ongoing inflation will persist long after the initiating shocks have disappeared and long after a reduction of demand has eliminated any excess demand-pull on the economy. Because of this inertia in an ongoing inflation rate, reduced demand results initially in excess capacity and high unemployment and only gradually in reduced inflation.

The inertia of inflation, the presence of cost-push as a source of inflation, and the variation of inflation along the short-run Phillips curve all discredit the idea that demand-pull is always the central source of inflation and that inflation can be eliminated efficiently simply by reducing demand to a point where labour and product markets are not excessively tight. Inflation is responsive to the strength of total demand, and avoiding levels of demand that would produce overly tight markets will avoid setting off a demand-pull inflation. However, in the face of either cost-push shocks or an ongoing inflation rate of whatever origin, reducing demand will mainly reduce output and employment in the short run, and only gradually reduce inflation, even if the economy is kept operating at rates well below the demand-pull region.

Because of these characteristics of economies, the demand management chore confronting policy-makers is not merely avoiding excess demand, as in the simplified Keynesian model, but rather choosing how much to accommodate inflation and how much to give up in output and employment in order to suppress inflation. As a related point, the identification of inflation with money in some models does not necessarily identify money as causing inflation in the sense of creating demand-pull, or 'too much money chasing too few goods'. Money may correlate with inflation whatever its cause unless policy-makers refuse to accommodate it at all and accept sharply lower output and employment levels instead.

Some models of inflation developed in the 1970s and 1980s maintain the essential features of the demand-pull inflation model but in a modified form that takes explicit account of the inertia in inflation. These models specify that there exists a natural rate of unemployment such that inflation accelerates when unemployment is below the natural rate and decelerates with unemployment above it. In their behavioural underpinnings, they assume wages are established in auction-like markets that would clear continuously except for uncertain expectations about inflation on the part of both firms and workers. They thus differ from models that are rooted in the original Keynesian insights about unemployment, which assume firms set wages in the context of more complicated long-run employment relations. In these models featuring long-term attachments between firms and workers, firms respond to reductions in demand primarily with layoffs rather than with market-clearing wage adjustments. Wasteful levels of unemployment can thus exist for extended periods.

The natural rate of unemployment is the analytical counterpart of full employment in Keynesian models, and unemployment rates below the natural rate correspond to the demand-pull region. These natural rate models can fit the historical data reasonably well if they make an important allowance for cost-push events and for inertia in inflation so that the response of inflation to economic slack is severely damped. Empirical testing thus far has been unable either to accept or reject some of the key features of the natural rate models. But there is little objective evidence that the natural unemployment rate that is empirically identified in these models corresponds to an optimal utilization of real resources in the same way that full employment represents an optimal operating level in the simpler Keynesian models. Avoiding excess demand, and thereby demand-pull inflation, is a clear prescription of both types of model; but because the presence

of inflation or even its worsening need not imply the existence of excess demand, avoiding inflation altogether while still fully utilizing labour and capital resources remains a difficult, and perhaps unattainable, goal for aggregate demand management.

GEORGE L. PERRY

See also COST-PUSH INFLATION; DEMAND MANAGEMENT; INFLATION.

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demand theory. The main purpose of demand theory is to explain observed consumer choices of commodity bundles. Market parameters, typically prices and income, determine constraints on commodity bundles. Given a combination of market parameters, a commodity bundle or a non-empty set of commodity bundles, which satisfies the corresponding constraints, is called a demand vector or a demand set. The mapping which assigns to every admissible combination of market parameters a unique demand vector (or a non-empty demand set) is called a demand function (or a demand correspondence). Traditional demand theory considers the demand function (or correspondence) as the outcome of some optimizing behaviour of the consumer. Its primary goal is to determine the impact on observed demands for commodities of alternative assumptions on the objectives and behavioural rules of the consumer, and on the constraints which he faces. The traditional model of the consumer takes preferences over alternative commodity bundles to describe the objectives of the consumer. Its behavioural rule consists in maximizing these preferences on the set of corresponding commodity bundles which satisfy the budget constraint imposed by the market parameters. If there is a unique preference maximizer under each budget constraint, then preference maximization determines a demand function. If there is at least one preference maximizer under each budget constraint, then preference maximization determines a demand correspondence.

Once the traditional view is adopted the occurrence of demand correspondences cannot be avoided. Compatibility of

observed demand, which is always unique, with some demand correspondence is a minor problem in general. However, the correspondence should be obtained through preference maximization. The last requirement leads to the main issues of modern demand theory: Which demand correspondences are compatible with preference maximization? Given any conditions necessary for demand correspondences to be compatible with preference maximization, are they sufficient? Which demand correspondences are compatible with a special class of preferences? What type of preferences yields a particular class of demand correspondences? When addressing these issues, modern demand theory attempts to link two concepts: preferences and demand.

Historically, the important concept was utility rather than preference. Before Fisher (1892) and Pareto (1896), utility was conceived as cardinal, that is it was assumed to be a measurable scale for the degree of satisfaction of the consumer. Fisher and Pareto were the first to observe that an arbitrary increasing transformation of the utility function has no effect on demand. Edgeworth (1881) had already written utility as a general function of quantities of all commodities and employed indifference curves. It is now widely accepted in demand theory that only ordinal utility matters. A utility function is merely a convenient device to represent a preference relation, and any increasing transformation of the utility function will serve this purpose as well.

The representability by utility functions imposes some restrictions on preferences. The problem of representability of a preference relation by a numerical function was solved by Debreu (1954, 1959, 1964) based on work by Eilenberg (1941), and by Rader (1963), and Bowen (1968).

While still assuming cardinal utility, Walras (1874) developed the first 'theory of demand': His demand was a function of all prices and endowment, obtained through utility maximization. Slutsky (1915) finally assumed an ordinal utility function with enough restrictions to yield a maximum under any budget constraint and testable properties of the resulting demand functions. In particular, he obtained negativity of diagonal elements and symmetry of the 'Slutsky matrix'.

