

Sharpening One's Diagnostic Skill By Simulating Students' Error Behaviors

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Good teaching requires understanding the learner's thinking. A good teacher's instructional efforts are not restricted to pre-planned behavior but are also led by a diagnosis and remediation of the learner's misconceptions. To do this successfully, the teacher must take the learner's place: that is, make a model of the student's current thinking and use this model to make decisions about pedagogical interventions.

From a teacher's point of view, the diagnosis of student errors is relevant only with respect to the remediation the teacher can give the student. The diagnostic procedure is regarded as a necessary first step toward planning appropriate interventions and choosing materials suited to facilitate the learning progress. The teacher is not in need of extensive diagnostic instruments being sharper and more comprehensive than the tools s/he can offer for a "therapy." Thus a teacher's diagnostic competence is related to his/her ability to intervene, it is one means for planning student activities.

In observing teachers in a diagnostic-prescriptive approach, one is faced with the fact that their diagnostic focus is led by the intervention material at their disposal and by the need to immediately apply some curricular unit. The shortcoming of this direct diagnostic-prescriptive method is obvious: changing a student's behavior from wrong to right at one task is perhaps sensible but possibly not lasting when the student is confronted with a different though similar problem. This attempt to modify behavior stemming from the practical needs of a teacher seems but a cure of symptoms instead of the disease underneath, the general effects being hoped for but remaining unclear.

From a more theoretical point of view, diagnosis is not the mere description of symptoms exhibited by a student. Rather, it tries to explain the origin of the symptoms by using the knowledge offered by a scientific model which describes errors as a necessary result of underlying cognitive features of the student. Such models may take different approaches as to which variables influence actual student behavior. As a comparison, for example, if someone's symptom is perpetual nose-scratching, the plain statement and description of this behavior may be sufficient for certain purposes. In such a case, the "therapy" would consist of a behavioral training to extinguish the scratching and establish a different behavior like crossing the legs, keeping the hands in the pocket, etc. Another approach would attempt to describe the overt behavior on the basis of underlying motives, its inhibitions and reinforcements and the feelings it avoids or enhances. The therapy obviously would be quite different. A psycho-physiological description would account for all muscular movements and their biochemical root, that is, the synthesis and decomposition of all substances involved. "Therapy" in the sense of behavior extinction would be by means of drugs or other medical intervention.

Models as discussed above are not to be termed right or wrong in the way that they describe a behavior, as they take different perspectives upon what questions are posed and answered concerning a given behavior. A debate between different diagnostic approaches does not refer to the consistency and validity of the models but to the question of whether a particular problem can be answered by a specific theory.

In the diagnostic-prescriptive approach in mathematics education, at least two different models can be distinguished: (a) the curriculum model and (b) the information processing model.

- (a) The traditional *curriculum model* constitutes a behavioristic approach as it attempts to explain student behavior as the outcome of the student's exposure to preceding curriculum units. That is, if certain prerequisites for a task have not been mastered by a student, erroneous solutions are resulting. The remediation proposed by this model is simple: to fill in gaps by means of additional exercises. This model principally follows the line of the mathematical subject matter to be learned as outlined in textbooks in a step-by-step fashion. The model tends to suggest remedial activities which are similar for all students, as mathematics learning is regarded as a steady progression from easy to difficult, from one unit to the next. The theoretical and practical aim of this model is thus the organization of an optimal path through the mathematical content.
- (b) The *information process model* which is finding increasing attention (Resnick & Ford, 1981) takes particular account of subject-matter knowledge structures to be constructed by the individual student. The student is regarded as a system that takes in and processes information, and student behavior is interpreted as the

result of such information processing. The remediation of a student's suboptimal behavior on a domain of tasks is targeted as a change in his/her knowledge base which is made to occur by the student's active interaction with remedial situations.

