JOURNAL OF MATHEMATICAL BEHAVIOR 5, 209–232 (1986)

The Role of Technology in the Cognitive Development of Mathematics Learners

IPKE WACHSMUTH
Universität Osnabrück, West Germany

JERRY P. BECKER
Southern Illinois University at Carbondale

INTRODUCTION

This article reports on information disseminated and discussions and recommendations elaborated in the theme group on "Technology and Cognitive Development" at the Fifth International Congress on Mathematical Education (ICME 5) which took place in Adelaide, Australia, August 1984. In a four-session working program, the group focused on three questions around which this report is organized:

1. The use of technology to further the cognitive development of learners of mathematics.
2. The use of technology to achieve a better understanding of the processes involved in the the development of children's cognitive structures relevant to mathematics.
3. Possible disadvantages connected with the use of technology.

It appeared that "technology" was almost exclusively thought of as (micro)computers. "Cognitive development" was taken to be a long-term process of change in children's mathematical intellectual behavior. Several group members listed in the references of this article gave introductory papers or catalyst statements which are amalgamated into this article together with information and opinions

---

The Co-Organizers and Co-Editors of this report wish to express their appreciation to the presenters and discussants in the Technology Theme Group for their attendance, contributions, and cooperation in preparation of this report. It is their attentive support, in all its forms, that has made the report possible. In addition, we express appreciation to Dr. Hartwig Meissner, West Germany, and Jury Mohyla, Australia, for their competent assistance in making arrangements for our sessions and helping out with all administrative and organizational matters related to our meetings.

The authors would have liked David Tall to contribute to our theme group, but he was unable to attend ICME 5. Having seen his fine program package *"Graphic Calculus"*, we feel this is the right place to give it special mention.

Correspondence and requests for reprints should be sent to Jerry P. Becker, Curriculum Instruction and Media, Southern Illinois University at Carbondale, Carbondale, IL 62901.

that were contributed in the discussions. A questionnaire completed by the group members on the last of the 4 working days provided further background information for the report. Student ages reflected in participants' contributions and discussions range from early elementary through the tertiary level.

THE USE OF TECHNOLOGY TO FURTHER THE COGNITIVE DEVELOPMENT OF LEARNERS OF MATHEMATICS

The Computer as an Aid to Develop Mathematical Concepts

We begin by presenting examples of work in which microcomputers are used to support concept development in content in the traditional mathematics curriculum. Ruth S. Hutter (Riverdale Country School, Bronx, New York) reported that at present most computer programs used in mathematics instruction fall into two categories, commonly called drill and practice and so-called teaching programs. The teaching programs furnish students with explanatory text and graphics in addition to having them answering questions and scoring their responses. Hutter (1984) viewed this procedure essentially as another form of drill which, though necessary, should be only a small part of the teaching process.

In contrast, the purpose of Hutter's (1984) presentation was to emphasize the invaluable role which the computer can play as a visual aid for clarifying mathematical concepts and facilitating understanding of conceptually difficult topics. While blackboard and textbook illustrations should remain primary tools for classroom instruction, these media are less helpful when a succession of drawings is required to present concepts such as the limiting processes of calculus or the changing images in transformations. Here, in contrast to films, the computer offers unique flexibility in that programs can be stopped at any time and parameters in equations can be varied at the will of the teacher or the students. While it has been said that a picture is worth a thousand words, Hutter felt inclined to say that computer graphs are worth a thousand pictures.

Ruth and Rudolf Hutter have developed a series of microcomputer programs for traditional mathematical content that are devised for use by an instructor in the classroom. The programs are not geared to any particular textbook but, rather, provide sufficient freedom for the teacher to determine the order and style of presentations. The programs include topics from calculus, pre-calculus, trigonometry, and intermediate algebra. Figure 1 illustrates how the sine curve is derived from the ratios of a side and hypotenuse in a right triangle with progressively enlarged angles in the unit circle (dynamically generated by the computer). After viewing the programs, group members had the impression that the *dynamics* of the images produced on the computer screen could potentially lead to good imagery supporting understanding.

Having used the programs in the classroom, Ruth Hutter (1984) reported that the attention and involvement of students of various abilities seems greatly enhanced. She also observed continuous mental involvement of students and a

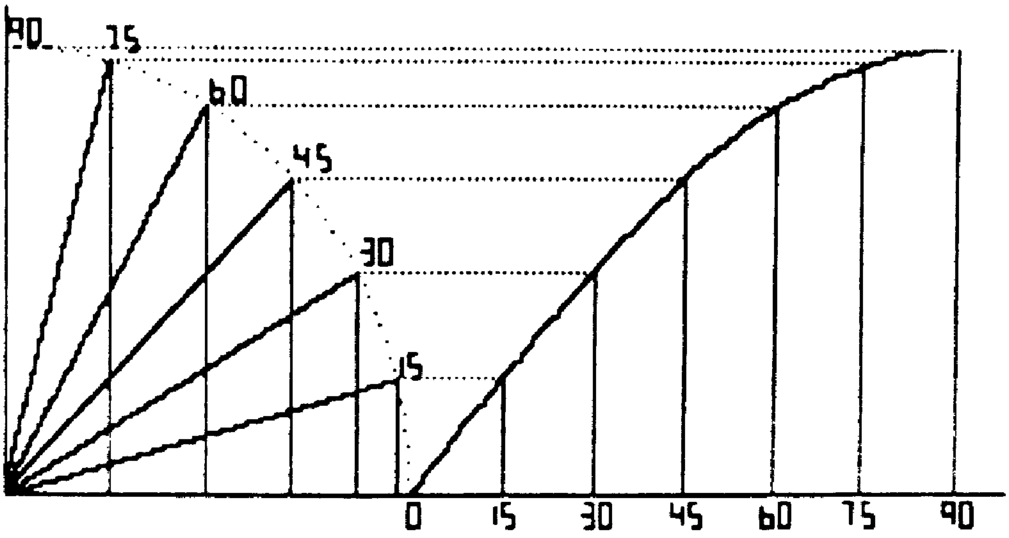

FIG. 1.

better retention of the topics discussed, as well as a facilitation of the use of mathematics in applications. Further work on such classroom use of the computer should concern the frequency of use, time limits (physical considerations like eye strain), required changes in textbooks, and production of new materials made necessary as a consequence of this use of new technology.

Kiyoshi Yokochi (Yamanashi University, Kofu, Japan) has developed microcomputer software concerned with the teaching of number for Japanese preschools and has also carried out several classroom experiments in order to locate critical parts of instruction where using such software could be beneficial. Yokochi (1984) identified a succession of three steps by which number teaching proceeds:

1. An introduction to the content in which children make good use of real objects.
2. The grasp of number concepts and rules which require that children make good use of the teacher's explanation and workbooks.
3. An application of what has been learned in real-life situations.

Yokochi pointed out the importance of the second step which is not reflected in the reality of many schools. Often emphasis is placed on the first and the third steps while in those schools placing emphasis on the second step instruction tends to be "dry as dust." Here Yokochi sees the point at which the microcomputer and software can be used that not only fascinates children, but also facilitates development of number concepts and rules in children.

The sequence of software lessons is ordered according to the development of children's cognition of number. The story line of lessons develops situations that are meaningful to the child's reality (e.g., playing soccer) where the "hero" is the child operating the computer. Through answering questions posed at many points in the lessons, the child is expected to grasp important aspects of the number concepts and to learn the rules to be mastered. Random sequences of questions, as well as music and graphics, are used to encourage the child to work through the lessons.

In most Japanese preschools, the number of students in a class is about 30 with, at most, 15 microcomputers available per classroom (sometimes fewer). The microcomputers are used in either an individualized or an all-students-together learning fashion. For an efficient use of the software Yokochi recommends that the software-assisted lessons be integrated into the usual course of lessons such that the computer is used about once a week to traverse the "grasp" step between the "introduction" and "application" steps. Weekly repetition of computer lessons are continued until all questions are answered correctly in the case of individualized learning, or until 90% of the children give correct answers to all questions in the case of classroom use of the software.

