# DLSP for Multi-Item Batch Production

by

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Abstract

The discrete lot-sizing and scheduling problem (DLSP) has been suggested for the simultaneous choice of lot sizes and production schedules. However, the common assumption of instantaneous availability of the manufactured units is not satisfied in practice if the units arrive in inventory in one batch no earlier than completion of the whole lot. Therefore, we present additional constraints for the case of batch production. The resulting extended DLSP is formulated as a mixed-integer linear program. The feasibility problem of this modification to the standard DLSP is again NP-complete. A two-phase simulated-annealing approach is suggested for solving the modified DLSP. Since DLSP is a finite-time-horizon model, sensible target inventories have to be determined. Numerical results are presented for different problem classes.

## 1. Introduction

One of the most challenging problems in production planning has been the simultaneous choice of lot sizes and production schedules in order to minimize cost. In the early 1960s, mixed-integer linear models were proposed in the economic literature to cope with this situation, e.g., Adam [1963] and Dinkelbach [1964]. Their principal idea is to divide the finite time horizon into (small) time intervals in which the machines can be used either for production of at most a single product, or can be setup for such a production. Recent interest in these models, now popular as the single- or multi-machine discrete lot-sizing and scheduling problem (DLSP), stems from the development of new algorithms to solve these problems. Fleischmann [1990], for example, suggests an exact algorithm for the single-machine multi-item case with zero setup times. A comprehensive reference to these algorithms is given by Salomon [1991]. After showing that generating feasible solutions for the single-machine multi-item DLSP with non-zero setup times is NP-complete, he suggests a heuristic based on Lagrangean relaxation.

All these modelling approaches pertain to the case of instantaneous availability of the manufactured units prior to the completion of the lot. However, this assumption is not satisfied in practice if the units arrive in inventory in one batch no earlier than completion of the whole lot. Therefore, we introduce additional constraints in order to model the case of batch production. The resulting extended DLSP is formulated as a mixed-integer linear program. For solving this problem a two-phase simulated-annealing (SA) approach is suggested because of the general applicability of SA (Kuik and Salomon [1990] apply SA to the related multi-level lot-sizing problem).

In many cases, there is no natural time limitation for production processes and demand will be stochastic. Hence, finite-time-horizon models are applied repeatedly (rolling application) in order to solve approximately the underlying infinite planning problem and to incorporate new estimates based on more available data in each planning instance. In this context, a special emphasis is put on the final inventories. Therefore, we determine sensible inventories by a parametric application of the modified DLSP-model to the expected demand. These inventories can be used as target (final) inventories for production planning in rolling application, while actual (final and therefore new initial) inventories result from the deviation of realized from estimated demand. Positive initial inventories yield more feasible production schedules with respect to the demand restrictions with additional cost saving potential. Therefore, optimal target inventories will also not be zero for all products simultaneously.

## 2. The DLSP in the Case of Batch Production

Ordinarily, DLSP is formulated for $N$ items and a finite-time horizon of $T$ time units. Here, production scheduling and demand should be considered on different time scales: Typically, demand can be estimated for example as demand per day, while the DLSP requires a less coarse discretization of the time axis due to the underlying idea that during one time unit (e.g., hours or 30-minute intervals), the machine can be used at most for setup or production of only one item (see constraints (4) below). Hence, the time units for DLSP will be chosen to be the greatest common divisor of setup times and minimal production times for all items. The different time scales of demand and scheduling yield a division of the planning period $T$ into $M$ (demand) subintervals of lengths $T_m$-$T_{m-1}$ ($m$=1,...,$M$; $T_0$=0; $T_M$=$T$) where demand $d_{it}$ for product $i$ is positive only at times $T_m$ ($d_{iT_m} \geq 0$ and $d_{it} = 0$ $t=1,...,T$; $t \neq T_m$; $m=1,...,M$; $i=1,...,N$).

A production schedule is a matrix $(v,y)=((v_{it}), (y_{it}))$ where

$v_{it}$ are zero-one variables indicating setup in period $t$ for production of product $i$ and

$y_{it}$ are zero-one variables indicating production of product $i$ in period $t$ with $y_{iT+1}$=0 for formal reasons.