Antonelli (1886) was the first to go the opposite way: construct indifference curves and a utility function from the so-called inverse demand function. Pareto (1980b) took the same route. Katzner (1970) reports on recent results in this direction. The construction of preference relations from demand functions was achieved in two ways: (1) Samuelson (1947) and Houthakker (1950) introduced the concept of revealed preference into demand theory. Considerable progress in relating utility and demand in terms of revealed preference was achieved by Uzawa (1960), further refinements being due to Richter (1966). (2) Hurwicz and Uzawa (1971) contributed to the following so-called integrability problem: construct a twice continuously differentiable utility representation from a continuously differentiable demand function which satisfies certain integrability conditions (including symmetry and negative semidefiniteness of the Slutsky matrix). Kihlstrom, Mas-Colell and Sonnenschein (1976) unified the two approaches under (1) and (2) in that they related the axioms of revealed preference to properties of the Slutsky matrix.

Since there exists a sizeable literature on demand theory, many of the concepts and results are well established and well known. These have become so much part of standard knowledge in economic theory that they are included in any microeconomic textbook today and other surveys. It would reduce the available space for a presentation of the new results of the last fifteen years substantially if an extended introductory account of demand theory were to be included here as well.

1. COMMODITIES AND PRICES

Consumers purchase or sell commodities, which can be divided into goods and services. Each commodity is specified by its physical quality, its location and the date of its availability. In the case of uncertainty, the state of the nature in which the commodity is available may be added to the specification of a commodity. This leads to the notion of a contingent commodity (see Arrow, 1953 and Debreu, 1959). We assume as in traditional theory that there exists a finite number l of such commodities. Quantities of each commodity are measured in real numbers. A *commodity bundle* is an l-dimensional vector $x = (x_1, \ldots, x_l)$. The set of all l-dimensional vectors $x = (x_1, \ldots, x_l)$ is the l-dimensional Euclidean space $\mathbb{R}^l$ which we interpret as the *commodity space*. $|x_h|$ indicates the quantity of commodity $h = 1, \ldots, l$. Commodities are assumed to be perfectly divisible, so that their quantity may be expressed as any (non-negative) real number. The standard sign convention for consumers assigns positive numbers for commodities made available to the consumer (inputs) and negative numbers for commodities made available by the consumer (outputs). Hence, any commodity bundle $x \in \mathbb{R}^l$ is conceivable.

The price p_h of a commodity $h, h = 1, \ldots, l$, is a real number which is the amount in units of account that has to be paid in exchange for one unit of the commodity. For the consumer, p_h is given and has to be paid now for the delivery of commodity h under the circumstances (location, date, state) specified for commodity h. A *price system* or *price vector* is a vector $p = (p_1, \ldots, p_l)$ in $\mathbb{R}^l$ and contains the prices for all commodities. The value of a commodity bundle x given the price vector p is $px = \sum_{h=1}^{l} p_h x_h$. This means that commodity bundles are *linearly priced*.

2. CONSUMPTION SETS AND BUDGET SETS

Typically, some commodity bundles cannot be consumed by a consumer for physical reasons. Those consumption bundles which can be consumed form the consumer's *consumption set*. This is a non-empty subset X of the commodity space $\mathbb{R}^l$. A consumer must choose a bundle x from his consumption set X in order to subsist. Traditionally, inputs in consumption are described by positive quantities and outputs by negative quantities. So in particular, the labour components of a consumption bundle x are all non-positive, unless labour is hired for a service. One usually assumes that the consumption set X is closed, convex, and bounded below. Vectors $x \in X$ are sometimes called *consumption plans*.

Given the sign convention on inputs and outputs and a price vector p, the value px of a consumption plan x defines the net outlay of x, that is the value of all purchases (inputs) minus the value of all sales (outputs) for the bundle x. Trading the bundle x in a market at prices p implies payments and receipts for that bundle. Therefore, the value of the consumption plan should not exceed the initial wealth (or income) of the consumer which is a given real number w. If the consumer owns a vector of initial resources $\omega \in \mathbb{R}^l$ and the price vector p is given, then w may be determined by $w = p\omega$. The consumer may have other sources of wealth: savings and pensions, bequests, profit shares, taxes or other liabilities. Given p and w, the set of possible consumption bundles whose value does not exceed the initial wealth of the consumer is called the *budget set* and is defined formally by

$$\beta(p, w) = \{x \in X \mid px \leqslant w\}.$$

The ultimate decision of a consumer is to choose a consumption plan from his budget set. Those vectors in $\beta(p, w)$ which the consumer eventually chooses form his *demand set* $\phi(p, w)$.

3. PREFERENCES AND DEMAND

The choice of the consumer depends on his tastes and desires. These are represented by his *preference relation* $\succsim$ which is a binary relation on X. For any two bundles $x, y \in X, x \succsim y$ means that x is at least as good as y. If the consumer always chooses a most preferred bundle in his budget set, then his demand set is defined by

$$\phi(p, w) = \{x \in \beta(p, w) | x' \in \beta(p, w)$$
$$\text{implies} \quad x \succsim x' \quad \text{or not} \quad x' \succsim x\}.$$

Three basic axioms are usually imposed on the preference relation $\succsim$ which are taken as a definition of a rational consumer.

Axiom 1 (Reflexivity). If $x \in X$, then $x \succsim x$, i.e. any bundle is as good as itself.

Axiom 2 (Transitivity). If $x, y, z \in X$ such that $x \succsim y$ and $y \succsim z$, then $x \succsim z$.

Axiom 3 (Completeness). If $x, y \in X$, then $x \succsim y$ or $y \succsim x$.

A preference relation $\succsim$ which satisfies these three axioms is a complete preordering on X and will be called a *preference order*. Already Axioms 2 and 3 define a preference order, since Axiom 3 implies Axiom 1. A preference relation $\succsim$ on X induces two other relations on X, the relation of strict preference, $\succ$, and the relation of indifference, $\sim$.