In summarizing the effects of this software on children, Yokochi (1984) noted that the children appear to be so absorbed by the lessons's stories that they do not want to miss class. Further, every child masters the content which, in general, is not expected from the usual approach to instruction. (For related literature see Yokochi [1983] and Yokochi and Machida [1982].)

Co-workers of Yokochi, Hirokazu Okamori and Izumi Nishitani have used Logo with a small number of fifth grade elementary school children. The project's aim was to develop children's comprehension of the similarity of figures, to have them learn a simple technique of measuring heights of well-known towers in Japan, to use the computer to calculate heights from reduced drawings produced on the screen, and to write Logo programs for magnifying or contracting figures. Problems children were asked to solve included writing Logo procedures that output an animal (shown in Figure 2) and one in which variables were used to output an "approaching" dog (supposedly in a succession of progressively magnified images). Okamori and Nishitani noted that while the children were not good at mathematics, by working with the computer they came to understand the property of similarity and the foundation of Logo programming. The computer was effective in motivating children and in helping to develop mathematical ideas in children.

Walter L. Fischer (University of Erlangen-Nürnberg, West Germany) noted that the introduction of computers into mathematics instruction has at least two dangers: On the one hand, computers could be used as mere calculators, and on the other, they could be used as high-styled playthings by teachers enraptured with programming. Using computers in such ways would occur on levels too high or too low, respectively, with respect to realistic educational aims and students' capacities. Fischer (1984) emphasized that for a systematic incorporation of computers into the curriculum, well-defined educational aims need to be identified, the achievement of which involve the use of computers. Whether

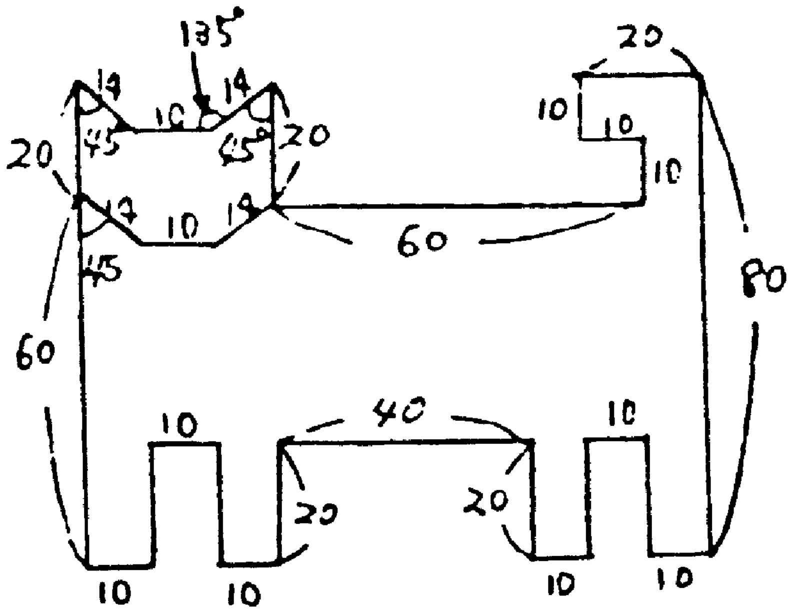

FIG. 2.

current curricula can be modified to incorporate use of computers or whether the *necessities of the computer age require curricula of a completely different structure* is a serious question which mathematics educators need to address.

Following the first line, Fischer has explored how computers can be incorporated into geometry instruction. The system consisting of computer and plotter (or screen) can be interpreted as a universal drawing device, the use of which is constrained by a set of propositions similar to those imposed on the use of compass and ruler. In pursuing the "Erlanger Programm" of transformational geometry proposed by Felix Klein, Fischer has found that the superior graphics capabilities of the microcomputer can help to overcome one of the shortcomings of this approach, namely, that instruction is usually confined to the group of Euclidean motions because of the complexity of drawings. As an example, Figure 3 shows a circular inversion produced by the computer, the genesis of which can be explained to students while the plotting is in progress.

Fischer (1984) concluded by saying that in lower grades computers can be used as a medium to impart knowledge of geometrical facts to students, whereas higher grade students should be able to conceptualize geometry programs for the computer on their own. The acquisition of programming skills will then be connected with educational objectives in geometry, and vice versa. In this stage, the computer can become an instructional tool as well as a medium to develop abilities and skills.

The last two examples presented in this section report on activities in developing countries. Dilip K. Sinha (Jadavpur University, Calcutta, India) illuminated some aspects of computer education from the perspective of a developing country. He reported that, besides the difficulties computer education faces in India due to its developing phase of industrialization, the possible erosion of computa-

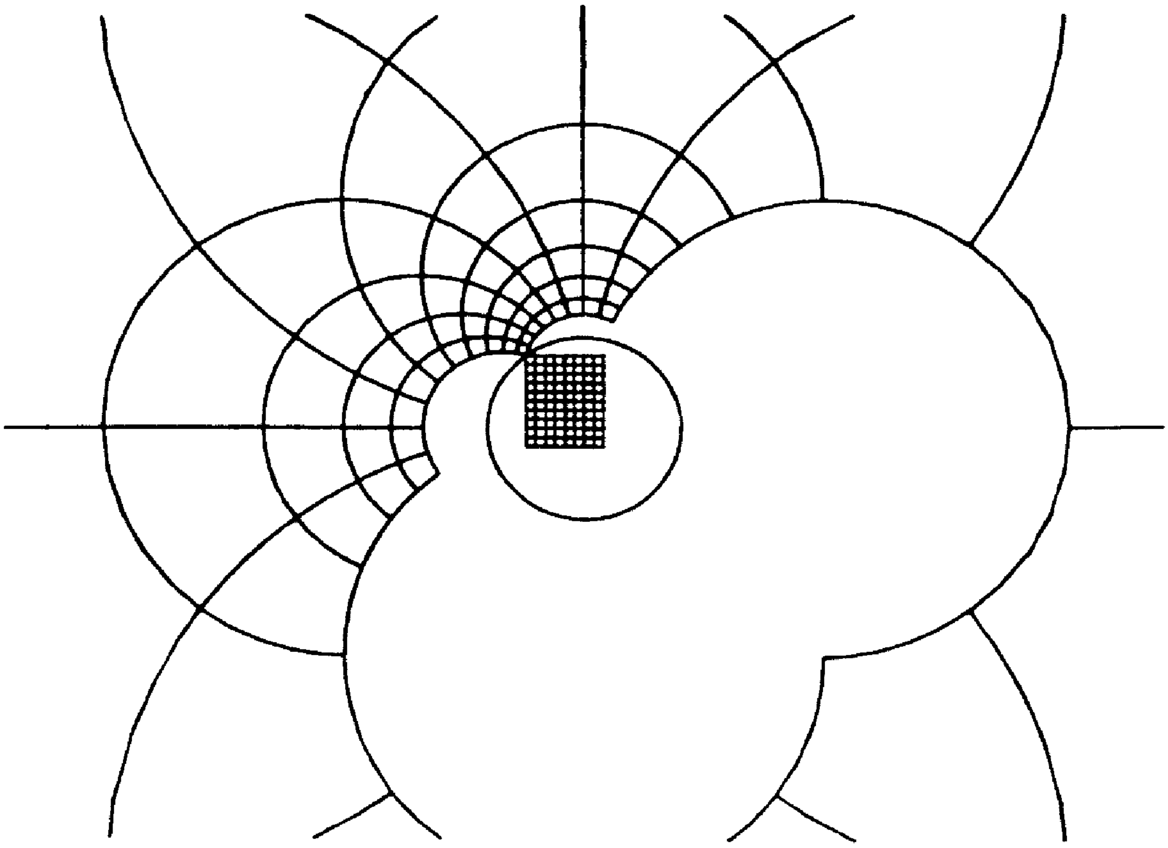

FIG. 3.

tional abilities is a purported weakness frequently expressed in this country of rich mathematical heritage. Sinha (1984) sought a general discussion of the point that understanding in mathematics can be reinforced through activities using the computer. He stated that no meaningful pedagogical program can bring about fruitful uses of innovative technology unless the issue of cognitive development is studied and related to a setting of mathematical education with an integrated computer component. Taking the instructional aim of achieving mathematical understanding and the consequent elimination of rote learning as prime imperatives, one has to look for appropriate uses of the computer toward these ends.

