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Resonance production and the approach to Feynman scaling

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In the framework of the uncorrelated jet model, we show that multiparticle production through intermediate resonances leads to late Feynman scaling in the central region. Our results are found to agree well with high-energy data on the "rise of the plateau" and on pion multiplicity behavior.

I. INTRODUCTION

Many dynamical descriptions of multihadron production predict or conjecture energy-independent normalized single-particle distributions: the spectrum

$$F\left(s;p_T,y=\tfrac{1}{2}\ln\frac{p_0+p_L}{p_0-p_L}\right)=\frac{1}{\sigma_{\text{inel}}}\frac{d\sigma}{dy\,dp_T^{\,2}} \quad (1)$$

should for $s\to\infty$ approach a scale-invariant limit ("Feynman scaling")

$$\lim_{s\to\infty} F(s;p_T,y)\simeq F(p_T,y)\,, \quad (2)$$

which, moreover, is generally independent of y ("plateau"),

$$F(p_T,y)\simeq F(p_T,0)\,, \quad (3)$$

provided $y\ll Y=\frac{1}{2}\ln s$. Here s denotes, as usual, the squared incident energy in the center-of-mass system (c.m.s.), p_T and p_L denote the transverse and longitudinal momenta of the secondary, respectively, with $m^2=p_0^{\,2}-p_L^{\,2}-p_T^{\,2}$.

Empirically, deviations from such scaling behavior have been observed in the central region ($y\approx 0$) up to the highest energies,[1-4] and for pion production already the gradient in that region ("rise from below")

$$\frac{\partial F}{\partial s}(y\approx 0)>0 \quad (4)$$

has led to difficulties for some models.[5] If we nevertheless assume relation (2) to become asymptotically fulfilled for all y we must account for the "late" Feynman scaling in the central region—in contrast to the fragmentation region $y=Y-\text{const}$, where both rate of approach and gradient ($\partial F/\partial s<0$) seem at least qualitatively understood.[6]

Scale-invariant spectra arise because an *asymptotic* multihadron system becomes essentially a one-dimensional free gas in rapidity space ("Feynman-Wilson gas"). A delay of asymptotic behavior can occur either if a significant fraction of the incident energy does not go into production ("leading-particle effect[7]") or if production proceeds through massive intermediate systems, which provide a new energy scale. It has recently become evident that the bulk of pion production takes place through intermediate resonances[8-10] ($\rho,\omega,\eta,\cdots$); hence a change in energy scale indeed takes place and should be taken into account. We want to investigate here the size and relative role of the two mechanisms mentioned.

For definiteness, we shall consider the reaction

$$pp\to\pi+\text{anything} \quad (5)$$

in the framework of the uncorrelated-jet model.[11,12] The final state of reaction (5) contains two fairly distinct components (cf. Fig. 1): a mostly pionic multihadron state and two leading baryons, which carry away about half the incident c.m.s. energy and which are sharply aligned along the beam. Meunier[7] has studied the nonasymptotic terms which arise from the leading particle effect alone—assuming the pionic system to be asymptotic. The uncorrelated-jet model (UJM) provides a rather good overall description of multihadron production, yielding (in common with Regge, dual, bremsstrahlung, and parton considerations) asymptotic scaling and logarithmic multiplicity growth. By describing multipion production in terms of the UJM, we can thus also calculate deviations from asymptotic behavior due to the central pionic component—in addition to the proton vertices. Since the convergence to scaling in the UJM is known to be slow,[13] such effects cannot be neglected *a priori*. The UJM can, moreover, be easily general-

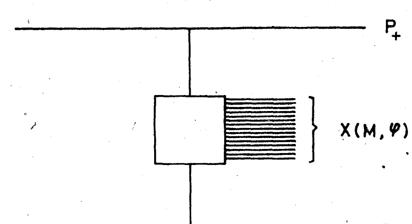

FIG. 1. Schematic pion production and definition of kinematical variables.

ized to the production of resonances, which subsequently decay isotropically into pions. We can thus compare the relative effect of three distinct mechanisms:

(i) the role of the leading protons,[7]

(ii) the nonasymptotic terms from the pion component, and

(iii) the nonasymptotic effects of intermediate resonance production.

In Sec. II, we shall study direct pion production in the UJM with leading protons, comparing our results with data for spectra and multiplicities; in Sec. III we shall do the same for pion production through intermediate resonances.

