THE INTEGRAL OF THE SPIN-DEPENDENT STRUCTURE FUNCTION g_1^p AND THE ELLIS–JAFFE SUM RULE

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A recent EMC experiment has found that the integral of the spin-dependent structure function g_1 of the proton violates the Ellis–Jaffe sum rule. It is shown here that this result can be strengthened when combined with older data from SLAC.

Recently, an experiment by the European Muon Collaboration (EMC) has measured [1] the asymmetry in deep-inelastic polarized muon–proton scattering at CERN. The asymmetry A_1 is shown in fig. 1 as a function of the Bjorken scaling variable x, together with older data [2,3] from polarized electron–proton scattering at lower beam energies at SLAC. The agreement of the CERN and SLAC data is very good. From A_1, the spin-dependent structure function g_1 of the proton was computed, using the relation

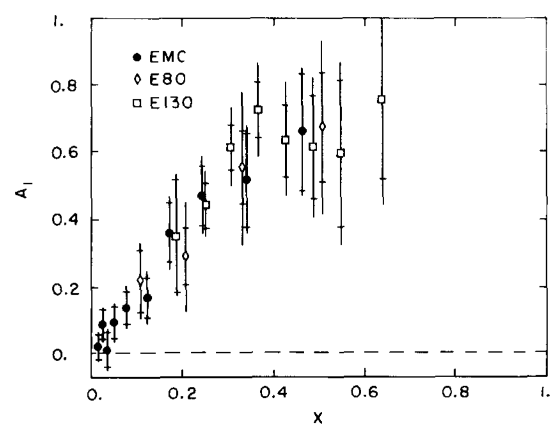

Fig. 1. Compilation of all the data on A_1^p as a function of x. The EMC points (ref. [1]) are shown as full circles while the SLAC points are shown as open diamonds (experiment E-80, ref. [2]) and open squares (E-130, ref. [3]). Inner error bars are the statistical errors and the outer error bars are the total errors (statistical plus systematic added in quadrature). The systematic errors include uncertainties in the values of R and A_2.

$$g_1(x)=\frac{A_1(x)F_2(x)}{2x(1+R)}, \qquad (1)$$

where F_2 is the usual spin-independent structure function and R is the ratio of longitudinal and transverse total cross sections. A_1 is related to the measured asymmetry A by $A=D(A_1+\eta A_2)\simeq DA_1$ where the kinematic factor D also depends on R. The quantity A_2 is a second asymmetry and η is a small kinematic factor.

In the EMC experiment the integral of g_1 over x was found to be

$$\int_0^1 g_1^p(x)\mathrm{d}x=0.114\pm0.012\pm0.026, \qquad (2)$$

where the first error is statistical and the second systematic. This result is in disagreement with the Ellis–Jaffe sum rule [4]. Without the QCD correction this sum rule predicts

$$\int_0^1 g_1^p(x)\mathrm{d}x = \frac{1}{12}\left|\frac{g_A}{g_V}\right|\left[1+\frac{5}{3}\left(\frac{3\mathscr{F}/\mathscr{D}-1}{\mathscr{F}/\mathscr{D}+1}\right)\right]$$

$$=0.200\pm0.005\,. \qquad (3)$$

Here $g_A/g_V = -1.254\pm0.006$ is the ratio of axial-vector and vector weak coupling constants in nucleon β-decay [5], $\mathscr{F}$ and $\mathscr{D}$ are the SU(3) couplings and the uncertainty in the sum rule is mainly due to the experimental error in the measurement [6] of the ratio $\mathscr{F}/\mathscr{D}=0.632\pm0.024$. The first-order QCD correction [7] reduces the value of the integral in (3) to 0.189 at $Q^2=10$ GeV$^2/c^2$. The full circles in fig. 2 show the values for the integral of g_1 from the low edge of each x bin to 1, plotted at the edge of the bin. The solid curve was computed using a fit to the EMC values for A_1. The difference between the theoretical and experimental values is $0.075\pm0.012\pm0.026$.

