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DYNAMICAL AND BOSE-EINSTEIN CORRELATIONS OF CENTRALLY PRODUCED PION PAIRS IN HADRON-HADRON COLLISIONS *

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We calculate dynamical and Bose-Einstein inclusive correlations of pion pairs (as functions of the invariant pair mass $M_{\pi\pi}$) from the decay of independently produced clusters, whose finite size is explicitly taken into account. We show that recent pp bubble chamber data from FNAL on the correlation functions $C_2(M_{\pi\pi})$ for centrally produced $\pi^+\pi^-$ and $\pi^-\pi^-$ pairs can be understood within this picture. Some implications for Hanbury-Brown–Twiss type analyses are discussed.

1. Introduction

Experiments have revealed strong short-range correlations in rapidity among centrally produced particle pairs at energies $E_{\text{lab}} \gtrsim 100$ GeV [1]. These rapidity correlations can be understood as being due to the intermediate production of clusters which subsequently decay into the observed final-state hadrons [2]. In rapidity space the most sensitive observable quantity from which to determine cluster properties from experiment is the *strength* of the short-range correlations, which is related to the ratio $\langle n(n-1)\rangle_c/\langle n\rangle_c$ of the first two moments of the cluster decay multiplicity distribution (averaged over the produced cluster spectrum). Assuming independent emission of clusters, one arrives at an average cluster mass $\bar{M}_c \simeq 1.3$ GeV and an average charge multiplicity $\langle n_{\text{ch}}\rangle_c \simeq 2$ ***. These numbers are consistent with other final-state densities observed, such as gap distributions, zone characteristics, semi-inclusive correlations as well as charge-transfer correlations measured in rapidity space [3].

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*** The most recent accounts of the present experimental status of cluster models can be found in ref. [3a]. A dissident view with regard to cluster properties is maintained in ref. [3b].

These small numbers and the very fact that high-statistics bubble chamber experiments at $E_{\rm lab} \lesssim 20$ GeV have revealed the importance of inclusive resonance produc tion [4] suggest the use of the invariant mass of the pair, $M_{\pi\pi}^2 = (p_1 + p_2)^2$, rather than its relative rapidity, $\Delta y = y_1 - y_2$, as the variable against which to plot the correlation C_2; while the shape of $C_2(\Delta y)$ is only determined by the (assumed) isotropy of cluster decay [5], the shape of $C_2(M_{\pi\pi})$ would be expected to exhibit additional information such as, e.g., resonance production [6,7].

Besides these *dynamical* effects due to intermediate cluster production, one expects identical pions emerging from hadron collisions to show *Bose-Einstein* (BE) correlations *. In fact it has been repeatedly suggested that this analogue of the Hanbury-Brown–Twiss effect be utilized to determine the size of the pion-emitting source in hadron collisions experimentally [9]. Here again, the variable $M_{\pi\pi}$ is much more suitable than Δy, because there is a one-to-one correspondence between the BE symmetry point $\boldsymbol{p}_1 = \boldsymbol{p}_2$ and the threshold mass $M_{\pi\pi}^0 = 2m$ (m being the pion mass). Although the BE effect is clearly established experimentally in azimuthal correlations [10] and in the Kopylov variables $\boldsymbol{q} = \boldsymbol{p}_1 - \boldsymbol{p}_2$ [11] as well as in $q^2 = 4m^2 - M_{\pi\pi}^2$ [12], the procedures to actually extract the interaction volume from data are plagued by the problem of how to disentangle background, (i.e., dynamical) from BE short-range correlations [10–12]. Moreover, the situation is confused by the expectation that abundant production of resonances of width $\Gamma \lesssim 100$ MeV would result in a tremendously narrow BE correlation width, $\Delta M_{\pi\pi} \simeq 10$ MeV [13,14], which is below present experimental resolution.

In this paper, therefore, we set out to investigate the relative importance of Bose-Einstein and dynamical correlations from cluster decay using the invariant pair mass $M_{\pi\pi}$ as the appropriate variable. As far as dynamics is concerned we need, of course, some theoretical prejudice. We shall adopt the attitude to work in a framework which is as simple as possible. Therefore we return to the old independent emission model of clusters with mean properties borrowed from "rapidity physics". We differ however from previous authors in the treatment of cluster decay. In a preceding paper we have shown that neither a factorized form of the two-particle density nor the thermodynamical manner of including Bose statistics is appropriate when evaluating the decay of a cluster of small mass such as 1.3 GeV [15]. Therefore in this paper we will carefully take account of the lightness of clusters by imposing energy-momentum conservation, isospin invariance and Bose statistics in cluster decay.

We should mention that this paper was motivated by the recent publication [6] of first high-energy two-particle correlations as functions of $M_{\pi\pi}$ and subsequent interpretations [6,16] of these data. The conclusion reached in ref. [16] was that the cluster model fails in explaining the large correlations observed in the region $M_{\pi\pi} \lesssim m_\rho$. Confirming our earlier preliminary results [17], we shall demonstrate in

* The implications of BE effects in the uncorrelated jet model have been investigated in ref. [8].

this paper that this conclusion is false and that the cluster picture can, indeed, accommodate the inclusive correlation data from the 205 GeV/c pp bubble chamber experiment * as presented in ref. [6].

2. The model

Intuitively, the most attractive way to view clusters is to describe them as fireballs à la Hagedorn [18], since his fireballs imply both resonances and their interactions in an average sense. Interactions of resonances are expected to occur due to the observed density of secondaries in rapidity space [19]. To put it differently, it is unlikely that established resonances are produced independently; they rather cluster into larger structures, i.e., fireballs.