II. THE JET MODEL FOR PIONS

The kinematical variables of the process shown in Fig. 1 will be denoted as follows in the c.m.s.: P_+ and P_- are the momenta of the leading nucleons, $x_\pm = 2P_{L\pm}/\sqrt{s}$ are their associated Feynman variables, M and φ are the mass and rapidity of the produced multihadron state X, and y is the rapidity of a single pion.

Using the assumption that the leading particles have no transverse momentum, energy-momentum conservation can be written as

$$P_{0+} + P_{0-} + M\cosh\varphi = \sqrt{s}\,, \qquad (6)$$

$$(x_+ - x_-)\tfrac{1}{2}\sqrt{s} + M\sinh\varphi = 0\,. \qquad (7)$$

From Eqs. (6) and (7) one obtains

$$x_\pm = \mp\frac{M}{\sqrt{s}}\sinh\varphi + \left(1 - \frac{M}{\sqrt{s}}\cosh\varphi\right)\left(1 - \frac{4m_p^2}{sa}\right)^{1/2}, \qquad (8)$$

where m_p is the proton mass and

$$a = 1 - \frac{2M}{\sqrt{s}}\cosh\varphi + \frac{M^2}{s}\,. \qquad (9)$$

In the following we assume that the multihadron state X depends only on its invariant mass M and that it consists of pions only. As a consequence, the one-particle inclusive y distribution for pions can be calculated from the corresponding normalized distribution in the center of mass of the pion system $(1/\sigma_{X\,\mathrm{TOT}})d\sigma_X/dy^*$ and the x distribution of the leading protons

$$\frac{d\sigma}{dy} = \int \frac{d\sigma}{dx_+dx_-}\frac{1}{\sigma_{X\,\mathrm{TOT}}}\frac{d\sigma_X}{dy^*}(M, y^*)dx_+dx_-\,, \qquad (10)$$

where $y^* = y - \varphi$ is the pion rapidity in the center of mass of the pion system. It is convenient to change the integration variables from x_+, x_- to M, φ:

$$dx_+dx_- = \frac{2M}{s(1 - 4m_p^2/sa)^{1/2}}\left(1 + \frac{4m_p^2M^2}{s^2a^2}\sinh^2\varphi\right)dMd\varphi\,. \qquad (11)$$

The ranges of M and φ are then

$$2m_\pi \le M \le \sqrt{s} - 2m_p\,, \qquad (12)$$

$$|\sinh\varphi| \le \frac{\sqrt{s}}{2M}\frac{1 - M^2/s}{1 - m_p/\sqrt{s}}\left(1 - \frac{4m_p}{\sqrt{s}}\frac{1 - m_p/\sqrt{s}}{1 - M^2/s}\right)^{1/2}. \qquad (13)$$

As explained in the Introduction, we calculate the pion distributions in the center of mass of the pion system X from an uncorrelated-jet model. In that model, the fully exclusive distribution for the production of N pions of four-momentum p_i is given by

$$\Gamma_N \sim \frac{\nu^N}{N!}\delta^{(4)}\left(\sum_{j=1}^N p_j - q\right)\prod_{i=1}^N \frac{d^3p_i}{2p_{i0}}f(p_{iT})\,, \qquad (14)$$

where q is the total four-momentum of the system of pions, $q^2 = M^2$. For simplicity we have assumed chargeless pions. The function $f(p_T)$ describes the transverse-momentum cutoff and is normalized as follows:

$$\pi\int_0^\infty f(p_T)p_Tdp_T = 1\,. \qquad (15)$$

The phase-space volume is then given by

$$\Omega(q) = \sum_{N=2}^\infty \frac{\nu^N}{N!}\int\prod_{i=1}^N \frac{d^3p_i}{2p_{i0}}f(p_{iT})\,\delta^{(4)}\left(\sum_{j=1}^N p_j - q\right) = \sum_{N=2}^\infty \frac{\nu^N}{N!}\Omega_N(q)\,, \qquad (16)$$

and the normalized single-particle distribution in the c.m. system of X is

$$\frac{2p_0}{\sigma_{X\,\mathrm{TOT}}}\frac{d^3\sigma_X}{d^3p} = \nu f(p_T)\frac{\Omega(Q - p)}{\Omega(Q)}\,, \qquad (17)$$

with $Q = (M, \vec{0})$. The asymptotic behavior of the average multiplicity in the UJM is found to be

$$\bar{N} \xrightarrow[M\to\infty]{} \nu\ln M^2\,. \qquad (18)$$

To obtain the asymptotic form of the inclusive y distribution one has to integrate the asymptotic expression for the single-particle distribution

$$\frac{2p_0}{\sigma_{X\,\mathrm{TOT}}}\frac{d^3\sigma_X}{d^3p} \approx \nu f(p_T)(1 - x_0)^{\nu-1}\,, \qquad (19)$$

where $x_0 = 2p_0/M = 2m_T\cosh y^*/M$, and m_T is the transverse mass $m_T = (m^2 + p_T^2)^{1/2}$. Since, how-