Definition. Let $\succsim$ be a preference relation on the consumption set X. A bundle x is said to be *strictly preferred to* a bundle y, i.e. $x \succ y$, if and only if $x \succsim y$ and not $y \succsim x$. A bundle x is said to be *indifferent to* a bundle y, i.e. $x \sim y$, if and only if $x \succsim y$ and $y \succsim x$.

LEMMA. *Suppose $\succsim$ is reflexive and transitive. Then*

(i) $\succ$ *is irreflexive, i.e. not $x \succ x$, and transitive;*

(ii) $\sim$ *is an equivalence relation on X, that means $\sim$ is reflexive, transitive, and symmetric, i.e. $x \sim y$ if and only if $y \sim x$.*

For $Z \subset X, x \in Z, x$ is called *maximal in Z*, if for all $z \in Z$, not $z \succ x$. x is called a *best element of Z* or *most preferred in Z*, if for all $z \in Z, x \succsim z$. Best elements are maximal; maximal elements are not necessarily best elements. If $\succsim$ is complete, then best and maximal elements coincide. Obviously for any price vector p and initial wealth w,

$$\phi(p, w) = \{x \in \beta(p, w) | x \quad \text{is maximal in} \quad \beta(p, w)\}.$$

Axioms 1-3 are not questioned in most consumer theory. However, transitivity and completeness may be violated by observed behaviour. Recent developments in the theory of consumer demand indicate that some weaker axioms suffice to describe and derive consistent demand behaviour (see, for example Sonnenschein, 1971; Katzner, 1971; Shafer, 1974; Kihlstrom et al., 1976; Kim and Richter, 1986).

In an alternative approach, one could start from a strict preference relation as the primitive concept. This may sometimes be convenient. However, the weak relation $\succsim$ seems to be the more natural concept. If the consumer chooses x, although y was a possible choice as well, then his choice can only be interpreted in the sense of $x \succsim y$, but not as $x \succ y$.

For the remainder of this section, let us fix a preference order $\succsim$ on X and a non-empty subset B of R^{l+1} such that for every $(p, w) \in B$, there is a unique $\succsim$-best element in $\beta(p, w)$; i.e. maximization of $\succsim$ defines a *demand function* $f: B \to X$, $\phi(p, w) = \{f(p, w)\}$ for all $(p, w) \in B$.

Let $x, x' \in X, x \neq x'$. We call x *revealed preferred to x'* and write xRx', if there is $(p, w) \in B$ such that $x = f(p, w)$ and $px' \leqslant px$. xRx' implies that both x and x' are in the budget set $\beta(p, w)$ and x is chosen. Since f is derived from $\succsim$-maximization, xRx' implies $x \succ x'$. We call x *indirectly revealed preferred to x'* and write xR^*x', if there exists a finite sequence $x_0 = x, x_1, \ldots, x_n = x'$ in X such that $x_0 R x_1, \ldots, x_{n-1} R x'$. Obviously, R^* is transitive. Since $\succ$ is transitive, xR^*x' implies $x \succ x'$. Consequently, the following must hold. (Otherwise $x \succ x!$)

(SARP) $xR^*x' \Rightarrow$ not $(x'R^*x)$.
(SARP) implies
(WARP) $xRx' \Rightarrow$ not $(x'Rx)$.

(SARP) is called the *strong axiom of revealed preference*; (WARP) is called the *weak axiom*. Hence $\succsim$-maximization implies the strong axiom. For the inverse implication, see Chapters 1, 2, 3, and 5 of Chipman et al. (1971).

4. CONTINUOUS PREFERENCE ORDERS AND UTILITY FUNCTIONS

The Axioms 1-3 have intuitive appeal. This is less so with the topological requirements of the following Axiom 4.

Axiom 4 (Continuity). For every $x \in X$, the sets $\{y \in X | y \succsim x\}$ and $\{y \in X | x \succsim y\}$ are closed relative to X.

Closedness of $\{y \in X | y \succsim x\}$ requires that for any sequence $y^n, n \in N$, in X such that y^n converges to $y \in X$ and $y^n \succsim x$ for all n, the limit in y also satisfies $y \succsim x$. If $\succsim$ is a preference order, then Axiom 4 is equivalent to:

For every $x \in X$, the sets $\{y \in X | y \succ x\}$ and $\{y \in Y | x \succ y\}$ are open in X.

Openness of $\{y \in X | y \succ x\}$ means that if $y \succ x$, then $y' \succ x$ for any y' close enough to y.

The sets $\{y \in X | y \succsim x\}$ are called *upper contour sets* of the relation $\succsim$ and the sets $\{y \in X | x \succsim y\}$ are called *lower contour sets* of $\succsim$. For $x \in X$, the set $I(x): = \{y \in X | y \sim x\}$ is called the *indifference class* of x with respect to $\succsim$ or the *$\succsim$-indifference surface* through x or the *$\succsim$-indifference curve* through x. In case $\succsim$ is reflective and transitive, $I(x)$ is the equivalence class of x with respect to the equivalence relation $\sim$.

There is a preference order $\succsim$ on $R^l, l \geqslant 2$, which does not satisfy Axiom 4, namely the *lexicographic order* defined by $(x_1, \ldots, x_l) \succsim (y_1, \ldots, y_l)$ if and only if there is $k, 1 \leqslant k \leqslant l$ such that $x_j = y_j$ for $j < k$ and $x_k > y_k$, or $x = y$.

Few studies of the relationship between the order properties of Axioms 1-3 and the topological property of Axiom 4 have been made. We emphasize the following result:

THEOREM (Schmeidler, 1971). *Let $\succsim$ denote a transitive binary relation on a connected topological space X. Assume that there exists at least one pair $\bar{x}, \bar{y} \in X$ such that $\bar{x} \succ \bar{y}$. If for every $x \in X$, (i) $\{y \in X | y \succsim x\}$ and $\{y \in X | x \succsim y\}$ are closed and (ii) $\{y \in X | y \succ x\}$ and $\{y \in X | x \succ y\}$ are open, then $\succsim$ is complete.*

Definition. Let X be a set and $\succsim$ be a preference relation on X. Then a function u from X into the reals R is a (*utility*) *representation* or a *utility function for* $\succsim$, if for any $x, y \in X, u(x) \geqslant u(y)$ if and only if $x \succsim y$.