It is important to note that the thermodynamic limit cannot be taken when treating fireballs of mass M_c = 1.3 GeV [15]. Therefore one is led to the microcanonical description of fireballs as given by Frautschi and Hagedorn [20]. Unfortunately, their statistical bootstrap scheme has not yet been solved with Bose statistics and isospin included. Being faced with fairly light clusters, however, one may well accept a phase-space description (à la Fermi) as a good approximation to fireball decay (à la Hagedorn).

So we actually calculate our distributions from the density of states $\tau(Q^2)$ for a finite ideal Bose gas of mass $M_c = \sqrt{Q^2}$. For isospin-zero pions, the formula for $\tau(Q^2)$ has been given by Chaichian et al. [21] in terms of a "cluster expansion":

$$B\tau(Q^2) = \sum_{k=1}^{\infty} \sum_{n_1,\dots,n_k=0}^{\infty} k\{n_i\}\, B^l \Omega^{(l)}(Q^2, \underbrace{m, \dots, m}_{n_1 \text{ times}}, \dots, \underbrace{km, \dots, km}_{n_k \text{ times}}), \qquad (1)$$

with

$$l = \sum_{r=1}^{k} n_r , \qquad h\{n_i\} = \prod_{r=1}^{k} \frac{r^{-3n_r}}{n_r!} .$$

Here Q stands for the cluster four-momentum, and $\Omega^{(l)}$ is the Lorentz-invariant momentum-space integral for l particles of the indicated masses. B is a constant of dimension $[L^2]$ that one would expect to be of the order

$$B \approx \frac{\pi}{h^2 m^2} = 4.1 \text{ GeV}^{-2} ,$$

if m = 140 MeV.

The terms in eq. (1) with $k > 1$ can be thought of as being composed out of one

* Unfortunately, the results of the ISR experiment published in ref. [7] are not useful for an independent analysis since they have not been corrected for acceptance biases.

or more *BE clusters* which are defined as "condensate droplets" consisting of i pions of identical momenta. Thus each contribution to $\tau(Q^2)$ is characterized by the number of times, n_i, a BE cluster appears in it, where $2 \leqslant i \leqslant k$. The partial sum corresponding to $k = 1$ is nothing but the Boltzmann-statistics expression for τ. The total pion number from the term described by the set $\{n_i\} \equiv \{n_1, ..., n_k\}$ is given by $\Sigma_{r=1}^{k} rn_r$. Therefore, if one wants to include all contributions up to a given pion number N, one is faced with a partition problem. For $N = 16$, e.g., one has 914 such partitions.

The above defined BE clusters should not be confused with the usual notion of clusters as intermediate structures in hadron–hadron collisions. To ensure clarity, we shall henceforth call the latter clusters *strong interaction* (SI) clusters.

The generalization of eq. (1) to the case of pions with isospin 1 has been derived by Kripfganz [22]. The result is an expansion in BE clusters that are still degenerate in their charge composition: instead of being characterized by one number i, the BE cluster is now characterized by a set $\{i\} = (i_+, i_-, i_0)$, the numbers of π^+, π^-, π^0, respectively, which it contains. The resulting number of partitions grows so rapidly with N that the problem exceeds the capacity of a big computer. We observe, however, that the coefficients $h\{n_i\}$ given in ref. [22] are small and even negative whenever they involve BE clusters that consist of pions of different charges. Therefore, we approximate the problem by retaining only the manifestly positive "diagonal" terms, whose BE clusters contain only equally charged pions. In this case we obtain from eqs. (22) and (23) of ref. [22] the following natural generalization of eq. (1) for the level density of an SI cluster with isospin (I, I_3):

$$B\tau_{I_3}^{I}(Q^2) = \sum_{k=1}^{\infty} \sum_{\{n(i,q)\}} h_{I_3}^{I}\{n(i, q)\}\, B^{l}\Omega^{(l)}(Q^2, \underbrace{m, ..., m}_{n'_1 \text{ times}}, ..., \underbrace{km, ..., km}_{n'_k \text{ times}}) \quad , \tag{2}$$

with

$$n'_r = \sum_{q=+,-,0} n(r, q) , \qquad l = \sum_{r=1}^{k} n'_r .$$

$n(r, q)$ denotes the number of times a BE cluster composed of r pions of charge q appears. The coefficients h depend on the whole set $\{n(i, q)\}$ and read

$$h_{I_3}^{I}\{n(i, q)\} = P_{\{N_t\}}^{I, I_3} \prod_{t=+,-,0} \prod_{r_t=1}^{k} \frac{r_t^{-3n(r, t)}}{n(r, t)!} \quad . \tag{3}$$

$P_{\{N_t\}}^{I, I_3}$ denotes the standard Cerulus coefficients for an isospin (I, I_3) object decaying into the set $\{N_t\} = (N_+, N_-, N_0)$ of N_+, N_-, N_0 pions of charges +, −, 0, respectively [23]. Here $N_t = \Sigma_{r=1}^{k} rn(r, t)$ and of course $I_3 = N_+ - N_-$. For $N \leqslant 16$

and $I = 1, I_3 = 0$ this leads to 9742 terms in the expansion, eq. (2)!