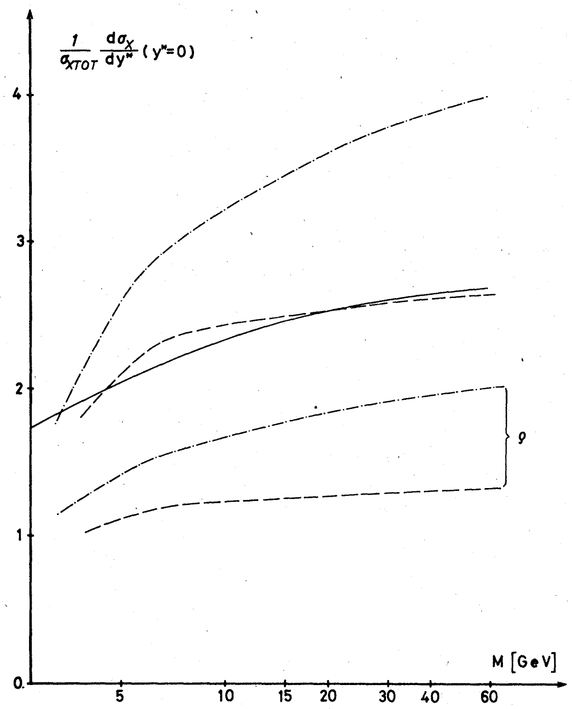

FIG. 2. Normalized y distributions in the system X at $y^*=0$ as a function of the invariant mass M of X. The solid line is the result of the pionic UJM ($\nu_\pi = 2.942$), the dashed (dashed-dotted) lines of the pion-production model with intermediate ρ resonances and $\bar{\nu}_\rho = 1.471$ (2.25). The curves marked ρ show the corresponding distributions for the ρ resonances directly.

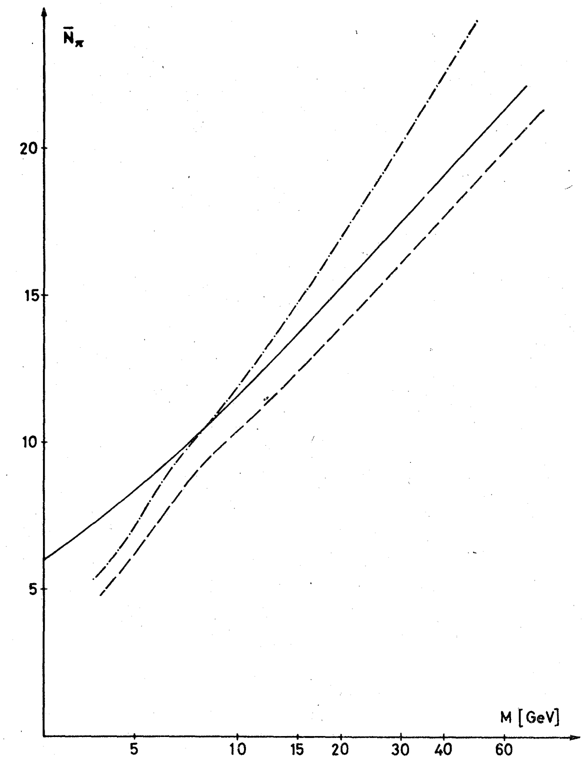

FIG. 3. Total average multiplicity $\bar{N}_\pi$ in the system X as a function of the invariant mass of X. Notation and parameter values are the same as in Fig. 2.

ever, in Eq. (10) the y distribution is also needed at moderate and low energies, we have used the Fourier-transform method, Ref. 14, to compute the phase-space volumes in the UJM.

