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PARTIAL WAVE RELATIONS IN $\pi\Lambda$ SCATTERING AND THE $\Sigma\Lambda\pi$ COUPLING CONSTANT

J. ENGELS

Department of Theoretical Physics, University of Bielefeld, Germany

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Abstract: An investigation of relations for the $P\frac{3}{2}$, $D\frac{3}{2}$ and $S\frac{1}{2}$ partial waves in $\pi\Lambda$ scattering leads to the value $G^2_{\Sigma\Lambda\pi}/4\pi = 12.9 \pm 0.8$. These partial wave relations are obtained from fixed-t finite-contour dispersion relations, which are also valid, if a normal dispersion relation has to be subtracted, by projection. The unknown f and ϵ Regge residues are then connected to coupling constants. As input information on the Σ-resonances and estimates of the asymptotic total $\pi\Lambda$ cross section are used. A small or even zero f-trajectory coupling to the A-amplitude together with an ϵ-trajectory coupling as determined from ϵ coupling constants is well compatible with the data. The D/F ratio for the tensor meson-bary on coupling of the B-amplitude is in the range $3.3 < D/F < 4.1$.

1. Introduction

There is no direct experimental information on elastic $\pi\Lambda$ scattering. Our knowledge on the $\pi\Lambda$ interaction comes mainly from two sources: analyses of $\overline{\text{K}}$N scattering in a multichannel formalism provide the $\pi\Lambda$ amplitude as a by-product and secondly from the decays of Σ-resonances into π and Λ. The first information is probably less reliable since it is not directly connected with experiment. We prefer therefore to evaluate the data on the Σ-resonances.

In this paper we want to determine the $\Sigma\Lambda\pi$ coupling constant. At the same time we would like to see whether a subtraction of the fixed-t dispersion relation for the A-amplitude is necessary or not. As pointed out by Renner and Zerwas [1] this question is important for the coupling of the tensor meson nonet to the baryon octet in the case of the A-amplitude.

To attack the last mentioned problem we start (sect.2) from finite-contour dispersion relations for fixed-t (FCDR). We assume that the high-energy behaviour of the amplitudes may be described by only Regge poles. An FCDR coincides then with the usual fixed-t dispersion relation if there no subtraction is necessary and it is still valid in the subtraction case - naturally the Regge-pole contribution is then enhanced. In sect. 3 we connect the Regge pole residues at the respective particle poles to coupling constants defined by SU(3) invariant Lagrangians.

The next step (sect. 4) is to derive partial wave relations from the FCDRs by projection, i.e. CGLN relations. Partial wave relations are the appropriate theoretical tools to extract information from resonance data. In that respect we follow earlier work by Martin [2], who used a partial wave dispersion relation for the $P_{\frac{3}{2}}$ wave. However, our relations have no trouble with the left-hand cut - the real part of the amplitude can be expressed by Σ-resonance and Regge contributions plus a Born term - though the energy region where we can use our relations is limited. We do not only evaluate the $P_{\frac{3}{2}}$ relation but also the $D_{\frac{3}{2}}$ and $S_{\frac{1}{2}}$ relations (sect. 5). It turns out that the $S_{\frac{1}{2}}$ partial wave relation is by far the most sensitive to the Born term and not the relation for the $P_{\frac{3}{2}}$ wave.

2. $\pi\Lambda$ Finite-contour dispersion relations

In the following we use the notation and definitions of the compilation of coupling constants by Ebel et al. [3] (but set $m_\pi = 1$, $m_\Lambda = M$). The kinematics of $\pi\Lambda$ scattering are identical with those of πN scattering. Elastic $\pi\Lambda$ scattering is in a pure $I = 1$ state and the crossing properties of the invariant amplitudes A and B are the same as those of the isospin even charge combination of πN scattering.

$$A(\nu, t) = A(-\nu, t), \tag{2.1a}$$

$$B(\nu, t) = -B(-\nu, t), \tag{2.1b}$$

where $\nu = (s-u)/4M$. Suppose that for $|\nu| \geqslant N$, Im $\nu > 0$ the invariant amplitudes satisfy a Regge-pole expansion

$$A(\nu, t) = -\sum_n \beta_n(t) \frac{1 + e^{-i\pi\alpha_n(t)}}{\sin \pi\alpha_n(t)} \left(\frac{\nu}{N_0}\right)^{\alpha_n(t)}, \tag{2.2a}$$

$$B(\nu, t) = -\sum_n \gamma_n(t) \frac{1 + e^{-i\pi\alpha_n(t)}}{\sin \pi\alpha_n(t)} \left(\frac{\nu}{N_0}\right)^{\alpha_n(t)-1}. \tag{2.2b}$$

The index n corresponds here to the pomeron, f and ϵ Regge poles. For fixed-t one can then integrate along a finite contour (see fig. 1) in the complex ν-plane and obtains

$$A(\nu, t) = A_{\mathrm{p}}(\nu, t) + \frac{1}{\pi} \int_{\nu_0}^{N} \operatorname{Im} A(\nu', t) \left[\frac{1}{\nu'-\nu} + \frac{1}{\nu'+\nu}\right] \mathrm{d}\nu'$$

$$-\sum_n \beta_n(t) \frac{1 + e^{-i\pi\alpha_n(t)}}{\sin \pi\alpha_n(t)} N_0^{-\alpha_n(t)} \frac{1}{2\pi i} \int_{C_N} \mathrm{d}\nu' \frac{2\nu'^{\alpha_n(t)+1}}{\nu'^2 - \nu^2}, \tag{2.3a}$$

$$B(\nu, t) = B_{\mathrm{p}}(\nu, t) + \frac{1}{\pi} \int_{\nu_0}^{N} \operatorname{Im} B(\nu', t) \left[\frac{1}{\nu'-\nu} - \frac{1}{\nu'+\nu}\right] \mathrm{d}\nu'$$

$$-\sum_n \gamma_n(t)\,\frac{1+e^{-i\pi\alpha_n(t)}}{\sin\pi\alpha_n(t)}\,N_0{}^{1-\alpha_n(t)}\,\frac{1}{2\pi i}\int_{C_N} d\nu'\,\frac{2\nu\nu'^{\alpha_n(t)-1}}{\nu'^2-\nu^2}\ , \tag{2.3b}$$

where C_N is that part of the circle of radius N, which lies in the upper half of the ν-plane and

$$A_p(\nu,t) = G^2_{\Sigma\Lambda\pi}\,\frac{M-m_\Sigma}{2M}\left[\frac{1}{\nu_p-\nu}+\frac{1}{\nu_p+\nu}\right] = G^2_{\Sigma\Lambda\pi}\left(1-\frac{m_\Sigma}{M}\right)\frac{\nu_p}{\nu_p^2-\nu^2}\ , \tag{2.4a}$$

$$B_p(\nu,t) = \frac{G^2_{\Sigma\Lambda\pi}}{2M}\left[\frac{1}{\nu_p-\nu}-\frac{1}{\nu_p+\nu}\right] = \frac{G^2_{\Sigma\Lambda\pi}}{M}\,\frac{\nu}{\nu_p^2-\nu^2}\ , \tag{2.4b}$$

$$\nu_p = \frac{1}{2M}\left(\tfrac{1}{2}t-1+m_\Sigma^2-M^2\right),\ \nu_0 = 1+\frac{t}{4M}; \tag{2.5}$$

m_Σ is the Σ-mass. Because of the Λ-Σ mass difference we have contributions from Σ-exchange to both invariant amplitudes.

