AN INTERREGIONAL POPULATION-EMPLOYMENT MODEL FOR THE FEDERAL REPUBLIC OF GERMANY: METHODOLOGY AND FORECASTING RESULTS FOR THE YEAR 2000

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1. INTRODUCTION

This paper describes a model which has the following characteristics:

a) it is based on a partition of the Federal Republic of Germany (FRG) into 79 Planning Regions, but it also contains two other regional levels: the level of the eleven States and the level of a partition of the FRG into two groupings of these regions, a northern group and a southern group of regions;

b) for each region a set of variables is defined which refers to the components of change in population and employment: these variables describe the supply and demand for labour in the regions;

c) the aim of the model is explanation as well as prediction. For prediction, a special method, based on linear programming, is used;

d) the period of analysis is the decade 1961-70. The forecasting period ranges from 1970 to 1990 and as a consequence the forecasting results are obviously not very up to date. For this reason an additional model has been developed which is based on data for the period 1961 to 1978. This second model starts from a partition of the FRG into 343 regional units, the so called 'Kreise' (see Figure 1), but containing only population-oriented variables with no employment variables. In order to combine the advantages of Model I with those of Model II, the basic results of Model I, the forecasted migration flows for the 79 regions, are used as a form of exogenous information by forecasting net migration for the 343 units. As Model I combines variables of the demand, as well as the supply side of the regional labour markets, it is called a *simultaneous model,* whereas Model II may be called a *partial model* because it contains only variables which describe the supply of labour. The basic differences between the models are:

	Model I (simultaneous)	*Model II (partial)*
Period of Analysis	1961-70	1961-78
Forecasting Period	1970-90	1978-2000
Regional Partition	79 Regions	343 Regions
Variables	supply and demand for labour	only population-oriented variables

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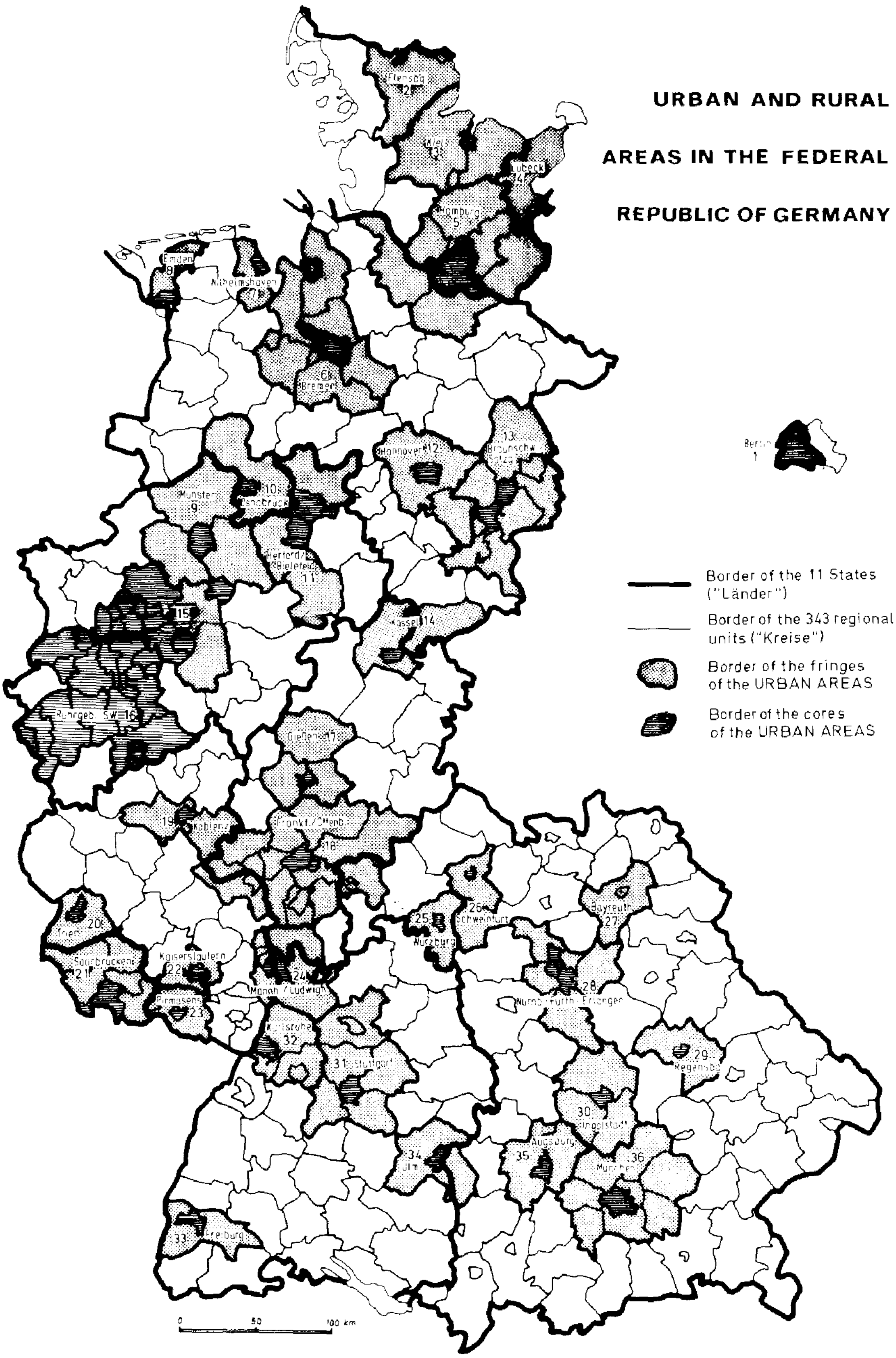

FIGURE 1. Urban and rural areas in the Federal Republic of Germany

This article concentrates on a description of the methodological characteristics of Model I. For Model II, which consists of various sub-models (births, deaths, migration), only the main forecasting results are presented. For a detailed description of Model II see Birg [4].

2. THE MAIN COMPONENTS OF POPULATION AND EMPLOYMENT CHANGE IN THE REGIONS

The components of population change between the two censuses in 1961 and 1970 are given in Table 1. The figures show that the growth component 'in-migration to region r' ($IND^r + INA^r$ in Table 1) is on average three times the growth component 'births.' If we set migration in relation to the growth of the regional labour force, rather than in relation to population growth, we have to set the migration in the period 1961-70 in relation to the number of births between 1945 and 1954 because the number of people which enter the labour market between 1961 and 1970 were born between 1945 and 1954. In this case the relation between births in region r and in-migration is 1 : 4.4. This relation is highly dependent on the degree of regional partitioning (see [3; p. 91]):

Regional Partition	*Births/In-Migration-Ratio*
National Level	1 : 1.2
Planning Regions	1 : 4.4
'Kreise'	1 : 5.5
Communities	1 : 10 (rough estimation)

We can conclude from these findings that a population-employment model should be designed according to the following principles:

a) in the specification of model equations, account should be taken to the fact that, at the regional level, in-migration is of much greater importance for

