THE ORDER OF THE DECONFINEMENT TRANSITION IN SU(3) YANG–MILLS THEORY

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We study the finite temperature deconfinement transition in SU(3) Yang–Mills theory, using a high statistics Monte Carlo evaluation on a $8^3 \times 3$ lattice. It is shown to be of first order: at the critical temperature, there is a clear two-state signal; above and below, we have hysteresis behaviour.

At sufficiently high physical temperatures, the SU(N) Yang–Mills system consists of deconfined gluons [1]; at sufficiently low temperatures, we have confinement, and the system consists of gluonium states. The deconfinement transition was first predicted by strong coupling lattice considerations [2]; for the SU(2) and SU(3) systems, it has in the past two years been studied extensively by Monte Carlo methods [3–7].

The transition is related to a global symmetry under the center Z_N of the SU(N) gauge group; this symmetry is realized in the "disordered" confinement phase, but broken in the "ordered" deconfinement phase [6]. Thus Z_N should characterize the universality class of the theory, and the critical behaviour of the SU(N) gauge system should parallel that of the corresponding Z_N gauge system. It is moreoever conjecture [8] that the universal finite temperature aspects of a (d + 1)-dimensional euclidean gauge theory are the same as those of an effective d-dimensional spin theory with only nearest neighbour interactions; this equivalence was shown to hold in the strong coupling limit [2] and is supported by recent Monte Carlo studies of the Z_2 and Z_3 systems [9]. For the SU(2) Yang–Mills system in three space dimensions we thus expect the critical behaviour of the three-dimensional Ising model, while the SU(3) system should correspond to the three-dimensional Z_3 or three-state Potts model [10]. In particular, this implies that the finite temperature deconfinement transition should be of second order for SU(2), but of first order for SU(3) gauge theory.

Previous Monte Carlo results on the SU(2) [3,5,7] and SU(3) [4,7] Yang–Mills systems are qualitatively in accord with this prediction: both order parameter and energy density drop at T_c much more rapidly for SU(3) than for SU(2), and low statistics SU(3) calculations gave some indications of hysteresis-like behaviour [4]. However, a rapid change of behvaviour can also be due to a sharp second order transition, and pre-equilibrium behaviour can simulate metastable states. The main aim of the present paper is therefore to establish the order of the deconfinement transition in SU(3)-Yang–Mills theory by a high statistics Monte Carlo analysis, comparing the development of the system when starting from completely ordered and completely disordered initial states at the same coupling value [11].

In addition, we shall see that such an evaluation provides a determination of the transition point, which is independent of spatial lattice size; this is to be contrasted to previous determinations using either the average order parameter or the specific heat; both do show such a dependence. Finally we note that the latent heat of the transition constitutes an additional physical observable for tests of scaling or of universality [12].

Let us begin by sketching the behaviour expected on a finite lattice for systems undergoing first and

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second order transitions [11]. Consider a system of q-valued spins situated on N^d sites of a d-dimensional lattice. Starting either from a completeley ordered or a completely random initial configuration, we pass site by site through the entire lattice, randomly changing each present spin orientation, with the criterium of maximizing a given distribution function. After each passage through the lattice ("iteration"), we calculate the average over the lattice of whatever physical observable we are interested in, e.g., the energy density ϵ. After sufficiently many iterations, the averages arising from the two different initial states will converge to *one* stable equilibrium value, provided we are not in a critical region. Directly at the critical temperature, in case of a first order transition the energy density ϵ will attain one stable value coming from the ordered start and a different one for the disordered start. On any finite lattice, there will be phase flips, whose likelihood decreases inversely to the size of the system; but even when they occur, the two-state structure generally persists [‡1]. In the case of a second order transition, the two averages will fluctuate but eventually converge to one equilibrium at T_c; after an initial relaxation time, there is no more two-state character. If we go a little below or above T_c, then in the case of a first order transition, the two-state character generally persists, while the probability of phase flips increases and flips predominate in the direction from metastable to stable state (hysteresis).