It is straightforward to obtain the inclusive two-particle density $F_2(p_1, p_2)$ of an SI cluster of isospin (I, I_3) into two pions of charges t, t':

$$F^{I,I_3}_{2t,t'}(p_1, p_2) = \tau^I_{I_3}(Q^2)^{-1} \sum_{k=1}^{\infty} \sum_{\{n(i,q)\}} h^I_{I_3}\{n(i,q)\} B^l$$
$$\times \Big[\sum_{j,j'=1}^{k} j^3 j'^3 n(j,t)(n(j',t') - \delta_{jj'}\delta_{tt'}) \Omega^{(l-2)}(\tilde{Q}^2, ..., \not{j}m, ..., \not{j}'m, ...)$$
$$+ 2p_{10}\delta^{(3)}(\boldsymbol{p}_1 - \boldsymbol{p}_2)\delta_{tt'} \sum_{j=1}^{k} j^3(j-1)n(j,t)\Omega^{(l-1)}((Q-jp_1)^2, ..., \not{j}m, ...)\Big] , \tag{4}$$

with

$$\tilde{Q} = Q - jp_1 - j'p_2 .$$

Note that the integral over F_2 is normalized to the second moment over the SI cluster decay multiplicity distribution

$$\int \prod_{i=1}^{2} \frac{d^3p_i}{2p_{i0}} F^{I,I_3}_{2t,t'}(p_1, p_2) = \langle n_t(n_{t'} - \delta_{tt'})\rangle^{I,I_3} . \tag{5}$$

The δ-function term in eq. (4) is the most prominent consequence of Bose-Einstein statistics: it is due to two equally charged pions emanating out of one BE cluster. This contribution will in the following be called the "condensation" term.

So far we have been only concerned with the SI cluster properties themselves. To make contact with experiment, we must formulate the independent emission hypothesis [24]. According to this hypothesis, the normalized inclusive one- and two-particle densities from a hadron-hadron collision are given in terms of an SI cluster yield function $\tilde{w}(Q^2)$ that is independent on the SI cluster momentum (we suppress isospin for the moment)

$$\rho_1(p_1) = \int d^4Q\, \tilde{w}(Q^2)\, F^Q_1(p_1) , \tag{6}$$

$$\rho_2(M_{\pi\pi}) = \int \prod_{i=1}^{2} \frac{d^3p_i}{2p_{i0}} \delta(\sqrt{(p_1+p_2)^2} - M_{\pi\pi})$$
$$\times \{ \int d^4Q\, \tilde{w}(Q^2)\, F^Q_2(p_1, p_2) + \rho_1(p_1)\, \rho_1(p_2) \} . \tag{7}$$

F^Q_1, F^Q_2 are inclusive one- and two-particle decay densities of an SI cluster at four-momentum Q. The structure of F^Q_2 for $\boldsymbol{Q} = \boldsymbol{0}$ has been given in eq. (4). The independent emission hypothesis is reflected in the structure of the last term in eq. (7).

Using the convolution

$$\rho_1 \otimes \rho_1(M_{\pi\pi}) = \int \prod_{i=1}^{2} \frac{d^3 p_i}{2p_{i0}} \, \delta(\sqrt{(p_1+p_2)^2} - M_{\pi\pi}) \, \rho_1(p_1)\rho_1(p_2) \,, \tag{8}$$

one defines the inclusive two-particle correlation

$$C_2(M_{\pi\pi}) = \rho_2(M_{\pi\pi}) - \rho_1 \otimes \rho_1(M_{\pi\pi}) \,. \tag{9}$$

Within the independent emission hypothesis, eq. (7), C_2 is simply given by

$$C_2(M_{\pi\pi}) = \int d^4Q \, \widetilde{w}(Q^2) \, \mathcal{F}_2(Q^2, M_{\pi\pi}) \,, \tag{10}$$

with

$$\mathcal{F}_2(Q^2, M_{\pi\pi}) = \int \prod_{i=1}^{2} \frac{d^3 p_i}{2p_{i0}} \, \delta(\sqrt{(p_1+p_2)^2} - M_{\pi\pi}) F_2^Q(p_1, p_2) \,. \tag{11}$$

Note that the Q dependence in $F_2(Q^2, M_{\pi\pi})$ has dropped out, since F_2 is an invariant function. The d^4Q integration in eq. (10) is essentially over longitudinal phase space. We thus arrive at the simple form for C_2:

$$C_2(M_{\pi\pi}) = \int dM_c \, w(M_c) \, \mathcal{F}_2(M_c^2, M_{\pi\pi}) \,, \tag{12}$$

where

$$w(M_c) = \int d^4Q \, \delta(\sqrt{Q^2} - M_c) \widetilde{w}(Q^2) \,. \tag{13}$$

It is now a trivial matter to include SI cluster isospin and pion charges

$$C_2^{tt'}(M_{\pi\pi}) = \int dM_c \{ \sum_{I, I_3} w^{I, I_3}(M_c) \, \mathcal{F}_{2t,t'}^{I,I_3}(M_c^2, M_{\pi\pi}) \} \,, \tag{14}$$

where t, t' denote the observed pion charges. To leave matters simple, we choose the shape of $w^{I,I_3}(M_c)$ to be independent of (I, I_3):

$$w^{I,I_3}(M_c) = \alpha_{I,I_3} w(M_c) \,.$$

This leads us to

$$C_2^{tt'}(M_{\pi\pi}) = \int dM_c \, w(M_c) \sum_{I,I_3} \alpha_{I,I_3} \, \mathcal{F}_{2t,t'}^{I,I_3}(M_c^2, M_{\pi\pi}) \,. \tag{15}$$

Here the function $w(M_c)$ contains the cluster production dynamics. In the following we shall choose a simple parametrization for it:

$$w(M_c) = (M_c - M_0)^\beta \exp(-\gamma M_c) \,. \tag{16}$$

We adjust the model to the empirical findings from rapidity space distributions, i.e., we adjust β, γ, M_0 to reproduce $\bar{M}_c \approx 1.3$ GeV. A reasonable choice consists in putting $M_0 = 0.5$ GeV, $\beta = 4$, $\gamma = 6$ GeV^{-1}. Note that this value of γ is also suggested by inclusive p_T spectra. The parameter B in eq. (4) was taken to be 4 GeV^{-2}, which yields $\langle n_{ch} \rangle_c = 2.1$, and therefore the reasonable average pion energy in the SI cluster rest system is 413 MeV. The normalizations α_{I,I_3} control the actual numbers of produced SI clusters; they must evidently depend on the primary energy and will be discussed in sect. 3.

