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The Foundation of the Theory of Monopolistic Competition Revisited

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It is shown why different price normalization rules influence oligopolistic equilibria in general equilibrium models. This is done for the Roberts and Sonnenschein duopoly and for a price duopoly in an exchange economy. *Journal of Economic Literature* Classification Numbers: D43, D51.

1. INTRODUCTION

Most of the modern literature on monopolistic competition has been influenced in one way or another by the contribution by Roberts and Sonnenschein [12]. In their seminal and highly regarded paper they show for a class of examples of convex economies that the behavior of profit maximizing imperfectly competitive firms cannot be mutually consistent. Firms' reaction curves typically may be discontinuous or not convex valued, thus Nash equilibria fail to exist. Since nothing in their set up seems to be pathological relative to the standard assumptions for competitive Arrow–Debreu economies, they conclude that their results suggest "... the need for a fundamental re-examination of the way our partial and general equilibrium models of monopolistic competition fit together."

It is quite straightforward to see for the examples by Roberts and Sonnenschein that the nonexistence result is due primarily to their particular choice of price normalization. It has been known since the work by Gabszewicz and Vial [7] and Cornwall [4] that price normalization influences noncompetitive equilibria in Arrow–Debreu economies. In a simple general equilibrium monopoly model Cornwall pointed out "... that a price normalization rule affects not only the magnitude of the outcome but also whether or not a solution exists." Dierker and Grodal [5] reemphasize the importance of normalization for existence. In a duopoly model in the spirit of Gabszewicz and Vial, they show by means of an example that Cournot–Walras equilibria fail to exist for all normalization rules. Their result depends in a crucial way on a double fold of the correspondence describing the competitive consumption sector. This feature excludes the possibility of continuous price selections everywhere. Since they can

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guarantee in their examples that firms' best replies in pure strategies are unique, they conclude that "... therefore the nonexistence of Cournot–Walras equilibrium is not due to reaction correspondences not being convex-valued but rather due to the multiplicity of market clearing prices, which forces price selections to be discontinuous." These findings do not contradict or enlighten but rather complement the results by Roberts and Sonnenschein. They construct their equilibrium price correspondence from an individual consumer's smooth utility function which, therefore, cannot possess a double fold. Thus the debate whether nonconcavities or noncontinuous selections cause nonexistence remains unresolved, as well as why normalization matters.

At this point two important open problems seem to remain with respect to normalization for noncompetitive Arrow–Debreu economies. First, it is unclear why an admittedly arbitrary choice of a numeraire, or more generally, of a normalization rule has such a decided influence on outcomes in economies formulated exclusively in real terms. It is sometimes argued that these results depend essentially on the fact that firms maximize profit. This reasoning may possibly explain the dependence of equilibria on different normalization rules but not the nonexistence. Besides, Dierker and Grodal partially invalidate these arguments through their second example where firms maximize their owner's indirect utility. Therefore, if profit maximization plays an essential part in the normalization story, oligopolistic equilibria in pure exchange economies should be independent of normalization.

Second, the general consequences of a dependence of equilibria on normalization for equilibrium theory are not yet recognized, since the full extent of the dependence has not been exhibited. Existence is one important issue but not the only one. It has been shown for two special cases that almost any allocation can be an equilibrium. Grodal [8] shows such a result for a Cournot–Walras oligopoly if the price selection is bijective, Böhm [1] contains a similar result for the monopoly with endogenous factor markets, except if demand elasticities are infinite. For the general oligopoly model it seems safe to conjecture the same type of result for any real Arrow–Debreu economy. More importantly, however, the normalization problem should be investigated in other noncompetitive general equilibrium models, for example those with externalities, nonconvexities, public goods, and taxation. In all of these areas the leading economic examples are using some numeraire or some price normalization. It is therefore important to understand fully why and when normalization matters. Only then it will be possible to determine whether real equilibrium allocations are subject to dependence on normalization.

