

Damping rate of a fast fermion in hot QED

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The self-consistent determination of the damping rate of a fast moving fermion in a hot QED plasma is reexamined. We argue how a detailed investigation of the analytic properties of the retarded fermion Green's function motivated by the cutting rules at finite temperature may resolve ambiguities related to the proper definition of the mass-shell condition.

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I. INTRODUCTION

Recent studies of damping rates of fast moving particles in a hot QED (or QCD) plasma have raised some interesting problems [1–11]. The difficulties start from the infrared sensitive behavior of these rates: A logarithmic divergence remains even after including the perturbative Braaten-Pisarski resummation [12, 13], which only screens the infrared sensitivity down to scales of $O(eT)$. In the case of QED, which has no magnetic mass to serve as infrared cutoff at the scale $O(e^2T)$, a self-consistent determination of the rate γ of a fast moving fermion in the plasma is the most elegant solution, as first suggested by Lebedev and Smilga [2]. Here γ also plays the role of the infrared cutoff. In the analysis considered until now it is always assumed (even explicitly) that the retarded propagator of the fast fermion (more precisely its analytically continued form) has a true complex pole in the lower energy half plane at the position $E - i\gamma$, with E the energy of the fermion. However, the presence of this pole does not allow an infrared stable solution for γ when the on-shell condition at this complex pole is required [6]: This is indeed the favorable and consequent condition in the case of a true pole on the “physical” Riemann sheet.

Based on the analytic structure of the retarded Green's functions as deduced from the general properties of spectral functions we argue in the following that the complex pole in question is actually not on the “physical” sheet, and we construct an explicit and simple example which should represent the physical (realistic) situation and which may help to clarify the problem of self-consistency for γ [14]. By this attempt we take the point of view that the damping rate of a fast moving fermion in a QED heat bath is a physical quantity, contrary to the

arguments given in Ref. [8]. Therefore an infrared finite result is to be aimed for it.

II. RETARDED GREEN'S FUNCTION AT FINITE TEMPERATURE

First it is convenient to illustrate some properties of the retarded Green's function $G^R(p_0, p)$ [16–20]. It is defined as the boundary value of a complex function $G(z, p)$ as the complex argument z approaches the real axis from above:

$$G^R(p_0, p) \equiv G(z = p_0 + i\epsilon, p), \quad (1)$$

where the function $G(z, p)$ is determined by a spectral function $\rho(p_0, p)$ as

$$G(z, p) = \int_{-\infty}^{+\infty} \frac{dp'_0}{p'_0 - z} \rho(p'_0, p). \quad (2)$$

If $\rho(p_0, p)$ is a Lorentzian of the form

$$\rho(p_0, p) = \frac{\gamma/\pi}{[p_0 - E(p)]^2 + \gamma^2}, \quad \gamma > 0, \quad (3)$$

for all values of p_0 , $-\infty < p_0 < +\infty$, then the retarded Green's function becomes

$$G^R(p_0, p) = \int_{-\infty}^{+\infty} \frac{dp'_0}{p'_0 - (p_0 + i\epsilon)} \frac{\gamma/\pi}{[p'_0 - E(p)]^2 + \gamma^2} = -\frac{1}{p_0 + i\epsilon - [E(p) - i\gamma]}, \quad (4)$$

where we first keep p_0 on the real axis. However, this explicit function may be analytically continued to complex values of p_0 : For $\text{Im } p_0 > 0$, G^R is regular as it should be from the definition (1), but for $\text{Im } p_0 < 0$ the analytically continued G^R has a true pole at $p_0 = E - i\gamma$. The origin of this pole is easily understood when considering the situation in the p'_0 plane [21, 22]. As illustrated in Fig. 1(a) for the continuous integration contour $-\infty < p'_0 < +\infty$,

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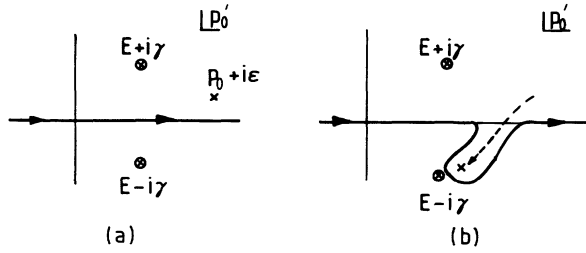


