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High-temperature fermion propagator: Resummation and gauge dependence of the damping rate

R. Baier
Fakultät für Physik, Universität Bielefeld, D-4800 Bielefeld 1, Federal Republic of Germany

G. Kunstatter
Physics Department and Winnipeg Institute for Theoretical Physics, University of Winnipeg, Winnipeg, Manitoba, Canada R3B 2E9

D. Schiff
Laboratoire de Physique Théorique et Hautes Énergies, Université Paris-Sud, Bâtiment 211, F-91405 Orsay, France

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The gauge-fixing dependence of the damping rate of the QED and QCD fermionic excitations at high temperatures and long wavelengths is reexamined in detail using the effective leading-order perturbation expansion developed by Braaten and Pisarski. In contrast with what is expected from recent formal discussions, explicit calculations in covariant gauges yield a fermionic damping rate that is gauge parameter dependent. This result has general implications for gauge theories at nonzero temperature.

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In order to understand the stability of hot matter, e.g., of the QED and the QCD plasma at high temperatures, it is of central importance to correctly calculate, in the framework of thermal perturbation theory [1], the damping rates of the plasma excitations (waves), the bosonic as well as the fermionic ones.

Indeed, during the last, more than ten, years many calculations with different disagreeing results, mainly on the gluon damping rate in hot QCD ("the plasmon puzzle"), have been published [2]. Only recently Pisarski [3], Kraemmer *et al.* [4], and by an independent argument Kobes, Kunstatter, and Rebhan [5] have shown that the apparent gauge-fixing dependence of the damping constants at one-loop level is due to the lack of consistency of the approximation used. Although Ref. [5] gives explicit arguments only for the gluon damping rate, the results apply to the fermion damping rate as well.

At high temperatures and at leading order in the gauge coupling constant, Braaten and Pisarski [6] have suggested that a consistent approximation scheme can be obtained by resumming the "hard thermal loops" into effective propagators and vertices for soft gluons and quarks. Moreover, they claim that to leading order this scheme yields the same damping constant in strict Coulomb and covariant gauges [7].

In this paper we reexamine the question of gauge-fixing independence by considering the class of covariant gauges in more detail. Following Braaten and Pisarski [6], we use effective propagators and vertices in the long-wavelength limit. The tree-level Ward identities satisfied by the effective propagators and vertices [6,8] can be used in order to evaluate the gauge-fixing dependence of the resummed self-energies, more precisely of their discontinuities. In the following we concentrate on the damping rate of the fermion excitation (at vanishing three-momentum), since it is technically simpler than the rate of the gluon. Braaten and Pisarski [7] used Ward identities to argue that the gauge-dependent parts of the fermionic damping rate vanish on the physical mass shell in strict Coulomb and covariant gauges. More recently, a calculation of the fermionic damping rate has been performed in a general class of linear gauges [9]. However, in this calculation also the gauge-dependent parts are dropped because they are shown to be proportional to the mass-shell condition. In the present work, the gauge-dependent parts are calculated explicitly in covariant gauge, and shown not to vanish.

The fermionic damping constant is related to the imaginary part of the effective fermion self-energy Σ. There are two contributions at leading order in the coupling constant g:

$$\Sigma(p)=\Sigma_a(p)+\Sigma_b(p)\,. \tag{1}$$

The first is the one-loop fermion self-energy contribution, familiar from the $T=0$ case; however, here the fermion and the gluon propagator Δ_f and $\Delta_{\mu\nu}$, respectively, and the gluon-fermion vertex Γ_μ are the effective thermal ones, including the hard thermal loops, as given in Ref. [6] (the superscript asterisk used in [6] is dropped). The term Σ_b contains the effective four-point vertex $\Gamma_{\mu\nu}$, in which two gluons are attached to the fermion line. Following closely the notation (and conventions) of [6,8] the terms are

$$\Sigma_a(p)=-g^2\int dk\,\Delta_{\mu\nu}(k)\Gamma^\mu(p,-p+k;-k)\times\Delta_f(p-k)\Gamma^\nu(p-k,-p;k) \tag{2}$$

and

$$\Sigma_b(p)=(ig^2/2)\int dk\,\Delta_{\mu\nu}(k)\Gamma^{\mu\nu}(p,-p;-k,k)\,. \tag{3}$$

Since we do not need the specific expressions for the vertex functions, the following analysis holds for QED as well as for QCD, apart from color factors which are suppressed.

