Deconfinement for SU(2) Gauge Theory in 2 + 1 Dimensions

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By calculating Polyakov-loop averages on a $60^2\times 2$ lattice, we determine the critical exponent of deconfinement for SU(2) gauge theory in 2 + 1 dimensions. Universality arguments predict it to be the same as the critical exponent $\beta=\frac{1}{8}$ for the spontaneous magnetization in the two-dimensional Ising model. Our results are in good accord with this prediction.

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SU(N) lattice gauge theory leads to a deconfinement transition at a temperature T_c,[1] at which the system passes from a state of invariance under global Z_N transformations to one where this symmetry is broken. The symmetry of the state at any given temperature is measured by the lattice average of the Polyakov loop $L(\mathbf{x})$, which thus serves as a deconfinement order parameter.[2] For the SU(2) case in the strong-coupling limit, it is possible to integrate out all degrees of freedom except the $L(\mathbf{x})$ at spatial sites $\mathbf{x}$; the result is an effective spin theory of the same spatial dimensions as the original gauge theory.[3] This has led to the conjecture[4] that, in general, SU(N) gauge systems and Z_N spin systems of the same spatial dimensions should exhibit the same finite-temperature behavior, provided that the transition is in both cases continuous. In particular, both transitions should then yield the same critical exponents.

For SU(2) lattice gauge theory in three space dimensions it was recently shown[5,6] that the critical exponent of deconfinement, β, with

$$\bar{L}\sim(T-T_c)^{\beta},\quad T>T_c, \tag{1}$$

is in accord with the value $\beta=0.33$ found numerically for the spontaneous magnetization in the three-dimensional Ising model. In the present note we want to study this question for the case of two space dimensions, where the Ising model has Onsager's celebrated analytic solution, yielding $\beta=\frac{1}{8}$ for the corresponding critical exponent. Similar studies for the Z_2 gauge system[7] in two space dimensions have led to critical exponents somewhat larger than this prediction[8]; they have at the same time elucidated some of the difficulties inherent in such checks.

We consider the Polyakov loop

$$L(\mathbf{x})=\frac{1}{2}\prod_{i=0}^{N_\tau-1}U_{\mathbf{x};i,i+1} \tag{2}$$

at a spatial site $\mathbf{x}$ on a (2+1)-dimensional lattice of size $N_\sigma^2\times N_\tau$, where N_σ and N_τ denote the total number of sites in space and temperature directions. The SU(2) matrix $U_{\mathbf{x};i,i+1}$ is associated with the link connecting the temperature sites i and $i+1$ along the lattice axis at the space point $\mathbf{x}$. With the Wilson action in 2 + 1 dimensions,

$$S(U)=K\sum_{\text{plaquettes}}(1-\tfrac{1}{2}\operatorname{Re}\operatorname{Tr}UUUU), \tag{3}$$

we calculate the average of $L(\mathbf{x})$ over all N_σ^2 spatial sites of the lattice for a given configuration i of the U's and a given coupling $K=4/g^2a$; here g denotes the bare coupling and a the lattice spacing. The absolute value of the result is then averaged over n_i successive configurations (iterations) produced by the

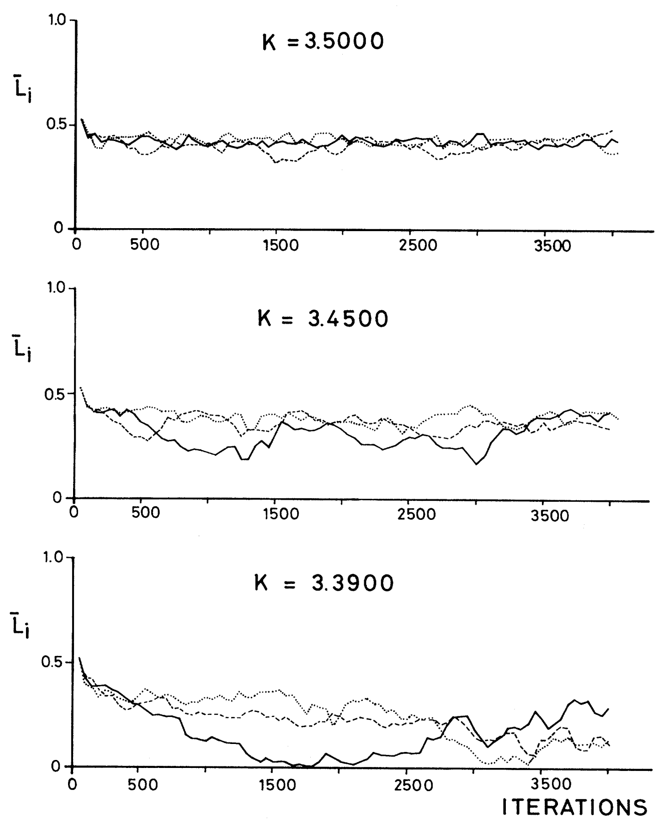

FIG. 1. The behavior of $\bar{L}_i$, Eq. (4), at $K=3.5$, 3.45, and 3.39 as a function of the number of iterations, for three cold start runs with different random numbers.