Sinha and his co-worker G. Bajani suggest that these aims can be achieved, in an ample measure, through studies in computer topics which are interwoven with mathematical topics. Seeking algorithmic approaches to strictly mathematical studies could add new perspectives to the cognitive development of mathematics learners. A pilot project on "Computer Literacy and Studies in Schools" (CLASS) testing a computer education program in some selected schools was recently initiated in India. It is expected to provide a substantial input to future investigations in this direction. As a first step, a workshop held by the National Council of Educational Research and Training (1984) in India has presented a report.

Porama Saengcharoenrat (Prince of Songkla University, Thailand) is involved in developing programs on computer-based learning in her country. One way in which she sees that computers can help university students learn mathematics is described as follows. Classes at the universities in Thailand tend to be overcrowded, so that teachers have on the average rather little time for the individual student. Furthermore, it is an unfortunate characteristic of Thai students that they usually lack courage to ask questions and to try to clarify unclear points. In both these respects the use of the computer appears to be helpful. When students study at their own terminals, they can proceed at their own pace and can go through the lessons as many times and as carefully as they like in order to understand the subject more completely. Difficulties arise from the fact that only teaching packages written in Thai language will facilitate a use of the computer by a wide group of people. Since Thai script has a very complex notation, many technical problems need to be solved before the situation can be improved (see also Saengcharoenrat, 1984).

Cognitive Development Through Active Engagement in Programming

The examples so far are concerned mainly with ways of using computers that were pre-programmed to provide a learner with demonstrations or instructional material. A very different approach, which is still somewhat controversial regarding what kind of mathematics learning is involved, is the "environmental approach" typified by Seymour Papert's Logo Laboratory. The observation is sometimes heard that overusing the computer in certain ways might in a sense lead to "programming" the child. A quotation from Papert's (1980) book *Mindstorms* serves as a response when he notes: "The child programs the computer. And in teaching the computer how to think, children embark on an exploration about how they themselves think. The experience can be heady: Thinking about thinking turns the child into an epistemologist, an experience not even shared by most adults" (p. 19).

Another criticism sometimes heard is that of a mechanical kind of thinking being produced in children as a result of working with the computer.

> By deliberately learning to imitate mechanical thinking, the learner becomes able to articulate what mechanical thinking is and what it is not. The exercise can lead to greater confidence about the ability to choose a cognitive style that suits the problem. Analysis of 'mechanical thinking' and how it is different from other kinds and practice with problem analysis can result in a new degree of intellectual sophistication. . . . Instead of inducing mechanical thinking, contact with computers could turn out to be the best conceivable antidote to it. . . . Through these experiences these children would be serving their apprenticeships as epistemologists, that is to say learning to think articulately about thinking. (Papert, 1980, p. 27)

Carrying this aspect further, it seems a sensible role for computers in mathematics education to enable children to observe, and become reflective about, *their own actions* (carried out by a computer they have programmed). Thus, we wonder whether programming a computer may stimulate thinking experiences that are beneficial to the learning of mathematics and, especially, problem solving. Children can observe and become reflective about their own thinking and thereupon use it more effectively (technology as a "thinking aid"). The dialogue style of interaction with some more advanced systems might even enhance that feature, as was reported by several participants.

The Computer as a Tool for Problem Solving and Exploration

Some presentations emphasized possible uses of computers as a tool in problem solving in mathematics. Jerry Becker (Southern Illinois University, Carbondale, Illinois) reported on such use of the microcomputer in the secondary mathematics classroom. In a project he co-directed (Becker & Pedersen, 1983) students were taught the rudiments of programming in Apple Pascal (Carmony, McGlinn, Becker, & Millman, 1984) and then attention was turned to solving problems. Problems posed were those which could be solved with the use of a microcomputer as a tool in conjunction with application of various problem-solving heuristics to which students were exposed earlier (see Schoenfeld, 1983; Burton, 1979). Two examples follow:

1. Note that 153 has the property that $1^3 + 5^3 + 3^3 = 153$. What other 3-digit numbers have this property? 4-digit numbers? 5-digit numbers? and so on.
2. A vicious dog (D) is at the southwest corner of a square garden 200 m on a side (see Figure 4). A man (M) is in the center of the garden. The dog will not attack the man in the garden, nor will it attack the man once he has left the garden. But the dog will run along the wall and try to bite the man as he crosses the wall. If the dog runs π times as fast as the man, is there a point(s) to which the man can run to escape the dog without being bitten?

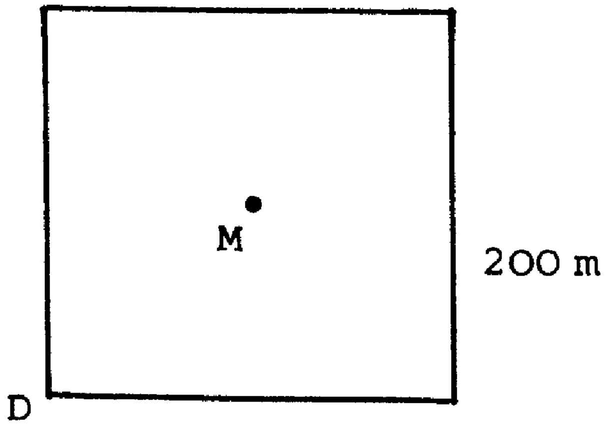

FIG. 4.

The microcomputer proved to be useful in the solution of these and many other such problems and may contribute to cognitive development in several respects: students tend to become *reflective* about their thinking and actions—they must organize their thinking and then take action in using the microcomputer as a tool; students' use of the microcomputer to explore and exhaust all possible cases may serve as an additional heuristic and students may evolve a new perspective on mathematical problem solving as a result of these experiences. Algorithmic thinking may also become part of the students mental processes and, thus, the development of problem-solving skills may be enhanced by use of the microcomputer.

In a separate part of the project, Papert's Turtlegraphics was used with elementary level students. Here it was found that, almost without exception, students reported that they were *always thinking* while interacting with the microcomputer. Related to cognitive development were the following observations: students thought through a sequence of actions, related well to visualization of their actions, and learned to see equivalences—several programming steps can be done in one step.

It was observed during discussion that we may not know exactly *how* the microcomputer contributes toward cognitive development, but there was general consensus that it may have benefit and that further efforts to study this issue are justified.

Grayson H. Wheatley (Purdue University, East Lafayette, Indiana) has also worked on using the computer as a problem-solving tool. Wheatley starts from the observation that beliefs and expectations play a major role in mathematics problem solving (Wheatley, 1984). If, as it has been argued by educators favoring the constructivist approach of learning, students must construct knowledge for themselves, then using the computer as a problem-solving tool is a valuable experience. By organizing ideas around a problem, students can build personal schemas that can be readily applied to a wide variety of new situations. A major strength of solving problems through programming is that the user is in control. This is in sharp contrast to other modes in which the user responds to questions displayed on the screen or is "programmed" through a set of tasks. Furthermore, a programming language can be learned effectively in the process of solving mathematical problems. Finally, computer problem solving encourages the student to develop a "relational" view of mathematics that draws understanding from observations of the cause–effect dependancy between the different concepts and rules involved in constructing the solution.

