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GEOMETRY IN EVERYDAY LIFE

Abstract

At the beginning there will be told some words about the sense of mathematics education and different conceptions of teaching mathematics as well as different approaches to mathematics. In connection with this the conception of the "practice-orientated-mathematics-education (PROM)" which the author formulated will be commented. This conception - which must be seen as one point of departure side by side with others - has the aim to qualify the pupils to master life and bases on the method of working on the problems true to life. In the second part it will be discussed how this conception fits with geometry teaching and it will be listed several problems and objects of everyday life in which geometry is of importance. After that experiences with some units of teaching geometry in everyday life in school will be explained.

I will begin with a very general question, the question of the relevance of mathematics education and of geometry teaching in particular. I mean, what do you answer when you are asked why you teach one subject rather than another. Of course we have to differentiate whether we are asked by a collegue or a pupil or a parent of a pupil. I don't mean that you have to ask this question every time you prepare a lesson and it is certainly inappropriate to use a technological system of objectives. However the question about the relevance of teaching should be borne in mind at all times. This means that at least a teacher should know why he or she deals with a certain topic and why it is presented in a particular way. I will deal with this problem later on but for the moment I would like to give some background information.

If you look at the curriculums of mathematics education over the last forty years, or rather until about 1975, you can find three main trends. First there was the traditional maths-education in the schools for the lower classes, the so called "Volksschule", Where the goal of education was to prepare the pupils for their coming life with pragmatic orientated skills and qualifications. Therefore besides the fundamental operations of arithmetic only the so-called "Bürgerliche Rechenarten" was taught, i.e. percentages, calculation of interest, alligation, etc., together

with the calculation of solids. Secondly there was the traditional grammar school, the so-called "Gymnasium", where besides giving the pupils basic knowledge and supplying essential cultural values the main emphasis was placed on general education and on general preparation for university. So the main task consisted of transmitting systematized elementary mathematics in its traditional form and training logical thinking. With the reform of maths-education in the 60's all types of schools were subordinated under a new system which was given the name of "new-maths". This was the main third trend. In its original version the task of mathematics teaching was to enable pupils to learn mathematics interpreted as a system of structures and to use the deductive method. One variant also accentuates the formation of general objectives, another variant attaches great importance to the activity of finding mathematical structures in life-problems, and a third variant tries to emphasize applied mathematics.

Practice orientated mathematics education

As a reaction to this third main trend, which was very closely orientated towards mathematical science, some maths-teaching specialists pleaded in the second half of the 70's for a conception which takes as its point of departure problems of everyday life. I shall now explain in detail the so-called "practice orientated mathematical education (PROM)" which I have formulated. But before doing so I would like to explain two points.

First my conception is different from the so-called "application orientated mathematical education" conception, widely held nowadays, where the problem of application of mathematics is seen from the viewpoint of mathematicians. In my conception we shall not start with mathematics and then look for applications in life. Conversely we have to use real life as point of departure and then look out how mathematics can help to solve problems of everyday life. Although in many cases teaching practice derived from "application orientated mathematics education" can be used in my conception. Secondly I don't plead for a mathematical education which is formed only by practice orientated mathematics teaching. This teaching method must also be combined with problems of pure mathematics and problems of other sciences as well as just playful

problems. Reverting to my opening remarks, of course it is necessary that the teacher explains to the pupils at least roughly why the problem being studied in lesson helps them to think in a addition problems from the history of science and several situations in the development of science offer very helpful points. Also the training of skills has to be done as it is necessary in normal life to train special skills. But if the pupils can see the training of skills in connection with problems of everyday life, they have worked on before, the sense of training skills becomes more clear.

In the following passage I want to talk about my conception of practice orientated mathematical education and practice orientated geometry education in particular. The point of departure is as already said the everyday world. That means the first criterion for selecting subjects is the importance of the lesson to the present or prognosticated future life of the pupils. In addition a general goal of education should be the promotion of personal development and the formation of men to master life and in particular the method of working on problems true to life. In respect to the term "everyday world" I mean also the causal and social interlockings with nature and with communities such as family, community of a house, municipality, nation or mankind; "everyday world" further means the mental and cultural goods of our own and other civilisations. In respect to later life of course we must concede that we can only make forecasts; but with this dilemma every pedagogical conception is concerned. More than that a pedagogic is alive and fruitful only when it can combine funded analysis about future with hope and confidence in the rightness of pedagogical handlings. Furthermore I must concede that different children have and will have different everyday worlds. A pedagogical conception therefore must be reducible to a common standpoint as well as give enough freedom for individual forming. Please do understand the term "orientation at everyday life" always in this global view. Also in this view you have to see the following general objectives with which I want to explain the general goal a little more precisely:

1) Every person shall be able to understand as far as possible the world in which he or she lives and from which he or she is directly or indirectly touched.

