WHAT KIND OF ORGANIZATION MAKES STRUCTURAL KNOWLEDGE EASILY PROCESSIBLE?

Ipke Wachsmuth, Universität Osnabrück (R.F.A.)

La construction des structures cognitives demande l'action, c'est le point commun des diverses théories sur l'acquisition des connaissances. Ce principe - autant que les méthodes de l'enseignement des mathématiques qui en dérivent - n'inclut pas a priori, que les connaissances qui s'apprennent de cette manière, soient utiles pour produire des processus cognitifs; cette à dire qu'elles seront capables de créer des activités pour appliquer les connaissances et à contribuer pour apprendre le savoir ultérieur. Un point essentiel de cela est, qu'on a besoin d'une suite d'événements bien dirigés, qui se mettent en scène par des actions consécutives d'un "agent". Nous allons discuter la question, quels principes d'organisation des connaissances pourraient favoriser la construction des structures cognitives orientées vers l'action.

1. STRUCTURE AND PROCESS

All human perception depends on certain grounds of shape, space and time, which are the very first formal principles of all phenomena, as was pointed out by Kant (Kant, 1975). Space and time are the schemata and conditions of cognition, not originating from but presupposed by our senses. Nevertheless, they are subject to the logical inferences of the intellect which takes account of space when considering objects and of time when considering states. We use space to express (statically) the interrelations between things, thus yielding *structure*, and we use time to express (dynamically) states and alteration, by this yielding *process*; and all kinds of mathematical considerations can be expressed in terms of structure and process. (What is called a category, for instance, is given by a set of objects together with a set of morphisms.)

Again, we find these principles of structure and process in various models of knowledge representation: To express interdependancies between contents of knowledge which in some respect refer to the same certain issue, we use the notion of *schema* which can be considered as to arrange certain sets of concepts within the "mental space" of all concepts already formed. So, when speaking of "knowledge" in the following, we shall always refer to *conceptual* structures, as *schemata* in the sense of (Skemp, 1979) and, in a respect somewhat different (see below), as *memory schemata* in the sense of (Bobrow/Norman, 1975), and to what is called the *information processing* level or likewise symbol processing level, where meanings become attached to the mental entities we process: It is the symbolic representation which enables man to deal with a manifold of phenomena without having to reiterate them in their totality, and it is the schema which is assumed, by many authors, to be the primary organizing unit of meaning and processing of information.

But, according to our view of structural and process-like aspects herein, we find two different types of "schemata" to be distinguished: a "relational" and an "operational" type; the first one, by use of descriptions, stating relations between the concepts involved, and thus giving rise to *understanding*. The second "active" type evolves orientation from descriptions ("structural knowledge") to process; e.g. in assimilating realities or conceptual interdependancies *to bring about understanding*, or in reorganizing structural knowledge to *integrate* new situations and experiences (accommodation), or to *produce action* by directory of the disposable knowledge: It needs process to make structural knowledge effective.

This second type of a schema would, for example, be called "schème" by Piaget (vs. "schéma" in case of type-1) or "operative schema" (vs. "figurative schema") by Inhelder; Bartlett would rather speak of "active, developing pattern" or "organized settings", Furth would speak of "operative plans", Lindsay, Norman, and Rumelhart of "action schema", and Neisser (who refers to the first type as "stored plans of actions" in the sense of Miller, Galanter, and Pribram) speaks of "stored plans *for* action which direct their execution". (All references see Kluwe, 1979, pp.20-23; see also Skemp, 1979, p.219.) Bobrow and Norman conceive schemata to be "active processing elements" which can become active if requested (Bobrow/Norman 1975, p.132); and what in (Skemp, 1979) is called a "director system" seems to be related to this second type of a schema.

All schema models of the first type commonly serve as structural representation units which organize knowledge and are *subject to accommodation* (or restructuring), while schemata of the second type are involved in process: they are models for "instances" *which* e.g. *accommodate* represented knowledge, and *which direct action*.

2. PROCESS *vs.* ACTION

A most important difference between considering mathematical processes, as far as mathematics itself is concerned on the one hand, and from the mathematician's or student's viewpoint on the other, lies in the fact that what is process in the first case appears as *action* of a person (an "agent") *evolving* the process, by a sequence of acts. And while,in difference to structure, process is coupled with a *direction* (of proceeding), is action coupled with *intention* (whatfor to proceed); while mathematical process relates to logical rules, does action submit to psycho-logical influences: The agent has to make decisions what intermediate goals are to run up to, and what could be "means" to reach "ends". And as a basis, and motive, for his decisions he uses what we have been calling type-1 resp. type-2 schemata.

