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Soft photon production rate in resummed perturbation theory of high temperature QCD

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Abstract. We calculate the production rate of soft real photons from a hot quark-gluon plasma using Braaten–Pisarski's perturbative resummation method. To leading order in the QCD coupling constant g we find a logarithmically divergent result for photon energies of order gT, where T is the plasma temperature. This divergent behaviour is due to unscreened mass singularities in the effective hard thermal loop vertices in the case of a massless external photon.

1 Introduction

Theoretical investigations predict the formation of a quark-gluon plasma (QGP) in high-energy heavy ion collisions. Many signatures for this new phase of nuclear matter have been proposed, in particular electromagnetic ones: photon production [1] by the QGP is expected to be an interesting signal, as the mean free path of the photon γ in the thermal medium is expected to be larger than the size of the plasma, at least when the energy of the γ is not too small.

The present paper is concerned with real direct photon production, assuming that the photon is not thermalized. The production rate of hard photons (with energy $E \sim T$, where T is the plasma temperature) has already been studied in great detail [1–7]. Especially when applying the framework of the resummed perturbative expansion of Braaten and Pisarski [8–11], it has been demonstrated that mass singularities due to the exchange of massless quarks are shielded by effects due to Landau damping [5–6]. In the following we are dealing with the soft photon production rate ($E \sim gT$, where g is the QCD coupling constant), whose calculation requires not only to use resummed quark propagators but also dressed vertices. This should allow us to extract the (finite) leading contribution to the production rate, thus completing the list of predictions of electromagnetic signals: real and virtual [12–14] photon rates.

The main result of the present work is that contrary to the hard photon case, for soft real photons the resummation advocated by Braaten and Pisarski does not succeed to screen mass singularities, i.e. in this case the resummed peturbative expansion fails to give a finite contribution at leading order for a physical quantity.

The outline of the paper is the following: in Sect. 2, we briefly review the situation for hard photon production. In Sect. 3, we deal with soft photons, showing the origin of mass singular terms and exhibiting them, then extracting the singular contribution using dimensional regularisation. Section 4 is devoted to a short discussion.

2 Hard photon production rate

The Born calculation of the hard photon production rate, which uses a bare internal quark propagator for the first order annihilation and Compton scattering amplitudes, gives as the leading term [2–7] in the limit of vanishing bare quark mass m:

$$E\frac{\mathrm{d}W}{\mathrm{d}^3\vec{p}} \simeq \frac{e_q^2\alpha\alpha_s}{2\pi^2}\,T^2\mathrm{e}^{-E/T}\ln\left(\frac{ET}{m^2}\right), \tag{1}$$

where $E > T$. α is the fine structure constant, e_q the quark charge and $\alpha_s = g^2/4\pi$.

The obvious problem with (1) is that the photon production rate is divergent when m tends to zero. Thus the leading order Born calculation is not satisfactory.

Indeed, when the momentum transfer of the exchanged quark, e.g. in the Compton process, is soft of $\mathcal{O}(gT)$ one has to resum an infinite set of diagrams contributing to the same order as the Born term. This resummation program has been proposed by Braaten and Pisarski [8–11]. In the case of the propagator it amounts to replace the bare by an effective one, whenever the momentum is soft.

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The main characteristic property of the effective propagator is that it is dynamically screened on the momentum scale of order gT for space-like momenta, due to the mechanism of Landau damping. The soft scale is characterized by the fermion mass induced by temperature, i.e. by $m_f=\sqrt{2\pi\alpha_s/3}\,T$. It acts as an infra-red cutoff, and in (1) one may replace m by m_f [3]. This heuristic manipulation yields in fact the right result in the leading-logarithm approximation.The rigorous calculation [5–6], which also allows to find the numerical coefficient appearing inside the log, leads to:

$$E\frac{\mathrm{d}W}{\mathrm{d}^3\vec{p}}\simeq\frac{e_q^2\alpha\alpha_s}{2\pi^2}T^2\mathrm{e}^{-E/T}\ln\left(\frac{c}{\alpha_s}\frac{E}{T}\right), \tag{2}$$

with $c\simeq 0.23$, when $E>T$.

One obtains a finite production rate for hard photons. The $\ln(1/\alpha_s)$ dependence in (2) is a reminiscence of the logarithmic divergence of the Born term, which indeed becomes dynamically screened after resummation.

