | 0.6113 (458) | 0.6205 (449) |
| 2.2800 | 40.20 | 0.6638 | 0.0186 | 0.6824 (459) | 0.6081 (441) |
| 2.2850 | 40.71 | 0.7753 | 0.0099 | 0.7852 (459) | 0.7458 (464) |
| 2.2900 | 41.22 | 0.8327 | 0.0211 | 0.8538 (459) | 0.7693 (458) |
| 2.2930 | 41.53 | 0.9020 | 0.0149 | 0.9169 (458) | 0.8573 (447) |
| 2.2950 | 41.74 | 0.8903 | 0.0217 | 0.9120 (458) | 0.8253 (466) |
| 2.2970 | 41.95 | 0.9316 | 0.0016 | 0.9332 (459) | 0.9269 (477) |
| 2.2985 | 42.11 | 0.9893 | 0.0360 | 1.0253 (323) | 0.8814 (382) |
| 2.3000 | 42.27 | 1.0400 | 0.0507 | 1.0907 (458) | 0.8878 (470) |
| 2.3020 | 42.48 | 1.0781 | 0.0287 | 1.1067 (458) | 0.9921 (468) |
| 2.3050 | 42.80 | 1.1102 | 0.0324 | 1.1426 (457) | 1.0129 (445) |
| 2.3070 | 43.02 | 1.1503 | 0.0346 | 1.1849 (373) | 1.0465 (407) |
| 2.3100 | 43.34 | 1.2060 | 0.0389 | 1.2449 (323) | 1.0893 (345) |
| 2.3150 | 43.89 | 1.2464 | 0.0450 | 1.2914 (324) | 1.1113 (340) |
| 2.3200 | 44.45 | 1.3822 | 0.0804 | 1.4625 (457) | 1.1411 (460) |
| 2.3250 | 45.01 | 1.4300 | 0.0807 | 1.5107 (458) | 1.1878 (451) |
| 2.3300 | 45.58 | 1.5164 | 0.0849 | 1.6014 (373) | 1.2616 (362) |
| 2.3350 | 46.15 | 1.5388 | 0.0872 | 1.6260 (456) | 1.2771 (397) |
| 2.3400 | 46.73 | 1.6626 | 0.1108 | 1.7734 (455) | 1.3302 (397) |
| 2.3450 | 47.32 | 1.7134 | 0.1190 | 1.8324 (454) | 1.3563 (340) |
| 2.3500 | 47.92 | 1.8277 | 0.1585 | 1.9862 (454) | 1.3520 (434) |
| 2.3600 | 49.14 | 1.8918 | 0.1734 | 2.0652 (455) | 1.3717 (379) |
| 2.3750 | 51.03 | 1.9475 | 0.2219 | 2.1694 (835) | 1.2818 (296) |
| 2.4000 | 54.34 | 2.1767 | 0.3278 | 2.5045 (826) | 1.1934 (285) |
| 2.4250 | 57.87 | 2.1277 | 0.3762 | 2.5039 (832) | 0.9990 (278) |
| 2.4500 | 61.63 | 2.3244 | 0.5013 | 2.8256 (828) | 0.8206 (271) |
| 2.4750 | 65.64 | 2.2262 | 0.5076 | 2.7338 (831) | 0.7035 (264) |
| 2.5270 | 74.85 | 2.2852 | 0.5908 | 2.8759 (638) | 0.5128 (389) |
| 2.6600 | 104.77 | 2.2659 | 0.6829 | 2.9489 (827) | 0.2171 (186) |

## 4 Interaction measure and critical behaviour

To study the effect of critical behaviour on the interaction measure $\Delta$ for $T > T_c$, we consider a simple model obtained by modifying an ideal gas of massless constituents. The partition function

$$\ln Z(T, V) = c V T^3 (1-t)^{2-\alpha}, \qquad t \leq 1 \tag{12}$$

with $t \equiv T_c/T$, constant $c$ and critical exponent $0 < \alpha < 1$, leads to an ideal gas for $T \to \infty$; for $T \to T_c$, it results in singular behaviour.