The transverse-momentum cutoff for the pions was taken to be

$$f_\pi(p_T) = \frac{\lambda^2}{\pi}\exp[-\lambda p_T]\,, \tag{20}$$

where the size of the parameter $\lambda = 6.2\ \text{GeV}^{-1}$ is obtained from a fit to the 90° pion spectra at the CERN ISR.[15] As in Ref. 13, we fixed the value of $\nu = \nu_\pi$ at about 3, as suggested by the measured asymptotic behavior of the average charged multiplicity at the ISR.[16] In Fig. 2 we show the result for $(1/\sigma_{X\,\text{TOT}})d\sigma_X/dy^*$ at $y^*=0$, and in Fig. 3 we show the total average multiplicity $\bar{N}_\pi$, both as a function of the energy M of the pion system. Clearly, in the region of interest, $M \lesssim 60$ GeV, we have a rising central plateau already for the pion system alone. To evaluate the convolution, Eq. (10), a realistic leading proton spectrum is needed. We follow Meunier[7] and choose a di-triple-Regge parametrization

$$\frac{d\sigma}{dx_+dx_-} = [R(x_+) + P(x_+)][R(x_-) + P(x_-)]\,, \tag{21}$$

where

$$R(x) = \frac{A\exp[b_R(x)t_{\min}]}{b_R(x)}(1-x)^{1-2\alpha_0}\,, \tag{22}$$

$$P(x) = \frac{B\exp[b_P(x)t_{\min}]}{b_P(x)}(1-x)^{-1}\,, \tag{23}$$

$$b_R(x) = [0.5 - 1.5\ln(1-x)]\ \text{GeV}^{-2}\,, \tag{24}$$

$$b_P(x) = [4.5 - 0.4\ln(1-x)]\ \text{GeV}^{-2}\,, \tag{25}$$

$$t_{\min} = -m_p^2(1-x)^2/x\,, \tag{26}$$

$$\alpha_0 = 0.45\,,\quad B = 0.0272\ A\,. \tag{27}$$

As can be seen from Eqs. (22) and (23), this parametrization contains an exponential $t_{\min}$ cutoff, ensuring that the triple-Regge formula is not used outside its region of validity. On integrating over x, Eq. (21) leads to the usual $RRP+PPP$ parametrization of the leading-particle spectrum. We have determined the parameter A from the condition

$$\int \frac{d\sigma}{dx_+dx_-}dx_+dx_- = \sigma_{\text{inel}}\,, \tag{28}$$

where

$$\sigma_{\text{inel}}(\sqrt{s}) = [17.99 \pm 4.43 \ln(\sqrt{s}/\text{GeV})] \text{ mb}, \quad (29)$$

in accord with Ref. 17. Equation (29) implies a 14% rise of σ_{inel} from 23 to 63 GeV.[18]

The results for the ratios

$$R_1(\sqrt{s}) = \frac{d\sigma}{dy}(\sqrt{s}) \Big/ \frac{d\sigma}{dy}(23 \text{ GeV}) \quad (y=0), \quad (30)$$

$$R_2(\sqrt{s}) = \frac{dN}{dy}(\sqrt{s}) \Big/ \frac{dN}{dy}(23 \text{ GeV}) \quad (y=0), \quad (31)$$

where

$$\frac{dN}{dy} = \frac{1}{\sigma_{\text{inel}}} \frac{d\sigma}{dy} \quad (32)$$

as obtained from Eq. (10), with the pion UJM as input, are displayed in Figs. 4(a) and 4(b). For comparison we show the corresponding experimental results obtained by integrating the fit

$$p_0 \frac{d^3\sigma}{d^3p}\Big|_{x=0} = As^{\alpha} \exp(Bm_T) \quad (33)$$

to the π^- data of the British-Scandinavian-MIT Collaboration.[2] As can be seen from Figs. 4(a) and 4(b), the predicted rise of the center of the y distribution is still below the experimental finding, although, as shown in Fig. 5, the result for the average charged multiplicity lies in the expected region. To calculate the charged multiplicity, we have used the formula

$$\bar{N}_{\text{ch}} = \tfrac{2}{3} \bar{N}_{\pi} + 2. \quad (34)$$

We conclude that a purely pionic jet model cannot explain the ~40% rise of the central plateau observed at the ISR in the energy range from 23 to 63 GeV. In the next section we will therefore formulate a jet model for intermediate clusters or resonances, which subsequently decay into pions.