But what is the origin of such knowledge structures, and how do they develop? Some of the theories dealing with the acquisition of knowledge agree in the point that to build up cognitive structures action is required ("to comprehend is to operate"), including mental action by symbolic operation, and basing on the mechanisms of what Piaget has called empirical and reflective abstraction (e.g. Piaget, 1975, pp.87-89). However, this principle, as well as derived methods for mathematics education, would it include the fact that the structural knowledge acquired from this is process-oriented, i.e. is able to produce actions which yield application and contribute to further acquisition of knowledge? Does comprehension already provide adequate action schemata?

Resulting from reflective abstraction, a conceptual schema is not of a special style as a sensori-motor schema normally is (e.g. typewriting schemata would hardly serve as, or be extendable to, practicable schemata for playing piano). But instead, one of its most important features is it to be *general*, and, in fact, very often it is the degree of generality of a conceptual schema that becomes extended in the process of learning mathematics. Action schemata must be general to apply to a broad range of situations, which is a requirement of *economy*, and they must be general to be applicable to situations not having appeared during the process of their abstraction, which is a basis for *transfer*. So, what can be done to pursue these principles of generality in comprehension during processes of learning mathematics?

The following two examples are chosen to elucidate the role of general action schemata not being focussed to special context, as well as of psychological concerns influencing decisions when knowledge is put into process (as an agent proceeds).

3. TWO EXAMPLES

1) Having learnt some relation between the side lengths of a right triangle, say, $a^2 + b^2 = c^2$, certainly a student will soon be able to determine b, if for example a and c are given, in generating process by use of an action schema like

first: isolate the unknown
then: insert known information

which, in such a general shape, usually was acquired earlier and perhaps has to be extended to the new context of quadratic equations.

2) But what about someone having recently learnt how to integrate certain classes of real functions, including the rule for partial integrating, and is given the task

$$\int \cos^2 x dx = ?$$

Analyzing this task, which soon turns out to be a problem, could give us some insight, so let us see. We'd try "to integrate" (wouldn't you?) in the *first approach:*

$$\int \cos^2 x dx = \int \overset{f'}{\cos x} \, \overset{g}{\cos x} \, dx$$

$$= [\overset{f}{\sin x} \, \overset{g}{\cos x}] + \int \sin x \sin x \, dx$$

$$= [\sin x \cos x] + [-\cos x \sin x] + \int \cos^2 x dx$$

which carries back solution of the task to the solution of the task. Problem! Remembering (from some part of structural knowledge) that $\sin^2 x = 1 - \cos^2 x$, we would probably come to the *second approach:*

$$\ldots = [\sin x \cos x] + \int (1 - \cos^2 x) dx$$

$$= [\sin x \cos x] + \int 1 dx - \int \cos^2 x dx$$

which again seems to yield a problem of self-reference. So, if we still follow the command (INTEGRATE!) of the symbol "$\int$" we are getting into a deep-end. What to do? - Now let us see how general our action schemata are! Do we remember that for example $\sqrt{t}$ could signalize: TRY TO RADICATE! or could indicate: TAKE IT AS AN OBJECT!; i.e. that the symbol "$\sqrt{\ }$" is of a twofold psychological nature, showing *command for proceeding*, as well as *declaring an object* (a number)? Then we would possibly be able to transfer this knowledge (which is processible in more than one respect) to the recently introduced symbol "$\int$", and switch from transformation of terms to equivalent-transformation in the *third approach:*

$$\int \cos^2 x dx = [\sin x \cos x + x] - \int \cos^2 x dx$$

$$\Leftrightarrow \quad 2\int \cos^2 x dx = [\sin x \cos x + x]$$

$$\Leftrightarrow \quad \int \cos^2 x dx = \tfrac{1}{2}[\sin x \cos x + x]$$

and there we are!