The EMC result is consistent with a determination based on the SLAC data, which gave [3]

$$\int_0^1 g_1^p(x)\mathrm{d}x = 0.17\pm0.05\,, \qquad (4)$$

with statistical and systematic errors combined in quadrature. The uncertainty is dominated by the extrapolation to low x (below 0.1) where no data were taken. Due to this large error, the result in (4) is also consistent with the Ellis–Jaffe sum rule.

Since the new and the old data are in agreement

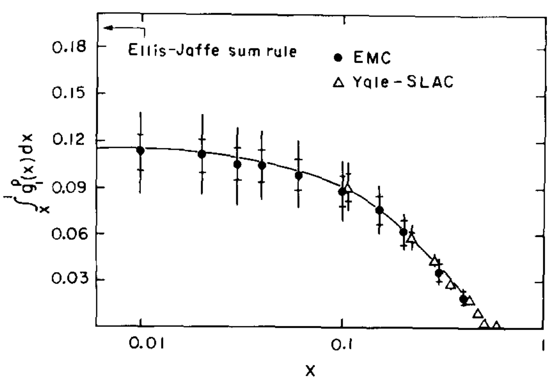

Fig. 2. Results for $\int g_1(x)\mathrm{d}x$. Full circles: computed from the EMC data [1]. Open triangles: computed from the SLAC data, merging the two experiments [2,3]. The solid curve was computed from a fit to the EMC A_1 data. Inner (outer) error bars are the statistical (total) errors.

over the region of overlap, $0.1\leqslant x\leqslant0.7$, the total error can be reduced by combining the two results in this region and using the EMC points at low x. For this purpose, it is important to treat the two sets of data in as similar a fashion as possible. Therefore, the SLAC data were re-examined in order to separate the statistical and systematic errors. Point-by-point statistical errors were added in quadrature when computing the integral, while systematic errors were added linearly, to allow for the possibility of common systematic errors affecting all the x bins in the same direction. In addition, uncertainties due to the values of R and A_2, which is unknown except that it is bounded by $|A_2|\leqslant\sqrt{R}$ were included in the systematic error [#1]. Finally, the data from ref. [2] which were obtained in narrow x bins, were merged into the bins of ref. [3].

The integrals were computed using a parametrization of F_2 taken from ref. [8], at $Q^2=10$ GeV$^2/c^2$, the mean Q^2 of the EMC data. Since no evidence for any Q^2 dependence of A_1 was found from the SLAC and EMC data [1–3] it is justified to assume that the A_1 values of the two experiments are valid at this Q^2. For R, a QCD calculation [9] was used. Since $R=0$ was assumed in extracting F_2 in ref. [8], F_2 was corrected for consistency for the non-zero R values used here (see eq. (2.7) in ref. [8]).

The two data sets give, over the region of overlap,

$$\int_{0.1}^{0.7} g_1^p(x)\mathrm{d}x = 0.091\pm0.008\pm0.0013 \quad \text{(SLAC)}\,,$$

$$\int_{0.1}^{0.7} g_1^p(x)\mathrm{d}x = 0.087\pm0.010\pm0.015 \quad \text{(EMC)}\,. \qquad (5)$$

The SLAC results for $\int g_1^p\mathrm{d}x$ are also shown in fig. 2 (open triangles). The SLAC and EMC results are in excellent agreement.

For the SLAC data, $R=0.25\pm0.10$ was used in computing A_1, as previously done in refs. [2,3]. This number is larger than the QCD prediction [9]. Using the same QCD calculation for the SLAC data, at

[#1] The exact expression for g_1 is $g_1(x)=\{F_2(x)/2x(1+R)\}[A/D+(\sqrt{Q^2}/\nu-\eta)A_2]$. The second term inside the square brackets gives the uncertainty due to the unknown value of A_2. The definition of all the relevant quantities can be found in refs. [1–3].

the appropriate Q^2, would reduce the A_1 values because of the R dependence of D and the first integral in (5) would decrease by 0.012. However, this calculation gives R values that are probably too small for the kinematic range of the SLAC data. A recent experiment at SLAC found [10] that perturbative QCD describes the data on R at these low Q^2 values only if corrections for target-mass effects are included. The parametrization given in ref. [10] implies that R is in the range 0.13–0.29 in the kinematic range of the data of ref. [2] and 0.08–0.16 in that of ref. [3], with the highest R values obtained for the lowest x points (0.10–0.22). With this parametrization of R, the SLAC result above (eq. (5)) is reduced by 0.002.