In the following we want to know whether the resummed perturbative expansion achieves the same screening of mass singularities in the case of soft photon production, i.e. for photon energies of $\mathcal{O}(gT)$.

3 Soft photon production rate

The production rate may be computed in a systematic way by evaluating the imaginary part of the photon polarization tensor:

$$E\frac{\mathrm{d}W}{\mathrm{d}^2\vec{p}}=-\frac{1}{(2\pi)^3}n_B(E)\,\mathrm{Im}\,\Pi^\mu_\mu(E,\vec{p}), \tag{3}$$

where Π^μ_μ is first calculated in the euclidean formalism*. The Bose–Einstein distribution is denoted by n_B.

When the photon energy E is of order gT, either k and k' (Fig. 1) are soft ($\sim gT$), and both quark propagators have to be resummed, or k and k' are hard ($\sim T$), but the latter contribution is suppressed by a factor g^2 and we shall neglect it. By evaluating the soft photon production rate according to (3) we thus consider both internal quark propagators as soft ones. As the photon momentum $P=k+k'$ is soft, vertices have also to be resummed and the relevant photon polarization tensor entering (3) is shown in Fig. 1.

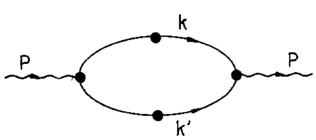

Fig. 1. One-loop diagram for the production of a real soft photon (weavy line) with momentum P. The effective quark propagator and the effective quark-photon vertex are indicated by a blob.

As the internal quark propagators are resummed, we expect that screening occurs as for the hard photons, and no divergence appears when k or k' are vanishing. However, the introduction of effective vertices, though necessary to take into account all diagrams contributing to the rate at leading order in g, will be shown to lead to unscreened collinear divergences.

3.1 *Resummed photon self energy*

In order to evaluate the production rate (3) we first consider Π^μ_μ:

$$\Pi^\mu_\mu(E,\vec{p})=e_q^2e^2N_cT\sum_{k_4}\int\frac{\mathrm{d}^3\vec{k}}{(2\pi)^3}\,\mathrm{tr}[{}^*\Delta(k){}^*\Gamma^\mu(k,k';-P)\cdot{}^*\Delta(-k'){}^*\Gamma_\mu(-k',-k;P)], \tag{4}$$

where N_c is the number of colours and ${}^*\Delta(k)$ is the effective quark propagator:

$${}^*\Delta(k)=\frac{1}{2}\left(\frac{\gamma\cdot k_+}{D_+(k)}+\frac{\gamma\cdot k_-}{D_-(k)}\right), \tag{5}$$

with

$$k_\pm=(1,\pm i\hat{k}),\quad \hat{k}=\vec{k}/|\vec{k}|.$$

The functions $D_\pm(k)$ are given in [13–17]. The effective quark-photon vertex [10–11, 13–14] is represented by:

$${}^*\Gamma^\mu=\gamma^\mu+m_f^2\int\frac{\mathrm{d}\Omega}{4\pi}\frac{Q^\mu Q}{(Qk)(Qk')},$$

$$Q=(i,\hat{Q}). \tag{6}$$

The second term in the r.h.s. of (6) is the hard thermal loop correction-in terms of an angular integral-to the bare vertex γ^μ. Q is a light-like vector, $Q^2=0$; the inner product $Q\cdot k=Q_4k_4+\hat{Q}\cdot\vec{k}$ is denoted by (Qk).

The Dirac trace in (4) is split into three terms according to the number of hard loop corrections, cf. (6): they are denoted by tr(0), tr(1), tr(2), respectively. We get:

$$\Pi^\mu_\mu=e_q^2e^2N_cT\sum_{k_4}\int\frac{\mathrm{d}^3\vec{k}}{(2\pi)^3}[\mathrm{tr}(0)+\mathrm{tr}(1)+\mathrm{tr}(2)], \tag{7}$$

where

$$\mathrm{tr}(0)=2\sum_{i,j=\pm}\frac{(k_ik'_j)}{D_iD'_j}, \tag{7a}$$

$$\mathrm{tr}(1)=-4m_f^2\int\frac{\mathrm{d}\Omega}{4\pi}\frac{1}{(kQ)(k'Q)}\sum_{i,j=\pm}\frac{(k_iQ)(k'_jQ)}{D_iD'_j}, \tag{7b}$$