TABLE 1. Components of Population and Employment Change 1961-70

Components of Population Change (millions of people)			
P^r	number of inhabitants in 1961	$\sum_r P^r(61)$	$= 56.2$
B^r	number of births 1961-70	$\sum_t \sum_r B^r(t)$	$= 9.0$
D^r	number of deaths 1961-70	$\sum_t \sum_r D^r(t)$	$= 6.2$
IND^r	number of in-migrations to region r from other regions	$\sum_t \sum_r IND^r(t)$	$= 20.2$
$OUTD^r$	number of out-migrations from region r to other regions	$\sum_t \sum_r OUTD^r(t)$	$= 20.2$
INA^r	number of in-migrations from other countries	$\sum_t \sum_r INA^r(t)$	$= 7.6$
$OUTA^r$	number of out-migrations to other countries	$\sum_r \sum_r OUTA^r(t)$	$= 6.0$
P^r	number of inhabitants in 1970	$\sum_r P^r(70)$	$= 60.7$
Components of Employment Change (millions of employees)			
A^r	number of employees (= supply of jobs) in 1961	$\sum_r A^r(61)$	$= 26.5$
ΔAI^r	number of jobs created by investment in firms which existed already in 1961	$\sum_t \sum_r \Delta A^r(t)$	≈ 11.6
ΔAD^r	number of jobs lost by depreciation of capital in firms which existed already in 1961	$\sum_t \sum_r \Delta AD^r(t)$	≈ 12.0
ΔAN^r	number of jobs created by the foundation of new firms	$\sum_t \sum_r \Delta AN^r(t)$	≈ 0.5
A^r	number of employees (= supply of jobs) in 1970	$\sum_r A^r(70)$	$= 26.3$

$r \in$ {79 Regions}, $t \in \{1961, \ldots, 1970\}$
Source: Birg [3, p. 95 and p. 114 f.]

population and employment change than natural increase by births. The same relation can be observed with regard to the number of out-migrations and deaths;

b) in- and out-migration should be treated as separate variables. As the age and sex structure of the migration flows are, as a rule, very different, their effects on the number of births and deaths are important even in the case of zero net migration.

As in the case of population change, employment change can be separated into two components:

a) 'natural increases' of employment in firms which existed already at the beginning of the period; and

b) net increases resulting from the foundation and closure of firms and from the migration of firms between regions.

The number of jobs increases because of investment and declines as a result of the depreciation of capital. Table 1 shows that the number of jobs created by investment and the number of jobs lost by depreciation are almost equal in size: for the measurement of investment and depreciation figures at the national level, see [1] and [7]. Compared to the absolute magnitude of these components, the number of jobs created by the foundation of new firms and by the migration of firms between the regions is rather small (12.0 million versus 0.5 million).

On the supply side of the labour markets, the role of 'natural increase' and migration is reversed: whereas migration of people is the major component for regional population change, the interregional migration of capital is, in most regions, an almost negligible component for the change in the supply of jobs, measured in terms of the change in employment.

Once again these empirical facts are important for specifying the model adequately: the model equations should, in the first instance, describe and explain the changes in firms which already existed at the beginning of the period of analysis.

3. THE INTERDEPENDENT DEVELOPMENT OF POPULATION AND EMPLOYMENT CHANGE IN THE PERIOD 1961-70

Employment and population change are interdependent for two reasons: most people who want to move from one region to another need a new job in the region they want to move to (except in the case of commuters or retiral); and the creation of jobs by investment is based on the expectation that the supply of labour, determined largely by migration, will meet the demand for labour caused by the investment decisions.

The variables of the demand and supply sides of the regional labour markets can be linked by the following *balance equations* which hold for every region:

Balance equation for population change between 1961 and 1970:

$$P^r(70) = \gamma_N^r P^r(61) + \gamma_{IND}^r \sum_{t=61}^{70} IND^r(t) + \gamma_{INA}^r \sum_{t=61}^{70} INA^r(t)$$
$$- \gamma_{OUTD}^r \sum_{t=61}^{70} OUTD^r(t) - \gamma_{OUTA}^r \sum_{t=61}^{70} OUTA^r(t) \qquad (1)$$

Balance equation for employment change between 1961 and 1970:

$$\xi_N^r \gamma_N^r P^r(61) + \xi_{IND}^r \gamma_{IND}^r \sum_{t=61}^{70} IND^r(t) + \xi_{INA}^r \gamma_{INA}^r \sum_{t=61}^{70} INA^r(t)$$
$$- \xi_{OUTD}^r \gamma_{OUTD}^r \sum_{t=61}^{70} OUTD^r(t) - \xi_{OUTA}^r \gamma_{OUTA}^r \sum_{t=61}^{70} OUTA^r(t) + C^r(70)$$
$$= A^r(70) + NE^r(70) \qquad (2)$$

The variables and parameters are defined as follows:

P^r = number of inhabitants in region r

IND^r, INA^r = in-migrations from other regions and from other countries respectively

$OUTD^r$, $OUTA^r$ = out-migrations to other regions and to other countries respectively

C^r = commuter balance

A^r = number of jobs (= employees)

NE^r = number of unemployed

γ_N^r = growth factor which describes population growth by natural increase

γ_{IND}^r, γ_{INA}^r = growth factors which describe population growth by natural increase for the two groups of people who migrated into the region

γ_{OUTD}^r, γ_{OUTA}^r = growth factors for the groups of people who migrated out of the region

ξ_N^r = activity rate for the *stock* of population at the end of the decade 1961-70 due to *natural increase*

$\left.\begin{array}{l}\xi_{IND}^r, \xi_{INA}^r \\ \xi_{OUTD}^r, \xi_{OUTA}^r\end{array}\right\}$ = activity rates for the corresponding population *flows* in the decade 1961-70

All γ-parameters in equations (1) and (2) refer to the period between the two censuses in 1961 and 1970. All activity rates ξ refer to the *end* of the period of analysis (May 27, 1970, the day of the census).

The first term in equation (1) gives the population at the end of the decade due to the natural increase of those inhabitants who already lived in the region at the beginning of the decade. The growth factor γ_N^r is a function of the age, sex and region specific mortality and fertility rates of these inhabitants. The parameters γ_{IND}^r, γ_{INA}^r, γ_{OUTD}^r and γ_{OUTA}^r are growth factors for the corresponding migration flows.

The left-hand side of the balance of employment change describes the supply of labour at the end of the decade which equals the demand for labour plus the number of unemployed (right-hand side). The different parameters ξ^r give the activity rates for the various components of population change.

The most important interdependencies between the components of population and employment change are described by the following two migration functions, which have been tested by multiple linear regression on the basis of cross-section analysis for the decade 1961-70 using the 79 regions as the sample:

The *in-migration function* (standard deviation in brackets, u = disturbance term, ρ^2 = coefficient of determination) is as follows:

$$\sum_{t=61}^{70} IND^r(t) = \underset{(5{,}444)}{17{,}603} + \underset{(0.097)}{1.593} \cdot \Delta A^r + \underset{(0.018)}{0.948} \sum_{t=61}^{70} OUTD^r(t) + u^r_{IND} \qquad (3)$$
$$\rho^2 = 0.98$$

The variable ΔA^r serves to measure the net increase of jobs between 1961 and 1970.