III. THE JET MODEL WITH INTERMEDIATE RESONANCES

A jet model for resonances of mass m_R is built in the same manner as the one for pions just described. In Eqs. (14) to (19), the pion mass has to be replaced by the resonance mass and the parameter ν by ν_R. In accord with the mass dependence of transverse-momentum distributions,[19] we have modified the cutoff function to

$$f_R(p_T) = \frac{\lambda^2}{\pi(\lambda m_R + 1)} \exp[-\lambda(m_{RT} - m_R)], \quad (35)$$

where m_{RT} is the transverse resonance mass. This form describes the experimental low-p_T be-

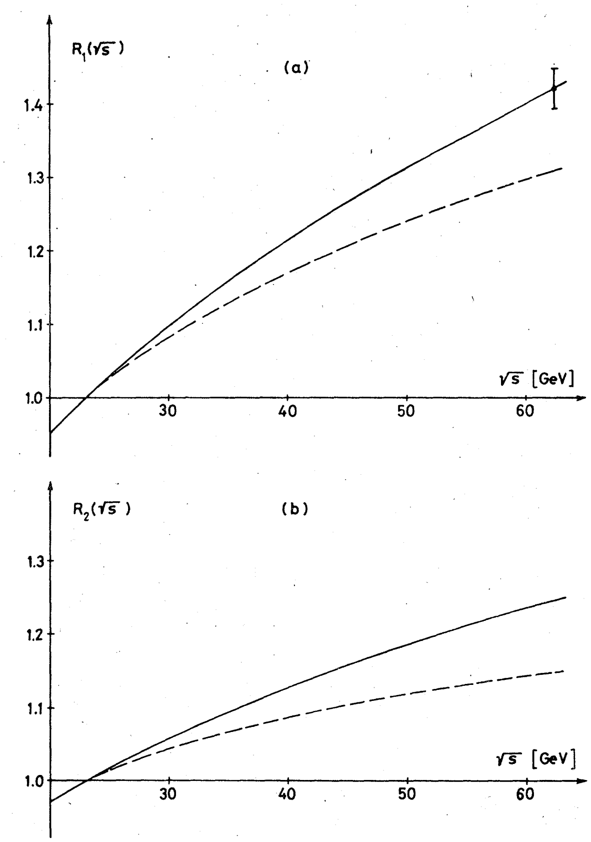

FIG. 4. (a) The ratio $R_1(\sqrt{s})$, for the pionic UJM (dashed line); the solid line shows the experimental result (obtained by integrating the fit for the data of Ref. 2). The point shown illustrates the typical error range. (b) The ratio $R_2(\sqrt{s})$, obtained by multiplying the curves of Fig. 4(a) by $\sigma_{\text{inel}}(23 \text{ GeV})/\sigma_{\text{inel}}(\sqrt{s})$, as calculated from Eq. (29).

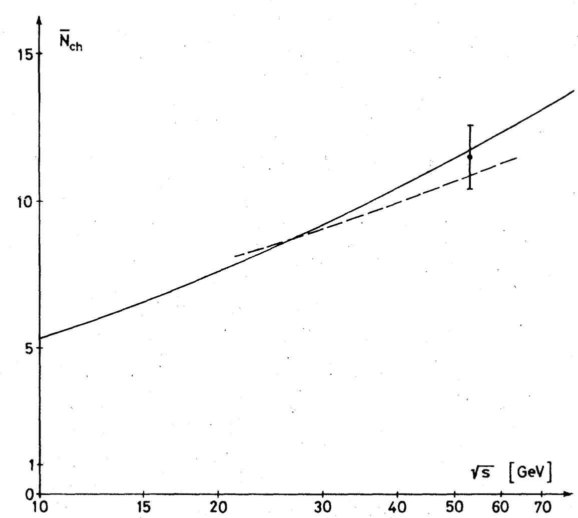

FIG. 5. The mean charged multiplicity $\bar{N}_{\text{ch}}$; the solid line is a fit of data [from Ref. 16; $\bar{N}_{\text{ch}} = 1.17 + 0.30 \ln s + 0.13 (\ln s)^2$, $\sqrt{s} = 3-152$ GeV]. The dashed curve shows the result of the pionic UJM.

havior of the single-particle cross sections from π to J/ψ (see Refs. 19 and 20), with the same value of the parameter λ as before. So there remains just one parameter, namely, ν_R, to be determined. If we want to obtain the same asymptotic number of pions, then we have to choose

$$\nu_R = \nu_\pi / \bar{n}\,, \tag{36}$$

where $\bar{n}$ is the average number of pions into which a resonance decays. As a consequence of the reduced resonance multiplicity, the single-particle distribution for the resonances will be much more asymptotic, i.e., plateaulike, than the corresponding pion distribution. This can be seen immediately from the factor $(1-x_0)^{\nu-1}$ in Eq. (19). If ν_R is less than one, i.e., $\bar{n}>3$, one even gets a dip in the central plateau region of the inclusive single-resonance distribution.