The second point emphasized by Wheatley is that computing devices, whether computers or calculators, can be important tools for exploration. Using a computer graphing program to explore the nature of functions can place the emphasis on mathematics rather than computation. Wheatley reported findings (Wheatley & Wheatley, 1982) which show that students make more exploratory moves in problem solving with the use of calculators. In a study, he found that in problem solving 11-year-olds performed more exploratory computations, used a wider variety of heuristics, and completed the same number of problems in half the time with better performance. When students expect to get stuck on a problem and expect to explore several approaches, they are more successful than students who expect to apply a rule.

Studies at Purdue University have also shown that bright 11- and 12-year-olds became quite adept at writing programs (in Basic) to solve mathematics problems like, "Find three positive integers such that their sum is 43 and the sum of their cubes is 17299." A 12-year-old with computer problem-solving experience, when confronted with an algebra word problem outside the context of using computers, immediately sketched a computer program based on exhaustive search which he could subsequently refine to obtain the correct solution. While this boy had been unable at age 11 to set up an equation and solve it in the traditional way, the new tool now enabled him to handle a problem reasonably challenging for 15-year-olds. Similar observations with other students from the same class substantiate the impression that the heuristics used in computer problem solving, of which exhaustive search is one, are quite powerful in that they apply to a broad range of problems.

Examined in the light of computer availability, Wheatley put forward the argument that the heavy emphasis which traditional courses place on learning procedures for computing quotients and roots, as well as solving systems of equations, for example, may no longer be appropriate. On the other hand, students relying too heavily on computer-related strategies may not develop the relationships which are acquired by looking for patterns and attempting to build a representation that characterizes the structure of the problem. While it has been shown that solving problems by constructing computer programs is an excellent setting for enhancing problem-solving abilities, Wheatley also calls for investigation of undesirable side effects like that of overusing computing facilities which might impair the development of pattern finding strategies.

Developing Imagery and "Big Ideas" Through Computers

It has been noted that imagery plays an important role in high-achieving mathematics students while less able students may experience more difficulty in understanding mathematics which requires imagery, and this may be especially true when it comes to "abstract" mathematical concepts and relationships. In this connection, Robert B. Davis (University of Illinois, Urbana, Illinois) made reference to two different views of teaching: one that views students learning by following explicit instruction, and one that views students "building up" mental representations in their mind while interacting with an environment. He made the point that a large amount of computer-assisted instruction treats knowledge as if it were co-extensive with vocabulary drill which is not entirely surprising since this approach has been a weakness of much education in pre-computer forms. By treating "knowledge" as nothing more than "knowledge about words," one confuses, as Davis put it, encyclopedias and dictionaries—the former being properly a book of knowledge, the latter a book of words.

Davis emphasized that learning often depends upon implicit knowledge that is not stored in one's mind in the form of actual language statements. A great deal of such knowledge is derived from experience of various types. Technology can offer ways of teaching children without using any words. An example of such teaching material aimed at building up a reasonable notion of fraction size is a computer game called DARTS. In this game, a vertical number line appears on the screen with indicated end marks and balloons attached at random positions. By typing in a fraction, a dart can be made to fly across to "pop" a balloon if the fraction is a close-enough estimate.

Davis continued by mentioning the role which technology could play as a mediator in building up "big ideas" in a learner's mind; for example, the important concept of convergence of an infinite sequence. He referred to the notion of "frame" structures that organize such knowledge in a way that the whole is more than the sum of its parts (cf. Davis, 1984). The understanding derived from such big ideas is often metaphorical, not logical in that one uses ideas already developed as a way to approach new ones which sometimes requires that slight modifications or adaptations be made. As an example from physics, Davis mentioned how Rutherford scattering became understandable thanks to the big idea of the solar system; a revised model of the solar system was further used to create an "atomic structure" model (cf. Gentner & Stevens, 1983; Davis, 1984).

Gerald Goldin (1983) has presented some thoughts which reinforce these ideas from which we quote a paragraph here:

> It is an interesting conjecture that 'genius' in mathematics or physics has to do mainly with the imagistic level. It seems unlikely that the unusually great mathematician achieves that greatness merely by superior encoding of verbal problem statements in formal symbols, or by more efficient processing of formal notations, or by much better and more sophisticated heuristic plans. It is more plausible that the innovator has somehow succeeded in constructing a superior imagistic (non-verbal) representation, within which mathematical relationships inaccessible to others can be 'seen.' The challenge of mathematics for such a person is to develop appropriate notations within which such relationships can be precisely described and axiomatized, so that theorems can be proven. (p. 117)

We conclude that it is a challenge for us to use technology to develop appropriate methods that can support imagistic thinking of "abstract" mathematical situations and relationships among all students. The graphic capabilities of microcomputers are a possible means to achieve visualizations. Examples shown and discussed in the working group emphasized the dynamical aspect of computer graphics, with the idea of creating moving images preparatory to the development of important abstract concepts (big ideas).

Ipke Wachsmuth (University of Osnabrück, West Germany) provided two examples from outside of the theme group that were discussed to outline further the far-reaching possibilities provided by professional computer graphics. Thomas Banchoff (1978) has become known for his computer animations of the geometry of surfaces in 3- and 4-space. At Brown University, he has used a high-speed graphics computer to display on the screen a sequence of images which the viewer readily interprets as the projections of an object rotating in three-dimensional space. By turning dials, one can investigate a curve or surface by having it rotate about different axes and stopping it at especially interesting positions. One can "fly inside" the object to focus on some local behavior or proceed to examine some specific singularity by deforming the object through a one-parameter family of curves or surfaces.

The other example concerns computer graphics from the theory of dynamical systems. At the University of Bremen, West Germany, H.O. Peitgen and P.H. Richter (1984) have graphics computers produce "Gedanken maps" to visualize characteristic features of mathematical feedback systems as in, for example, the Newton iteration procedure for certain complex equations. Pictures never seen before illuminate the situation in regions of dramatic change in the system's behavior of convergence, like the "Julia sets" separating the domains of competing attractors, or like the "Mandelbrot sets." Figure 5 shows a very modest graphic (reproducable in black-and-white) of a Julia set; other graphics are very colorful and richer in terms of coded information. A case has been reported where a result obtained theoretically was shown to be false by looking at such a map.

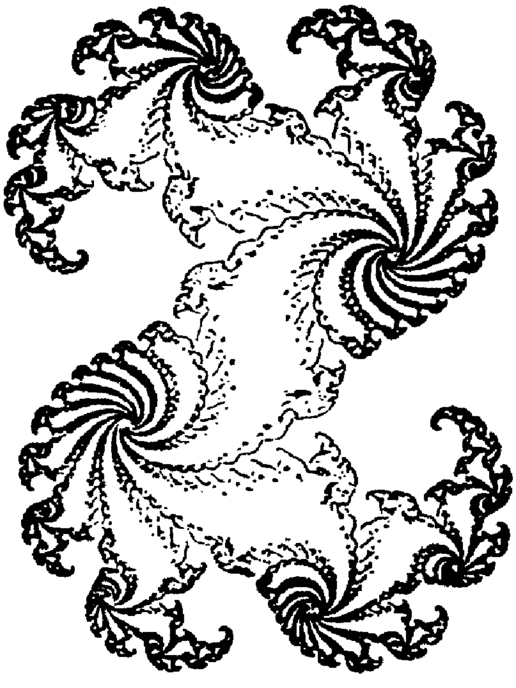

FIG. 5.

A kind of "experimental mathematics" has evolved from these approaches which may be progressively more important for an understanding and mathematization of complex processes. A group, working around W. Metzler at the University of Kassel, West Germany, is exploring visualizing mathematics and has started to explore possible uses of such technology, also within the reach of microcomputers, for didactical purposes (see Metzler, Beau, & Überla, 1985).