From (12) we obtain

$$P = c T^4 (1-t)^{2-\alpha}, \tag{13}$$

$$\varepsilon = 3 c T^4 [(1-t)^{2-\alpha} + 1/3(2-\alpha)\, t (1-t)^{1-\alpha}], \tag{14}$$

**Table 2.** Numerical results from the $12^3 \times 4$ lattice

| $4/g^2$ | $T$ | $\varepsilon/T^4$ | $P/T^4$ | $s/T^3$ | $\Delta$ |
|---|---|---|---|---|---|
| 2.1980 | 32.74 | 0.0929 | 0.0186 | 0.1115 (355) | 0.0371 (355) |
| 2.2600 | 38.24 | 0.1837 | 0.0204 | 0.2041 (250) | 0.1225 (297) |
| 2.2650 | 38.72 | 0.2507 | 0.0227 | 0.2735 (249) | 0.1826 (285) |
| 2.2700 | 39.21 | 0.2440 | 0.0123 | 0.2563 (250) | 0.2071 (292) |
| 2.2750 | 39.70 | 0.2843 | 0.0141 | 0.2984 (249) | 0.2421 (289) |
| 2.2800 | 40.20 | 0.3694 | 0.0199 | 0.3893 (250) | 0.3097 (307) |
| 2.2850 | 40.71 | 0.3779 | 0.0122 | 0.3901 (250) | 0.3414 (345) |
| 2.2880 | 41.02 | 0.4672 | 0.0171 | 0.4843 (249) | 0.4159 (344) |
| 2.2900 | 41.22 | 0.5162 | 0.0209 | 0.5371 (176) | 0.4534 (244) |
| 2.2930 | 41.53 | 0.6032 | 0.0225 | 0.6257 (176) | 0.5358 (208) |
| 2.2950 | 41.74 | 0.6149 | 0.0257 | 0.6406 (204) | 0.5377 (316) |
| 2.2970 | 41.95 | 0.6215 | 0.0229 | 0.6444 (250) | 0.5530 (344) |
| 2.2985 | 42.11 | 0.6934 | 0.0270 | 0.7204 (249) | 0.6123 (314) |
| 2.3000 | 42.27 | 0.7239 | 0.0291 | 0.7531 (203) | 0.6365 (339) |
| 2.3020 | 42.48 | 0.7897 | 0.0266 | 0.8163 (249) | 0.7099 (407) |
| 2.3050 | 42.80 | 0.8524 | 0.0228 | 0.8753 (249) | 0.7839 (311) |
| 2.3070 | 43.02 | 0.9175 | 0.0386 | 0.9561 (140) | 0.8016 (279) |
| 2.3100 | 43.34 | 0.9984 | 0.0380 | 1.0365 (204) | 0.8844 (312) |
| 2.3150 | 43.89 | 1.1215 | 0.0545 | 1.1760 (249) | 0.9580 (382) |
| 2.3200 | 44.45 | 1.2614 | 0.0701 | 1.3315 (287) | 1.0511 (320) |
| 2.3250 | 45.01 | 1.3756 | 0.0885 | 1.4641 (249) | 1.1102 (370) |
| 2.3300 | 45.58 | 1.5122 | 0.0995 | 1.6118 (288) | 1.2136 (323) |
| 2.3400 | 46.73 | 1.6597 | 0.1244 | 1.7841 (287) | 1.2863 (295) |
| 2.3500 | 47.92 | 1.8032 | 0.1546 | 1.9577 (286) | 1.3395 (302) |
| 2.3620 | 49.39 | 1.9849 | 0.2122 | 2.1971 (452) | 1.3483 (217) |
| 2.3750 | 51.03 | 2.1119 | 0.2663 | 2.3783 (451) | 1.3130 (241) |
| 2.4000 | 54.34 | 2.1629 | 0.3471 | 2.5101 (453) | 1.1215 (232) |
| 2.4150 | 56.43 | 2.2521 | 0.4066 | 2.6587 (350) | 1.0324 (300) |
| 2.4250 | 57.87 | 2.2787 | 0.4303 | 2.7090 (452) | 0.9879 (226) |
| 2.4500 | 61.63 | 2.2959 | 0.4941 | 2.7900 (451) | 0.8135 (222) |
| 2.4750 | 65.64 | 2.3638 | 0.5679 | 2.9317 (452) | 0.6601 (216) |
| 2.5500 | 79.32 | 2.3688 | 0.6584 | 3.0272 (448) | 0.3937 (155) |
| 2.6600 | 104.77 | 2.4429 | 0.7557 | 3.1986 (451) | 0.1759 (143) |