The systematic errors in the two results above have very different origins, and therefore they can be combined as if they were statistical. This gives

$$\int_{0.1}^{0.7} g_1^p(x)\mathrm{d}x = 0.089 \pm 0.006 \pm 0.010 . \qquad (6)$$

In addition, EMC alone gives

$$\int_{0.01}^{0.1} g_1^p(x)\mathrm{d}x = 0.024 \pm 0.007 \pm 0.008 . \qquad (7)$$

In combining (6) and (7), care must be taken regarding the correlation in the uncertainties of the low- and high-x EMC data. If the systematic errors in (6) and (7) were uncorrelated, they should be added in quadrature while if they were correlated they should be added linearly. Since (6) was obtained with approximately equal contributions from SLAC and EMC, the mean of the systematic errors obtained by the two approaches is taken. Adding the contributions of the extrapolations of the EMC data to unmeasured regions $x=0$–0.01 and $x=0.7$–1.0 (0.002 and 0.001, respectively), we obtain

$$\int_0^1 g_1^p(x)\mathrm{d}x = 0.116 \pm 0.009 \pm 0.019 \quad \text{(world average)} , \qquad (8)$$

with the systematic error containing an additional 10% uncertainty arising from the value of F_2 (see ref. [11] for an up-to-date discussion of the experimental situation on F_2).

The combination of the two results makes more pronounced the difference between the experimental and the QCD corrected theoretical values. Combining all errors in quadrature, this difference is

$$\int_0^1 g_1^p(x)\mathrm{d}x|_{\text{theory}} - \int_0^1 g_1^p(x)\mathrm{d}x|_{\text{experiment}} = 0.073 \pm 0.022 , \qquad (9)$$

or about 3.5 standard deviations.

One potential explanation for the failure of the sum rule could be that the Q^2 of the experiments is not high enough for asymptotic arguments to apply, therefore the Q^2 evolution of the structure function might conceivably be larger than the one predicted by perturbative QCD. The Drell–Hearn–Gerasimov sum rule [12] for real photoproduction requires that the asymmetry be negative in the limit $Q^2=0$ over at least part of the range of the photon energy ν. Hence g_1^p may vary rapidly with Q^2 until its integral reaches the positive value predicted by eq. (3). However the comparison [1] between the low-energy ($\langle Q^2 \rangle \simeq 4\ \mathrm{GeV}^2/c^2$) SLAC data and the higher-energy ($\langle Q^2 \rangle \simeq 10\ \mathrm{GeV}^2/c^2$) EMC data failed to detect any strong Q^2 dependence.

In addition to the data in refs. [2,3], there exist data from SLAC on the asymmetry in the resonance region (missing-mass range $W=1.1$–1.9 GeV) at even lower Q^2, 0.5 and 1.5 GeV^2/c^2 [13]. The asymmetry is positive practically everywhere except in the region of the $\Delta(1232)$ resonance and is in good agreement with the deep-inelastic data, indicating that the transition from real to virtual photoproduction is essentially complete in the kinematic range of the experiments [1–3]. This is also supported by a partial-wave analysis [14] of unpolarized pion-electroproduction data at $Q^2=0.3$–$1.0\ \mathrm{GeV}^2/c^2$.

In conclusion, we have shown that the violation of the Ellis–Jaffe sum rule found by EMC [1] becomes more significant when all the available data [2,3] are included. In addition, the comparison of data taken in different kinematic ranges seems to exclude the possibility that the effect is due to a strong Q^2 dependence.

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