We note the use of the imaginary-time formalism at temperature T, i.e., boson [fermion] energies are discrete $k_0=2\pi nT$ $[p_0=(2n+1)\pi T]$, where n is an integer. Later, in order to calculate the discontinuity of Σ the analytic continuation [6], $k_0[p_0] \to -i(\omega+i\epsilon)$, to a continuous Minkowski energy ω is performed. The loop integral is denoted by $T\sum_{k_0}\int d^3k/(2\pi)^3 \equiv \int dk$, having in mind that this integral includes only soft momenta of $O(gT)$. The metric is Euclidean, e.g., $k^2=(k_0)^2+\mathbf{k}^2$.

The effective vertices Γ_μ and $\Gamma_{\mu\nu}$ obey the Ward identities [6,8]

$$k_\mu \Gamma^\mu(p,q;k) = -i[\Delta_f^{-1}(p)+\Delta_f^{-1}(q)] \quad (4)$$

(it has the same structure as at the tree level) and

$$k_\mu k_\nu \Gamma^{\mu\nu}(p,-p;-k,k) = -i[2\Delta_f^{-1}(p)-\Delta_f^{-1}(p+k) - \Delta_f^{-1}(p-k)] , \quad (5)$$

where the effective inverse fermion propagator is decomposed as $\Delta_f^{-1}(p) = -i\not{p} - \Sigma(p)$, for a fermion with zero bare mass.

Next we investigate the gauge fixing dependence of the discontinuity in the resummed self-energy $\Sigma(p)$. All the gauge dependence comes from the boson propagator $\Delta_{\mu\nu}$, and we consider gauge variations $\delta\Delta_{\mu\nu}$ about an arbitrary ("fiducial") gauge [10], restricting to the class of covariant gauges, such that the gauge-dependent part of the boson propagator is given by

$$\delta\Delta_{\mu\nu}(k) = \xi \frac{k_\mu k_\nu}{(k^2)^2} , \quad (6)$$

with ξ an arbitrary parameter.

Using the Ward identities, Eqs. (4) and (5), we find

$$\delta\Sigma(p) = -\xi g^2 \Delta_f^{-1}(p) \int \frac{dk}{(k^2)^2} + \xi g^2 \Delta_f^{-1}(p) \int \frac{dk}{(k^2)^2} \Delta_f(p-k) \Delta_f^{-1}(p) . \quad (7)$$

Note that Eq. (7) is consistent with the analyses of Refs. [5] and [6], which formally prove that the gauge variation of Σ is proportional to the lowest-order inverse propagator $\Delta_f^{-1}(p)$ and hence vanishes on shell, since, after analytic continuation,

$$\Delta_f^{-1}(p)|_{\text{on shell}} \equiv 0 . \quad (8)$$

As emphasized in Ref. [5], gauge independence only follows if the corresponding coefficients in Eq. (7) do not develop poles on the effective mass shell. Despite the fact that arguments have been given [5] as to why such poles should not occur in general, we now show that the imaginary part of the integral

$$\int \frac{dk}{(k^2)^2} \Delta_f(p-k) \quad (9)$$

does in fact develop an on-shell double pole, so that the second term in (7) yields a finite, gauge-dependent contribution to the resummed damping rate. Thus, although the gauge-dependence identities themselves are not violated, the conclusion that the damping rate is gauge independent appears to break down in covariant gauge.

We focus on the damping rate of the excitation at rest (at vanishing three-momentum), which after analytic continuation becomes proportional to $\text{disc}\Sigma(p_0,\mathbf{p}=0)$—the same label p_0 is used for the discrete Euclidean as well as for the continued continuous Minkowski energy of the external fermion line.

The effective fermion propagator is described in Refs. [11–13]. The hard thermal loop is gauge-fixing independent and gives the leading term in T dominating at soft momentum p; exhibiting the Dirac structure the effective propagator is expressed by two functions as

$$\Delta_f^{-1}(p) = \gamma_0 D_0(p) + i\boldsymbol{\gamma}\cdot\hat{\mathbf{p}} D_s(p) , \quad (10)$$

with $\hat{\mathbf{p}} \equiv \mathbf{p}/|\mathbf{p}|$. The scale in the functions $D_{0,s}$ (their specific form may be found in [13]) is of $O(gT)$ and determined by the fermion mass induced by temperature, $m_f^2 = g^2T^2/8$.

For positive energy the fermionic excitations contain two modes (denoted by the subscripts $\pm$), corresponding to positive as well as a negative helicity-chirality ratio, respectively. They are solutions of the real parts of the dispersion equations $D_0(p) = \pm D_s(p)$. Introducing two projectors for the $(\pm)$ modes, $P_\pm(p) \equiv (\gamma^0 \pm i\boldsymbol{\gamma}\cdot\hat{\mathbf{p}})$, and correspondingly $D_\pm = D_0 \pm D_s$, we define the gauge variations for the two modes by

$$\delta\Sigma_\pm(p) \equiv \tfrac{1}{4}\,\text{Tr}[P_\pm \delta\Sigma(p)] , \quad (11)$$

where Tr denotes the trace with respect to the Dirac matrices and $\delta\Sigma$ is given by Eq. (7).