Section 2 of this note shows first, for the duopoly model considered by Roberts and Sonnenschein, that every feasible allocation may be a Cournot equilibrium for an appropriately chosen normalization rule. This supports the general conjecture on dependence and it reflects a revised and different implication of the original nonexistence result by Roberts and Sonnenschein. Though the result may not seem too surprising, the method of proof reveals the essential role of a normalization rule. Changes of normalizations do not change the objective functions of the firms. However, they change the constraint set against which the duopolist maximizes. Hence profit maximization is not responsible for normalization dependence. Moreover, nonconcavities of the payoff functions and/or discontinuous price selections may be sources for nonexistence, but they are never the cause for dependence on normalization. In Section 3 a simple class of exchange economies with price setting behavior of two consumers is used to exhibit the role of different normalization rules. Two examples indicate how and when normalization matters.

2. Monopolistic Competition and Cournot–Walras Equilibria

Consider an economy with three commodities $h=1, 2, 3$, one consumer, and two firms which produce commodities one and two costlessly with amounts $0 \leqslant x_1 \leqslant \bar{x}_1$ and $0 \leqslant x_2 \leqslant \bar{x}_2$. The consumer has a twice differentiable, strictly monotonic, and strictly quasiconcave utility function $U: \mathbb{R}^3_+ \to \mathbb{R}$ which is of the form $U(x_1, x_2, x_3) = u(x_1, x_2) + x_3$. The consumer's endowment is $\omega = (0, 0, \omega_3)$ with $\omega_3 > 0$. Strict quasi-concavity of U and the separability implies strict concavity of u.

A price normalization rule for this economy with three commodities is a function $f: \mathbb{R}^2_+ \to \mathbb{R}$ such that for any pair of relative prices (α_i, α_j), $i \neq j$, $i, j \in \{1, 2, 3\}$,

$$p_k = \frac{1}{f(\alpha_i, \alpha_j)} \qquad k \neq i, j$$

$$p_i = \frac{\alpha_i}{f(\alpha_i, \alpha_j)}$$

$$p_j = \frac{\alpha_j}{f(\alpha_i, \alpha_j)}.$$

Roberts and Sonnenschein choose $i=1$, $j=2$, $k=3$ with $f(\alpha_1, \alpha_2)$ identically equal to one. Rather than choosing commodity three as a numeraire, define the relative prices $p_2/p_1 = \alpha_2$ and $p_3/p_1 = \alpha_3$ and a price normalization rule $f: \mathbb{R}_+ \to \mathbb{R}_+$ which depends on α_2 only. Thus,

$$p_1 = \frac{1}{f(\alpha_2)}$$

$$p_2 = \frac{\alpha_2}{f(\alpha_2)}$$

$$p_3 = \frac{\alpha_3}{f(\alpha_2)}.$$

Since the consumer's utility is separable with respect to commodity three and since the duopolists produce only commodities one and two, it is clear that only α_2 matters for the consumer's demand.

Denote $u_1 = \partial u/\partial x_1$, $u_2 = \partial u/\partial x_2$, and define $\mathbf{R}(x_1, x_2) = u_2(x_1, x_2)/u_1(x_1, x_2)$. Then the consumer's inverse demand function is

$$\alpha_2 = \mathbf{R}(x_1, x_2).$$

Producer i's profit is $\pi_i = p_i x_i$, $i = 1, 2$, since production costs are zero. Therefore, given the normalization rule f the best response correspondence for the two producers are

$$\mu_f^1(x_2) = \arg\max \left\{ \frac{x_1}{f(\mathbf{R}(x_1, x_2))} \,\middle|\, 0 \leqslant x_1 \leqslant \bar{x}_1 \right\}$$

and

$$\mu_f^2(x_1) = \arg\max \left\{ \frac{x_2 \mathbf{R}(x_1, x_2)}{f(\mathbf{R}(x_1, x_2))} \,\middle|\, 0 \leqslant x_2 \leqslant \bar{x}_2 \right\}.$$

$(\tilde{x}_1, \tilde{x}_2)$ is a Nash equilibrium if $\tilde{x}_i \in \mu_f^i(\tilde{x}_j)$, $i, j = 1, 2$, $i \neq j$.