3. Numerical results

We first admitted both isoscalar and isovector SI clusters but the correlations turned out to depend little on isospin I. Therefore we shall only present results for isovector SI clusters in the following. The calculations were done on a computer, using a program of Kajantie and Karimäki to evaluate the momentum-space integrals [25]. We allowed for SI cluster decay multiplicities up to 16.

We first display in fig. 1 the correlation functions $C_2^{+-}(M_{\pi\pi})$ and $C_2^{--}(M_{\pi\pi})$ due to single SI clusters of various charges and discrete representative mass $M_c = 1.32$ GeV. The quantum number dependence of the shapes of these distributions demonstrates our previous claim that the thermodynamic limit is too rough an approximation in this context. Note that C_2^{--} has been multiplied by factors of 4 and 16 in figs. 1a, b, respectively.

C_2^{--} is clearly narrower than C_2^{+-} for neutral and positive clusters. The suppression of C_2^{--} can be understood as being mainly due to charge conservation: it needs at least a four-particle state to accommodate two negatives in an $I_3 = 0$ configuration and a five-particle state to get two negatives out of a $I_3 = +1$ configuration. Thus the suppression factors 4 and 16 are consequences of the decay multiplicity distribution of the SI clusters. For negatively charged SI clusters, it is equally probable to find pion pairs of the same and opposite charges. Therefore, the correlations C_2^{+-} and C_2^{--} are fairly much the same. Note that the "condensate" terms have not been added in fig. 1. Their inclusion would in fact render the integrated correlations from negatively charged SI clusters exactly equal.

Next we compare the relative strength $\langle n_t(n_{t'} - \delta_{tt'})\rangle / \langle n_t \rangle$ per cluster of definite mass M_c as a function of M_c (fig. 2). The relative ordering of the different curves is again expected from charge conservation and charge symmetry. Since all these mean values soon show a straight-line behaviour, the relative magnitudes of C_2^{+-} and C_2^{--} become sensitively dependent on SI cluster mass for both neutral and positively charged SI clusters. More interesting information is contained in fig. 3, where we have plotted the integrals of the F_2 densities, again per SI cluster of given M_c *versus* M_c, fig. 3a, and the "condensate" terms contained in these integrals (in per cent) in the case of identical pions (see fig. 3b). Typically, this contribution is of the order of 20–30% in the region of the observed average SI cluster mass, and it becomes

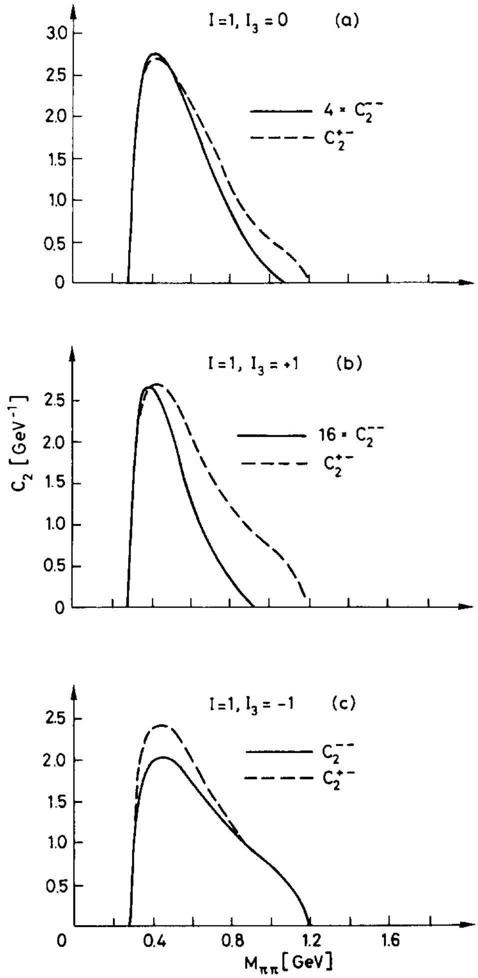

Fig. 1. Correlation functions $C_2^{+-}(M_{\pi\pi})$ (dotted curves) and $C_2^{--}(M_{\pi\pi})$ (full curves) from single isovector SI clusters of mass M_c = 1.32 GeV and (a) zero charge, (b) positive charge, (c) negative charge. The correlations C_2^{--} have been scaled up by factors 4 and 16 in cases (a) and (b), respectively.

the dominant term in the region of low SI cluster masses. The dependence on the SI cluster charge is appreciable.

