Approaching Vagueness
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TWO APPROACHES TO VAGUENESS: THEORY OF INTERACTION AND TOPOLOGY*

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This paper deals with a new perspective in handling the problem of vagueness. On the one hand it will be shown that an interactional theory can give a suitable framework for modelling some global dynamic properties of vague expressions, on the other hand it will be argued that a microanalysis of semantic processes should be based on a theory using concepts of mathematical topology.

1. INTRODUCTORY REMARKS

Developing a theory of vagueness is one of the central aims of present semantics. Moreover, the explanatory power of semantic theories is sometimes judged only with respect to their success in solving the problems of vagueness. On the one hand, the resulting negative evaluation of many semantic theories may be unjustified since much of the semantic research of the last twenty years has provided important results concerning the set-up of explicit semantic theories. On the other hand, criticising the euphoric reception and adoption of the first outlines of formal semantics must be accepted, since these outlines provide only a rough theoretic framework and cannot solve serious problems of empirical semantics. In contrast, an empirically adequate and powerful semantic theory would have to explain among other things, how participants in communication – although using vague expressions – can come to an agreement on their interpretations of verbal utterances.

In this paper I want to argue for developing extensions of hitherto known theories of vagueness systematically, particularly with respect to properties of vagueness which can be modelled in a natural way by topological techniques (e.g. the non-transitivity of vague similarity relations). In order to motivate this I would like to sketch some deficiencies of existing approaches. This discussion will show that using a topological framework will

be fruitful for handling some of the problems of vagueness unsolved up to now. In the present paper I can only present a shortened version of my suggestions however (for further discussions cf. Kindt 1982a,b).

2. ON THE EXPLICATION OF VAGUENESS

Some of the crucial problems of vagueness can already be studied within a formal language with partially defined predicates as follows. Assume for instance, that the interpretation of a vague adjective like "old" will be learned by perceiving situations in which this expression or its negation is unequivocally associated with some persons. In the case of primary language acquisition, the child will at first learn that, e.g., his grandmother can be called "old" whereas he himself must be designated by "not old". Later on the child's extensional interpretation of "old" will be successively expanded to other clear cases of old/not old persons. This expansion, however, will not lead to a totally defined predicate.

The given reconstruction of the acquisition process does not take into account the contextual dependency of interpretation. Nevertheless, we may accept it as an admissible idealization that there are unspecific everyday contexts in which the adjective "old" is usually associated with certain agegroup contexts. For instance, persons of 70 or more may be judged as old and persons younger than 30 as not old; in other cases "old" will not be unequivocally applicable.

Different ways have been suggested for using formal languages with partially defined predicates in order to develop a logic of vagueness. Mainly two conceptions were considered, namely three-valued logic and supervaluation logic. Both approaches are suitable for enlightening, e.g., the Sorites paradox. Besides this the adherents of the supervaluation approach claim that, among other things, they can handle sentences like "Peter is old and he is not old" more adequately. They argue that this sentence should always be associated with the truth-value "false" independently of whether the sentence "Peter is old" is true, false or indefinite. In three-valued logic, however, the first sentence gets the value "indefinite", if the second itself has this value. The seeming advantage of the supervaluation approach in handling the above sentence does not yet compensate, I think, the loss of the property of truth functionality (with respect to a single structure). In fact, a more detailed analysis of the problems concerning the above sentence shows that in the argumentation mentioned two different truth concepts are mixed up, namely logical truth and situational truth. Whereas logical truth and general truth (i.e. truth in all situations) coincide in classical logic, this is not necessarily the case in

nonclassical logics. This claim can easily be demonstrated by the example of an extended Liar sentence ("I am valid and I am not valid" must generally take the value "indefinite", if we want to avoid inconsistency; cf. Kindt 1976).

A factual advantage of the supervaluation approach is that within its framework the concept of precisification is explicitly discussed. However, this discussion is based on the inadequate assumption that making a vague expression more precise means expanding or completing its extension (i.e. the sets of the positive resp. negative clear cases). In contrast to this the observation of everyday communication makes it plausible that precision may also result by weakening extensional interpretations. Moreover, it is surprising that the possibility or better: the social admissibility of precisifying vague expressions has hardly been considered resp. used as the essential condition for an exact definition of vagueness[1]. In my opinion it is this condition, which allows us to distinguish between the vagueness and the indefiniteness of expressions. For example, the German adjective "geradzahlig" (meaning "is an even number") is applicable to natural numbers but not to men. And it would scarcely be accepted if anybody tried to extend its domain of application. On the other hand, the adjective "old" may not be unequivocally applicable in some contexts to persons aged between 35 and 65 (it neither holds true that such persons are old nor that they are not old). However, the participants of a conversation may come to agree on temporarily extending the application domain of "old"; perhaps they negotiate to use "old" for persons older than 60 and "not old" for persons younger than 40. Conversely, it is also allowed to weaken a given interpretation in the case of vagueness but not in the case of indefiniteness. A participant usually interpreting "old" as "50 or more" will be prepared to modify temporarily this interpretation, if another participant objects to it: "A person of 50 is not yet old!" In contrast to this, one could hardly imagine that anybody would demand to restrict the interpretation of "geradzahlig", e.g., to numbers greater than 17.

Profitting from the discussed distinction between vagueness and indefiniteness I will now suggest an explication for vagueness. But before I can do this, I have to introduce some other definitions and notions.

A *structure* is an ordered pair $S = \langle X,f \rangle$ where X represents the *universe* (i.e. the domain of *individuals*) and f the *meaning function* which interprets each *constant* of the given language (the set of these constants is denoted by CON). More precisely, f assigns to each *individual constant* an element of X and to each n-ary *predicate constant* a partially defined function from X^n to 2 (= {0,1}).

The vagueness of an expression must always be seen with respect to some specific context. Hence, we need a suitable notion of context. The most simplest way for defining contexts in extensional semantics is to regard *contexts* as sets of structures. However, the usual extensional semantics has the disadvantage that it does not allow for a distinction between changes of interpretation and changes of properties of individuals depending upon structures of context. In order to explicate vagueness we rely upon contexts preserving all the properties of individuals; in particular, we assume that the structures of a context have the same universe. The *domain* of a function g is denoted by DMg and $g \restriction Y$ is the *restriction* of g to Y. The set of meaning functions in a context C is denoted by F(C), and for each predicate constant P we define:

$$F(C,P) := \{f(P) : f \in F(C)\},$$

$$MDM(C,P) := \bigcap_{m \in F(C,P)} DMm$$

$$VD(C,P) := \bigcup_{m \in F(C,P)} DMm - MDM(C,P)$$

MDM(C,P) is called the *minimally common domain* (for meanings) of P in C, and VD(C,P) is called the *vagueness domain* of P in C. If l and m are meanings belonging to F(C,P), we define:

$$DIFF(l,m) := (l \cup m) \restriction ((DM\, l \cap DMm) - (l \cap m))$$

DIFF(l,m) is the part of $l \cup m$ where l and m are both defined but take different values.

We now have all the concepts necessary to proceed to an explication of vagueness for the case of predicates.

2.1 DEFINITION:

The n-ary predicate constant P is *vague* in the context C if the following conditions hold true:

(i) $VD(C,P) \neq 0$;

(ii) $f(P) \restriction MDM(C,P) = g(P) \restriction MDM(C,P)$ for all $f, g \in F(C)$;

(iii) for all $f, g \in F(C)$ with $f(P) \neq g(P)$ there is an $h \in F(C)$ such that $h \restriction (CON - \{P\}) = f \restriction (CON - \{P\})$ and $h(P) = g(P)$;

(iv) for all $f, g \in F(C)$ there are $h_1, h_2, h_3 \in F(C)$ such that

$h_1(P) = f(P) \cap g(P) \neq 0$,

$h_2(P) = f(P) - DIFF(f(P), g(P))$

$h_3(P) = f(P) \cup g(P)$, if $DIFF(f(P), g(P)) = 0$.

The condition (i) says that in C there are meanings of P differing with respect to their domains; according to (ii) however, all meanings of P take the same values in the minimally common domain, i.e. here they have a *global 'common kernel'* ($DM \cap F(C,P) = MDM(C,P)$). However, it may be the case that the global common kernel is empty; consider, for example, the open interval $(0, \frac{1}{n+1})$ for each natural number n as an interpretation for the predicate "very small real number". The condition (iii) guarantees that differences of the meanings of P can be regarded as independent of the interpretation of other constants (concerning the empirical claim of (iii) the proposed version of independence property is a bit too strong). In (iv) three *closure properties* of F(C,P) are formulated which represent different types of modifying the meaning of P. If the two participants of a conversation differ in their meanings of P, they may come to agree on a common modified meaning which results by intersecting the meanings introduced at the beginning.

According to (iv) such an *interpretation assimilation* — I want to speak here of the *'minimal consensus model'* of negotiating — always exists but does not lead to a 'thinning out' of the meaning. The adequacy of this postulate is immediately plausible, since in natural languages there are nearly always positive and negative prototypical objects assigned to a vague term by means of which this interpretation of this term is learned. For our instance of "old" we may assume that in most contexts participants agree on regarding persons of 70 or more as old and children younger than 10 as not old.