THEOREM. *Assume $\partial\mathbf{R}/\partial x_1 > 0$ and $\partial\mathbf{R}/\partial x_2 < 0$; i.e., both goods are strictly normal. Then, for every $0 \leqslant (x_1^0, x_2^0) \leqslant (\bar{x}_1, \bar{x}_2)$ there exists a normalization rule $f: \mathbb{R}_+ \to \mathbb{R}_+$ such that (x_1^0, x_2^0) is a Nash equilibrium; i.e., $x_i^0 \in \mu_f^i(x_j^0)$, $i, j = 1, 2$, $i \neq j$.*

Proof. Let $\mathbf{R}_0 = \mathbf{R}(x_1^0, x_2^0)$ and choose the normalization rule

$$f_0^n(\mathbf{R}) = \max\{\mathbf{R}_0, \mathbf{R}\} + n\,|\mathbf{R} - \mathbf{R}_0|$$

for $n \geqslant 0$. This implies the price functions, i.e., the inverse demand functions,

$$p_1^n(x_1, x_2^0) = \frac{1}{f_0^n((x_1, x_2^0))}$$

$$p_2^n(x_1^0, x_2) = \frac{\mathbf{R}(x_1^0, x_2)}{f_0^n(\mathbf{R}(x_1^0, x_2))},$$

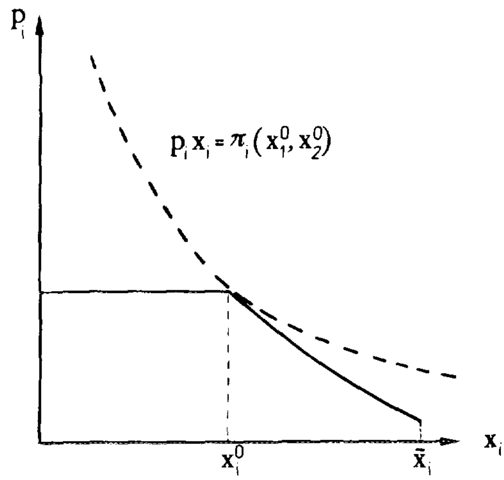

FIGURE 1.

against which the two firms maximize their profits

$$\pi_i^n(x_1, x_2) = x^i p_i^n(x_1, x_2), \qquad i = 1, 2.$$

The diagram displays the inverse demand function for firm i given x_j^0, $j \neq i$, and $n = 0$. The dashed line is the profit contour $\{(x_i, p_i) | p_i x_i = \pi_i(x_1^0, x_2^0)\}$, which may intersect the inverse demand function to the right of x_i^0. However, the construction guarantees that there always exists an $n \geq 0$ such that the whole inverse demand function lies below the profit contour (Fig. 1). For all $n \geq 0$ one has

$$p_1^n(x_1^0, x_2^0) = 1/\mathbf{R}_0 \qquad \text{and} \qquad p_2^n(x_1^0, x_2^0) = 1.$$

Moreover, $p_i^n(x_i, x_j^0)$ is nondecreasing for $0 \leq x_i \leq x_i^0$, strictly decreasing for $x_i^0 \leq x_i \leq \bar{x}_i$, and

$$\lim_{n \to \infty} p_i^n(x_i, x_j^0) = 0 \qquad \text{if} \quad x_i \neq x_i^0, \quad \text{for} \quad i, j = 1, 2; \quad i \neq j.$$