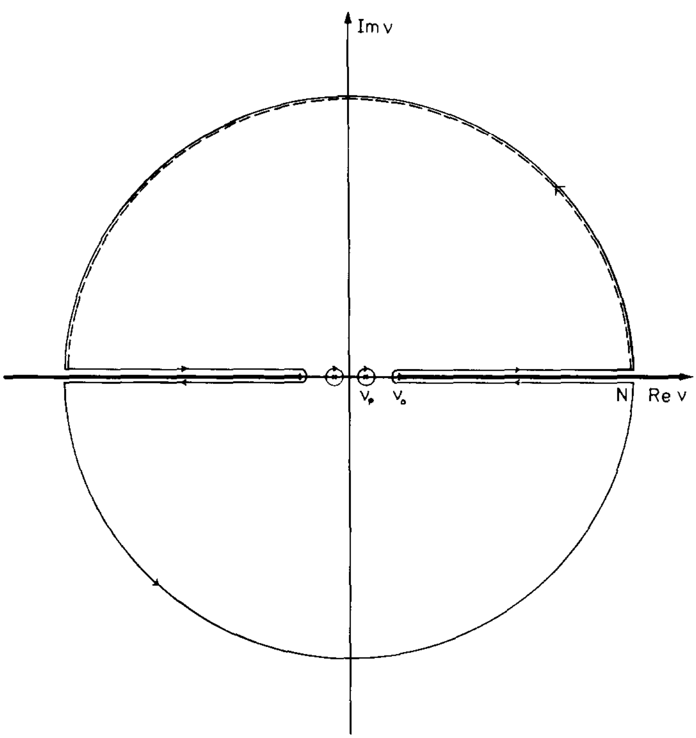

Fig. 1. Integration contour in the complex ν-plane, $---C_N$.

The integral over C_N is easily evaluated after expanding it in powers of ν ($|\nu| < N$)

$$\frac{1}{2\pi i}\int_{C_N}\frac{d\nu'\nu'^{\alpha_n(t)\pm 1}}{\nu'^2-\nu^2}=\frac{1}{2\pi i}N^{\alpha_n(t)\pm 1-1}\sum_{k=0}^{\infty}\left(\frac{\nu}{N}\right)^{2k}\frac{e^{i\pi\alpha_n(t)}-1}{\alpha_n(t)\pm 1-1-2k} \tag{2.6}$$

Inserting eq. (2.6) into eqs. (2.3a) and (b) yields

$$A(\nu,t)=G^2_{\Sigma\Lambda\pi}\left(1-\frac{m_\Sigma}{M}\right)\frac{\nu_p}{\nu_p^2-\nu^2}+\frac{2}{\pi}\int_{\nu_0}^{N}\mathrm{Im}A(\nu',t)\frac{\nu'd\nu'}{\nu'^2-\nu^2}$$
$$+\frac{2}{\pi}\sum_n\beta_n(t)\left(\frac{N}{N_0}\right)^{\alpha_n(t)}\sum_{k=0}^{\infty}\frac{(\nu/N)^{2k}}{2k-\alpha_n(t)}, \tag{2.7a}$$

$$B(\nu,t)=\frac{G^2_{\Sigma\Lambda\pi}}{M}\frac{\nu}{\nu_p^2-\nu^2}+\frac{2\nu}{\pi}\int_{\nu_0}^{N}\mathrm{Im}B(\nu',t)\frac{d\nu'}{\nu'^2-\nu^2}$$
$$+\frac{2}{\pi}\sum_n\gamma_n(t)\left(\frac{N}{N_0}\right)^{\alpha_n(t)-1}\sum_{k=0}^{\infty}\frac{(\nu/N)^{2k+1}}{2k+2-\alpha_n(t)}. \tag{2.7b}$$

Each term of (2.7a) and (b) has separately the same crossing properties as the respective full amplitude. The t-channel poles, which come from Regge exchange, are explicitly given. Since

$$\nu=\frac{p_tq_t}{M}\cos\theta_t, \tag{2.8}$$

where $p_t=\sqrt{\frac{1}{4}t-M^2}$, $q_t=\sqrt{\frac{1}{4}t-1}$, θ_t are the t-channel c.m. three-momenta of the lambda and pion and the angle, the pole residues have as ν-dependence the leading term of $P_\alpha(\cos\theta_t)$ and at the pole position they do not depend on N (refs. [4, 5]).

For $\nu=0$ relation (2.7a) coincides with the finite-energy sum rule (FESR) of Renner and Zerwas [1] and eq. (2.7b) leads after division by ν and in the limit $N\to\infty$ to

$$\lim_{\nu\to 0}\frac{1}{\nu}B(\nu,t)=\frac{G^2_{\Sigma\Lambda\pi}}{M\nu_p^2}+\frac{2}{\pi}\int_{\nu_0}^{\infty}\frac{d\nu'}{\nu'^2}\mathrm{Im}B(\nu',t). \tag{2.9}$$

In πN scattering the FCDRs corresponding to eqs. (2.7a) and (b) have been successfully tested (see ref. [6] for a discussion).

3. Connection between residue functions and coupling constants

A comparison of eqs. (2.7a) and (b) to Feynman-graph calculations allows the determination of the Regge residues at the pole position in terms of coupling constants [4, 5]. As far as possible we take in this chapter SU(3) symmetry into account.

3.1. *The pomeron coupling*

Usually the pomeron is regarded as an SU(3) singlet - if it is understood as particle at all. Its coupling constants should therefore be the same as in πN scattering. If, in particular, it is assumed that the A-amplitude in πN scattering satisfies an unsubtracted fixed-t dispersion relation [7], one has

$$\beta_{\mathrm{p}} = 0. \tag{3.1}$$

Another – SU(3) independent – argument for the vanishing of β_{p} while $\gamma_{\mathrm{p}} \neq 0$ would be the postulate of s-channel helicity conservation for $\pi\Lambda$ scattering.

3.2. *The coupling of the* f*-meson*

We define the coupling constants of the tensor mesons to the pseudoscalar mesons and the baryons by the Lagrangians

$$L_{\mathrm{TPP}} = 2\,\frac{G_{\mathrm{TPP}}}{m_{\mathrm{T}}}\,T_i^{\mu\nu}\,\partial_\mu P_j\,\partial_\nu P_k\,d_{ijk}, \tag{3.2}$$

$$\begin{aligned} L_{\mathrm{TBB}} &= 2i\,\frac{G_{\mathrm{TBB}}^{(1)ijk}}{m_{\mathrm{B}}}\,T_i^{\mu\nu}\,\bar{B}_j(\gamma_\mu\partial_\nu + \gamma_\nu\partial_\mu)B_k \\ &\quad + 4\,\frac{G_{\mathrm{TBB}}^{(2)ijk}}{m_{\mathrm{B}}^2}\,T_i^{\mu\nu}\,\partial_\mu\bar{B}_j\,\partial_\nu B_k, \end{aligned} \tag{3.3}$$

with

$$G_{\mathrm{TBB}}^{(\kappa)ijk} = -F^{(\kappa)}if_{ijk} + D^{(\kappa)}d_{ijk} + S^{(\kappa)}\delta_{i0}\delta_{jk}, \qquad \kappa = 1,2, \tag{3.4}$$

in analogy to the f-meson couplings to the pions and nucleons given in ref. [3]. The indices i, j and k in eqs. (3.2) and (3) are SU(3) indices and f_{ijk} and d_{ijk} are the SU(3) structure constants generalized from $i = 1, \ldots, 8$ to $i = 0, 1, \ldots, 8$ by [8]

$$d_{0jk} = \sqrt{\tfrac{2}{3}}\,\delta_{jk}, \qquad f_{0jk} = 0. \tag{3.5}$$

The Lagrangian L_{TPP} is constructed in such a way that the coupling of the f′ meson to two pions is zero, if one considers f and f′ as an ideal mixture of an SU(3) singlet $|0\rangle$ and SU(3) octet $|8\rangle$ component

$$\begin{aligned} |\mathrm{f}\rangle &= \sqrt{\tfrac{2}{3}}|0\rangle + \sqrt{\tfrac{1}{3}}|8\rangle, \\ |\mathrm{f}'\rangle &= \sqrt{\tfrac{1}{3}}|0\rangle - \sqrt{\tfrac{2}{3}}|8\rangle. \end{aligned} \tag{3.6}$$