A second model of interpretation assimilation, the *'minimally change model'*, is given by the procedure that one or each of the participants weakens his interpretation of P in all cases where it definitely contradicts the interpretation of the other participant. This procedure may not lead to a common meaning of P, since the two resulting meanings may nevertheless have different domains.

In the third model — I want to call it *'maximal consensus model'* — the procedure of the second will be supplemented in so far as each of the participants takes over the values of the meaning of the other participant for all arguments where his own meaning was undefined at the beginning.

In reality, an interpretation assimilation will be evoked on having noticed that the respective meanings of the participants take contradictory values for the same arguments. However, the suggested models are idealized in so far as participants will presumably modify their interpretations only for arguments where contradictory values were taken. In a sense, the second model is the most realistic one, since it needs only a minimal change of meanings. I suppose that, for reasons of effectiveness, participants in a conversation will most of the time choose the simplest way of coming to an understanding. However, in cases of necessarily higher cooperativeness or of greater tolerance span the maximal consensus model may be applied approximately. According to my own observations, the use of evaluative expressions like "beautiful" is accompanied with greater tolerance; in communication on art for instance, it has not so many negative consequences if a communicator is temporarily taking over the positive judgement of another participant's for some work of art although it possibly lies in his own vagueness domain of "beautiful".

By the way, I do not claim that in reality only the three suggested models are (approximately) applied. As may be expected, we have also to account for the case where on participant will (partially) take over the total meaning of another participant although his own initial meaning contradicts it. Such cases can be observed in situations of language learning or, more generally, in situations with asymmetrical dominance relations among the participants. However postulating axiomatically an interpretation assimilation according to this model is not necessary.

Which mathematical structure is defined by 2.1 on F(C,P) for an n-ary predicate constant P^2? Before answering this question I want to recall the definitions of some lattice-theoretical notions (see also, e.g., Gierz et al. 1980).

2.2 DEFINITION:

Consider a set X equipped with a transitive relation $\leq$. $a \in X$ is a *lower (upper) bound* of $Y \subset X$ iff $a \leq y$ ($y \leq a$) for all $y \in Y$. If the set of lower (upper) bounds of Y has a unique greatest (smallest) element, we call it the inf (sup) of Y (for *infimum* resp. *supremum*).

2.3 DEFINITION:

Let $\leq$ be a partially ordering on X (i.e. $\leq$ is a reflexive, transitive and antisymmetric relation). $\langle X, \leq \rangle$ is a *semilattice* iff every nonempty finite subset of X has an inf. $\langle X, \leq \rangle$ is a *lattice* iff X is a semilattice and every nonempty finite subset of X has a sup. $\langle X, \leq \rangle$ is a *complete lattice* iff every subset of X has an inf and a sup.

With respect to $\subset$, F(C,P) is a semilattice with partially existing sups for nonempty subsets. A subset G of F(C,P) is said to be *consistent*, if DIFF(l,m) = 0 for all l, m $\in$ G. Accordingly, each maximal consistent subset of F(C,P) is a lattice. Hence, F(C,P) can more exactly be described as a semilattice which is the union of those lattices which are established by maximal consistent subsets of F(C,P). In the set of partially defined functions from X^n to 2 (in the following designated by $[X^n \to 2]$) F(C,P) has a *minimum (bottom) element*, namely $\cap$ F(C,P) and it has *maximal (top) elements*, namely $\cup$ G, for every maximal consistent subset G of F(C,P). I want to call a semilattice complete, if each maximal sublattice is complete in the usual sense. Hence, the bottom element and the top elements belong to F(C,P), if F(C,P) is complete.

A further interesting task of mathematical description is to characterize exactly the potential for making precise a meaning in F(C,P). For this task we need some further definitions (cf., e.g., Császár 1978).

2.4 DEFINITION:

A *topology* on a set X is a set $\overline{T}$ of ('*open*') subsets of X such that

(i) X and the empty set belong to $\overline{T}$,

(ii) any union of sets belonging to $\overline{T}$ also belongs to $\overline{T}$,

(iii) any intersection of finitely many sets belonging to $\overline{T}$ also belongs to $\overline{T}$.

A subset N of X is called a *neighborhood* of x $\in$ X, if x $\in$ N $\supset$ T for some T $\in$ $\overline{T}$.

2.5 DEFINITION:

A *filter base* on a set X is a nonempty set $\overline{B}$ of subsets of X such that

(i) the empty set does not belong to $\overline{B}$,

(ii) any intersection of two sets belonging to $\overline{B}$ contains a set of $\overline{B}$.

2.6 DEFINITION:

Let $\overline{B}$ be a filter base and $\overline{T}$ a topology on X. $\overline{B}$ *converges* to

$x \in X$ iff every neighborhood of x contains a set belonging to $\bar{B}$.

2.7 DEFINITION:

For $m \in F(C,P)$ and $G \subset F(C,P)$, with $m \in G$, we set

$$\uparrow m := \{l \in F(C,P): m \subset l\}$$

$$\uparrow_G m := \{\uparrow l: l \in G \text{ and } m \subset l\}$$

$$\downarrow m := \{l \in F(C,P) : l \subset m\}$$

$$\Downarrow m := \{\downarrow l: l \in F(C,P) \text{ and } l \subset m\}$$

$\uparrow m$ is the set of extensions of m ('*upper meanings*') and $\downarrow m$ is the set of reductions of m ('*lower meanings*'). $\Downarrow m$ is a filter base ('the *lower filter base*'); the same applies to $\uparrow_G m$ for each maximal consistent subset G. If $F(C,P)$ is complete, then $\cap F(C,P)$ belongs to $\downarrow m$ and $\cup G$ belongs to $\uparrow m$ for maximal consistent $G \subset F(C,P)$ such that $m \in G$. That the bottom element and some of the top elements are possibly accessible from m can also be expressed topologically.

2.8 THEOREM:

Let the n-ary predicate constant P be vague in the context C and suppose that the bottom element and the top elements of $F(C,P)$ belong to $F(C,P)$. If m is a meaning of P in C, then $\Downarrow m$ converges to $\cap F(C,P)$ and, for any maximal consistent subset $G \subset F(C,P)$, $\uparrow_G m$ converges to $\cup G$ with respect to each toplogy on $F(C,P)$.

At this stage however, the advantage of a topological view on vagueness is not yet very high. That advantage will be more evident if we consider topologies (resp. uniformities) on the universe in the next sections.

3. THE INTRODUCTION OF RICHER STRUCTURE CONCEPTS

The approaches for modelling vagueness mentioned above, namely the supervaluation logic and three-valued logic, suggest minimal solutions for some problems of vagueness, but they cannot handle all the crucial phenomena in this field. For example, within their frameworks the following fact cannot be explained. Sometimes a communicator might not be able to decide whether two objects x and y should be called "chairs"; however, if another participant claims: "y *is* a chair!", then in order to judge x at least as typical as a chair as y, the first participant may answer: "Okay, then x is a chair too.". The problem pointed out by

this example again shows that a satisfactory treatment of vagueness must take into account the interaction leading to negotiating interpretation assimilations. The respective interactions can be reconstructed for our example in a generalized form as follows. At the beginning the first participant chose an interpretation f with the property that f(P) is not defined for x and y. After the claim of the other participant, he wants to extend f(P) later on such that y gets the value 1. In order to find a suitable extension he will follow an upward path in ↑f(P) (cf. Definition 2.7) according to the presupposed context. But before his search is successful he reaches an extension which assigns the value 1 to x; and consequently the same value is assigned to the extension searched.

However, the suggested reconstruction is empirically inadequate in so far as judging x at least as typical as a chair as y need not be the result of an additional activity of extending meanings but may be forced by the interpretation chosen for P from the beginning. This deficiency of the structure concept used so far could be amended if we associated, e.g., the pair <↓m, ↑m> with the extensional meaning of P instead of m. However, a more plausible solution is given by the fuzzy logic approach or by the concept of structures with graded predicates (cf. Definition 3.1). Hence, I will now turn to discuss the problems tied up with the fuzzy logic approach.

Among other things, the problem with the above example may lead resp. has led to the assumption that a theory of vagueness would require a logic with more than three truth-values. In principle, the degree concerning the application of a predication may be arbitrarily high. Hence, a logic with finitely many truth-values will not suffice. The step from a logic with an infinite domain of truth-values to the use of the whole interval [0,1] of real numbers is not too far, since the latter choice can be justified as a theoretical idealization. Although the fuzzy logic approach seems to be very promising at first sight, it has been severely criticized. Here I want to discuss only two objections. First, there is the argument that the so-called '*vagueness dilemma*' (cf. Blau 1978) not only remains unsolved but that it is even made worse: in contrast to the assumption that vagueness leads to an indeterminacy in assigning truth-values, the fuzzy logic approach presupposes the sharpest assignment imaginable (are you able, e.g., to tell which agegroup should be judged old to degree of 0,781?). This criticism is indeed justified. But, on the other hand, it can easily be seen that – at least in a weak sense – the vagueness dilemma will be solved if we use subintervals instead of points in [0,1] as truth-values. If, e.g., the sentence "Peter is old" gets the value [0.75,1], then it cannot unequivocally be judged as true but it can be estimated as belonging to the upper quarter of positive ratings. Suggestions concerning to this choice of truth-value domain were already been made by Grattan-Guiness (1976).