The *out-migration function* is:

$$\sum_{t=61}^{70} OUTD^r(t) = \underset{(5{,}663)}{18{,}601} + \underset{(0.013)}{0.189} \cdot p^r(61) + \underset{(0.040)}{0.403} \sum_{t=61}^{70} IND^r(t) + u^r_{OUTD} \qquad (4)$$
$$\rho^2 = 0.97$$

The in-migration function (3) can be interpreted as follows: given the number of out-migrations from a region, in-migration is higher the greater the net increase of jobs, ΔA^r. The regression coefficient of the variable $OUTD^r$ in equation (3) is very close to 1. This may be interpreted as follows: jobs and dwellings which have become vacant as a result of out-migration are occupied directly or indirectly almost completely by inmigrants in the same period. This means that there is an important *rotation of jobs and inhabitants.*

The out-migration function (4) shows that the greater the number of inhabitants, the greater the amount of out-migration, for a given level of in-migration.

As the variables IND^r and $OUTD^r$ occur in both functions, the two migration functions should be interpreted simultaneously. This can be done by showing the dependency of net migration, $IND^r - OUTD^r$ on the variables ΔA^r and P^r. If we put $OUTD^r$ on the left-hand side of equation (3) we get the inverse of the in-migration function:

$$\sum_t OUTD^r(t) = -18{,}569 - 1.680\Delta A^r + 1.055 \sum_t IND^r(t) \pm (1/0.948)u^r_{IND} \qquad (3.1)$$

The graphs of the inversed in-migration function (3.1) and the out-migration function (4) are shown in Figure 2. The figure can be interpreted in the following way: for a given number of inhabitants, (positive) net migration becomes greater the bigger the increase in jobs ΔA^r (shift of function (3.1) to the right); for a given increase in jobs ΔA^r, (positive) net migration is found to be higher the lower the number of inhabitants (downward shift of function (4)). This means

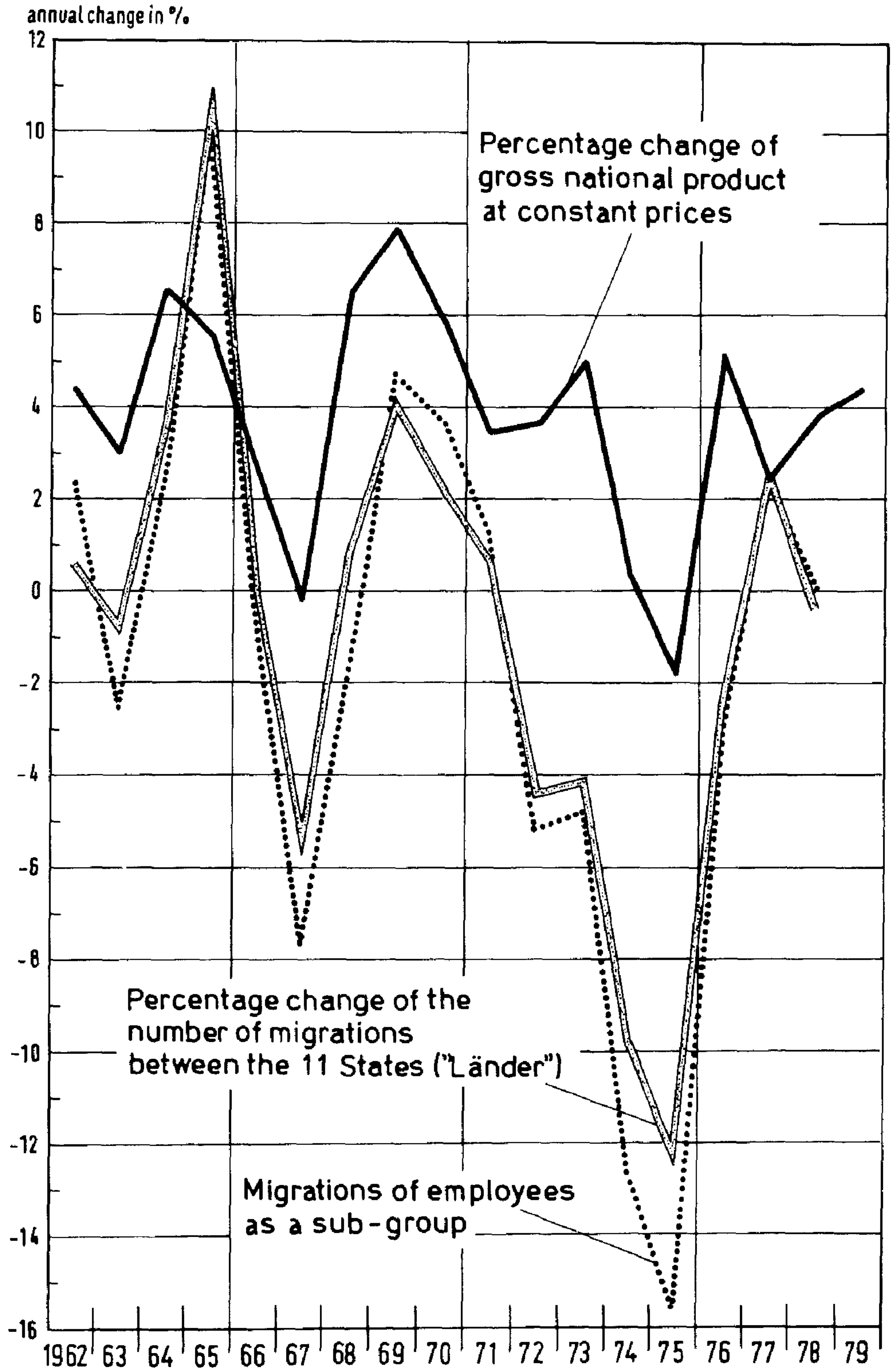

FIGURE 2. Functions for in- and out-migration

that a *deconcentration process* is caused by the interregional distribution of population: ceteris paribus, (positive) net migration is lower the greater the number of inhabitants in the region.

Alternative migration functions can be specified by using the *flows* M^{rs} of the migration matrix rather than the sums of the rows ($OUTD^r$) and columns (IND^r) of the matrix as dependent variables. However, tests of these functions yielded significant but, for prediction purposes, too low coefficients of determination ($\rho^2 = 0.43$; see [3; p. 99 f.]).

For in-migration from other countries and for out-migration to other countries, similar functions as in (3) and (4) have been tested. They show that migration of foreigners causes — contrary to domestic migration — a *concentration process* (see equation (10) and [3, p. 110 and p. 175]).

To describe the *supply of jobs*, various hypotheses have been tested by cross-section analysis using the 79 regions as the sample, as in (3) and (4). For this purpose A^r has been split into 6 manufacturing sectors A_i^r, $i = 1, \ldots, 6$, and in various tertiary sectors. Among the 6 functions for the manufacturing sectors, only the function for the sector wood, paper, leather and textiles (sector 5) met the statistical test criteria (for test results for the other sectors see [3, p. 126]):

$$\begin{aligned}\Delta A_5^r = {} & \underset{(641)}{1{,}786} - \underset{(0.004)}{0.028}\, A_5^r + \underset{(1.996)}{6.226}\, I_5^r - \underset{(0.080)}{0.209}\, W_5^r + u_5^r \\ \rho^2 = {} & 0.68\end{aligned} \tag{5}$$

In this equation, I_5^r is gross investment and W_5^r the wage rate in sector 5 in region r. The variable A_5^r, the absolute number of employees (= number of occupied jobs), has a negative sign, which means that net increase in jobs, ΔA_5^r, is smaller the higher the number of jobs in this sector, for a given wage rate.