In contrast to behavioristic approaches, the main advantage of the information processing approach lies in a detailed analysis of problem solving processes which make specific assumptions about components of mental processes. A central role is attributed to the way in which knowledge is represented in memory. It seems that problem solving skill depends a great extent on problem understanding as supported by the quality of mental representation and organization of knowledge.

Information processing models of a student's cognitive behavior in a specific mathematical domain follow certain *a priori* assumptions outlined below.

- (a) A student's problem solving behavior is "logical" and stable within a restricted area of knowledge, that is, errors are seldom arbitrary or random;
- (b) the model is consistent/deterministic in the sense that it contains knowledge sufficient to produce error behaviors by way of simulation even when the behavior, from the teacher's point of view, may seem inconsistent and arbitrary;
- (c) the model resembles the student's cognitive processes (at least to some extent) with respect to a given task domain, that is, it can predict the outcome of the student's performance on a similar task;
- (d) the model is idiosyncratic as it refers to an individual student: the errors of his/her classmate have to be explained by another instantiation of the model;
- (e) the validity of such a model is to be explored in the individual case: one can possibly study/probe the model's predictions by posing problems to the individual student modeled.

The construction and use of formal learner models which are based on the information processing approach is expected to have payoff in improving the quality of instruction, by better understanding the organization of the learner's knowledge in a specific subject domain.

The general point to be made in this paper is that computerized learner models can be a step further in the diagnostic-prescriptive approach which goes beyond the cure of symptoms. In particular, the construction of models which can simulate students' error behaviors on the basis of assumptions of underlying cognitive features may contribute to a more profound diagnosis and will thus permit better prescriptions. A special observation is that as one goes ahead constructing such a model, the ability to sense what is going on with an unsuccessful student is sharpened.

Learning about student knowledge from cases of pathology

What makes students' error behaviors so particularly interesting? Successful problem solving of average students rarely, if ever, gives hints on computational strategies used, e.g., if a student gives the right answer, nobody knows why and how (and typically nobody asks either). The study of "pathological" cases, ranging from systematic error patterns to complete disability to solve arithmetic operations sheds light on the cognitive process necessary for coping with a task and the procedural structure of mathematics acquisition which otherwise is too easily taken for granted.

The concept of "pathology" is used as a metaphor guiding observations in the sense of carefully studying the mechanisms of students' unusual problem solving processes and looking for cracks in algorithmic procedures or for unexpected use of mathematical concepts (Lorenz, in press). The idea is not that "pathological" behavior be distinguished from "normal" behavior but that one can learn much about the normal structure of numerical computation through the "study of the breakdown (i.e., pathology) of the number concept of the computational operations" (Luria, 1969 - p.37). This perspective corresponds to the fact that learning is not a continuous process progressing smoothly "upwards" but that it must be characterized more appropriately by steps, sudden jumps and unforeseen shifts and changes in concepts. Standardized tests render little information about children's thinking in this respect. Instead, clinical interviews and protocol analyses of children's thinking aloud are more likely warranted to "reveal regularities of behavior - especially regularities that can be related to theories about how internal information processing proceeds" (Resnick & Ford, 1981).

Nantais-Martin, Bergeron, and Herscovics (1983) have undertaken training with teachers in doing clinical interviewing with mathematics students. This instruction sensitized the teachers toward subtleties in the students' work which might put the teachers in a better position to help students construct their mathematical knowledge. But when confronted with student errors teachers would usually prescribe remedial units that are based on their subjective experience, diagnostic competence and knowledge about the individual child. The next logical step would be to give the teacher models at hand by which they can arrive at well-founded remedial plans in terms of the model. Computer models that can simulate error behaviors of individual students seem to be a good exercise in this direction.

But what is the justification of simulating student behavior? The assumptions entering a model of a student's cognitive structure underneath may in general be tentative and implicit. Should it not be sufficient to think carefully about the processes to be modeled and describe them verbally? Stating assertions about students' cognitive structures as verbal descriptions are but a first step in the modeling.