THE USE OF TECHNOLOGY TO ACHIEVE A BETTER UNDERSTANDING OF THE PROCESSES INVOLVED IN CHILDREN'S DEVELOPMENT OF COGNITIVE STRUCTURES RELEVANT TO MATHEMATICS

Investigating Cognitive Development in the Context of Computers

Elmar Cohors-Fresenborg (University of Osnabrück, West Germany) reported on research aimed at introducing a fundamental understanding of computers to children (ages 10–13) which is based on elementary actions, and, secondly, on cognitive strategies observable in children's construction and analysis of computer programs. Concerning the first topic, he described a teaching project introducing the concept of functions on the basis of algorithms (Cohors-Fresenborg, Griep, & Schwank, 1982). A fundamental hypothesis of the project is that the central problem in programming is to *organize* a sequence of actions to be executed by the computer. In this approach the set of whole numbers is used as an initial data structure where the elementary operations performed on it are counting forward and backwards. The control structures are comprised by concatenation of partial algorithms and the iteration of algorithms (while a storage remains non-empty).

The approach includes three different ways of representing an algorithm: (1) The whole numbers are embodied by a heap of sticks, then the elementary operations in an algorithm are adding or taking away a single stick; (2) an algorithm is represented by a so-called computing network which constitutes a functionable flow chart and is constructed with a special manipulative called "Dynamical Mazes"; (3) an algorithm is represented as a program for a special model computer, called "Registermachine," which is implemented on a usual microcomputer.

Classroom experiments have proved that, by this *non-verbal* approach to algorithms, 13-year-olds easily get a fundamental understanding of constructing and analyzing algorithms and a first introduction to a programming language which, from the mathematical point of view, has the complexity of Pascal.

A second part of the project is devoted to the role which the three different ways of representing algorithmic concepts play in concept formation of 13-year-olds (Cohors-Fresenborg, in press) and to different cognitive strategies exhibited by children when they construct or analyze algorithms (Cohors-Fresenborg & Kaune, 1984). A number of case studies have shown that the above-mentioned three ways of representation do not form a hierarchy. Some children prefer to begin at the level of handling sticks, while others prefer to first construct a computing network.

The case studies have also shown that there exist at least two different cognitive strategies in constructing algorithms for the Registermachine. One strategy may be described as follows: children reflect on how they would organize the acting of the Registermachine sequentially. Questionned to describe what a program does, they use words indicating that they are imagining a Registermachine acting step-by-step. But there are also children that begin inventing a computer program by structuring the given problem beforehand, using *concepts* they know from earlier problem-solving experiences. In the first case one could speak of a sequential and in the second case of a conceptual strategy.

Patricia F. Campbell (University of Maryland) has focused her recent studies on the young child in a microcomputer environment. She mentioned that there was concern in the U.S. that the microcomputer could destroy the environment of preschool and primary school. In particular, would microcomputers discourage or distort the social and emotional development of young children, replacing active manipulation and experiential learning? On the other hand, the microcomputer has the potential of providing the young child with an environment replete with symbolic and spatial experiences. Campbell and her colleagues at the University of Maryland are exploring how young children interact with a microcomputer and how their level of development may influence their interaction.

In a first study (Fein, Campbell, & Schwartz, 1984) computer graphics were regarded as early symbolization. At the Center for Young Children (CYC) at the University of Maryland, 4-year-olds had access to microcomputers four days a week during their free choice period. Initially the children viewed the microcomputer as a toy, as they learned to use commercially produced graphic art packages. However, after first exposures had produced an understanding of the cause and effect relationship between the controls and the screen display, manipulations became more functional in nature as the children learned to control the display and make the cursor move on the screen. The play became constructive rather than functional as the children drew images on the screen in order to produce a product. Sometimes children had a final product in mind prior to the initiation of the drawing. This type of play expanded as the children began to attach a symbolic meaning to a dynamic image on the monitor screen, a meaning which they could easily change in their "pretend" world.

Observations of dramatic play at the microcomputer also revealed instances of collective symbolism as pairs of children collaborated to define, share and modify the meanings of created objects and scenes in their make-believe world on the monitor screen. Rather than promoting isolation, the microcomputer led to discussion and social interaction (Wright & Samaras, in press). One key difference between a computer graphics environment and that available via crayons and paper was dynamic control. With the microcomputer, misplaced lines could be quickly erased, colors could be changed, and images were easily modified. For a 4-year-old experiencing the frustration of inefficient motor control, the potential of the illusion of movement and the option of change made this an art project that was more interesting to play with and to share.

A second study which Campbell reported explored preschoolers' dealing with Logo, from a syntactic and spatial perspective (Campbell, Fein, Scholnick, Frank, & Schwartz, 1985). While it has been argued that learning to program a computer will promote cognitive benefits such as facility with problem-solving heuristics and the development of procedural thinking, some researchers have questioned these conclusions, as Campbell pointed out. At the most primitive level, programming requires a comprehension of the syntactic potential of the programming language. Although prior studies have documented that children can program in the Logo language, there is little research on what the constructs of programming in Logo are or how they are mastered. The study conducted recently at the CYC examined 5-year-olds' emerging competence with the syntax and semantics of the Logo language (using a simplified version, "Instant Logo") and, further, their ability to take the perspective of the triangular cursor (turtle) and to reorient accordingly in a two-dimensional space. The project did not view the learning of Logo as the development of programming skills which would be beneficial for transfer to other areas; rather, learning Logo was viewed as a positive goal in itself.

A preliminary data analysis reveals "findings" that the reflective control of two succesive commands involving distance and direction emerged gradually for the children. They had a definite bias for the forward command for distance and the right command for direction, and utilization of the four graphic commands seemed to occur more efficiently when the cursor was in the HOME position (center screen, heading up). Many of the children exhibited revision skills that seemed to expand into monitoring skills as they began to interpret and compensate for the subsequent movement of the cursor. Some children seemed to be able to estimate a unit of measure, primarily for distance, and they were able to apply this as they moved the cursor to a specified location or reproduced a given line segment.

An hypothesis offered by Campbell describes the process of children's learning of Logo in the following manner: The meaning of the four graphic commands is mastered readily. Awareness of the need to order direction commands prior to distance commands emerges gradually, producing a primitive system of commands based on isolated movement. Recognition of the reversible relationship between left/right commands and forward/back commands permits revision of the prior entry. When this level is reached, the system of Logo commands is not only based on isolated movement, but it is also a compensatory system. Eventually there is an understanding that a single command or move may be embedded in a whole series of commands in order to produce a pre-determined image or a component of a figure.

The Role of Artificial Intelligence Research and Technology for Understanding and Furthering Cognitive Development

In what ways can technology be used to achieve a better understanding of the processes involved in children's cognitive development with respect to mathematics? Discussions in the theme group placed special emphasis on computer systems evolving from artificial intelligence (AI) research and technology that attempts to model students' knowledge of mathematics and behavior in mathematical situations, as well as making use of it in instruction.

In this section we will discuss examples of AI-influenced activities. Particular questions addressed are the following:

- What can we learn from AI computer models about the nature of mathematics learning?
- In what ways can AI technology assist instruction and promote cognitive development?

The central idea of the theoretical study of "artificial intelligence" is "to proceed on the basis of the conjecture that every aspect of learning or any other feature of intelligence can in principle be so precisely described that a machine can be made to simulate it" (original wording of Rockefeller proposal for the Dartmouth conference in 1956). Following the paradigm that the human is an information processing system, AI is the study of how to organize processes to bring about "intelligent" behavior. Information processing models constitute an approach of extending Piaget's research questions (How is knowledge structured at different stages of cognitive development?) to reach for an understanding of the process of change of cognitive structures which occurs as a result of an individual's active interaction with the outside world. "Learning to generate is learning to understand." The rigid restrictions imposed on computer simulations of cognitive processes require not only the product (i.e., the "intelligent" behavior exhibited), but also the processes giving rise to such behavior to be consistent with observations of human performers.

