

# DEMOGRAPHIC ASPECTS OF LABOUR MARKET EFFICIENCY

by

HERWIG BIRG  
University of Bielefeld

## 1. INTRODUCTION

Economic theories of the labour market emphasize the importance of the wage rate and of the price of capital as the basic links between the supply of and the demand for labour. The following analysis, however, concentrates on demographic factors. Demographic variables are often overlooked but they have an important explanatory power. In order to show this it has to be initially observed that the main function of the labour market is to attach individual workers to individual jobs. For the "attachement of workers to jobs" the term matching-process will be used.

The outcome of the matching-process on the supply side of the labour market is employment and unemployment and on the demand side occupied or vacant jobs. In order to explain these outcomes the matching-process itself has to be investigated. It is the matching-process which causes the various fluctuations in the status of a job (vacant, occupied) and in the status of those willing to work (employed, unemployed).

The variations in the status of employees are linked with the components of change of the labour stock due to the renewal process caused by retirement and natural increase. Similarly, the variations in the status of the jobs are linked with the components of change of the stock of jobs due to the renewal process caused by the depreciation of capital and by investment.

The aim of the analysis is to show that:

- (1) the changes in the status of (potential) employees and jobs are interdependent,
- (2) the intensity of the matching-process, measured by the number of changes in the status of jobs and (potential) employees, as well as the efficiency of the matching-process are important factors for the explanation of the imbalances in labour markets.
- (3) the intensity as well as the efficiency of the matching-process are influenced by important demographic factors.

## 2. DESCRIPTION OF THE OUTCOMES OF THE MATCHING-PROCESS

Considering a time period  $[0, t]$ , the magnitude of the labour force on a certain regional labour market at the beginning and at the

end of the period is linked by the following equation

$$L(t) = L(0) - L_D(t) + L_N(t) \quad (1)$$

where  $L(0)$  is the number of the (potential) employees at the beginning and  $L(t)$  the number at the end of the period.  $L_D(t)$  denotes those lost by retirement, by death, by out-migration and by commuting out of the region.  $L_N(t)$  is the labour force gained by natural increase, by immigration and by commuting into the region. Equation (1) can be expressed in an alternative way by defining the difference  $L(0) - L_D(t)$  as the stock present on the labour market at the beginning and at the end of period  $[0,t]$ , thus

$$L(t) = L_S(t) + L_N(t) \quad (2)$$

where  $L_S(t) = L(0) - L_D(t)$ .

The number of jobs  $A$  at the beginning and at the end of the period  $[0,t]$  is linked by the analogous equation

$$A(t) = A(0) - A_D(t) + A_N(t) \quad (3)$$

where  $A_D(t)$  is the number of jobs lost by the depreciation of capital and  $A_N(t)$  is the number of jobs created by investment. For the number of jobs existing at the beginning and the end of period  $[0,t]$  the symbol  $A_S(t)$  is used, so that equation (3) can be reformulated as

$$A(t) = A_S(t) + A_N(t) \quad (4)$$

where  $A_S(t) = A(0) - A_D(t)$ .

The three labour components  $L_D(t)$ ,  $L_S(t)$  and  $L_N(t)$  and the three job components  $A_D(t)$ ,  $A_S(t)$ ,  $A_N(t)$  are linked by variables describing the different possibilities of combination of the 6 components. For example: Among the number  $A_D(t)$  of jobs lost in the period  $[0,t]$  through the depreciation of capital there is a subset of jobs which were occupied by a subset of the number  $L_D(t)$  of employees who retired in the same period. The number of jobs in the subset of  $A_D(t)$  and the number of employees in the subset of  $L_D(t)$  which occupied these jobs are equal; this number is denoted by  $Z_{DD}(t)$ . The variable  $Z_{DD}(t)$  is represented by the element in the first row and the first column of the matrix given in TABLE 1

The system of notation for the variables in TABLE 1 is the following:

- F denotes the number of vacant jobs.
- U denotes the number of those unemployed.
- X is the symbol for the number of employees (which equals the number of occupied jobs).
- The first index of a variable refers to the row (labour), the second to the column (jobs). The meaning of the indices D, S and N is the same as for A and L in TABLE 1.

Substituting the variables of TABLE 1 into the balance equations (2) and (4) the following equations are obtained:

$$L(t) = X_{SS}(t) + X_{SN}(t) + X_{NS}(t) + X_{NN}(t) + U_{SD}(t) + U_{ND}(t) + U_S(t) + U_N(t) \quad (5)$$

$$A(t) = X_{SS}(t) + X_{SN}(t) + X_{NS}(t) + X_{NN}(t) + F_{DS}(t) + F_{DN}(t) + F_S(t) + F_N(t) \quad (6)$$

For a more comprehensive notation the following sums of components are defined:

$$X(t) = X_{SS}(t) + X_{SN}(t) + X_{NS}(t) + X_{NN}(t) \quad (7)$$

$$U(t) = U_{SD}(t) + U_{ND}(t) + U_S(t) + U_N(t) \quad (8)$$

$$F(t) = F_{DS}(t) + F_{DN}(t) + F_S(t) + F_N(t) \quad (9)$$

With the definitions (7) - (9) the equations (5) and (6) can be rewritten as:

$$L(t) = X(t) + U(t) \quad (10)$$

$$A(t) = X(t) + F(t) \quad (11)$$

T A B L E 1

Matrix of the State Variables of the Matching-  
Process at the End of the Period [0,t]