From the quark model one expects f′ to decouple from the nucleons too. This condition can be expressed as

$$0 = F^{(\kappa)} - D^{(\kappa)} - \sqrt{\tfrac{2}{3}}\,S^{(\kappa)}, \qquad \kappa = 1,2. \tag{3.7}$$

For the process baryon a + pseudoscalar meson b → baryon a′ + pseudoscalar meson b′ with $m_{\mathrm{a}} = m_{\mathrm{a}'} = M$, $m_{\mathrm{b}} = m_{\mathrm{b}'} = 1$ one derives in Born approximation for tensor-meson exchange from eqs. (3.2) and (3) the following contributions to the A-and B-amplitudes, omitting non-pole terms

$$A_{\mathrm{T}} = \frac{16}{m_{\mathrm{T}}} \frac{G_{\mathrm{TPP}}}{m_{\mathrm{T}}^2 - t} d_{i_{\mathrm{T}}, i_{\mathrm{b}'}, i_{\mathrm{b}}} \left\{ G_{\mathrm{TBB}}^{(1) i_{\mathrm{T}}, i_{\mathrm{a}'}, i_{\mathrm{a}}} \tfrac{1}{3} q_t^2 + G_{\mathrm{TBB}}^{(2) i_{\mathrm{T}}, i_{\mathrm{a}'}, i_{\mathrm{a}}} \frac{2}{3M^2} (q_t p_t)^2 P_2(\cos\theta_t) \right\}, \tag{3.8a}$$

$$B_{\mathrm{T}} = \frac{16}{m_{\mathrm{T}} M} \frac{G_{\mathrm{TPP}} G_{\mathrm{TBB}}^{(1) i_{\mathrm{T}}, i_{\mathrm{a}'}, i_{\mathrm{a}}}}{m_{\mathrm{T}}^2 - t} q_t p_t \cos\theta_t \, d_{i_{\mathrm{T}}, i_{\mathrm{b}'}, i_{\mathrm{b}}}. \tag{3.8b}$$

With eq. (2.8) one finally arrives at

$$A_{\mathrm{T}} = \frac{16}{m_{\mathrm{T}}} \frac{G_{\mathrm{TPP}}}{m_{\mathrm{T}}^2 - t} d_{i_{\mathrm{T}}, i_{\mathrm{b}'}, i_{\mathrm{b}}} \left\{ G_{\mathrm{TBB}}^{(2) i_{\mathrm{T}}, i_{\mathrm{a}'}, i_{\mathrm{a}}} \nu^2 \right.$$

$$\left. + \tfrac{1}{3} q_t^2 \left[G_{\mathrm{TBB}}^{(1) i_{\mathrm{T}}, i_{\mathrm{a}'}, i_{\mathrm{a}}} - \frac{p_t^2}{M^2} G_{\mathrm{TBB}}^{(2) i_{\mathrm{T}}, i_{\mathrm{a}'}, i_{\mathrm{a}}} \right] \right\}, \tag{3.10a}$$

$$B_{\mathrm{T}} = \frac{16}{m_{\mathrm{T}}} \frac{G_{\mathrm{TPP}} G_{\mathrm{TBB}}^{(1) i_{\mathrm{T}}, i_{\mathrm{a}'}, i_{\mathrm{a}}}}{m_{\mathrm{T}}^2 - t} \nu \, d_{i_{\mathrm{T}}, i_{\mathrm{b}'}, i_{\mathrm{b}}}, \tag{3.10b}$$

which is the most convenient form for a comparison to eqs. (2.7a) and (b). In our case the tensor meson is the f, the baryons a and a′ are lambdas and b and b′ are pions. We write the f-trajectory as

$$\alpha_{\mathrm{f}}(t) = 2 - \alpha_{\mathrm{f}}'(m_{\mathrm{f}}^2 - t), \tag{3.11}$$

and retain in eq. (3.10a) only the leading term in ν. The connection between the residues and the coupling constants is then

$$\beta_{\mathrm{f}}(m_{\mathrm{f}}^2) = 8\pi N_0^2 \frac{\alpha_{\mathrm{f}}'}{m_{\mathrm{f}}} G_{\mathrm{f}\pi\pi} G_{\mathrm{f}\Lambda\Lambda}^{(2)}, \tag{3.12a}$$

$$\gamma_{\mathrm{f}}(m_{\mathrm{f}}^2) = 8\pi N_0 \frac{\alpha_{\mathrm{f}}'}{m_{\mathrm{f}}} G_{\mathrm{f}\pi\pi} G_{\mathrm{f}\Lambda\Lambda}^{(1)}, \tag{3.12b}$$

where

$$G_{\mathrm{f}\pi\pi} = G_{\mathrm{TPP}} \, d_{i_{\mathrm{f}}, i_{\pi^\circ}, i_{\pi^\circ}} = G_{\mathrm{TPP}}, \tag{3.13}$$

$$G_{\mathrm{f}\Lambda\Lambda}^{(\kappa)} = F^{(\kappa)} - \tfrac{2}{3} D^{(\kappa)}, \qquad \kappa = 1, 2, \tag{3.14}$$

and eq. (3.7) was used to obtain relation (3.14).

3.3. *The coupling of the* ϵ*-meson*

The interaction of the ϵ-meson with the pseudoscalar mesons and the baryons will be described by the Lagrangians

$$L_{\epsilon\mathrm{PP}} = \tfrac{1}{2} G_{\epsilon\mathrm{PP}} \, m_\epsilon \, \epsilon P_i P_j \delta_{ij}, \tag{3.15}$$

$$L_{\epsilon BB} = G_{\epsilon BB}\, \epsilon\, \bar{B}_i B_j\, \delta_{ij}, \tag{3.16}$$

i.e. we assume the ϵ to be an SU(3) singlet. The constants $G_{\epsilon PP}$ and $G_{\epsilon BB}$ coincide with $G_{\epsilon\pi\pi}$ and $G_{\epsilon NN}$ as defined by Ebel et al. [3]. In Born approximation we have for ϵ-exchange, omitting non-pole terms

$$A_\epsilon = G_{\epsilon\pi\pi} G_{\epsilon NN} \frac{m_\epsilon}{m_\epsilon^2 - t}, \tag{3.17a}$$

$$B_\epsilon = 0 \tag{3.17b}$$

The equation which corresponds to (3.11) is

$$\alpha_\epsilon(t) = -\alpha'_\epsilon(m_\epsilon^2 - t). \tag{3.18}$$

However, a comparison of eqs. (3.17a) and (b) to (2.7a) and (b) at the ϵ-pole position results only in one equation

$$\beta_\epsilon(m_\epsilon^2) = \tfrac{1}{2}\pi\, \alpha'_\epsilon\, m_\epsilon\, G_{\epsilon\pi\pi}\, G_{\epsilon NN}. \tag{3.19}$$

The reason is of course that the expansion of $B(\nu, t)$ in t-channel partial wave amplitudes contains no S-wave contributions.