Clearly, if u is a utility representation for $\succsim$ and $f: R \to R$ is an increasing transformation, then $f \circ u$ is also a representation of $\succsim$. If $u: X \to R$ is any function, then $\succsim$, defined by $x \succsim y$ if and only if $u(x) \geqslant u(y)$ for $x, y \in X$, is a preference order on X and u is a utility representation for $\succsim$.

Most utility functions used in consumer theory are continuous. If u is continuous and $\succsim$ is represented by u, then by

necessity $\gtrsim$ is a continuous preference order. In our case where $X \subset \mathbb{R}^l$, the opposite implication also holds. If $\gtrsim$ is a continuous preference order, then it has a continuous utility representation.

THEOREM (Debreu, Eilenberg, Rader). *Let X be a topological space with a countable base of open sets (or a connected, separable topological space) and $\gtrsim$ be a continuous preference order on X. Then $\gtrsim$ has a continuous utility representation.*

A preference order $\gtrsim$ on X, which is not continuous, need not have a utility representation. For instance, the lexicographic order on $\mathbb{R}^l$ does not have a utility representation, not even a discontinuous one. As an immediate consequence of the representation theorem for preference relations, one obtains one of the standard results on the non-emptiness of the demand set $\phi(p, w)$ since any continuous function attains its maximum on a compact set (Weierstrass' Theorem).

COROLLARY. *Let $X \subset \mathbb{R}^l$ be bounded below and closed, $\gtrsim$ be a continuous preference order on X, $p \in \mathbb{R}^l_{++}$ (i.e. $p \gg 0$), and $w \in \mathbb{R}$. Then $\beta(p, w) \neq \emptyset$ implies $\phi(p, w) \neq \emptyset$.*

5. SOME PROPERTIES OF PREFERENCES AND UTILITY FUNCTIONS

Some of the frequent assumptions on preference relations correspond almost by definitions to analogous properties of utility functions, while other analogies need demonstration. We discuss the assumptions most commonly used.

MONOTONICITY

Definition. A preference order $\gtrsim$ on $X \subset \mathbb{R}^l$ is *monotonic*, if $x, y \in X$, $x \geqq y$, $x \neq y$ implies $x \succ y$.

This property means desirability of all commodities. If a monotonic preference order has a utility representation u, then u is an increasing function (in all arguments). Inversely, if $\gtrsim$ is represented by an increasing function, then $\gtrsim$ is monotonic.

NON-SATIATION

Definition. Let $\gtrsim$ be the preference relation of a consumer over consumption bundles in X and let $x \in X$.

(i) x is a *satiation point* for $\gtrsim$ if $x \gtrsim y$ for all $y \in X$, i.e. x is a best element in X.
(ii) The preference relation is *locally not satiated at x*, if for every neighbourhood V of x there exists $z \in V$ such that $z \succ x$.

Consider a utility representation u for $\gtrsim$. $x \in X$ is a satiation point if and only if u has a global maximum at x. $\gtrsim$ is locally not satiated at x if and only if u does not attain a local maximum at x. Local non-satiation excludes that u be constant in a neighbourhood of x. If $\gtrsim$ is locally not satiated at all x, then $\gtrsim$ cannot have thick indifference classes or satiation points.

CONVEXITY

Definition. A preference relation $\gtrsim$ on $X \subset \mathbb{R}^l$ is called

(i) *convex*, if the set $\{y \in X \mid y \gtrsim x\}$ is convex for all $x \in X$;
(ii) *strictly convex*, if X is convex and $\lambda x + (1 - \lambda)x' \succ x'$ for any two bundles $x, x' \in X$, such that $x \neq x', x \gtrsim x'$, and for any λ, such that $0 < \lambda < 1$;
(iii) *strongly convex*, if X is convex and $\lambda x + (1 - \lambda)x' \succ x''$ for any three bundles $x, x', x'' \in X$ such that $x \neq x', x \gtrsim x'', x' \gtrsim x''$, and for any λ such that $0 < \lambda < 1$.

Definition. $u : X \to \mathbb{R}$ is called

(i) *quasi-concave*, if $u(\lambda x + (1 - \lambda)y) \geqq \min\{u(x), u(y)\}$ for all $x, y \in X$ and any λ, $0 \leqslant \lambda \leqslant 1$;

(ii) *strictly quasi-concave*, if $u(\lambda x + (1 - \lambda)y) > \min\{u(x), u(y)\}$ for all $x, y \in X$, $x \neq y$, and any λ, $0 < \lambda < 1$.

Let u be a representation of the preference order $\gtrsim$. u is (strictly) quasi-concave if and only if $\gtrsim$ is (strictly) convex. Quasi-concavity is preserved under increasing transformations, i.e. it is an ordinal property. In contrast, concavity is a cardinal property which can be lost under increasing transformations. With respect to the difficult problem to characterize those preference orders which have a concave representation, we refer to Kannai (1977).

Clearly, if $\gtrsim$ is locally not satiated at all x, then $\gtrsim$ does not have a satiation point. In general, the inverse implication is false. If, however, $\gtrsim$ is strictly convex and does not have a satiation point, then $\gtrsim$ is locally not satiated at all x. Moreover, if $\gtrsim$ is strictly convex, then $\gtrsim$ has at most one satiation point. An immediate implication is the following lemma.