After this prelude on cluster properties, let us smear out the SI cluster mass as described in sect. 2, eqs. (15) and (16). We thus have three parameters, α_{1I_3}, to adjust the model to the mean central pion multiplicities $\bar{n}_{\pi^+} = 3.8$ and $\bar{n}_{\pi^-} = 2.7$ measured in the $E_{\text{lab}} = 205$ GeV pp bubble chamber experiment [6] ("central" was defined in ref. [6] by a cut in Feynman x, $|x| \leqslant 0.6$). These numbers suffice to fix

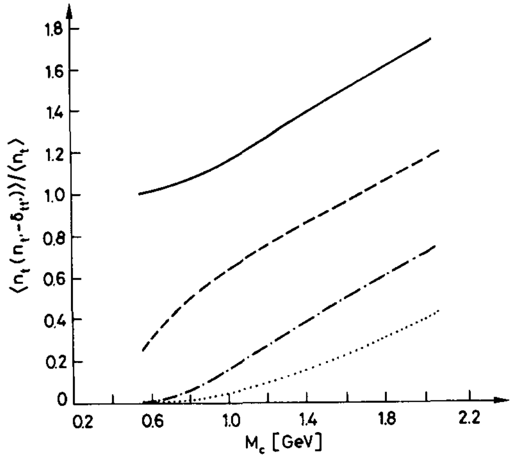

Fig. 2. The correlation strength $\langle n_t(n_{t'} - \delta_{tt'})\rangle/\langle n_t\rangle$ from a single SI cluster as a function of its mass M_c. Full curve: $t = +, t' = -$, neutral clusters. Dashed curve: $t = +, t' = -$, non-neutral clusters, and $t = -, t' = -$, negatively charged clusters. Dashed-dotted curve: $t = -, t' = -$, neutral clusters. Dotted curve: $t = -, t' = -$, positively charged clusters.

two of our parameters, or to put it differently, we can still play (within certain limits) with the number of, say, negative SI clusters. Setting their production rate equal to zero clearly minimizes C_2^{--}. In this case, we obtain $\bar{\nu}_0 = 1.97$ and $\bar{\nu}_+ = 1.1$ for the mean number of neutral and positive SI clusters, respectively. The overall cluster distribution function $\alpha w(M_c)$, $\alpha = \Sigma_{I_3}\alpha_{1 I_3}$, is plotted in fig. 4. It corresponds to an average charge multiplicity per average SI cluster of $\langle n_{ch}\rangle_c = 2.12$ and $\bar{M}_c = 1.28$ GeV.

The comparison with the experimental data [6] is contained in fig. 5. The full curves shown there are the model predictions without the "condensation" term in eq. (4). We find that the observed experimental ratio

$$\int \mathrm{d}M_{\pi\pi}\, C_2^{+-}(M_{\pi\pi}) / \int \mathrm{d}M_{\pi\pi}\, C_2^{--}(M_{\pi\pi}) \approx 5$$

is correctly borne out by the model. Furthermore, the strong peaking of $\pi^+\pi^-$ correlations at $M_{\pi\pi} < m_\rho$ is described by the model as well. The $\pi^-\pi^-$ correlation, however, is evidently much larger in the low-$M_{\pi\pi}$ region than the full curve predicts. In fact, the width of this curve is only slightly narrower (by about 40 MeV) than for $\pi^+\pi^-$. So we do need the "condensation" terms, to be discussed below. As far as the large $M_{\pi\pi}$ is concerned, on the other hand, we have to remember that our ideal gas approximation to fireball decay becomes poor at large M_c. In fact, the large-mass tail of $C_2(M_{\pi\pi})$ is somewhat sensitive to the size of γ in eq. (16) which is not the case for the proper fireball model [17]. Therefore one should not attach too much weight to the large-$M_{\pi\pi}$ behaviour of our approximation. In any case, one can

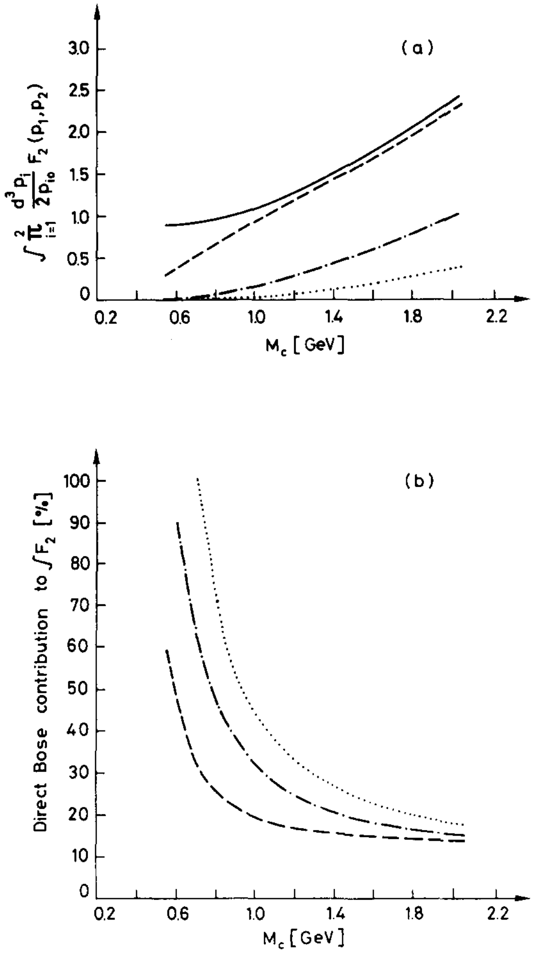

Fig. 3. (a) The integral

$$\int \prod_{i=1}^{2} \frac{d^3p_i}{2p_{i0}} F_2(p_1, p_2)$$

from single-cluster decay plotted *versus* SI cluster mass M_C. The meaning of the various distributions is the same as in fig. 2. (b) The percentage of the "condensation" term (see text after eq. (5)) contained in the curves of fig. 3a in the case of $\pi^-\pi^-$ distributions. (dashed curve) negatively charged; (dotted curve) positively charged; (dash-and-dot curve) neutral SI clusters.

easily understand the background under the ρ signal in $C_2^{+-}(M_{\pi\pi})$ as coming from three- and more-pion decay of SI clusters.