To explain the number of employees (= occupied jobs) in the *tertiary sector*, A_T^r, the following function was specified on the basis of a Christaller-type model:[1]

$$\begin{aligned}A_T^r = {} & \underset{(3{,}910)}{187} + \underset{(0.021)}{0.116}\, P_1^r + \underset{(0.058)}{0.103}\, P_2^r + \underset{(0.036)}{0.179}\, P_3^r + \underset{(0.031)}{0.203}\, P_4^r + u_T^r \\ \rho^2 = {} & 0.90\end{aligned} \tag{6}$$

In this equation, P_i^r is the number of inhabitants living in communities of size class i ($i = 1, \ldots, 4$):

size class	*number of inhabitants*	*variable*
1	less than 2,000	P_1^r
2	2,000 — 5,000	P_2^r
3	5,000 — 20,000	P_3^r
4	20,000 — 100,000	P_4^r

[1] For a detailed description of the model and for alternative estimation functions, see [5].

The estimation function (6) meets the test requirements quite well. Furthermore, the parameters show, in most cases, the ascending order suggested by the theoretical arguments of the model.

With respect to prediction, the functions (5) and (6) cause the same difficulty as the migration functions on the basis of the flows, M^{rs}: the exogenous variables cannot be forecasted without specifying adequate additional estimation functions for these variables.

To avoid these difficulties, the following functions have been used to determine the supply of jobs:

$$A_i^r(70) = \frac{\sum_r A_i^r(70)}{\sum_r A_i^r(61)} A_i^r(61) + VA_i^r(61/70); \qquad \begin{array}{l} i = 1, \ldots, 44 \\ \text{production and} \\ \text{service sectors} \end{array} \tag{7}$$

In this equation, the first term may be interpreted, in terms of the shift-and-share analysis, as the national share in the shift of regional employment change and the residual, VA_i^r, as the regional share. Attempts to specify an empirical estimation function for the variable VA_i^r have so far failed for the FRG [2, p. 48] as well as for the U.S. [6], but χ^2-tests show that the sign of the residual is stable over time in many regions [2, p. 33 f.]. This stability assumption has been used as part of the information within the prediction framework. If (7) is summed over all 44 sectors we obtain:

$$A^r(70) = \psi^r(61/70)\, A^r(61) + VA^r(61/70) \tag{7.1}$$

$$\psi^r(61/70) = \sum_{i=1}^{44} \psi_i(61/70)\, A_i^r(61) \,/\, \sum_{i=1}^{44} A_i^r(61)$$

where ψ_i are the sectoral growth factors. The regional growth factor, ψ^r, is the weighted sum of the sectoral growth factors, ψ_i, at the national level.

4. METHODOLOGICAL PRINCIPLES OF PREDICTION

Dynamic, Comparative-Static and Comparative-Dynamic Forecasting Functions

From a theoretical point of view, the prediction period 1970-1990 should be treated as a series of time intervals, e.g., years, in order to describe the values of the model variables by recursive difference-equations (Rogers, 1968 and 1979; Willekens, 1979). However, this would require the specification of the migration functions (3) and (4) and of all other estimation functions on the basis of the ten years in the decade 1961-70 rather than on the basis of the decade as one single period. In this case the parameters of the in-migration functions (3) and the parameters of the other functions must be estimated for every year separately:

$$IND^r(t) = a_0(t) + a_1(t)\,\Delta A^r(t) + a_2(t)\,OUTD^r(t) + u_{IND}^r(t) \qquad t = 1961, \ldots, 1970 \tag{3.2}$$

For the following reasons this approach seems not to be appropriate. Firstly, there is a strong correlation between the migration variables and the business cycle (see Figure 3). From this correlation we must expect instability in the parameters a_i, so that $a_i(t) \neq a_i(t-1)$. Secondly, using time-dependent parameters, $a_i(t)$, would require the forecasting of these parameters. This means forecasting the business cycle for a period of twenty years — a hopeless proposition.