TABLE I. Average Polyakov loops on a $60^2 \times 2$ lattice at different couplings $K = 4/g^2a$.

K	$\bar{L}$
3.39	0.253 83 ±0.013 57
3.40	0.325 50 ±0.009 95
3.418	0.354 30 ±0.008 29
3.45	0.388 01 ±0.004 75
3.4625	0.408 45 ±0.004 39
3.475	0.409 16 ±0.004 16
3.4875	0.423 21 ±0.004 44
3.50	0.430 00 ±0.003 31
3.525	0.441 50 ±0.002 80
3.55	0.448 28 ±0.004 50
3.60	0.472 40 ±0.002 60
3.65	0.487 21 ±0.002 56
3.70	0.505 60 ±0.003 68
3.75	0.519 52 ±0.001 26
3.80	0.529 40 ±0.002 00
4.00	0.566 40 ±0.002 60
5.00	0.676 40 ±0.001 60

Metropolis algorithm,

$$\bar{L} = n_i^{-1} \sum_i \bar{L}_i, \quad \bar{L}_i = |N_\sigma^{-2} \sum_{\mathbf{x}} L(\mathbf{x})|. \tag{4}$$

We take $\bar{L}_i$ in this final average in order to avoid cancellations which would occur if the system flips from one ordered state to another.

The actual evaluation was performed on a $60^2 \times 2$ lattice, starting from an ordered initial configuration (cold start). For each value of the coupling K, we ran in general 4000 iterations. In Fig. 1, we show the behavior of $\bar{L}_i$ as a function of the number of iterations, for several runs per coupling. We start with a K value well in the ordered (deconfinement) region. It is seen that $\bar{L}_i$ then first converges quite rapidly to a stable equilibrium value, but as we lower K towards the critical point, the fluctuations become larger and larger, as expected for a continuous transition.

In Table I we list the results obtained for $\bar{L}$ at the couplings studied. They are in general based on 3500 iterations; we discard the first 500 iterations in each case to reduce transient effects. The errors in Table I were obtained in the following way. First the average over the results of n consecutive iterations was taken. Then the linear correlation coefficient of each two successive n blocks was measured. Whereas for $n = 10$ there was a clear linear correlation, the correlation of neighboring blocks of size $n = 100$ for $K \geq 3.436$ and $n \simeq 350$ for $K < 3.436$ showed no significant deviation from zero on the 95% confidence level. From these large bins the usual estimate of the standard deviation was calculated.

We now fitted the data by the form

$$L = A(K - K_c)^\beta [1 + \beta (K - K_c)^b], \tag{5}$$

with open A, B, K_c, and β. The second term inside the square brackets describes corrections to scaling; since it is presumably small near K_c, we begin with $b = 1.6$ as obtained for the two-dimensional Ising model.[9] Since not much is known about renormalization and scaling behavior in two space dimensions,[10] we study $\bar{L}$ as a function of the coupling rather than

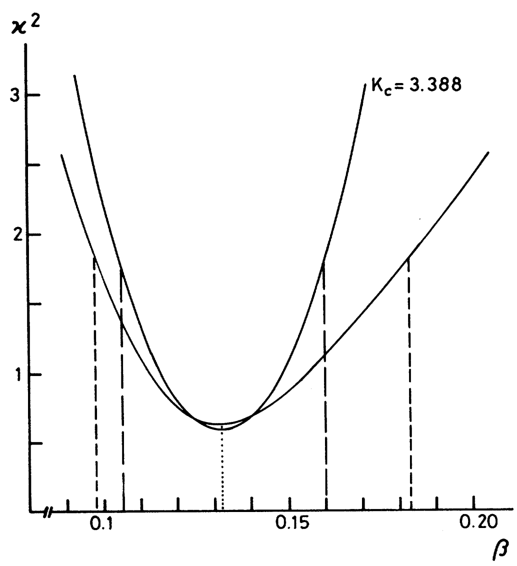

FIG. 2. The quantity χ^2 with and without fixed K_c, Eq. (6), as a function of the critical exponent β, for the fit Eq. (5) in the interval $3.39 \leq K \leq 3.75$; the dashed lines indicate the 95%-confidence-level errors.

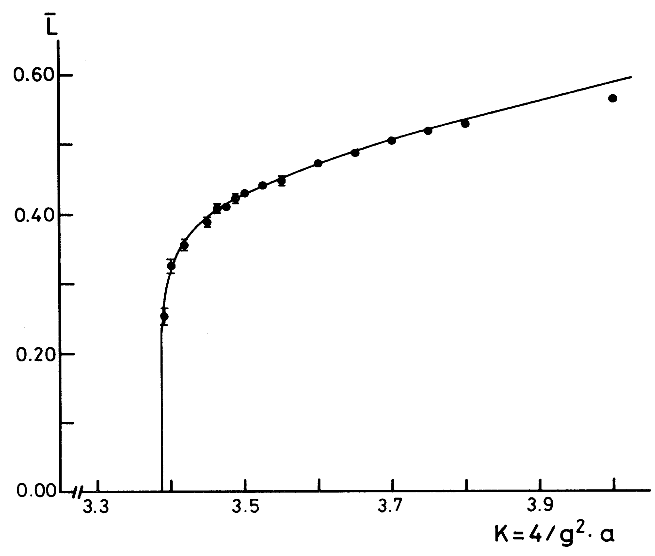

FIG. 3. The calculated values of the average Polyakov loop $\bar{L}$ as function of the coupling $K = 4/g^2a$, together with the fit (solid line) of Eq. (5) using the parameter values of Eq. (7).