The requirement to simulate the information processing by a computer program demands a precision of the description of cognitive features which is not always present in teachers' and researchers' diagnostic thinking. Writing a computer program which is capable of reproducing behavior causes one to confront a host of issues possibly ignored when describing hypothetical cognitive features in vague language statements. In order to specify the computer model, explicit statements are needed of all assertions made to explain a child's misconception and to formulate a model which is comprehensive, consistent, and "logical."

In the section which follows an example is presented which demonstrates a fifth grade student's performance on a mathematical problem solving task which could be termed pathological as defined above. This example has already been discussed and a computer model reproducing aspects of this particular individual's behavior was presented (Wachsmuth, 1985). We present it again here to illustrate how the intent to make a computer generate the particular behavior contributed to concretizing initial vague ideas about the origin of the behavior and to discuss the possible contribution that is offered by a logical analysis of mathematical behavior based on hypotheses of the student's cognitive structures in terms of the model.

A computer model of a student's error behavior

As means to describe and analyze the representation and utilization of domain-specific knowledge in a concise way, a computerized knowledge representation model LAKOS was developed at the University of Osnabrück. (LAKOS is an acronym which stands for the German translation for Logical Analysis of Cognitive Organizational Structures.) The model is capable of simulating aspects of cognitive behavior of individual learners by making assumptions about their inner environment, that is, specify hypothetical knowledge structures with respect to a given task domain. It is based on the technique of logical programming and models learner knowledge in terms of network structures as formulated by a human experimenter.

A central paradigm for the LAKOS model is that the cognitive behavior exhibited by an individual is a *knowledge-based process* which evolves from relatively simple component processes of an inferential nature. The complexity of observed behavior depends on the numerosity and organizational structuring of the facts and rules which form the knowledge base of the individual with respect to the subject domain.

A second paradigm in the approach is that the behavior exhibited by an individual in a task situations is generally not supported by the total body of long-term knowledge acquired by the individual. Rather it is assumed that knowledge to be used in a given situation has to be *activated* in order to be accessible where the accessibility of particular knowledge depends on contextual cues inherent in the

situation. The degree to which certain knowledge is contextually bound with respect to a set of specific situation is referred to by the term, *situation specificity*. While the activation of particular knowledge may depend on various kinds of contextual information, the present approach assumes that significant context information is carried by language as a primary carrier of instructional transactions. To avoid the need to deal with various linguistic matters concerning the analysis of natural language sentences, attention is restricted to the context-dependent meaning of words.

The central idea of this modeling approach is that the potential actions an individual is able to perform is determined by his/her knowledge network. The explanatory power of the model thus lies in the fact that its actions in the course of a simulated dialogue are determined by the organizational structuring of its knowledge base.

The computer model consists of a user interface, a knowledge base referred to as long-term memory (to be instantiated for the individual student modeled), and three particular component processes evoked in the reaction to an input; i.e., (a) understanding (that is, constructing a semantic representation of) an input sentence, (b) searching the knowledge base for information relevant to answering the question, and (c) synthesizing a language answer. Further components of the system are a semantic short-term memory which keeps results of the most recent inferences, and a mechanism referred to as focus which regulates the activation of knowledge coded in long-term memory. The knowledge in long-term memory is organized in the form of a network referred to as a knowledge network. The nodes in this network contain lexical language records and knowledge of a particular field of subject matter in the form of rules which are interpreted as abstract ways to think and act. While the focus can shift along the links in the network during the processing of an input sentence, the focus remains at the current network node at the conclusion of a response: this is the starting focus for the next input (i.e., "mind set").