Then the objective is to minimize the sum of setup and inventory holding cost

$$\text{Min } C((v,y)) = \sum_{i=1}^{N} \sum_{t=1}^{T} r_i v_{it} + h_i I_{it} \tag{1}$$

s.t.

$$I_{it-1} + o_i y_{it} - d_{it} = I_{it} \qquad i=1,...,N;\ t=1,...,T \tag{2}$$

$$v_{it-a_i+\tau} \geq y_{it} - y_{it-1} \qquad i=1,...,N;\ t=a_i+1,...,T;\ \tau=0,...,a_i-1 \tag{3}$$

$$y_{it} = 0 \qquad i=1,...,N;\ t=1,...,a_i \tag{3a}$$

$$\sum_{i=1}^{N} (y_{it}+v_{it}) \leq 1 \qquad t=1,...,T \tag{4}$$

$$e_{it} \geq y_{it} - y_{it+1} \qquad i=1,...,N;\ t=1,...,T \tag{5}$$

$$e_{it} \leq y_{it} \qquad i=1,...,N;\ t=1,...,T \tag{6}$$

$$e_{it} \leq (1-y_{it+1}) \qquad i=1,...,N;\ t=1,...,T \tag{7}$$

$$E_{im} = \sum_{t=1}^{T_m} e_{it} \qquad i=1,...,N;\ m=1,...,M \tag{8}$$

$$V_{im} = \sum_{t=1}^{T_m} v_{it} \qquad i=1,...,N;\ m=1,...,M \tag{9}$$

$$D_{im} = V_{im} - a_i E_{im} \qquad i=1,...,N;\ m=1,...,M \tag{10}$$

$$\tilde{y}_{im} \le y_{it} \qquad i=1,...,N;\ m=1,...,M;\ t=1,...,T_m \qquad (11)$$

$$\tilde{y}_{im} \le \left( V_{im} - \sum_{\tau=1}^{t} v_{i\tau} \right) + (a_i - D_{im}) \qquad i=1,...,N;\ m=1,...,M;\ t=1,...,T_m \qquad (12)$$

$$\tilde{I}_{im} = \sum_{\tau=1}^{T_m} (o_i \tilde{y}_{i\tau m} - d_{i\tau}) + I_{i0} \qquad i=1,...,N;\ m=1,...,M \qquad (13)$$

$$I_{it},\ I_{i0},\ \tilde{I}_{im} \ge 0 \qquad i=1,...,N;\ t=1,...,T;\ m=1,...,M \qquad (14)$$

$$y_{iT+1} = 0, \quad e_{it},\ v_{it},\ y_{it},\ \tilde{y}_{im} \in \{0,1\} \qquad i=1,...,N;\ t=1,...,T;\ m=1,...,M \qquad (15)$$

The objective (1) and constraints (2), (3) and (4) are taken from the standard DLSP [Salomon 1991, pp. 34 and 43]. Again, $r_i$ in (1) are sequence independent setup costs per setup period and the inventory holding costs are given by the product of parameter $h_i$ (costs per unit and period) with nonnegative inventories $I_{it}$. The demand-satisfaction constraints (2) of standard DLSP describe the dependence of current inventories on the inventories of the preceding period, the quantity of the corresponding item produced (where $o_i$ is the production speed) and the demand. The correct sequence of setup and production periods is modelled in equation (3) where $a_i > 0$ is the corresponding number of setup periods. Constraints (3a) are additional to the DLSP formulation due to Salomon [1991] and prohibit production of item $i$ in periods $1,...,a_i$ with no preceding setup. Constraints (4) are used to prevent simultaneous action (setup or production) on the machine.

These constraints alone do not pertain to the case of batch production. While the inventory including work in process $I_{it}$ (as described above) is required for computing holding costs in (1), the last batch begun before time $t$ is not available for satisfying demand if the production process of this batch is not finished by $t$. Therefore, we introduce inventory $\tilde{I}_{im}$ which is already available in demand instance $T_m$. In order to model this available inventory, some auxiliary variables and constraints (5) - (12) are needed: Here, zero-one end-of-batch variables $e_{it}$ indicate the production of the last unit of the current batch in $t$ by $e_{it}=1$. End-of-batch constraints (5) - (7) are similar to setup constraints (3). $E_{im}$, $V_{im}$ and $D_{im}$ are set in equations (8) - (10) to the cumulated number of completed batches, number of setup time units and a correction term, respectively. This correction term is required to determine zero-one variables $\tilde{y}_{im}$ which are then used in (13) to define the auxiliary inventory $\tilde{I}_{im}$ of available units. It should be noted that $\tilde{y}_{im}$ vanishes even if the decision variable $y_{it}$ is equal to one, if period