	$A_D(t)$	$A_S(t)$	$A_N(t)$	
$L_D(t)$	$Z_{DD}(t)$	$F_{DS}(t)$	$F_{DN}(t)$	
$L_S(t)$	$U_{SD}(t)$	$X_{SS}(t)$	$X_{SN}(t)$	$U_S(t)$
$L_N(t)$	$U_{ND}(t)$	$X_{NS}(t)$	$X_{NN}(t)$	$U_N(t)$
		$F_S(t)$	$F_N(t)$	

Definitions:

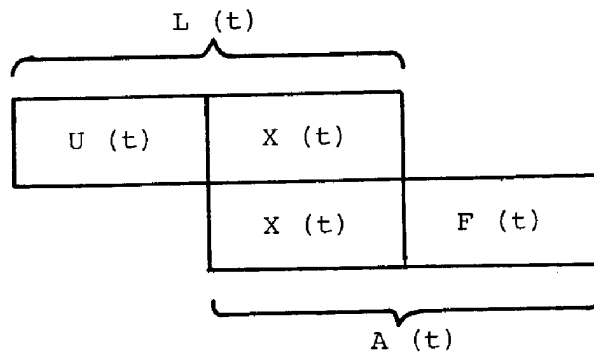
number of jobs ...

- $A_D(t)$  ... lost by capital depreciation in [0,t]
- $A_S(t)$  ... still existing at the end of period [0,t]
- $A_N(t)$  ... gained by investment in [0,t]

number of members of labour force ...

- $L_D(t)$  ... lost by retirement, out-migration etc.
- $L_S(t)$  ... still existing at the end of period [0,t]
- $L_N(t)$  ... gained by natural increase, in-migration etc.

The normal situation of labour markets can be characterized as the co-existence of unemployed workers and vacant jobs, which is symbolically represented in the following diagramm:



where  $L(t)$  = total size of working force  
 $U(t)$  = number of unemployed  
 $X(t)$  = number of employed = number of occupied jobs  
 $A(t)$  = total number of jobs  
 $F(t)$  = number of vacant jobs.

Different kinds of imbalances on the labour market can now be distinguished:

(1) If the size of the labour force exceeds the number of jobs (i.e.  $L(t) > A(t)$ ) the job deficit or labour surplus is

$$L(t) - A(t) = U(t) - F(t) \quad (11.1)$$

(2) If the number of jobs exceeds the size of the working force (i.e.  $A(t) > L(t)$ ) the labour deficit or job surplus is

$$A(t) - L(t) = F(t) - U(t) \quad (11.2)$$

(3) If the number of jobs is equal to the size of the labour force (i.e.  $L(t) = A(t)$ ) there is no job deficit and no labour deficit, but the coexistence of unemployed and vacant jobs still justifies a characterization of this case as a "disequilibrium on the labour market".

Clearly, the labour-surplus as well as the labour deficit are, to a considerable extent, due to demographic factors. But the aim of this paper is to show also that the third case, the zero-deficit-disequilibrium-case, is caused by demographic factors because these factors lead to a decline in the intensity and in the efficiency of the matching-process in many industrialized countries.

### 3. BASIC TYPES OF THE MATCHING PROCESS

The number of changes of the status of the members of the working force (jobs) during the whole period  $[0, t]$  normally considerably exceeds

the number of members of the working force (jobs). In order to determine the difference between the size of the working force (jobs) and the number of their status changes two large classes of different kinds of job-labour-matching, "permanent-stock-matching" (abbreviated by the symbol "PSM") and "renewal-matching" (abbreviated by "RM") are distinguished.

Permanent-stock-matching takes place between jobs and workers who are present on the labour market at the beginning and at the end of the period  $[0,t]$ , i.e. between the individual jobs included in  $A_S(t)$  and the individual members of the labour force included in  $L_S(t)$  (TABLE 2).

Renewal-matching takes place between individual jobs included in  $A_D(t)$  or in  $A_N(t)$  and individual members of the labour force included in  $L_D(t)$  or in  $L_N(t)$  (TABLE 2).

For the permanent-stock-matching the following three basic variables are defined: The first variable is  $y(t)$  which denotes the number of state transitions from the state "employed" to the state "unemployed" for the individuals per unit of time:

$$y = y(t) \tag{12}$$

The second variable is  $v(t)$  which defines the number of state transitions between the state "vacant" and "occupied" for the jobs per unit of time:

$$v = v(t) \tag{13}$$

The third variable is  $m(t)$  which is equal to the number of successful job-labour-matchings per unit of time:

$$m = m(t) \tag{14}$$

To quantify the variables over the whole period  $[0,t]$  the rates can be summed over the time intervals within  $[0,t]$  which leads to the following corresponding definitions:

$$Y(t) = \int_0^t y(t) dt \tag{15}$$

$$V(t) = \int_0^t v(t) dt \tag{16}$$

$$M(t) = \int_0^t m(t) dt \tag{17}$$

T A B L E 2

Classification of Types of Matching-Processes

		$A_D(t)$	$A_S(t)$		$A_N(t)$
			$F_P(o)$	$X_P(o)$	
$L_D(t)$		RM <sub>1</sub>	RM <sub>2</sub>		RM <sub>3</sub>
$L_S(t)$	$U_P(o)$	RM <sub>4</sub>	Permanent Stock - Matching PSM		RM <sub>5</sub>
	$X_P(o)$				
$L_N(t)$		RM <sub>6</sub>	RM <sub>7</sub>		RM <sub>8</sub>