for pressure and energy density; hence the specific heat $c_V = (\partial \varepsilon / \partial T)_V$ diverges as $(1-t)^{-\alpha}$ for $t \to 1$. For the interaction measure $\Delta$ we get

$$\Delta = c(2-\alpha)\, t(1-t)^{1-\alpha}; \tag{15}$$

it vanishes for $T \to T_c$ as well as for $T \to \infty$, and it has a peak at

$$T/T_c = t^{-1} = (2-\alpha). \tag{16}$$

While the position of the peak is model-dependent (it is shifted by the addition of a non-singular term to $\ln Z$), the slope of $\Delta$ for $T \to T_c$ has a universal form,

$$\frac{d\Delta}{dT} \sim (1-t)^{-\alpha}, \tag{17}$$

diverging at $T = T_c$ with the same critical exponent as the specific heat.

The occurence of a peak in $\Delta$ just above $T_c$ is thus a direct consequence of the second order deconfinement trasition. Universality arguments [11] predict for the $\alpha$ in $SU(2)$ gauge theory the same value as in the three-dimensional Ising model, $\alpha \simeq 0.11$. This suggests that near $T_c$, $\Delta$ should have the form

$$\Delta = A\, t(1-t)^{0.89} + Bt, \qquad A, B = \text{const}. \tag{18}$$

In contrast to (15), we have also added a non-singular term $B$, since in the actual calculations $\varepsilon$ and $P$ do not vanish for $T = T_c$, as is the case for our simple model (12). We would like to compare this form with our data. To do so, we have to take into account the difference between a continuum form and results obtained on a finite lattice. These differences occur both because of the lattice cut-off [12] and because $g^2 \neq 0$ in the lattice evaluation. To correct our measured results for $\varepsilon$ and $P$, we divide the Monte Carlo values for these quantities by the corresponding weak

**Table 3.** Numerical results from the $18^3 \times 4$ lattice

| $4/g^2$ | $T$ | $\varepsilon/T^4$ | $P/T^4$ | $s/T^3$ | $\Delta$ |
|---|---|---|---|---|---|
| 2.1500 | 29.04 | 0.0316 | 0.0153 | 0.0469 (290) | −0.0142 (268) |
| 2.2700 | 39.21 | 0.1701 | 0.0244 | 0.1945 (191) | 0.0969 (227) |
| 2.2800 | 40.20 | 0.2181 | 0.0160 | 0.2341 (192) | 0.1702 (219) |
| 2.2850 | 40.71 | 0.3080 | 0.0227 | 0.3307 (191) | 0.2400 (271) |
| 2.2900 | 41.22 | 0.3572 | 0.0232 | 0.3804 (111) | 0.2877 (281) |
| 2.2950 | 41.74 | 0.4474 | 0.0235 | 0.4709 (136) | 0.3769 (248) |
| 2.2965 | 41.90 | 0.4754 | 0.0247 | 0.5002 (111) | 0.4013 (183) |
| 2.2970 | 41.95 | 0.5016 | 0.0266 | 0.5282 (93) | 0.4217 (276) |
| 2.2980 | 42.06 | 0.5258 | 0.0249 | 0.5507 (96) | 0.4511 (201) |
| 2.3000 | 42.27 | 0.5600 | 0.0258 | 0.5858 (90) | 0.4826 (202) |
| 2.3020 | 42.48 | 0.6579 | 0.0298 | 0.6877 (96) | 0.5685 (214) |
| 2.3050 | 42.80 | 0.7613 | 0.0371 | 0.7984 (111) | 0.6499 (270) |
| 2.3070 | 43.02 | 0.8267 | 0.0391 | 0.8659 (96) | 0.7094 (190) |
| 2.3100 | 43.34 | 0.9597 | 0.0458 | 1.0055 (111) | 0.8222 (235) |
| 2.3150 | 43.89 | 1.1208 | 0.0579 | 1.1787 (135) | 0.9471 (320) |
| 2.3170 | 44.11 | 1.1817 | 0.0623 | 1.2440 (111) | 0.9949 (285) |
| 2.3200 | 44.45 | 1.2896 | 0.0669 | 1.3566 (192) | 1.0888 (332) |
| 2.3300 | 45.58 | 1.5226 | 0.0938 | 1.6164 (191) | 1.2412 (261) |
| 2.3400 | 46.73 | 1.7214 | 0.1329 | 1.8543 (191) | 1.3226 (251) |
| 2.3500 | 47.92 | 1.8666 | 0.1577 | 2.0243 (443) | 1.3935 (160) |
| 2.3630 | 49.51 | 1.9772 | 0.2010 | 2.1782 (346) | 1.3741 (194) |
| 2.3750 | 51.03 | 2.0719 | 0.2554 | 2.3272 (395) | 1.3058 (205) |
| 2.4000 | 54.34 | 2.2000 | 0.3541 | 2.5541 (468) | 1.1378 (204) |
| 2.4250 | 57.87 | 2.2841 | 0.4305 | 2.7146 (409) | 0.9927 (195) |
| 2.4500 | 61.63 | 2.2908 | 0.4992 | 2.7900 (656) | 0.7932 (208) |
| 2.4750 | 65.64 | 2.3412 | 0.5568 | 2.8980 (368) | 0.6708 (184) |
| 2.5300 | 75.41 | 2.3923 | 0.6475 | 3.0397 (905) | 0.4498 (202) |
| 2.5780 | 85.14 | 2.4306 | 0.7060 | 3.1366 (288) | 0.3125 (170) |
| 2.6210 | 94.92 | 2.4396 | 0.7398 | 3.1795 (294) | 0.2201 (171) |
| 2.6600 | 104.77 | 2.4643 | 0.7622 | 3.2266 (339) | 0.1776 (115) |