LEMMA. *Let $X \subset \mathbb{R}^l$ be bounded below, convex, and closed. Let $\gtrsim$ be a strictly convex, continuous preference order on X, $p \in \mathbb{R}^l_{++}$, and $w \in \mathbb{R}$. Then $\beta(p, w) \neq \emptyset$ implies that $\phi(p, w)$ is a singleton.*

SEPARABILITY. Separable utility functions were used in classical consumer theory long before associated properties of preferences had been defined. All early contributions to utility theory assumed without much discussion an additive form of the utility function over different commodities. It was not until Edgeworth (1881) that utility was written as a general function of a vector of commodities. The particular consequences of separability for demand theory were discussed well after the general nonseparable case in demand theory had been treated and generally accepted. Among the many contributors are Sono (1945), Leontief (1947), Samuelson (1947), Houthakker (1960), Debreu (1960), Koopmans (1972). We follow Katzner (1970) in our presentation.

Let $N = \{N_j\}_{j=1}^k$ be a partition of the set $\{1, \ldots, l\}$ and assume that $X = S_1 \times \cdots \times S_k$. Let $J = \{1, \ldots, k\}$ and for any $j \in J, y \in X, y = (y_1, \ldots, y_k) \in \Pi_{i \in J} S_i$ write $y_{-j} = (y_1, \ldots, y_{j-1}, y_{j+1}, \ldots, y_k)$ for the vector of components different from j. For any y_{-j}, a preference order $\gtrsim$ on X induces a preference order $\gtrsim_{y_{-j}}$ on S_j which is defined by $x_j \gtrsim_{y_{-j}} x_j'$ if and only if $(y_{-j}, x_j) \gtrsim (y_{-j}, x_j')$ for $x_j, x_j' \in S_j$. In general, the induced ordering $\gtrsim_{y_{-j}}$ will depend on y_{-j}. The first notion of separability states that for any j, the preference orders $\gtrsim_{y_{-j}}$ are independent of $y_{-j} \in \Pi_{i \neq j} S_i$. The second notion of separability states that for any proper subset I of J, the induced preference orders $\gtrsim_{y_N}$ on $\Pi_{i \in I} S_i$ are independent of $y_N \in \Pi_{i \notin I} S_i$.

Definition. Let $\gtrsim$ be a preference order on $X = \Pi_{j \in J} S_j$.

(i) $\gtrsim$ is called *weakly separable* with respect to N if $\gtrsim_{y_{-j}} = \gtrsim_{z_{-j}}$ for each $j \in J$ and any $y_{-j}, z_{-j} \in \Pi_{i \neq j} S_i$.
(ii) $\gtrsim$ is called *strongly separable* with respect to N if $\gtrsim_{y_N} = \gtrsim_{z_N}$ for each $I \subset J, I \neq \emptyset, I \neq J$ and any $y_N, z_N \in \Pi_{i \notin I} S_i$.

Definition. Let $u : \Pi_{j \in J} S_j \to \mathbb{R}$. u is called

(i) *weakly separable* with respect to N, if there exist continuous functions
$v_j : S_j \to \mathbb{R}, j \in J$, and $V : \mathbb{R}^k \to \mathbb{R}$ such that $u(x) = V[v_1(x_1), \ldots, v_k(x_k)]$.
(ii) u is called *strongly separable* with respect to N, if there exist continuous functions
$v_j : S_j \to \mathbb{R}, j \in J$, and $V : \mathbb{R} \to \mathbb{R}$ such that $u(x) = V[\Sigma_{j \in J} v_j(x_j)]$.

The two important equivalence results on separability are due

to Debreu and Katzner. The version of Debreu's Theorem given here is slightly weaker than his original result.

THEOREM (Katzner, 1970). *Let* $\succsim$ *be a continuous, monotonic preference order on* $X = \Pi_{j \in J} S_j$ *with* $S_j = \mathbb{R}^N j$ *for all* $j \in J$. *Then* $\succsim$ *is weakly separable if and only if every continuous representation of it is weakly separable.*

THEOREM (Debreu, 1960). *Let* $\succsim$ *be a continuous, monotonic preference order on* $X = \Pi_{j \in J} S_j$ *with* $S_j = \mathbb{R}^N j$ *for all* $j \in J = \{1, \ldots, k\}$ *and* $k \geqslant 3$. *Then* $\succsim$ *is strongly separable if and only if every continuous representation is strongly separable.*

Under the assumptions of this theorem, if $\succsim$ is strongly separable with representation $u(x) = V[\Sigma_{j \in J} v_j(x_j)]$, then V must be increasing or decreasing. Therefore,

$$v(x) = \begin{cases} \Sigma_{j \in J} v_j(x_j) & V \text{ increasing} \\ -\Sigma_{j \in J} v_j(x_j) & V \text{ decreasing} \end{cases}$$

is also a representation of $\succsim$. This is the form of separable utility used by early economists who thought that each commodity h had its own intrinsic utility representable by a scalar function u_h. The overall utility was then simply obtained as the sum of these functions, $u(x) = \Sigma_h u_h(x_h)$. Such a formulation is given by Jevons (1871) and Walras (1874) and implicitly contained in Gossen (1854).

For $k = 2$, weak and strong separability of preferences coincide. But there are separable preferences which do not admit a strongly separable utility representation, for instance $X = \mathbb{R}^2_+$, $N_j = \{j\}$ for $j = 1, 2, \succsim$ given by $u(x_1, x_2) = \sqrt{x_1} + \sqrt{x_1 + x_2}$.

Separability of preferences imposes restrictions on demand correspondences and on demand functions (for details see Barten and Böhm, 1982, sections 9, 14, and 15).

6. CONTINUOUS DEMAND

Given any price-wealth pair $(p, w) \in \mathbb{R}^{l+1}$, the budget set of the consumer was defined as $\beta(p, w) = \{x \in X | px \leqslant w\}$. Let $S \subset \mathbb{R}^{l+1}$ denote the set of price-wealth pairs for which the budget set is non-empty. Then β describes a correspondence from S into X, i.e. β associates to any $(p, w) \in S$ the non-empty subset $\beta(p, w)$ of X. There are two standard notions of continuity of correspondences, upper hemi-continuity and lower hemi-continuity (see Hildenbrand, 1974).