The "condensation" term that should be added to the full curve drawn in fig. 5b for C_2^{--} amounts to $0.17\delta(M_{\pi\pi} - 2m)$. In order to visualize this term, we remember

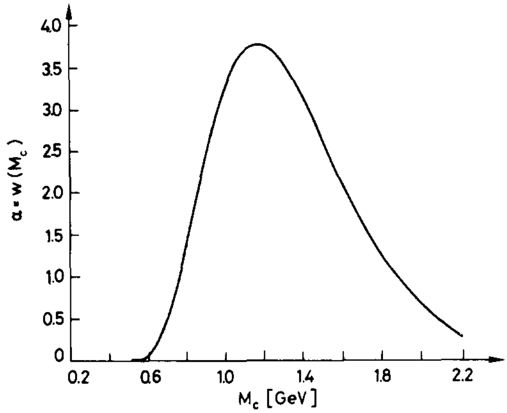

Fig. 4. SI cluster distribution assumed to be produced during collision (see eq. (16)). It corresponds to $\beta = 4$, $\gamma = 6\ \mathrm{GeV}^{-1}$, $M_0 = 500$ MeV. Average cluster mass is $\bar{M}_c = 1.28$ GeV. Mean SI cluster numbers: $\bar{\nu}_- = 0$, $\bar{\nu}_+ = 1.1$, $\bar{\nu}_0 = 1.97$.

that its δ-function character stems from the usual limit of continuous counting of states in the derivation of the level density τ [21]. With a proper choice of a finite quantization volume, the δ function becomes smeared out to a function $C_{\mathrm{BE}}^{--}(M_{\pi\pi})$ with a width σ related to the spatial extension of an SI cluster. Hence we make a properly normalized Gauss ansatz for it:

$$C_{\mathrm{BE}}^{--}(M_{\pi\pi}) = 0.17\frac{2}{\sqrt{2\pi\sigma^2}}\exp(-(M_{\pi\pi} - 2m)^2/2\sigma^2)\,. \tag{17}$$

Naturally one would expect $\sigma^2 = m^2 = 0.02\ \mathrm{GeV}^2$. The corresponding prediction for the $\pi^-\pi^-$ correlations is shown as a dashed curve in fig. 5b. For the purpose of illustration, we have also included in this figure the cases $\sigma^2 = 0.01\ \mathrm{GeV}^2$ and $\sigma^2 = 0.03\ \mathrm{GeV}^2$. We see that the "condensation" terms add appreciably but not sufficiently to the low $M_{\pi\pi}$ peaking of C_2^{--}. Roughly speaking the SI cluster decay accounts for 80% of the observed low-mass enhancement. One might be tempted to increase the number of negative clusters in order to better reproduce the peak in C_2^{--}. In fig. 6, we display the situation for the (arbitrary) choice $\bar{\nu}_0 = \bar{\nu}_-$. As expected, the C_2^{--} correlation does show a stronger enhancement for small $M_{\pi\pi}$ than previously found; but the general behaviour of the $\pi^-\pi^-$ correlation is now significantly above the data.

The data indicate a dip structure at $M_{\pi\pi} = 700$ MeV, which, if it persisted, would be a serious problem for such simple SI cluster emission models as ours.

To give an impression of the importance of energy-momentum conservation in SI cluster decay, we have included in fig. 6a the results of the most simple-minded

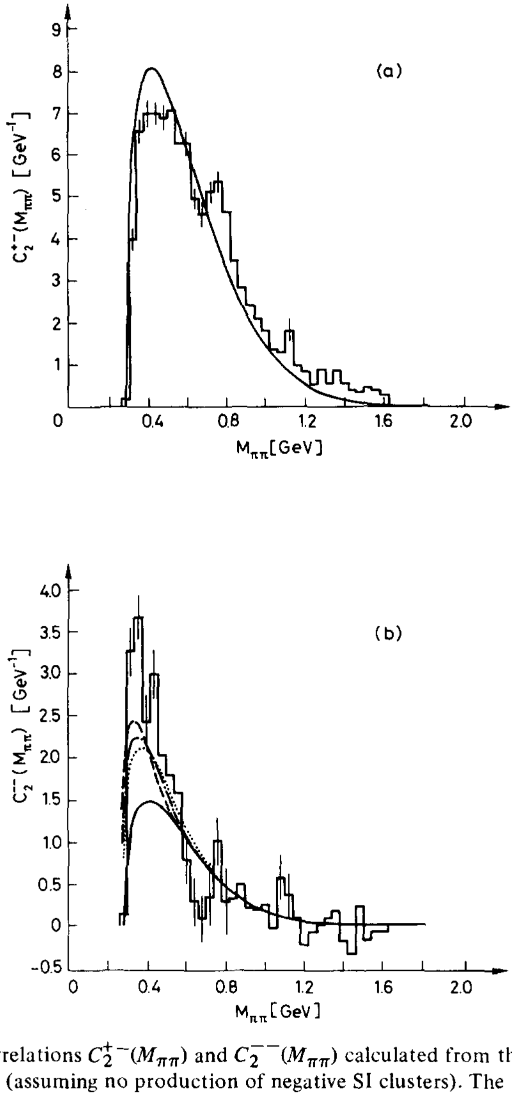

Fig. 5. Inclusive correlations $C_2^{+-}(M_{\pi\pi})$ and $C_2^{--}(M_{\pi\pi})$ calculated from the SI cluster distribution shown in fig. 4 (assuming no production of negative SI clusters). The full curve in fig. 5b does not contain the "condensation" term. The latter has been added with various widths (see eq. (17)): (dashed curve) $\sigma^2 = 0.01$ GeV^2, (dash-and-dot curve) $\sigma^2 = 0.02$ GeV^2, (dotted curve) $\sigma^2 = 0.03$ GeV^2. Data are taken from ref. [6].