2) Every person shall be able to master life, that is, to solve or to digest in a positive manner the problems coming from his or her life and his or her world around.
3) Every person shall be able to have enough possibilities to form his or her life in future.
Because of the comprehension of the world as complex related unity and because of the latest scientific findings of the psychology of learning I must add a didactic principle:
4) The acquisition of isolated bits of knowledge, skills and abilities are not enough, they rather must be based on concrete experiences and combined with situations true to life. In addition the limitation of mathematical auxiliaries as well as the interlockings of the various aspects problems of life have constantly must be kept in mind.

To serve the above mentioned objectives and this principle as well as possible I suggest for a practice-orientated teaching to use the following method:
5) The subjects are more or less complex problems which are represented in form of situations of life. In addition a global method is necessary, where mathematical and non-mathematical aspects will not be separated and where the accentuation of a theme doesn't lie on mathematics but on the points you normally see in such a situation. The practice-orientated conception therefore permanently has a tendency to interdisciplinary teaching.

Since I want to realize this conception in a teaching system where mathematics are taught in special lessons as usual it is necessary for the maths-teacher on one hand to select problems respectively situations which consist essential aspects which can be solved by mathematics and on the other hand to become acquainted with considering non-mathematical aspects in mathematical lessons.

For teaching geometry in everyday life you have to apply the acquivalent comment in respect to geometry.

In the following I want to explain this further by showing and commentating exemplary several situations which are useful for such geometry teaching. But before doing that I would like to schedule the methodical run in such a practice and problem

orientated course.

6) The methodical run can be organized in the following five steps:

a) A unit begins, as already mentioned, with a situation which occurs or has occurred or could occur in everyday life. In the first step therefore the pupils must be made familiar with the given situation if it is not immediately clear for them or even coming from them directly. What I mean is that all pupils must understand the circumstances of the situation such that the situation comes to life. The ways for that of course depend on several factors, especially the situation of the class. I only want to give some ideas for the method of presentation: Word problem or description with words, picture or sequence of pictures (mostly without text), narration (by the teacher, a pupil or any other person), radio play, movie or video (without instruction), talk or discussion and improved or planned role-playing. I can't consider the pros and cons of these representations here, but I would like to remark that you shouldn't choose always the same representation and that you should look for the most fitting representation in respect to the given situation.

b) In the second step it is the task to define the problem more precisely and to find out the specific questions for the problem. Should the occasion arise the specialization of the main point could be decided by the pupils. But in any case it is the pupils task to dissect the complex situation into separated problems.

c) In connection with this point stands the third step on which the students have to separate essential and unessential informations and to trace out missing informations if necessary. This step once can be solved immediately and at another time needs a lot of effort. Sometimes a new description of the situation is advisable.

d) In the fourth step the several partial problems have to be solved. It is not the place to enter into methodic helps for this point, I only want to hint at papers along with the theme "heuristics" or "problem solving" or "learning by discovery".

e) In the fifth step the single solutions must be arranged so that the solution of the entire problem comes out. And after all the final solution has to be integrated into the given situation of life whereat the limitation of the solution must be reflected.

Supplementary I would like to remark that this sequence of steps

is not to understand as a strict scheme, moreover the orientation on reality at all times dominates.

As you may already have seen this conception doesn't fit with a systematic-orientated geometric instruction where theorems and proofs are in the foreground. I rather plead for a geometry teaching where the formation of secured and aspectful geometric concepts and the ability of solving problems within the geometry or with the application of geometry should be promoted. The aspect of proofing then later on is added when the attributes of special geometric objects are no longer evidently. Thus a main factor of geometry teaching is next to the clarification of geometric concepts through logic the clarification and deepening of geometric concepts through application and demonstration of the relevance of geometric concepts in situations true to life. In such a geometry teaching therefore the problems about "geometry in everyday life" play an important role.

Geometry teaching within PROM

Now I shall pass on to explain teaching "geometry in everyday life" within the scope of my practice-orientated conception more in detail. For that I will first schedule for you some fields and situations of everyday life in which geometry has a specific relevance. And afterwards I will show you exemplary some tested units. Being on the look-out for situations which are useful for geometric applications in the sense of the practice-orientated conception you shouldn't start with mathematical concepts as usual and then search for situations of application. On the contrary you have to inspect fields of life to see whether they contain problems important for life, are understandable for the students, and can be solved by the help of geometry. In accordance with categories of sociology we can look out for such problems under the aspect of job, public affairs, family and leisure. In all of that four categories you can find situations which are suitable for teaching "geometry in everyday life".

