Representation-finite tree algebras

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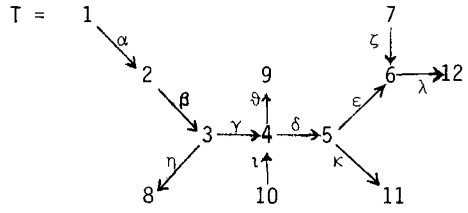
It is well-known, how the representation theory of representation-finite quivers can be reduced to representations of posets (see [2]). We show that this can be generalized to representation-finite trees with arbitrary relations. This generalization was conjectured and partially proved by the first author using a quite technical inductive argument. During ICRA III, the second author observed that there is a direct proof which is based on results of [3] and which is given here.

1. Modules having peaks; statement of the theorem

Throughout the paper, k denotes a commutative field. In this paragraph, we have to give a lot of notations and definitions. Instead of doing it formally, we illustrate them by examples. We are sure that this is easier to read.

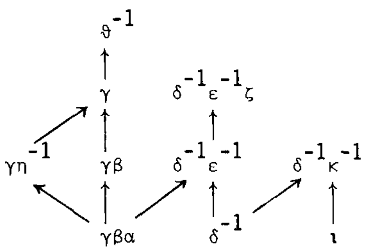
Let T be a finite quiver, whose underlying graph is a tree. Denote the corresponding path algebra by kT. It contains the ideal kT^+ generated by the arrows, this is just the radical of kT. Any quotient algebra $A = kT/R$ with $R \subseteq (kT^+)^2$ is called a tree algebra. Of course, the category of all finite-dimensional left A-modules can be identified with the full subcategory of all finite-dimensional representations of T which satisfy the relations in R. (If M is a representation of T, we denote by M(i) the vectorspace associated to the point i of T, and by $M(\alpha)$ or just by α the map associated to the arrow α.) As an example, we will consider the following tree T



with R being generated by the relations $0 = \eta\beta\alpha = \varepsilon\delta\gamma\beta\alpha = \lambda\zeta = \vartheta\gamma = \lambda\varepsilon\delta = \kappa\delta\iota$.
A walk w in T from i to j of length n is a finite sequence $w = \alpha_1 \dots \alpha_n$, with α_i or α_i^{-1} an arrow, such that α_n starts at i, α_1 ends at j and the remaining starting and ending points fit together well. Moreover, we don't allow w to have a subsequence of the form $\alpha\alpha^{-1}$, $\alpha^{-1}\alpha$, r or r^{-1} with $r \in R$. On the finite set S_j of all walks with end point j we define a particular ordering by:

$$w_1 \leq w_2 \iff \begin{cases} w_1 = v\xi w_1', \; w_2 = v\eta w_2' & \text{with } \eta^{-1}\xi \in R \;\; \underline{\text{or}} \\ w_2 = w_1 w_2', \; w_1 = v\alpha, & \text{where } \alpha^{-1} \text{ is an arrow } \underline{\text{or}} \\ w_1 = w_2 w_1', \; w_2 = v\alpha, & \text{where } \alpha \text{ is an arrow } \underline{\text{or}} \\ w_1 = w_2 \end{cases}$$

For instance, S_4 in our example has the following shape:



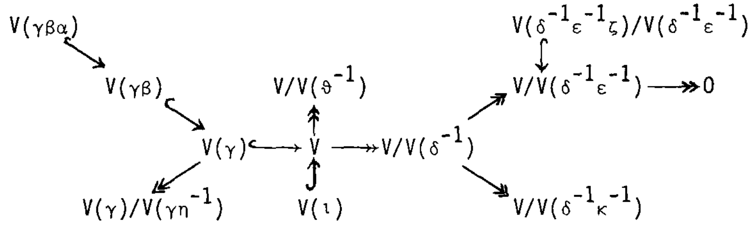
Each walk $w = \alpha_1 \ldots \alpha_n \in S_j$ defines a subfunctor denoted by $w[-]$ of the functor $\mathrm{Hom}(P_j,-) : \mathrm{mod}\; kT/R \longrightarrow \mathrm{mod}\; k$ (here, P_j denotes the indecomposable projective module corresponding to the point j, and note that for any module M, $\mathrm{Hom}(P_j,M)$ is nothing else but the vectorspace $M(j)$). Namely, in case α_n is an arrow, say starting at i, let $w[M] = w(M(i))$, whereas in case α_n^{-1} is an arrow let $w[M] = w(0)$. In our example $\gamma\beta\alpha[M] = \gamma\beta\alpha(M(1))$ and $\gamma\eta^{-1}[M] = \gamma\eta^{-1}(0)$, both being subspaces of $M(4)$. We have $w_1 \leq w_2$ if and only if $w_1[M] \subseteq w_2[M]$ for all modules M. Of course, this gives rise to a functor F_j from $\mathrm{mod}\; A$ to the category $R(S_j)$ of all finite-dimensional representations of the poset S_j, where $F_j(M)$ has as total space $M(j)$ and where $(F_jM)(w)$ equals $w[M]$ for $w \in S_j$.

A representation M of T has <u>peak</u> j, if each arrow leading to j is represented by an injection, each arrow going away from j by a surjection. (By definition, an arrow $i_1 \xrightarrow{\alpha} i_2$ is said to lead to j provided i_2 and j belong to the same connected component of $T \setminus \{\alpha\}$, otherwise α is said to go away from j. In our example, $\alpha,\beta,\gamma,\iota$ and ζ are leading to 4, the others are going away from 4.) Denote by $\mathcal{P}_j$ the full subcategory of all representations of A having peak j.

<u>Lemma</u>: The functor F_j induces an equivalence between $\mathcal{P}_j$ and $R(S_j)$.

<u>Proof</u>: We define a functor $G : R(S_j) \longrightarrow \mathcal{P}_j$ which gives the inverse of $F_j|\mathcal{P}_j$. For simplicity, we give the construction only in the above example. Let V be an object of $R(S_4)$, i.e. V is a vectorspace with a family of subspaces $V(w)$, $w \in S_4$.