$$\mathrm{tr}(2)=-m_f^4\int\frac{\mathrm{d}\Omega_1}{4\pi}\int\frac{\mathrm{d}\Omega_2}{4\pi}\frac{(Q_1Q_2)}{(kQ_1)(kQ_2)(k'Q_1)(k'Q_2)}\cdot\sum_{i,j=\pm}\frac{1}{D_iD'_j}[(Q_1k_i)(Q_2k'_j)+(Q_1k'_j)(Q_2k_i)-(Q_1Q_2)(k_ik'_j)]. \tag{7c}$$

The primed quantities depend on the momentum $k'=P-k$.

* Π^μ_μ is evaluated for $p_4=2\pi nT$ and then continued according to $ip_4\to E$. In the imaginary time formalism, the euclidean Dirac algebra $\{\gamma^\mu,\gamma^\nu\}=2\delta^{\mu\nu}$ is used

To obtain the imaginary part of Π^μ_μ we use the identity [13–14]:

$$\operatorname{Im} T \sum_{k_4=2\pi n T} f(\mathrm{i}k_4) f'(\mathrm{i}(p_4-k_4))$$
$$=\pi(1-\mathrm{e}^{E/T}) \int_{-\infty}^{\infty} \mathrm{d}\omega \int_{-\infty}^{\infty} \mathrm{d}\omega' n_F(\omega) n_F(\omega')$$
$$\cdot \delta(E-\omega-\omega')\rho(\omega)\rho'(\omega'), \tag{8}$$

where n_F is the Fermi-Dirac distribution, and ρ, ρ' are the spectral densities associated with f, f', respectively:

$$\rho(\omega)=\lim_{\varepsilon\to 0} \frac{1}{\pi} \operatorname{Im} f(\omega+\mathrm{i}\varepsilon). \tag{9}$$

Equation (8) can be used only if the dependence of Π^μ_μ on ik_4 and ik_4' is factorized. Thus, as proposed by Wong [14], it is convenient to take the discontinuity of Π^μ_μ in the factorized form (7), before integrating over $\mathrm{d}\Omega$.

When the continuation $ik_4 \to \omega + \mathrm{i}\varepsilon$ is performed, the functions with non-vanishing discontinuities are $[D_\pm(k)]^{-1}$, $[(Qk)D_\pm(k)]^{-1}$, $[(Q_1k)\,(Q_2k)D_\pm(k)]^{-1}$ appearing in the terms tr(0), tr(1), tr(2) of (7). The associated spectral densities are denoted by $\rho_\pm, \sigma_\pm, \tau_\pm$, respectively. For space-like momentum k, $\omega<|\vec{k}|$, they read:

$$\rho_\pm=\frac{1}{\pi} \operatorname{Im}[D_\pm]^{-1}=\beta_\pm(\omega,|\vec{k}|), \tag{10a}$$

$$\sigma_\pm=\frac{1}{\pi} \operatorname{Im}[(Qk)D_\pm]^{-1}=\mathbf{P}\left(\frac{1}{Qk}\right)\beta_\pm-\delta(Qk)\alpha_\pm, \tag{10b}$$

$$\tau_\pm=\frac{1}{\pi} \operatorname{Im}[(Q_1k)(Q_2k)D_\pm]^{-1}$$
$$=\left[\mathbf{P}\left(\frac{1}{Q_1k}\right)\mathbf{P}\left(\frac{1}{Q_2k}\right)-\pi^2\delta(Q_1k)\delta(Q_2k)\right]\beta_\pm$$
$$-\left[\mathbf{P}\left(\frac{1}{Q_1k}\right)\delta(Q_2k)+\mathbf{P}\left(\frac{1}{Q_2k}\right)\delta(Q_1k)\right]\alpha_\pm, \tag{10c}$$

where

$$\lim_{\varepsilon\to 0} \frac{1}{D_\pm(ik_4\to\omega+\mathrm{i}\varepsilon)}=\alpha_\pm+\mathrm{i}\pi\beta_\pm. \tag{10d}$$

$\mathbf{P}$ denotes the principal part prescription. The detailed expressions for the functions $\alpha_\pm$ and $\beta_\pm$ can be found in [13–17]. We show below that the mass singularities arise only when both k and k' are space-like momenta.