The following example shows how the computer model was used to explain a student's error behavior by making assumptions about underlying cognitive features, that is, by explaining what the student does in relation to what s/he *knows*. The illustration stems from clinical research in the realm of rational number learning carried out by the Rational Number Project². One fifth-grade subject's behavior in dealing with a complex problem solving situation, requiring the size comparison of several fractions, gave reason to believe that different repertoires of rules were used by the subject on the same task and that use of a particular repertoire apparently depended upon particular wordings, involved in the formulation of questions. Since some rules produced a kind of "pathological" behavior, it was possible to detect the subject's use of these rules from the behavior exhibited. When contradictory answers given in the different contexts were contrasted in the interview, the inconsistency in the subject's knowledge base caused a cognitive conflict to occur.

A segment from a video-taped interview with this fifth grader, Terri (age 11:6) presents explanations she gave about her solution for the problem solving task. In this task, students were individually shown a scale with 11 gray swatches increasing in grayness from white to black. Further they were given a set of 12 fractions, written as symbols a/b on little cards, which were said to represent ink mixtures with a parts black ink in b parts solution. They were then asked to "arrange these mixtures in order from lightest to darkest and put one at the color card were you think it belongs." Terri arrived at the solution shown in Figure 1.

Figure 1 Terri's order arrangement of cards

0	10	20	30	40	50	60	70	80	90	100%	
$\frac{0}{20}$	$\frac{1}{5}$	$\frac{2}{4}$	$\frac{2}{5}$	$\frac{2}{7}$	$\frac{4}{6}$	$\frac{4}{8}$	$\frac{4}{10}$	$\frac{6}{9}$	$\frac{6}{15}$	$\frac{6}{20}$	$\frac{12}{15}$

Note: Fifth grader Terri's (11:6) order arrangement of 12 fraction cards along scale of gray levels from white (0%) to black (100%). Percent marks were not present on scale.

In earlier interviews, Terri had persistently called pairs of fractions - when presented to her as written or spoken symbols - *equal* if they had the same denominator. The original wording in these interviews was like "One-fifth and one-sixth, are they equal or is one less? - Which one is less? - Tell me how you know!" Mention of this fact is important since it is asserted that the wordings involved in these stereotype situations presumably had attained the role of key words triggering certain behavior. In contrast to this behavior exhibited in stereotype circumstances, Terri had now attached the fractions $6/15$ and $12/15$ to different gray levels (at 90% and right adjacent to 100%, respectively; see Figure 1). Apparently she followed some kind of "lexicographical" ordering strategy bearing on the whole number symbols in the 12 fractions. This happened, although initially the task situation seemed to have been clear to her as well as seen from the following dialogue. After she had placed all fraction cards at the gray level scale, Terri was asked at first why she had put $0/20$ at the beginning of the scale (white, i.e. 0%). Terri explained: 0. TERRI: Because there'd be no black ink, no black ink so it would be clear water.

After a short dialogue about $4/8$ and $4/6$ which Terri called about equally dark, but $4/8$ still a little bit darker than $4/6$, Terri was asked about the two fractions $6/15$ and $12/15$, without specific references to her placement of these fractions at the scale. Here now follows the dialogue:

1. INTERVIEWER: Now, Terri, what about 6/15 and 12/15?
2. TERRI: They're equal, like (laughs).
3. INTERVIEWER: OK, but you put them in different positions, though, why did you do that?
4. TERRI: Because! That's the way I thought I should do it! (moves and messes up chart; a short dispute following this was excluded from the transcript.)
5. INTERVIEWER: I would still like to know: you say six-fifteenths and twelve-fifteenths are equal?
6. TERRI: Right.
7. INTERVIEWER: But you put them on different parts...
8. TERRI: 'Cause six comes before twelve so I thought that's the way you do it...
9. INTERVIEWER: OK, did you think in terms of darkness when you did that?
10. TERRI: Yeah, sorta like...
11. INTERVIEWER: Which would be darker? Six-fifteenths or twelve-fifteenths?
12. TERRI: Twelve-fifteenths.
13. INTERVIEWER: OK, and which fraction would be bigger?
14. TERRI: Twelve-fifteenths.
15. INTERVIEWER: And if I ask you, six-fifteenths, twelve-fifteenths, are they equal or is one less?
16. TERRI: It's less.
17. INTERVIEWER: Which one is less?
18. TERRI: Six...um...fifteenths.
19. INTERVIEWER: And why did you say it's less?
20. TERRI: 'Cause it...oh! (puts head in hand and sighs) No, they're equal. Because they have the same denominator.

