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Relations for hadron pairs at large p_T in the elastic quark-quark scattering model

R. Baier, J. Cleymans, and B. Petersson
Department of Theoretical Physics, University of Bielefeld, Germany
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Relations between cross sections for producing hadrons pairs with large transverse momenta are presented. They follow from the isospin and charge-conjugation properties of the elastic quark-quark scattering model. These relations are independent of any specific parametrization for the quark-distribution and quark-fragmentation functions and of any specific form for the hard-scattering cross section.

In the recent past, experimental evidence[1,2] has become available, which indicates that particles with large transverse momentum to the beam direction in hadron-hadron collisions are produced from a basic two-body hard-collision mechanism.[3] The quantum-number structure of this hard-collision mechanism is still not completely clear. The mechanism may, e.g., be due to elastic quark-quark scattering[4,5] or to the interchange of quark constituents.[6] It is the purpose of this article to present a number of linear relations which test certain basic assumptions of the quark-quark elastic-scattering model. These relations concern inclusive cross sections $A+B\to C+D+\cdots$, where C and D have large and opposite transverse momenta. They follow from the isospin and charge-conjugation properties of the model and from the absence of the exchange of flavor quantum numbers in the basic interaction. They are *independent* of any specific parametrization of the quark-distribution functions, the quark-fragmentation functions, and of the chosen form for the quark-quark elastic cross section. They are also independent of the transverse-momentum spread of the partons in the hadrons and of the transverse-momentum spread in the outgoing jets. Still, we must of course be in the validity domain of the hard-scattering model. We choose therefore to work mainly with the configuration in Fig. 1 for which the arguments given below are expected to be valid for particles with transverse momenta larger than 2 GeV/c. In this case, the main contributions to the correlations come from subprocesses having large $\hat{s}$ and $\hat{t}$ *even* if the transverse momentum of the partons is appreciable. Deviations from the relations proposed in this paper can therefore be traced back immediately to the basic structure of the elastic quark-quark scattering model. They could, for example, show that in the basic interaction there is an exchange of flavor quantum numbers or indicate substantial amounts of interference between identical partons (such terms have not been taken into account in previous applications of the model[4,5]).

The relations we present are, in general, not valid in the constituent-interchange model[6] or in the quark-fusion model.[7] This is due to the fact that some of the constituents considered there have isospin 1 and, furthermore, an exchange of quantum numbers in the t channel occurs.

In the general hard-scattering model the relevant correlations

$$E_C E_D \frac{d^6\sigma}{d^3p_C d^3p_D}$$

are given by integrals, over the constituent momenta, of the following expression:

$$\sum_{\substack{k_1,k_2\\k,k'}} f^A_{k_1}(x_1,k_{1T}) f^B_{k_2}(x_2,k_{2T}) \frac{d\sigma^{k_1+k_2\to k+k'}}{d\hat{t}} \times D^C_k(z,h_T) D^D_{k'}(z',h'_T), \qquad (1)$$

where $f^A_k(x)$ is the momentum distribution for a consitituent of type k in hadron A whereas $D^C_k(z,h_T)$ is the fragmentation function for constituent k into hadron C. k_T is the transverse momentum of

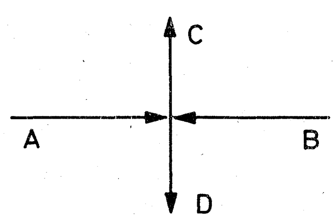

FIG. 1. Kinematical configuration of correlations discussed in text.

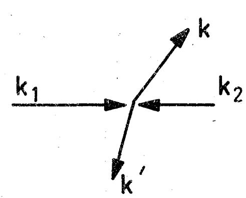

FIG. 2. Constituent-constituent collision relevant for the description of the discussed correlations.

constituent k with respect to the beam axis while h_T is the transverse momentum of hadron C with respect to the jet axis k (see Fig. 2). The notations correspond to those presented in Ref. 4.

We now assume explicitly that the constituent-constituent scattering amplitude is flavor independent (it turns out that this assumption is unnecessary when comparing pp collisions with pn collisions), and that there is no flavor-quantum-number exchange in the t channel. Within the framework of hard-collision models these are the first basic assumptions we make. Expression (1) can then be written as

$$\frac{d\sigma}{d\hat{t}}\left(\sum_{k_1} f_{k_1}^A(x_1,k_{1T})D_{k_1}^C(z,h_T)\right)\left(\sum_{k_2} f_{k_2}^B(x_2,k_{2T})D_{k_2}^D(z',h'_T)\right)+\frac{d\sigma}{d\hat{u}}\left(\sum_{k_1} f_{k_1}^A(x_1,k_{1T})D_{k_1}^D(z',h'_T)\right)\left(\sum_{k_2} f_{k_2}^B(x_2,k_{2T})D_{k_2}^C(z,h_T)\right). \tag{2}$$

The first (second) term denotes the scattering of constituent k_1 (see Fig. 2) into $k(k')$ and k_2 into $k'(k)$. Note that in (2) we have disregarded the interference term to which we will come later. Field and Feynman[5] noted that neglecting the interference terms one has $\sigma(\pi^+p\to\pi^0+\cdots)=\sigma(\pi^-p\to\pi^0+\cdots)$. Experimental results[8] on π^0 production with pion beams suggest that this is the case within error bars.