3.2 Mass singular contributions

Taking the imaginary part of Π^μ_μ leads to products of the type $\rho_\pm\rho'_\pm$, $\sigma_\pm\sigma'_\pm$ and $\tau_\pm\tau'_\pm$ in (7a), (7b) and (7c). The next step is to integrate over $\mathrm{d}\Omega$, and over $\mathrm{d}\Omega_1$, $\mathrm{d}\Omega_2$, respectively.

Only the $1/(Qk)$ factors can produce singularities: e.g. in the product of terms $\sigma_\pm\sigma'_\pm$ arising from (7b) we have

$$\mathbf{P}\left(\frac{1}{Qk}\right)\delta(Qk')=\mathbf{P}\left(\frac{1}{QP}\right)\delta(Qk')=\frac{1}{(QP)}\delta(Qk'), \tag{11}$$

(as $(QP)\geq 0$ the $\mathbf{P}$ prescription is dropped); i.e. when $(QP)=0$, i.e. $\hat{Q}\to-\vec{p}/E$, $(Q^2=P^2=0)$, a non-integrable singularity appears. In the following we regularize this singularity by using dimensional regularisation of the angular integral over $\mathrm{d}\Omega$ in $D=3+2\hat{\varepsilon}$ dimensions, with $\hat{\varepsilon}>0$, but keeping only the singular parts:

$$\int \frac{\mathrm{d}\Omega}{4\pi} \to \int \frac{\mathrm{d}\Omega}{4\pi}\bigg|_{\mathrm{reg}} \overset{\mathrm{def}}{=} \frac{1}{2}\int_0^\pi \mathrm{d}\theta \sin^{D-2}\theta=\frac{1}{2}\int_{-1}^{1} \mathrm{d}\cos\theta(1-\cos^2\theta)^{\hat{\varepsilon}}. \tag{12}$$

By taking the discontinuity of (7) using (8), we thus retain only the products $\mathbf{P}(1/Qk)\,\delta(Qk')$ in order to compute the leading (singular) contribution to the soft photon production rate. The singularity arises when $(Qk)=(Qk')=0$, which is possible only for space-like k and k'. For this reason we restrict ourselves to this domain, since all other contributions are regular*.

In some more detail we describe the procedure for the term tr(1), (7b). The part giving rise to the singularity reads-after regularisation according to (12):

$$\operatorname{Im} \Pi^\mu_\mu|_{1,\mathrm{reg}}=-4m_f^2 \int [\mathrm{d}k] \int \frac{\mathrm{d}\Omega}{4\pi}\bigg|_{\mathrm{reg}} \frac{1}{(QP)}$$
$$\cdot\{\delta(Qk')[\beta_+(1+\hat{k}\cdot\hat{Q})+\beta_-(1-\hat{k}\cdot\hat{Q})]$$
$$\cdot[\alpha'_+(1+\widehat{k'}\cdot\hat{Q})+\alpha'_-(1-\widehat{k'}\cdot\hat{Q})]$$
$$+\mathrm{sym}(k\leftrightarrow k')\}, \tag{13}$$

where the continuation e.g. $(Qk)\Rightarrow\omega+\hat{Q}\cdot\vec{k}$ is implied. $\alpha_\pm$, $\beta_\pm$ are functions of ω and $k=|\vec{k}|$. The integrations with respect to k, ω, ω' are indicated by the short-hand notation:

$$\int [\mathrm{d}k] \equiv e^2 e_q^2 N_c \pi(1-\mathrm{e}^{E/T}) \int \frac{\mathrm{d}^3\vec{k}}{(2\pi)^3}$$
$$\cdot \int_{-|k|}^{|k|} \mathrm{d}\omega \int_{-|k'|}^{|k'|} \mathrm{d}\omega' n_F(\omega) n_F(\omega')\delta(E-\omega-\omega'). \tag{14}$$