4. CGLIN relations

In our opinion the CGLN relations are particularly suited to extract information from resonance data. They have been described and used in a variety of papers [9]. Let us shortly repeat the idea. One inserts in the dispersion relations – in our case FCDRs – for the invariant amplitudes the partial wave expansions for Im A and Im B. From the resulting real parts Re A and Re B the partial wave amplitudes are projected out again. So, one finally has for each partial wave amplitude a relation which connects its real part to the imaginary parts of all partial wave amplitudes and to the projections of the Σ-pole term and the Regge contributions:

$$\begin{aligned}\mathrm{Re} f_{l+}(W) = f_{l+,\,\Sigma\text{-pole}}(W) + \frac{\mathrm{P}}{\pi} \int_{M+1}^{W_N} \mathrm{d}W' \sum_{l'=0}^{\infty} [K_{ll'}(W, W')\, \mathrm{Im} f_{l'+}(W') \\ + K_{ll'}(W, -W')\, \mathrm{Im} f_{(l'+1)-}(W')] + \mathrm{Re} f_{l+,\,\mathrm{Regge}}(W),\end{aligned} \tag{4.1a}$$

$$\begin{aligned}\mathrm{Re} f_{(l+1)-}(W) = f_{(l+1)-,\Sigma\text{-pole}}(W) - \frac{\mathrm{P}}{\pi} \int_{M+1}^{W_N} \mathrm{d}W' \sum_{l'=0}^{\infty} [K_{ll'}(-W, W')\, \mathrm{Im} f_{l'+}(W') \\ + K_{ll'}(-W, -W')\, \mathrm{Im} f_{(l'+1)-}(W')] + \mathrm{Re} f_{(l+1)-,\,\mathrm{Regge}}(W),\end{aligned} \tag{4.1b}$$

where $W = \sqrt{s}$. To obtain eqs. (4.1a) and (b) we have chosen

$$N = \omega_{\rm L}^{\rm N} + \frac{t}{4M}\,, \tag{4.2}$$

where $\omega_{\rm L}^{\rm N}$ is constant and W_N the W-value corresponding to a lab energy $\omega_{\rm L}^{\rm N}$ of the incoming pion. Naturally, the CGLN method may be applied for other functions $N(t)$ too. The kernels $K_{ll'}\,(W, W')$ are well-known (see ref. [9]) and

$$f_{l\pm,\Sigma\text{-pole}}(W) = \frac{G^2_{\Sigma\Lambda\pi}}{8\pi W}\left\{(E+M)\left[\frac{-\delta_{l0}}{W+m_\Sigma} + \frac{m_\Sigma - 2M + W}{2q^2}\,Q_l(z)\right]\right.$$
$$\left. + (E-M)\left[\frac{-\delta_{l\pm1,0}}{W-m_\Sigma} + \frac{2M - m_\Sigma + W}{2q^2}\,Q_{l\pm1}(z)\right]\right\}, \tag{4.3}$$

$$z = 1 + \frac{2M^2 - m_\Sigma^2 - s + 2}{2q^2}\,,$$

where q and E are the c.m. three-momentum and energy of the lambda and $Q_l(z)$ is the Legendre function of the second kind.

The Regge contribution is obtained by numerical projection of the corresponding terms in eqs. (2.7a) and (b).

The CGLN relations are no longer valid if the energy W becomes larger than some $W_{\rm max}$. This is because the partial wave expansions of the imaginary parts of the invariant amplitudes diverge in a W' interval, if t becomes less than a certain $t_{\rm M}$. If the Mandelstam representation holds for the $\pi\Lambda$ invariant amplitudes, then $t_{\rm M} = -26.54$. The corresponding $W_{\rm max} = 1560$ MeV is obtained from the condition that $t_{\rm M} = -4q^2_{\rm max}$, where $q_{\rm max}$ is the momentum belonging to $W_{\rm max}$. For a discussion of these problems see ref. [10].

From the experience made in πN scattering, where the highest allowed value for the kinetic energy of the pion is $T_\pi \approx 400$ MeV, but the applicability of the method extends to $T_\pi \approx 850-1000$ MeV (refs. [9,11]) we expect the equivalent to be true in $\pi\Lambda$ scattering. Our $T_{\rm max}$ is 380 MeV, so if we believe that the CGLN method is practicable up to $T_\pi \approx 800$ MeV we come to $W \approx 1835$ MeV. This is sufficient for our purpose.

5. Numerical results

The relations (4.1a) and (b) have been evaluated for the three partial waves $P_{\frac{3}{2}}$, $D_{\frac{3}{2}}$ and $S_{\frac{1}{2}}$, each in the energy region of its first resonance; i.e. for the $P_{\frac{3}{2}}$ wave the $\Sigma(1385)$, for the $D_{\frac{3}{2}}$ wave the Σ (1670) and for the $S_{\frac{1}{2}}$ wave the Σ (1750) resonance. So, we are in the above mentioned energy region.

5.1. *The resonance contributions*

In table 1 we show the resonances and their parameters which we have used [12]. The first six resonances are quite well established, their masses and widths are at least approximately known. However, the existence of the other four resonances listed in table 1 is doubtful and their parameters are very uncertain. They help us nevertheless in estimating the error of the overall resonance contribution.

Table 1
First part: established resonances; second part: uncertain resonances

Resonance	Mass (MeV)	Γ_{tot} (MeV)	$\Gamma_{\pi\Lambda}$ (MeV)	Wave
$\Sigma(1385)$	1384.25	35.9	32.6	$P_{\frac{3}{2}}$
$\Sigma(1670)$	1668.30	57.4	18.4	$D_{\frac{3}{2}}$
$\Sigma(1750)$	1750.0	80.0	18.4	$S_{\frac{1}{2}}$
$\Sigma(1765)$	1764.6	104.6	15.7	$D_{\frac{5}{2}}$
$\Sigma(1910)$	1908.5	67.4	5.2	$F_{\frac{5}{2}}$
$\Sigma(2030)$	2030.0	140.0	25.1	$F_{\frac{7}{2}}$
$\Sigma(1620)$	1619.4	41.3	10.0	$P_{\frac{1}{2}}$
$\Sigma(1880)$	1880.0	181.7	7.3	$P_{\frac{1}{2}}$
$\Sigma(1940)$	1940.0	235.1	20.3	$D_{\frac{3}{2}}$
$\Sigma(2080)$	2080.0	170.0	15.0	$P_{\frac{3}{2}}$

The partial waves are calculated from a Breit-Wigner ansatz

$$f_l = \frac{\frac{1}{2}x\Gamma_l}{q(M_R - W - \frac{1}{2}i\Gamma_l)}\,, \tag{5.1}$$

where M_R is the resonance position, x is the ratio of the partial width $\Gamma_{\pi\Lambda}$ and the total width Γ. The quantity Γ_l is

$$\Gamma_l = \Gamma k_l, \tag{5.2}$$

$$k_l = \frac{1 - \exp\left[-(q/q_R)^{2l+1}\right]}{1 - \exp\left[-1\right]}\,, \tag{5.3}$$

and the index R of q means that q has to be taken at the resonance position. As an example we show in figs. 2a and b the real and imaginary parts of the $\Sigma(1385)$ resonance as calculated from eq. (5.1).