Therefore, for every $x_i \neq x_i^0$, there exists a number $N(x_i)$ and a neighborhood $V(x_i)$ such that $n \geq N(x_i)$ and $x_i' \in V(x_i)$ implies

$$\pi_i^{n+1}(x_i', x_j^0) < \pi_i^n(x_i', x_j^0) < \pi_i(x_i^0, x_j^0).$$

Since

$$\frac{\partial \pi_1^n}{\partial x_1}(x_1^0, x_2^0) < 1 - (n+1)(x_1^0/\mathbf{R}_0)\frac{\partial \mathbf{R}}{\partial x_1}(x_1^0, x_2^0)$$

and

$$\frac{\partial \pi_2^n}{\partial x_2}(x_1^0, x_2^0) = 1 + (n+1)(x_2^0/\mathbf{R}_0)\frac{\partial \mathbf{R}}{\partial x_2}(x_1^0, x_2^0),$$

it follows that, for all $x_i \in [0, \bar{x}_i]$, there exists a number $N(x_i)$ and a neighborhood $V(x_i)$ such that $n \geqslant N(x_i)$ and $x_i' \in V(x_i)$ implies

$$\pi_i^{n+1}(x_i', x_j^0) \leqslant \pi_i^n(x_i', x_j^0) \leqslant \pi_i(x_i^0, x_j^0).$$

Since $\{V(x_i)\}$, $i = 1, 2$, is a cover of a compact interval, there exists a finite subcover and consequently a number N independent of $x_i \in [0, \bar{x}_i]$, $i = 1, 2$, such that for all $n \geqslant N$,

$$\pi_i^n(x_i, x_j^0) \leqslant \pi_i(x_i^0, x_j^0)$$

for all $x_i \in [0, \bar{x}_i]$. Hence (x_1^0, x_2^0) is a Nash equilibrium for the normalization f_0^N. Q.E.D.

To conclude this section, different normalization rules imply different objective demand functions against which firms maximize. Thus, in many cases, these can be chosen to generate any allocation as an equilibrium outcome. Conversely, nonexistence may be an outcome since different normalization rules may imply nonconvex-valued, discontinuous best responses, the consequences of which have been amply demonstrated in the literature. Finally, if the two goods are perfect substitutes, i.e., if $\mathbf{R}(x_1, x_2)$ is constant, then normalization does not matter.

3. Price Duopoly in Exchange Economies

Consider an exchange economy with three commodities indexed $h = 1, 2, 3$, two duopolists indexed $i = 1, 2$, and a "large" competitive sector represented by a differentiable excess demand function $z\colon \mathbb{R}_+^3 \to \mathbb{R}^3$ with the usual properties, i.e., zero homogeneity and Walras law. Each duopolist controls completely the market of one of the commodities in the sense that he owns all of the available initial endowment and that he is allowed to set the price in the market. Without loss of generality for the purpose here, assume that duopolist $i = 1, 2$ controls market $h = i$ and that i owns $e_i > 0$ units of commodity i and nothing of the other goods. Preferences of duopolist $i = 1, 2$ are such that he desires commodity 3 and his own but none of the commodity of the other duopolist. Then, his utility function can be written as $u^i\colon \mathbb{R}_+^2 \to \mathbb{R}$. In this situation each duopolist $i = 1, 2$ faces a demand function $z_i(p_1, p_2, p_3)$ for his own commodity. For whatever amount $0 \leqslant s_i \leqslant e_i$ he supplies to the market at price p_i, he is able to buy the amount $p_i s_i/p_3$ of commodity 3 on the competitive market. A model with a similar structure is used by Codognato and Gabszewicz [2, 3], but with an attempt to model the strategic setting as a Cournot–Walras quantity oligopoly.

The formal description of the price duopoly is now straightforward. For $i = 1, 2$, define the payoff functions

$$V^i(p_1, p_2, p_3) = u^i(e_i - s_i, p_i s_i/p_3)$$

with $s_i = z_i(p_1, p_2, p_3)$, where duopolist i controls price p_i, $i = 1, 2$. Since V^i is homogeneous of degree zero, some normalization for the set of prices is required in order to close the model and to define a best response by the duopolist. For the purposes here a normalization rule is best described by a monotonically increasing linear homogeneous function $F: \mathbb{R}^3_+ \to \mathbb{R}_+$ and a scaling factor $c > 0$ (see Böhm [1]). This implies as the set of feasible nominal prices