In the limit $\hat{\varepsilon}\to 0$ $\operatorname{Im} \Pi^\mu_\mu|_{1,\mathrm{reg}}$ behaves as $1|\hat{\varepsilon}$: the residue is determined by replacing $\hat{Q}$ by $-\vec{p}/E$. The integral over $\mathrm{d}\Omega$ is then computed for $\hat{\varepsilon}\to 0$:

$$\int \frac{\mathrm{d}\Omega}{4\pi}\bigg|_{\mathrm{reg}} \frac{1}{(QP)} \simeq \frac{1}{2E\hat{\varepsilon}}. \tag{15}$$

The leading divergent behaviour expressed in terms of the factor $1/\hat{\varepsilon}$ finally becomes:

$$\operatorname{Im} \Pi^\mu_\mu|_{1,\mathrm{reg}}=-2m_f^2 \frac{1}{\hat{\varepsilon}} \int [\mathrm{d}k]\delta(Pk)$$
$$\cdot\left\{\left(\beta_+\left(1-\frac{\omega}{k}\right)+\beta_-\left(1+\frac{\omega}{k}\right)\right)\left(\alpha'_+\left(1-\frac{\omega'}{k'}\right)\right.\right.$$
$$\left.\left.+\alpha'_-\left(1+\frac{\omega'}{k'}\right)\right)+\mathrm{sym}(\omega,k\leftrightarrow\omega',k')\right\}. \tag{16}$$

Next we discuss the contribution including two vertex corrections, i.e. the tr(2) term of (7c). Let us focus in the

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* No singularity is produced by the $\mathbf{PP}$ or $\delta\delta$ products present in the terms proportional to $\sigma_\pm\sigma'_\pm$ and $\tau_\pm\tau'_\pm$

following on the "+ +" terms (the others being obtained by symmetry) and extract the potentially singular part from the $\tau_+\tau'_+$ term. The expression of τ_+ (cf. (10c)) shows that terms in $\alpha_+\beta'_+$, $\alpha'_+\beta_+$, $\alpha_+\alpha'_+$ and $\beta_+\beta'_+$ will appear in the singular contribution, because each of the latter quantities can be associated with a $\mathbf{P}(1/Qk)\,\delta(Qk')$ product.

The contribution is:

$$\mathrm{Im}\,\Pi^{\mu}_{\mu}|_{2,\mathrm{reg},++}=2m_f^4\int[\mathrm{d}k]\int\frac{\mathrm{d}\Omega_1}{4\pi}\bigg|_{\mathrm{reg}}\int\frac{\mathrm{d}\Omega_2}{4\pi}\bigg|_{\mathrm{reg}}(Q_1Q_2)$$
$$\cdot[(Q_1k_+)(Q_2k'_+)+(Q_1k'_+)(Q_2k_+)-(Q_1Q_2)(k_+k'_+)]$$
$$\cdot\left\{\frac{1}{(Q_1P)}\mathbf{P}\left(\frac{1}{Q_2k'}\right)\mathbf{P}\left(\frac{1}{Q_2k}\right)\delta(Q_1k)\alpha_+\beta'_++\frac{1}{(Q_1P)}\right.$$
$$\cdot\mathbf{P}\left(\frac{1}{Q_2k}\right)\mathbf{P}\left(\frac{1}{Q_2k'}\right)\delta(Q_1k')\alpha'_+\beta_+-\frac{1}{(Q_1P)}\frac{1}{(Q_2P)}$$
$$\left.\cdot(\delta(Q_1k)\delta(Q_2k')\alpha_+\alpha'_+-\pi^2\delta(Q_1k')\delta(Q_2k')\beta_+\beta'_+)\right\}. \tag{17}$$

We find the $1/\hat{\varepsilon}$ contribution for $\widehat{Q_1}\to-\vec{p}/E$ or $\widehat{Q_2}\to-\vec{p}/E$ (the latter is obtained by symmetry $k\leftrightarrow k'$). No double pole has to be considered because of the presence of the (Q_1Q_2) factor.