RM = Renewal-Matching

An upper limit for the number of matchings in the permanent stock case is<sup>1)</sup>

$$M(t) \leq Y(t) = V(t) \tag{18}$$

Introducing a parameter  $\alpha$  into the inequality (18) the following equation is obtained:

$$M(t) = \alpha Y(t) = \alpha V(t) ; \quad 0 \leq \alpha \leq 1 \tag{19}$$

This equation is called the "PSM-function" because it contains only matchings between the individuals in the labour force in  $L_S$  and the

1) For a detailed deduction of this relation see: H. Birg: "Demographic Aspects of Labour Market Efficiency." In: Materials of the Institute of Population Research and Social Policy, No. 9, University of Bielefeld, 1983.

jobs in  $A_S$  which are already present at the beginning and at the end of the period  $[0,t]$ . Matchings between, for example, individuals and jobs which occur on the labour market during the period  $[0,t]$  are not contained in  $M(t)$ .

During the period some individuals leave the labour market and some appear on the labour market. The same holds for the jobs. In order to take account of the effects of the corresponding RM-matchings given in TABLE 2 on the number of job vacancies and on the number of unemployment cases the variables  $V_{RM}(t)$  and  $Y_{RM}(t)$  are defined so that the sum of job vacancies and unemployment cases of both the PSM- and the RM-matchings is

$$Y^*(t) = Y(t) + Y_{RM}(t) , \quad Y_{RM}(t) \geq 0 \quad (20)$$

$$V^*(t) = V(t) + V_{RM}(t) , \quad V_{RM}(t) \geq 0 \quad (21)$$

Correspondingly the total number of successful matchings is defined as the sum of the successful matchings of the PSM-class and of the RM-class:

$$M^*(t) = M(t) + M_{RM}(t) , \quad M_{RM}(t) \geq 0 \quad (22)$$

For the PSM-class it can be shown that  $Y(t)$  equals  $V(t)$ <sup>1)</sup>. But for the RM-class the relation between  $Y_{RM}(t)$  and  $V_{RM}(t)$  is not known. As a consequence the relation between  $Y^*(t)$  and  $V^*(t)$  may be  $Y^*(t) \geq V^*(t)$  or  $Y^*(t) \leq V^*(t)$ . Therefore a preliminary formulation for the total matching-function  $f$  is

$$M^*(t) = f(Y^*(t), V^*(t)) \quad (23)$$

Normally  $M(t)$  does not exceed either  $Y(t)$  or  $V(t)$ . An adequate specification of the total matching-function can therefore be given as

$$M^*(t) = \alpha Y^*(t) \quad \text{for } Y^*(t) < V^*(t) \quad (24)$$

$$M^*(t) = \alpha V^*(t) \quad \text{for } V^*(t) < Y^*(t) \quad (25)$$

or as

$$M^*(t) = \alpha \min (Y^*(t), V^*(t)) \quad (26)$$

1) See full version of this article, cited in the previous note.



The derivatives of the variables in (24) - (26) give the values of the variables per unit of time rather than for the whole period [0,t]:

$$y^*(t) = \frac{d}{dt} Y^*(t) \quad (27)$$

$$v^*(t) = \frac{d}{dt} V^*(t) \quad (28)$$

$$m^*(t) = \alpha \min (y^*(t), v^*(t)) \quad (29)$$

#### 4. DEMOGRAPHIC FACTORS OF LABOUR MARKET EFFICIENCY

The question of how demographic change influences the imbalance in the labour market and the efficiency of the matching-process will now be considered on the basis of a minimum of plausible assumptions.

The first assumption is that  $V^*(t)$  is less than or equal to  $Y^*(t)$  so that the matching-function can be written as

$$M^*(t) = \alpha V^*(t) , \quad 0 \leq \alpha \leq 1 \quad (30)$$

The second assumption made is that the number of jobs vacancies  $V^*(t)$  is, among other factors, a function of the number of jobs at the beginning of the period [0,t], i.e.

$$V^*(t) = \beta A(0) , \quad 0 \leq \beta \quad (31)$$

The last and central assumption is that the change in the number of the employed  $\Delta X$  is a function of the number of successful matchings:

$$\Delta X = \gamma M^*(t) , \quad \gamma \leq 1 \quad (32)$$

Since  $\Delta X$  can be negative and  $M^*(t)$  is greater than or equal to zero, the range for the parameter  $\gamma$  is  $\gamma \leq 1$ . The parameter  $\alpha$  determines the number of successful matchings and therefore it is called the "efficiency parameter". For the parameter  $\beta$  the term "intensity parameter" is suggested because  $\beta$  measures the relation between the number of fluctuations and the stock of jobs which is, among other factors, the main source of these fluctuations. For  $\gamma$  the term "employment parameter" is suggested.