Define GV to be the following:



with all maps being the canonical ones. By construction, GV lies in P_4.

This lemma shows that for a representation-finite tree algebra A all posets S_i are representation-finite. The converse is also true, it is the main result of this paper:

Theorem: Let kT/R be a tree-algebra. Then kT/R is representation-finite if and only if all S_i are representation-finite. Moreover, in that case each indecomposable has a peak.

2. Proof of the theorem

The proof of the theorem rests on the results from [3] on tilted algebras. To apply these results, we have to know that any tree-algebra we are interested in, has a preprojective component in its Auslander-Reiten quiver. This follows from a paper of Bautista-Larrion [1] and, for the convenience of the reader, we give here a direct proof.

Recall that a component C of the Auslander-Reiten quiver of an algebra A is called preprojective, provided C contains no oriented cycle and each module in C has the form $\tau^{-t}P$ for some natural number t and some indecomposable projective P. Here τM denotes the Auslander-Reiten translate of the indecomposable M. For instance, the preprojectives of a hereditary connected algebra form a preprojective component. An indecomposable M is called a predecessor of another indecomposable N, if there is a chain

$$M \longrightarrow M_1 \longrightarrow M_2 \longrightarrow \dots \longrightarrow M_n \longrightarrow N$$

of irreducible maps. Denote this by $M \rightsquigarrow N$.
A preprojective component C is closed under predecessors and each $M \in C$ has only finitely many predecessors. Moreover, C contains all indecomposables U with $\mathrm{Hom}(U,X) \neq 0$ for some $X \in C$. (see [3] for details).

If $\bar{T}$ is a connected subquiver of T, denote by $\bar{R}$ the ideal generated by the paths of R which lie inside of $\bar{T}$. We call $k\bar{T}/\bar{R}$ a branch-algebra of kT/R.

Clearly, mod $k\overline{T}/\overline{R}$ can be identified with the full subcategory of mod kT/R, whose objects are the representations vanishing outside of $\overline{T}$.

Proposition Each tree algebra $A = kT/R$ has a preprojective component.

Proof: Let $x \longrightarrow y$ be an arrow in T and U the indecomposable direct summand of the radical of P_x with $U(y) \neq 0$. Consider the connected component $\overline{T}$ of y in $T \setminus \{x\}$ and the corresponding ideal $\overline{R}$. Then $\overline{A} = k\overline{T}/\overline{R}$ is a branch algebra of A and we denote by $\overline{\tau}$ the corresponding Auslander-Reiten translate.

Claim Let $\mathcal{C}$ be a preprojective component of $\overline{A}$. For each $X \in \mathcal{C}$, $X \neq U$, which has not U as a predecessor (with respect to $\overline{A}$), we have $\overline{\tau}^{-1}X = \tau^{-1}X$.

Proof of the claim: By induction on the number of predecessors of X. Thus we start with the case, where X is simple projective in mod $\overline{A}$, hence in mod A. Then the middle term of the Auslander-Reiten sequence $0 \longrightarrow X \longrightarrow P \longrightarrow \tau^{-1}X \longrightarrow 0$ has to be projective. By assumption we have $X \neq U$, hence $P \in \text{mod}\,\overline{A}$ and $\tau^{-1}X = \overline{\tau}^{-1}X$.

For the induction step consider first the case, where X is not projective. In the Auslander-Reiten sequence

$$e:\ 0 \longrightarrow \overline{\tau}X \longrightarrow \oplus Y_i \longrightarrow X \longrightarrow 0$$

of mod $\overline{A}$, the Y_i denote indecomposables. By induction, we have $\tau^{-1}\overline{\tau}X = \overline{\tau}^{-1}\overline{\tau}X = X$. This implies, that e is an Auslander-Reiten sequence of mod A. Again by induction, we have $\tau^{-1}Y_i = \overline{\tau}^{-1}Y_i \in \text{mod}\,\overline{A}$ and this gives us the non-projective heads of all irreducible morphisms in mod A starting at X. On the other hand, the assumption $X \neq U$ implies $j \in \overline{T}$ for each irreducible morphism $X \longrightarrow P_j$, $j \in T$. Therefore X is non-injective in mod A iff it is so in mod $\overline{A}$ and then $\tau^{-1}X = \overline{\tau}^{-1}X$ holds.

The induction step is even easier, in case X is projective.

Now we prove the proposition by induction on the number of points of T. Suppose first, that there is an arrow $x \longrightarrow y$ in T as in the beginning of the proof, such that U does not belong to a preprojective component of $\overline{A}$. By induction, there is a preprojective component of $\overline{A}$, which is eaven a preprojective component of A, since our claim holds for each point of that component, and since a module in mod $\overline{A}$ is projective in mod $\overline{A}$ iff it is so in mod A.

In the remaining case, we construct by induction full subquivers $\mathcal{C}_n$ of the Auslander-Reiten quiver of A satisfying the following conditions:

(1) $\mathcal{C}_n$ is finite, connected, without oriented cycles, closed under predecessors and contains only modules of the form $\tau^{-t}P_j$, $j \in T$, $t \in \mathbb{N}$.