$$P = \{(p_1, p_2, p_3) \in \mathbb{R}^3_+ \mid c = F(p_1, p_2, p_3)\}.$$

A best price response of consumer i given a choice p_j of the other consumer is defined by

$$\mu^i(p_j) = \arg\max_{p_i} \{V^i(p_i, p_j, p_3) \mid \exists p_3 : c = F(p_1, p_2, p_3)\},$$

which implies the first-order conditions

$$\frac{\partial V^i}{\partial p_i} = \lambda \frac{\partial F}{\partial p_i}$$
$$\frac{\partial V^i}{\partial p_3} = \lambda \frac{\partial F}{\partial p_3}.$$

It is therefore obvious that, in general, different normalization rules imply different best price responses and therefore different Nash equilibria. Applying these arguments to two specific normalization rules, suppose first that the price of the competitive commodity is fixed at $\bar{p}_3 > 0$; i.e., $F(p_1, p_2, p_3) = \bar{p}_3$. Let $(\bar{p}_1, \bar{p}_2) > 0$ denote a Nash equilibrium. Then, one must have

$$\frac{\partial V^i}{\partial p_i}(\bar{p}_1, \bar{p}_2, \bar{p}_3) = 0 \qquad i = 1,2.$$

Now assume that prices are normalized on the simplex

$$\Delta = \{(p_1, p_2, p_3) \in \mathbb{R}^3_+ \mid p_1 + p_2 + p_3 = \bar{p}_1 + \bar{p}_2 + \bar{p}_3 = K\}.$$

In general, then, $(\bar{p}_1, \bar{p}_2)$ can no longer be a Nash equilibrium since duopolist i now maximizes $V^i(p_i, \bar{p}_j, K - p_i - \bar{p}_j)$ which implies the first-order condition

$$\frac{\partial V^i}{\partial p_i}(p_1, p_2, p_3) - \frac{\partial V^i}{\partial p_3}(p_1, p_2, p_3) = 0.$$

It is clear that both sets of conditions hold at $(\bar{p}_1, \bar{p}_2, \bar{p}_3)$ in special cases only. Hence different normalizations yield also different Nash equilibria in exchange economies in general.

The reduced form of the duopoly model with the payoff functions V^i as defined above provides the ultimate explanation why normalization matters in oligopoly models. If an oligopolistic economy contains a competitive sector represented by some excess demand function or by an inverse demand function *and* if not all prices are controlled by oligopolists, then some normalization is required to make the model, i.e. the Nash equilibria, determinate. But different normalizations imply different directional changes of the market clearing prices in the competitive sector. Therefore best responses by oligopolists are evaluated along different directions. Therefore, Nash equilibria for some normalization can be made to disappear by an appropriate (local) change of the normalization, unless

$$\partial V^i / \partial p_k(\bar{p}_1, \bar{p}_2, \bar{p}_3) = 0 \qquad \text{for all} \quad k = 1, 2, 3.$$

Obviously, this is a very special case which follows neither by construction nor from the zero homogeneity of the payoff functions V^i.

Since the arguments using the first-order conditions may look a little too facile, the following two examples should be more convincing.

EXAMPLE 1. Assume as aggregate demand functions of the competitive sector for the two monopolized commodities

$$z_i(p_i, p_j, p_3) = \frac{\delta_i p_3^2 e_3}{p_i(p_i + p_j + p_3)}, \qquad i, j \in \{1, 2\}, \quad i \neq j,$$

where $0 < \delta_i < 1$ and $e_3 > 0$. Let $\omega_1 = (e_1, 0, 0)$ and $\omega_2 = (0, e_2, 0)$ denote the endowments of the two duopolists with $e_i > 0$, and assume as utility functions

$$u^i(y_i, y_3) = y_i y_3 \qquad i = 1, 2.$$

Consider first the case when prices are normalized on the simplex

$$\Delta = \{(p_1, p_2, p_3) \in \mathbb{R}^3_+ \mid p_1 + p_2 + p_3 = 3\}.$$

It is a straightforward exercise to show that $p=(1,1,1)$ is a Nash equilibrium for $e_i=4$ and $\delta_i=1/3$, $i=1,2$, $e_3=9$. One finds that $V^i(.,p_j,.)$ is strictly quasi-concave for $p_j=1$ and, as required,

$$\frac{\partial V^i}{\partial p_i}(1,1,1)=\frac{\partial V^i}{\partial p_3}(1,1,1)>0.$$