If the definitions  $\Delta X = \Delta L - \Delta U$  and  $\Delta X = \Delta A - \Delta F$  (equations 10 and 11) the  $\Delta X$  from equation (32) is substituted, the increase of the number of unemployed and of the number of vacant jobs is obtained as a function of the parameter  $\gamma$ :

$$\Delta U = \Delta L - \gamma M^*(t) \quad (33)$$

$$\Delta F = \Delta A - \gamma M^*(t) \quad (34)$$

Using equation (30) and (31) in (33) und (34)

$$\Delta U = \Delta L - \alpha\beta\gamma A(0) \quad (35)$$

$$\Delta F = \Delta A - \alpha\beta\gamma A(0) \quad (36)$$

are obtained.

As a consequence of the three assumptions it can be stated that the higher the intensity and the efficiency of the matching-process the lower - ceteris paribus - is the increase in unemployment and job vacancy. This result is quite plausible, but it has been deduced on the basis of the condition that the matching-parameters are constant.

The condition of constancy will now be replaced by other assumptions which can be derived on the basis of the results of section 3.

For a determination of the efficiency parameter  $\alpha$  the following function is suggested:

$$\alpha(t) = \alpha(L_D(t) - A_D(t), L_S(t) - A_S(t), L_N(t) - A_N(t)) \quad (37)$$

The idea of the specification of (37) is the following: The renewal of the labour force and of the capital stock are interpreted to be two interdependent processes. Interdependency means that the components  $L_D(t)$  and  $A_D(t)$  and the components  $L_N(t)$  and  $A_N(t)$  are linked by the matching-process so that the variables in each pair of components should be of a similar magnitude.

In fact the figures for both processes show a high similarity between the age of the capital stock and the age of the labour force: A young labour force is, as a general rule, associated with a high proportion of new capital equipment. Figures for the censuses of 1961 and 1970 in the Federal Republic of Germany show a strong similarity of the components of change in both processes<sup>1)</sup>:

---

1) Sources for these figures can be found in: H. Birg: "An Inter-regional Population Employment Model for the Federal Republic of Germany: Methodology and Forecasting Results for the Year 1000" in: Papers of the Regional Science Association, Vol. 47, 1981, p. 97f.

Components of Change  
in the Labour Force

L (61)	= 26,8 Mill.
L <sub>D</sub> (61-70)	= 12,6 Mill.
L <sub>S</sub> (61-70)	= 14,2 Mill.
L <sub>N</sub> (61-70)	= 12,3 Mill.
L (70)	= 26,5 Mill.

Components of Change  
in the Stock of Jobs

A (61)	= 26,5 Mill.
A <sub>D</sub> (61-70)	= 12,0 Mill.
A <sub>S</sub> (61-70)	= 14,5 Mill.
A <sub>N</sub> (61-70)	= 11,8 Mill.
A (70)	= 26,3 Mill.

If the assumption is accepted that the higher the congruency between the components of the labour-capital-renewal-process the higher is the efficiency of the matching-process, then the differences  $L_D - A_D$ ,  $L_S - A_S$  and  $L_N - A_N$  are appropriate variables in the function of equation (37) which determines  $\alpha$ . These assumptions may be expressed in terms of the partial derivatives of  $\alpha$  as:

$$\frac{\partial \alpha}{\partial (L_D - A_D)} < 0 \quad (38.1)$$

$$\frac{\partial \alpha}{\partial (L_S - A_S)} < 0 \quad (38.2)$$

$$\frac{\partial \alpha}{\partial (L_N - A_N)} < 0 \quad (38.3)$$

For the intensity parameter  $\beta$  the following function  $\beta$  is suggested:

$$\beta(t) = \beta \left( \frac{L_D(t)}{L(0)}, \frac{L_N(t)}{L(0)}, \frac{A_D(t)}{A(0)}, \frac{A_N(t)}{A(0)} \right) \quad (39)$$

The assumption made in equation (39) is that the magnitudes of the renewal components of individuals and jobs in relation to the magnitudes of their initial stocks determine, among other factors, the number of job vacancies  $V^*(t)$ , given the number of jobs and members of the labour force at the beginning of the process. The assumption can be expressed in the following way:

$$\frac{\partial \beta}{\partial (L_D(t)/L(0))} > 0 \quad \left. \vphantom{\frac{\partial \beta}{\partial (L_D(t)/L(0))}} \right\} \quad (40.1)$$

$$\frac{\partial \beta}{\partial (L_N(t)/L(0))} > 0 \quad \left. \vphantom{\frac{\partial \beta}{\partial (L_N(t)/L(0))}} \right\} \quad (40.2)$$

- demographic influence

$$\left. \begin{aligned} \frac{\partial \beta}{\partial (A_D(t)/A(0))} > 0 & \quad (40.3) \\ \frac{\partial \beta}{\partial (A_N(t)/A(0))} > 0 & \quad (40.4) \end{aligned} \right\} \text{ - economic influence}$$

How can the condition of constancy for the parameter  $\gamma$ , introduced in (32), be replaced by a more realistic assumption? This is a central question because answering it implies establishing a hypothesis of the determinants of the change in employment. The following hypothesis is suggested:

$$\gamma(t) = \gamma (\alpha(t), \beta(t), D(t), p_L(t)/p_C(t)) \quad (41)$$

whereby

$$\frac{\partial \gamma}{\partial \alpha} > 0 \quad , \quad \frac{\partial \gamma}{\partial \beta} > 0 \quad (41.1)$$

$$\frac{\partial \gamma}{\partial D} > 0 \quad , \quad \frac{\partial \gamma}{\partial (p_L/p_C)} < 0 \quad (41.2)$$

In (41)  $D(t)$  is the expected demand for the products of the firms.  $p_L(t)$  and  $p_C(t)$  are the prices per unit of labour and per unit of capital respectively.