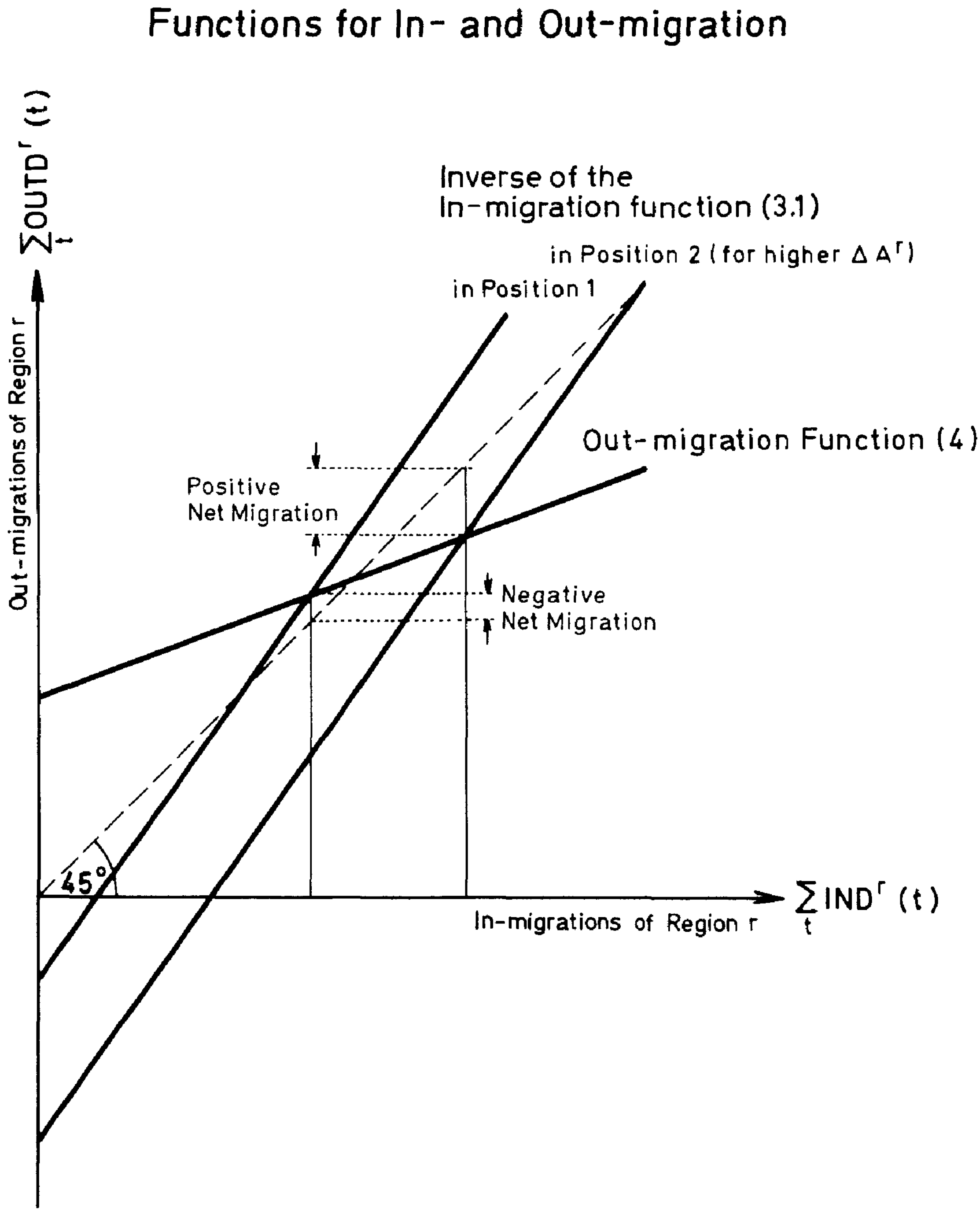

FIGURE 3. The influence of the business cycle on migration in the Federal Republic of Germany

Using twenty prediction periods rather than one single large period requires us to multiply the number of prediction variables by twenty. This would exceed the capacity of the numerical calculations if the prediction problem is to be solved by linear programming, a solution method which has many advantages.[2]

For this reason, the period 1970-90 has been treated as one single forecasting period. As the prediction period is twice the length of the test period the parameters of the estimation functions had to be adjusted to the length of the prediction period. This can be done by dividing the estimation function by the number of years between 1961 and 1970 and by multiplying it by the number of years in the prediction period. In the case of the in-migration function we obtain, for example:

$$\sum_{t=70}^{90} IND^r(t) = 35{,}206 + 1.593\Delta A^r(70/90) + 0.948 \sum_{t=70}^{90} OUTD^r(t) + 2u_{IND}^r(t) \quad (3.3)$$

Equation (3.3) may be called a *comparative-static function* because it connects variables of the same twenty year period. However, the model also contains functions which combine population variables for the beginning and the end of the prediction period [3, p. 169 f.]. These functions may be called dynamic functions or *comparative-dynamic* functions as they combine variables of different periods. A recursive *dynamic function* is used to estimate the number of births and deaths resulting from the number of inhabitants living in the regions at the beginning of the prediction period. The mixture of these functions suggests classifying the model as a *semi-dynamic* or quasi-dynamic model.

The Relevance of the Disturbance Terms in the Forecasting Functions

In time series analysis it is normally assumed that the expectation of the disturbance term is zero:

$$E(u(t)) = 0, \qquad t = 1, \ldots, T$$

In interregional cross-section regression, the same assumption is made for a given period t:

$$E(u^r(t)) = 0, \qquad r = 1, \ldots, R$$

If interregional cross-section regression is repeated for several periods we observe, at least in the case of the migration functions (3) and (4), that the values of $u^r(t)$ for a given region r generally have the same sign in different periods, so that the expectation is not zero:

$$E(u^r(t)) \neq 0, \qquad \begin{aligned} &r = \text{given} \\ &t = 1, \ldots, T \end{aligned}$$

Furthermore, we observe that the absolute values of the disturbance terms are relatively constant over time. This means that, for example, in the region

[2] See the next section of this paper.

of Munich, the actual in-migration variable, $IND^r(t)$, is always above the value estimated by (3.2) and that in the region of Essen, this value is always below the estimated value. This regularity is due to the fact that not all variables which are relevant to explain in-migration occur in function (3). As these omitted variables are different from region to region it seems to be impossible to specify the function in such a way that it contains all factors relevant for all regions. Another reason against specifying these region-specific variables explicitly is the already relatively high coefficient of determination ($\rho^2 = 0.98$).

Despite the high coefficient of determination, the relative share of the disturbance term u^r/IND^r is, for several regions, very high (50 percent and more). To disregard the disturbance term by setting it as zero would mean neglecting information which is very relevant for forecasting. For this reason every estimation function used for prediction was split into two sub-functions. One function serves to compute the upper boundary of the forecasted variable by adding the empirical disturbance term in the period of analysis to the functional value; the other function serves to compute the lower boundary, setting the disturbance term zero.[3]

Solution Techniques

As all forecasting functions (equations and inequalities) are linear, they form a convex space whose dimensions equal the number of forecasting variables used in the model. The task of forecasting is then to choose one point of the space (one vector) as the most probable one. If the interval for a variable, defined by splitting each forecasting function into two inequalities, is widened, the probability of the variable having a value inside the interval will increase and the probability of having a value outside the interval will decrease. The magnitudes of all intervals for all variables are defined in such a way that the probabilities of the variables lying within the intervals are all equal. In this case it is not possible to choose one point of the solution space as the most probable one, except when there is additional information that restricts the solution space.

In this model, an *objective function* is introduced in order to choose among the alternative points of the solution space. Thus, linear programming can be used as a solution technique. For alternative methods see [3, p. 62].

5. THE PREDICTION VARIABLES, THE OBJECTIVE FUNCTION AND THE CONSTRAINTS

The regional labour market balance for the end of the prediction period (middle of the year 1990) can be written as follows (using equations (2) and (7.1) and putting the unemployment variable to the left):

[3] For a detailed description of this approach see [3, p. 56 and 167 f.].

$$
\begin{aligned}
NE^r(90) = \; & \xi^r_N(90)\, P^r_N(90) \\
& + \eta^r_{IND}(90)\, IND^r(K) + \eta^r_{INA}(90)\, INA^r(K) \\
& - \eta^r_{OUTD}(90)\, OUTD^r(K) - \eta^r_{OUTA}(90)\, OUTA^r(K) \\
& + C^r(90) \\
& - [\psi^r(70/90)\, A^r(70) + VA^r(70/90)] \\
& r = 1, \ldots, 79
\end{aligned} \tag{8}
$$

In this equation the following definitions and abbreviations are used:

$P^r_N(90)$ = natural increase of the population in 1970

$\eta^r_{IND} = \xi^r_{IND}\gamma^r_{IND}$, $\quad \eta^r_{INA} = \xi^r_{INA}\gamma^r_{INA}$

$\eta^r_{OUTD} = \xi^r_{OUTD}\gamma^r_{OUTD}$, $\quad \eta^r_{OUTA} = \xi^r_{OUTA}\gamma^r_{OUTA}$

$IND^r(K) = \sum_{t=70}^{90} IND^r(t)$, $\quad INA^r(K) = \sum_{t=70}^{90} INA^r(t)$

$OUTD^r(K) = \sum_{t=70}^{90} OUTD^r(t)$, $\quad OUTA^r(K) = \sum_{t=70}^{90} OUTA^r(t)$

Summing in equation (8) gives the national labour market balance:

$$
\begin{aligned}
\sum_r NE^r(90) = \; & \sum_r \xi^r_N(90)\, P^r_N(90) \\
& + \sum_r \eta^r_{IND}(90)\, IND^r(K) + \sum_r \eta^r_{INA}(90)\, INA^r(K) \\
& - \sum_r \eta^r_{OUTD}(90)\, OUTD^r(K) + \sum_r \eta^r_{OUTA}(90)\, OUTA^r(K) \\
& + \sum_r C^r(90) \\
& - \sum_r [\psi^r(70/90)\, A^r(70) + VA^r(70/90)]
\end{aligned} \tag{9}
$$

Equation (9) has been chosen as the *objective function* of the model: the prediction problem is that of determining the variables of this function in such a way that the sum of the numbers of unemployed in the 79 regions is minimized. By defining a constraint which sets the condition that in each region the demand for labour (= value of the last row in (8)) must not exceed the supply of labour (= value of the sum of the first 4 rows in (8)) it is guaranteed that the regional imbalances in (9) do not cancel out.