Since computer simulations of human problem-solving processes were carried out by Newell and Simon in the 60s and 70s, a growing body of knowledge is accumulating which helps to elucidate processes of cognitive development in light of the information processing paradigm. Techniques have evolved which can serve to make better use of knowledge in specific domains, for example, expert systems. Such "intelligent" computer programs can, for example, retrieve information necessary to answer questions, perform dialogs with a human partner, assign individual problems, and analyze errors. They are expected to assist instruction in more than a merely primitive way ("intelligent computer-assisted instruction"). Educational institutions are exploring these possibilities and companies are pushing such technology to the point of application in the classroom. From this evolves the need to discuss the chances, disadvantages, and far-reaching ways in which such technology can affect our children learning mathematics in school or, perhaps, in a home computer environment.

As one example, primarily concerning the first question, Ipke Wachsmuth reported on work that is done at the University of Osnabrück, West Germany. As a central concern of the Lakos Project ("Logical Analysis of Cognitive Organizational Structures"), a knowledge representation model has been specified which is implemented in a Prolog machine. (A brief demonstration of the model was given in the Technology Summary Session.) The model stresses the organization of knowledge represented in memory and can perform dialogs which are based on individual children's knowledge of specific mathematical domains (e.g., fractions). Based on analyses of interview protocols obtained from long-term clinical research, the hypothesized knowledge structures of individual subjects are modeled in networks which link "knowledge units" that are expressed as logical propositions and stored in indexed memory nodes. In performing dialogs with the computer, the behavior of the model in answering particular queries about mathematical facts can be explored and compared with behaviors that were actually observed with the subject. Situations where certain knowledge is not accessed in a particular context can be modeled as well as misconceptions and cognitive conflicts due to inconsistencies. Differences occuring between the model's and the subject's actual behavior can lead to a refined understanding of the subject's cognitive structures, since in order to generate certain behavior in the model, the knowledge units and memory links need to be specified precisely.

Based on such an hypothesis of what are the current cognitive structures of a pupil, the user of the model is in the position to obtain a diagnosis of the origins of misbehavior, and to evaluate instructional procedures which can bring about progress in the cognitive development of the pupil. With future versions of the model being planned to be capable of self-modifications under interactions in dialogs (i.e., learning), the potential of such a model might even include exploring the effects of alternative remedial interventions in the model before actually intervening in the classroom (though probably at considerable cost). At present, the major pay-off of the model is found in the way it sensitizes the user to the subtleties in the child's understanding of, knowing (or retrieving), and being able to express particular mathematical issues; further, it helps to develop a fine-tuned feeling for performing (teaching or interview) dialogs with the pupil.

The next example concerns the second question of using AI technology in instruction. Betty P. Travis (University of Texas at San Antonio) is involved in a research effort on applying AI techniques to educational software. Here learning to program a computer is viewed as a problem-solving task, in which a computer tutor shall interact with the individual student in a manner so as to diagnose the errors in programs written by the student and to lead the student in a Socratic manner to an understanding of the specific errors. For doing this, the system uses the collected knowledge base of experienced teachers of computer programming. As background, the project uses the general model for an ICAI system which is composed of three components: (1) an expert-system component which is charged with the task of generating problems and evaluating the correctness of the student's solutions; (2) a student-model component which is to represent the student's current understanding of the material to be taught and the update of his/her particular error history; and (3) a teaching-strategies (tutoring) component which must integrate knowledge about natural-language dialogues, teaching methods, and the subject area. This is the component which communicates with the student, selecting problems for him/her to solve and monitoring and critizing his/her performance. Embedded in a larger framework, the core of this approach is to compare, in a given problem situation, the student's actual response with an ideal interaction generated by the expertise component (see Figure 6). The difference will then be evaluated in order to make a decision about appropriate tutoring strategies. Work is still in a preliminary stage with the main efforts so far being devoted to constructing the expertise component.

In relating the approaches discussed above to the current teaching reality, Ronald H. Wenger (University of Delaware) noted that discussion about the content and objectives of the U.S. precollege mathematics curriculum has increased due, in part, to the existence of computer technology. Software such as muMath, PRESS, and MACSYMA can effectively accomplish tasks like equation solving. Such powerful software utilities motivate some to hope that instructional time devoted to such algorithms and procedures can be decreased. While such expectations are often based on exaggerations and little information of the

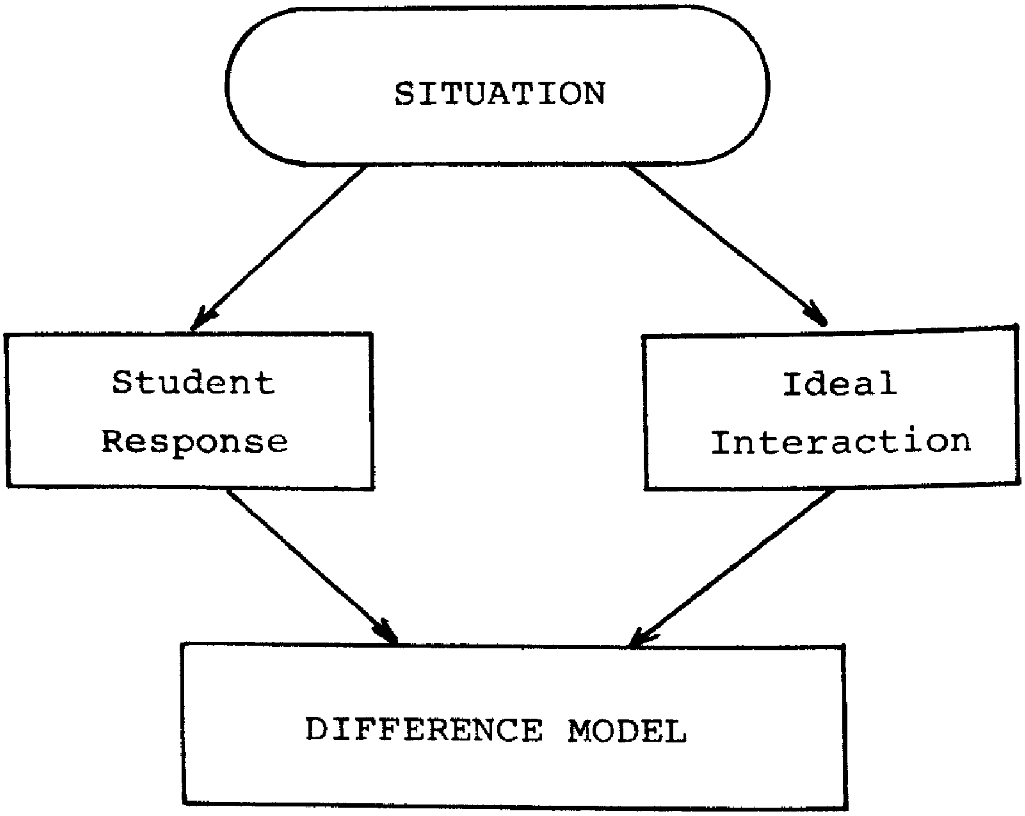

FIG. 6.

power of the existing software, the scenario does help to focus attention upon serious weaknesses in the way mathematics is often taught, and upon weaknesses of many existing textbooks. Virtually all textbooks focus on the "object" or procedural levels of mathematical content and leave discovery of relevant "meta-level" heuristics and strategies to the student. Little improvement is expected until new methodologies are created, tested, and used to analyze the higher level forms of cognitive development which mathematicians wish the curriculum to serve.