Using:

$$\int\frac{\mathrm{d}\Omega}{4\pi}\,\delta(Qk)=\frac{\theta(-k^2)}{2|\vec{k}|},$$
$$\int\frac{\mathrm{d}\Omega}{4\pi}\,\delta(Qk')(Qk)=\frac{\theta(-k'^2)}{2|\vec{k}'|^2}(\omega|\vec{k}'|-\omega'\vec{k}\cdot\widehat{\vec{k}'}), \tag{18}$$

and [11]:

$$\int\frac{\mathrm{d}\Omega}{4\pi}\mathbf{P}\left(\frac{1}{kQ}\right)=L(k)\equiv L=\frac{1}{2k}\ln\left|\frac{\omega+k}{\omega-k}\right|,$$
$$\int\frac{\mathrm{d}\Omega}{4\pi}\mathbf{P}\left(\frac{k'Q}{kQ}\right)=\omega'L-\frac{\vec{k}\cdot\vec{k}'}{\vec{k}^2}(\omega L-1), \tag{19}$$

we obtain after combining the contributing terms:

$$\mathrm{Im}\,\Pi^{\mu}_{\mu}|_{2,\mathrm{reg},++}=m_f^4\frac{1}{\hat{\varepsilon}}\int[\mathrm{d}k]\frac{\delta(Pk)}{E}\left\{(\alpha_+\beta'_++\alpha'_+\beta_+)\right.$$
$$\cdot\left[2(1-\omega/k)(1-\omega'/k')L+((E-k)/k'-1)\right.$$
$$\left.\cdot\left(\left(1-\frac{\omega^2}{k^2}\right)L+\frac{\omega}{k^2}\right)\right]+(-\alpha_+\alpha'_++\pi^2\beta_+\beta'_+)$$
$$\cdot\frac{1}{2k}\left[2(1-\omega/k)(1-\omega'/k')+\left(1-\frac{\omega^2}{k^2}\right)\right.$$
$$\left.\left.\cdot((E-k)/k'-1)\right]+\mathrm{sym}(\omega,k\leftrightarrow\omega',k')\right\}. \tag{20}$$

Similar contributions come from $\tau_+\tau'_-$ and $\tau_-\tau'_-$, respectively.

The functions L (and L') in (20) are eliminated by using the definitions for $\alpha_\pm$ and $\beta_\pm$, (10d):

$$m_f^2(1-\omega/k)L\alpha_+=\left(\omega-k-\frac{m_f^2}{k}\right)\alpha_+-\pi^2\frac{m_f^2}{2k}(1-\omega/k)\beta_++1,$$
$$m_f^2(1-\omega/k)L\beta_+=\left(\omega-k-\frac{m_f^2}{k}\right)\beta_++\frac{m_f^2}{2k}(1-\omega/k)\alpha_+. \tag{21}$$

Thus (20) contains terms proportional to $\alpha_+\alpha'_+$, $\alpha_+\beta'_+$, $\alpha'_+\beta_+$ and $\beta_+\beta'_+$, and terms linear in β_+, β'_+. The $\alpha_+\alpha'_+$ and $\beta_+\beta'_+$ terms vanish in (20), whereas the $\alpha_+\beta'_+$ and $\alpha'_+\beta_+$ terms compensate with those of (16). All what remains are the linear terms in β_+ and β'_+. The final result (including all contributions from $\tau_\pm\tau'_\pm$) is:

$$\mathrm{Im}\,\Pi^{\mu}_{\mu}|_{\mathrm{reg}}=2m_f^2\frac{1}{\hat{\varepsilon}}\int[\mathrm{d}k]\frac{\delta(Pk)}{E}$$
$$\cdot\left\{\beta'_+\left(1-\frac{\omega'}{k'}\right)+\beta_+\left(1-\frac{\omega}{k}\right)\right.$$
$$\left.+\beta'_-\left(1+\frac{\omega'}{k'}\right)+\beta_-\left(1+\frac{\omega}{k}\right)\right\}. \tag{22}$$

This expression may still be simplified by following a procedure familiar from the hard photon case [6]. Since the functions β_+ and β_- as given by [13–17] are peaked for $\omega\to0$ (k being fixed) we may replace $n_F(\omega)\,n_F(E-\omega)$ by $n_F(0)n_F(E)=\frac{1}{2}n_F(E)$. After performing the angular integration in (22), the remaining integrals $\int\mathrm{d}\omega(1\mp\omega/k)\beta_\pm(\omega,k)$ are evaluated using the sum rule [6]:

$$\int_{-\infty}^{\infty}\mathrm{d}\omega(1\mp\omega/k)\rho_\pm(\omega,k)=0. \tag{23}$$

The dominant contribution of the integral over k comes from $m_f<k<T$. The leading contribution for $g\to0$ then reads:

$$E\frac{\mathrm{d}W}{\mathrm{d}^3\vec{p}}\simeq\frac{1}{\hat{\varepsilon}}\frac{e_q^2\alpha\alpha_s}{2\pi^2}T^2\,n_F(E)\left(\frac{m_f}{E}\right)^2\ln\left(\frac{1}{\alpha_s}\right). \tag{24}$$

This result shows that the Braaten–Pisarski resummation does not yield a finite soft real photon production rate: a logarithmic divergence remains.