Secondly, one may object against the usual version of the fuzzy logic approach that through the meaning f(P) of an n-ary predicate constant P a total ordering on X^n is established, if f(P) is a mapping from X^n to [0,1]. But to presuppose a total ordering is empirically inadequate, since the respective n-tuples of individuals may not always be comparable. For example, two objects which are to be categorized may deviate from a chair with respect to different dimensions. Thus, it is not clear which of them is more deviant. Although this objection is correct too, I think that studying at first the idealized case of total orderings is legitimate in an initial stage of research (however, e.g. Goguen 1969 had already introduced a more general notion of a fuzzy predicate). Nevertheless, it would be interesting to know more exactly how strong this idealization theoretically is. I would like to deal with this question now. In order to accomplish this object I will first introduce a new structure concept.

3.1 DEFINITION:

$S = \langle X,f \rangle$ is a *structure with graded predicates* iff f is a function with DMf = CON such that

(i) $f(a) \in X$ for each individual constant $a \in$ CON;

(ii) for every n-ary predicate constant $P \in$ CON the following conditions are satisfied:

$f(P) = \langle f(P)_0, f(P)_1 \rangle$;

$f(P)_0$ is a function belonging to $[X^n \to 2]$;

$f(P)_1$ is a reflexive and transitive relation on a subset of X^n such that $DMf(P)_0 \subset DMf(P)_1$ and

for all $x, y \in X^n$ with $\langle x,y \rangle \in f(P)_1$:

if $f(P)_0(x) = 1$, then $f(P)_0(y) = 1$;

if $f(P)_0(y) = 0$, then $f(P)_0(x) = 0$.

Instead of $\langle x,y \rangle \in f(P)_1$ I shall also write $x \overset{\leq}{P} y$.

In condition (ii) it is not demanded that $f(P)_1$ is an antisymmetric relation; hence, by $f(P)_1$ not a partial ordering but a quasi-ordering is given. According to the considerations in section 2 (ii) does not presuppose that $f(P)_0$ is a total predicate on X^n.

To each structure $S = \langle X,f \rangle$ in the sense of section 2 a structure $S = \langle X,F' \rangle$ with graded predicates corresponds in a canonical way. In order to show this we have only to define:

d_P is a partially defined metric on $DMf(P)_1$. By means of d_P we define the distance between arbitrary objects and the positive resp. negative cases of P for i = 0,1:

$$d_{P,i}(x) := \inf\{d_P(x,y) : f(P)_0(y) = i\};$$

$d_{P,i}(x) := 0$ if there is no y such that $f(P)_0(y) = i$ and $\langle x,y\rangle \in DM\ d_P$.

Finally, the intended extension $f'(P)_0$ is given by:

$$f'(P)_0(x) := \begin{cases} f(P)_0(x) & \text{if } x \in DM\ f(P)_0 \\ \frac{1}{2}(1 + d_{P,0}(x) - d_{P,1}(x)) & \text{otherwise} \end{cases}$$

Case 2 The way of defining an extension according to case 1 can be taken over in a slightly modified version, if the length of $\overset{<}{P}$-chains is at least finite. Here we define:

$d_P(x,y) := 1 - \frac{1}{n}$ iff n is the minimal length of maximal $\overset{<}{P}$-chains connecting x and y.

Case 3 The way of defining an extension will be more complicated, if there are infinite chains. For reasons of theoretical completeness I would like to sketch the essential steps of a suitable procedure.

It may be supposed that a well-ordering $\{Y_\alpha : \alpha<\beta\}$ for an ordinal β can be defined on the set of maximal $\overset{<}{P}$-chains. For reasons of simplicity, furthermore, we assume that $\overset{<}{P}$ is complete in $DMf(P)_1$.

Now we define recursively for each natural number n a partition of $Y_\alpha - DMf(P)_0$ into 2^n disjoint subchains $Y^0_{\alpha,n},\ldots,Y^{2n-1}_{\alpha,n}$:

$$Y^0_{\alpha,0} := Y_\alpha - DMf(P)_0 \text{ for all } \alpha<\beta$$

Let already $Y^m_{\alpha,n}$ be defined for all $\alpha<\beta$, $m<2^{n-1}$ and $Y^m_{\alpha,n+1}$ for all $\alpha<\gamma<\beta$, $m<2^{n+1}-1$ and $Y^m_{\gamma,n+1}$ for all $m<2^l-1$ (with $l<n$). Then we set

$$Y^{2^{l-1}}_{\gamma,n+1} := \{y \in Y^l_{\gamma,n} : y \overset{<}{P} z\}$$

$$Y^{2\,1}_{\gamma,n+1} := \{y \in Y^{1}_{\gamma,n} : y \geq_P z\}.$$

Here $z \in Y^{1}_{\gamma,n} \cup \{\inf Y^{1}_{\gamma,n}\}$ is chosen as follows:

(i) If $y \leq_P \inf Y^{2\,1}_{\alpha,n+1}$ and $y \not\leq_P \inf Y^{2\,1}_{\alpha,n+1}$ for all $y \in Y^{1}_{\gamma,n} - \{\inf Y^{1}_{\gamma,n}\}$, $\alpha<\gamma$ then we choose $z \in Y^{1}_{\gamma,n} - \{\inf Y^{1}_{\gamma,n}\}$ arbitrarily[4] in the case of $Y^{1}_{\gamma,n} \neq \{\inf Y^{1}_{\gamma,n}\}$ and we set $z := \inf Y^{1}_{\gamma,n}$ otherwise.

(ii) If the presupposition of (i) is not satisfied, we set

$$Z := \{y \in (Y^{1}_{\gamma,n} - \inf Y^{1}_{\gamma,n}\}) : \text{there is } \alpha<\gamma \text{ with } y \leq_P \inf Y^{2\,1}_{\alpha,n+1}\} \text{ and}$$

$$z := \begin{cases} \sup Z & \text{if } Z \neq 0 \\ \inf Y^{1}_{\gamma,n} & \text{otherwise} \end{cases}$$

Similar to this procedure we can define for each n a partition of the open interval (0,1) into 2^n disjoint subintervals $I^0_n,\ldots,I^{2n-1}_n$:

$$I^0_n := (0,\tfrac{1}{2^n}),\ I^m_n := [\tfrac{m}{2^n},\ \tfrac{m+1}{2^n}) \text{ for } 0<m<2^n$$

An infinite sequence j of natural numbers ($j \in \omega^\omega$) defines a sequence of nested set intervals for $Y^0_{\alpha,o}$ resp. I^0_o iff $j(n+1) = 2^{j(n)}-1$ or $= 2^{j(n)}$ for every n. Finally, we obtain the intended extension $f'(P)_o$ as follows: if $y \in DMf(P)_1 - DMf(P)_o$, then there are α,j such that $y \in \bigcap \{Y^{j(n)}_{\alpha,n} : n \in \omega\}$. Now we define:

$$f'(P)_o(y) := \bigcap \{I^{j(n)}_n : n \in \omega\}$$

It can be shown that this assignment is independent of the choice of α.

$$\langle x,y\rangle \in f'(P)_1 \text{ iff } f(P)(x) \leq f(P)(y)$$
$$\text{or } (f(P)(x) = 0 \text{ and } y \notin DMf(P))$$
$$\text{or } (f(P)(y) = 1 \text{ and } x \notin DMf(P)).$$

Conversely, each structure with graded predicates determines a structure (in the sense of section 2) of course.

The new structure concept yields a new logical rule, if we define a language extension by associating a new 2n-ary predicate constant $P^{\leq}$ (the '*comparative*') with each n-ary predicate constant P. Furthermore, among other things, the following semantic rules have to be established:

(1) $f(P^{\leq})_0(x_0,\ldots,x_{2n-1}) = 0$ iff $\langle x_0,\ldots,x_{n-1}\rangle \in DMf(P)_1$, $\langle x_n,\ldots,x_{2n-1}\rangle \in DMf(P)_1$ and $\langle x_0,\ldots,x_{n-1}\rangle \overset{\not\leq}{P} \langle x_n,\ldots,x_{2n-1}\rangle$

(2) $f(P^{\leq})_0(x_0,\ldots,x_{2n-1}) = 1$ iff $\langle x_0,\ldots,x_{n-1}\rangle \overset{\leq}{P} \langle x_n,\ldots,x_{2n-1}\rangle$.