$t$ is used for production of item $i$ in a batch which was started before but has not been completed by $T_m$. This behaviour of the auxiliary variables $\tilde{y}_{im}$ is guaranteed by constraints (11) and (12), where the latter consists of two parts both of which are zero in the case of incomplete production at time $T_m$: In this case, the difference between cumulated number of setup periods before $T_m$ and before $t \leq T_m$ is greater than zero for all batches that are completed before $T_m$, while the second bracket is positive only in the case of no production extending in time over $T_m$. Moreover, $\tilde{y}_{im}$ may also vanish despite completion of the batch by $T_m$ if not all of the units produced are needed to satisfy demand in $T_m$. Note, that the auxiliary inventory $\tilde{I}_{im}$ will be smaller than the actually available inventory only in this case. Constraints (14) and (15) are the standard non-negativity and binary constraints, respectively, for the decision and auxiliary variables. Initial inventories can be thought of as given constants while final (target) inventories are implicitly included in the demand of period $T$.

Salomon [1991] shows NP-completeness of the feasibility problem of the DLSP with non-zero setup times. The modified DLSP is also in NP, because feasibility of any given structure (production schedule) for any problem instance can be checked in polynomial time. Since any instance of the standard DLSP can be transformed to an instance of the modified DLSP (again in polynomial time) and a feasible production schedule for the modified DLSP is also feasible for its standard version given essentially by constraints (2) - (4), the standard DLSP reduces to the modified DLSP which is then also NP-complete [Florian et al. 1980].

## 3. The Two-Phase Simulated Annealing

Simulated annealing is a multi-purpose heuristic for the solution of combinatorial optimization problems that was first suggested by Kirkpatrick et al. [1983] and Černý [1985]. A neighbourhood structure is superimposed on the (usually finite but large) space of feasible solutions (configurations or in this context production schedules). Given such a feasible configuration $(v_{cur}, y_{cur})$ a candidate solution $(v_{can}, y_{can})$ is drawn randomly from the corresponding neighbourhood. This new configuration will be accepted subject to either improvement of the objective function or another random experiment with acceptance probability given by $e^{-\Delta C/\gamma}$ where $\Delta C = C(v_{can}, y_{can}) - C(v_{cur}, y_{cur})$ is the difference of the cost function values of the candidate and the current configuration. $\gamma$ is a control parameter corresponding to temperature in the original physical analogue in thermodynamics [Metropolis et al. 1953]. Infinite repetition of this procedure with a fixed value of control parameter $\gamma$ can be viewed as one realization of a homogeneous Markov chain where the current state of the Markov chain is the last accepted configuration. Iterative reduction of the temperature (i.e., $\gamma$) yields a sequence of such Markov chains and it can be shown [Mitra et al. 1986] that, roughly spoken, the sequence of configurations converges asymptotically to a globally optimal solution, almost surely. Besides this convergence behaviour (although efficiency compared to tailored algorithms is usually poor), the main advantage of SA relative to other (tailored) solution methods suggested is its general applicability. Solving a specific problem with SA requires only determination of a neighbourhood structure and an appropriate cooling schedule (i.e., choice of the sequence of control parameters $\gamma$ and number and length of the finite approximations of the homogeneous Markov chains). Van Laarhoven [1988] discusses different cooling schedules and application of SA to selected combinatorial optimization problems. However, it is important to point out that the neighbourhood choice is usually performed on the set of feasible configurations only. Feasibility in the context of the modified DLSP is mainly given by the demand-satisfaction under batch production (13) and setup constraints (3) - (4). For a general problem instance, it is therefore necessary to construct an initial feasible solution disregarding costs and to optimize with respect to the objective function in a second phase. The overall structure of the two-phase algorithm outlined in the following two sections is similar to the two-phase simplex method for the solution of linear programs.