At first sight this episode is surprising. Thinking about it for an extended time left the following question unanswered: What happened to Terri when she arrived at the conclusion that 12/15 is greater than 6/15? Apparently, she did so on the basis of an inference which was supported by the interpretation of fraction sizes via gray levels (line 11-14) and no longer by her lexical ordering strategy. Did this experience result in an insight stable enough to persist through a situation in which when Terri had normally called these fractions "equal" (line 15-18)? The answer is "no" since Terri was asked to give a reason (line 19), her opinion collapsed and she reverted to her earlier answer (line 20).

The wish to find the logical explanation for this seemingly irrational behavior crystallized in two questions. Which cognitive features of Terri might have produced her behavior? Which assumptions of Terri's knowledge structure would make the computer model reproduce such behavior?

In general the reproduction of some aspects of Terri's behavior should give a better sense for *possible* causes. What Terri is doing here seems to be following rules rather than being irrational. Her

correct "lexicographical" ordering of the fraction cards is a definite support thereof. Were it possible to generate this kind of a behavior on the basis of a determinable set of "laws" in Terri's thinking, one certainly would understand better the way she acted.

Using the LAKOS model and its conceptual framework, Terri's behavior was analyzed in detail. Particular aspects were simulated for making the analysis more precise. To this end, the current hypothetical knowledge structure of Terri concerning size comparisons of fractions was modeled in a network. Based on her performance and on protocols of other clinical interviews, including one from the same day, her knowledge was organized in single elements. Each knowledge element was formulated as a logical proposition and was stored in an indexed memory node. Thereby, knowledge elements observed to be jointly accessible in a certain context were put in the same node. As for the language records, those words playing the role of *key words* for the activation of a knowledge element were put in the node holding that element. Words that were understood across situations were put in higher nodes. (For further information cf. Wachsmuth, in press).

The incompatible rules producing the inconsistencies in Terri's behavior observed in the interview were put in two nodes that could not be activated simultaneously. Which of the two nodes, (F1) or (F2), is focused on, depends on the use of key words in the dialogue. To this end, certain language records are only available in one of either nodes, that is, the knowledge in these nodes is situation-specific.

In Figure 2 is presented an original protocol of a dialogue conducted with the computer model. The user asked questions and probed behavior in a similar way as in a diagnostic interview. The computer answered the questions or executed commands using the specific knowledge base instantiated as Terri's hypothetical cognitive structure. Asking "WHY?" made the computer model give the reason for its most recent answer.

Inconsistent behavior is observed in the dialogue in that TERRI's opinion of the size relation between $6/15$ and $12/15$ changes several times. At about the middle of the protocol the contents of short-term memory (STM) are listed. From these can be seen that TERRI's current opinion of $6/15$ being less than $12/15$ is still present in STM when context (F1) is, cued by the key words "equal or one less," accessed another time. But in this context, TERRI's answer statement cannot be justified and the recognition of this inconsistency results in a conflict (see Figure 2).