We now turn to the SU(3) Yang–Mills system, whose euclidean action in Wilson form

$$S(U) = \frac{6\xi^{-1}}{g_\sigma^2} \sum_{\{P_\sigma\}} (1 - \tfrac{1}{3}\,\mathrm{Re\,tr}\,UUUU) + \frac{6\xi}{g_\sigma^2} \sum_{\{P_\beta\}} (1 - \tfrac{1}{3}\,\mathrm{Re\,tr}\,UUUU) \quad (1)$$

is a sum over space-like (P_σ) and space-temperature (P_β) plaquettes [5]; we consider a completely periodic lattice with N_σ^3 (N_β) sites in the spatial (temperature) directions with corresponding spacings a_σ and a_β, and with $\xi \equiv a_\sigma/a_\beta$. The associated space-like and temperature-like couplings are denoted by g_σ and g_β, respectively. The euclidean partition function is given by

$$Z_E(N_\sigma, N_\beta, g_\sigma^2, g_\sigma^2, \xi) = \int \prod_{\text{links}} dU \exp[-S(U)] , \quad (2)$$

where the product runs over all links of the lattice. In our evaluation, we shall generally chose $\xi = 1$, which makes [13] $g_\sigma^2 = g_\beta^2 \equiv g^2$, so that we have $Z_E(N_\sigma, N_\beta, g^2)$.

The order parameter for confinement in this system [3,6] is the average value of the thermal Wilson loop $L(\boldsymbol{x})$:

$$L(\boldsymbol{x}) = \tfrac{1}{3}\,\mathrm{tr} \prod_{\tau=1}^{N_\beta} U_{x;\tau,\tau+1} . \quad (3)$$

It is related to the free energy F_q of an isolated colour charge

$$\langle L \rangle \sim \exp(-\beta F_q) , \quad (4)$$

and hence (on an infinite lattice) vanishes in the confined phase, while attaining non-zero values above T_c. Integrating out all gauge degrees of freedom except the $L(\boldsymbol{x})$ is conjectured [8] to reduce the (3 + 1)-dimensional euclidean SU(3) gauge form to that of an effective three-dimensional Z_3 spin system with nearest neighbour interaction; this provides the basis for the equivalence of the critical behaviour of the corresponding systems.

We evaluate

$$\langle L \rangle \equiv \int \prod dU\, L(U) \exp(-S) \Big/ \int \prod dU \exp(-S), \quad (5)$$

by calculating the lattice average $\bar{L}$ for a given configurration of U's, and then average over successive iterations of different configuraitons. For SU(3), order implies one of the three physically equivalent Z_3 modes

$$(\bar{L}/|\bar{L}|) = 1, \exp(2\pi i/3), \exp(4\pi i/3) . \quad (6)$$

If for any specific configuration we obtain a complex $\bar{L}$, then a corresponding transformation of the link matrices is carried out, so that the system is always kept in the sector which is connected to the continuum limit $U \to 1$. The iteration average of $\bar{L}$ obtained in this way is denoted by $\langle L \rangle$

To determine $\langle L \rangle$ as function of the physical temperate $T = \beta^{-1} = (N_\beta a)^{-1}$, where we have taken $\xi = 1$, we make use of the renormalization group relation

[‡1] See e.g. the behaviour of the Z_3 gauge system studied in refs. [9].