In (8) and (9), the expression, $P^r_N(90)$, gives the number of inhabitants in the regions at the end of the period due to births and deaths from the *initial* population stock in 1970 (no migration). P^r_N was computed outside the model using age and sex specific mortality and age and region specific fertility rates. The term

$$\psi^r(70/90)\, A^r(70), \qquad r = 1, \ldots, 79$$

was also computed outside the model, using the sectoral growth factors ψ_i at the national level which have been estimated in a special national model [7]. The η-parameters in (9) may be designated as *net activity rates*, which have also been estimated outside the model (see [3, p. 180 f.]).

Thus we get the following list of coefficients in the objective function, together with the endogenous variables to be determined in the model:

Endogenous Variables:	*Coefficients:*	
$\xi_N^r(90)$	$P_N^r(90)$	
$IND^r(K)$	$\eta_{IND}^r(90)$	
$INA^r(K)$	$\eta_{INA}^r(90)$	
$OUTD^r(K)$	$\eta_{OUTD}^r(90)$	$r = 1, \ldots, 79$
$OUTA^r(K)$	$\eta_{OUTA}^r(90)$	
$C^r(90)$	1	
$VA^r(70/90)$	-1	

The coefficients of $C^r(90)$ equal 1, those of $VA^r(70/90)$ equal -1. As the term $\psi^r(70/90)A^r(70)$ is computed outside the model, it can be excluded from the objective function. But, for technical reasons, $\psi^r(70/90)$ is treated as an endogenous variable with fixed values and coefficients $-A^r(70)$.

The most important group of constraints is based on the *econometric relationships* between the endogenous variables, e.g., the

— functions (3) for in-migration from other regions, IND^r

— functions (4) for out-migration to other regions, $OUTD^r$

— functions for in-migration from other countries, INA^r

— functions for out-migration to other countries, $OUTA^r$.

There are various other types of constraints which can be grouped as follows:[4]

— Constraints by means of direct empirical boundaries for special variables

— Normative constraints (regional unemployment rate $\leq x$ %)

— Constraints on the basis of an econometric model at the national level

— Constraints on the basis of definitions

6. METHODOLOGICAL CHARACTERISTICS OF THE SOLUTION

The model contains 1,519 variables and 2,940 constraints. The variables can be grouped into 753 endogenous variables and 766 exogenous variables and service variables. Variables which are computed on the basis of definitions, as for example the population stock $P^r(90)$ in the balance equation (1) for 1990, are designated as 'service-variables.'

The forecasting task, minimizing equation (9) subject to 2,940 constraints, showed that there was no solution. This means that the information incorporated in the constraints was not consistent. This is a normal situation in linear programming for large systems. There are two ways to make the problem solvable: by changing parameters in the econometric relationships; or by widening the intervals of the constraints.

After numerous simulations it turned out that some of the parameters in the

[4] For a detailed description see [3, p. 162 f.].

migration functions had to be slightly altered. The necessary changes can be interpreted in a plausible way (see [3, p. 198-199]):

a) The parameters in the prediction function for migration to other countries had to be altered in the following way:

$$\begin{array}{l} OUTA^r(K) = 7{,}647 - \underset{\substack{\downarrow \\ 0.030}}{0.046}P^r(70) + \underset{\substack{\downarrow \\ 0.990}}{0.739}INA^r(K) + u^r_{OUTA} \\ \text{altered parameters:} \end{array} \tag{10}$$

In the period of analysis there was an extremely high net immigration from abroad. Contrary to this, for the period of prediction a very low net immigration from other countries was assumed. It seems to be plausible that the system had no solution for the initial parameters which reflect a migration behaviour that is not compatible with this assumption.

b) In the prediction function for in-migration from other regions, the rotation-parameter had to be altered:

$$\begin{array}{l} IND^r(K) = 35{,}206 + 1.593\Delta A^r + \underset{\substack{\downarrow \\ 0.910}}{0.948}OUTD^r(K) + u^r_{IND} \\ \text{altered parameter:} \end{array} \tag{3.4}$$

This means that the effect of free jobs and dwellings (set vacant by out-migration) on the in-migration variable in the prediction period is smaller than in the period of analysis. This outcome may be a consequence of the increased demand for jobs resulting from natural increase of population in the regions: In the prediction period 1970-90 we have a *declining population* but a *rising labour force*.

The solution has two important characteristics:

— *Maximizing* the objective function gives forecasting results which are similar to the results yielded by *minimization*. This means that the solution space is considerably restricted by the amount of information incorporated in the constraints: The smaller the solution space, the smaller the *predicted intervals* of the endogenous variables.

— The upper and lower boundaries for the directly restricted variables were, in most cases, not effective. This means that the *interdependencies* between the variables, described by the econometric functions, are *intensive*.

In the FRG linear programming has so far been used only in the framework of decision models [11]. But this technique seems to be very effective in extracting a consistent sub-set of all information which is potentially relevant for prediction.

7. SUMMARY OF THE EMPIRICAL RESULTS

A selection of the main empirical results of Model I is given in Table 2. As noted in the introduction, the forecasting results for Model I are not so up to date as the results of Model II. For this reason the empirical results of Model II are presented as well (Table 3). The outcome of Model II may be summarized in the following way: total population in the FRG will decline from 62.0 million in 1975 to 55.8 million in 2000. The relative decline in urban and rural areas is about the same, namely 10 percent. But within the urban areas there is a considerable decline of 4.2 million in the cores (17 percent) and an almost constant number of inhabitants in the fringes of the rural areas. 70 percent of the decline in the cores is caused by the birth deficit and 30 percent by net migration, especially to the fringes.