For comparison the correction factor k_l was once changed into 1 and the second time the usual threshold factor $(q/q_R)^{2l+1}$ was taken. Eq. (5.3) was chosen because at low energies one has the correct threshold behaviour and for high energies the imaginary part decreases faster than usual

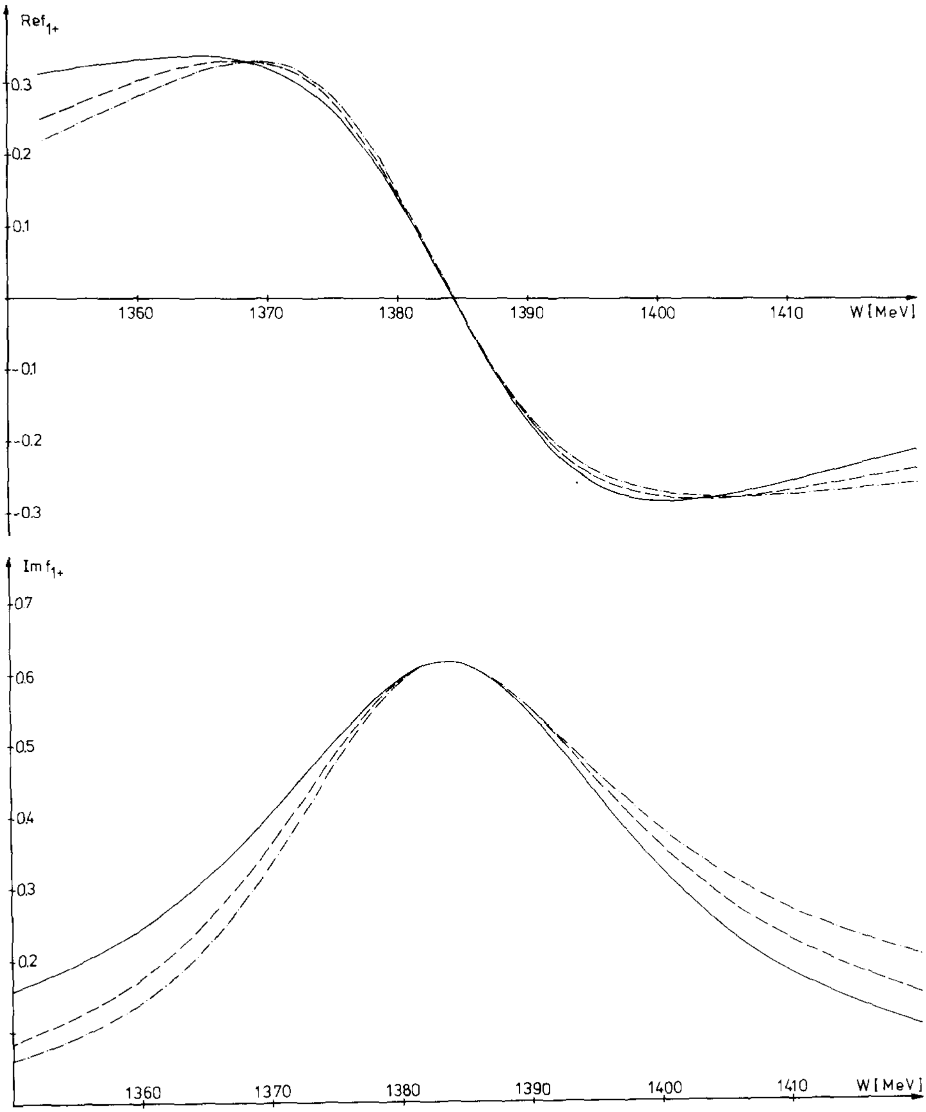

Fig. 2. a. Real part of f_{1+} according to eq. (5.1) for the $\Sigma(1385)$ resonance with different correction factors: —— $k = 1$, – – – k from eq. (5.3) and —·—· $k = (q/q_R)^3$. b. Imaginary part of f_{1+}, same notation as in fig. 2a.

$$\mathrm{Im}\, f_l \sim \begin{cases} q^{-1}, \\ q^{-3}, \end{cases} \quad \text{for } k_l = \begin{cases} (q/q_R)^{2l+1}, \\ \text{from eq.(5.3)}. \end{cases} \tag{5.4}$$

Among the kernels $K_{ll'}(W, W')$ only $K_{ll}(W, W')$ has a singularity in the integration region $M + 1 \leqslant W \leqslant W_N$ and requires a principal-value integration. The corresponding terms in eqs. (4.1a) and (b) are

$$f_{l+,\mathrm{PVI}}(W) = \frac{\mathrm{P}}{\pi} \int_{M+1}^{W_N} \mathrm{d}W' \, \frac{\mathrm{Im}\, f_{l+}(W')}{W' - W} \, \frac{W'}{W} \left(\frac{q}{q'}\right)^{2l} \frac{E+M}{E'+M}, \tag{5.5a}$$

$$f_{(l+1)-,\mathrm{PVI}}(W) = \frac{\mathrm{P}}{\pi} \int_{M+1}^{W_N} \mathrm{d}W' \, \frac{\mathrm{Im}\, f_{(l+1)-}(W')}{W' - W} \, \frac{W'}{W} \left(\frac{q}{q'}\right)^{2l} \frac{E-M}{E'-M}. \tag{5.5b}$$

In figs. 3, 4 and 5 we present, in the respective resonance regions, the real parts, the principle value integrals and the sum of the remaining resonance contributions to the CGLIN relations for the $P_{\frac{3}{2}}$, $D_{\frac{3}{2}}$ and $S_{\frac{1}{2}}$ partial waves as calculated from the six established resonances. The difference

$$\begin{aligned} \Delta_{l\pm}(W) &= \mathrm{Re}\, f_{l\pm} - f_{l\pm,\mathrm{PVI}} - (\text{remaining resonance contributions}) \\ &= f_{l\pm,\Sigma\text{-pole}} + f_{l\pm,\mathrm{Regge}}, \end{aligned} \tag{5.6}$$

is also shown. As expected, it is a slowly varying function. The situation in the cases of the $\Sigma(1385)$ and $\Sigma(1670)$ resonances is quite similar: $\Delta_{l\pm}$ is smaller than $\mathrm{Re}\, f_{l\pm,\mathrm{PVI}}$; except in a small neighborhood of the resonance position, the difference between $\Delta_{l\pm}$ and $f_{l\pm,\mathrm{PVI}}$ is of the order of $\Delta_{l\pm}$. We have a quite different picture for the $\Sigma(1750)$ resonance: Δ_{0+} is at least a factor 30–40 times $\mathrm{Re}\, f_{0+}$; $\mathrm{Re}\, f_{0+} - f_{0+,\mathrm{PVI}}$ is practically zero. The huge Δ_{0+} is produced to 97% by the $\Sigma(1385)$ contribution and it is independent of a variation of the total and partial widths of the $\Sigma(1750)$ resonance.

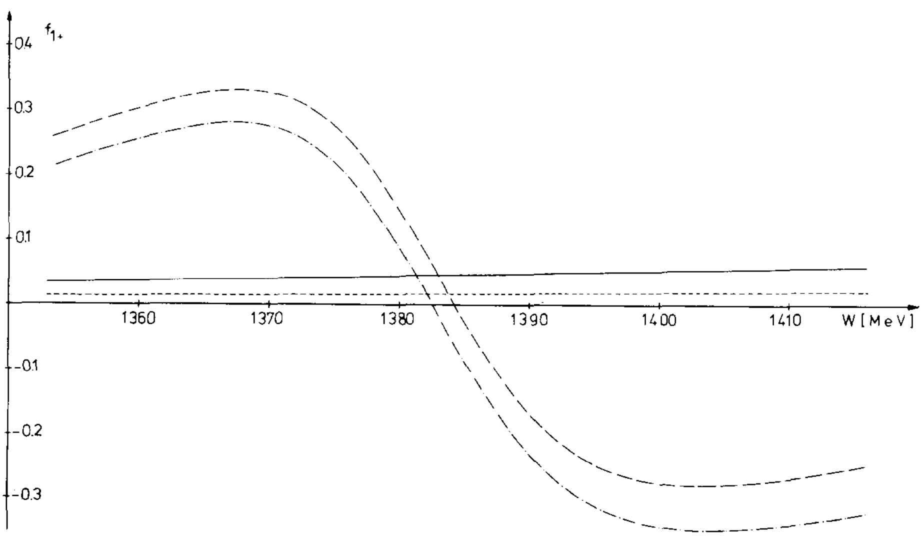

Fig. 3. Real part f_{1+} — —, Principal value integral $f_{1+,\mathrm{PVI}}$ —·—·, remaining resonance contributions ----, Δ_{1+}—— all in the region of the $\Sigma(1385)$ resonance.