FIG. 1. Integration contour for the retarded Green's function. Poles of the Lorentzian form of Eq. (3) are indicated by \otimes . In (a) p_0 is real, while (b) shows the pole singularity for $\text{Im} p_0 < 0$ which arises from the pinching of the contour between the two singularities $p'_0 = p_0$ and $p'_0 = E - i\gamma$ of the integrand.

$$G^R(p_0, p) = \int_{p_{\text{th}}}^{\infty} \frac{dp'_0}{p'_0 - (p_0 + i\epsilon)} \frac{\gamma/\pi}{[p'_0 - E(p)]^2 + \gamma^2} = \frac{1}{2\pi i} \left\{ \frac{1}{p_0 - [E(p) - i\gamma]} \ln \left[\frac{p_{\text{th}} - p_0 - i\epsilon}{p_{\text{th}} - E(p) + i\gamma} \right] - \frac{1}{p_0 - [E(p) + i\gamma]} \ln \left[\frac{p_{\text{th}} - p_0 - i\epsilon}{p_{\text{th}} - E(p) - i\gamma} \right] \right\}. \quad (5)$$

In the p'_0 plane the situation looks as in Fig. 2. One immediately sees that when $p'_0 = p_0 + i\epsilon$ approaches $p'_0 = E - i\gamma$, for example, the p'_0 -integration contour is not necessarily pinched, and so there is *no* pole at $p_0 = E - i\gamma$ in the analytically continued G^R on the first (“physical”) sheet. Instead, there is an end point singularity at $p_0 = p_{\text{th}}$, which is a logarithmic branch point, and the discontinuity across this cut on the real axis is given (by construction) by the Lorentzian $\rho(p_0, p)$ of Eq. (3). With respect to this cut the pole has been moved onto the other sheets obtained after continuation of the logarithm, $\ln(p_{\text{th}} - p_0 - i\epsilon)$: This amounts to deforming the contour in such a way that a pinch is present [cf. Fig. 1(b)].

The behavior with respect to $p_0 = E + i\gamma$ is symmetric with respect to the one at $p_0 = E - i\gamma$; on the first sheet $G^R(p_0, p)$ of Eq. (5) is regular for $\text{Im} p_0 > 0$ and for $\text{Im} p_0 < 0$, with the only singularity on this sheet that of the logarithmic branch point at $p_0 = p_{\text{th}}$.

The case just described is in close analogy to the discussion of resonances (at zero temperature): With re-

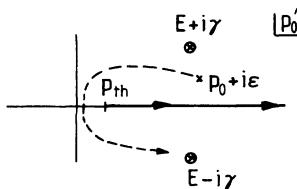


FIG. 2. Integration contour for Eq. (5), $p_{\text{th}} \leq p'_0 < \infty$, indicating that there is no pinch singularity on the first sheet at $p'_0 = p_0 - E \mp i\gamma$ with respect to the end point singularity at $p_0 = p_{\text{th}}$.

the singularities of the integrand are at $p'_0 = p_0 + i\epsilon$, and, due to the Lorentzian at $p'_0 = E \pm i\gamma$, indicated by crosses. The contour for the retarded function is below the pole at $p'_0 = p_0 + i\epsilon$, indicated by $\epsilon > 0$. In order to continue G^R into the lower half plane, $\text{Im} p_0 < 0$, the contour has to be deformed [Fig. 1(b)], and when $p'_0 = p_0 + i\epsilon$ approaches $p'_0 = E - i\gamma$ it is pinched, resulting in the pole of G^R at $p_0 = E - i\gamma$. For the continuous contour (Fig. 1) this pinch is always present when the continuation to values for $\text{Im} p_0 < 0$ is performed.

However, the situation becomes different when, for instance, it is assumed that the spectral function ρ is non-vanishing at some threshold $p_0 \geq p_{\text{th}}$, but otherwise it is zero. If, for example, ρ is given for $p_0 \geq p_{\text{th}}$ by Eq. (3) with $E(p) > p_{\text{th}}$, the retarded function is

spect to the variable of the energy squared resonances are on the unphysical sheet, and the branch point is determined by the threshold properties of scattering amplitudes.