The resulting logical rule is given by:

$$\frac{Pa_0\ldots a_{n-1} \quad P^{\leq}a_0\ldots a_{2n-1}}{Pa_n\ldots a_{2n-1}}$$

We now turn to the question whether for each predicate constant the function $f(P)_0$ of a structure with graded predicates can be extended to a totally defined and monotone function on $DMf(P)_1$ in a natural way[3]. I shall distinguish three cases. At the outset we define:

$$x \overset{<}{P} y \text{ iff } x \overset{\leq}{P} y \text{ but } y \overset{\not\leq}{P} x.$$

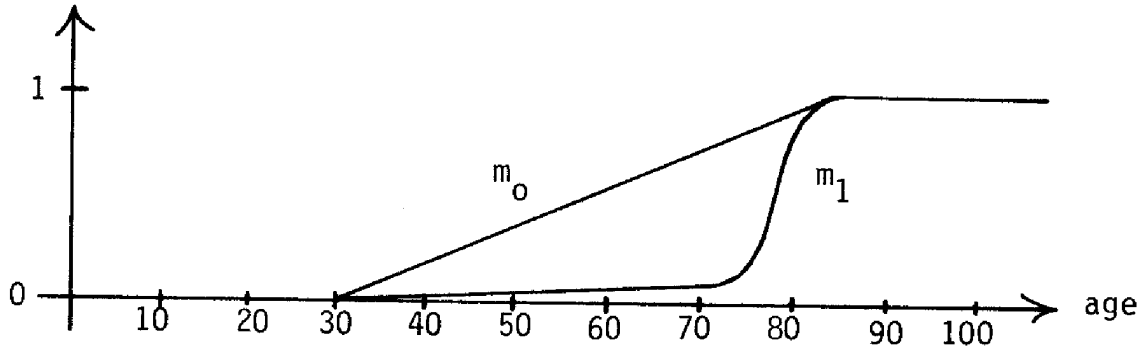
Case 1 We assume that the length of (proper) $\overset{<}{P}$-chains in $DMf(P)_1$ is limited and the least upper bound is $u>2$. This assumption is empirically justifiable, since there is only a finite number of relevant property-dimensions and for each dimension we need only a finite scale. We now introduce a partially defined distance function:

$$d_P(x,y) := \frac{n-1}{u-1} \text{ iff there is a maximal } \overset{<}{P}\text{-chain } k : n \to DMf(P)_1$$

connecting x and y, and the length of other such chains is not smaller than n (note that $n = \{0,1, \ldots, n-1\}$).

In a sense, the suggested extensions in all three cases might be empirically inadequate, since the chosen distance values don't reflect the state of affairs in reality but depend on the arbitrarily given individuals of a structure. In other words, it would be necessary to presuppose externally defined property-distances.

This point is the object of the criticism of Todt's (1980) as well; he justly objects to the fuzzy logic approach that no conditions restricting the meanings of predicate constants (i.e. the functions of $[X^n \rightarrow [0,1]]$) are stated. Without any restictions, however, we get inadequate results of this kind: a person of 70, usually judged as fairly old, might get a truth-value near 0 for certain interpretations (cf. the diagram).



The problem of the meaning m_1 is that it does not represent the agegroups true to scale. For a more adequate representation we need a richer structure concept which allows us to talk about proximity relations between individuals with respect to their properties. In my opinion, the assumption that communicators have a metric for each property is too strong. Hence, I will discuss the more general case of uniformities (for the definitions of topological standard concepts used in the following cf., e.g., Czászár 1978).

3.2 DEFINITION:

A *pseudo-metric* on a set X is a function d from X^2 to the set of nonnegative real numbers such that for all $x,y,z \in X$:

(i) $d(x,x) = 0$;

(ii) $d(x,y) = d(y,x)$ ('*symmetry*')

(iii) $d(x,z) \leq d(x,y) + d(y,z)$ ('*triangle inequality*')

In particular, by a pseudo-metric d on X a filter base (cf. Definition 2.5) on X^2 is given which consists of all ε-surroundings

$$B_\varepsilon = \{\langle x,y\rangle \in X^2 : d(x,y) < \varepsilon\} \text{ for } \varepsilon > 0.$$

Three of the properties of this filter base are used for the definition of generalized surroundings.

3.3 DEFINITION:

A filter base $\bar{B}$ on X^2 is a (symmetric) *uniform base* on X iff for all $x \in X$, $B \in \bar{B}$:

(i) $\langle x,x\rangle \in B$;

(ii) $B^{-1} = B$;

(iii) there exists $B' \in \bar{B}$ such that $B' \circ B' \subset B$.

Here B^{-1} is the *inverse* of B and $\circ$ symbolizes the operation of *composition*:

$$B^{-1} = \{\langle x,y\rangle : \langle y,x\rangle \in B\};$$

$$B \circ B' = \{\langle x,z\rangle : \text{there is a } y \text{ with } \langle x,y\rangle \in B \text{ and } \langle y,z\rangle \in B'\}.$$

A uniform base $\bar{B}$ on X is said to be a *uniformity* on X, if every subset of X^2 containing a set of $\bar{B}$ also belongs to $\bar{B}$.

By means of uniform bases we can describe characteristic topological properties of predicates. Firstly, a uniform base $\bar{B}$ on X provides a general concept for indiscernibility. Two elements x,y of X are *indiscernible* iff $\langle x,y\rangle$ belongs to $\cap \bar{B}$. It can easily be shown that $\cap \bar{B}$ is an equivalence relation. Furthermore, with $\bar{B}$ a partially defined function is given in a natural way which measures the distance between elements of X with respect to the underlying predicate.

3.4 DEFINITION:

Let $\bar{B}$ be a uniform base on X. For all $x,y \in X$ such that $x,y \in \cup \bar{B}$ we set

$$d_{\bar{B}}(x,y) := \cap \{B \in \bar{B} : \langle x,y\rangle \in B\}.$$

3.5 DEFINITION:

A uniform base $\bar{B}$ on X is called *normal*, if $\cap \bar{B} \circ B = B \circ \cap \bar{B} = B$ for all $B \in \bar{B}$. $\bar{B}$ is said to be *additive*, if $B \circ B' \in \bar{B}$ for all $B, B' \in \bar{B}$.

I state the following theorem without proof.

3.6 THEOREM:

Let $\bar{B}$ be a normal uniform base on X. For all $x,y,z,w \in X$:

(i) $d_{\bar{B}}(x,y) \supset \cap \bar{B}$, if $d_{\bar{B}}$ is defined for $\langle x,y\rangle$;

(ii) $d_{\bar{B}}(x,y) = d_{\bar{B}}(z,w)$, if $\langle x,z\rangle, \langle y,w\rangle \in \cap \bar{B}$; (*'invariance'*)

(iii) $d_{\bar{B}}(x,y) = \cap \bar{B}$ iff $\langle x,y\rangle \in \cap \bar{B}$;

(iv) $d_{\bar{B}}(x,y) = d_{\bar{B}}(y,x)$;

(v) $d_{\bar{B}}(x,z) \subset d_{\bar{B}}(x,y) \circ d_{\bar{B}}(y,z)$, if $\bar{B}$ is additive and $d_{\bar{B}}$ is defined for $\langle x,y\rangle$ and $\langle y,z\rangle$.

The theorem shows that the distance function $d_{\bar{B}}$ has properties similar to a pseudo-metric. In contrast to a pseudo-metric, the values of $d_{\bar{B}}$ are not necessarily totally ordered.

In the sequel we will assume that the uniform bases needed are normal. Hence, they show the invariance property 3.6 (ii). This assumption does not imply a loss of generality, which is shown by the following consideration. If $\bar{B}$ is not a normal uniform base on X, we can define a normal uniform base $\bar{C}$ equivalent to $\bar{B}$ (i.e. $\bar{B}$ and $\bar{C}$ generate the same uniformity):

$$\bar{C} := \{\cap \bar{B} \circ B \circ \cap \bar{B} : B \in \bar{B}\}.$$

Now we can introduce an appropriate topological structure concept.

3.7 DEFINITION:

$S = \langle X,f\rangle$ is a *structure with uniformly graded predicates* iff f is a function with $DMf = CON$ such that $f(a) \in X$ for each individual constant $a \in CON$ and that for every n-ary predicate constant $P \in CON$ the following conditions are satisfied:

(i) $f(P) = \langle f(P)_0, f(P)_1, f(P)_2\rangle$;

(ii) $f(P)_0$ and $f(P)_1$ fulfill the respective postulates 3.1 (ii) for structures with graded predicates;

(iii) $f(P)_2$ is a normal uniform base on a subset of X^n;

(iv) $\cup f(P)_2 \supset f(P)_1$ and $\cap f(P)_2 \subset f(P)_1 \cap f(P)_1^{-1}$;

(v) for all $u,v,x,y \in X^n$ with $u \overset{\leq}{p} x, y \overset{\leq}{p} v$:

(1) $d_{f(P)_2}(u,x) \subset d_{f(P)_2}(y,v)$ or $d_{f(P)_2}(u,x) \supset d_{f(P)_2}(y,v)$;

(2) if $d_{f(P)_2}(u,v)$ contains $d_{f(P)_2}(u,x)$, $d_{f(P)_2}(x,v)$, $d_{f(P)_2}(u,y)$ and $d_{f(P)_2}(y,v)$, then $x \overset{\leq}{p} y$ is equivalent to

$d_{f(P)_2}(u,x) \subset d_{f(P)_2}(u,y)$ and to $d_{f(P)_2}(y,v) \subset d_{f(P)_2}(x,v)$.