TABLE 2. Components of the Labour Market Balance in 1990 (Results of Model I)

No.	Forecasting Regions	Number of inhabitants 1990 in 1,000 (1)	Activity rate 1990 in % (2)	Commuter balance 1990 (3)	Demand for Jobs 1990 without commuters (4)	Demand for Jobs 1990 including commuters in 1,000 (5)	Supply of jobs 1990 (6)	Number of unemployed 1990 (7)
1	Flensburg	423	40	− 5	170	165	158	7
2	Itzehoe	224	40	− 3	89	86	79	7
3	Kiel	545	42	3	228	231	231	0
4	Lübeck	380	41	3	154	157	157	0
5	Bad Oldesloe	691	46	− 88	320	232	232	0
	Schleswig-Holstein	2,262	42	− 89	961	872	857	15
6	Hamburg	1,486	52	169	772	941	941	0
7	Bremen	653	51	75	330	405	405	0
8	Emden	416	38	− 12	157	145	141	4
9	Oldenburg	777	42	− 24	326	302	302	0
10	Bremervörde	486	44	− 57	214	157	157	0
11	Lingen	401	37	− 3	147	144	127	17
12	Werden	398	43	− 17	172	155	154	1
13	Uelzen	693	45	− 57	310	253	253	0
14	Osnabrück	522	42	− 6	217	211	211	0
15	Hannover	988	51	68	506	574	574	0
16	Hildesheim	683	46	− 28	314	286	286	0
17	Braunschweig	1,050	47	− 3	497	494	494	0
18	Göttingen	549	44	− 2	241	239	239	0
	Niedersachsen	6,963	45	−141	3,100	2,959	2,938	21
19	Münster	1,173	40	− 3	467	464	422	42
20	Bielefeld	1,328	43	6	565	571	571	0
21	Duisburg	1,005	37	− 15	377	362	329	33
22	Essen	2,663	43	− 41	1,135	1,094	1,095	0
23	Dortmund	1,085	40	− 3	434	431	431	0
24	Paderborn	364	37	− 10	134	124	112	12
25	Mönchengladbach	1,160	44	− 30	509	479	479	0
26	Düsseldorf	1,799	51	92	923	1,015	1,015	0
27	Hagen	901	44	− 6	394	388	388	0
28	Arnsberg	546	37	− 5	202	197	197	0
29	Aachen	901	40	− 20	361	341	341	0
30	Köln	2,463	46	48	1,131	1,179	1,179	0
31	Siegen	424	41	3	173	176	159	17
	Nordrhein-Westfalen	15,812	43	16	6,804	6,820	6,719	101
32	Kassel	782	42	5	332	337	337	0
33	Marburg	387	42	− 5	163	158	158	0
34	Fulda	586	42	− 44	246	202	202	0
35	Gießen	614	44	− 3	269	266	266	0
36	Frankfurt	2,483	54	116	1,352	1,468	1,467	1
37	Darmstadt	938	50	− 72	467	395	395	0
	Hessen	5,790	49	− 3	2,828	2,825	2,825	0
38	Montabaur	445	42	− 30	186	156	156	0
39	Koblenz	600	45	− 29	270	241	242	0
40	Bitburg	159	36	− 10	57	47	47	0

TABLE 2. (continued)

No.	Forecasting Regions	Number of inhabitants 1990	Activity rate 1990	Commuter balance 1990	Demand for Jobs 1990 without commuters	Demand for Jobs 1990 including commuters	Supply of jobs 1990	Number of unemployed 1990
		in 1,000	in %			in 1,000		
		(1)	(2)	(3)	(4)	(5)	(6)	(7)
41	Trier	317	40	− 5	126	121	122	0
42	Bad Kreuznach	322	41	− 9	132	123	123	0
43	Mainz	510	46	− 5	235	230	230	0
44	Kaiserslautern	495	45	− 23	223	200	200	0
45	Ludwigshafen	543	46	3	252	255	255	0
46	Landau	247	47	− 20	116	96	97	0
	Rheinland-Pfalz	3,638	44	−125	1,598	1,473	1,473	0
47	Saarland	1,078	38	18	414	432	433	0
48	Mannheim	764	45	62	343	405	405	0
49	Tauberbischofsheim	333	43	− 20	142	122	123	0
50	Heilbronn	558	49	5	275	280	280	0
51	Karlsruhe	796	48	21	384	405	405	0
52	Stuttgart	2,609	54	26	1,413	1,439	1,439	0
53	Heidenheim	413	48	3	197	200	182	18
54	Offenburg	430	45	− 8	195	187	188	0
55	Pforzheim	432	49	− 11	213	202	202	0
56	Tübingen	611	47	− 3	288	285	285	0
57	Ulm	449	50	7	223	230	230	0
58	Freiburg	418	45	6	189	195	195	0
59	Lörrach	369	47	− 21	173	152	152	0
60	Donaueschingen	460	49	3	227	230	230	0
61	Konstanz	622	48	2	299	301	301	0
	Baden-Württemberg	9,264	49	73	4,561	4,634	4,615	19
62	Aschaffenburg	306	44	− 12	135	123	123	0
63	Würzburg	428	41	− 5	174	169	169	0
64	Schweinfurt	432	43	− 3	185	182	176	6
65	Bayreuth	1,053	47	− 18	492	474	474	0
66	Ansbach	285	46	− 9	130	121	121	0
67	Nürnberg	1,046	48	53	507	560	559	1
68	Regensburg	645	43	− 11	275	264	264	0
69	Weiden	398	41	− 9	161	152	139	13
70	Ingolstadt	416	46	− 11	192	181	182	0
71	Landshut	404	44	− 9	176	167	167	0
72	Passau	501	42	− 3	210	207	188	19
73	Neu-Ulm	259	49	− 21	126	105	105	0
74	Augsburg	848	51	− 8	429	421	421	0
75	München	2,152	56	44	1,208	1,252	1,252	0
76	Kempten	523	48	− 3	252	249	249	0
77	Garmisch-Partenkirchen	482	45	− 15	218	203	203	0
78	Traunstein	421	46	− 6	193	187	187	0
	Bayern	10,599	48	− 45	5,064	5,019	4,980	39
79	Berlin (West)	1,757	56	3	990	993	993	0
	Federal Republic of Germany	59,303	46	− 50	27,422	27,372	27,178	194

TABLE 3. Population Forecasts for West German Urban Areas (Cores and Fringes) (Results of Model II)

	Number of Inhabitants 1975	Number of Inhabitants 2000	Total Change	Components of Population Change: Birth Deficit	Net Migration to: Total	Net Migration to: Other Regions	Net Migration to: Other Countries
				1,000 Persons			
1 *Berlin (West)*	2,024	1,502	− 522	−361	−162	−157	− 5
2 *Flensburg*							
Core	95	76	− 18	− 7	− 11	− 10	− 1
Fringe	176	169	− 8	− 16	9	9	0
Total	271	245	− 26	− 24	− 2	− 2	− 1

TABLE 3. (continued)