Wenger also outlined one methodology, the Global Task Analysis (GTA), and one project, LP (Learning PRESS, a PRolog Equation Solving System), which are complementary efforts aimed at a diagnosis of relevant instructional objectives and which use equation solving as the prototypic domain (at the precalculus level in the U.S. or the A-level in Britain). The GTA leads to a grammar describing the heuristics, strategies, and procedures hypothesized to be used by the expert when confronted with a mixed list of equations. LP is a Prolog computer program developed in the AI Department at the University of Edinburgh which extends the earlier equation-solving program, PRESS, to include means of inferring new solution methods from worked examples. In particular, these efforts illustrate the importance of heuristic and other "meta-level" strategies and skills, and they can help to clarify the relevant component skills and their relationships to diagnosing difficulties at strategic and at procedural levels. Some references annotated by Wenger which are relevant to the specific focus of this work include Silver (1984), Bundy (1983), and Carry, Lewis, and Bernard (1980). In addition, a brief and critical summary with respect to the artificial intelligence theme is found in Snow (1984).

It is expected that this work will have implications for the evaluation and design of textbooks and will provide useful foundations for building student models (possibly instantiated in computer simulations) which help to clarify the relationships among concepts and skills used by successful solvers. Furthermore, one hopes to obtain insights as to how much emphasis should be given to the algorithmic skills currently so heavily emphasized in the curriculum, in light of the fact that now computers can often carry out such tasks more skillfully than many students. Further, these efforts will help to identify those algebraic skills which are important for using powerful computer utilities with understanding. Finally, we note that it is hoped that such models can help to obtain a more precise notion of "cognitive structure" which would provide a basis for a more profound understanding of the process of building-up such structures, that is, cognitive development.

POSSIBLE DISADVANTAGES CONNECTED WITH THE USE OF TECHNOLOGY

Loss of basic skills and deprivation from natural environments were among the disadvantages noted in discussions in the theme group. The fact that the computer might be overused, misused by inexperienced teachers, used for inappropriate topics, or to repeat things which can be done in other ways were viewed as undesirable. Tendencies to "simulate" real-life teachers and an over-reliance on mechanical teaching systems were thought of likewise. The premature pushing of some software projects to classrooms reminded some participants of occurences in the new math movement.

Although some of these remarks were expressed with sometimes great emotional involvement of participants, little evidence was given in terms of findings substantiating these views, though this does not preclude such substantiation in the future. Results of research directed to this issue should be collected in order to obtain a clearer notion of the existence, or non-existence, of disadvantages in the use of technology.

Concerning the question of a possible loss of basic skills through the use of technology, a reference was found of a recently published 3-year study with several thousand subjects, Grades 7–9, which does *not* substantiate disadvantages with respect to computing skills (Wynands, 1984, p. 25): There is no substantial difference in computing skill between the groups of subjects who used calculators from seventh to ninth grade and the control groups who never used calculators. Another reference includes some remarks related to the aspect of deprivation and inappropriate computer uses which we quote here:

> The greatest source of concern about having microcomputers in classrooms for young children is that the microcomputer activities will *supplant* the many activities children do with 'real' materials. . . . There is not doubt that working with materials is important for young children, and it would be unimaginable, not to speak of absurd, to have a microcomputer replace the water table, block corner or pet rabbit. What seems to underlie this concern is a sense that the microcomputer innovation has a life of its own proceeding at an intense, unstoppable pace . . . this view implies that the technology will take over, that what teachers do or believe will not matter. Whatever research knowledge we have on this issue suggests quite the opposite—that what school systems and teachers do with computers—what they use them for, how they interpret them, how they present them to children—has an enormous effect on what happens in a particular system or classroom. . . . The technology does not have a life of its own, nor does it stand on its own. (Sheingold, in press)

Very probably, it will depend to a great extent on the teacher whether the use of technology is disadvantageous to the cognitive development of children. But it also appears that great differences exist in the quality and suitability of educational software. It would seem that approaches not emphasizing interaction or individual exploration, as well as CAI programs that rely more on rote than on reason, are counterproductive to rather than enhancing of cognitive development. One thing seems sure: Monotonous work which does not provide for self-directed engagement will not foster creativity nor will it contribute to cognitive development of mathematics learners.

SUMMARY

It was realized that technology will play an increasingly important role in the learning and teaching of mathematics. There was accord in the feeling that students may develop a different perspective on mathematics and on learning mathematics as a consequence of working with the computer. However, it is not yet clear what cognitive structures are being developed via experiences stemming from technological stimuli. The computer may provoke approaches which are new and promising but we do not yet know enough about how to capitalize on it. There is doubt as to what technology or how much will be needed. Examples have demonstrated that technology can be very useful, *depending on how it is used*. While it should not necessarily replace any other effective mediator of cognitive development, the computer may certainly play an important complementary role. Being in the early stages of exploring modes for the use of technology, there is a need for us to search for a "balanced" use of computers in teaching and learning mathematics.

The opinion expressed most frequently in the theme group discussions was that computers can help children in *visualizing mathematics* and help them *learning to think* and becoming reflective about their own thinking and acting. Perhaps we could say that the theme group was unanimous in this feeling.

An opinion also expressed repeatedly is that the benefits from using the computer in mathematics learning can be doubly advantageous in that mutual progress is achieved in both operating the computer and mastering subject matter. For example, it was said that interweaving studies in computer topics with mathematics topics can be beneficial, or that a programming language can be learned effectively in the process of solving mathematical problems with the computer, or that when students conceptualize their own programs (e.g., for geometry demonstrations), the acquisition of programming skills will be connected with educational objectives in the subject area and vice versa. This seems to us an important point, in addition to the beneficial influence of the computer on visualizations and on learning to think as mentioned earlier.

Concerning the way in which technology can contribute to a clearer picture of the processes involved in the cognitive development of mathematics learners, it seems an important point that insights can be obtained about the development and differentiation of cognitive strategies via the learners' progressive mastery of symbolic systems provided by a computer. Ultimately desirable is an understanding of the development of cognitive structures in learners' minds in order to understand the origins of their behavior and strategies and be able to account for these in instruction. The use of AI techniques in modeling cognitive structures and their development, if complemented/guided by empirical work, may be a promising approach that deserves further research attention.

While the group feels that new technology can be used in many ways in mathematics teaching and thereby may take on a new perspective on mathematics and mathematics learning, it appears necessary to take note of possible disadvantages in order to maximize the value of technology. *Teachers* need to know how to handle the microcomputer—its value may to a great extent depend on the teacher and the subsequent interaction of the teacher and the student. At the same time, the group feels that we need to learn more about how to embed what appears beneficial in use of computers into classroom-usable form.

If the development of certain cognitive structures and strategies is what happens in the intellectual development of humans that has long-term implications in mathematics learning, then what activities seem to most enhance cognitive development and what cognitive strategies do students need to combine with the use of computers? We did not come up with the answer(s), but we feel we have pulled together ideas and research which move us a little in the direction of (a) understanding better the use of technology in cognitive development of mathematics learners, and (b) knowing more about uses of technology for better understanding of the processes involved in children's cognitive development.

Participants in the Theme Group on Technology and Cognitive Development were:

Australia	G. Ball, A Ballard, M. Baones, M. Brown, A. Elliott, W. Ransley
France	D. Lavallade
India	A.K. Das, D.K. Sinha, S.P. Yadav
Japan	H. Hirabayashi, I. Nishitani, H. Okamori, K. Yokochi
New Zealand	J. Sauer
Papua, New Guinea	R. Selden
South Africa	P. Laridon, J. Vermeulen
Thailand	P. Saengcharoenrat
U.S.A.	J. Becker, P. Campbell, R.B. Davis, R. & R. Hutter, D. Kressen, D. Resek, B. Travis, R. Wenger, G. Wheatley
West Germany	E. Cohors-Fresenborg, W.L. Fischer, K. Haussmann, W. Lorbeer, M. Reiss, I. Wachsmuth

REFERENCES: THEME GROUP CONTRIBUTIONS

Becker, J.P. (1984, August). *The microcomputer as a tool in problem solving.* Paper presented at the Fifth International Congress on Mathematical Education (ICME 5), Adelaide, Australia.

Campbell, P. (1984, August). *The young child in a microcomputer environment.* Paper presented at the Fifth International Congress on Mathematical Education (ICME 5), Adelaide, Australia.