Definition. A compact-valued correspondence Ψ from S into an arbitrary subset T of $\mathbb{R}^l$ is *upper hemi-continuous* (u.h.c.) at a point $y \in S$, if for all sequences $(y^n, z^n) \in S \times T$ such that $y^n \to y$ and $z^n \in \Psi(y^n)$ for all n, there exist $z \in \Psi(y)$ and a subsequence z^{n_k} of z^n such that $z^{n_k} \to z$.

Definition. A correspondence Ψ from S into an arbitrary subset T of $\mathbb{R}^l$ is *lower hemi-continuous* (l.h.c.) at a point $y \in S$, if for any $z \in \Psi(y)$ and any sequence y^n in S with $y^n \to y$ there exists a sequence z^n in T such that $z^n \to z$ and $z^n \in \Psi(y^n)$ for all n.

Definition. A correspondence is *continuous* if it is both lower and upper hemi-continuous.

For single-valued correspondences, the notions of lower and upper hemi-continuity coincide with the usual notion of continuity for functions. For proofs of the following lemmas, see Debreu (1959) or Hildenbrand (1974).

LEMMA. *Let* $X \subseteq \mathbb{R}^l$ *be a convex set. Then the budget correspondence* $\beta : S \to X$ *has a closed graph and is lower hemi-continuous at every point* (p, w) *for which* $w > \min\{px | x \in X\}$ *holds.*

Combining a previous Corollary on the non-emptiness of the demand set and a fundamental theorem of Berge (1966) yields the next result.

LEMMA. *Let* $X \subseteq \mathbb{R}^l$ *be a convex set. If the preference relation has a continuous utility representation, then the demand correspondence is defined (i.e. non-empty valued), compact-valued, and upper hemi-continuous at each* $(p, w) \in S$ *such that* $\beta(p, w)$ *is compact and* $w > \min\{px | x \in X\}$.

It follows immediately from the definitions, that $\phi(\lambda p, \lambda w) = \phi(p, w)$ for any $\lambda > 0$ and any price-wealth pair (p, w), i.e. demand is homogeneous of degree zero in prices and wealth. For convex preference orders, the demand correspondence is convex-valued. For strictly convex preference orders, the demand correspondence is single-valued, that is one obtains a demand function. The results of this section and of section 4 are summarized in the following lemma which uses the weakest assumptions of traditional demand theory to generate a continuous demand function.

LEMMA. *Let* $S' := \{(p, w) \in S | \beta(p, w)$ *is compact and* $w > \min\{px | x \in X\}\}$. *If* $\succsim$ *denotes a strictly convex and continuous preference order, then* $\phi(p, w)$ *defines a continuous demand function* $\phi : S' \to X$ *which satisfies:* (i) *homogeneity of degree zero in prices and wealth and* (ii) *the strong axiom of revealed preference.*

7. CONTINUOUS DEMAND WITHOUT TRANSITIVITY

Transitivity is often violated in empirical studies. This excludes utility maximization, but not necessarily preference maximization. However, as the next theorem indicates, existence and continuity of demand do not depend on transitivity as crucially as one may expect. The theorem follows from a result by Sonnenschein (1971).

THEOREM. *Let* $S^* = \{(p, w) \in S | \phi(p, w) \neq \emptyset\}$, X *compact, and* $\succsim$ *complete and with closed graph.*

(i) *If* $\{x' \in X | x' \succ x\}$ *is convex for all* $x \in X$, *then* $\phi(p, w) \neq \emptyset$ *whenever* $\beta(p, w) \neq \emptyset$ *(i.e.* $S^* = S$*).*

(ii) *If* $S^* = S$ *and* $(p^0, w^0) \in S$ *such that* β *is continuous at* (p^0, w^0), *then* ϕ *is u.h.c. at* (p^0, w^0).

The assumption that X is compact is not necessary. For case (i) it suffices that all budget sets $\beta(p, w)$ under consideration be compact. For case (ii) it is sufficient that there are a compact subset X^0 of X and a neighbourhood S^0 of (p^0, w^0) such that $\phi(S^0) \subset X^0$.

To complete this section we state a lemma on the properties of a demand function obtained under preference maximization without transitivity. This contrasts with the lemma at the end of the previous section. Nontransitivity essentially implies that the strong axiom of revealed preference need not hold. The lemma follows from the theorem by Sonnenschein and from the result by Shafer (1974).

LEMMA. *Let* $X = \mathbb{R}^l_+$, $B = \mathbb{R}^{l+1}_{++}$. *Suppose continuity and strong convexity of* $\succsim$ *(in addition to completeness). Then preference maximization yields a continuous demand function* $f : B \to X$ *which satisfies* (i) *homogeneity of degree zero in prices and wealth and* (ii) *the weak axiom of revealed preference.*

The converse statement of the lemma does not hold. For $l = 2$, $X = \mathbb{R}^2_+$, $B = \mathbb{R}^3_{++}$, there is a C^1-function $f : B \to X$ which fulfills (i), (ii), and (iii) $pf(p, w) = w$ for all $(p, w) \in B$, but which cannot be obtained as the demand function for a continuous, complete and strictly convex preference relation (John, 1984; Kim and Richter, 1986).

8. SMOOTH PREFERENCES AND DIFFERENTIABLE UTILITY FUNCTIONS

Due to the representation theorem of section 5, continuity of a utility function and continuity of the represented preference order are identical under the perspective of demand theory. When continuous differentiability of demand is required, continuity of the preference relation will not suffice in general. The first rigorous attempt to study 'differentiable preference orders' goes back to Antonelli (1886). We follow the more direct approach of Debreu (1972) to characterize 'smooth preference orders'. Smoothness of preferences is closely related to sufficient differentiability of utility representations and the solution of the integrability problem (see Debreu, 1972, also Debreu, 1976, Hurwicz, 1971, and section 12 below).

For the purpose of this and subsequent sections, let P denote the interior of $\mathbf{R}^l_+$ and assume that $X = P$. Let $\succsim$ be a continuous and monotonic preference order on P which we may consider as a subset of $P \times P$, i.e. $(x, y) \in \succsim \Leftrightarrow x \succsim y$ for $(x, y) \in P \times P$. Also, the associated indifference relation $\sim$ will be considered as a subset of $P \times P$. To describe a smooth preference order, differentiability assumptions will be made on the (graph of the) indifference relation in $P \times P$.