III. FERMION DAMPING RATE

As the simplest case we consider the damping rate of a heavy fermion (“muon”) of mass M in a hot QED plasma of (massless) electrons and photons. The energy, the momentum, and the velocity of the “muon” are denoted by E , \mathbf{p} ($p = |\mathbf{p}|$), and v , respectively, but the main interest is in the limit $v \rightarrow 1$ [6, 9, 10].

Since we are only concerned with the leading order behavior $e \rightarrow 0$ at high temperature T , for a fast muon ($E \gg M > T$) a couple of approximations which simplify the calculations and the discussion may be applied [1–10]. With $-(G^R)^{-1} = p_0 - \Sigma^R$ and Eq. (4) we relate in the usual way γ to the imaginary part of the muon self-energy by $\gamma(p_0, p) = -\text{Im} \Sigma^R(p_0, p)$, with p_0 real. The explicit expressions are derived, for example, in detail in [10], which we closely follow concerning conventions and notation, and we start off with Eq. (17) of this reference. To leading order $v \rightarrow 1$ we have

$$\gamma(p_0, p) \simeq +e^2 T \int_{\text{soft}} \frac{d^3 q}{(2\pi)^3} \int_{-q}^{+q} \frac{dq_0}{q_0} \rho_t(q_0, q) \times \text{Im} \hat{G}^R(p_0 - q_0, \mathbf{p} - \mathbf{q}). \quad (6)$$

The following remarks summarize the results for γ obtained so far in the literature.

(i) Equation (6) is derived in the one-loop approximation, in which the hard energetic “muon” emits or absorbs one soft photon of four-momentum q^μ . As well,

corrections of order $1/E$ are neglected.

(ii) Only the dominant transverse photon contribution has to be taken into account, which following the Braaten-Pisarski hard thermal loop resummation method [12] is determined by its spectral density $\rho_t(q_0, q)$; its explicit form is given in [23], but for the following we only require the (approximate) integral

$$\int_{-q}^{+q} \frac{dq_0}{q_0} \rho_t(q_0, q) \simeq 1/q^2 \quad (7)$$

when $q \leq eT$ [6, 9, 10].

(iii) All the complications due to the spin of the fermion are suppressed in Eq. (6), in the sense that the heavy fermion propagator is described by a (retarded) scalar function \hat{G}^R .

(iv) Inserting for a bare fermion $-(\hat{G}^R)^{-1} = p_0 + i\epsilon - E(p)$, $E(p) = \sqrt{\mathbf{p}^2 + M^2}$, into Eq. (6), we have

$$\begin{aligned} \text{Im } \hat{G}^R(p_0 - q_0, \mathbf{p} - \mathbf{q}) \Big|_{q_0 \rightarrow 0} \\ \simeq \pi \delta(p_0 - \sqrt{(\mathbf{p} - \mathbf{q})^2 + M^2}), \end{aligned} \quad (8)$$

and one immediately finds the infrared divergent result

$$\gamma(p_0 \simeq E, p) \simeq \frac{e^2}{4\pi} T \int \frac{e^T dq}{q}. \quad (9)$$

This is mainly due to soft photon exchange, and it is the origin of the problems with the hard fermion damping rate. It also shows that to leading order only the infrared sensitive region $|q_0| \leq q \rightarrow 0$ has to be considered. It is worth noting that the coefficient in Eq. (9) is gauge parameter independent [3, 5].

(v) In hot QED there is no (nonperturbative) magnetic mass to provide a cutoff to the logarithmic infrared divergence in Eq. (9) [19]. In QCD the magnetic mass is expected to be on the scale $m_{\text{mag}} \simeq g^2 T$, with g the strong coupling constant: Consequently using m_{mag} and evaluating γ on the real axis ($p_0 \simeq E$) a finite value may be, and has been, argued for quarks in QCD [3, 4, 7].

(vi) From Eq. (9) γ is “anomalous,” in that its magnitude is on the scale $e^2 T$ (neglecting $\ln e$ factors for the moment). Therefore it has been first conjectured by Lebedev and Smilga [2] to use γ itself as a possible infrared cutoff. This opens the possibility of a self-consistent calculation of γ by replacing \hat{G}^R in Eq. (6) by the simple Lorentzian of Eq. (4) [2]. In Ref. [2] it is shown that only the fermion propagator should be modified, but not the photon one; also *no* vertex corrections are required.