(2) $\tau^{-1}\mathcal{C}_n \cup \mathcal{C}_n \subseteq \mathcal{C}_{n+1}$. (Here $\tau^{-1}\mathcal{C}_n = \{\tau^{-1}X$: X is not injective and belongs to $\mathcal{C}_n\}$)

Put $\mathcal{C}_0 = \{S\}$, where S is simple projective, and suppose $\mathcal{C}_n$ has already been constructed. Number the modules $M_1, M_2, \ldots, M_t$ of $\mathcal{C}_n$ with $\tau^{-1}M_i \notin \mathcal{C}_n$ for $1 \leqslant i \leqslant t$, in such a way that $M_i \rightsquigarrow M_j$ implies $i < j$. (If we have t=0 put $\mathcal{C}_{n+1} = \mathcal{C}_n$). Once more, we construct by induction full subquivers $\mathcal{D}_i$ of the Auslander-Reiten quiver of A with $\mathcal{D}_0 = \mathcal{C}_n$ and $\mathcal{D}_i \cup \{\tau^{-1}M_{i+1}\} \subseteq \mathcal{D}_{i+1}$ for $0 \leqslant i \leqslant t-1$, such that the $\mathcal{D}_i$ satisfy condition (1). Of course, $\mathcal{C}_{n+1} = \mathcal{D}_t$ does the job.
If $\tau^{-1}M_{i+1} \in \mathcal{D}_i$, put $\mathcal{D}_{i+1} = \mathcal{D}_i$. In the other case let

$$0 \longrightarrow M_{i+1} \longrightarrow (\bigoplus_{1}^{u} X_j^{n_j}) \oplus (\bigoplus_{1}^{s} P_j^{m_j}) \oplus (\bigoplus_{s+1}^{r} P_j^{m_j}) \longrightarrow \tau^{-1}M_{i+1} \longrightarrow 0$$

be the Auslander-Reiten sequence starting at M_{i+1}. Here we have $P_j \in \mathcal{D}_i$ iff $1 \leqslant j \leqslant s$ and $X_j \in \mathcal{D}_i$ by construction. To get $\mathcal{D}_{i+1}$, add $\tau^{-1}M_{i+1}$ and all its predecessors to $\mathcal{D}_i$ and view it as a full subquiver of the Auslander-Reiten quiver. We show first, that $\mathcal{D}_{i+1}$ has no oriented cycle. Each such cycle has to contain $\tau^{-1}M_{i+1}$ or some P_j, $s+1 \leqslant j \leqslant r$. Therefore we have only to prove, that there is no arrow (in $\mathcal{D}_{i+1}$) starting at $\tau^{-1}M_{i+1}$ and that all arrows starting at some P_j, $s+1 \leqslant j \leqslant r$, fly to $\tau^{-1}M_{i+1}$.
Let $M_{i+1} \longrightarrow Y$ be an arrow in $\mathcal{D}_{i+1}$. Since $\tau^{-1}M_{i+1} \notin \mathcal{D}_i$, which is closed under predecessors, we have $Y \rightsquigarrow P_j$ for some $j \geqslant s+1$, thus $P_j \rightsquigarrow P_j$, what is impossible by our claim.
Next, take an arrow $P_j \longrightarrow Y$ with $Y \neq \tau^{-1}M_{i+1}$. Again we have $Y \rightsquigarrow P_k$ for some $s+1 \leqslant k \leqslant r$, thus $P_j \rightsquigarrow P_k$. Let $P_j \rightarrow V_1 \rightarrow V_2 \ldots\ldots V_q \rightarrow U \rightarrow P_k$ be a chain of irreducible morphisms. By the claim, P_j belongs to mod $\overline{A}$, where $\overline{A}$ is the branch algebra defined by P_k and U. Since there is an arrow $M_{i+1} \rightarrow P_j$, we infer that $U = M_{i+1}$, i.e. $P_j \rightsquigarrow M_{i+1}$, a contradiction.
It is easy to see, that $\mathcal{D}_{i+1}$ satisfies all other conditions. To finish the proof of the proposition, one has to observe that $\mathcal{C} = \bigcup \mathcal{C}_n$ is a preprojective component.

The only result which we will need from [3] is the following:

<u>Lemma 1</u> Let kT/R be a tree algebra having a preprojective component $\mathcal{C}$ and a sincere representation $N \in \mathcal{C}$. If $\alpha_n \ldots \alpha_1$ is a path in T, then $N(\alpha_n) \ldots N(\alpha_1)$ is injective, surjective or zero.
Recall that N is <u>sincere</u> iff $N(j) \neq 0$ for all $j \in T$.

<u>Proof of Lemma 1</u> : This follows directly from theorem 8.5 of [3] .

Furthermore we need the following little lemma on representations of partially ordered sets.

Lemma 2. Let S be a partially ordered set, $a \in S$ a point and V an indecomposable representation such that $0 \neq V(a) \neq V$. Then there exist b and $c \in S$, such that the spaces $V(a)$, $V(b)$, $V(c)$ are pairwise incomparable.

Proof: Suppose not. Then the set $\{b_1,\ldots,b_n\}$ of elements, such that $V(b_i)$ is incomparable to $V(a)$, can be numbered such that $i \leq j$ implies $V(b_i) \subseteq V(b_j)$. It is well-known and easy to see, that the indecomposable representations of the set

$$S' = \{1', 1 \leq 2 \leq 3 \leq \ldots \leq n\}$$

are 1-dimensional, i.e. have k as total space. The restriction of V to $\{a, b_1, \ldots, b_n\}$ can be considered as a representation of S' and decomposed into $V_1 \oplus V_2$, such that $V(a) = V_1(a) = V_1$. A short computation shows that this is a decomposition of V in the category of all S-spaces, a contradiction.

Finally, we prove the remaining part of the theorem. Let a tree algebra $A = kT/R$ be given, such that all partially ordered sets S_i are representation-finite. We have to show, that A is representation-finite and that each indecomposable has a peak. By induction, this is true for each branch algebra of A.

Take an indecomposable N belonging to a preprojective component of A. If N is not sincere, it has a peak by induction, so we may assume N to be sincere, hence Lemma 1 applies. In particular each arrow is represented by an injection or a surjection. Now choose a point p, such that $\dim N(p)$ is maximal. We will show that p is a peak and we need the following

Claim: Let w be a walk form i to j through k with $i \neq k \neq j$. Then $\dim N(i) \gneq \dim N(k) \lneq \dim N(j)$ cannot occur.