At vanishing three-momentum, $\mathbf{p}=0$, the two solutions are degenerate, since after continuation $D_s(p_0,\mathbf{p}=0)=0$, and $D_0(p_0,\mathbf{p}=0) = (p_0^2 - m_f^2)/p_0$. Also the damping rates coincide for the two modes. Therefore the quantity of interest becomes

$$\text{disc}[\delta\Sigma_\pm(p_0,\mathbf{p}=0)] \simeq \xi \frac{[p_0^2-m_f^2]^2}{p_0^2} \text{disc}\left\{ g^2 \int \frac{dk}{(k^2)^2} \left[\frac{1}{D_+(p_0-k_0,\mathbf{k})} + \frac{1}{D_-(p_0-k_0,\mathbf{k})} \right] \right\} . \quad (12)$$

Irrelevant numerical factors are suppressed. We note that the integral $\int dk/(k^2)^2$ in Eq. (7) does not contribute after analytic continuation to the discontinuity in the self-energy. In order to evaluate the double poles in $1/k^2$ which appear in Eq. (12) we use the prescription

$$\frac{1}{(k^2)^2} \to \left(-\frac{\partial}{\partial m^2} \right) \frac{1}{k^2+m^2} \Bigg|_{m^2 \to 0} ; \quad (13)$$

consequently, we first take $m^2 \to 0$ before the effective on-shell limit, $p_0 \to m_f$. We denote this procedure by

$$\lim \triangleq \lim_{p_0 \to m_f} \lim_{m^2 \to 0} . \quad (14)$$

The discontinuity in Eq. (12) is computed after performing the sum over the energy k_0 of the intermediate boson using the identity (cf. [14])

$$\mathrm{disc}\, I(ip_0 \to p_0 + i\epsilon) \equiv \mathrm{disc}\left\{ T \sum_{n=-\infty}^{+\infty} f_{\mathrm{boson}}(k_0) f_{\mathrm{Dirac}}(p_0 - k_0) \right\}\Bigg|_{ip_0 \to p_0 + i\epsilon}$$

$$= \pi (e^{\beta p_0} + 1) \int_{-\infty}^{+\infty} dw\, dw'\, \delta(p_0 - w - w') n_B(w) \rho_B(w) n_F(w') \rho_F(w') \,, \qquad (15)$$

where $\rho_{B,F}$ are the spectral densities for $f_{\mathrm{boson,Dirac}}$. The statistical distribution functions are $n_{B,F}(w) = (e^{\beta w} \mp 1)^{-1}$, respectively, with $\beta = 1/T$.

The spectral densities are, for the boson, $\rho_B(w) = \epsilon(w) \delta(w^2 - \mathbf{k}^2 - m^2)$, and for the fermion, the density ρ_F is given in Refs. [13,14] in terms of $\rho_\pm \simeq \mathrm{disc}(1/D_\pm)$. For the damping rate under consideration $\rho_\pm$ receive two contributions: the timelike ones $\rho_\mp^{\mathrm{res}}$ from the quasiparticle excitations, and the spacelike ones $\rho_\mp^{\mathrm{disc}}$ due to Landau damping. Therefore it is convenient to treat these two contributions separately, when evaluating the gauge variation of the fermion damping rate, Eq. (12).

Let us first examine in detail the discontinuity due to the quasiparticles, i.e., the expression

$$\mathrm{disc}[\delta\Sigma_\pm(p_0, \mathbf{p} = 0)]|_{\mathrm{res}} \simeq \xi g^2 \frac{[p_0^2 - m_f^2]^2}{p_0^2} I^{\mathrm{res}}(p_0) \,, \qquad (16)$$

where the integral is given by

$$I^{\mathrm{res}}(p_0) \simeq \pi n_F^{-1}(p_0) \lim \left(\frac{\partial}{\partial m^2} \right) \int \frac{d^3k}{(2\pi)^3} \int_{-\infty}^{+\infty} dw\, \epsilon(w) n_B(w) \delta(w^2 - k^2 - m^2)$$

$$\times \left\{ \left(\frac{w_\mp^2(k) - k^2}{4m_f^2} \right) [n_F(w_+) \delta(p_0 - w - w_+(k)) + (1 - n_F(w_+)) \delta(p_0 - w + w_+(k))] + [(+) \leftrightarrow (-)] \right\} ; \quad (17)$$

it includes the sum over the positive and negative modes. Here and in the following $k = |\mathbf{k}|$. Without restriction for the general case we choose $p_0 - m > m_f$, before performing lim in the sense of Eq. (14).