(vii) The self-consistency requirement for the damping rate requires, however, clarification of the “on-shell” condition used to evaluate Eq. (6). On the one hand, as used in Ref. [2], one can keep in Eq. (6) p_0 on the real axis, in which case $p_0 = E(p)$ is used. As follows from the remarks after Eq. (9), such a self-consistent determination of the damping rate will lead to a gauge invariant result to leading order. On the other hand, one can generalize Eq. (6) to complex p_0 by demanding that it hold at the complex pole $p_0 = E(p) - i\gamma$ under the narrow width

assumption $\gamma \ll E(p)$. It is crucial to point out that under the assumption that G^R is given by Eq. (4) the complex pole is on the first (and only) sheet in the energy plane; therefore the second case of self-consistency could be argued as the physical one, and is in any case a point at which gauge invariance can formally be proved to hold [24]. However, as shown in Ref. [6], in this case the infrared divergence is not screened by a nonvanishing γ . Therefore, this attempt fails for QED when Eq. (4) is used as a model for the “dissipative” retarded fermion propagator, and as such the narrow width condition in the form $\text{Im } \Sigma^R(p_0 = E) \simeq \text{Im } \Sigma^R(p_0 = E - i\gamma)$ does not hold.

In addition to this question of the proper on-shell condition to use, it is not clear that this particular form of the self-consistent determination of the damping rate will lead to a gauge invariant result beyond the leading order considered here. This and other considerations have led some to conclude that the fermion damping rate in QED is not directly observable [8, 33]. We however take the point of view that the ambiguity associated with the on-shell condition indicates that the ansatz of Eq. (4) for the retarded fermion propagator does not reflect the proper physical conditions, and that instead it is realistic to use as a simple model of the spectral function the form given by Eq. (5), which allows for branch cuts on the real axis in the energy plane. Implicit in this viewpoint is the assumption that a self-consistent scheme to determine the damping rate in a gauge invariant manner can be found which also works beyond the leading order.

IV. SPECTRAL FUNCTION AND CUTTING RULES

When discussing the location of branch points and cuts in self-energy functions at nonvanishing temperatures it is useful to recall that Weldon [25] has shown, starting from the one-loop approximation and extrapolating to the many-particle case, that the branch points are determined by the $T = 0$ masses of the particles in the heat bath. One can see this by considering the spectral function directly, where one notes that it is evaluated, independent of $T = 0$ or $T \neq 0$, by using the energy eigenstates of the (full) Hamiltonian, $H|n\rangle = E_n|n\rangle$. For example, for fermions the resulting ρ at finite T has the structure [17, 18]

$$\begin{aligned} \rho(p_0, \mathbf{p}) \doteq \sum_{m,n} (2\pi)^4 \delta(p_0 - (E_n - E_m)) \delta(\mathbf{p} - (\mathbf{p}_n - \mathbf{p}_m)) \\ \times e^{-E_m/T} (1 - e^{-p_0/T}) |\langle m|\psi|n\rangle|^2, \end{aligned} \quad (10)$$

where ψ denotes the Dirac field operator. Obviously the energies E_n and momenta \mathbf{p}_n appearing in the δ functions are temperature independent, and therefore so are the positions of the branch points according to the cutting rules [25, 26].

Although the positions of the cuts do not depend on temperature, the discontinuities across the cuts become temperature dependent. Let us consider the one-loop $g\bar{\psi}\psi\phi$ self-energy example of Weldon [25]. The cut structure for the fermion self-energy with mass M is reproduced in Fig. 3 in terms of the variable $s = p_0^2 - \mathbf{p}^2$, as

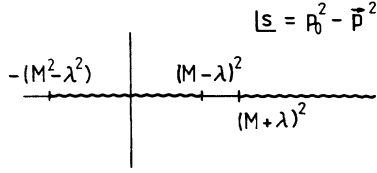


FIG. 3. Location of the branch cuts (in the one-loop approximation) for the thermal fermion propagator [25]. λ denotes the $T = 0$ photon mass ($\lambda \rightarrow 0$), while M is the fermion mass.