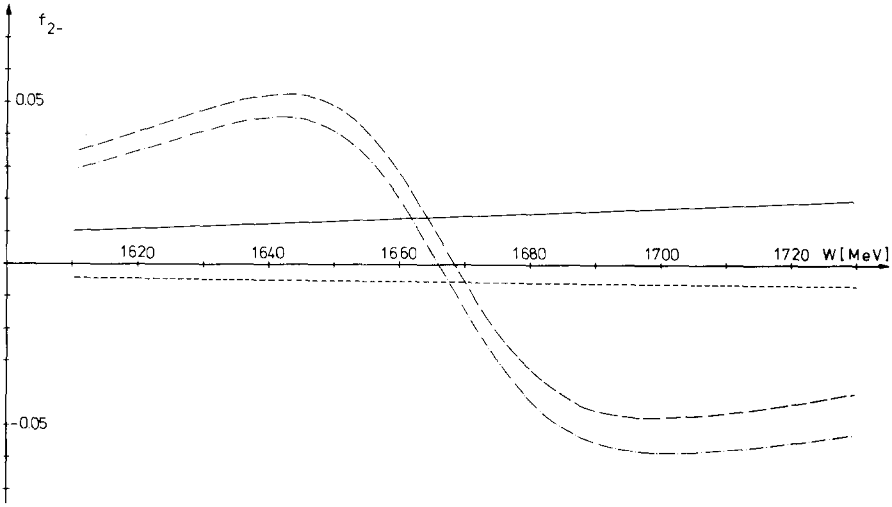

Fig. 4. The same notation as in fig. 3, amplitude f_{2-} in the region of the $\Sigma(1670)$ resonance.

5.2. *The Σ-pole term and the Regge contributions*

The energy dependence of Δ_{0+} is equal to that of the Σ-pole term $f_{0+,\Sigma\text{-pole}}$. Let us therefore assume for a moment that there are no Regge contributions to the $S\frac{1}{2}$ wave. The result for the $\Sigma\Lambda\pi$ coupling constant is then

$$\frac{G^2_{\Sigma\Lambda\pi}}{4\pi} = 11.51\ (10.90), \tag{5.7}$$

where the number in parenthesis is obtained if the less certain resonances are included in the calculation. Determining now the Σ-pole contributions to the $P\frac{3}{2}$ and $D\frac{3}{2}$ waves it turns out that they can not explain Δ_{1+} and Δ_{2-}. It is clear that only a simultaneous fit of Δ_{1+}, Δ_{2-} and Δ_{0+} by the Σ-pole and Regge terms can solve this problem.

For simplicity we assume in the following that all Regge residues are constants except for β_f

$$\beta_f(t) = \alpha_f(t)\bar{\beta}_f, \tag{5.8}$$

with $\bar{\beta}_f$ = const. By eq. (5.8) we ensure that no ghost state appears at $\alpha_f = 0$. Throughout the calculation we fix $N_0 = 1$, $\alpha_p(t) = 1$, $\alpha'_f = \alpha'_\epsilon = 0.0175 = 0.9\ (\text{GeV}/c)^{-2}$ and we take $W_N = 2.5$ GeV, i.e. $N(t = 0) = 16$, which is about the end of the resonance region. So, finally the six free constants γ_p, $\bar{\beta}_f$, γ_f, β_ϵ, γ_ϵ and $G^2_{\Sigma\Lambda\pi}/4\pi$ remain. However, there exists some information (see table 2) on these parameters:

(i) Merlani and Violini [13] applied FESR to $\pi\Lambda$ forward scattering with an input from a $\bar{K}N$ zero-range K-matrix and obtained for the asymptotic total cross section $\sigma_{\pi\Lambda} = 17 \pm 2.0$ mb; Queen [14] evaluated a $\pi\Lambda$ forward dispersion relation for the inverse amplitude and found 15 mb $< \sigma_{\pi\Lambda} <$ 50 mb. This is in agreement with the quark-model prediction [15]

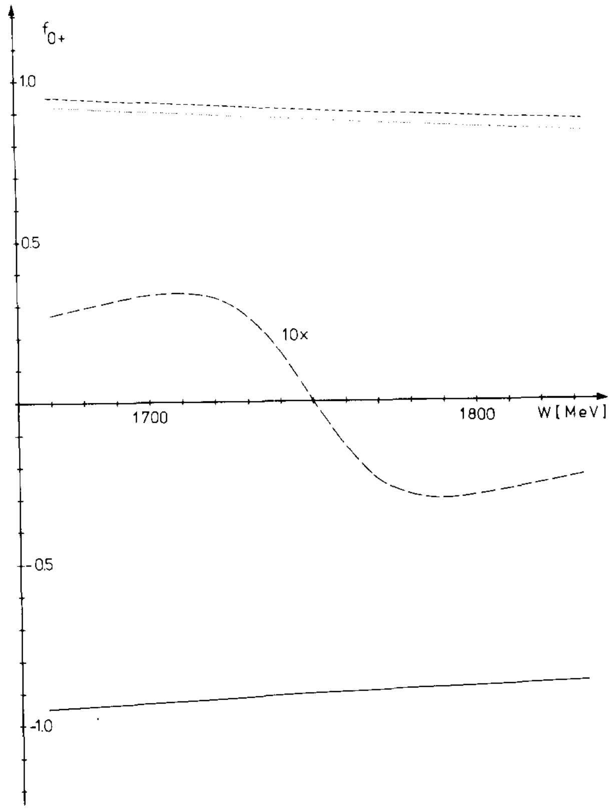

Fig. 5. Amplitude f_{0+} in the region of the $\Sigma(1970)$ resonance, the real part coincides with the principal value integral, it was multiplied by 10 — —, remaining resonance contributions ---, $\Sigma(1385)$ contribution ...., Δ_{0+} ——.

$$\sigma_{\pi\Lambda} = \tfrac{1}{3}\,\sigma_{\pi N} + \tfrac{2}{3}\,\sigma_{KN}, \tag{5.9}$$

which gives $\sigma_{\pi\Lambda}$ = 19 mb, if $\sigma_{\pi N}$ = 23 mb and σ_{KN} = 17.5 mb are inserted in eq. (5.9) (the hypothesis that the pomeron is an SU(3) singlet leads to the prediction $\sigma_{\pi\Lambda} = \sigma_{\pi N} = \sigma_{KN}$). From these numbers we may deduce γ_p – remember that we take $\beta_p = 0$.

(ii) The πN Regge fit of Barger and Philips [16] yields a t-dependent γ_p, if we regard the pomeron again as an SU(3) singlet. The result has the same order of magnitude as the one which was determined from $\sigma_{\pi\Lambda}$. Identifying the P″ trajectory of ref. [16] with our ϵ-trajectory we find a negligible γ_ϵ.

(iii) The value of β_ϵ may be obtained via eq. (3.19) from various estimates on the ϵ coupling constants [17–21].