Instead of $d_{f(P)_2}$ I shall also write d_p.

Evidently, each structure with uniformly graded predicates is a structure with graded predicates too. The condition (iv) says that any two elements of X^n comparable with respect to $f(P)_1$ have a welldefined distance. Furthermore, indiscernible elements are equivalent in the sense of $f(P)_1$ as well. It is not required, however, that the converse holds; for instance, a green-blue object and a green-yellow object may have the same grade of greenness with respect to the colour of green. According to (v) the distances given by d_p must be compatible with the ordering $f(P)_1$.

The intuition behind the uniform base given by $f(P)_2$ is that it can be defined in terms of the respective bases for the *relevant conceptual dimensions* of a predicate. The assumption that we can construct a suitable uniform base out of those underlying bases is not too strong. In particular, it can be shown that there always exists the supremum of any set of uniformities; a suitable base for it consists of the elements of the underlying bases and of all finite intersections of such elements. However other bases and hence different distance functions can be defined.

Using the structure concept of 3.7 I would like to suggest a solution for the problem concerning the intended restriction of meanings in the fuzzy logic approach. This solution is given by introducing the concept of admissible fuzzy meanings.

3.8 DEFINITION:

Let $S = \langle X,f\rangle$ be a structure with uniformly, e.g. graded predicates and let P be a predicate constant. The function g from $DMf(P)_1$ to [0,1] is an *admissible fuzzy meaning* of P iff $g \upharpoonright DMf(P)_0 = {}^1 f(P)_0$ and g preserves distances within $DMf(P)_1$-$DMf(P)_0$, i.e. for all $x,y,z,w \in DMf(P)_1 - DMf(P)_0$:

$$|g(x) - g(y)| = |g(z) - g(w)|, \text{ if } d_P(x,y) = d_P(z,w).$$

Similar to the above discussion we may now ask whether $f(P)_0$ can be extended in a natural way to a totally defined monotone ^{0}function from $DMf(P)_0$ to [0,1] resp. whether such an extension is an admissible fuzzy meaning. If $f(P)_2$ is countable, we can provide an answer to the first part of this question using the wellknown topological fact that the existence of a countable uniform base implies pseudo-metrizability. However, the pseudo-metric constructed accordingly would not necessarily be empirically adequate resp. lead to an admissible fuzzy meaning. Hence, we have to consider a different type of extensions.

3.9 DEFINITION:

Let $S = \langle X,f\rangle$ be a structure with uniformly graded predicates and let P be a predicate constant. For x,y with $x \leqslant_P Y$ we set

$$\delta_P(x,y) := \cap\{d_P(z,w) : x \approx_P z \text{ and } y \approx_P w\}.$$

Here $u \approx_P v$ means that $u \leqslant_P v$ and $v \leqslant_P u$. For $x \in DMf(P)_1$ and $i = 0,1$ we set

$$\delta_{P,i}(x) := \cap\{\delta_P(x,y) : f(P)_0(y) = i \text{ and } (y \leqslant_P x \text{ or } x \leqslant_P y)\},$$

if $\delta_P(x,y) \subset \delta_P(x,z)$ or $\delta_P(x,y) \supset \delta_P(x,z)$ for all y,z with $y,z \leqslant_P x$ in the case of $i = 0$ and $x \leqslant_P y,z$ in the case of $i = 1$.

For $x \in DMf(P)_1$, $n > 0$, $i,j \leq 1$ and $m \leq n+i$ we set

$$\text{grade}_{P,n}(x) := \left[\frac{m-j}{n+i}, \frac{m}{n}\right],$$

if i,j,m are the smallest numbers such that there exist B, $\bar{B}$ satisfying $\bar{B} \subset f(P)_2$, $B = \cap \bar{B}$ and the following:

$$B^n \subset \delta_{P,0}(x) \circ \delta_{P,1}(x) \subset B^{n+i},$$

$$B^{m-j} \subset \delta_{P,0}(x) \subset B^m.$$

Here B^k is defined by: $B^0 := \cap f(P)_2$ and $B^{k+1} := B^k \circ B$.

Finally, we set for $x \in DMf(P)_1$

$$grade_P(x) := \cap\{grade_{P,n}(x): n \in \omega \text{ and } x \in DMgrade_{P,n}\}.$$

By $grade_P$ (resp. by $g(x) := \cup\, grade_P(x)$) is defined an extension of $f(P)_0$, but not necessarily an admissible fuzzy meaning. For having this property $grade_P$ must satisfy some additional conditions. Concerning this point I will not go into details and I want to give only the following hints.

First, $grade_P$ is totally defined, only if for all $x \in DMf(P)_1$ there are y,z such that $y \lesssim_P x \lesssim_P z$, $f(P)_0(y) = 0$ and $f(P)_0(z) = 1$. Furthermore, it is required that the distances between x and the respective elements of $f(P)_0^{-1}(0)$ and $f(P)_0^{-1}(1)$ are comparable.

Secondly, $grade_P$ must take intervals as values consisting of a unique real number. This condition will only be satisfied, if $f(P)_2$ is additive and sufficiently fine.

Thirdly, δ_P must have a special additivity property: if $x \lesssim_P y \lesssim_P z$, then $\delta_P(x,z) = \delta_P(x,y) \circ \delta_P(y,z)$. For this it is required that the universe X is sufficiently complete and d_P is additive in the following sense: if $x \lesssim_P y \lesssim_P z$, then there should always exist x',y',z' such that $x \approx_P x'$, $y \approx_P y'$, $z \approx_P z'$ and $d_P(x',z') = d_P(x',y') \circ d_P(y',z')$.

Finally, I want to take up an idea formulated at the beginning of the present section. The structure concepts defined hitherto are still inadequate in so far as clear cases of predication cannot exactly be distinguished from not so clear cases and, within the structure itself, no information whatever is given about the vagueness domain. However, it would be too strong an assumption to demand that the whole vagueness profile in the underlying context must be represented within the structure at hand. It will be sufficient, if that part of information is given, which reflects the possible meaning modifications provided by the associated uniform base. In other words, I suggest to extend definition 3.7 in the following way. $f(P)_0$ should now represent the *clear cases* of predication and $f(P)$ contains a further component $f(P)_3$; this component is a subset of the uniform base $f(P)_2$ and specifies the admissible deviation distance form clear cases. The *internal vagueness profile* can then be characterized by the set of deviation standards accessible from $f(P)_3$.

3.10 DEFINITION:

$S = \langle X, f \rangle$ is an *extended structure with uniformly graded predicates* iff f is a function with $DMf = CON$ such that $f(a) \in X$ for each individual constant $a \in CON$ and that for every n-ary predicate constant $P \in CON$ the following conditions are satisfied:

(i) $f(P) = \langle f(P)_0, f(P)_1, f(P)_2, f(P)_3 \rangle$;

(ii) $f(P)$ fulfills the postulates 3.7 (ii)-(v);

(iii) $f(P)_3$ is subset of $f(P)_2$;

(iv) there are no $x,y,z \in X^n$, $B, B' \in f(P)_3$ such that $f(P)_0(x) = 0$, $f(P)_0(y) = 1$, $\langle x,z \rangle \in B$ and $\langle z,y \rangle \in B'$.

By means of $f(P)_3$ we extend $f(P)_0$ to $\bar{f}(P)_0$ via:

$\bar{f}(P)_0(x) = i$ iff $f(P)_0(x) = i$ or there are $y \in DMf(P)_0$ and $B \in f(P)_3$ such that

$f(P)_0(y) = i$ and $\langle x,y \rangle \in B$.

The condition 3.10 (iv) guarantees that there are no contradictory truth-value assignments to $\bar{f}(P)_0$. This condition must also be satisfied if we consider extensions of $\bar{f}(P)_0$.

The final task of my theoretical considerations would be to bring together the definition of structures in the sense of 3.10 and the definition of vagueness in section 2. For example, one could demand that with each structure all possible reductions and extensions of $f(P)_3$ must be contained in the considered context. Since the results of executing this task are not yet explicitly required in the next section, I would like to skip that point.

4. CONCLUDING DISCUSSION

In the final section I would like to outline some of the merits of a semantic theory based on the previously introduced theoretical concepts.