The original dialogue with Terri stems from a more complex situation and which probably requires further analysis in terms of attitudinal patterns in the interaction of interviewer and subject. It must not be overlooked that such patterns also contribute to the behavior of a subject. That is, the *decisions to act* made by Terri certainly cannot be explained only on a cognitive basis. However, Terri's possible *ways to act* are constrained by her knowledge repertoire: "false" actions

Figure 2 Sample dialogue from LAKOS1 system

I'M TERRI> 6 15 and 12 15. EQUAL OR ONE-LESS?
 6 15 AND 12 15. EQUAL! _____ FOCUS (F 1: *1)
 I'M TERRI> 6 15 AND 12 15. EQUAL?
 YES. 6 15 AND 12 15. EQUAL! _____ FOCUS (F 1: *1)
 I'M TERRI> WHY?
 BECAUSE (EQ 15 15)! _____ FOCUS (F 1: *1)
 I'M TERRI> 12 15 AND 6 15. ARRANGE-IN-ORDER!
 6 15 AND 12 15.! _____ FOCUS (F 2: *1)
 I'M TERRI> 6 15 AND 12 15. WHICH-ONE-LESS?
 6 15 AND 12 15. FIRST-LESS! _____ FOCUS (F 2: *1)
 I'M TERRI> WHY?
 BECAUSE (LESS 6 12)! _____ FOCUS (F 2: *1)
 I'M TERRI> STM!
 1 STM (! (BECAUSE TRUE (< (6 15) (12 15)) < = (LESS 6 12))) (? (WHY)).
 2 STM (! (< (6 15) (12 15))) (? ((OR < >) (6 15) (12 15))).
 3 STM (! (DUMMY (6 15) (12 15))) (! (ORDER (12 15) (6 15))).
 4 STM (! (BECAUSE TRUE (= (6 15)) < = (EQ 15 15))) (? (WHY)).
 I'M TERRI> 6 15 AND 12 15. EQUAL OR ONE-LESS?
 6 15 AND 12 15. ONE-LESS! _____ FOCUS (F 1: *1)
 I'M TERRI> 6 15 AND 12 15. WHICH-ONE-LESS?
 6 15 AND 12 15. FIRST-LESS! _____ FOCUS (F 1: *1)
 I'M TERRI> WHY?
 < < < CONFLICT > > > < < (6 15) (12 15)) BUT (=
 (6 15) (12 15))
 BECAUSE (EQ 15 15)! _____ FOCUS (F 1: *1)
 I'M TERRI> 6 15 AND 12 15. ONE-LESS?
 NO. 6 15 AND 12 15. EQUAL! _____ FOCUS (F 1: *1)
 I'M TERRI> WHY?
 BECAUSE (EQ 15 15)! _____ FOCUS (F 1: *1)

Note: System prompt "I'M TERRI" refers to knowledge network currently loaded
 Indented: System's responses. FOCUS information refers to knowledge sub-
 structure activated by processing an input.

exhibited by Terri must rely on a (possibly inadequate) application of knowledge she possesses.

Based on the model, Terri's behavior can now be explained as follows. In the context of "ordering" fractions (which to her apparently does not mean "ordering by size"), Terri calls the fractions 6/15 and 12/15 different because her criterion is the order relationship

context of "equal-or-one-less questions" Terri calls the fractions equal according to the criterion of the equalness of the denominators. On questions that were meant to have the same meaning, Terri arrives at two different interpretations for which she has different repertoires to act.

We note the crucial role of the linguistic dimension in this episode. As can be assumed, and the computer model can verify the consistency of this assumption, certain key words bring about changes in Terri's interpretation of the situation. The surprising behavior close to the end of the interview excerpt (line 16-18 in the transcript), however, is of a more complex origin. Presumably the opinion Terri reached directly before (lines 16-18), namely, that $12/15$ is greater than $6/15$, is still present to her in STM when she is reminded, through deliberate use of key words, of the "equal-or-one-less" context. According to her preceding answer she still states that $6/15$ is less. But in this context there is no basis to support this answer as she is unable or unwilling to reproduce the chain of inferences she was led through before. Thus, it seems cogent that she reverts to her earlier opinion for which she can offer a justification.