We will now simplify notations by defining a function $F(A\to C)$ in the following way:

$$F(A\to C)=\sum_k f_k^A(x,k_T)D_k^C(z,h_T), \tag{3}$$

where the sum runs, as usual, over all possible types of constituents. In the framework of the elastic quark-quark scattering model we will restrict this sum to quark constituents, thus explicitly excluding diquark states and quark-antiquark states. Processes involving gluons (e.g., $qG\to qG$) may, however, be included. This is the second of our basic assumptions.

By using the isospin and charge-conjugation properties of the quark-distribution and quark-fragmentation functions one can derive the following relations for the functions F defined in (3):

$$F(p\to\pi^+)=F(n\to\pi^-)=F(\bar{p}\to\pi^-), \tag{4a}$$

$$F(p\to\pi^-)=F(n\to\pi^+)=F(\bar{p}\to\pi^+), \tag{4b}$$

$$F(p\to K^+)=F(n\to K^0)=F(\bar{p}\to K^-), \tag{4c}$$

$$F(\pi^+\to\pi^-)=F(\pi^-\to\pi^+), \tag{4d}$$

etc. . . . .

From the symmetry properties of the functions F we deduce immediately from (2):

$$d\sigma(p\bar{p}\to\pi^+\pi^++\cdots)=d\sigma(pn\to\pi^+\pi^++\cdots). \tag{5}$$

This relation depends only on the isospin and charge-conjugation properties of the model and is not restricted to the symmetric configuration of Fig. 1. Similarly, with the same degree of validity one has

$$d\sigma(pn\to\pi^+\pi^-+\cdots)=d\sigma(p\bar{p}\to\pi^+\pi^-+\cdots), \tag{6a}$$

$$d\sigma(pn\to\pi^+\pi^0+\cdots)=d\sigma(p\bar{p}\to\pi^+\pi^0+\cdots), \tag{6b}$$

$$d\sigma(pn\to\pi^0\pi^0+\cdots)=d\sigma(p\bar{p}\to\pi^0\pi^0+\cdots)=d\sigma(pp\to\pi^0\pi^0+\cdots), \tag{6c}$$

$$d\sigma(\pi^+p\to\pi^-\pi^0+\cdots)+d\sigma(\pi^+p\to\pi^+\pi^0+\cdots)=d\sigma(\pi^-p\to\pi^-\pi^0+\cdots)+d\sigma(\pi^-p\to\pi^+\pi^0+\cdots), \tag{6d}$$

etc.

Many other relations can be written but involve several charge combinations, however, for the symmetric configuration at 90° they simplify considerably; so, e.g., in general the relation

$$d\sigma(pn\to\pi^+\pi^++\cdots)+d\sigma(pn\to\pi^-\pi^-+\cdots)=d\sigma(pp\to\pi^+\pi^-+\cdots)+d\sigma(pp\to\pi^-\pi^++\cdots) \tag{7}$$

becomes

$$d\sigma_s(pn\to\pi^+\pi^++\cdots)=d\sigma_s(pp\to\pi^+\pi^-+\cdots) \tag{8}$$

(the index s should simply remind us of the symmetry of the configuration). It should be clear to the reader that a relation like (8) is not expected to be true in, e.g., the quark-fusion model[7] where the basic hard-collision process is the fusion of a quark-antiquark pair in a pair of mesons ($q\bar{q}\to M\bar{M}$) since then opposite-charge correlations are expected to be much bigger than like-charge correlations. Within the framework of the elastic quark-quark scattering model deviations from relations (5)–(8) are only expected to arise from interference terms, present when two identical quarks scatter off each other. These relations could therefore play an important role in getting further information on the magnitude and sign of the interference terms.

We have, furthermore,

$$d\sigma_s(pn\to\pi^+\pi^-+\cdots)=\tfrac{1}{2}[d\sigma_s(pp\to\pi^+\pi^++\cdots)+d\sigma_s(pp\to\pi^-\pi^-+\cdots)],\tag{9a}$$

$$d\sigma_s(pn\to\pi^+\pi^0+\cdots)=\tfrac{1}{2}[d\sigma_s(pp\to\pi^+\pi^0+\cdots)+d\sigma_s(pp\to\pi^-\pi^0+\cdots)].\tag{9b}$$

From these relations follows that for proton-induced reactions on any isoscalar nuclear target N:

$$d\sigma_s(pN\to\pi^+\pi^-+\cdots)=\tfrac{1}{2}[d\sigma_s(pN\to\pi^+\pi^++\cdots)+d\sigma_s(pN\to\pi^-\pi^-+\cdots)].\tag{10}$$