Cohors-Fresenborg, E. (1984, August). *Representations of algorithms and cognitive styles.* Paper presented at the Fifth International Congress on Mathematical Education (ICME 5), Adelaide, Australia.

Davis, R.B. (1984, August). *Frames, metaphors, and computer-assisted instruction.* Paper presented at the Fifth International Congress on Mathematical Education (ICME 5), Adelaide, Australia.

Fischer, W.L. (1984, August). *Computer and transformational geometry.* Paper presented at the Fifth International Congress on Mathematical Education (ICME 5), Adelaide, Australia.

Hutter, R. (1984, August). *Using the computer in classroom teaching of high-school and undergraduate mathematics--Concept development, not drill.* Paper presented at the Fifth International Congress on Mathematical Education (ICME 5), Adelaide, Australia.

Sinha, D.K. (1984, August). *Computer education in a developing country: Some conceptual and cognitive aspects.* Paper presented at the Fifth International Congress on Mathematical Education (ICME 5), Adelaide, Australia.

Travis, B. (1984, August). *A computer tutor for error diagnosis in student programming.* Paper presented at the Fifth International Congress on Mathematical Education (ICME 5), Adelaide, Australia.

Saengcharoenrat, P. (1984, August). *Computer based learning in Thailand.* Paper presented at the Fifth International Congress on Mathematical Education (ICME 5), Adelaide, Australia.

Wachsmuth, I. (1984, August). *The role of artificial intelligence research and technology for the learning of mathematics.* Paper presented at the Fifth International Congress on Mathematical Education (ICME 5), Adelaide, Australia.

Wenger, R. (1984, August). *Diagnostic uses of computers—some metaphors and methodologies from cognitive science.* Paper presented at the Fifth International Congress on Mathematical Education (ICME 5), Adelaide, Australia.

Wheatley, G. (1984, August). *Computers and calculators as tools for exploration and problem solving.* Paper presented at the Fifth International Congress on Mathematical Education (ICME 5), Adelaide, Australia.

Yokochi, K., & Okamori, H. (1984, August). *Developing preschool and elementary school children's mathematical idea through educational software.* Paper presented at the Fifth International Congress on Mathematical Education (ICME 5), Adelaide, Australia.

REFERENCES

Banchoff, T.F. (1978). Computer animation and the geometry of surfaces in 3- and 4-space. *Proceedings of the International Congress of Mathematicians,* Helsinki, Finland.

Becker, J.P., & Pedersen, K. (1983, June). *Integrating Problem Solving into Mathematics Teaching.* (Proposal funded by the Illinois State Board of Education (ISBE): Springfield, IL). Southern Illinois University at Carbondale, Illinois.

Bundy, A. (1983). *The computer modelling of mathematical reasoning.* London: Academic.

Burton, L. (1979). *The Skills and Procedures of Mathematical Problem Solving.* Report of an S.S.R.C.-Sponsored Project at the Polytechnic of the South Bank, London, England.

Campbell, P.F., Fein, G.G., Scholnick, E.K., Frank, R.E., & Schwartz, S.S. (1985, April). *Initial mastery of the syntax and semantics of Logo.* Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, IL.

Carmony, L., McGlinn, R., Becker, J.P., & Millman, A. (1984). *Problem solving in Apple Pascal.* Rockville, MD: Computer Science Press.

Carry, L.R., Lewis, C.H., & Bernard, J.E. (1980). *Psychology of equation solving: An information processing study.* Department of Curriculum and Instruction, University of Texas at Austin.

Cohors-Fresenborg, E. (in press). On the representation of algorithmic concepts. In F. Lowenthal (Eds.), *Pragmatics and education.* New York: Plenum.

Cohors-Fresenborg, E., Griep, M., & Schwank, I. (1982). *Register-machines and functions—A schoolbook introducing the concepts of function on the basis of algorithms. Osnabrücker Schriften zur Mathematik,* Osnabrück, West Germany.

Cohors-Fresenborg, E., & Kaune, C. (1984). Sequential versus conceptual—Two modes in algorithmic thinking. In B. Southwell, R. Eyland, M. Cooper, J. Conroy, & K. Collis (Eds.), *Proceedings of the Eighth International Conference for the Psychology of Mathematics Education* (pp. 261-267). Sydney, Australia.

Davis, R.B. (1984). *Learning mathematics: The cognitive science approach to mathematics education.* Norwood, NJ: Ablex.

Fein, G.G., Campbell, P.F., & Schwartz, S.S. (1984). *Microcomputers in the preschool: Effects on cognitive and social behavior.* Unpublished manuscript.

Gentner, D., & Stevens, A.L. (1983). *Mental models.* Hillsdale, NJ: Erlbaum.

Goldin, G.A. (1983). Levels of language in mathematical problem solving. In J.C. Bergeron & N. Herscovics (Eds.), *Proceedings of the Fifth Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Montreal, Canada.

Metzler, W., Beau, W., & Überla, A. (1985). Anschaulichkeit bei der Modellierung und Simulation dynamischer Systeme. In H. Kautschitsch & W. Metzler (Eds.), *Mathematische modelle und visualisierungen.* Schriftenreihe Didaktik der Mathematik, Universität für Bildungswissenschaften in Klagenfurt. Wien: Hölder, Pichler u. Temtsky; Stuttgart: Teubner.

National Council of Educational Research and Training (1984). *CLASS-Computer Literacy and Studies in Schools* (Report of the National Workshop on Computer Literacy Curriculum held on 26–27 March 1984, New Delhi). National Council of Educational Research and Training (NCERT), New Delhi, India.

Papert, S. (1980). *Mindstorms: Children, computers and powerful ideas.* New York: Basic Books.

Peitgen, H.O., & Richter, P.H. (1984). *Morphologie komplexer grenzen—Bilder aus der theorie dynamischer Systeme.* Bremen, West Germany: Universität Bremen.

Schoenfeld, A.H. (1983). *Problem solving in the mathematics curriculum.* Washington, DC: Mathematical Association of America.

Sheingold, K. (in press). The microcomputer as a symbolic medium. In P.F. Campbell & G.G. Fein (Eds.), *Young children and microcomputers.* Reston, VA: NCTM.

Silver, B. (1984). *Using meta-level inference to constrain search and to learn strategies in equation solving.* Unpublished doctoral dissertation, Department of Artificial Intelligence, University of Edinburgh.

Snow, R.E. (1984). Human and artificial intelligence: Some English–Dutch contrasts. *European Scientific News, 38*(2), 63–66.

Wachsmuth, I. (1985, April). *Logical analysis of cognitive organizational structures—The LAKOS project.* Paper presented at the Annual Meeting of the American Educational Research Association, Chicago, IL.

Wheatley, G. (1984). The importance of beliefs and expectations in the problem solving performance of sixth grade pupils. *Proceedings of the Sixth Annual Meeting of the North-American Chapter of the International Group for the Psychology of Mathematics Education.* Madison, WI.

Wheatley, G., & Wheatley, C. (1982). *Calculator use and problem solving strategies of grade six pupils* (Final Report). National Science Foundation. (ERIC Document Reproduction Service No. ED 219 250)

Wright, J.L., & Samaras, A.S. (in press). Play and Mastery. In P.F. Campbell & G.G. Fein (Eds.), *Young children and micro-computers.* Reston, VA: NCTM.

Wynands, A. (1984). Rechenfertigkeit und taschenrechner—ergebnisse einer dreijährigen untersuchung in den klassen 7–9. *Journal für Mathematik-Didaktik, 5,* 3–32.

Yokochi, K. (1983). Educational meaning of personal computers—Around some development of instructional software. *Bulletin for Mathematics Education Study* (Mathematics Education Society of Japan), *24,* 35–51.

Yokochi, K., & Machida, S. (1982). *Personal computer and program—Calculator for mathematics education.* Tokyo: Gyosei.