The δ constraints lead to the following conditions: $p_0 - E(k) = w_\pm(k)$ with respect to the two modes, where we denote $E(k) = (k^2 + m^2)^{1/2}$.

Inserting the solutions the integral simplifies as

$$I^{\mathrm{res}}(p_0) \simeq n_F^{-1}(p_0) \lim \left(\frac{\partial}{\partial m^2} \right) \sum_{+,-} \left\{ \frac{k}{E(k)} n_B(E(k)) n_F(w(k)) \left(\frac{w^2(k) - k^2}{m_f^2} \right) \frac{1}{|\partial E/\partial k^2 + \partial w/\partial k^2|} \right\}_{k = k_\pm,\, w \equiv w_\pm(k)} . \qquad (18)$$

In order to perform the interesting limit, Eq. (14), we realize that the solutions of the constraint equations vanish linearly as $k_\pm = \mathrm{const} \times (p_0 - m_f)$, which is derived from the behavior of the positive and negative modes in the limit of vanishing three-momentum [13]: i.e.,

$$w_\pm(k) \underset{k \to 0}{\simeq} m_f \pm \frac{k}{3} + \frac{1}{3} \frac{k^2}{m_f} + \cdots \,. \qquad (19)$$

For the dominant term we finally obtain in the appropriate limit the surprising result

$$I^{\mathrm{res}}(p_0) \simeq -3T \left[\frac{1}{4k_+^2} + \frac{1}{2k_-^2} + O\left(\frac{1}{k_\pm} \right) \right]$$

$$\simeq -\frac{2T}{(p_0 - m_f)^2} \,; \qquad (20)$$

that is, the integral becomes "singular" when approaching the fermionic excitation mass shell (at vanishing three-momentum).

Next we study the gauge-fixing parameter-dependent contributions coming from the branch cut of the effective quark propagator associated with Landau damping. In this case, from first inspection, it seems safe to work directly on shell and to investigate the integral $I^{\mathrm{disc}}(m_f)$ defined by

$$\mathrm{disc}[\delta\Sigma_\pm(p_0 = m_f, \mathbf{p} = 0)]|_{\mathrm{disc}} \simeq \xi g^2 \frac{[p_0^2 - m_f^2]^2}{m_f^2} I^{\mathrm{disc}}(m_f) \,. \qquad (21)$$

We find that $I^{\mathrm{disc}}(m_f)$ has a well-behaved finite value, leading to a ξ-dependent contribution in Eq. (21), which, however, vanishes on shell when $p_0 = m_f$.

This completes the proof that the covariant gauge variation of the damping rate does not vanish on shell. Indeed, we find a contribution to the fermionic rate due to the quasiparticle excitations, which reads

$$\mathrm{disc}[\delta\Sigma_\pm(m_f, \mathbf{p} = 0)] \propto \xi g^2 T \neq 0 \,. \qquad (22)$$

A similar result [10] is true also for the damping rate of the gluonic excitations.

The above calculation has potentially important implications not only for resummed thermal QCD: The analysis uses only the Ward identities satisfied by the resummed n-point functions, and does not depend on the detailed structure of the fermion propagator in (9). Moreover, the integral in Eq. (9) is representative of the mass-shell behavior of more general cases, such as a massive field coupled to a massless one at finite temperature. Thus, the problem may not be specific to the resummation techniques used, but instead gauge dependence may be generic to any gauge theory at finite temperature, in covariant gauges at least.

The origin of this problem may lie in the infrared behavior of the theory, i.e., possible infrared singularities near the mass shell. Thus, a suitable regularization scheme may lead to a resolution of the problem of gauge dependence.[1] In particular, the problematic integral in (9) gives a vanishing contribution to the damping rate provided an infrared regulator is introduced, and kept nonzero until after the mass-shell condition is imposed. This has been verified using both a cutoff [15] and dimensional regularization [16]. It should be noted that such a regulator is, in principle, distinct from the mass term used in (13) to treat the double pole. The need for such a regulator is familiar at zero temperature, when, for example, logarithmic divergences occur near the mass shell in the (real) wave-function renormalization constant. The surprising aspect of the present analysis is that the terms in question at finite temperature are not divergent so that there is no obvious reason for keeping an infrared regulator in the discontinuity. [In the effective resummed framework, the real part of the self-energy in Eq. (7) has no direct significance, since one expects that higher-loop diagrams will also contribute to the real part to the same order (i.e., g^2T).]

It is unclear whether extra higher-loop contributions to the imaginary part, as conjectured in Ref. [17], have any bearing on this problem. In any case, it is hoped that the consequences of our result are worthy of further attention.

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[1]This possible resolution was first pointed out to us by Rebhan [15], and is reinforced by calculations of Braaten [16].

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