Since present experiments suggest that the bulk of pion production occurs through intermediate vector mesons,[8-10] we shall take the ρ meson as a representative resonance. This also simplifies the decay treatment, because we only have to calculate two-pion decays.

We thus consider in the following an uncorrelated jet model for spinless ρ mesons ($m_\rho = 0.77$ GeV), which decay isotropically into two pions. The normalized single-particle distribution for the pions in the c.m.s. of X is then obtained by folding the corresponding distribution for the ρ's with the distribution of the pions coming from the decay in the c.m.s. of the ρ:

$$\frac{2p_0}{\sigma_{X\,\mathrm{TOT}}}\frac{d^3\sigma_X}{d^3p} = \int \frac{d^3k}{2k_0}\frac{2k_0}{\sigma_{X\,\mathrm{TOT}}}\frac{d^3\sigma_X^\rho}{d^3k}\frac{2p_0'}{\sigma_{\rho\,\mathrm{TOT}}}\frac{d^3\sigma_\rho^\pi}{d^3p'}\,, \tag{37}$$

where p and k are the momenta of the π and the ρ in the X system, respectively, p' is the pion momentum in the ρ rest frame, and

$$m_\rho p_0' = p_0 k_0 - \vec{p}\cdot\vec{k}\,, \tag{38}$$

$$m_\rho \vec{p}\,'\cdot\vec{k} = k_0(\vec{p}\cdot\vec{k}) - p_0\vec{k}^2\,. \tag{39}$$

The isotropic decay distribution is given by

$$\frac{2p_0'}{\sigma_{\rho\,\mathrm{TOT}}}\frac{d^3\sigma_\rho^\pi}{d^3p'} = \frac{2\delta(m_\rho - 2p_0')}{\pi(m_\rho^2/4 - m_\pi^2)^{1/2}}\,, \tag{40}$$

which leads to

$$\frac{2p_0}{\sigma_{X\,\mathrm{TOT}}}\frac{d^3\sigma_X}{d^3p} = \frac{m_\rho}{\pi(m_\rho^2/4 - m_\pi^2)^{1/2}}\int k_T dk_T \int dy_\rho [p_T^2 k_T^2 - (p_0 k_0 - p_L k_L - m_\rho^2/2)^2]^{-1/2}\frac{2k_0}{\sigma_{X\,\mathrm{TOT}}}\frac{d^3\sigma_X^\rho}{d^3k}\,, \tag{41}$$

and finally to

$$\frac{1}{\sigma_{X\,\mathrm{TOT}}}\frac{d\sigma_X}{dy^*} = \frac{\pi m_\rho^2}{(1-4m_\pi^2/m_\rho^2)^{1/2}}\int_0^{k_{TM}} k_T dk_T \int_{-y_{\rho M}}^{y_{\rho M}} dy_\rho \frac{m_{\rho T}\cosh(y^*-y_\rho)}{[m_\rho^2 + m_{\rho T}^2\sinh^2(y^*-y_\rho)]^{3/2}} \times\theta\left[\frac{(m_\rho^4 + 4m_\pi^2 k_T^2)^{1/2}}{2m_{\rho T}m_\pi} - \cosh(y^*-y_\rho)\right]\frac{2k_0}{\sigma_{X\,\mathrm{TOT}}}\frac{d^3\sigma_X^\rho}{d^3k}\,, \tag{42}$$