The neighbourhood structure for a given production schedule $(v,y)$ is defined by reducing $(v,y)$ to a "pure" production schedule $y^{pure}$ by physically eliminating setup periods. An element $(v_{can}, y_{can})$ out of the neighbourhood of $(v,y)$ is then obtained by exchanging the activities of two arbitrary periods in $y^{pure}$ and expanding (i.e., inserting setup periods in front of each production batch and shifting later production by the corresponding number of time units) $y_{can}^{pure}$ to $(v_{can}, y_{can})$. During expansion of $y_{can}^{pure}$ to $(v_{can}, y_{can})$ a necessary condition for feasibility (the last batch must be finished in or before period $T$) is checked and if this condition is violated another configuration is drawn out of the neighbourhood of $(v,y)$.

### 3.1 Phase 1: Search for a Feasible Production Plan

The procedure described above does not ensure feasibility in the sense of (2) to (15) since the demand-satisfaction constraints might not be satisfied. Therefore, a first production schedule is chosen to consist of a single batch for each item where the batch size is determined to satisfy the cumulated demand in $T$ and production is carried out immediately. For this situation, initial inventories are computed in order to fulfil the demand satisfaction constraints (13) in each demand instance. Afterwards the sum of the positive deviations of these hypothetically needed inventories from the actual inventories form the objective function and are minimized in phase 1. As in the simplex method, a feasible production schedule is found if this sum vanishes. SA will yield more feasible (in the sense that production is finished by $T$) production schedules by restricting the exchange to active production periods only contrary to the exchange of activities which also include the final idle periods.

### 3.2 Phase 2: Search for a Cost-Optimal Production Plan

The actual optimization with respect to the cost function is carried out in phase 2. Here, the same neighbourhood structure is used, but exchanges are now carried out between arbitrary (active and idle) periods of $y^{pure}$. Production schedules that are not feasible in the sense of (13) are not considered as candidates. Thus, feasibility is preserved before each acceptance in phase 2 subject to the acceptance criterion corresponding to the original cost function (1).

## 4. Parametric Optimization of Target Inventories

In chapter 2 the modified DLSP was introduced for the situation of deterministic (possibly estimated) demand and given inventories. In most practical situations, the scheduling problem arises in the context of rolling application with stochastic demand. Especially here, it is important to find a sensible set of inventories which can be used as target inventories at each planning instance. Actual final inventories are the initial inventories of the next interval. It should be noted that these inventories will differ from the target inventories in practice due to the deviation of realized from estimated demand. In this situation, demand is to be modelled by random demand variables. In order to find a set of sensible target inventories, the stochastic demand variables are replaced by their expected values. For this demand structure, final and initial inventories are assumed to be equal and varied parametrically in a modified DLSP application in order to find a cost optimal set of target inventories. Additionally, sensible target inventories for the stochastic application will usually contain a safety stock component in order to compensate for the deviations of the realized from the estimated demand.

## 5. Numerical Results

Demand schedules are generated randomly for the numerical evaluation of the modified DLSP solution procedure suggested in the preceding paragraphs. We consider planning horizons of maximal length of 60 periods and up to six products which are distinguished into up to three different categories depending on their demand expectations and variances. These consist of products with high expected values and low variances, intermediate moments and finally of products with low expected demand and high variances. Every tenth period is a demand instance for all products. Demand for each product is measured in how many units can be produced during one production period. Thus, all production speeds are equal to one. Similarly, inventory costs for all products are assumed to be equal and unity. Setup of one period is required for each product and the corresponding costs are set to 60.

The solution procedure is programmed in FORTRAN 77 and implemented on an IBM PC with an 80 486 / 25 MHz processor for the parametric application of DLSP, and on an IBM compatible NEC PC with an 80 486 / 20 MHz processor for the case of random demand. The computation times required for solving the parametric DLSP on the first computer turn out to be approximately 80 % of the computation times on the slower processor. We use a geometric cooling schedule, the number of repetitions is given by "acceptances max constant" and no acceptances at one temperature stage is used as the stopping criterion (for notation see Collins et al. [1988]; the parameter values are given in table 1). Note that SA parameters used for the numerical experiments below are constant while they should be problem-size dependent for application to general problems. In most practical applications, finding a feasible

| Parameter | Phase 1 | Phase 2 |
|---|---|---|
| Initial Temperature | 10 | 100 |
| Reduction | 0.9 | 0.9 |
| Max Number of Acceptances | 120 | 70 |
| Max Number of Repetitions | 10 000 | 1000 |
| Number of Reductions | 50 | 50 |