Proof: Let $w = \alpha_1 \ldots \alpha_n$ be a counterexample of minimal length. Then $N(\alpha_1)$, $N(\alpha_n)$ are not bijective, but all the other $N(\alpha_i)$ are bijective.

1st case w or w^{-1} is a path.

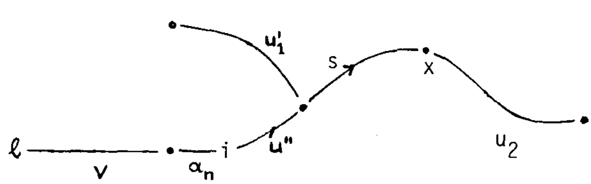
Dualizing, if necessary, we can assume that w is a path. Then $N(\alpha_2)\, N(\alpha_3)\ldots N(\alpha_n)$ is proper surjective, $N(\alpha_1)$ proper injective, but the composition neither injective, nor surjective nor zero, a contradiction to Lemma 1.

2nd case w changes the direction.

Let ℓ be an intermediate point, where w changes the direction. By duality we may assume, that ℓ is a source, i.e. the picture is as follows

$$j \xrightarrow{\alpha_1} \cdot \;\cdots\; \cdot \longleftarrow \ell \underbrace{\longrightarrow \ell' \longrightarrow \cdot \cdots \cdot}_{v} \xrightarrow{\alpha_n} i$$

Let $T_{\ell'}$ be the connected component of $T \setminus \{\ell\}$ containing ℓ' and let $\bar{T}$ be the full subtree of T with point set $T_{\ell'} \cup \{\ell\}$. Let $\bigoplus_{q=1}^{r} N_q$ be a decomposition of $N|\bar{T}$ into indecomposables of $\bar{A} = k\bar{T}/\bar{R}$. We have $N_q(\ell) \neq 0$ for each q, for otherwise N decomposes. Moreover, we have $\dim N_q(i) \gneqq \dim N_q(\ell)$ for some q, say $q = 1$. By induction, N_1 has a peak $x \in \bar{T}$. Let u be the walk from ℓ to x. Clearly, u has the form $u = u'\alpha_n^{-1}v^{-1}$. By construction, we have $0 \neq u'\alpha_n^{-1}[N_1] \neq N_1(x)$. By Lemma 2, there exists $u_1, u_2 \in \bar{S}_x$ (the ordered set with respect to $\bar{A}$), such that $u'\alpha_n^{-1}[N_1]$, $u_1[N_1]$ and $u_2[N_1]$ are pairwise incomparable.



Write $u' = su''$, $u_1 = su_1'$ such that u'' and u_1' are disjoint and do the same for u_2, $u' = tu'''$, $u_2 = tu_2'$. Put $\bar{u}_1 = v\alpha_n u''^{-1}u_1'$, $\bar{u}_2 = v\alpha_n u'''^{-1}u_2'$ Then $\bar{u}_1$, $\bar{u}_2$ are walks belonging to S_ℓ, which are incomparable.
The same argument gives two walks $\bar{u}_3$, $\bar{u}_4$ in S_ℓ, which come through j. Therefore S_ℓ contains four incomparable elements, the final contradiction.

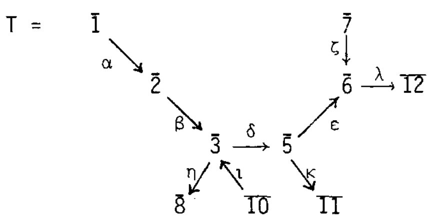
The claim together with the fact, that each arrow is represented by an injection or surjection, immediately implies that p is a peak.

Now, by the proposition there is a preprojective component C. Since each module in C has a peak and since T has only finitely many points, C is finite and contains therefore all indecomposables.

3. Applications and examples

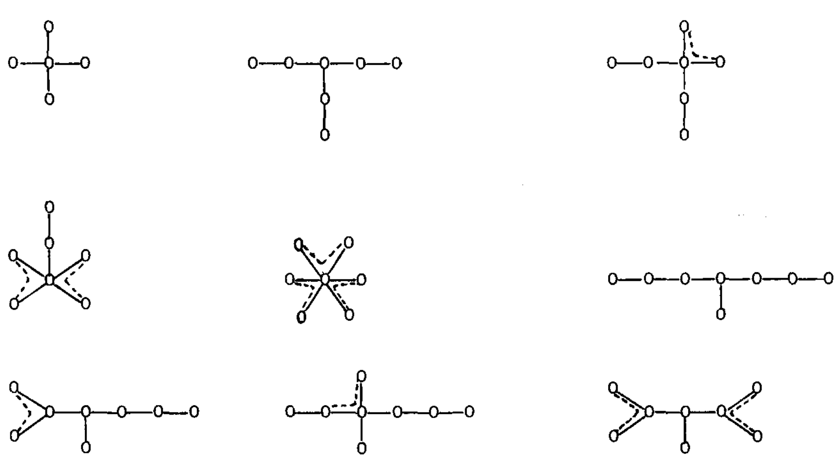
Besides the branch-algebras, there is another type of tree algebras, which can be derived from a given tree algebra kT/R. Let $x \xrightarrow{\beta} y$ be a fixed arrow in T. Denote by $x_1,\dots,x_n$ all points with arrow $x_i \xrightarrow{\alpha_i} x$ and by $y_1,\dots,y_m$ those with arrow $y \xrightarrow{\gamma_i} y_i$. Furthermore, suppose that $\beta\alpha_i = 0$ iff $1 \le i \le r$ and similarly $\gamma_i\beta = 0$ iff $1 \le i \le s$. Denote by T_{x_i} (resp. T_{y_i}) the connected components of