at zero temperature a small (zero temperature) photon mass λ is introduced, having in mind the limit $\lambda \rightarrow 0$ whenever allowed. The cut starting at $s \geq (M + \lambda)^2$ is familiar from zero temperature. The cut between $-(M^2 - \lambda^2) \leq s \leq (M - \lambda)^2$ is due to the absorption or emission of photons from the heat bath, and vanishes for $T \rightarrow 0$; in the limit $M > T$ its discontinuity is exponentially suppressed, and therefore this cut is neglected in the following discussion. In Fig. 4 we plot the discontinuity in this example for both hard and soft regions of external momenta — we consider the two contributions $A(p_0, p)\gamma_0 p_0$ and $MD(p_0, p)$ to $\text{Im}\Sigma^R(p_0, p)$. We have kept the photon mass λ small but finite in these figures in order to differentiate between the two regions $s > (M + \lambda)^2$ and $s < (M - \lambda)^2$ — this is indicated by the break in the curves. In the soft regime of Fig. 4(a) we find that the Landau damping contribution for $s < (M - \lambda)^2$ dominates, as expected, but in the hard regime of Fig. 4(b) the cut coming from $s > (M + \lambda)^2$ starts to dominate.

From this we deduce that the thermal spectral function for the heavy, energetic fermion propagator $-(G^R)^{-1} = p_0 - \Sigma^R(p_0, p)$ has to have a contribution from at least the branch cut for $s \geq (M + \lambda)^2$, i.e., for $|p_0| \geq \sqrt{p^2 + (M + \lambda)^2}$. Note that, in this region, such a contribution comes entirely from the self-energy Σ^R , since there is no pole contribution. Consequently we take as a realistic ansatz for the determination of \hat{G}^R in Eq. (6) the following.

(i) For positive energy there is a single cut starting at $p_{\text{th}} = \sqrt{p^2 + (M + \lambda)^2}$; this is smaller than the energy of the thermally excited heavy fermion, which receives contributions of $O(eT)$ [23, 27, 28], such that $E(p) - p_{\text{th}} \simeq O(e^2 T^2/E)$ for $\lambda \rightarrow 0$ and $E \geq T$. Except for this, in the following the thermal contribution of $O(eT)$ to the mass

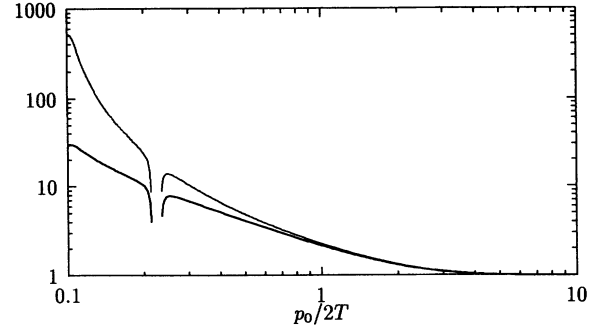
$$\rho(p_0, \mathbf{p}) = \frac{\gamma}{2\pi E(p)} \left[\frac{1}{[p_0 - E(p)]^2 + \gamma^2} - \frac{1}{[p_0 + E(p)]^2 + \gamma^2} \right] \Theta(|p_0| - p_{\text{th}}). \quad (11)$$

The temperature dependence only shows up in $\gamma = \gamma(T)$.

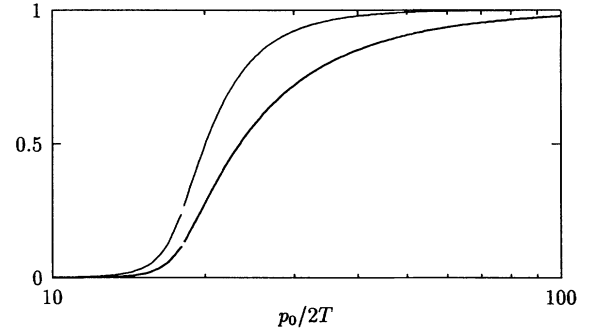
(iv) Under the strong assumption, which we accept for simplicity in the following, that Eq. (11) dominates ρ for all values of p_0 , and not only in the neighborhood of $E(p)$, one may require the sum rule to be satisfied by the ansatz (11):

$$1 = \int_{-\infty}^{+\infty} \rho(p_0, \mathbf{p}) p_0 dp_0; \quad (12)$$

this fixes the normalization given in Eq. (11) when terms of $O(\gamma/E)$ are neglected. This assumption leads to an



(a)



(b)

FIG. 4. The discontinuity of the one-loop fermion self-energy example of Weldon [25]. In (a) we consider the soft region, with $M/2T = 0.2$ and $p/2T = 0.1$, while in (b) we consider the hard region, with $M/2T = 15$ and $p/2T = 10$. The upper line in both figures denotes $A(p_0)/A(p_0 = \infty)$, while the lower line denotes $D(p_0)/D(p_0 = \infty)$.

is neglected because of the limit $M > T$.

