THE LATENT HEAT OF DECONFINEMENT IN SU(3) YANG–MILLS THEORY

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We calculate the latent heat of deconfinement in SU(3) Yang–Mills theory from the difference in energy density between the deconfined and the confined phases at the critical temperature T_c. The calculation is based on a high statistics Monte Carlo evaluation using lattices with 8^3 and 10^3 spatial sites and 2, 3 and 4 temperature sites. Both T_c and the latent heat $\Delta\epsilon$ are shown to satisfy scaling. We find $T_c = 208 \pm 20$ MeV, $\Delta\epsilon/T_c^4 = 3.75 \pm 0.25$; the latter is in accord with bag model arguments.

SU(N) Yang–Mills systems exhibit deconfinement at sufficiently high physical temperature, when the global symmetry under the center Z_N of the gauge group is broken [1]. Using Z_N to specify the universality class of the theory, one can attempt to gain information about the critical behaviour of gauge systems by studying related spin models [2,3]. For the SU(3) Yang–Mills system, these considerations lead to the prediction of a first order deconfinement transition [3], and Monte Carlo studies on the lattice confirm this prediction [4].

The method we used in ref. [4] consists in comparing the development of the system when starting from completely random (Z_N symmetric) and completely ordered (broken Z_N) initial states at the same coupling value. It allows a determination of the deconfinement temperature, T_c, which is essentially independent of the spatial lattice size. For a first order transition, it provides in addition the possibility to calculate the latent heat $\Delta\epsilon$, i.e., the discontinuity in the energy density ϵ at T_c. The main aim of this note is to carry out such a calculation, again using a high statistics Monte Carlo analysis. Evaluating ϵ at the critical coupling for both ordered and random starts, we obtain, respectively, the upper (ϵ_c^P) and lower (ϵ_c^G) values at T_c; then $\Delta\epsilon = \epsilon_c^P - \epsilon_c^G$. We calculate $\Delta\epsilon$ using lattices of different temporal sizes; with T_c and $\Delta\epsilon$ we then have two independent physical observables to use in verifying that we are in the scaling regime of the coupling.

The plan of our paper is as follows. We first consider the scaling behaviour of the deconfinement temperature, by comparing results for T_c obtained on lattices with different numbers N_β of temporal sites. Next we calculate $\Delta\epsilon/T_c^4$, again for lattices with different N_β, and study the scaling behaviour of the latent heat. Finally we obtain on an $8^3 \times 3$ lattice the overall energy density ϵ as function of T, including finite coupling corrections.

The euclidean partition function of the SU(3) Yang–Mills system is given by [5]

$$Z_E(N_\sigma, N_\beta, g_\sigma, g_\beta, \xi) = \int \prod_{(\text{links})} dU \exp[-S(U)] \,, \qquad (1)$$

where the action $S(U)$ is in Wilson form

$$S(U) = \frac{6}{g_\sigma^2}\frac{1}{\xi}\sum_{\{P_\sigma\}}(1 - \tfrac{1}{3}\,\mathrm{Re\,tr}\,UUUU) + \frac{6}{g_\beta^2}\,\xi\sum_{\{P_\beta\}}(1 - \tfrac{1}{3}\,\mathrm{Re\,tr}\,UUUU)\,. \qquad (2)$$

Here the summations run over space-like (P_σ) and space–temperature (P_β) plaquettes; we consider a completely periodic lattice with N_σ^3 (N_β) sites in the spatial (temperature) directions, with corresponding spacings

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a_σ and a_β, and with $\xi \equiv a_\sigma/a_\beta$. The associated space-like and temperature-like couplings are denoted by g_σ and g_β, respectively [6,7]. In the final numerical evaluation, we generally take $\xi = 1$, which makes $g_\sigma = g_\beta \equiv g$ and $a_\sigma = a_\beta \equiv a$.

To determine T_c, we calculate the order parameter $\bar{L}$ of the system: the thermal Wilson loop $L(\boldsymbol{x})$ averaged over all spatial sites $\boldsymbol{x}$ of the lattice [1,8]. For $\xi = 1$ and a given coupling g, we start (a) from a completely random and (b) from a completely ordered initial configuration of U's and then iterate. For non-critical couplings, (a) and (b) rapidly converge, whereas near g_c there is a clear two-state pattern [4]. In fig. 1, we show the behaviour of $\bar{L}$ for $N_\beta = 2, 3, 4$ at the critical couplings. Note that for systems of finite spatial extent, phase flips such as seen in fig. 1c are expected to occur.