**Table 1**: Parameters of Simulated Annealing Cooling Schedule

production plan will be of primary interest when cost parameters are not easily estimated. Therefore, the cooling schedule for phase 2 (optimization) is chosen to be coarser than for phase 1 (feasibility). Suboptimal solutions obtained by this rough procedure are improved by shifting batches to the right in order to fill unnecessary gaps.

To determine a set of sensible target inventories, we use the rounded expected values for the demand in the demand instances. Due to computational restrictions (e.g., there are $10^6$ different inventory combinations for six products if inventories vary from 0 to 9), only preselected initial and final inventories are tested in the parametric application of modified DLSP. For each inventory configuration, this problem is solved 25 times with different initializations of the random number generators due to the stochastic nature of the solution procedure. The results of the best five of all investigated inventory configurations are given in table 2.

| Inventory | Average Cost | CV*) of Cost | Number of Best | Average CPU Time (sec) | CV*) of CPU Time |
|---|---|---|---|---|---|
| 2 0 2 0 0 0 | 1026.84 | 3.58 | 6 | 301 | 9.9 |
| 2 0 1 0 0 0 | 1027.52 | 3.15 | 5 | 298 | 9.9 |
| 2 0 1 1 0 0 | 1041.96 | 3.35 | 3 | 295 | 11.0 |
| 3 0 2 0 0 0 | 1045.60 | 2.23 | 1 | 309 | 7.9 |
| 4 0 2 0 0 0 | 1048.80 | 3.15 | 1 | 301 | 11.7 |

**Table 2**: Parametric application of the modified DLSP to selected inventory configurations

*) (Coefficient of Variation) 100

Typically, asymmetric initial inventories yield the lowest costs despite the underlying symmetric demand structure where each product category consists of two identical elements. This result can be explained by the trade-off between the size of the set of feasible solutions (which is larger for higher initial inventories) and the contribution to the final objective-function value from the holding costs for the initial inventory.

The inventory vector (2,0,2,0,0,0) yields the lowest average cost value and outperforms the other invetigated inventory configurations in 6 out of 25 repetitions. Safety stocks are set to one unit for every product which yields target inventories for the DLSP of (3,1,3,1,1,1) for products 1 to 6. This inventory vector is used as initial and target inventories for the modified DLSP.

During the numerical experiments for the modified DLSP with randomly generated demand structures it turns out to be necessary to differentiate between hard and easy problems. As an elementary indicator of difficulty, we use the ratio of the total demand plus one setup period per product and the length of the planning period. Problems with an 85 % or higher degree of difficulty are considered to be hard. Note that two problems with the same elementary degree of difficulty can be quite different with respect to actual difficulty due to different demand distributions over time.

One 3-product / 30-period (3/30) problem with a 63.3 % elementary degree of difficulty is solved exactly by LINDO-386 on the IBM 80486 / 25 MHz after more than 48 hours of computation. The same optimal solution is found much more rapidly (with an average CPU time on the 20 MHz machine of 95.2 seconds) by the two-phase simulated annealing algorithm in five out of ten repetitions while the other five solutions are near optimal with a coefficient of variation of 2.63 %. Note that the CPU times for our solution procedure can be improved by adapting the parameters of the cooling schedule (Table 1) to the actual problem size. Due to the extremely high computation time required for the exact solution, we use the coefficient of variation of the final objective function value for ten different realizations as a criterion of stability and quality of the solutions found instead of the true optimum.

| 3/30 | Degree of Difficulty | Number of Problems | Average CV*) of Cost | Average CPU Time (sec) | CV*) of average CPU Time | Unsolved Problems |
|---|---|---|---|---|---|---|
| Easy | .70 - .84 | 15 | 2.27 | 103 | 9.4 | - |
| Hard | .85 - .92 | 15 | 2.07 | 103 | 16.7 | - |