(ii) Because of the narrow width condition $\gamma \ll E(p)$, near $p_0 \simeq E(p)$, the cut's discontinuity is dominated by the nearby "pole." We assume that the pragmatic parametrization of this discontinuity is given by the Lorentzian of Eq. (3), assuming γ to be momentum independent (as a calculational simplification).

(iii) In order to respect the symmetry properties of the fermion spectral density [29], $\rho(-p_0, \mathbf{p}) = -\rho(p_0, \mathbf{p})$, the following ansatz is suggested:

overestimate of the damping rate when determined self-consistently.

Considering these points, we take as the retarded Green's function for this simple toy model that following from the spectral function of Eq. (11) [30]:

$$G^R(p_0, \mathbf{p}) = \frac{1}{4\pi E(p)} \left\{ \left[\frac{i}{p_0 + i\epsilon - E(p) + i\gamma} \ln \left(\frac{p_{\text{th}} + E(p) - i\gamma}{p_{\text{th}} - E(p) + i\gamma} \frac{p_{\text{th}} - p_0 - i\epsilon}{p_{\text{th}} + p_0 + i\epsilon} \right) - (\gamma \rightarrow -\gamma) \right] + [E(p) \rightarrow -E(p)] \right\}. \quad (13)$$

This function, more precisely, its analytic continuation in p_0 , does not have poles at $p_0 = \pm E \pm i\gamma$ on the first "physical" sheet with respect to the branch points at $p_0 = \pm p_{\text{th}}$. The discontinuity across the cut starting at $p_0 = \pm p_{\text{th}}$ is given, by construction, by the Lorentzian of Eq. (11) for real values of p_0 , and near $p_0 \simeq E(p)$ we find, for $\gamma \ll E(p)$ and $E \geq T$, the narrow width condition

$$\frac{\text{Im } G^R[p_0 = E(p)]}{\text{Im } G^R[p_0 = E(p) - i\gamma]} \simeq 1. \quad (14)$$

V. SELF-CONSISTENCY FORMULATION

Although we have assumed the spectral function of the propagator has the form of Eq. (11) for all values of p_0 , in the true situation we might expect that this would be only in a neighborhood of $p_0 \simeq E(p)$. Thus, for a self-consistent determination of γ we insert G^R of Eq. (13) into Eq. (6) and evaluate the result at the point $p_0 = E(p)$; to leading order we then have

$$\begin{aligned} \gamma(p_0 \simeq E(p), \mathbf{p}) &\simeq \frac{e^2}{4\pi^2} T \int_0^{eT} dq \int_{-1}^{+1} d \cos \theta \frac{\gamma}{\gamma^2 + q^2 \cos^2 \theta} \\ &\simeq \frac{e^2}{2\pi^2} T \int_0^{eT} \frac{dq}{q} \arctan(q/\gamma) \simeq \frac{e^2}{4\pi} T \int_{\gamma}^{eT} \frac{dq}{q} \simeq \frac{e^2}{4\pi} T \ln \frac{eT}{\gamma} \simeq \frac{e^2}{4\pi} T \ln \frac{1}{e}, \end{aligned} \quad (15)$$

where the infrared "screening" by γ is explicitly exhibited. This then reproduces the original self-consistent derivation of the fast damping rate in Ref. [2], but in this case without a singularity if one had used instead the point $p_0 = E - i\gamma$ on the physical sheet to evaluate Eq. (6). This is consequently consistent with the narrow width assumption of the form $\text{Im } \Sigma^R[p_0 = E(p)] \simeq \text{Im } \Sigma^R[p_0 = E(p) - i\gamma]$.

With minor modifications the preceding mechanism should be applicable to the case of QCD fast damping rates and color relaxation times [31], without having to introduce a magnetic mass as an infrared cutoff.