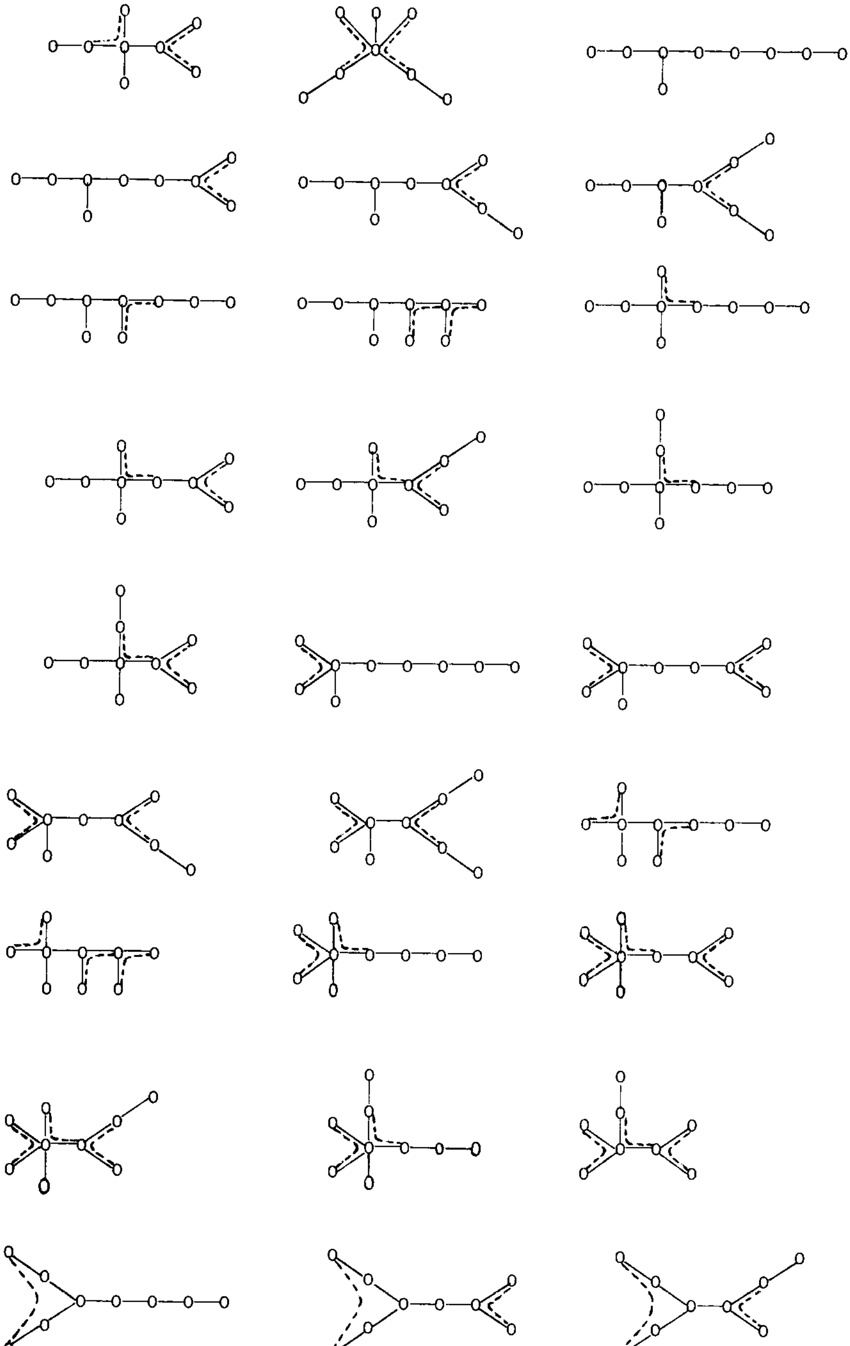
x_i (resp. y_i) in $T \setminus \{x \xrightarrow{\beta} y\}$. Let $\bar{T}$ be the tree obtained from $\tilde{T} = T \setminus \left(\bigcup_{i=1}^{r} T_{x_i} \cup \bigcup_{i=1}^{s} T_{y_i} \right)$ by shrinking $x \xrightarrow{\beta} y$ to a point z. Consider the ideal $\bar{R}$ of $k\bar{T}$ generated by the paths w, such that either w is a path in $\tilde{T}$ belonging to R not containing β or such that w can be written as $w_1 w_2$, where $w_1 \beta w_2$ is a path in $\tilde{T}$ belonging to R. For instance if we start with the arrow $3 \xrightarrow{\gamma} 4$ in our example, we get:

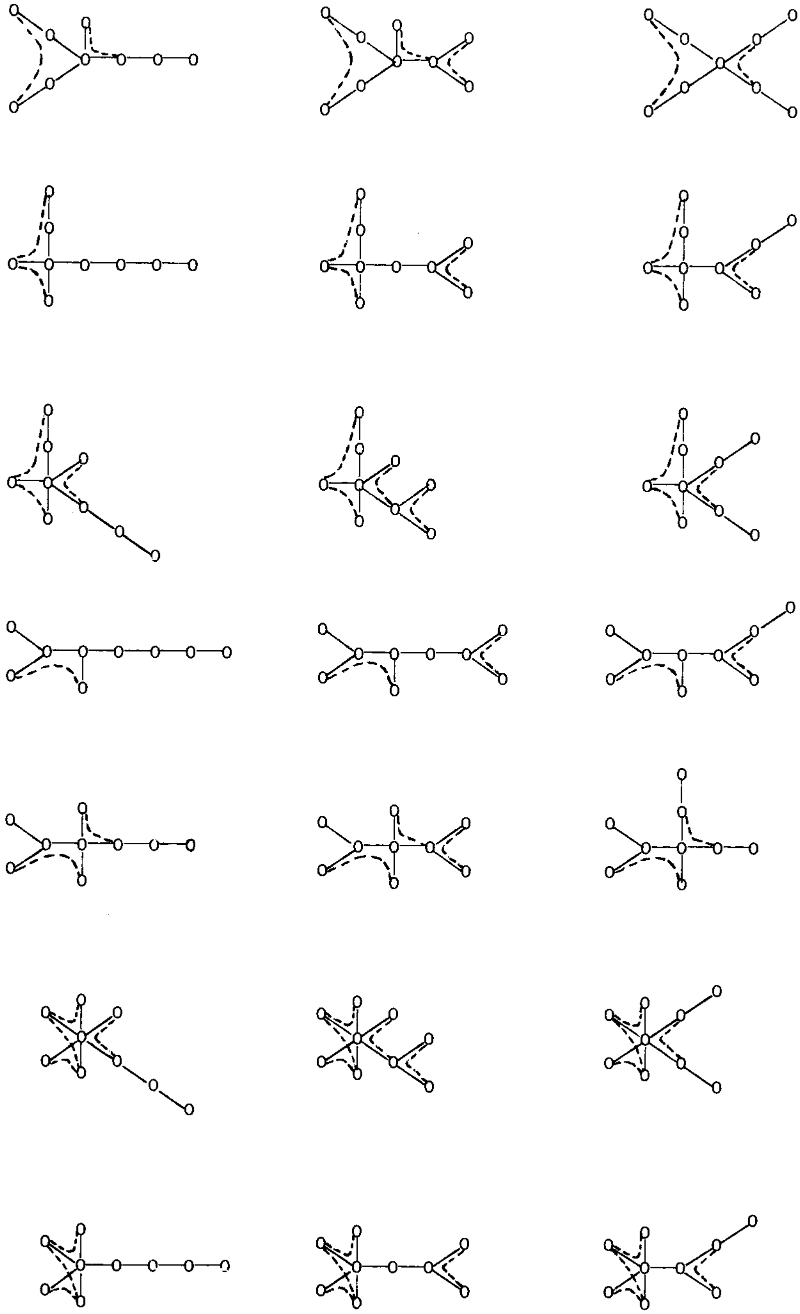


With $\bar{R}$ generated by $\eta\beta\alpha = \varepsilon\delta\beta\alpha = \lambda\zeta = \lambda\varepsilon\delta = \kappa\delta\iota = 0$. The algebra $k\bar{T}/\bar{R}$ is called a shrinked algebra of kT/R and mod $k\bar{T}/\bar{R}$ can be interpreted as full subcategory of mod kT/R, containing only modules where β is represented by a bijection. Finally, an algebra A is said to be contained in B, if there is a finite sequence $A_0, A_1, \ldots, A_n$ of algebras, such that $A = A_0$, $B = A_n$ and A_i is a branch-algebra or a shrinked algebra of A_{i+1} for $0 \leq i \leq n-1$.

Corollary 1. A tree-algebra kT/R is representation-finite if and only if it does not contain one of the following algebras:







Here, we do not specify the orientations. A dotted line marks a (zero-) relation (of course, the arrows along a dotted line have to point in one direction). These algebras are concealed quiver algebras [4,7] of type $\tilde{D}_4$, $\tilde{E}_6$, $\tilde{E}_7$ or $\tilde{E}_8$ (depending on the number of points), and all their representations have a peak.

Proof: The proof consists in a verification, that these are the 'minimal' algebras which contain one of the minimal representation-infinite posets in some S_i.

In practice, the above criterion is not so easy to apply, if one wants to know whether a given tree-algebra A is representation-finite or not. Quite often it is more convenient to compute the dimension-vectors of the indecomposables starting with the simple projectives. In this connection, the following remark is useful.

Corollary 2. Let U be an indecomposable representation of a representation-finite tree-algebra kT/R. Then $\dim U(x) \leq 6$ for all $x \in T$.

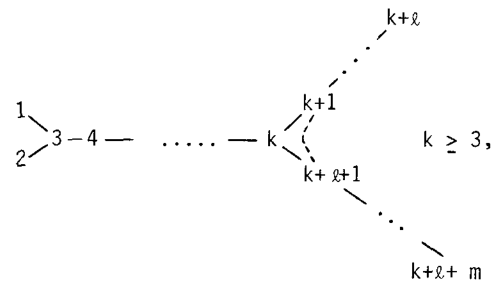
Proof: This follows from Kleiner's theorem ([5]).

Kleiner's list of posets having an exact indecomposable representation even yields a list of all indecomposables of all representation-finite trees. The original partial proof of the theorem used this list. To convince the reader of the arising combinatorial difficulties, we give the list of all representation-finite tree-algebras kT/R, such that $\dim U(x) \leq 4$ for all $x \in T$ and U indecomposable, and such that there exists at least one sincere indecomposable V.

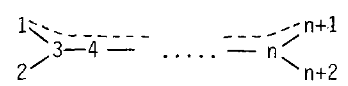
Again, we do not specify the orientation, and the dotted lines describe the generating relations.

No relation: The Dynkin-diagrams with the exception of E_8.