(iv) Renner and Zerwas [1] suggest from $\pi\Lambda$ and $\pi\Sigma$ FESRs combined with the Adler conditions non-negligible values of $\bar{\beta}_f$ and β_ϵ in contrast to Engels and Pilkuhn

Table 2

Ref.	Method	$G^2_{\Sigma\Lambda\pi}/4\pi$	$\bar{\beta}_f$	β_ϵ	γ_p	γ_ϵ
[13]	$\pi\Lambda$ FESR, input: KN–K–matrix				0.85 ± 0.17 a)	
[14]	inverse forward $\pi\Lambda$ dispersion rel.	$\approx 10 \pm 10$			> 0.75 < 2.5 a)	
[15]	quark model				0.95 a)	
[16]	πN Regge fit α) P+P′+P″ β) P+P′				α) 0.83–0.4 β) 0.60–0.35 b)	α) 0.0– –3.5 β) 0.0 b)
[17]	$\pi\pi$ and πN N/D calculations and analytic continuation			9.8 c) 7.0 d)		
[18]	πN fixed-angle dispersion relations with δ^0_0 up-down			6.9		
[19]	πN fixed-u dispersion relations			8.6		
[20]	πN backward dispersion relations			3.1		
[21]	πN backward dispersion relations			9.5		
[1]	FESR plus Adler condition for $\pi\Lambda$ and $\pi\Sigma$	11.5 f)	3.3 e)	19.5 e)		
[22]	unsubtracted fixed-t dispersion relations and Adler condition for $\pi\Lambda$ and $\pi\Sigma$ scattering	12.5	0.0	0.0		

a) For $\beta_p = 0$. b) P or P″ treated as SU(3) singlet. c) δ^0_0 down-up or up-up. d) δ^0_0 down-down or up-down. e) For $N(0) = 16$. f) Was used as input from Goldberger–Treiman relation for $\Sigma \to \Lambda e\bar{\nu}$ (ref. [24]).

[22], who worked with unsubtracted A dispersion relations, i.e. $\beta_p = \overline{\beta}_f = 0$ and neglected β_ϵ.

In table 3 we show the result of various simultaneous fits of Δ_{1+}, Δ_{2-} and Δ_{0+}. The numbers in parenthesis were used as input. For each input we have made two fits with slightly different weights for the three quantities. The chi squared are

$$\chi^2_{l\pm} = \sum_{i=1}^{n} (\Delta_{l\pm}(W_i) - f_{l\pm,\Sigma\text{-pole}}(W_i) - f_{l\pm,\text{Regge}}(W_i))^2 , \tag{5.10}$$

where the sum extends over about fifty points W_i in the respective resonance region of a partial wave.

It is clear that the more parameters we leave free the more we get unreasonably big and compensating single contributions to the fit curve. In particular γ_p and γ_ϵ tend to become too big. Therefore we have fixed $\gamma_p = 0.85$ and $\gamma_\epsilon = 0.0$ in most fits. A variation of γ_ϵ in the range $0.0 > \gamma_\epsilon > -10.0$ has anyhow no effect. As it should be – to fulfill for instance the Adler condition – the magnitudes of $\overline{\beta}_f$ and β_ϵ are strongly correlated; a big β_ϵ implies also a big $\overline{\beta}_f$. However, for the β_ϵ values derived from ϵ coupling constants, $\overline{\beta}_f$ is small and compatible with zero, as would be expected if the A-amplitude satisfies an unsubtracted fixed-t dispersion relation. The parameter values proposed by Renner and Zerwas [1] can, on the other hand, not be excluded, though they do not give such good fits as with a small $\overline{\beta}_f$.

The output value for $G^2_{\Sigma\Lambda\pi}/4\pi$ is remarkably invariant against different parameter inputs and its N-dependence is unimportant. If we include the less certain resonances the value of $G^2_{\Sigma\Lambda\pi}/4\pi$ is always lowered by about 0.6 just as in eq. (5.7). Our final result with a crude error estimate is then

$$\frac{G^2_{\Sigma\Lambda\pi}}{4\pi} = 12.9 \pm 0.8. \tag{5.11}$$

Keeping γ_p and γ_ϵ fixed as mentioned above the result for γ_f is in the range $6 < \gamma_f < 15$ (the less certain resonances give a somewhat lower value). From eq. (3.10b) evaluated for πN scattering and eq. (3.14) we have

$$G^{(1)}_{f\Lambda\Lambda} = G^{(1)}_{fNN} \, \frac{2(1 - \frac{2}{3}D^{(1)}/F^{(1)})}{3 - D^{(1)}/F^{(1)}} . \tag{5.12}$$

Since $G^{(1)}_{fNN}$ is rather well-known [21, 23] we may estimate $D^{(1)}/F^{(1)}$

$$3.3 < \frac{D^{(1)}}{F^{(1)}} < 4.1. \tag{5.13}$$

Finally we show in figs. 6–8 a decomposition of the contributions to $\Delta_{l\pm}$ for a representative simultaneous fit.

Table 3

$G^2_{\Sigma\Lambda\pi}/4\pi$	$\bar{\beta}_f$	β_ϵ	γ_p	γ_f	γ_ϵ	χ^2_{1+}	χ^2_{2-}	χ^2_{0+}
(11.5)	(3.3)	(19.5)	2.52	223.0	-5963.0	3.87×10^{-3}	2.93×10^{-4}	1.81×10^{-2}
			3.71	322.0	-8798.0	3.47×10^{-3}	3.97×10^{-4}	2.27×10^{-2}
13.0	(3.3)	(19.5)	(0.85)	13.0	(0.0)	2.46×10^{-3}	7.41×10^{-5}	4.81×10^{-1}
11.9				13.4		4.30×10^{-3}	7.28×10^{-5}	2.14×10^{-2}
(11.5)	(3.3)	(19.5)	(0.85)	8.3	(0.0)	5.95×10^{-3}	3.82×10^{-4}	1.99×10^{-2}
				13.9		4.96×10^{-3}	8.12×10^{-5}	6.24×10^{-2}
(12.5)	(0.0)	(0.0)	6.78	656.0	-18070.0	4.90×10^{-3}	6.07×10^{-4}	1.79×10^{-2}
			8.72	820.0	-22780.0	4.16×10^{-3}	8.40×10^{-4}	2.55×10^{-2}
(12.5)	(0.0)	(0.0)	(0.85)	11.3	(0.0)	1.19×10^{-2}	1.63×10^{-3}	1.19×10^{-2}
				21.3		9.43×10^{-3}	8.73×10^{-6}	1.24×10^{-1}
13.2	(0.0)	6.0	(0.85)	13.1	(0.0)	1.06×10^{-4}	2.26×10^{-5}	5.60×10^{-3}
13.2 [a]		6.3		12.6		9.97×10^{-5}	2.50×10^{-5}	4.16×10^{-3}
13.3	-0.304	(4.5)	(0.85)	13.5	(0.0)	8.52×10^{-5}	1.76×10^{-5}	4.36×10^{-3}
13.3	-0.343			13.2		8.47×10^{-5}	1.84×10^{-5}	3.59×10^{-3}
13.2	0.148	(6.9)	(0.85)	12.7	(0.0)	1.08×10^{-4}	2.65×10^{-5}	6.42×10^{-3}
13.1	0.109			12.4		1.05×10^{-4}	2.74×10^{-5}	4.36×10^{-3}
13.1	0.648	(9.5)	(0.85)	11.8	(0.0)	1.35×10^{-4}	3.79×10^{-5}	1.38×10^{-2}
12.9	0.600			11.6		1.32×10^{-4}	3.90×10^{-5}	5.30×10^{-3}

[a] Results are shown in figs. 6–8.