	Number of Inhabitants			Components of Population Change			
					Net Migration to		
	1975	2000	Total Change	Birth Deficit	Total	Other Regions	Other Countries
				1,000 Persons			
3 *Kiel*							
Core	264	221	-44	-27	-17	-15	-1
Fringe	435	406	-29	-54	25	25	0
Total	699	627	-72	-81	9	10	-1
4 *Lübeck*							
Core	235	186	-49	-31	-18	-17	-1
Fringe	186	181	-5	-27	22	22	-0
Total	421	367	-54	-58	4	5	-1
5 *Hamburg*							
Core	1,734	1,295	-438	-320	-118	-115	-3
Fringe	1,091	1,186	95	-88	183	187	-4
Total	2,825	2,482	-343	-408	65	72	-7
6 *Bremen*							
Core	579	465	-114	-85	-29	-24	-5
Fringe	934	939	5	-59	64	67	-3
Total	1,514	1,405	-109	-144	35	42	-8
7 *Wilhelmshaven*							
Core	104	86	-19	-13	-6	-6	1
Fringe	95	91	-4	-7	3	4	-0
Total	199	177	-23	-20	-3	-2	-1
8 *Emden*							
Core	54	49	-5	-2	-4	-4	-0
Fringe	85	82	-3	-7	4	4	0
Total	139	131	-9	-8	-0	-0	-0
9 *Münster*							
Core	263	264	1	-11	13	14	-2
Fringe	776	772	-4	-26	22	25	-3
Total	1,039	1,036	-3	-37	34	39	-5
10 *Osnabrück*							
Core	164	141	-23	-10	-13	-11	-1
Fringe	281	277	-4	-17	13	14	-1
Total	445	418	-26	-27	0	3	-2
11 *Herford/ Bielefeld*							
Core 1	320	278	-42	-40	-2	3	-5
Core 2	236	195	-41	-36	-5	-3	-2
Fringe	783	738	-45	-79	34	41	-7
Total	1,338	1,211	-128	-155	27	40	-13
12 *Hannover*							
Core	563	456	-107	-68	-39	-31	-8
Fringe	748	778	30	-25	55	61	-7
Total	1,311	1,235	-76	-93	16	30	-14
13 *Braunschweig/ Salzgitter*							
Core 1	271	233	-38	-30	-8	-6	-2
Core 2	120	93	-27	-11	-17	-15	-2
Fringe	795	678	-117	-87	-30	-26	-4
Total	1,186	1,004	-182	-127	-55	-47	-8
14 *Kassel*							
Core	210	145	-65	-36	-29	-27	-3
Fringe	465	491	26	-21	47	48	-1
Total	675	636	-39	-57	18	21	-3
15 *Ruhrgebiet — Nordost*	5,610	4,488	-1,123	-648	-475	-483	8
16 *Ruhrgebiet — Südwest*	6,254	5,837	-416	-460	44	126	-83
17 *Gießen*							
Core	76	68	-8	-2	-6	-5	-1
Fringe	523	482	-40	-48	7	11	-4
Total	599	550	-49	-50	1	6	-5

TABLE 3. (continued)

	Number of Inhabitants			Components of Population Change			
					Net Migration to		
	1975	2000	Total Change	Birth Deficit	Total	Other Regions	Other Countries
				1,000 Persons			
18 *Frankfurt/ Offenbach*							
Core 1	652	438	−214	−103	−111	−94	−17
Core 2	118	83	−35	−12	−23	−20	−3
Fringe	2,862	2,849	−13	−246	233	264	−31
Total	3,632	3,370	−262	−361	99	150	−51
19 *Koblenz*							
Core	119	103	−16	−8	−8	−8	−0
Fringe	311	278	−33	−31	−3	−2	−1
Total	431	381	−49	−39	−10	−9	−1
20 *Trier*							
Core	101	83	−18	−9	−10	−10	−0
Fringe	121	125	4	−0	5	5	−0
Total	222	208	−14	−9	−5	−5	−0
21 *Saarbrücken*							
Core	385	311	−73	−45	−29	−29	1
Fringe	719	629	−90	−58	−32	−31	−1
Total	1,103	940	−163	−103	−60	−60	−1
22 *Kaiserslautern*							
Core	102	89	−14	−13	−0	1	−1
Fringe	97	94	−4	−3	−0	−0	−0
Total	200	182	−17	−17	−1	0	−1
23 *Pirmasens*							
Core	55	40	−14	−7	−8	−7	−0
Fringe	138	115	−24	−12	−11	−11	−0
Total	193	155	−38	−19	−19	−18	−1
24 *Mannheim / Ludwigshafen*							
Core 1	174	135	−39	−15	−25	−21	−4
Core 2	321	249	−71	−34	−38	−32	−6
Fringe	1,105	1,092	−13	−62	50	64	−15
Total	1,599	1,476	−124	−111	−13	12	−25
25 *Würzburg*							
Core	113	100	−13	−6	−7	−7	−0
Fringe	145	153	8	−9	16	17	−1
Total	258	253	−6	−15	9	10	−1
26 *Schweinfurt*							
Core	57	43	−14	−6	−8	−8	−1
Fringe	181	170	−11	−9	−2	−2	−0
Total	238	213	−25	−15	−10	−9	−1
27 *Bayreuth*							
Core	67	65	−2	−6	4	4	−0
Fringe	101	88	−13	−12	−1	−1	−0
Total	168	152	−16	−18	3	3	−1
28 *Nürnberg / Fürth-Erlangen*							
Core 1	510	407	−103	−64	−38	−27	−11
Core 2	204	179	−24	−19	−5	−2	−3
Fringe	448	484	36	−15	50	50	1
Total	1,162	1,070	−92	−99	7	21	−14
29 *Regensburg*							
Core	133	118	−15	−12	−3	−3	−0
Fringe	134	146	11	−3	14	15	−1
Total	268	264	−4	−15	11	12	−1
30 *Ingolstadt*							
Core	90	82	−9	−2	−7	−5	−1
Fringe	244	237	−7	−8	2	3	−1
Total	334	319	−15	−10	−5	−3	−3

TABLE 3. (continued)

Population Forecasts for West German Urban Areas (Cores and Fringes)

	Number of Inhabitants			Components of Population Change			
						Net Migration to	
	1975	2000	Total Change	Birth Deficit	Total	Other Regions	Other Countries
				1,000 Persons			
31 *Stuttgart*							
Core	613	465	– 148	– 66	– 82	– 64	–18
Fringe	1,873	1,805	– 68	–112	44	93	–49
Total	2,486	2,270	– 216	–178	– 39	28	–67
32 *Karlsruhe*							
Core	266	226	– 40	– 28	– 12	– 8	– 4
Fringe	633	582	– 51	– 67	15	26	–10
Total	899	807	– 92	– 95	3	17	–14
33 *Freiburg*							
Core	179	173	– 6	3	– 9	– 10	0
Fringe	185	220	35	5	31	31	– 1
Total	364	393	29	8	21	22	– 0
34 *Ulm*							
Core	97	82	– 15	– 7	– 8	– 7	– 1
Fringe	296	288	– 8	– 18	10	13	– 3
Total	393	370	– 23	– 25	2	6	– 3
35 *Augsburg*							
Core	254	200	– 54	– 29	– 24	– 20	– 4
Fringe	268	270	3	– 23	26	29	– 3
Total	522	471	– 51	– 52	1	9	– 8
36 *München*							
Core	1,323	1,173	– 150	–111	– 40	– 19	–21
Fringe	844	1,062	217	– 24	241	251	–10
Total	2,168	2,235	67	–135	202	233	–31
Urban Areas	43,186	38,878	–4,308	–4,061	–247	122	–369
Rural Areas	18,806	16,936	–1,870	–1,634	–235	–123	–112
Federal Republic of Germany	61,991	55,814	–6,178	–5,696	–482	– 1	–481

Source: H. Birg, "Berechnungen zur langfristigen Bevölkerungsentwicklung in den 343 kreisfreien Städten und Landkreisen der Bundesrepublik Deutschland." In: No. 2 of the "Vierteljahrshefte zur Wirtschaftsforschung," Deutsches Insitute für Wirtschaftsforschung, Berlin (W), 1980.

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