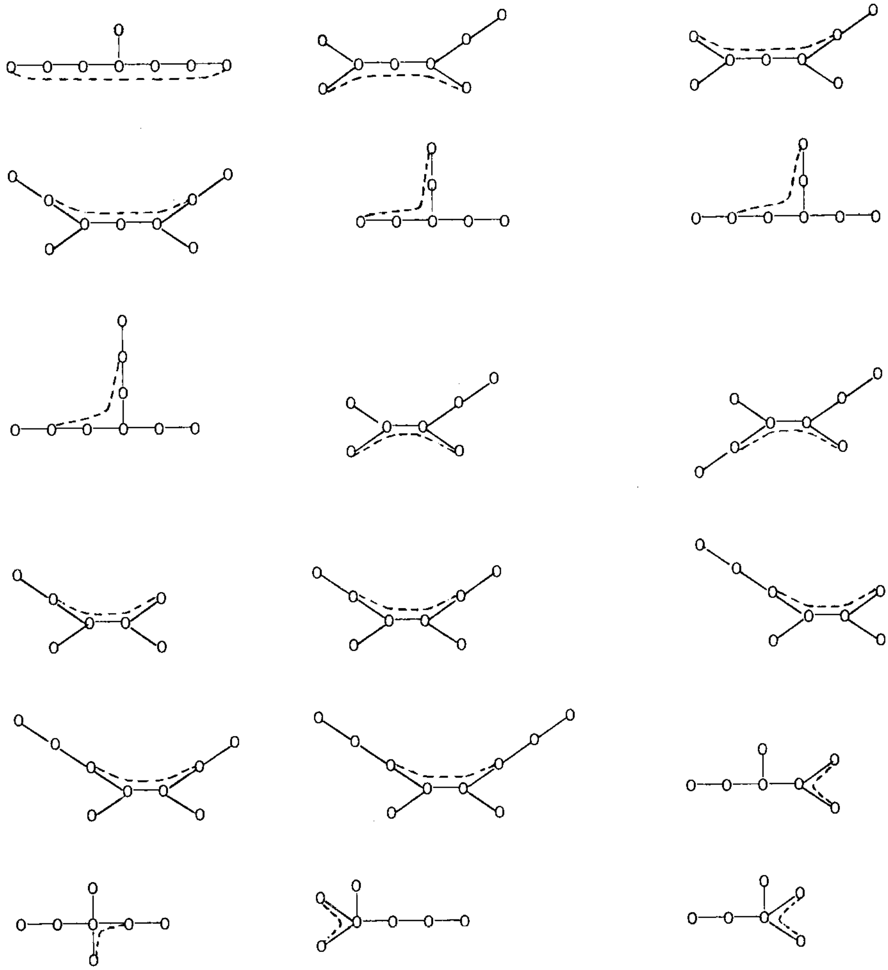
1 Relation



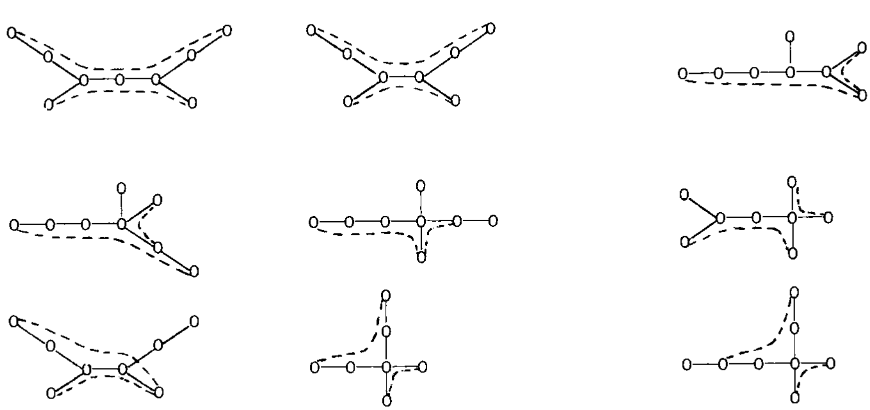
$k \geq 3,\ \ell \geq 1,\ m \geq 1$

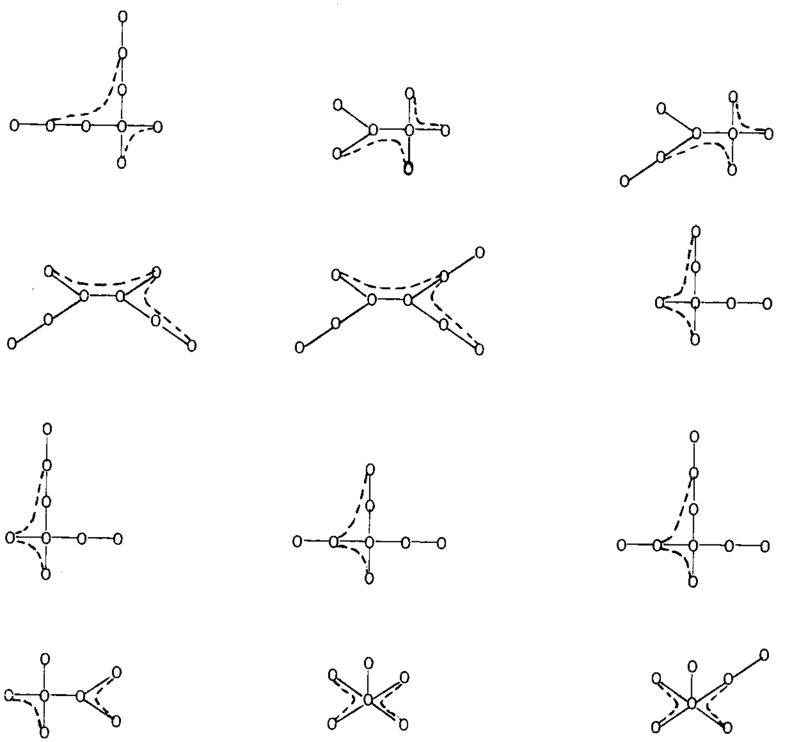


, $n \geq 4$

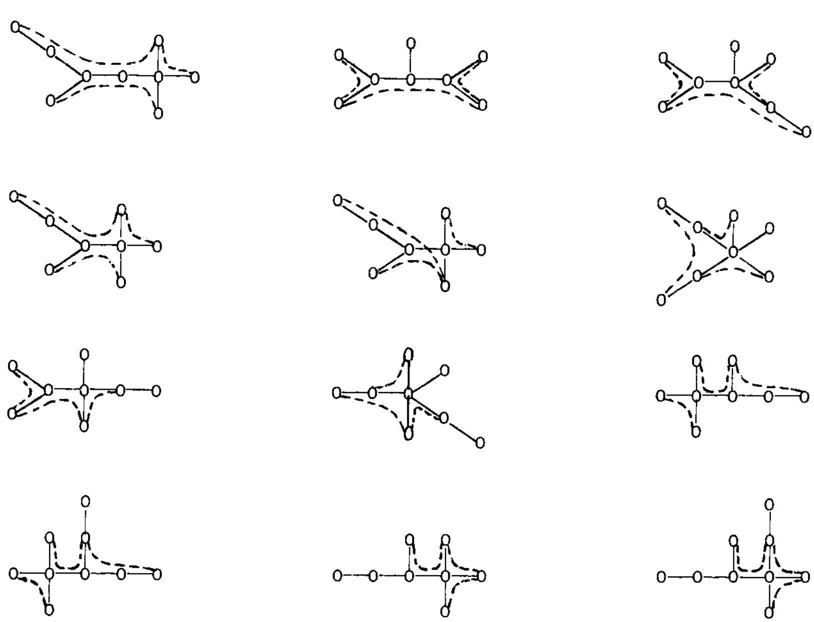


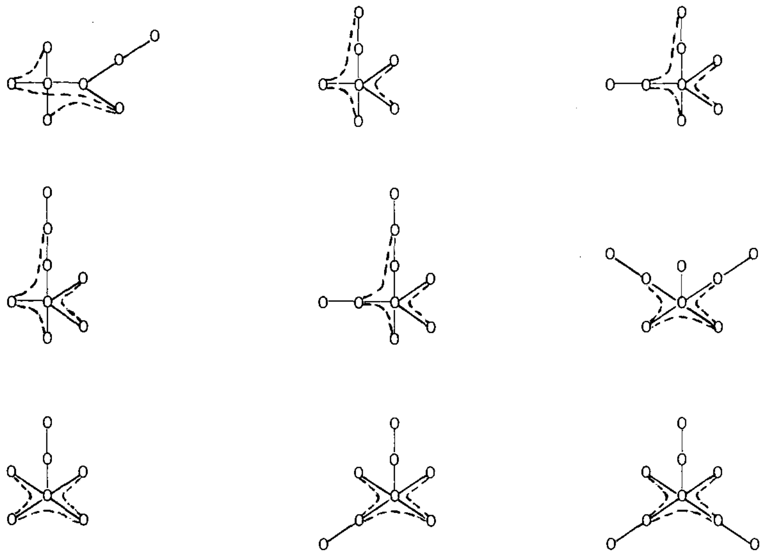
2 Relations



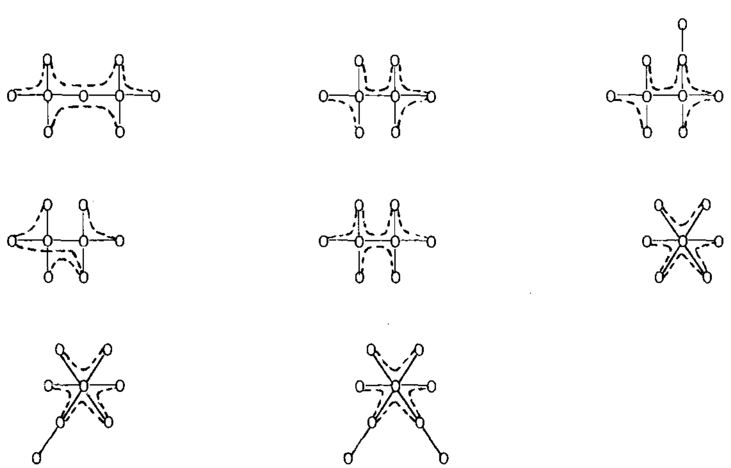


3 Relations





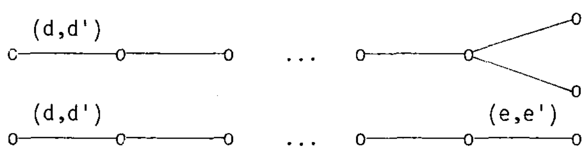
4 Relations



5. Generalization to species

Let $S = (F_a, {}_aN_b)_{a,b}$ be a k-species with underlying graph a tree, kS the tensor algebra of S, and R an ideal inside the square of the radical of kS. Then: if $A = kS/R$ is of finite representation type , then all indecomposable A-modules have a peak. Here, the point j of the underlying graph of S is said to be a peak for the representation $M = (M_a, {}_b\varphi_a : M_a \otimes {}_aN_b \longrightarrow M_b)$ of S if and only if for every $0 \neq n \in {}_aN_b$, the k-linear map ${}_b\varphi_a \otimes n : M_a \approx M_a \otimes n \longrightarrow M_b$ is injective in case $a \rightarrow b$ is an arrow leading to j, and is surjective in case $a \rightarrow b$ is an arrow going away from j.