Numerical results from the $26^3 \times 4$ lattice

| $4/g^2$ | $T$ | $\varepsilon/T^4$ | $P/T^4$ | $s/T^3$ | $\Delta$ |
|---|---|---|---|---|---|
| 2.2750 | 39.70 | 0.1607 | 0.0270 | 0.1877 (390) | 0.0796 (199) |
| 2.2975 | 42.01 | 0.4276 | 0.0270 | 0.4545 (90) | 0.3466 (273) |
| 2.3000 | 42.27 | 0.5459 | 0.0295 | 0.5754 (157) | 0.4574 (318) |

Table 4. Results from the $18^3 \times 4$ lattice corrected by weak coupling

| $4/g^2$ | $T$ | $\varepsilon/\varepsilon_{wc}$ | $P/P_{wc}$ | $\Delta_{corr.}$ |
|---|---|---|---|---|
| 2.1500 | 29.04 | 0.0129 | 0.0186 | −0.0112 (212) |
| 2.2700 | 39.21 | 0.0686 | 0.0294 | 0.0774 (181) |
| 2.2800 | 40.20 | 0.0878 | 0.0192 | 0.1355 (174) |
| 2.2850 | 40.71 | 0.1240 | 0.0272 | 0.1910 (216) |
| 2.2900 | 41.22 | 0.1438 | 0.0278 | 0.2288 (223) |
| 2.2950 | 41.74 | 0.1800 | 0.0282 | 0.2995 (197) |
| 2.2965 | 41.90 | 0.1912 | 0.0297 | 0.3189 (146) |
| 2.2970 | 41.95 | 0.2018 | 0.0320 | 0.3351 (219) |
| 2.2980 | 42.06 | 0.2115 | 0.0299 | 0.3584 (159) |
| 2.3000 | 42.27 | 0.2252 | 0.0310 | 0.3833 (160) |
| 2.3020 | 42.48 | 0.2645 | 0.0358 | 0.4515 (170) |
| 2.3050 | 42.80 | 0.3060 | 0.0445 | 0.5161 (214) |
| 2.3070 | 43.02 | 0.3323 | 0.0469 | 0.5632 (150) |
| 2.3100 | 43.34 | 0.3856 | 0.0550 | 0.6526 (186) |
| 2.3150 | 43.89 | 0.4502 | 0.0694 | 0.7516 (254) |
| 2.3170 | 44.11 | 0.4746 | 0.0747 | 0.7894 (226) |
| 2.3200 | 44.45 | 0.5178 | 0.0802 | 0.8637 (263) |
| 2.3300 | 45.58 | 0.6108 | 0.1123 | 0.9840 (207) |
| 2.3400 | 46.73 | 0.6901 | 0.1591 | 1.0482 (198) |
| 2.3500 | 47.92 | 0.7477 | 0.1886 | 1.1037 (127) |
| 2.3630 | 49.51 | 0.7913 | 0.2402 | 1.0878 (154) |
| 2.3750 | 51.03 | 0.8284 | 0.3049 | 1.0335 (162) |
| 2.4000 | 54.34 | 0.8781 | 0.4220 | 0.9004 (161) |
| 2.4250 | 57.87 | 0.9101 | 0.5122 | 0.7854 (154) |
| 2.4500 | 61.63 | 0.9111 | 0.5929 | 0.6281 (165) |
| 2.4750 | 65.64 | 0.9296 | 0.6602 | 0.5317 (146) |
| 2.5300 | 75.41 | 0.9465 | 0.7651 | 0.3580 (160) |
| 2.5780 | 85.14 | 0.9588 | 0.8319 | 0.2505 (136) |
| 2.6210 | 94.92 | 0.9598 | 0.8696 | 0.1782 (138) |
| 2.6600 | 104.77 | 0.9674 | 0.8939 | 0.1450 (94) |