To get a first impression of situations of everyday life which can be used as point of departure for our practice orientated geometry teaching I want to show you the following (not at all complete) schedule where are listed several activities of everyday life.

1. Handicraft and planning
- building of houses, e.g. building out a loft, planning a layout of a house, renovation of a facade;
- styling of rooms, e.g. repapering a room, planning a furniture layout, designing lamps;
- cultivating of gardens, e.g. paving a path, planting a patch, sprinkling a lawn;
- planning of landscape, e.g. surveying a region, setting a footpath, calculating a wooded area;
- arts and crafts, e.g. designing of jewelleries, analyzing precious stones, picturing ornaments.

2. Industry
- metal-working, e.g. constructing gearings and waves, comparing cylinder capacities, optimizing transport routes and charges, programming industrial robots;
- packaging, e.g. producing cardboards, designing packages, calculating the falling off at punching special forms, comparing sizes of packages.

3. Trade and traffic
- deliveries, e.g. loading a furniture van, stacking drinking bags, loading containers;
- advertising, e.g. administrating advertising pillars, calculating the costs of sheets of paper for different sizes, making geometric advertising designs (e.g. for TV);
- traffic, e.g. working out city and road maps, comparing flight routes, navigating coastal shipping.

4. Public affairs
- municipality, e.g. planning a sports field, drawing a development plan, laying a supply network;
- state, e.g. planning motorroad crossings, constructing tunnels and bridges, renewing dikes, surveying and calculating storage lakes, planning solar energy stations.

5. Culture
- art and architecture, e.g. planning the scenery on a revolving stage, analyzing of church buildings, drawing famous facades;
- popular science, e.g. determining calendars and planetary

orbits, analyzing shadows and sundials, computing and comparing areas of leaves,calculating curved reflectors.

6. Private affairs
- hobby, e.g. building models, making inlaid work, constructing kites, designing knitting patterns;
- holiday, e.g. stowing luggage into the car, testing a range finder of a bike, computing the slope of a footpath or a road;
- games, e.g. calculating a billiards match, solving a geometric puzzle, programming a computer game.

The selection of a subject for a special class of course requires still several thoughts. But I hope you have already got an impression of which situations in life are suitable as subject and what geometric concepts and theorems are met with in these situations. In addition the subjects must be chosen in respect to the previous knowledge of the students. But I don't mean that all knowledge which is necessary to solve the task has to be available before. The relevance of mathematics, and here in the practise-orientated conception the tool-character of mathematics, can be shown to the pupils better when some special mathematical concepts and relations which are important for the solution are developed in connection with finding the solution. On the other hand it must be restricted that the most important previous knowledge should be available before and that not at all times it is possible to develop new mathematical concepts during solving a problem. But because of the above named relevance of mathematic teaching it is very helpful to introduce mathematical concepts or work out mathematical theorems in connection with solving problems from time to time.

Now I want to describe four units already realized in school. I think that some of the named points become so more clear.

1. Building out a loft

This unit was taught by teachers to about 14 year-old children in a basic course of a 9 year elementary school. It was worked out in a seminar of mine concerning practical experiences within the teacher education. Only four lessons had been planned for this unit though six would have been better.

In the 1. lesson a role-playing in which the situation of two boys in a family with a little house was used to introduce the situation of life. The boys struggle with each other because of having friends in the room of the two. This play was first read aloud and then played with casted roles. In the following the problem of building out a loft was discussed. Among other things it was recorded that because of financial problems the family would do the work by its own and that making a plan including calculation of cost is necessary. Also it was recorded that the most important work is the isolation. In addition different forms of a roof were discussed and after that the teacher chose the form of a hip-roof. Also a picture of "that" house was shown. (The situation above named was shamed by the teaching students but for better getting into the spirit of the situation it was presented to the children as real. And I think this was ok, because the situation was true to life. Besides the children had been well-know with such a situation as we could notice.)

In the 2. lesson the situation was enlived again. Then the partial problem of calculating the covering of the floor and the walls was worked out. The solution was worked out in groups. As a positive by-product different ways of solution came out and have been compared. (In this connection I have to say that a supplementary lesson had become necessary. The mathematical problem of computing the surface of a hip-roofed solid turned out as a difficulty for the children. They already have had difficulties with computing rectangles and triangles. A general repetition was not done because the children should learn to come along with such problems by their own. Therefore different individual and also unorthodox solutions were admitted.)