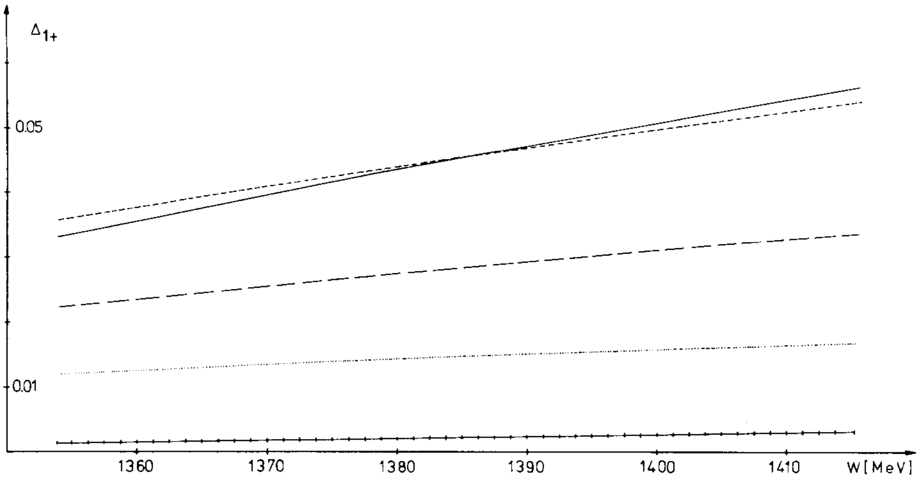

Fig. 6. Δ_{1+} —— and the fit curve ------, $f_{1+,\Sigma\text{-pole}}$ — —, ϵ-trajectory contribution from the A-amplitude ·········, f-trajectory contribution from the B-amplitude +++++, the pomeron B-amplitude contribution ·—·—· was too small to be shown.

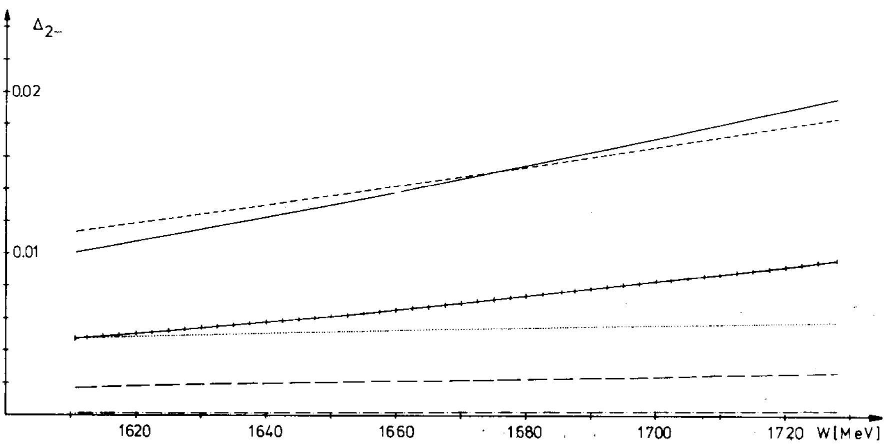

Fig. 7. The same notation as in fig. 6, amplitude f_{2-}.

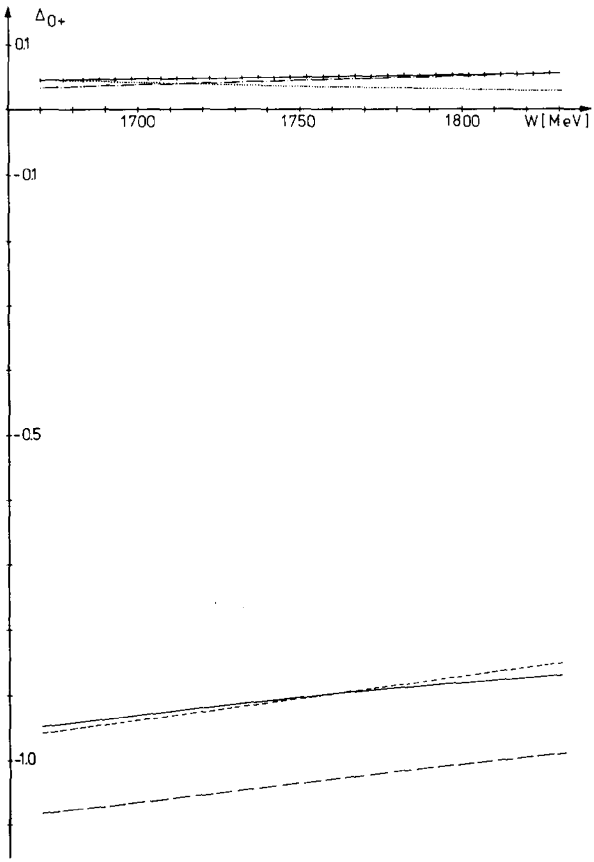

Fig. 8. The same notation as in fig. 6, amplitude f_{0+}.

6. Summary and conclusion

We have derived partial wave relations from finite-contour dispersion relations at fixed $-t$ in $\pi\Lambda$ scattering. The high-energy behaviour of the full amplitudes was described in terms of the pomeron, f and ϵ Regge poles. As input we have used experimental information on the Σ resonances [12] and some estimates [13–16] on the magnitude of the asymptotic total $\pi\Lambda$ cross section. Finally it was assumed that the pomeron contribution to the A-amplitude is zero, which may be justified when either the pomeron is treated as SU(3) singlet or s-channel helicity conservation for $\pi\Lambda$ scattering is true. The evaluation of the relations for the $P\frac{3}{2}$, $D\frac{3}{2}$ and $S\frac{1}{2}$ partial waves in the regions of their respective first resonances has led to the following results:

(i) The value of the $\Sigma\Lambda\pi$ coupling constant is

$$\frac{G^2_{\Sigma\Lambda\pi}}{4\pi} = 12.9 \pm 0.8. \tag{5.11}$$

This number is determined up to about 10% already by the experimentally well-known contribution of the $\Sigma(1385)$ resonance to the $S_{\frac{1}{2}}$ wave relation. The prediction (5:11) is in good agreement with the result obtained from the Goldberger-Treiman relation for the $\Sigma \to \Lambda e\bar{\nu}$ decays ($G^2_{\Sigma\Lambda\pi}/4\pi = 11.4 \pm 1.2$) (ref. [24]) and other, less accurate determinations [2, 25, 26, 27].

(ii) It was not possible to decide definitely whether the fixed-t dispersion relation for the A-amplitude has to be subtracted or not – in our formulation of the problem we have an unsubtracted dispersion relation for A, when the f-trajectory decouples from the A-amplitude. However, if we take the ϵ Regge residue β_ϵ as determined from the ϵ coupling constants as input to our fits, we get as fit output a negligible f-trajectory coupling $\overline{\beta}_f$ to the A-amplitude. The input $\overline{\beta}_f = 0$ produces on the other hand a β_ϵ output, which is well compatible with the known ϵ coupling constants. Moreover, both fits are better than those obtained with the $\overline{\beta}_f$ and β_ϵ values of Renner and Zerwas [1]. So, it may well be that the SU(3) covariant coupling of the tensor meson nonet to the baryon octet is zero for the A-amplitude. Engels and Pilkuhn [22] combined an unsubtracted fixed-t dispersion relation for A with the Adler condition and obtained – with resonance input only – $G^2_{\Sigma\Lambda\pi}/4\pi = 12.5$. The difference to our result (5.11) is now easily explained: Engels and Pilkuhn did not include the ϵ Regge contribution.

(iii) The D/F ratio for the tensor meson-baryon coupling in the case of the B-amplitude is in the range $3.3 < D/F < 4.1$.

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