Discussion

The model sketched in this paper takes into account that patterns of thinking which students develop in their exposure to elementary mathematics instruction are highly individualized and often do not follow the orthodox instructional models of textbooks and the classroom (Lankford, 1972). The problem arising with the orthodox approach is that the use of classical text issues does not include ways to cope with students' specific and highly individualized problem solving behavior. The practical and theoretical approaches to diagnosis and remediation are quite similar as they progress by studying student error patterns and comparing them to the "undisturbed" case of successful problem solving. The necessary step between behavior descriptions and remedial interventions is the assumption of cognitive causes giving rise to the behavior, that is, assumptions about the student's knowledge base.

To arrive at elaborated assumptions about the cognitive structure of students one has to carefully observe "pathological" cases. For example, since some rules in the case of Terri described above produced a kind of "pathological behavior," it was possible to detect the subject's use of these rules from the behavior exhibited. Specification of a simulation model renders an account of the assumptions as to what are cognitive causes of the behavior. Assumptions which in the everyday teaching situations often would remain implicit become the matter of discourse. They permit more explicit predictions on the expected outcome of remedial interventions by sketching the changes to be targeted in the knowledge base.

The simulation model specified for Terri constitutes an explicit hypothesis of her current cognitive structure with respect to size comparisons of fractions. Specifically, the knowledge network includes

two disparate knowledge repertoires (set of rules) which become activated in a mutually exclusive fashion when certain key words suggest a particular interpretation of the task situation. The detailed formulation of this knowledge network provides a concise diagnosis of the origins of Terri's pathological behavior. The instability in Terri's performance is due to the fact that she activates different knowledge repertoires when trying to respond to a question on the basis of her changing interpretation of it. As long as the knowledge network remains as is, Terri's performance cannot become stable since there is no basis in the network providing for that. Thus, the knowledge network needs to be changed.

Based on this diagnosis it is possible to sketch the kind of changes in Terri's knowledge base that would support more correct performance in application situations like the one she was assessed on and to speculate about instructional procedures which can bring about such progress in Terri's cognitive development. In doing so, we must be aware of the fact that one certainly cannot erase a faulty part in Terri's knowledge base and establish a new one as one could do with a computer. The reason to assume that "false" rules cannot be simply neutralized by learning more correct ones: it will also need efforts to "outrule" false rules. Thus, it seems more adequate to target changes which modify the existing knowledge network in such a way that inadequate behavior is "overridden" by more adequate behavior.

On sketching changes to be targeted, the model suggests that performance can be improved if a new node is created which contains information on which type of rule *not* to use (i.e., intercept application of such a rule). Actually in Terri's case, her notions of the word "order" and the word "equal" need differential interpretations with respect to fractions. That is, she needs the insight that other criteria than a direct comparison of numerals in denominators or numerators are relevant in the context of comparing the size of fractions. Remedial instruction for Terri would thus have to include that she learns to compare fractions by more than one criterion (i.e., by number or size of parts as well as by the criterion of fraction size) and to *distinguish* between these criteria. The fact that Terri's behavior seems likely to be triggered by key words should be discussed with her. The general implication is that the effect of alternate interpretative frameworks can only be controlled when a new higher order node is established in the knowledge network which is access to *all* alternative ways to interpret the data in a situation ("awareness of the range"), together with a rule achieving appropriate discrimination (inhibition of "false" interpretations).

But one should be cautious with such an argument. The installation of an inhibition in the human mind is not just like "putting in another rule!" It may well be that access structures can be instable themselves. Furthermore, even when Terri's performance can be improved in the described fashion, it is likely that stable performance

is still restricted to specific situations and does not necessarily generalize to other situations not met before.

On the basis of the model it seems that a possible way to achieve correct performance in a broader range is the following: Many nodes representing specific situations to which certain knowledge is relevant need to become subordinately linked to a node containing rules which can support correct performance across this class of situations. Those rules should then become "visible" for any subordinate node activated. The more specific situations are represented in nodes linked to a node describing an abstract way to act in a class of situations, the greater is the chance that this knowledge becomes activated by a specific situation.