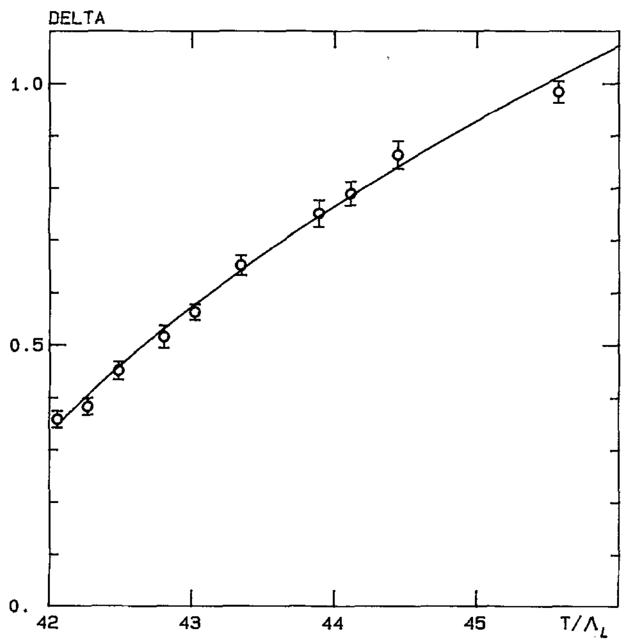


Fig. 5. The behaviour of $\Delta$ just above $T_c$, compared to the universal critical form, (18), with $A=7.461$, $B=0.325$

coupling values on the lattice [13]. The detailed form of the correction factors is given in the Appendix, and the corrected values of $\varepsilon$, $P$, and $\Delta$ are listed in Table 4. The results for $\Delta$ are shown in Fig. 5 for temperatures just above $T_c$, together with the form predicted by (18); the constants $A$ and $B$ are fitted. We see that the lattice results indeed follow the expected pattern.

## 5 The evolution of gluon deconfinement

In this section, we want to consider a possible dynamical scheme leading to the observed behaviour of $\varepsilon$, $P$ and $\Delta$. The decrease of $\Delta$ sufficiently far above $T_c$ has been related to the presence of remnant massive modes by a number of authors. Our aim here will be to account as well for the increase of $\Delta$ just above $T_c$, and to understand how massive modes can lead to the calculated large deviations from ideal gas behaviour. Since both the energy density and the pressure of an ideal glueball system (with $M \simeq 1$ GeV) are small compared to those of an ideal gluon gas, the mere presence of such modes above $T_c$ cannot result in a $\Delta$ of the measured size. On the other hand, if the momentum spectrum of the constituents contains massive modes at low momenta and massless gluons at high momenta, then the absence of low momentum gluons, together with the temperature-dependence of the spectrum, results in a large and rapidly varying $\Delta$, as we shall see.