According to the definition of extended structures with uniformly graded predicates we can distinguish four empirically important types of variation for the interpretation of vague expressions. First, there may be differences concerning the '*extensional standard meaning*' $f(P)_0$ determining for which arguments the given predicate is unequivocally applicable. Empirically, one can assume

that this determination is relatively stable for each communicator (with respect to standardized contexts); but, naturally, different persons may also differ in their assignments to a certain degree. According to our assumptions in section 2, however, the extensional standard meanings of different participants will normally have a nonempty common kernel; otherwise they could not come to a mutual understanding.

Secondly, we have to consider that differing grading relations are established by $f(P)_1$. On the one side, for the choice of $f(P)_1$ it is relevant where the '*centre*' (maximum) of the predicate is localized. For instance, the decision whether or not an object is judged as more green than another one depends on the presupposed prototypes of green. On the other side, the relation $f(P)_1$ may be chosen coarser or finer. For instance, the decision whether the colour of an object can be categorized as red or orange depends on such a choice: if we apply a coarse colour scale consisting only of the ground-colours, an orange object might be designated as red[5].

Here we have to consider however, that the degree of fineness of $f(P)_1$ is determined by $f(P)_2$ (cf. condition 3.7 (v)). This can more clearly be demonstrated by the following example. If we compare the height of towers it will usually suffice to give the height differences (i.e. the distance) in metres. Hence, two towers will be regarded as being of equal height, although one of them may be 10 centimetres higher.

The last example already shows that, thirdly, the uniform base $f(P)_2$ and the distance measure thereby induced may have different degrees of fineness. Furthermore, there are other ways of varying $f(P)_2$, namely by choosing bases for relevant conceptual dimensions and by composing them. This can be done in quite different ways. I will discuss this point in greater detail later on.

Among other things, the degree of fineness chosen for $f(P)_1$ and $f(P)_2$ is regulated by a '*principle of distance*': the greater the distance is, which lies between a communicator and the object perceived by him, the coarser the degree of fineness may be (e.g., an old person will usually not perceive the age-differences of juveniles exactly).

In general, there are restrictions with respect to the fineness degree of that choice depending on perceptual, social, situational etc. conditions. In a sense, it would usually be regarded as ridiculous to give the age of an adult by the exact amount of years etc., e.g. 32 years 3 weeks and 7 days; in contrast, the age of babies is given by the number of weeks. On the other hand, the choice of a uniform base may differ with respect to the relevance/dominance of the underlying meaning dimensions and meaning aspects.

For example, besides the numerical date, the adjective "old" may also refer to typical properties such as grey-hairedness, sunken eyes, forgetfulness etc.; in this sense, e.g., a scientist of 45 having overworked himself may already be called old.

Fourthly, the '*internal vagueness zone*' given by $f(P)_3$ may be chosen smaller or larger. I assume that communicators are most quickly prepared for changing their interpretations with respect to that vagueness zone and that they always choose it so to say in a provisional way. Hence, I would think that within the suggested theoretical framework the vagueness dilemma mentioned in section 3 can be regarded as solved.

I suppose that communicators actually distinguish among the four components of interpretation. Hence, the models of negotiating interpretation assimilations suggested in section 2 should be differentiated accordingly. Concerning this point I only want to give some hints: In general, the participants of a conversation will at first presuppose that they assign approximately the same standard meanings to each expression. Rarely ever, and only if they notice that they don't come to an agreement otherwise, will they explicitly investigate the adequacy of this assumption. This will frequently be carried out by choosing some crucial examples and by comparing the results of applying the underlying interpretations to them. Naturally, negotiating on different standard meanings takes most pains and may even lead to conflicts. In contrast, the determination of the other three components of interpretation can be observed more frequently and is executed more directly. The speaker often explicitly introduces relevant meaning aspects and indicates the intended degrees of fineness. Concerning distances and grading I suppose that the hearer will accept such specifications most of the time. Likewise, there are typical strategies for indicating the latitude of the internal vagueness zone. On the one hand, the speaker often demarcates this zone by an '*inclusion/exclusion procedure*' namely by making precise the intended meaning of the respective expression. On the other hand, there exist typical sayings by means of which the vagueness zone can be enlarged (e.g., "or something like that", "anyhow", "in a sense" etc.).

If we measure distances in everyday practise, we cannot describe the real states of affairs exactly but at best approximately. Hence, it should not be astonishing that there may arise some problems, if the measuring procedure applied is too coarse. One of these problems is the non-transitivity of the perceptual indiscernibility relation. For example, our perceptual ability to distinguish among colours is not very high. Thus, it may happen that we judge some objects x and y resp. y and z as indiscernible with respect to their yellow colour. Furthermore, we may then infer that z is as yellow as x. But if we compare x and z later

on, we might notice that, e.g., z is darker than x. Just like other semantic paradoxes this inconsistency is no object of discussion in everyday communication. The reason is not that communicators put up with unavoidable contradictions resulting from the use of vague expressions; I rather think that they observe the consistent use of such expressions. But in which way do they master that problem? In a situation of the kind described above they presumably choose a new, finer measure with respect to which x and y resp. y and z can no longer be regarded as indiscernible; and even if their own perceptual ability for discriminating colours is not sufficient for realizing the boundaries of such a finer measure, they will presuppose the existence of a suitable one. In other words, I assume that the participants do not confuse the indiscernibility concept with their own perceptual approximation of it. Within the theoretical framework of extended structures with uniformly graded predicates the relevant situation might be reconstructed as follows.

If we could choose for $f(P)_2$ an ideal uniform base $\bar{B}$, then, in particular, $\cap f(P)_2$ would be identical with the set $\cap \bar{B}$ of really indiscernable objects with respect to P and $\bar{f}(P)_0$ would have the nice closure property that $\bar{f}(P)_0(u) = i$ and $\langle u,v\rangle \in \cap f(P)_2$ imply $\bar{f}(P)_0(v) = i$. But in reality we are only able to approximate B by a finite or countable discrete filter base. If we use for this base a subset of $\bar{B}$, then $\cap \bar{B}$ will be approximated by a proper superset B_1 of $\cap \bar{B}$, where B_1 is the smallest set belonging to $f(P)_2$. Hence, $\cap f(P)_2$ is not transitive and has not the above closure property. Consequently, if we have $f(P)_0(x) = 1$ and $\langle x,y\rangle$, $\langle y,z\rangle \in B_1$, then $\bar{f}(P)_0(y) = 1$ holds, but not necessarily $\bar{f}(P)_0(z) = 1$. Nevertheless, using the closure property for practical reasoning can be justified as long as this does not lead to contradictions. But if we obtain inconsistent results, this will indicate that we should choose a better approximation for $\bar{B}$. Accordingly in the case of $\bar{f}(P)_0(z) = 0$ we might be inclined to introduce a new approximation $B_0 \subset B_1$ for $\cap \bar{B}$ such that $\langle x,y\rangle \notin B_0$ or $\langle y,z\rangle \notin B_0$. Then the incorrect conclusion can no longer be drawn.

I want to illustrate the suggested reconstruction by the example of the adjective "tall". For most purposes it will suffice to characterize the height of a person by an amount of centimetres. Hence, we can assume that a height difference of one half a centimetre will be neglected. With respect to persons approximately indiscernible in this sense a set B_1 can be defined as follows

$$B_1 := \left\{ \langle x,y\rangle \;:\; \begin{array}{l}\text{the height difference between} \\ \text{x and y is smaller than 0.5 cm}\end{array} \right\}$$

Since 300 cm might be regarded as an upper limit for height differences, it will suffice to construct a filter base out of the composition products of B_1 until B_1^{600} is reached

$$f(P)_2 := \{B_1^n : 1 \leq n \leq 600\}.$$

Surely, a person x of 190 cm of height must be referred to as tall ($f(P)_0(x) = 1$). If we have set $f(P)_3 = \{B_1^{30}\}$, then a second person y of 175 cm is rated as tall too ($f(P)_0(y) = 1$). However, according to the given definition for a third person z of 174,6 cm $f(P)_0(z)$ might be $\neq 1$, although y and z are regarded as indiscernible. In order to avoid this result we must either introduce a finer base (e.g. by judging only height differences of 0.1 cm as neglectable) or we must choose a smaller vagueness zone (e.g. via $f(P)_3 = \{B_1^{20}\}$).

The problem of non-transitivity of $\bigcap f(P)_2$ can be solved, however, by a different method of approximation. This solution corresponds to the procedure of measuring the difference in height by assigning an approximate value to each object first and calculate the difference among values secondly. Now I will describe this type of approximation in a suitable general form.

Assume that there are given a subset $\bar{C}$ of $\bar{B}$, $C_1 \in \bar{C}$ and a subset $\tilde{X}$ of $DM \cap \bar{B}$ such that the following conditions are satisfied:

(i) $\bar{C}$ is a filter base and $\bigcup \bar{C} = \bigcup \bar{B}$;

(ii) $C_1 \supset \bigcap \bar{B}$ and $C_1^2 \subset C$ for all $C \in \bar{C}$ with $C \neq C_1$;

(iii) $d_{\bar{B}}(x,y) \not\supseteq C_1$ for all $x,y \in \tilde{X}$;

(iv) for every $x \in DM \cap \bar{B}$ exists $y \in \tilde{X}$ such that $d_{\bar{B}}(x,y) \subset C_1$ and $d_{\bar{B}}(x,z) \supset C_1$ for all $z \in \tilde{X}$ with $z \neq y$.