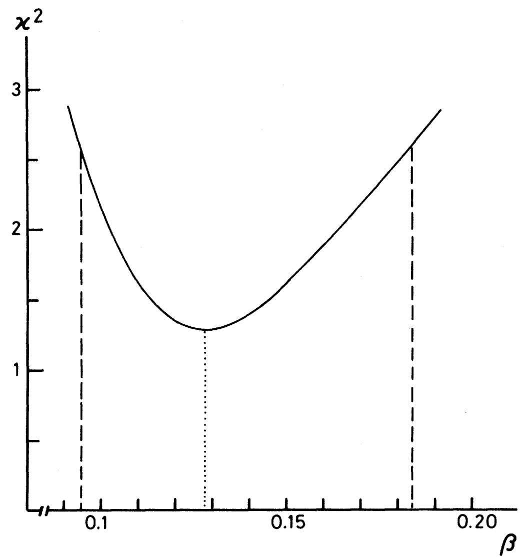

FIG. 4. The quantity χ^2, Eq. (6), as function of the critical exponent β, for the fit Eq. (5) with $B=0$ in the interval $3.39 \leqslant K \leqslant 3.475$; the dashed lines indicate the 95%-confidence-level errors.

the temperature. For the interval $3.39 \leqslant K \leqslant 3.75$, we obtain the χ^2 behavior shown in Fig. 2; here

$$\chi^2 = \sum_K \left[\frac{L(K) - f(K)}{\Delta L(K)}\right]^2 \Big/ \text{d.o.f.}, \tag{6}$$

with $f(K)$ denoting the form (5) and with the sum over all data points K. The minimum of χ^2 is reached for

$$\beta = 0.132, \quad K_c = 3.388, \quad A = 0.567, \quad B = 0.242. \tag{7}$$

To obtain an estimate of the systematical error for the value of the critical exponent, we have varied the upper limit of the fitted K range between 3.65 and 3.80; we find that $\Delta\beta = 0.008$. The statistical error can be read from Fig. 2, where the dashed lines indicate the β range at 95% confidence level, resulting in

$$\beta = 0.132^{+0.055}_{-0.034}. \tag{8}$$

If it were possible, as, e.g., in $3+1$ dimensions, by the measurement of the energy density on the same lattice (here we would need an additional plaquette measurement on a symmetric lattice) to obtain an independent K_c determination, then we would have a smaller error, as can be seen in Fig. 2, where χ^2 for the corresponding fit at fixed K_c is also plotted. The fit to the data using Eqs. (5) and (7) is shown in Fig. 3.

So far, we had fixed the power b of $K - K_c$ in the correction to scaling. To test the importance of this for the critical exponent, we now restrict our fit to a smaller range of K, where the correction term should become negligible. In Fig. 4, we show the χ^2 results for $3.39 \leqslant K \leqslant 3.475$ with $B=0$. The fit is now best for

$$\beta = 0.128, \quad K_c = 3.388, \quad A = 0.561, \tag{9}$$

in very good agreement with the result (7). Finally, we have attempted a fit with all parameters, including b, left open. The resulting χ^2 behavior for $3.39 \leqslant K \leqslant 3.75$ is shown in Fig. 5; it is minimal for

$$\beta = 0.133, \quad K_c = 3.388, \quad A = 0.568, \quad B = 0.245, \quad b = 1.64 \tag{10}$$

in accord with our previous results. The comparatively slow variation of χ^2 with β shows that it is difficult to fit both β and b in the given rather small K interval.

We can thus conclude that our results support well the proposed universality relation[3,4] between SU(2) gauge theory and the Ising model for the case of two space dimensions.

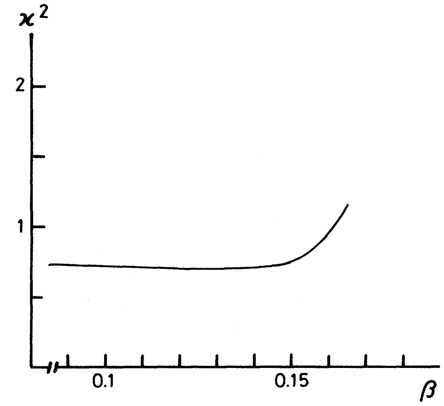

FIG. 5. The quantity χ^2, Eq. (6), as a function of the critical exponent β, for the fit Eq. (5) with open exponent b.

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