To illustrate the effect, let us first consider an ideal Boltzmann gas of massless constituents, with a constant low momentum cut-off $K$. Its partition function is

$$\ln Z_G = \frac{dV}{2\pi^2} \int_K^\infty dk\, k^2 e^{-k/T}, \tag{19}$$

and it leads to

$$\Delta = \frac{dK^3}{2\pi^2 T^3} e^{-K/T}. \tag{20}$$

This $\Delta$ has a peak at $T=K/3$, vanishes as $T^{-3}$ for large $T$ and goes to zero exponentially as $T \to 0$. The reason for this behaviour is evident from the momentum spectrum in (19),

$$f(k) = k^2 e^{-k/T}. \tag{21}$$

It peaks at $k=2T$, so that for $K>2T$, the integral includes only the exponentially falling high-momentum part of the spectrum; with increasing $T$ at fixed $K$ (or with decreasing $K$), more and more of the full spectrum is covered in the integration, and hence $\Delta$ approaches the ideal gas value.

While the overall functional form (20) is not unlike that found is Sect. 4, it yields a value of $\Delta$ which at the peak is much too small. Moreover, it contains no information about $T_c$ and hence cannot reproduce the rapid variation in the critical region. To incorporate the $T_c$ dependence, we make the momentum cut-off $K$ temperature-dependent. Since we expect no free gluons below $T_c$, a natural form for the cut-off is

$$K/T_c = p[(T-T_c)/T_c]^{-q}; \quad p, q = \text{const.} > 0. \qquad (22)$$

With this cut-off, $\ln Z_G$ vanishes at $T_c$, while at high $T$ the full gluon spectrum is recovered.

We now want to check if a description of this type, based on a rapid opening of the massless gluon sector just above $T_c$, can indeed account for the observed behaviour of $\Delta$. To obtain a full model, we make the ansatz

$$\ln Z = -\frac{dV}{2\pi^2}\left\{\int_0^K dk\,k^2 \ln(1-e^{-\sqrt{k^2+M^2}/T}) + \int_K^\infty dk\,k^2 \ln(1-e^{-k/T})\right\}, \qquad (23)$$

with $K$ given by (22). At $T=T_c$, (23) reduces to a massive glueball gas; for $T>T_c$, the massless gluons quickly take over. As degeneracy factor, we take $d=6$; this corresponds to two spin and three colour degrees of freedom for the gluons, and to degenerate $0^+/2^+$ states for the glueballs. The critical temperature is $T_c/\Lambda_L = 42$, and we write the glueball mass $M = rT_c$. The remaining three parameters $p, q$ and $r$ are determined by fitting $\varepsilon, P$ and $\Delta$ obtained from (23) to the measured values listed in Table 2. The result is shown in Figs. 6 and 7 – we see that our simple model can indeed describe the behaviour quite well. One can certainly improve the agreement by constructing a model including further expected features. Bag pressure, perturbative corrections, colour corrections – these all will bring $\varepsilon$ down slightly from its ideal gas value and thus improve the fit. We have not done this, since any further terms with open parameters will obviously allow a good fit. Our essential point is to emphasize that the rapid growth of $\Delta$ just above $T_c$ can be related to a rapid opening of the massless gluon sector; once the bulk of this sector is included, the corrections become smaller and $\Delta$ eventually goes to zero. The presence of massive modes near $T_c$ is necessary to have non-vanishing thermodynamic quantities there, but its actual effect on the functional behaviour of the $\Delta$ obtained from (23) is quite small.

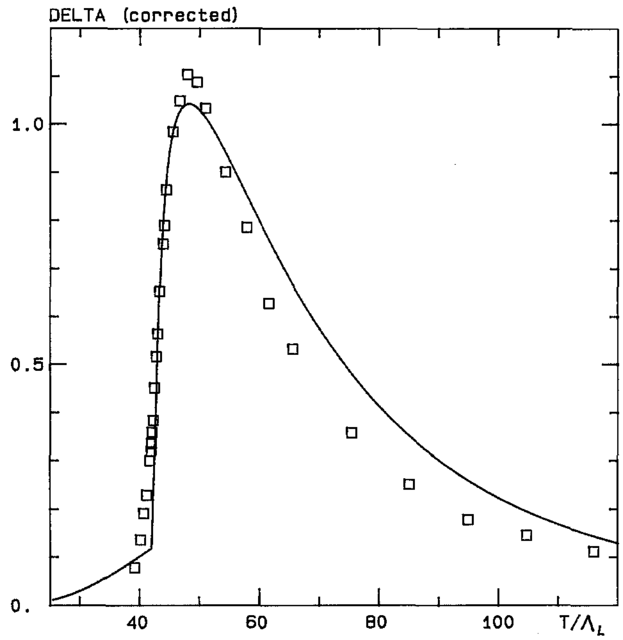


**Fig. 6.** The corrected interaction measure $\Delta_{\text{corr.}}$, compared to the phenomenological form obtained from (23) with $M=5.5\,T_c$, $p=2.859$ and $q=0.297$