The critical coupling g_c can be converted into a critical temperature T_c by use of the renormalization group relation

$$a\Lambda_L = \exp[-8\pi^2/11g^2 - \tfrac{51}{121}\ln(11g^2/16\pi^2)], \quad (3)$$

provided g_c is small enough to neglect higher powers of g^2 when integrating the Callan–Szymanzik equation to obtain eq. (3). In fig. 2, we compare our results with

$$T_c a_c = 1/N_\beta$$
$$= (T_c/\Lambda_L)\exp[-8\pi^2/11g_c^2 - \tfrac{51}{121}\ln 11g_c^2/16\pi^2], \quad (4)$$

where $a_c \equiv a(g_c^2)$; this scaling test is seen to be reasonably well fulfilled, with $T_c \simeq 80\ \Lambda_L$. When we look in more detail, however, deviations appear: in table 1, we show the values for $T_c \equiv 1/(N_\beta a_c)$ obtained by use of eq. (3), and the results for $N_\beta = 3$ and 4 differ by about 12%. It is known that such discrepancies can be caused by higher order terms in the renormalization group relation, without any violation of general scaling behaviour [9]. To test if this is the case here, we use recent high precision Monte Carlo data for the string tension [10] to express T_c directly in units of a physical observable. As seen in table 1, no measurable deviation remains; we thus obtain

$$T_c = (0.519 \pm 0.050)\sqrt{\sigma} \simeq 208 \pm 20 \text{ MeV} \quad (5)$$

for the critical temperature and conclude that we are indeed within the scaling region, at least for $N_\beta = 3$ and 4. – We note incidentally that this value of T_c agrees well with that obtained for the SU(2) system [5].

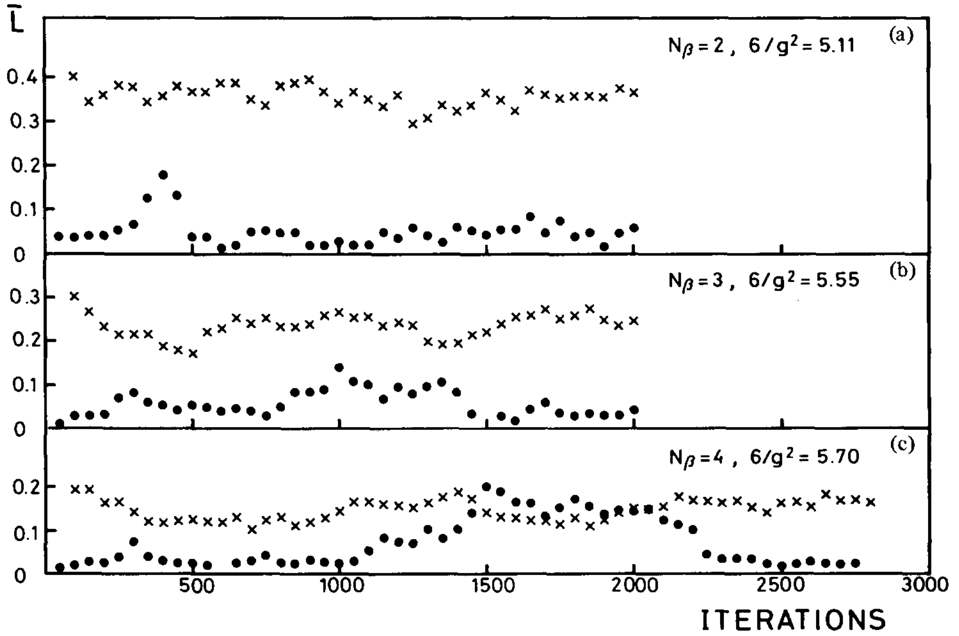

Fig. 1. Lattice average $\bar{L}$ of the order parameter, as function of the number of iterations, in bins of 50, after ordered (crosses) and random (dots) starts, calculated on the following lattices: (a): $8^3 \times 2$, (b): $10^3 \times 3$, (c): $10^3 \times 4$ (crosses), $8^3 \times 4$ (dots).