Now we choose for each $x \in DM \cap \bar{B}$ a $\tilde{x}$ such that (iv) is satisfied and define a new uniform base $\bar{\tilde{C}}$ in the following way. For each $C \in \bar{C}$ we define a set $\tilde{C}$ via:

$$\langle x,y\rangle \in \tilde{C} \quad \text{iff} \quad \langle \tilde{x},\tilde{y}\rangle \in C.$$

Now we set:

$$\bar{\tilde{C}} := \{\tilde{C} : C \in \bar{C}\}.$$

It can easily be shown that, in fact, $\bar{\tilde{C}}$ is a uniform base. Identifying $f(P)_2$ with $\bar{\tilde{C}}$ we can avoid the negative consequence that

$f(P)_2$ is not a uniform base and that, in particular, $\bigcap f(P)_2$ is not ^{2}transitive. However, this approximation has the disadvantage of distorting real distances. In the example discussed above, the difference in height between a person of 174,4 cm and a person of 174,6 cm will now have the approximate value of 1 cm, while the difference in height between a person of 174,6 cm and a person of 175,4 cm is neglected.

Until now we have only compared different interpretations of one predicate constant. However, we could also want to compare the interpretations of different predicate constants. Clearly, this can be done, if, e.g., some of the meaning dimensions in the respective uniform bases are identical. For example, the depreciatory judgement on a Great Dane "he is rather a calf than a dog" is based on such a comparison. But moreover, the sample sentence "Harold is more talkative than intelligent" shows that distances measured with respect to quite different uniform bases may also be comparable. Hence, communicators must dispose of a general grading concept. This assumption has also indirectly been proved by the success of psychometric methods in social psychology where mostly scales up to 7 degrees are used. By means of the concept of structures with uniformly graded predicates it can easily be seen how a general grading concept is established. We have only to partition the ordering $f(P)_1$ for each predicate constant P in a uniform way. In addition to ^{1}3.9, this can be done with the following definition.

4.1 DEFINITION:

Let $S = \langle X,f\rangle$ be an extended structure with uniformly graded predicates and let P be a predicate constant. For $x \in DMf(P)_1$ and $i = 0,1$ we set

$$\delta^+_{P,i}(x) := \cup \{\delta_P(x,y): f(P)_0(y) = i \text{ and } (y \leqq_P x \text{ or } x \leqq_P y)\} .$$

For $x \in DMf(P)_0$ we set

$$\delta^-_P(x) := \cup\left\{\delta_P(x,y): \begin{array}{l}(f(P)_0(x) = f(P)_0(y) = 0 \text{ and } x \leqq_P y) \text{ or}\\ (f(P)_0(x) = f(P)_0(y) = 1 \text{ and } y \leqq_P x)\end{array}\right.$$

Now we define two functions $grade^*_P$ and $grade^+_P$ in an analogous way as in 3.9. For the definition of $grade^*_P$ we replace $\delta_{P,0}(x)$ by $\delta^+_{P,0}(x)$ and $\delta_{P,1}(x)$ by $\delta^+_{P,1}(x)$ in 3.9; for the definition of $grade^+_P$ we replace $\delta_{P,0}(x)$ by $\delta^-_P(x)$ and $\delta_{P,1}(x)$ by $\delta^+_{P,i}(x)$ for $f(P)_0(x) = i$.

By means of 4.1 we can specify interpretations for some of the usual grading expressions. An object x is said to be *neutral* (with respect to P), if $\text{grade}_P(x) = \left[\frac{1}{2}, \frac{1}{2}\right]$; x is called *nearly* P (non P), if $x \notin \text{DMf}(P)_0$ and $\text{grade}_P(x) \geq \left[\frac{n-1}{n}, \frac{n-1}{n}\right]$ (resp. $\leq \left[\frac{1}{n}, \frac{1}{n}\right]$) for any chosen $n > 1$. Here $\leq$ is defined by: $[r,s] \leq [r',s']$ iff $r \leq r'$ and $s \leq s'$. x is called *very* P (non P), if $\text{grade}_P^+(x) \geq \left[\frac{n-1}{n}, \frac{n-1}{n}\right]$ for any chosen $n > 1$ and $f(P)_0(x) = 1$ (resp. $f(P)_0(x) = 0$). x is said to be *more* P *than* Q, if $\text{grade}_P^*(x) \geq \text{grade}_Q^*(x)$. In a similar way we could continue the definition procedure. But notice that I do not claim that communicators interpret the expressions "neutral", "nearly", "very", "more" in exactly this manner. I rather wanted to show how such interpretations can be defined in principle. Moreover, I assume that these and similar expressions can be interpreted differently, in particular, that they are vague above all (as far as the interpretation of "nearly" and "very" is concerned vagueness phenomena are partially accounted for by the choice of n).

An important and hitherto not sufficiently solved problem of semantics is it to find a clear demarcating like for ambiguity and vagueness. To say that an expression is ambiguous, if it has more than one meaning would not be reasonable, since every expression is ambiguous in this sense. Hence, similarly to the explication of vagueness we have to introduce a context dependent notion of ambiguity. However, it's not adequate either to call an expression ambiguous with respect to a given context C, if it has more than one meaning in C; for this condition is satisfied even for vague expressions. In order to find a clear separation one has to postulate additionally the existence of meanings which are sufficiently disjoint. A suitable condition expressing this for a predicate constant P would be:

(i) There are structures $S = \langle X,f\rangle$ and $S' = \langle X,f'\rangle$ in C such that

$$f(P)_0^{-1}(1) \cap f'(P)_0^{-1}(1) = 0$$

However, this condition does not exclude that the positive domain of the first reading of P (i.e. $f(P)_0^{-1}(1)$) is contained in the vagueness domain of the second reading or conversely. As for me I would prefer to postulate a stronger separability property like this:

(i*) There are structures $S = \langle X,f\rangle$ and $S' = \langle X,f'\rangle$ in C such that for all structures $T = \langle X,g\rangle$ and $T' = \langle X,g'\rangle$ in C:
if $f(P)_0 \subset g(P)_0$ and $f'(P)_0 \subset g'(P)_0$, then

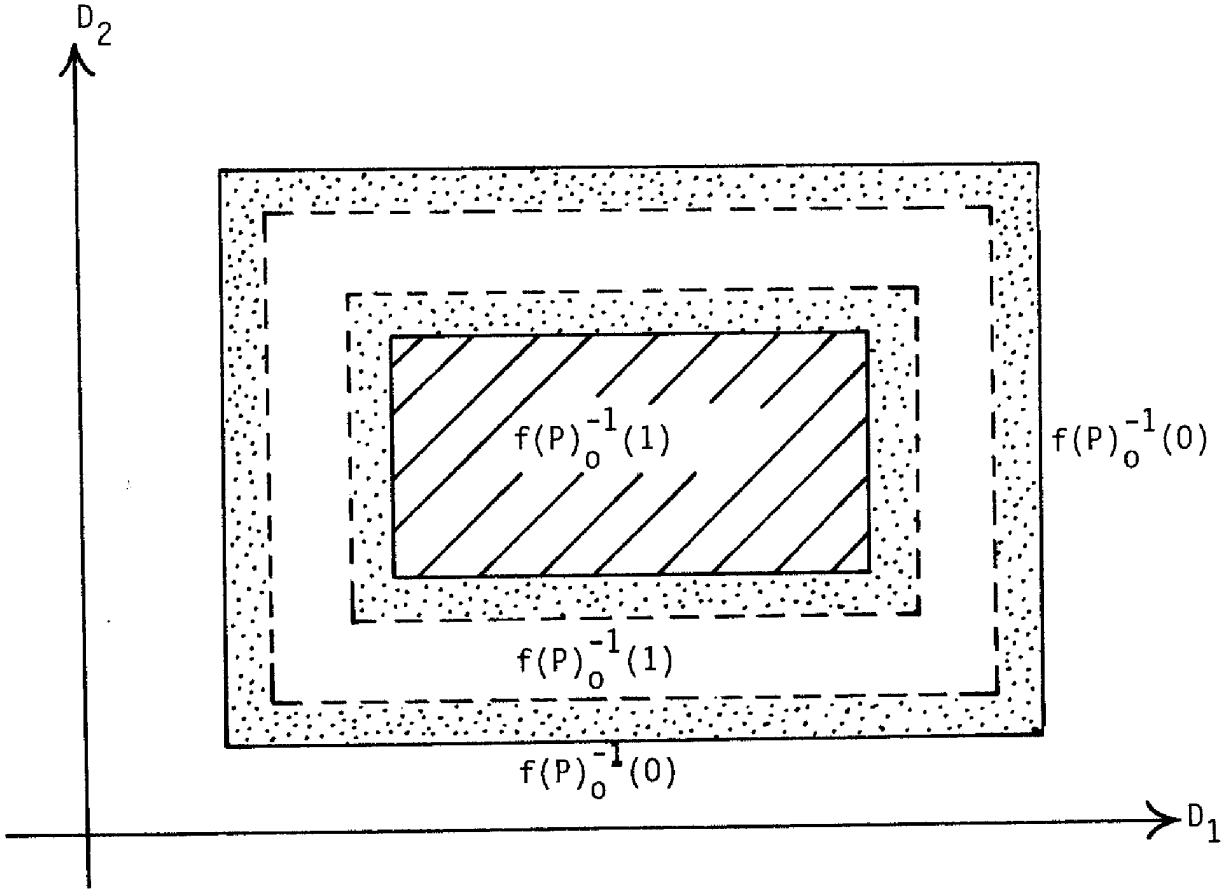
$$\bar{g}(P)_0^{-1}(1) \cap \bar{g}'(P)_0^{-1}(1) = 0.$$