**Table 3**: Results for the 3-product / 30-period DLSP

*) (Coefficient of Variation) 100

The results obtained for the two different classes of difficulty are given in table 3 for the 3/30 problem, and in table 4 for the 6/60 problem.

| 6/60 | Degree of Difficulty | Number of Problems | Average CV*) of Cost | Average CPU Time (sec) | CV*) of average CPU Time | Unsolved Problems |
|---|---|---|---|---|---|---|
| Easy | .65 - .84 | 15 | 3.25 | 409 | 10.9 | - |
| Hard | .85 - .92 | 15 | 2.92 | 747 | 41 | 13.3 % |
| Subclass 1 | .85 - .92 | 10 | 3.48 | 540 | 15 | - |
| Subclass 2 | .85 - .92 | 5 | 1.81 | 1185 | 8.8 | 40 % |

**Table 4**: Results for the 6-product / 60-period DLSP

*) (Coefficient of Variation) 100

For each of the 15 demand situations we compute ten repetitions with new seeds for the random number generator. In both cases, the coefficients of variation for the objective are at most 5 %, indicating stable results. Computation times tend to be higher for problems with a high elementary degree of difficulty. In particular, phase 1 can require a significant amount of computation time for the 6/60 problems. The two-phase solution procedure seems to be sensitive to higher demands at the beginning of the planning period. This observation might be an explanation for the substantial differences in computing times of the hard problem instances. A further natural subdivision of this class is found into problems with an average computation time similar to the class of easy problems and on the other hand problems which exceed 1000 seconds of required CPU time. In this latter class only, it happens that phase 1 of the algorithm is unable to find a feasible solution on several occasions. This may again be due to the fact that the set of feasible solutions is in general smaller for a high demand density towards the beginning of the planning period. This hypothesis is supported by the small coefficients of variation of the final objective function value. A specific problem instance is given in table 5 as an example for high demand distribution during the first two subintervals.

| Demand | 10 | 20 | 30 | 40 | 50 | 60 |
|---|---|---|---|---|---|---|
| 1 | 1 | 4 | 1 | 2 | 3 | 1 |
| 2 | 3 | 2 | 1 | 2 | 2 | 2 |
| 3 | 4 | 2 | 0 | 0 | 1 | 0 |
| 4 | 1 | 2 | 2 | 1 | 1 | 1 |
| 5 | 0 | 0 | 1 | 2 | 0 | 1 |
| 6 | 0 | 0 | 3 | 1 | 0 | 0 |

**Table 5:** Demand structure for a hard 6/60 problem

The elementary degree of difficulty is 88.3 % and a feasible solution is given in figure 1 (R stands for setup periods while production is indicated by 1).

```
        **************************************************
        *               Produktionsplan                  *
        **************************************************

             1         2         3         4         5         6
    123456789012345678901234567890123456789012345678901234567890
 1 |               R111               R11111        R1111       |
 2 |     R1111               R111             R11111            |
 3 |R1111                                                   R111|
 4 |          R11111                        R1           R11    |
 5 |                             R1111                          |
 6 |                    R1111                                   |

              Auslastungsgrad:       100.0 %
              Zielfunktionswert:    1441.0
```

**Figure 1:** Production Schedule for the demand structure of table 5

This is the only feasible solution that is found by phase 1 in ten repetitions after 1204 seconds of CPU time.

## 6. Summary and Conclusions

The problem presented here differs from the standard DLSP given for example by Salomon [1991] in considering the case of batch production, not necessarily vanishing inventories, and in using different time scales for production scheduling and demand. The feasibility problem for this modified DLSP is shown to be again NP-complete. The suggested two-phase simulated-annealing solution method is based on intuitive ideas. The optimization procedure is separated into phase 1 searching for a feasible solution and optimizing cost in phase 2. Production schedules are generated by dividing, combining and shifting batches. The numerical experiments presented indicate that our heuristic solution method yields stable results in short computation times compared to the exact solution obtained by LINDO-386. This is not surprising because of the large number of integer variables required in the proposed mixed-integer linear model. The solution procedure can be readily applied to larger problems where stable solutions will be obtained with reasonable computation times by a problem-size dependent choice of the cooling schedule. In contrast to the prohibitive computation times required for the exact solution, our SA approach allows the numerical sensitivity analysis of cost parameters which are often not easily estimated in practice.

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