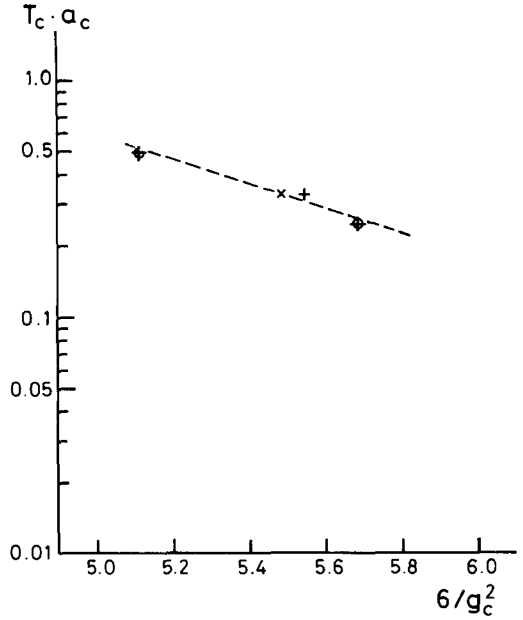

Fig. 2. Critical temperature $T_c a_c = 1/N_\beta$ as function of the critical coupling $6/g_c^2$; plusses: our data; crosses: from Montvay and Pietarinen, ref. [18]; open circles: from Kogut et al., ref. [18]. The dashed line is the renormalization group form with $T/\Lambda_L = 80$.

The energy density for the SU(3) Yang–Mills system on a lattice with $\xi = 1$ is given by [5]

$$\epsilon a^4 = 18[g^{-2}(\bar{P}_\sigma - \bar{P}_\beta) - c'_\sigma(\bar{P}_\sigma - \bar{P}) - c'_\beta(\bar{P}_\beta - \bar{P})] , \quad (6)$$

where $\bar{P}_\sigma$ and $\bar{P}_\beta$ denote plaquette averages with space–space and space–temperature links, respectively. $\bar{P}$ is the plaquette average on a sufficiently large symmetric lattice; it provides the zero point normalization necessary in the euclidean formulation [11]. The terms proportional to

$$c'_\sigma = (\partial g_\sigma^{-2}/\partial\xi)_{\xi=1} = 0.20161 ,$$
$$c'_\beta = (\partial g_\beta^{-2}/\partial\xi)_{\xi=1} = -0.13189 , \quad (7)$$

are finite g^2 corrections to ϵ [7]; for lattices of the size to be used here, they are expected to be non-negligible in the vicinity of the deconfinement transition [5,7].

The form (6) of the energy density is still subject to finite lattice size corrections: for fixed N_σ and fixed temperature, the low momenta are lost in the lattice evaluation, to be recovered only when $N_\sigma \to \infty$ [12] To compensate this, we measure the energy density relative to that of an ideal gas on a lattice of the same size. This means that eq. (6) must be multiplied by [12]

$$R \equiv (\epsilon_{\rm ideal}^{\rm continuum}/\epsilon_{\rm ideal}^{\rm lattice}) , \quad (8)$$

where in $R = R(N_\sigma, N_\beta)$ the same lattice size is used as in eq. (6). The values of R range from about 1.45 to 1.75; for an $8^3 \times 3$ lattice, we have $R = 1.7441$. From eqs. (6) and (8) we obtain the latent heat

$$\Delta\epsilon/T_c^4 = 18RN_\beta^4[g_c^{-2}\Delta(\bar{P}_\sigma - \bar{P}_\beta) - c'_\sigma\Delta\bar{P}_\sigma - c'_\beta\Delta\bar{P}_\beta] , \quad (9)$$

where Δ refers always to the difference between the ordered (cold) and random (hot) start iteration results of the respective quantity. Eq. (9) gives us the latent heat directly in terms of another physical observable, T_c. In fig. 3, we show our results for $N_\beta = 2$, 3 and 4; again we conclude that scaling is well satisfied and

Table 1
The deconfinement temperature.

N_β	$6/g_c^2$	$T_c[\Lambda_L]$	$T_c[\sqrt{\sigma}]$
2	5.11 ± 0.01	78 ± 1	
3	5.55 ± 0.01	86 ± 1	0.519 ± 0.015 a)
4	5.70 ± 0.01	76 ± 1	$0.519^{+0.050}_{-0.030}$

a) We have here used the value at $6/g^2 = 5.4$ from ref. [10].