4. ACTION-FAVORING ORGANIZATION OF KNOWLEDGE

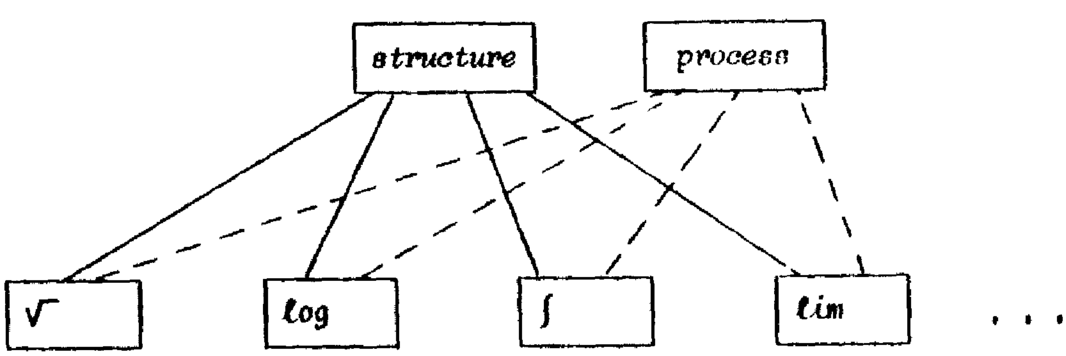
What does it mean to induce process from structural knowledge? At first, it could mean that, determined from a given task, a certain "goal state" is given to be reached, starting from some "present state", in a sequence of intermediate states, a "path" (Skemp, 1979, p.168). We could likewise say that a sequentialized, directed grouping of events, carrying on from state to state, is needed; in this making up the process which an agent has to evolve (see above), in a certain sequence of acts.

Not in every case, however, is a goal state actually known from the task formation: If the task is, say, to prove a certain assertion (and supposed it is valid), one has to find a sequence of inferences, the application of which being the events figuring the process. But, for (counter-)example, in the task above of integrating $\cos^2 x$, the goal state has to be found *during the process* (which is not merely a straight-forward computation determined by an algorithm as e.g. for x^2). So in this case, one has to use "means", not knowing to what actual ends! To evolve process, here, does *not* mean to an agent: to compare a goal state with some present state, make a plan, and figure out action, but "merely": to do *something* to perform the task. Hence, to progress to an unknown goal state, it needs various knowledge how to put knowledge into process at all, *how to move from* a position (present state) rather than how to reach a final order (goal state).

So what kind of organization of an agent's knowledge could make it easily processible? As a *first organizational principle*, we suggest to establish *twin* representations of mathematical concepts in a schema, using descriptions showing process as well as structure (e.g. "$\sqrt{2}$ stands for a process: to seak for a non-negative number which, being multiplied by itself, gives 2" *and* "$\sqrt{2}$ declares an object: *it is* the non-negative number which, being multiplied by itself, gives 2" etc.). In paragraph 3. we saw that within the same task, "$\int$" calls process where $\cos^2 x$ is the operand, as well as, a few steps later, it declares an object to be operated on as a whole, and which could be envisioned to be the result of the above process. Very similar situations would in fact appear upon $\sqrt{\ }$, $\cap$, *log*, *lim*, μ (from μ-recursion), etc., and lead to similar decisions, though occuring in quite different mathematical contexts.

As a *second organizational principle* we thus suggest to arrange *laterally* such special-context schemata, which are consecutively acquired in the course of mathematical instruction; i.e. to evolve *links* between relational descriptions being of similar shape, independent of context. (In the cases above, it is the context, not the functional aspects of symbols, which really differs.) Such links can be revealed by comparing discussion, and by reflective mental activity, yielding insight into general action structures and thus give rise to *general descriptions* for action schemata as postulated in the second paragraph. And this principle can immideatety be used in further acquisition of knowledge. (When teaching mathematics at school, in following this line, I once used as a key sentence for the formation of some lessons: "In what respect is *log* for *exp* the same as is $\sqrt{\ }$ for the power?")

In the whole proceeding, keeping aware of structure and of process should yield links to a higher-order view of the potentials involved, leading to a schema as sketched below, and allowing economical representation of knowledge, and transfer, by identifying analogies and similarities, which is a process of *abstraction* (from special-context schemata), and of *re-concretization* (in a particular special-context schema).



References:

Bobrow, D.G. and Norman, D.A.: Some Principles of Memory Schemata, in: Representation and Understanding, ed. by D.G. Bobrow and A. Collins, New York/London Academic Press, 1975, pp.131-149

Kant, I.: De mundi sensibilis atque intelligibilis forma et principiis, Schriften zur Metaphysik und Logik Band III, ed. by W. Weischedel, Darmstadt 1975

Kluwe, R.: Wissen und Denken, Stuttgart Kohlhammer, 1979

Piaget, J.: Biologische Anpassung und Psychologie der Intelligenz, Stuttgart Klett, 1975

Skemp, R.R.: Intelligence, Learning, and Action, Chichester Wiley, 1979