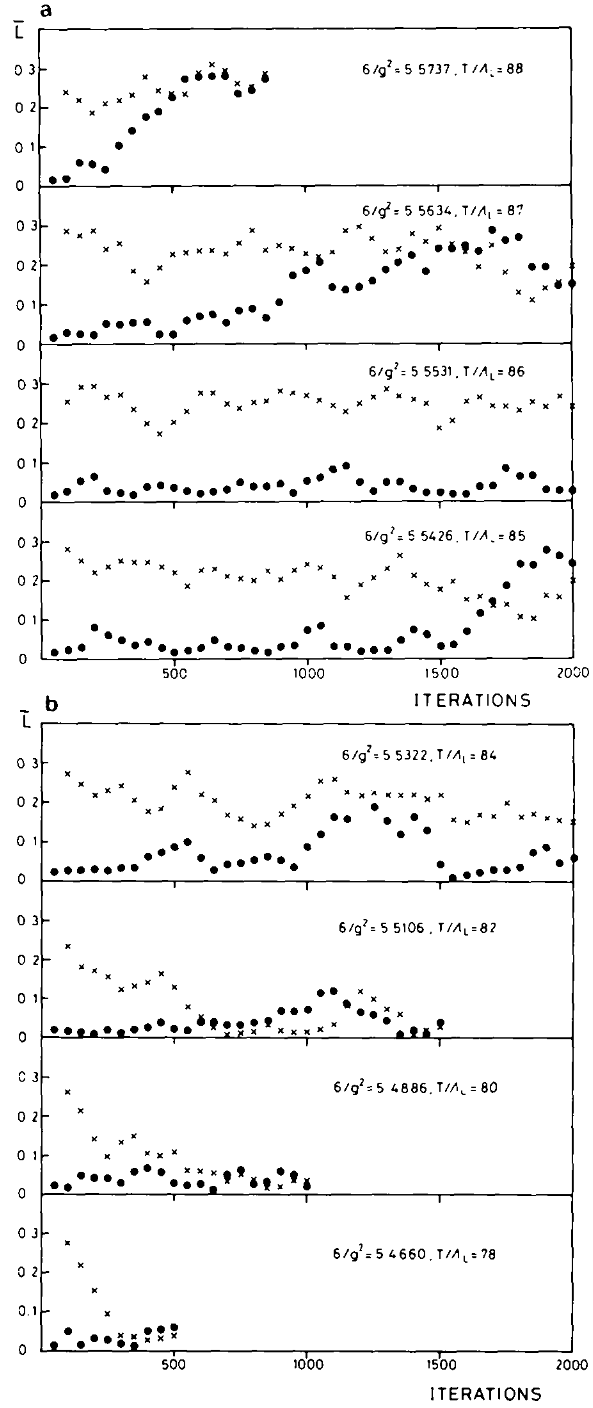

Fig. 1. Lattice average $\bar{L}$ of the order parameter, as function of the number of iterations after ordered and random starts, calculated on a $8^3 \times 3$ lattice for various values of the coupling $6/g^2$; also shown is the associated temperature, using the renormalization group relation.

$$a\Lambda_L = \exp[-8\pi^2/11g^2 - \tfrac{51}{121}\ln(11g^2/16\pi^2)] . \quad (7)$$

This of course implies that our data in fact fall into the region of validity of the scaling form (7); we shall see that this appears to be reasonably well the case, by comparing data from lattices with $N_\beta = 3$ and 4.

Let us now consider the results obtained for L on a lattice of $N_\sigma^3 \times N_\beta = 8^3 \times 3$ sites, with $\xi = 1$. We concentrate on the temperature range $T/\Lambda_L = 75$–90, since that is where the transition is expected to occur [4,7]. Using relation (7), this implies for $N_\beta = 3$ a range of couplings $5.4310 \leqslant 6/g^2 \leqslant 5.5937$.

In fig. 1, we show at various temperatures the behaviour of L as function of the number of iterations, starting in each case from a completely ordered and a completely random configuration. For $T/\Lambda_L = 78$, we have typical non-critical behaviour: $\bar{L}$ converges rapidly to one equilibrium value, close to zero and thus inindicating confinement (L will vanish in the confinement region only for an infinite spatial lattice). Increasing T leads to increased fluctuations and a greater relaxation length, until at $T/\Lambda_L = 86$ we have a clear and quite stable two-state signal. Above 86, fluctuations increase once more, and at 88 we are back to non-critical behaviour. At $T/\Lambda_L = 85$ and 87, we note that a two-state structure still persists, but phase-flips occur. This allows us to use the metastable states at these temperatures, and at $T/\Lambda_L = 84$ as well, to obtain a hysteresis-pattern shown in fig. 2. We thus conclude that the deconfinement transition is of first order.