Consequently, the goal that inadequate behavior becomes "overridden" by more adequate behavior is certainly not achieved by a single intervention. For remedial instruction this would imply that the use of adequate rules must be reinforced through practice in many situations which include not only those that application of false rules were observed in but also diverse new ones. Alternate ways of interpretation must become the matter of explicit discussion, and the classification of situations must be practiced.

Conclusions

Using an information processing approach, one is restricted to models of children's cognitive structures as outlined in theoretical works (e.g., Farnham-Diggory, 1974; Carpenter, Moser, & Romberg, 1982). The knowledge about an individual's ability to handle mathematical problems is at most as precise as the general model specifying instances like short-term memory, network of nodes and interactions of instances, etc. Thus the model cannot provide for an answer to all research and practical questions but it can help to specify the problems such as those listed below.

1. Which minimal steps are necessary to acquire a certain mathematical concept on the basis of a given knowledge structure, i.e., which additional nodes have to be built up, which meta-nodes help to discriminate between different sets of applications handled in the same way by the child, which meta-node can connect similar structures previously treated as dissimilar, etc.,
2. which features of the cognitive structure impede the child from learning specific facts or strategies, and
3. which specific cognitive unit is to be influenced by the material given, which effects are expected by certain visualization aids for this child, which are the effects for his/her classmate, etc.

One thus gets clues for remedial teaching not only in the individual case, but as well, in a prophylactic sense, an awareness of the implicit instructional pitfalls which together with the idiosyncrasies of the learner's information processing structure may lead to misconceptions and faulty strategies. And what else is diagnostic and remedial competence?

Footnotes

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³Name was changed

References

- Carpenter, T., Moser, J., & Romberg, T. (Eds.) (1982). *Addition and subtraction. A developmental perspective*. Hillsdale: Erlbaum.
- Farnham-Diggory, S. (1972). *Information processing in children*. New York: Academic Press.
- Lankford, F.G. (1972, October). *Some computational strategies of seventh grade pupils* (Final Report, Project No. 2-C-013). Charlottesville, VA: University of Virginia, HEWE/OE National Center for Educational Research and Development and the Center for Advanced Studies.
- Lawler, R.W. (1981). The progressive construction of mind. *Cognitive Science*, 5, 1-30.
- Lorenz, J.H. (in press). Pathologies in children's learning and its theoretical implication. In H.G. Steiner (Ed.), *Proceedings. Foundations and methodology of the discipline mathematics education*. Bielefeld, West Germany: University of Bielefeld.
- Luria, A.R. (1969). On the pathology of computational operations. In J. Kilpatrick and I. Wirszup (Eds.), *Soviet studies in the psychology of learning and teaching mathematics. Volume I* (pp. 37-74). Chicago, Il: University of Chicago.
- Nantais-Martin, N., Bergeron, J.C., & Herscovics, N. (1983). La minientrevue: Un nouvel outil d'évaluation de la compréhension en mathématique au primaire. In J.C. Bergeron & N. Herscovics (Eds.), *Proceedings of the fifth annual meeting of the North-American Chapter of the International Group for the Psychology of Mathematics Education. Volume 2* (pp. 205-214). Montreal, Quebec, Canada: The organizing committee.
- Resnick, L.B., & Ford, W.W. (1981). *The psychology of mathematics for instruction*. Hillsdale, NJ: Erlbaum.
- Wachsmuth, I. (1985, April) *Logical analysis of cognitive organizational structures. The LAKOS project (Part A). A computer model of student performance (Part B)*. Paper presented at the annual meeting of the American Educational Research Association, Chicago. (ERIC Document Reproduction Service No. ED 257635)
- Wachsmuth, I. (in press). Modeling the knowledge base of mathematics learners: Situation-specific and situation-unspecific knowledge. In H. Mandl & A. Lesgold (Eds.), *Learning issues for intelligent tutoring systems*. New York: Springer.