It seems to be plausible that the higher the efficiency and intensity of the matching-process, the higher the congruency between the characteristics of each specific job and the characteristics of each specific employee. The higher the congruency, the higher the probability that the workers remain employed for a long time interval. Because  $\alpha$  and  $\beta$  determine the number of matchings, the sub set of durable matchings will be higher with increasing  $\alpha$  and  $\beta$ . An empirical argument for equation (41) is the following: In the Federal Republic of Germany approximately one out of four jobs becomes vacant and is re-occupied every year. This means that there were roughly speaking 60 million successful matchings in the period 1961-70, i.e. between the last two censuses. The unemployment rate was extremely low in this period, sometimes below 1%. After 1970 the number of matchings decreased and the unemployment rates increased considerably.

## 5. EMPIRICAL EVIDENCE AT THE MACRO LEVEL

In the Federal Republic of Germany the annual number of matchings is estimated to be about 10 million including 5 million intra-firm-matchings and 5 million inter-firm matchings. This is a rough estimation<sup>1)</sup>. Exact figures for the annual number of matchings are available only for a special sub-group of all matchings, namely for those who obtained or changed their jobs with the assistance of the Federal Employment Agency.

The annual figures of the officially registered number of matchings vary between 3 million and 1,9 million, exhibiting a steady decline from 1970 corresponding to the growing number of unemployed.

A proxy variable for the number of matchings is the number of migrations between the 11 states of the Federal Republic. As most inter-state migrations involve a distance of 100 km or more, the proportion of residence changes which are combined with a change of job can be estimated to be at least 54%<sup>2)</sup>.

The decline of mobility is due to the diminishing growth rates of the GNP (GRAPH 1) but there is an additional factor which causes a reduction of mobility, namely the increase in the activity rates of married women. The link between mobility and activity rates is suspected to be an outcome of the factors which determine the probability of a job change. If the probability of a job change which is combined with a change of residence for an unmarried person  $i$  is  $p_i$ , then the probability of a job-change  $q_{ij}$  for married people  $i$  and  $j$  who want to change their residential location is less than  $p_i$  and less than  $p_j$ , namely  $q_{ij} \approx p_i p_j$ .

---

1) L. Reyher/D. Mertens: Zum Beschäftigungsproblem in den nächsten Jahren. In: Gewerkschaftliche Monatshefte, Vol. 11, 1977.

2) H. Birg: Zum Stand der Wanderungsforschung in der Bundesrepublik Deutschland. In: S. Rupp (ed.): Demographische Forschung heute - Bestandsaufnahme und Perspektiven. Jahrestagung der Deutschen Gesellschaft für Bevölkerungswissenschaft, Wiesbaden 1981, p. 129.

Wachstumsrate  
in v.H.