thermodynamical ansatz

$$F_2(p_1, p_2) \sim \{\exp(E_1/kT) - 1\}^{-1}\{\exp(E_2/kT) - 1\}^{-1} , \qquad (18)$$

arbitrarily normalized, and for the two temperatures $T = 120$ MeV and $T = 180$ MeV

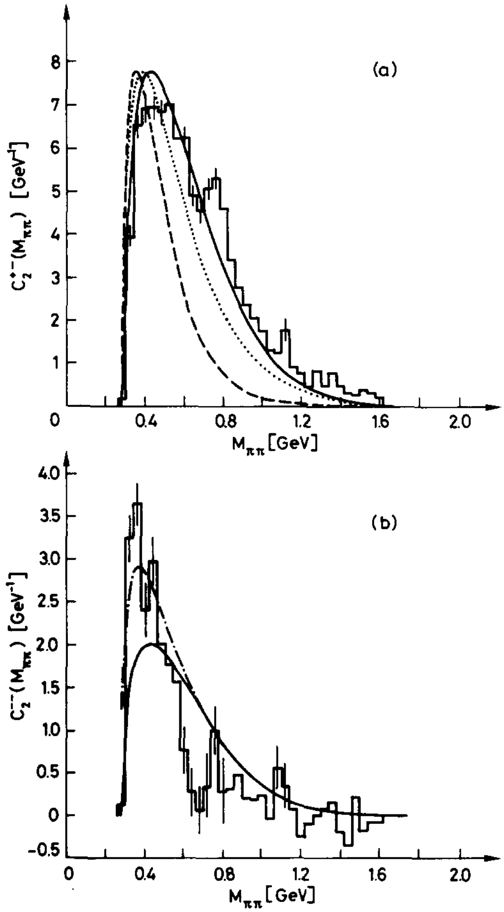

Fig. 6. (a) Inclusive correlation $C_2^{+-}(M_{\pi\pi})$. Full curve: prediction from SI cluster decay, with equal amounts of neutral and negative clusters, $\bar{\nu}_0 = \bar{\nu}_- = 0.6$, $\bar{\nu}_+ = 1.7$. Dashed (dotted) curve: calculated shapes from simple thermodynamical ansatz eq. (18) for $T = 120$ (180) MeV. (b) Inclusive correlation $C_2^{--}(M_{\pi\pi})$ calculated from SI cluster decay as in fig. 6a. The full curve does not contain the "condensation" term. The dashed-dotted curve included the "condensation" term with $\sigma^2 = 0.02\ \mathrm{GeV}^2$.

4. Discussion and summary

We have demonstrated that the main features of the present inclusive correlation data for $C_2^{+-}(M_{\pi\pi})$ and $C_2^{--}(M_{\pi\pi})$ can be naturally understood and economically described in a conventional independent emission model for fireballs of isospin one and average properties known from rapidity space investigations. The dynamical correlations for $\pi^+\pi^-$ and $\pi^-\pi^-$ pairs are predicted to be very similar in shape, with

$\pi^-\pi^-$ suppressed by about a factor of 5.5. This result suggests that we use the experimental $C_2^{+-}(M_{\pi\pi})$, normalized to match the experimental $C_2^{--}(M_{\pi\pi})$ for $M \gtrsim 700$ MeV as dynamical background when analyzing data to extract the Hanbury-Brown–Twiss effect.

The "condensation" terms from SI cluster decay amount to about 30% of the observed $\pi^-\pi^-$ correlations for pair masses below 450 MeV. They seemingly need some augmentation by BE correlations between equal pions out of different clusters. Such correlations have been evaluated in resonance approximations by Thomas [13] and Grassberger [14]. Inserting decay widths and presently known production rates of established resonances they arrive at effects which are concentrated within the mass region 280 MeV $\lesssim M_{\pi\pi} \lesssim$ 300 MeV. Taking clusters instead, we could expect a broadening of this region. However, the evaluation of these effects within our present statistical framework would need a number of additional assumptions on details of the cluster emission mechanism (such as long-range correlations and overall energy-momentum conservation) and should only be pursued in connection with semi-inclusive data *. Therefore, as far as the Bose-Einstein effect is concerned, our present treatment is complementary to refs. [13,14] **.

To make further progress in the understanding of the short-range correlation phenomena, one evidently needs:

(i) semi-inclusive correlations as functions of the invariant masses of two and more particles (including neutrals) to observe higher resonances directly and test production mechanisms more severely;

(ii) higher-statistics data of good mass resolution to see details of resonance interference and to settle the above-mentioned dip problem in $C_2^{--}(M_{\pi\pi})$;

(iii) relevant ISR data to see possible energy dependences of correlation functions $C_2(M_{\pi\pi})$.