4 Discussion

The above analysis allows to identify the diagrams which are responsible for the singularities as they originate from terms proportional to the product $\mathbf{P}(1/Qk)\,\delta(Qk')$. One example of such a diagram is shown in Fig. 2, where the singularity is due to the massless quark exchange present in the hard thermal loop effective vertices. The massless exchange is transparent in the two→three amplitude of Fig. 2b. The singularity arises from the configuration $Q\cdot P=0$: it corresponds to a collinear singularity when the photon is allowed to stay massless, $P^2=0$.

At present we do not know how to screen this mass singularity by a consistent prescription. Therefore a pragmatic approach is to introduce a soft cut-off of $\mathcal{O}(gT)$ in

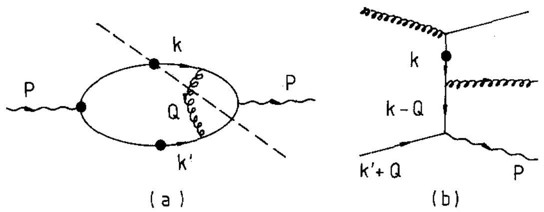

Fig. 2a, b. a Cutting the effective one-loop diagram through the effective hard thermal loop vertex gives rise to **b** the amplitude with a collinear singularity for $P \cdot Q = 0$. The curly line denotes the gluon

order to regularize the soft photon production rate at logarithmic accuracy:

$$E \frac{dW}{d^3 \vec{p}} \simeq e_q^2 \, \alpha \alpha_s \, T^2 \ln^2 \left(\frac{1}{\alpha_s} \right), \tag{25}$$

where the photon energy is assumed $E \sim m_f$.

The presented result is valid for soft massless, i.e. non-thermalized photons. This implies that the quark-gluon plasma has to have a finite size; its characteristic length is denoted by L. As already mentioned in the Introduction the mean free path l_γ of the detected photon has to be larger than L. Since only photons with wave lengths less than L are radiated, the dimension of the plasma becomes constrained [18]:

$$\frac{2\pi}{E} < L < l_\gamma(E). \tag{26}$$

Suppressing the logarithmic factors in (25) we estimate the photon's mean free path to be given by:

$$l_\gamma \simeq \frac{E}{\alpha \alpha_s T^2}. \tag{27}$$

This order of magnitude estimate is in agreement with $l_\gamma \simeq 1/n_q \sigma_{\mathrm{Compton}}(E)$, where n_q is the quark density and $\sigma_{\mathrm{Compton}}(E) \sim \alpha \alpha_s / ET$ is the high energy Compton cross section in the QGP, which is responsible for the photon absorption.

For soft photon energies $E \sim m_f \sim \mathcal{O}(gT)$ the constraint (26) becomes:

$$\mathcal{O}(1) < LT \sqrt{\alpha_s} < \mathcal{O}\left(\frac{1}{\alpha} \right), \tag{28}$$

i.e. for typical values of $T \sim 400$ MeV and $\alpha_s \sim 0.25$ the constraint reads: 1 fm $< L <$ 100 fm. This size is compatible with the expectations for a realistic QGP produced in heavy-ion collisions.

In summary it seems reasonable to foresee experimental situations where soft $\mathcal{O}(gT)$ non-thermalized photons would be emitted from a QGP. However, our present understanding does not allow us to derive their finite production rate.

Note added in proof. Recently our result (24) has been confirmed by an independent calculation within the real-time formalism by P. Aurenche, T. Becherrawy and E. Petitgirard, "Retarded/Advanced Correlation Functions and Soft Photon Production in the Hard Thermal Loop Approximation", preprint ENSLAPP-A-452/93, NSF-ITP-93-155 (December 1993).

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