Parallelity between Migration and Production

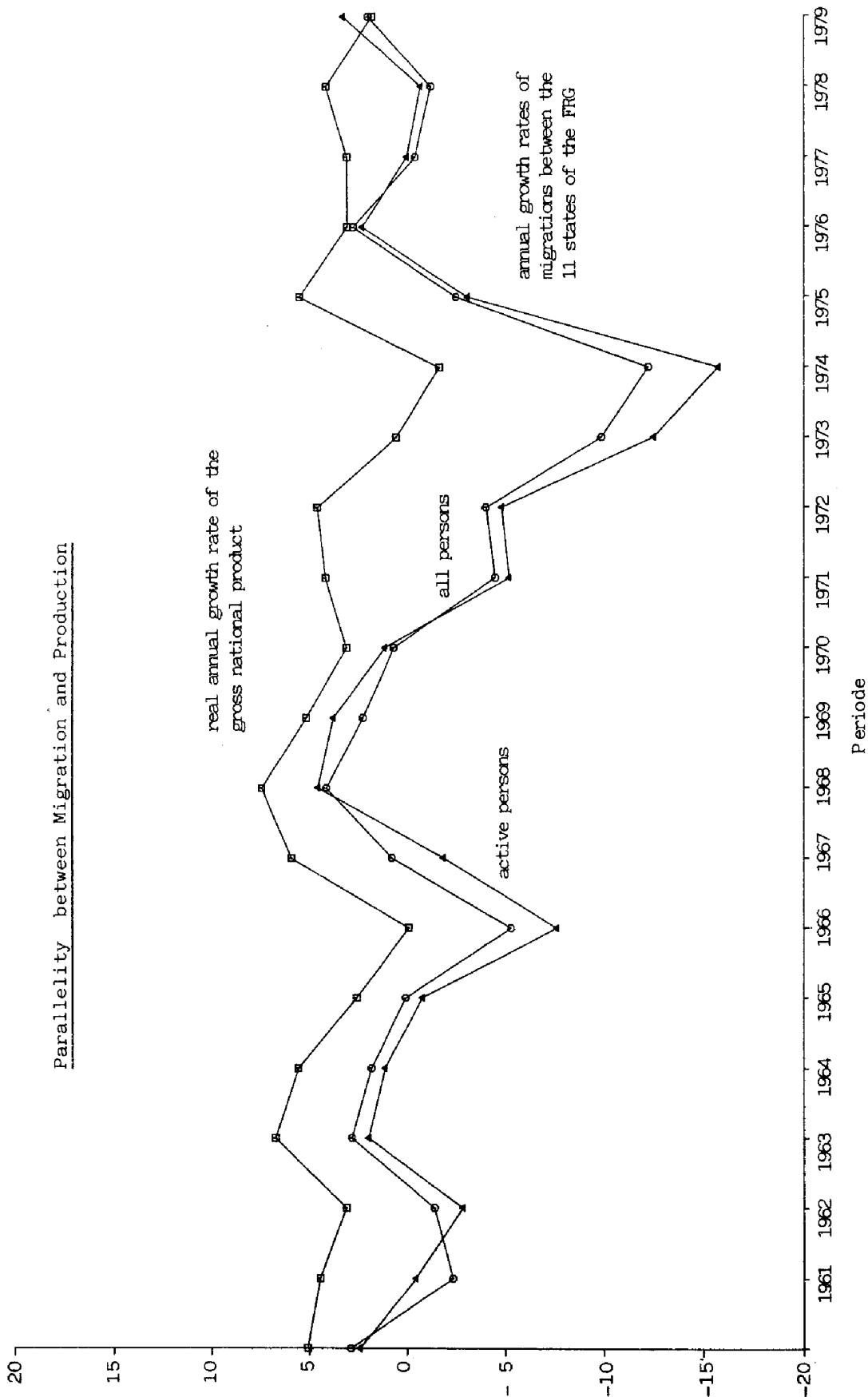
real annual growth rate of the  
gross national product

all persons

active persons

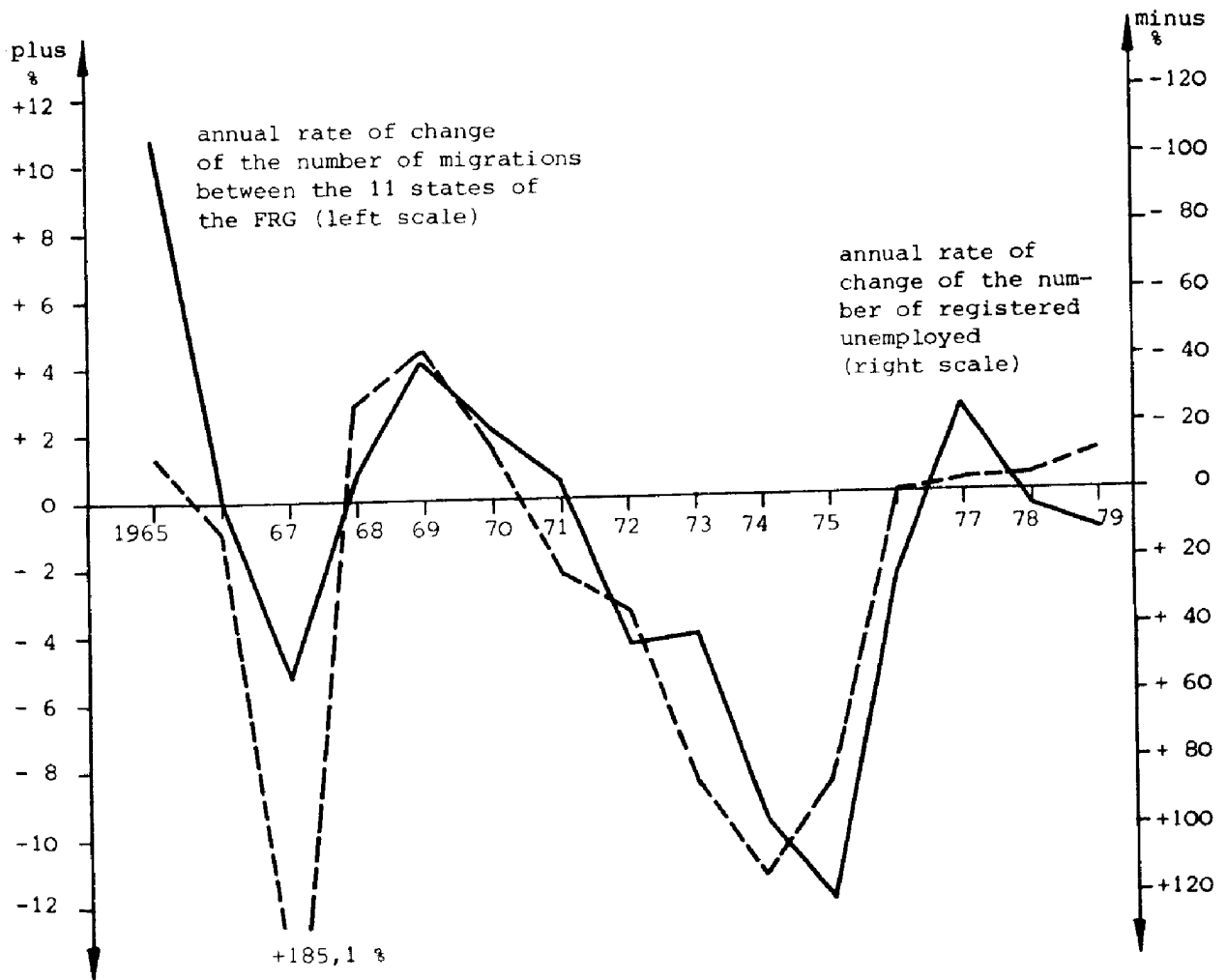
annual growth rates of  
migrations between the  
11 states of the FRG