The proof is an obvious generalization of the arguments given above. First, one notes that the radical rad P of any indecomposable projective A-module P is the direct sum of (at most three) indecomposable modules, and this implies that the Auslander-Reiten quiver of A has no oriented cycles (see [1] or the proof of the proposition in section 2). Again, using theorem 8.5 of [3], one knows that for indecomposable M, any composition of maps of the form ${}_b\varphi_a \otimes n : M_a \approx M_a \otimes n \rightarrow M_b$ is injective, surjective, or zero. In particular, for M indecomposable, $0 \neq n \in {}_aN_b$, the map ${}_b\varphi_a \otimes n$ is injective or surjective. Now assume M is indecomposable, and does not have a peak. In the tree case considered above, we have used lemma 2 in order to construct a full embedding of the module category of a quiver of type $\tilde{D}_n$ into mod A. In the general case, one similarly obtains a full embedding of the module category of a hereditary algebra with underlying graph of the form $\tilde{D}_n$, or



, or

where $dd' \geq 2$ and $ee' \geq 2$. Thus, A cannot be of finite representation type, contrary to the assumption.

As a consequence, it follows that if $A = kS/R$ is of finite representation type and M an indecomposable A-module, then <u>the components of the dimension vector $\underline{\dim}$ M all are</u> ≤ 6. (Recall that $\underline{\dim}$ M has as components $(\underline{\dim}\, M)_a = \dim(M_a)_{F_a}$).

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