For $k \geqslant 1$, let C^k denote the class of functions which have continuous partial derivatives up to order k, and consider two open sets X and Y in $\mathbf{R}^n$. A bijection $h : X \to Y$ is a C^k-diffeomorphism if both h and h^{-1} are of class C^k. $M \subset \mathbf{R}^n$ is a C^k-hypersurface, if for every $z \in M$, there exist an open neighbourhood U of z, an open subset V of $\mathbf{R}^n$, a hyperplane $H \subset \mathbf{R}^n$ and a C^k-diffeomorphism $h : U \to V$ such that $h(M \cap U) = V \cap H$. A C^k-hypersurface has locally the structure of a hyperplane up to a C^k-diffeomorphism. Considering the indifference relation $\sim$ as a subset of $P \times P$, the set $\tilde{I} = \{(x, y) \in P \times P \,|\, x \sim y\}$ gives the 'indifference surface' of the preference relation. Then $\succsim$ is called a C^2-preference order (or smooth preference order), if $\tilde{I}$ is a C^2-hypersurface.

THEOREM (Debreu, 1972). *Let $\succsim$ be a continuous and monotonic preference order on P and $\tilde{I}$ be its indifference surface. Then $\succsim$ is a C^2-preference order if and only if it has a monotonic utility representation of class C^2 with no critical point.*

9. PROPERTIES OF DIFFERENTIABLE UTILITY FUNCTIONS

Utility functions of class C^2 provide the truly classical approach to demand theory (see, for example, Slutsky, 1915; Hicks, 1939; Samuelson, 1947).

Let $\succsim$ be a monotonic, strictly convex C^2-preference order on P and $u : P \to \mathbf{R}$ be a C^2-utility representation of $\succsim$ with no critical point. Then u is continuous, increasing in all arguments, and strictly quasi-concave. Moreover, all second-order partial derivatives $u_{ij}(x) = (\partial^2 u/\partial x_i \partial x_j)(x)$, $i, j = 1, \ldots, l$, $x \in P$, exist, all u_{ij} are continuous functions of x and $u_{ij} = u_{ji}$ for $i, j = 1, \ldots, l$. Let $D^2 u = (u_{ij})$ denote the Hessian matrix of u. Then $D^2 u$ is symmetric. The first-order derivatives $u_i(x) = (\partial u/\partial x_i)(x)$, $i = 1, \ldots, l$ are continuous functions of x. Assume that $u_i(x) > 0$ for $i = 1, \ldots, l$, $x \in P$ and define

$$Du(x) = \begin{bmatrix} u_1(x) \\ \vdots \\ u_l(x) \end{bmatrix}$$

as the gradient of u at x. For many $m \times n$-matrix M, let M' denote the transpose of M.

THEOREM. *If $u : P \to \mathbf{R}$ is a strictly quasi-concave utility function of class C^2, then*

$$z' D^2 u(x) z \leqslant 0 \quad \text{for all} \quad x \in P \quad \text{and} \quad z \in \{\tilde{z} \in \mathbf{R}^l \,|\, \tilde{z} Du(x) = 0\}.$$

(For a proof, see Barten and Böhm, 1982).

It will be shown in the next section that the conclusion of this theorem does not guarantee the existence of a differentiable demand function. The following definition strengthens the property of strict quasi-concavity.

Definition. u is called *strongly quasi-concave* if

$$z' D^2 u(x) z < 0 \quad \text{for all} \quad x \in P, \quad z \neq 0$$
$$\text{and} \quad z \in \{\tilde{z} \in \mathbf{R}^l \,|\, \tilde{z} Du(x) = 0\}.$$

Consider the bordered Hessian matrix

$$H(x) = \begin{bmatrix} D^2 u(x) & Du(x) \\ [Du(x)]' & 0 \end{bmatrix}.$$

Then u is strongly quasi-concave whenever u is strictly quasi-concave and $H(x)$ is non-singular. (For a proof see Barten and Böhm, 1982).

The properties of strict and strong quasi-concavity are invariant under increasing C^2-transformations. For other results and consequences of differentiable utility functions the reader may consult Barten and Böhm (1982) and the references listed there, or Debreu (1972), Mas-Colell (1974).

10. DIFFERENTIABLE DEMAND

Section 7 gave sufficient conditions on preferences for the existence of a continuous demand function which is homogeneous of degree zero in prices and wealth and satisfies the strong axiom of revealed preference. In this section, the implications of smooth preferences for differentiability of demand will be studied.

Consider an assumption (D), consisting of the following three parts:

(D1) $X = P$
(D2) $\succsim$ is a monotonic, strictly convex C^2-preference order on X and the closure relative to $\mathbf{R}^l_+ \times \mathbf{R}^l_+$ of its indifference surface $\tilde{I}$ is contained in $P \times P$.
(D3) The price-wealth space is $B = \mathbf{R}^{l+1}_{++}$.

Given (D), there exists a demand function $f : B \to X$ with $p \cdot f(p, w) = w$ for all $(p, w) \in B$. Let u be an increasing strictly quasi-concave C^2-utility representation for $\succsim$. The following key result on the differentiability of demand was first given by Katzner (1968). For a detailed proof see Barten and Böhm (1982).

THEOREM. *Let $(\bar{p}, \bar{w}) \in B$ and $\bar{x} = f(\bar{p}, \bar{w})$. Then the following assertions are equivalent:*

(i) f is C^1 in a neighbourhood of $(\bar{p}, \bar{w})$.

(ii) $\begin{bmatrix} D^2 u(\bar{x}) & \bar{p}' \\ \bar{p} & 0 \end{bmatrix}$ is non-singular.

(iii) $H(\bar{x}) = \begin{bmatrix} D^2 u(\bar{x}) & Du(\bar{x}) \\ [Du(\bar{x})]' & 0 \end{bmatrix}$ is non-singular.