In the 3. lesson it was dealed with installation of heating elements. Because the size and the prize of the heating depends on the volume of the loft the computation of the volume was the main task in this lesson. The lesson was held by five teacher students simultaneous where each student taught to a small group of children. For each group a model of the hip-roofed solid made of syntethic moss was available. The children themselves have had the idea to cut the model into pieces which easily was done with a knife. Then they could compute the middle part, a prism. With the peaked solids several reflections and trials were done by the

children until some of them cut those solids into two halves and puzzled three of these halves together to a cube. (The children didn't know the formula for peaked solids yet; and it wasn't intended too. It rather was intended to find a solution of the given problem and to give previous experiences with such formulas.)

In the 4. lesson the calculation for the heating was finished and then the solution of the entire problem was worked out. A test about this subject was made some days later.

2. Shape and weight of packings

The unit was taught in two cases parallel in the eight grade of a 9 year elementary school. It consists of 6 lessons and a test. It also was worked out and held by teacher students.

In the 1. lesson different consumer goods were presented to the children. They first had to classify the objects in respect to material property, shape, size and weight as well as price. Afterwards the different shapes were discussed nearer. (In this lesson the children should become acquainted to several packings in context with their relevance in life and in respect to the different qualities. Besides the knowledge about solids should be increased.)

In the 2. lesson the volume of rectangular and cylindrically formed detergent boxes were computed. The question was whether the volumes of boxes with equal weight are equal too.

In the 3. lesson the partial problem was to calculate the cardboard mass. For this the children had to compute the surface of the detergent boxes in particular.

In the 4. lesson the subject was called "deceptive packings". First the outside and the inside volume of the detergent were compared. After that a cylindrical formed cream box was treated in the same way. (The subject of deceptive packing was thought of as the main point of the unit, but during the preparation we saw that it didn't hold for more than one or two lessons.)

In the 5. lesson the volume of an oil tin with the form of a conical frustum was computed.

In the 6. lesson finally some boxes whose shape can be seen as a composition of two or three basic solids were calculated. Such boxes are for example fish-tins, oil-tins and several boxes for sweets. In the test then some other objects of composed shapes such as a greenhouse, television aerial mast, space capsule, treasure chest and goods van had to be computed.

3. Dyke raising

This unit was first worked out and tested by a student teacher in connection with an exam and later on also realized by students with their practical course. It both times was taught to 14 and 16 years old children.

In the 1. lesson the introduction into the problem was done with the subject "stormy tidal wave". The lesson began with pictures and discussion about stormy tidal waves and their destructive effects as well as their prevention. At the end of the lesson the children should put themselves into the situation of a planner or member of parlament of a special state where the dyke has to be renewed and raised up.

In the 2. lesson which followed the first immediately a discussion about the points of cost took place. Afterwards each child had to compute the volume of the old dyke for 1 km distance. The data for that have been given by the teacher.

In the 3. lesson the volume and costs of the sand kernel for the new raised dyke for 1 km distance was worked out in groups.

In the 4. lesson also in groups the children calculated the costs for the bran layer of the new dyke including the costs for the transport of sand and bran. For one way one lorry there was given a flat rate to the children.

In the 5. lesson the children went over to the task to calculate area and costs of the grass layer and to estimate the costs for the workers and machines. For that the children also got standard

prices which the teacher students had asked for before. At the end of this lesson finally the total costs for a 12 km long dyke were determined.

In the 6. lesson several skills were trained once more. In the 7. lesson there was made a test about the total problem.

4. Geometry and arts

A unit about geometry and arts once was taught by myself in a project week of a 9 years elementary school and another time was worked out and held by students in two parallel classes of sixth grade in a 10 years elementary school.

In the 1. lesson of this unit the different shapes of buildings from different ages were discussed using slides.

In the 2. lesson which followed the first immediately, the children were asked to copy some fronts of such buildings, seen in the first lesson by hand. The fronts were given as a picture for each child. Especially the many round arches of the Porta Nigra in Trier took a lot of time and effort. As homework, the children had to visit buildings in the town and make a description of at least two buildings.

In the 3. and 4. lesson being a double lesson too the concepts of a rectangle and a circle were introduced. After that they had to practice handling a pair of compasses and a ruler by making figures which were composed of rectangles and circles or parts of them.

In the 5. and 6. lesson these skills were applied to design ornaments.

In the 7. and 8. lesson, finally, the children produced nice pictures made of coloured yarn and cardboard where figures of rectangles and circles were used as a basis. The children could work creatively at it.

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