Communications

Research Problems in Mathematics Education — III

More responses to an enquiry. Previous selections appeared in Volume 4, Numbers 1 and 2.

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1. The problem of dealing with mathematical symbolism appears to be a continuing obstacle to the access of "higher", formal mathematics for many students. A reason for this might be that many students never really learn to read mathematical symbol strings with comprehension and that their skills in reading written language with comprehension do not necessarily transfer to skills in reading mathematical symbol strings with comprehension. First, mathematical symbols are generally not written records of the sound of spoken language and thus cannot evoke meaning through the auditory system. Second, the construction of mathematical symbol strings often involves organizational principles different from those of natural language. "Parsing" of a symbol string into meaningful subunits for comprehension requires certain knowledge about the organizational principles.

Question: Can an instruction in reading mathematical symbolism with comprehension improve students' success with formal mathematics?

2. In a discussion of scientific literacy, Hawkins identified a class of key concepts he called "critical barriers": seemingly elementary concepts that may be exceedingly unobvious and difficult for those who have not yet assimilated them, but that are essential for further science learning (e.g., mirror vision, size and scale, elementary mechanics). "Critical barriers" seem to exists in mathematics as well. Understanding certain key ideas, such as ratio/proportion, linear order, equivalence, variable, and limit, may be essential for continued mathematics learning. Research is needed to identify such concepts, to characterize students' difficulties with them, and to design effective instruction for them.

3. In educational research, we assess successful and unsuccessful students in order to find out about characteristics that give rise to or impede learning success. On the other hand, the part of the teacher is commonly considered as an important variable in the mathematics classroom which can even outweight the influence of carefully designed instructional materials.

Question: Can an assessment of successful/unsuccessful *teachers* (e.g., their skills and habits, major instructional paradigms, metaphors, etc.) contribute to instructional improvement, perhaps tackle the so-called teacher variable? (Individual differences could exist, different instructional approaches could be relevant for different teachers,...)

4. What are the characteristics of a good mathematics teacher?

5. There has always been considerable interest both in trying to determine the characteristics of a "good" teacher and in identifying effective *non-human* instruction (e.g., games, textbooks, computer assisted instruction, films). What are the characteristics of "good" non-human instruction?

6. What ways are there for measuring the effects of the instruction provided in mathematics? In particular, how can the processes that children learn for doing mathematics be attributed to the particular instruction provided? How can the affective notions of mathematics be attributed to particular aspects of instruction? How could one measure the involvement of students in the mathematics?

7. The reductionistic approach of mathematics seeks to construct chains of definitions in which each new thing depends only on other things that have been previously defined, and to construct mathematical knowledge in that "logical" fashion. Current school curricula, especially at the secondary level, are more or less structured in accordance with such an approach of mathematics. On the other hand, while many (most?) children seem to enjoy mathematics in grade school, it seems to be the case that many children come to dislike mathematics under the exposure to the secondary curriculum.

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Question: Can a totally different approach to secondary mathematics teaching with increased emphasis on rich connections of mathematics to "imaginable" fields, like arts and music, physics (e.g., crystals), and decreased emphasis on reductionism be found that

(a) provides adequate mathematical instruction,

(b) attracts and emotionally involves students in mathematics,

(c) provides rich imagery components as a basis for a later reductionistic approach to mathematics for those who decide to go into mathematics or science?

8. The recent availability of microcomputers in schools has generated considerable interest in the uses of machines and other technologies in teaching mathematics. As these technologies develop, it will be possible to construct special-purpose devices for teaching specific skills or concepts. The availability of these devices may alter the environment in which children can learn the standard skills and concepts now taught in the schools.

Question: Which skills and concepts in mathematics would appropriately be taught by such devices?

9. The advent of computer technology raises concerns about what constitutes mathematics itself. For example, there is apparently now a program for the Apple which does algebraic "computations" such as solving linear or quadratic equations. The famous four-color problem was solved with the assistance of a computer.

Questions:

a. How will the definition of "computation" change to accommodate this new type of mechanical computability?

b. How will the nature of proof change with the use of computer and information technology?

c. Will we need to communicate a different kind of proof technique as part of the high school curriculum?

10. The availability of microcomputers and calculators has important implications for mathematics curricula on all levels. At the present time, in many texts there are supplementary activities which require micros or calculators. There has been no major revamping of the curricula. This revamping is in danger of being done in a haphazard manner, unless there is a coordinated effort by mathematics educators to undertake revision of mathematics curricula.

Question: How do we proceed in revamping mathematics curricula in the light of the availability of microcomputers?

11. What cognitive differences influence mathematics ability? What learning style differences affect the learning of mathematics? Is the ATI paradigm the best way to find these differences?

12. Through methods courses, in-service programs, and professional journals teachers are shown ways to teach mathematics for understanding (e.g., how to use popsicle sticks and multibase arithmetic blocks to teach place value). Yet, few of these methods appear to be used in elementary classrooms. Instead, teachers tend to teach mathematics the way they were taught–as an abstract set of rules. It has been reported that even teachers who helped created innovative curricula revert back to traditional methods in the everyday classroom.

Question: How can we get teachers to deviate from their prototypes of mathematics instruction and to implement new methods and curricula?

12b. (In the context of Problem 12) Would it be promising to implement instructional findings in teaching programs (that cannot revert back to old habits)?

13. Given the pool of interested teacher education candidates that make their way through teacher training programs in mathematics education, what adjustments might be made in such programs and/or in the certification procedures to fully develop and utilize their talents with assurance of professional level competence in the fundamental areas of ***communication*** skills, ***mathematical knowledge,*** and ***class-room management*** skills?

14. Current developmental and learning theories support the realities of readiness, partial understanding, and forgetting/decay of knowledge and skills, and call into question lock-step, logical, and technical aspects of the mathematics curriculum. What alternative presentation models might be developed compatible with these theories to teach the key ideas of mathematics to a wider range of students in a way that lays a firm, meaningful foundation for application and further technical and theoretical learning?

15. How is mathematics learning affected by what children think mathematics *is?* If children believe that mathematics is a collection of rules, for example, then is their learning influenced by their search for rules to memorize and attempt to apply?

16. What changes in children's ability to deal with ***proportion*** can be expected if decimals are taught (in the context of calculators) before fractions have been taught?

17. The *"frame"*-paradigm entertained in cognitive science seems to be a very useful concept in explaining phenomena encountered in the learning of mathematics; for example, it provides an explanation for the lack of interconnectedness of areas of mathematical knowledge that is often found in students, or for the phenomenon of "backsliding", that is, reverting to faulty procedures that were acquired in the process of learning but were already "replaced" by correct ones. These examples give rise to the general *Question*: What paradigms of cognitive research seem to be fruitful for adoption in mathematics education?

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Here are some pieces from my bottom drawer which bear on the question you ask.

How can perpetual reconstrueing be fostered and sustained?

The standard paradigm for mathematical instruction is

> Exposition: telling people what is true and, occasionally, what is false.