GRAPH 1



G R A P H 2

The Reciprocal Parallellity of Migration and Unemployment  
in the Federal Republic of Germany 1964-79<sup>\*)</sup>

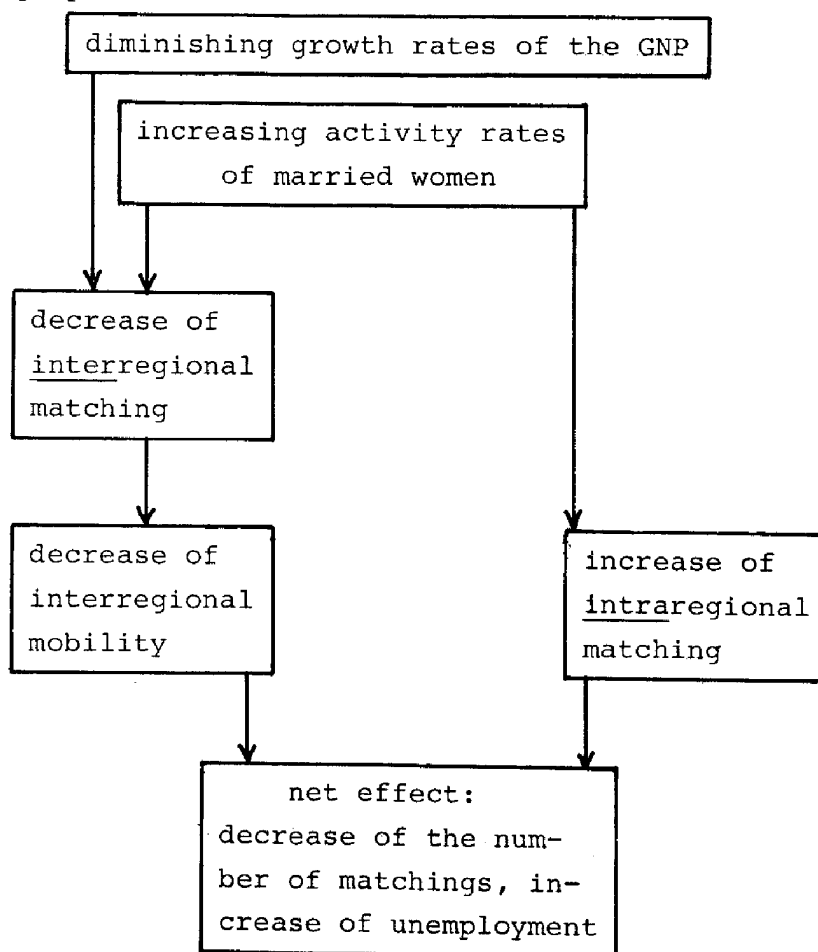


\*Right scale = reversed left scale

The interpretation of mobility as a matching-indicator is supported by the strong correlation between the number of unemployed and the number of migrations. The graphical representation of the unemployment and migration figures in GRAPH 2 shows a strong reciprocal relation between the annual rates of change of employment and mobility: The higher the unemployment, the lower the mobility.

After 1976 the relationship between female activity rates, mobility and unemployment is weakened by the acceleration of the decrease of the propensity of young people for marriage: Cohort analysis of the rates "number of persons fo age "a" which marry for the first time in year t" in percent of the "number of unmarried persons of age "a" at the beginning of year t" shows that the cohort curves for younger cohorts are always below the curves for the elder cohorts (see full version of this paper, p. 48).

It can be concluded that at least in the period 1964-77 demographic variables such as the number of migrations and the activity rates of married women contributed to the decline in the number of matchings and to the increase in unemployment. The relationship between demographic and economic variables in the period 1964-77 may be virtualized by the following graph:





The influence of the growth rate of the GNP on the number of matchings is, in the first place, a cyclical one, whereas the influence of the rising activity rate of married women is not cyclical but based on a stable trend. The cyclical influence of the business cycle is stronger than the trend influence of the activity rate.

#### 6. EMPIRICAL EVIDENCE AT THE REGIONAL LEVEL

In this section the effects of the size of a labour market on the efficiency of the matching-process will be discussed on a theoretical basis. Consider a regional labour market with  $n$  members of the labour force and  $n$  jobs. For simplicity it is assumed that all members of the labour force are employed and all jobs are occupied. Let it be assumed that the employees have the same profession but different abilities and characteristics which make them different with respect to their suitability for the jobs available. The individuals are numbered in such a way that individual 1 has the highest qualification, individual 2 the second best and so on. Let it be assumed that the jobs 1, ...,  $n$  are also different and that the employees rank the jobs according to their attractiveness, which may be a function of the income associated with each job as well as a function of various other factors.