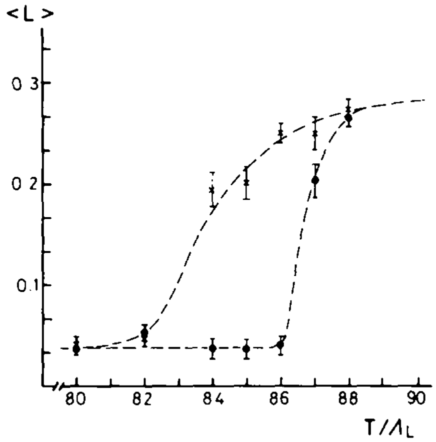

Fig. 2. Hysteresis pattern for the order parameter $\langle L \rangle$ of the SU(3) Yang–Mills system, calculated on a $8^3 \times 3$ lattice.

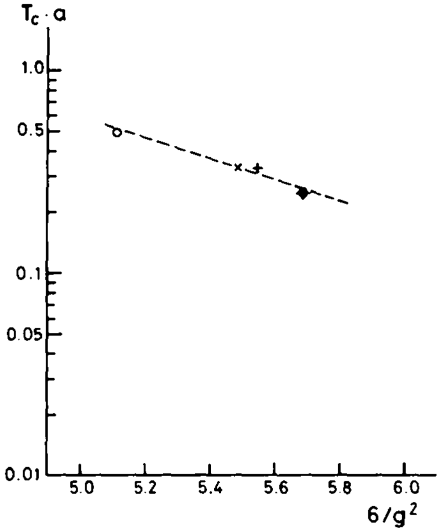

Fig. 3. Critical temperature $T_c = 1/N_\beta a_c$ as function of the critical coupling $6/g_c$; +: our data, ×: from Montvay and Pietarinen [4] ○: from Kogut et al. [7]. The dashed line is the renormalization group form with $T/\Lambda_L = 80$.

As already mentioned, the study of the order parameter as function of the number of iterations provides a determination of the transition point independent of spatial lattice size. Decreasing or increasing N_σ increases or decreases the probability of a phase-flip at the critical point; it does not, however, shift the point. To test this, we have repeated our calculations for a $10^3 \times 3$ lattice, using the critical coupling from the $8^3 \times 3$ evaluation. Again we observe a clear two-state signal. We therefore conclude that for $N_\beta = 3$ the critical point is $6/g_c^2 = 5.5531 \pm 0.0104$.

Finally let us look at the scaling behaviour of our results. We have carried out the same procedure for determining the critical coupling also for $N_\sigma = 8$, $N_\beta = 4$, where we find $6/g_c^2 = 5.6877 \pm 0.0120$. In fig. 3 we plot $T_c a$ for $N_\beta = 3$ and 4, together with the scaling prediction

$$T_c a = (T_c/\Lambda_L)\exp[-8\pi^2/11g^2 - \tfrac{51}{121}\ln(11g^2/16\pi^2)]. \tag{8}$$

Included in fig. 3 are also points from refs. [4] and [7]. We conclude that the scaling relation (7) appears reasonably well satisfied. To convert fully the precision of $6/g_c^2$ into a temperature value more extensive studies at different N_β are needed, since the critical g^2 values for $N_\beta = 2$ and 3 are rather close to the cross-over point from strong to weak coupling: for $6/g^2 \lesssim 5$, we are in the strong coupling regime. Such studies, as well as corresponding ones for the latent heat, are in progress.

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