For correlations involving kaons, one has

$$d\sigma_s(p\bar{p}\to K^+\pi^-+\cdots)=\tfrac{1}{2}[d\sigma_s(pp\to K^+\pi^++\cdots)+d\sigma_s(pp\to K^-\pi^-+\cdots)],\tag{11a}$$

$$d\sigma_s(p\bar{p}\to K^+\pi^++\cdots)=\tfrac{1}{2}[d\sigma_s(pp\to K^-\pi^++\cdots)+d\sigma_s(pp\to K^+\pi^-+\cdots)],\tag{11b}$$

$$d\sigma_s(p\bar{p}\to K^+K^++\cdots)=d\sigma_s(pp\to K^+K^-+\cdots),\tag{11c}$$

$$d\sigma_s(p\bar{p}\to K^+K^-+\cdots)=\tfrac{1}{2}[d\sigma_s(pp\to K^+K^++\cdots)+d\sigma_s(pp\to K^-K^-+\cdots)],\tag{11d}$$

$$d\sigma_s(pn\to K^+\pi^-+\cdots)=\tfrac{1}{2}[d\sigma_s(pp\to K^+\pi^++\cdots)+d\sigma_s(pp\to K^0\pi^-+\cdots)],\tag{11e}$$

$$d\sigma_s(pn\to K^+\pi^++\cdots)=\tfrac{1}{2}[d\sigma_s(pp\to K^+\pi^-+\cdots)+d\sigma_s(pp\to K^0\pi^++\cdots)],\tag{11f}$$

$$d\sigma_s(pn\to K^-\pi^++\cdots)=\tfrac{1}{2}[d\sigma_s(pp\to K^-\pi^-+\cdots)+d\sigma_s(pp\to \bar{K}^0\pi^++\cdots)],\tag{11g}$$

$$d\sigma_s(pn\to K^-\pi^-+\cdots)=\tfrac{1}{2}[d\sigma_s(pp\to K^-\pi^++\cdots)+d\sigma_s(pp\to \bar{K}^0\pi^-+\cdots)],\tag{11h}$$

$$d\sigma_s(pn\to K^+K^-+\cdots)=\tfrac{1}{2}[d\sigma_s(pp\to K^+K^0+\cdots)+d\sigma_s(pp\to K^0K^-+\cdots)],\tag{11i}$$

etc.

Combining relations (11e), (11h) and (11f), (11g) it is possible to obtain relations for K_s^0. Relations involving π^0 can be obtained in a similar way. For the symmetric correlations in pp collisions a hierarchy for the production of pairs of different mesons is expected to hold. It appears, from lepton-hadron data, that "favored" fragmentation functions, like, e.g., $D_u^{\pi^+}(z)$, are larger than "disfavored" fragmentation functions, like, e.g., $D_d^{\pi^+}(z)$. We expect therefore that $F(p\to\pi^+)$ will be larger than $F(p\to\pi^-)$ and, similarly $F(p\to K^+)>F(p\to K^-)$, so that for the correlations in pp collisions we expect

$$\begin{aligned}&\pi^+\pi^+>\pi^+\pi^->\pi^-\pi^-,\\&\pi^+K^+>\pi^-K^+>\pi^-K^-,\\&K^+K^+>K^+K^->K^-K^-.\end{aligned}\tag{12}$$

For isoscalar nuclear targets we expect the same hierarchy to hold (barring effects from anomalous nuclear enhancements).

In the case of pion beams we obtain

$$d\sigma_s(\pi^-p\to\pi^+\pi^-+\cdots)=\tfrac{1}{2}[d\sigma_s(\pi^+p\to\pi^-\pi^-+\cdots)+d\sigma_s(\pi^+p\to\pi^+\pi^++\cdots)],\tag{13a}$$

$$d\sigma_s(\pi^+p\to\pi^+\pi^-+\cdots)=\tfrac{1}{2}[d\sigma_s(\pi^-p\to\pi^-\pi^-+\cdots)+d\sigma_s(\pi^-p\to\pi^+\pi^++\cdots)],\tag{13b}$$

etc.

For kaon beams similar relations can be obtained, e.g.,

$$d\sigma_s(K^+p\to\pi^+\pi^-+\cdots)=\tfrac{1}{2}[d\sigma_s(K^-p\to\pi^+\pi^++\cdots)+d\sigma_s(K^-p\to\pi^-\pi^-+\cdots)].\tag{14}$$

The virtue of relations (5)–(14) is that they are independent of any specific parametrization of the quark-distribution and quark-fragmentation functions, of the specific form chosen for the hard-scattering cross section, of the transverse momentum of partons in hadrons and of the transverse momentum of the outgoing hadrons with respect to the jet axis. Deviations from these relations are thus expected to arise only from interference terms or from a breakdown of the model. In the former case we expect a systematic pattern of deviations which can be checked. We believe therefore that experimental tests of these relations will give us direct information on some of the basic assumptions made in the theoretical description of the production of large-transverse-momentum particles.

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