Suppose alternatively that commodity 3 is taken as a numeraire and $\bar{p}_3=1$. Then, $p=(1,1,1)$ cannot be a Nash equilibrium since this requires $\partial V^i/\partial p_i=0$. The best response of each duopolist at $(1,1)$ is unique and strictly larger than one. However, some further calculations show that $p_i=p_j=1$ is a Nash equilibrium for some level p_3 between 0.89 and 0.91. The crucial feature generating the dependence stems from the fact that for different normalization rules an agents price choice influences relative prices in different ways. If, in addition, the payoff function depends on all relative prices, as in the case here, there is room for benificial manipulation and an agent will exploit it in his best response.

EXAMPLE 2. Assume that a single competitor has the utility function

$$u^3(x_1,x_2,x_3)=\frac{1}{\rho}(x_1^\rho+x_2^\rho)+x_3$$

with $\rho<1$. Endowments are $\omega_3=(0,0,e_3)$ with $e_3>0$. This yields demand functions

$$x_i=\left(\frac{p_i}{p_3}\right)^\sigma,\qquad i=1,2,$$

where $\sigma=1/(\rho-1)<0$ is the elasticity of substitution. Note that the two demand functions have no cross price effects and that they are independent of the endowment e_3. If the two duopolists have the same characteristics as in the previous example, one obtains the payoff functions

$$V^i(p_1,p_2,p_3)=e_i\left(\frac{p_i}{p_3}\right)^{\sigma+1}-\left(\frac{p_i}{p_3}\right)^{2\sigma+1}.$$

It is straightforward to verify that $\bar{p}=(1,1,1)$ is the unique Nash equilibrium for the normalization rule where $p_3=1$ if $e_i=3$, $i=1,2$, and $\sigma=-2$. Since V^i is independent of p_j, it follows that in this case for all $\lambda>0$

$$\frac{\partial V^i}{\partial p_k}(\lambda,\lambda,\lambda)=0,\qquad i=1,2;\quad k=1,2,3,$$

holds. As a consequence $p = (c, c, c)$ is the Nash equilibrium for all normalization rules defined by monotonically increasing linear-homogeneous functions $F: \mathbb{R}^3_+ \to \mathbb{R}$ and a scaling factor $c > 0$ with $c = F(p_1, p_2, p_3)$. Note that this result is due primarly to the lack of a cross price effect in demand which implies a cross price effect of zero for the payoff functions as well.

4. Conclusions

It has been shown why any oligopoly model of general equilibrium which contains some price taking agents and some competitive markets suffers from an indeterminacy implied by a necessary but arbitrary choice of a numeraire or more generally of a normalization rule. Existence of equilibria as well as their qualitative properties depend on a technical assumption which has no economic foundation within static Arrow–Debreu models. In general this dependence cannot be avoided, which raises the more fundamental question whether the interaction of oligopolistic and perfectly competitive agents through markets can be discussed in a meaningful way in models where the notion of price and value is not uniquely defined. For many oligopoly models of game theory and of industrial organizations there is a natural numeraire to be chosen which causes the issue of this note to disappear. In many general equilibrium models the same is true, in particular when there is money in the economy. However, whenever this is not the case, any model, whether partial or general equilibrium, should be scrutinized with respect to the normalization issue. It seems that the models most likely to suffer from dependence on normalization in the sense of this paper are to be found in public economics, international trade, and in the theory of the second best.

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