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References

[1] G. Belletini, Invited talk presented at 2nd Int. Conf. on elementary particles, Aix-en-Provence, 1973;

* The method used in ref. [26] for investigating local strangeness conservation in $p\bar{p}$ annihilation might then be useful.

** Recently, Giovannini and Veneziano [27] suggested the elimination of correlations from within clusters (named jets in their paper) by forming certain combinations of differently charged pion-pair distributions. Unfortunately, though, they assume their jets to decay independently of quantum numbers (like charge), which certainly does not hold for short-range correlation phenomena.

J. Whitmore, Phys. Reports 10 (1974) 273;
P. Darriulat, Invited talk presented at 6th Colloquium on multiparticle reactions, Oxford, 1975;
L. Foà, Phys. Reports 22 (1975) 1;
J. Ranft, Fortschr. Phys. 23 (1975) 467;
J. Whitmore, Phys. Reports 27 (1976) 187.
[2] A. Białas, Proc. 4th Int. Colloquium on multiparticle hydrodynamics, Pavia, 1973;
E.L. Berger, Proc. 1973 Erice Summer School.
[3] (a) A. Arneodo and G. Plaut, Nucl. Phys. B107 (1976) 262; B113 (1977) 156;
F.W. Bopp, The cluster model, Siegen University preprint (July 1977).
(b) T. Ludlam and R. Slansky, Phys. Rev. D12 (1975) 56, 65;
R. Slansky, Phys. Reports 11 (1974) 99.
[4] D.R.O. Morrison, Invited talk presented at 7th Int. Colloquium on multiparticle reactions, Tutzing, Munich, 1976;
K. Böckmann, Invited talk presented at Conf. on multiparticle production processes and inclusive reactions, Serpukhov, 1976.
[5] A. Białas, K. Fiałkowski and K. Zalewski, Phys. Lett. 45B (1973) 337.
[6] E.L. Berger, R. Singer, G.H. Thomas and T. Kafka, Phys. Rev. D15 (1977) 206.
[7] G. Jansco et al., Nucl. Phys. B124 (1977) 1.
[8] J. Kripfganz, Acta Phys. Pol., to appear;
E.H. de Groot and H. Satz, Quantum statistics and the uncorrelated jet model at high energies, Bielefeld preprint BI-TP 77/15 (June 1977).
[9] G.I. Kopylov and M.I. Podgoretsky, Sov. J. Nucl. Phys. 15 (1972) 219; 18 (1974) 336; 19 (1975) 215;
E.V. Shuryak, Phys. Lett. 44B (1973) 387;
G. Cocconi, Phys. Lett. 49B (1974) 459;
M.I. Podgoretsky, Invited talk presented at Int. Conf. on high-energy physics, Tbilisi, 1976.
[10] G. Ranft et al., Nucl. Phys. B111 (1976) 297 and references therein;
A. Zieminski, Rapporteur's talk at European Conf. on particle physics, Budapest, 1977.
[11] A. Firestone et al., Nucl. Phys. B101 (1975) 19;
F. Grard et al., Nucl. Phys. B102 (1976) 201;
M. Deutschmann et al., Nucl. Phys. B103 (1976) 198;
J.S. Pearson and P.M. Yager, Univ. of California preprint, Davis, March 1977, UCD-PPL-77-02-10.
[12] N.N. Biswas et al., Phys. Rev. Lett. 37 (1976) 175.
[13] G. Thomas, Phys. Rev. D15 (1977) 2636.
[14] P. Grassberger, Nucl. Phys. B120 (1977) 231.
[15] J. Engels, K. Fabricius and K. Schilling, Phys. Lett. B59 (1975) 477; Phys. Rev. D16 (1977) 189.
[16] G.H. Thomas, Invited talk presented at Bielefeld Summer Institute on statistical methods and dynamical models, Bielefeld, 1976; Argonne preprint ANL-HEP-PR-77-01.
[17] K. Schilling, Invited talk presented at Symp. on hadron structure and multiparticle production, Kazimierz, Warsaw, 1977.
[18] R. Hagedorn, Nuovo Cim. Suppl. 3 (1965) 147.
[19] E.L. Feinberg, Lebedev Institute preprint no. 172 (1976).
[20] S. Frautschi, Phys. Rev. D3 (1971) 2821;
E.M. Ilgenfritz and J. Kripfganz, Nucl. Phys. B56 (1973) 241;
R. Hagedorn and I. Montvay, Nucl. Phys. B59 (1973) 45;
J. Engels, H. Satz and K. Schilling, Nuovo Cim. 17A (1973) 535;
I. Montvay, Nucl. Phys. B53 (1973) 521.

[21] M. Chaichian, R. Hagedorn and M. Hayashi, Nucl. Phys. B92 (1975) 445.
[22] J. Kripfganz, Nucl. Phys. B100 (1975) 302.
[23] F. Cerulus, Nuovo Cim. 19 (1961) 528.
[24] P. Pirilä and S. Pokorski, Nuovo Cim. Lett. 8 (1973) 141;
P. Pirilä and S. Pokorski, Phys. Lett. 43B (1973) 502;
F. Hayot and A. Morel, Nucl. Phys. B68 (1974) 323;
E.L. Berger and G.C. Fox, Phys. Lett. 47B (1973) 162.
[25] K. Kajantie and V. Karimäki, Comp. Phys. Comm. 2 (1971) 207.
[26] R. Hagedorn and J. Ranft, Nuovo Cim. Suppl. I, 6 (1968) 169.
[27] A. Giovannini and G. Veneziano, The Bose-Einstein effect and the jet structure of hadronic final states, CERN preprint TH.2347 (July 1977).