Once the demand function f is continuously differentiable, it is straightforward to derive all of the well-known comparative statics properties, for the proof of which we refer again to Barten and Böhm (1982). Let $f = (f^1, \ldots, f^l)$ be a demand

function of class C^1 and define

$$f_w = (f_w^1, \dots, f_w^l) = \left(\frac{\partial f^1}{\partial w}, \dots, \frac{\partial f^l}{\partial w}\right),$$

$$f_j^i = \frac{\partial f^i}{\partial p_j}, \qquad i, j = 1, \dots, l,$$

$$s_j^i = f_j^i + f_w^i f^j, \qquad i, j = 1, \dots, l.$$

We obtain the Jacobian matrix of f with respect to prices, $J = (f_j^i)$, and the *Slutsky matrix* $S = (s_j^i)$.

THEOREM.

(i) $p f_w = 1$, $pJ = -f$,
(ii) $Sp' = 0$,
(iii) S is symmetric,
(iv) $ySy' < 0$, if $y \in \mathbf{R}^l$, $y \neq \alpha p$ for all $\alpha \in \mathbf{R}$,
(v) *rank* $S = l - 1$.

Property (iv) implies that all diagonal elements of S are strictly negative, i.e. $s_i^i = f_i^i + f_w^i f^i < 0$. If $f_w^i \geqslant 0$, then $(\partial x_i / \partial p_i) < 0$, i.e. commodity i is a *normal good*. $f_w^i < 0$, i.e. commodity i is an *inferior good*, is a necessary, but not sufficient condition that $(\partial x_i / \partial p_i) > 0$, i.e. that commodity i is a *Giffen good*.

11. DUALITY APPROACH TO DEMAND THEORY

With the notion of an expenditure function, an alternative approach to demand analysis is possible which was suggested by Samuelson (1947). For the further development and details, we refer to Diewert (1974, 1982).

As a matter of convenience and for ease of presentation assumption (D) will be imposed on the preference relation $\succsim$. Let u denote a strictly quasi-concave increasing C^2-utility representation for $\succsim$ and let $f : B \to X$ be the demand function derived from preference maximization. Let us further assume that $u(X) = \mathbf{R}$. (This requirement can always be fulfilled by means of an increasing transformation.) Define the *indirect utility function* $v : B \to \mathbf{R}$ associated with u by $v(p, w) = u(f(p, w))$ for $(p, w) \in B$.

Given a price system $p \in \mathbf{R}_{++}^l$ and a utility level $c \in \mathbf{R}$, let $e(p, c) = \min\{p \cdot x \mid x \in X, u(x) \geqslant c\}$. Since u is strictly quasi-concave and increasing, there exists a unique minimizer $h(p, c)$ of this problem such that $e(p, c) = ph(p, c)$. $h : \mathbf{R}_{++}^l \times \mathbf{R} \to \mathbf{R}_{++}^l$ is called the *Hicksian (income-compensated) demand function* and $e : \mathbf{R}_{++}^l \times \mathbf{R} \to \mathbf{R}_{++}$ is called the *expenditure function* for u.

Since assumption (D) holds, preference maximization and expenditure minimization imply the following properties and relationships:

(1) $c = v[p, e(p, c)]$ for all (p, c).
(2) $w = e[p, v(p, w)]$ for all (p, w).
(3) $v(p, \cdot)$ and $e(p, \cdot)$ are inverse functions for any p.
(4) $h(p, c) = f[p, e(p, c)]$ for all (p, c).
(5) $f(p, w) = h[p, v(p, w)]$ for all (p, w).
(6) e is strictly increasing and continuous in c.
(7) e is non-decreasing, positive linear homogeneous, and concave in prices.
(8) v is strictly increasing in w, and continuous.
(9) v is non-increasing in prices and homogeneous of degree zero in income and prices.

Moreover, some interesting and important consequences of these properties can be obtained if the functions are sufficiently differentiable.

THEOREM.

(i) e is C^k *if and only if v is C^k.* $(k = 1, 2)$.
(ii) *If e is C^1, then $\partial e / \partial p = h$.*

If f is C^1, then

(iii) v is C^2.
(iv) $f = -(\partial v / \partial p)/(\partial v / \partial w)$ (*Roy's identity*).
(v) 'e is C^1 *if and only if h is C^1*'.
(vi) h is C^1 *and e is C^2, and $\partial h / \partial p = S$ (Slutsky equation) with $\partial h / \partial p$ evaluated at $[p, v(p, w)]$ and S at (p, w).*

12. INTEGRABILITY

A review of the previous discussions and analytical results involving the concepts of:

$\succsim$ preference
u utility
h income-compensated demand
e expenditure function
v indirect utility
f (direct) demand

makes apparent their relationships which can be characterized schematically by the following diagram:
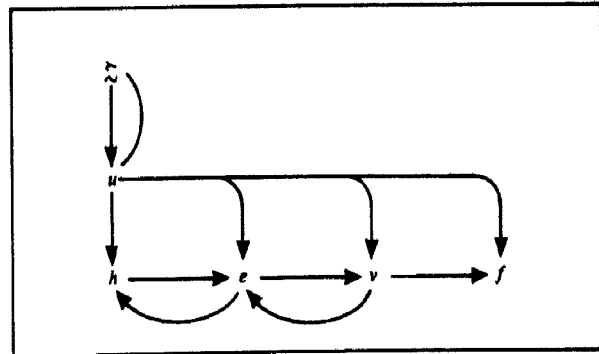
where $a \to b$ indicates that concept b can be derived from concept a under certain conditions.

The integrability problem is to establish $f \to u$, i.e. to recover the utility function from the demand function f. For details see the separate entry of Sonnenschein.

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See also CHARACTERISTICS; COMPENSATED DEMAND; DUALITY; ELASTICITY; INCOME EFFECT; INTEGRABILITY OF DEMAND; REVEALED PREFERENCE; SEPARABILITY; SUBSTITUTES AND COMPLEMENTS.

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