If every job becomes vacant within a certain period  $[0,t]$ , the individual 1 can choose among  $n$  alternative jobs within the period  $[0,t]$  because individual 1 has the best qualification. Assume that individual 1 chooses job 1, which is the most attractive. Then individual 2 can choose among  $n-1$  alternatives, individual 3 among  $n-2$  and so on:

employee	number of alternatives in the choice set of jobs
1	$n$
2	$n-1$
:	:
:	:
$n$	1

The total number  $Q$  of alternatives in all choice sets of all employees in the period  $[0,t]$  is equal to the sum  $Q = 1 + 2 + \dots + n$

$$Q = \frac{n(n+1)}{2} \tag{42}$$

and the average number of alternatives per employee is then  $Q/n = (n+1)/2$ .

Now the assumption is made that (1) the probability for a single worker to find a suitable job, (2) the number of job vacancies  $V$  and

(3) the number of successful matchings  $M$  are functions of  $Q$ :

$$V = \beta Q \quad (43)$$

$$M = \alpha V = \alpha \beta Q = \alpha \beta \frac{n(n+1)}{2} \quad (44)$$

An important aspect of this result is that the matching function (44) is not linear and as a consequence the number of matchings per worker

$$\frac{M}{n} = \alpha \beta ((n+1)/2) \quad (45)$$

is higher the bigger the labour market.

On the basis of this consideration it is possible to specify an out-migration function for regions or cities which can be empirically tested. Let it be assumed that all members  $n^r$  of the labour force in a region  $r$  are potential emigrants, denoted by the symbol  ${}^*E^r$ . Then the effective number of out-migration from region  $r$  to other regions in the country is the difference between  ${}^*E^r$  and the number of members of the labour force who remain on the labour market in  $r$ . If it is assumed that the number of those remaining is proportional to the number of matchings an out-migration function can be specified as:

$$E^r = E^r ({}^*E^r - M^r) \quad (46)$$

where  $E^r$  is the number of individuals who migrate from region  $r$  or city  $r$  to other cities or regions within the country. Assuming a linear form

$$E^r = a_0 + a_1 n^r - a_2 M^r \quad (47)$$

and by substituting (44) into (47):

$$E^r = a_0 + a_1 n^r - a_2 \alpha \beta \frac{(n^r)^2 + n^r}{2} \quad (48)$$

Equation (48) can be tested by the use of population figures if it is assumed that the activity rates  $n^r/P^r$  for regions or for cities respectively are of similar size ( $P^r$  denotes the number of inhabitants):

$$E^r = -380.5 + 604.4 P^r - 0.959 (P^r)^2 + u^r; r=1, \dots, 58 \text{ cities (49)}$$

(11.6)      (-3.55)

$$\rho^2 = 0.923; \text{ corrected } \rho^2 = 0.920; \quad F(2,55) = 330$$

$P^r$  = population in units of 10,000 inhabitants in the year 1970

$E^r$  = number of out-migrations from city  $r$  to other cities in 1970

From (49) the number of out-migrations per person is

$$\frac{E^r}{P^r} = 604.4 - \frac{380.5}{P^r} - 0.959 P^r \quad (50)$$

Function (49) has a positive slope whereas function (50) has a negative slope over the range  $P^r \geq 100,000$ :

$$\frac{dE^r}{dP^r} = 604.4 - 1.92 P^r > 0; \quad P^r \text{ in units of } 10,000 \quad (49.1)$$

$$\frac{d\left(\frac{E^r}{P^r}\right)}{dP^r} = -0.959 + \frac{380.5}{(P^r)^2} < 0 \quad (51)$$

It can consequently be stated that (a) the number of out-migrations increases with increasing population and (b) the number of out-migrations per person decreases with increasing population.

The results of the analysis can be summed up as follows:

(1) The size of a labour market, measured by demographic variables as the number of the members of the labour force or the number of inhabitants, has important effects on the number of matchings: the bigger the labour market, the greater the number of matchings per member of the labour force. As a consequence the number of out-migrations is higher and the number of the out-migrations per person is lower the higher the number of inhabitants (or members of the labour force).

(2) Diminishing regional labour markets due to the decreasing number of inhabitants or due to the decreasing number of jobs per profession caused by the specialization of skills will lead to a reduction in the number of successful matchings and to diminishing interregional migration flows. As a consequence also the employment rates of the regional labour markets will be influenced negatively by diminishing market sizes.

## 7. SUMMARY

The study is based on a special analysis of the labour market and of the matching process whose function is to attach individual members of the labour force to individual jobs. The interdependencies between the different components of change in the labour force and the different components of change in the stock of jobs (capital) are shown to be sources of the different types of the matching-process. Two classes of matching-processes are distinguished: renewal-matching and permanent-stock-matching.

Demographic change influences renewal-matching as well as permanent-stock-matching. Size effects and structural effects of demographic change are distinguished and studied separately. In many industrialized countries demographic change is likely to be an important cause for the decrease in both the intensity and the efficiency of the matching-process. In the Federal Republic of Germany growing unemployment figures and decreasing interregional matching and interregional mobility are interpreted to be caused by demographic variables to a considerable extent.