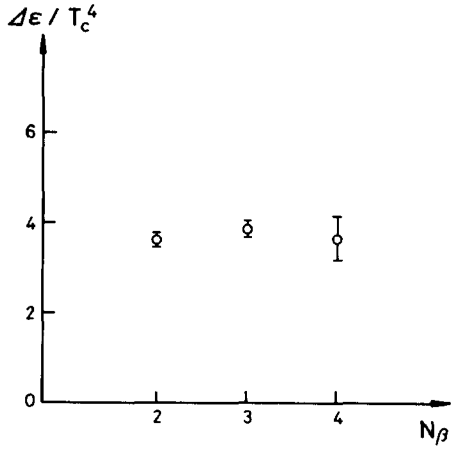

Fig. 3. Latent heat $\Delta\epsilon/T_c^4$ as function of temperature lattice size N_β.

find ‡1

$$\Delta\epsilon = (3.75 \pm 0.25)T_c^4 = 875 \pm 80 \text{ MeV/fm}^3 \qquad (10)$$

for the latent heat of deconfinement.

Let us try to get some feeling for this value. Simple bag model considerations define T_c as the temperature at which the bag pressure B equals the kinetic pressure of the gluons. This gives

$$\Delta\epsilon = 4B\,. \qquad (11)$$

Using the value (10) for $\Delta\epsilon$, we have $B^{1/4} \simeq T_c$, which agrees quite well with the range of B values obtained in hadron spectroscopy [13]. Our lattice result is thus compatible with an interpretation of deconfinement as bag fusion.

A first estimate of the latent heat was recently given by the group of Kogut et al. [14]; they study the full QCD system, with gluons and (quenched) quarks. Considering the small g^2 limit of the energy density [corresponding here to the first term only of eq. (6)], they observe an abrupt variation around T_c, from which they then obtain an estimate for the value of the latent heat. Let us see how this estimate compares to our result. With the same approximation as in ref. [14], we find for $N_\beta = 4$

$$\Delta\epsilon/T_c^4 \simeq 4^4 \times 18Rg_c^{-2}\Delta(\bar{P}_\sigma - \bar{P}_\beta) \simeq 2.60 \pm 0.50\,. \qquad (12)$$

This value, which is in accord with the discontinuity for the Yang–Mills sector shown in ref. [15], falls about 30% below the value obtained from the complete form (9); the finite g^2 terms can thus not be neglected here.

Finally, we want to consider the energy density itself over the temperature range from confinement to asymptotic freedom. For this, we need in eq. (6) the plaquette averages on a large symmetric lattice (in $\Delta\epsilon$, they drop out in the subtraction), and this requires considerable additional calculations. Combining our results on an 8^4 lattice with those of other authors [10,16], we obtain from eqs. (6) and (8) the behaviour shown in fig. 4. The energy density of the SU(3) system, in accord with previous results based on lower statistics [15], approaches the asymptotically free form considerably faster than that of the SU(2) case [5]; the latter is expected to undergo a continuous (second order) deconfinement transition [3,17]. Both for SU(2)

‡1 Not including the error in T_c.

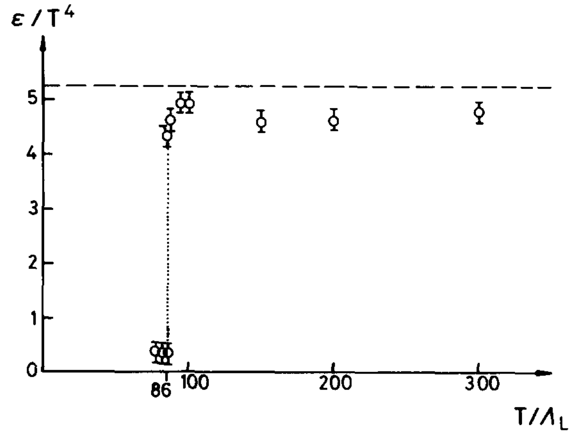

Fig. 4. Energy density ϵ/T^4 as function of temperature, calculated on an $8^3 \times 3$ lattice, using the renormalization group relation.

and SU(3) does the energy density appear to approach the Stefan–Boltzmann limit

$$\epsilon/T^4 = (N^2 - 1)\pi^2/15\,, \quad N = 2, 3\,, \qquad (13)$$

from below, in agreement with a perturbation expansion retaining the exchange term only [18].

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