VI. LARGE TIME BEHAVIOR AND DISCUSSION

In order to interpret γ of Eq. (15) in the context of this toy model, we study the time dependence of the Green's

function $G^R(t)$. This is given by the Fourier transform of the retarded Green's function of Eq. (13),

$$G^R(t) = \int_{-\infty}^{+\infty} dp_0 G^R(p_0) e^{-ip_0 t}, \quad (16)$$

where we now suppress the reference to the spatial momentum \mathbf{p} . Using the spectral representation of $G^R(p_0)$ we obtain, for times t real and positive,

$$\begin{aligned} G^R(t) &= \int_{-\infty}^{+\infty} dp'_0 \rho(p'_0) \int_{-\infty}^{+\infty} \frac{e^{-ip_0 t}}{-p_0 + p'_0 - i\epsilon} dp_0 \\ &= 2\pi i \int_{-\infty}^{+\infty} dp_0 \rho(p_0) e^{-ip_0 t}. \end{aligned} \quad (17)$$

Inserting now the spectral function under consideration, Eq. (11), into Eq. (17), we find

$$\begin{aligned} G^R(t) &= 2\pi i \frac{\gamma}{2\pi E} \int_{p_{\text{th}}}^{\infty} dp_0 \frac{e^{-ip_0 t}}{[p_0 - E(p)]^2 + \gamma^2} + \text{c.c.} \\ &= \frac{2}{E} \left\{ \pi \sin(Et) e^{-\gamma t} - \int_{(E-p_{\text{th}})/\gamma}^{(E+p_{\text{th}})/\gamma} dx \frac{\sin[(E-\gamma x)t]}{x^2 + 1} \right\} \\ &\simeq \frac{1}{E} \left\{ \pi \sin(Et) e^{-\gamma t} + \cos(Et) [e^{-\gamma t} \text{Ei}(\gamma t) - e^{\gamma t} \text{Ei}(-\gamma t)] \right\}, \end{aligned} \quad (18)$$

where $\text{Ei}(x)$ is the exponential integral [32] and the approximation $E \gg T$, for which $(E - p_{\text{th}}) \ll \gamma$ and $(E + p_{\text{th}}) \gg \gamma$,

has been used. One can consider Eq. (18) in two limits — if we assume $\gamma t \gg 1$, then we find

$$G^R(t) \sim \frac{2}{E} \frac{\cos(Et)}{\gamma t} + O\left(\frac{1}{\gamma t}\right)^2, \quad (19)$$

while if we assume $\gamma t \ll 1$, then we obtain

$$G^R(t) \sim \frac{\pi}{E} \sin(Et) + \frac{\gamma t}{E} \{2[\ln(\gamma t) + 1 - \gamma_E] \cos(Et) - \pi \sin(Et)\} + O(\gamma t)^2, \quad (20)$$

where γ_E is Euler's constant. Thus, as noted in Ref. [17] and stressed in Ref. [8], the time dependence of $G^R(t)$ may not necessarily be of an exponential form, even for very large times, and so care must be taken in these cases in characterizing γ as an exponential “damping” rate. Even so, it is still a parameter within the context of the ansatz for the Green's function which remains to be determined, and for this we can use the self-consistent condition derived from Eq. (11): $\gamma \sim -\text{Im} \Sigma^R(p_0 = E)$; the relation $\text{Im} \Sigma^R(p_0 = E) \simeq \text{Im} \Sigma^R(p_0 = E - i\gamma)$ assures us to this order that using the “complex” on-shell condition $p_0 = E - i\gamma$ will lead to the same self-consistent determination of γ .

This toy model thus suggests that a detailed analysis of the singular nature of the propagator could provide a resolution to the ambiguity associated with the on-shell condition to use in this approach for a self-consistent determination of damping rates of moving particles. However, although a gauge-independent result was obtained here to leading order, it is not obvious that using this “pole position” on-shell condition is a gauge invariant procedure to all orders in cases where the propagator involves branch cuts [9, 24]. As well, to go beyond this leading order would require a systematic algorithm which, among other things, most probably involves a (timelike) damp-

ing term for the internal “effective” propagator with a more complicated momentum dependence than the constant term assumed here [34, 35]. Whether such a scheme exists, which already at this leading order goes beyond the hard thermal loop resummation of Braaten and Pisarski [12], and can give a gauge invariant result for the damping rate in principle to all orders, remains to be seen.

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