with

$$k_{TM}^2 = M^2/4 - m_\rho^2\,, \tag{43}$$

$$y_{\rho M} = \left|\mathrm{Arcosh}\frac{M}{2m_\rho}\right|\,, \tag{44}$$

and θ is the usual step function. As a first guess for ν_ρ, we chose $\nu_\rho = \nu_\pi/2 \approx 1.5$. In Fig. 2 we compare the resulting $(1/\sigma_{X\,\mathrm{TOT}})d\sigma_X/dy^*$ ($y^*=0$) and in Fig. 3 the resulting $\bar{N}_\pi$, with the corresponding results for the direct pion UJM. It is clear from our choice of ν_ρ that the $\bar{N}_\pi$ curves have to be parallel asymptotically, which is fulfilled for $M \geq 10$ GeV. The pions coming from the decay will, however, on the average get more energy than those directly produced, since in the ρ rest frame they get always half of the energy of a ρ. This effect leads to a smaller $\bar{N}_\pi$. As a consequence, after folding the result from Eq. (42) for $\nu_\rho = \nu_\pi/2$ with the proton spectrum, the average number of charged particles predicted at ISR energies is smaller. The $\bar{N}_{\mathrm{ch}}$ curve lies even below the experimental data (see Fig. 6), though the predicted rise of the plateau center is the same as for the pionic UJM, which is seen in Fig. 7. One must therefore allow a higher ρ production than assumed in Eq. (36). We have changed ν_ρ to the new value 2.25, which fixes $\bar{N}_{\mathrm{ch}} = 8.24$ at $\sqrt{s} = 23$ GeV. The resulting $\bar{N}_{\mathrm{ch}}$ curve in Fig. 6 is in excellent agreement with the $(\ln s)^2$ parametrization[3,16] of experimental data in the energy range from $\sqrt{s} = 23$ to 63 GeV.

The necessary increase in ν_ρ leads also to a stronger increase of the central plateau, see Fig. 7. This is, of course, due to the stronger rise with s of the y distributions of the ρ mesons.

Finally, we note that not only dN/dy ($y=0$), but

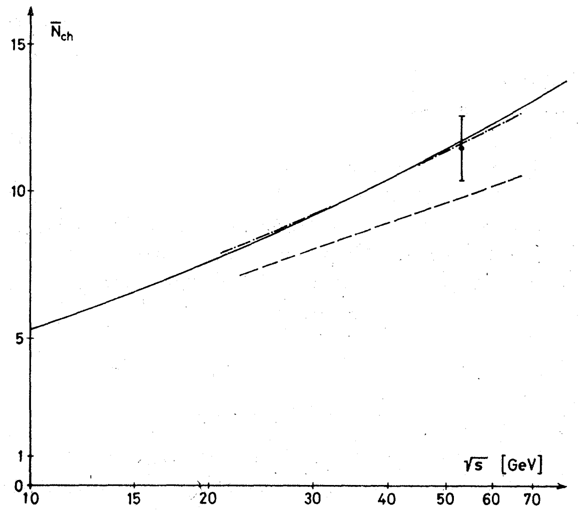

FIG. 6. The mean charged multiplicity $\bar{N}_{ch}$; the solid line is a fit of data (as in Fig. 5), the dashed and dashed-dotted lines are the results of the UJM with intermediate ρ resonances (with $\nu_\rho = 1.471$ and 2.25, respectively).

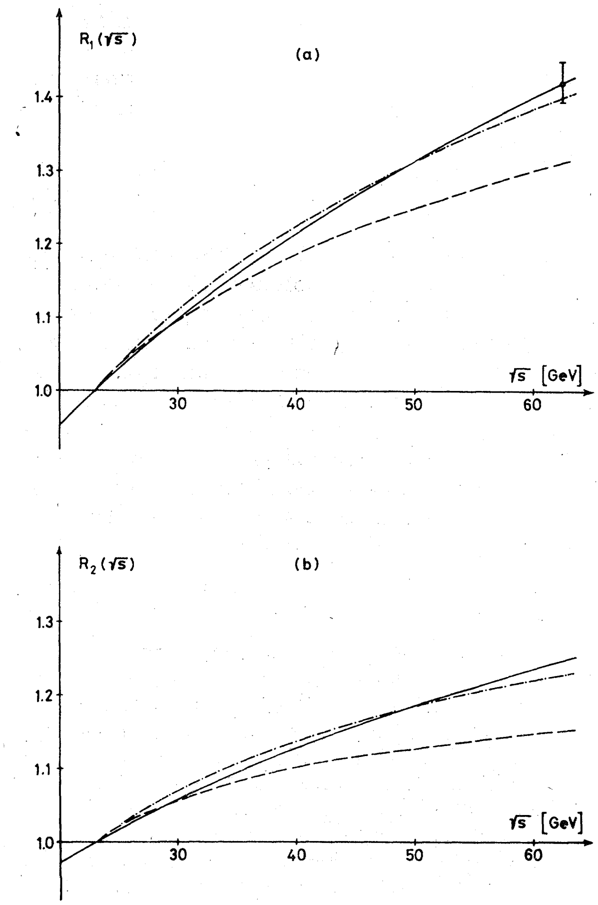

FIG. 7. (a) The ratio $R_1(\sqrt{s})$ for pion production through intermediate resonances. The solid line is a fit of data [as in Fig. 4(a)], the notation for the other curves is the same as in Fig. 6. (b) The ratio $R_2(\sqrt{s})$ for pion production through intermediate ρ resonances. The solid line is the same as in Fig. 4(b), the notation for the other curves is the same as in Fig. 6.