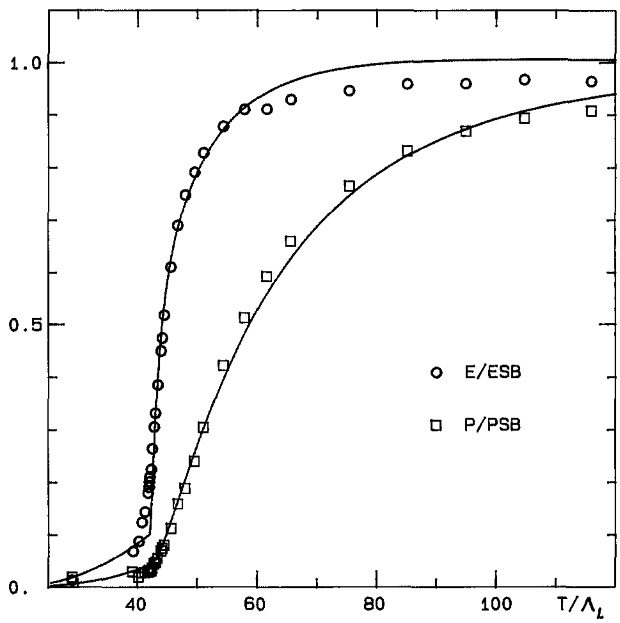


**Fig. 7.** The same comparison as in Fig. 6, but for the corrected energy density and pressure

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## Appendix A

The weak coupling forms of the energy density and pressure are obtained from (3)–(4) by inserting the weak coupling expansions of the plaquettes. They were calculated up to order $g^4$ in [13] and have the structure

$$P_{wc}=g^2\frac{N_c^2-1}{N_c}P^{(2)}+g^4(N_c^2-1)P^{(4a)}+g^4\frac{(2N_c^2-3)(N_c^2-1)}{N_c^2}P^{(4b)}, \quad (A.1)$$

where $N_c$ is the number of colours and $P^{(2)}$, $P^{(4a)}$ and $P^{(4b)}$ are colour-independent, but lattice size dependent. Denoting the differences between plaquettes by double indices

$$P_{\alpha\beta}=P_\alpha-P_\beta, \quad (A.2)$$

one finds (only up to $O(g^2)$ because of the factor $g^{-2}$) for $N_c=2$

$$\varepsilon_{wc}/T^4=18N_\tau^4\{P_{\sigma\tau}^{(2)}+g^2\cdot[2P_{\sigma\tau}^{(4a)}+\tfrac{5}{2}P_{\sigma\tau}^{(4b)}+c_\sigma P_{0\sigma}^{(2)}+c_\tau P_{0\tau}^{(2)}]\}, \quad (A.3)$$

$$P_{wc}/T^4=\varepsilon_{wc}/(3T^4)-8N_\tau^4(c_1+c_2)g^2(P_{0\sigma}^{(2)}+P_{0\tau}^{(2)}). \quad (A.4)$$

For the weak coupling corrected $\Delta$ one gets then

$$\Delta_{\text{corr.}}=\left(\varepsilon\frac{\varepsilon_{SB}}{\varepsilon_{wc}}-3P\frac{P_{SB}}{P_{wc}}\right)\Big/T^4, \quad (A.5)$$

or

$$\Delta_{\text{corr.}}=\left(\frac{\varepsilon}{\varepsilon_{wc}}-\frac{P}{P_{wc}}\right)\frac{\pi^2}{5}, \quad (A.6)$$

i.e., up to a constant factor, $\Delta_{\text{corr.}}$ is the difference of the corrected $\varepsilon/\varepsilon_{SB}$ and $P/P_{SB}$ values.

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