in fact the entire distribution is far from being asymptotic, i.e., plateaulike. In Fig. 8 we show dN/dy together with data[21] for $\sqrt{s} = 19.7$ GeV. It is evident that both exhibit essentially no flat region.

IV. CONCLUSIONS

We have shown that an uncorrelated-jet model including leading protons as well as intermediate resonances can account for the deviations from asymptotic behavior observed in the ISR energy range. Since inclusion of the leading particle effect alone[7] or together with nonasymptotic jet-model corrections for direct pions (Sec. II) cannot

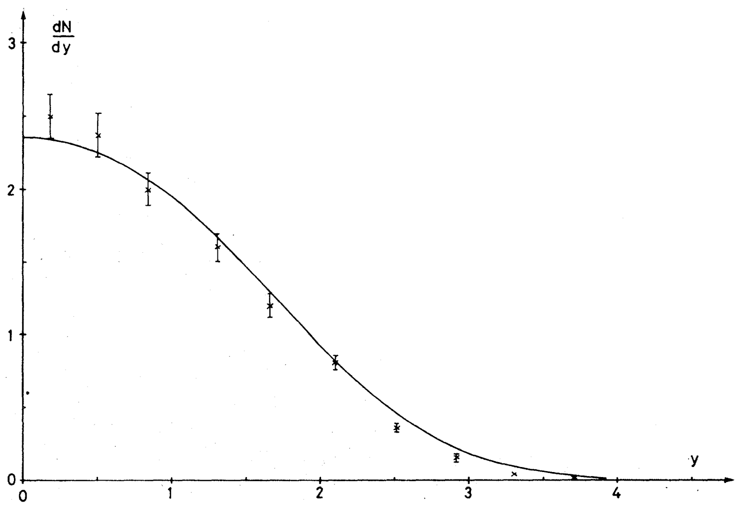

FIG. 8. The normalized y distribution at $\sqrt{s} = 19.7$ GeV calculated from the ρ resonance UJM with $\nu_\rho = 2.25$. The data points are from Ref. 21, for π^- at 19.7 GeV, with an arbitrary normalization.

account for the observed rise of dN/dy $(y=0)$, it is clear that the presence of resonances and the associated shift in energy scale is critical.

It should also be noted that we arrive at our conclusion only if we consider the finite-energy behavior of both dN/dy $(y=0)$ and

$$\bar{N}_\pi(s)=2\int_0^Y \frac{dN}{dy}\,dy \tag{45}$$

for the *observed* secondaries. This is theoretically necessary, since the convolution (10) in general does not provide a constant ratio of pion to resonance multiplicity. It is also experimentally necessary, since the observed deviations from scaling for dN/dy $(y=0)$ are accompanied by observed deviations of $\bar{N}_\pi(s)$ from the asymptotic $\bar{N}_\pi(s)=a\ln s+b$.[22] We have here considered only intermediate ρ mesons, but we do not expect any qualitative changes to arise through the introduction of other resonances, both of higher mass and with more-body decays. The correlation between the approach to scaling and the intermediate resonance or cluster mass will be studied in a subsequent paper.

A very striking experimental feature remains completely open in our present considerations. It is observed[4] that the deviations from scaling for $F(p_T, y)$ are greatest at small p_T. Such a behavior is expected from a jet model with explicit Bose-Einstein statistics[23]; it would certainly be of great interest to know whether intermediate resonances can also provide this effect.

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[22] After completion of this work, we received a paper by A. Białas and A. Kotański, [Acta Phys. Pol. B8, 779 (1977)], who also study pion production through intermediate clusters. They obtain a smaller increase of dN/dy, but they do not attempt to fit simultaneously the observed pion multiplicity $\bar{N}_\pi(s)$ at the highest present energies.
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