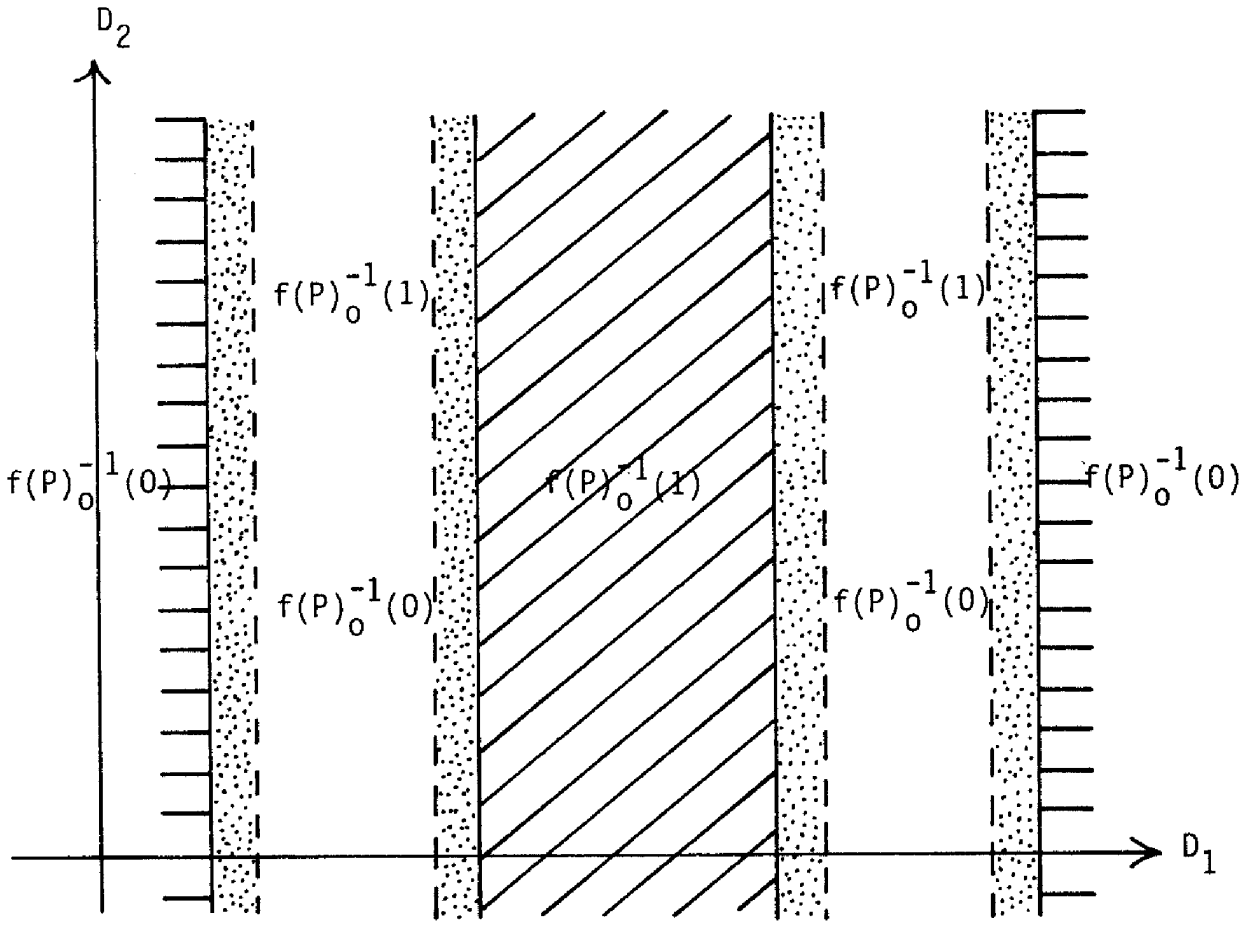
The suggested definition for a context dependent notion of ambiguity does not yet seem to explicate the original intuition which amounts to saying, e.g., that a word like "bank" is ambiguous but "banjo" isn't. I think ambiguity in this absolute sense can be identified with ambiguity with respect to a special context which I would call the 'empty context'; to explain what is meant by that notion would however require the introduction of a more adequate concept of context. In my opinion there is an additional, hitherto unnoticed condition which should enter the definition of ambiguity. For I suppose that, in general, the different, well-separable readings of an expression may have different degrees of preference. Hence, one should refer in condition (i) resp. (i*) only to interpretations dominant in the given context. Postulating this would presuppose, however, to equip the context with a suitable preference ordering.

Although it is necessary to draw a clear line of demarcation between ambiguity and vagueness, it should not be overlooked that there is an important link between these concepts. In traditional word semantics this link is indirectly taken up by the somewhat problematical distinction between homonymy and polysemy. Roughly speaking, the crucial point of this distinction is that the existence of different readings of a word can sometimes be explained as being the result of a historical process of meaning change. I want to conclude my paper by discussing this point and illustrating that vagueness must be regarded as an essential precondition for the whole range of dynamic and creative processes in meaning constitution.

In this section I mentioned the four types of interpretation variation. Whereas the first component $f(P)_0$ may be conceived of as being relatively stable, the three other components may be the subject of negotiation. Studying processes of meaning constitution empirically one will soon discover that, at one's own surprise, new meanings can very easily be created by suitable context operations. Of course, these meanings are only short-lived in general; but under special historical conditions they can be socially stabilized (in the contemporary situation, e.g., the German word "Hausbesetzung" ("house-occupation") gets a qualitatively new meaning-component "Instandbesetzung" ("occupation for repairing houses"). Hence, the potential for meaning changes can be studied independently of whether such changes lead to socially stabilized meanings or only to temporary and instable ones. But in which way is it possible to create new meanings? Modifying the internal vagueness zone in a given interpretation does not lead to essential meaning changes. However, this may happen if we modify the standard extension and the underlying uniform base such that some meaning dimensions will be diminished/eliminated and some assumptions about characteristic properties in such dimensions changed/deleted.

I want to illustrate the ensuing type of meaning modification processes by an example. In German the noun "Schlafmütze" has besides its original meaning ("night cap") also the meaning of "sleepy head". Probably the second reading is derived from the first by a meaning shift. Let us imagine a situation where a person A pointing at a person B says to a person C: "Diese Schlafmütze" ("this night cap"). We will assume that C does not know the shifted meaning and B is not wearing a night cap. How might C proceed for creating a suitable interpretation of the utterance? Since there is no evident referent for "diese Schlafmütze" with respect to the standard meaning of "Schlafmütze", C might be trying to lower or even to nullify the relevance degree of some dimensions being associated with that standard meaning. For which dimensions modifications should be executed will depend on the result of a situative evaluation. In our example C might already assume that B is the referent meant and thus C starts with diminishing the relevance of dimensions concerning colour, form etc. Because of the closure-property in the last condition of 3.1 (ii) the original extensional standard meaning might thereby be strongly modified. This holds true especially for elimination processes, as the following diagrams illustrate.





In the first diagram the dimensions D_1 and D_2 are relevant for the interpretation of P; in the second one the characteristic property in D_1 is weakened and D_2 is deleted.

This yields an enormous expansion of the positive part $f(P)_o^{-1}(1)$ of the extensional standard meaning, whereas the negative part $f(P)_o^{-1}(0)$ will be reduced. Notice that such an elimination process goes beyond meaning modifications exhausting the vagueness domain; but it can be regarded as a borderline-case of such modifications. As dimensions maintained in our example we might consider, e.g. function and time of wearing, associated type of users etc. The obtained modification, however, will not yet suffice for creating a suitable meaning and, in particular, for identifying the referent meant of "diese Schlafmütze". I assume that, in addition, new dimensions and new characteristic properties in some dimensions will be introduced and that we dispose of standard strategies for that task. In our example, e.g. a transition from the article of clothing to the wearer (a similar standard case is the transition from the ownee to the owner) will be executed.

Furthermore an acoustic dimension might be introduced which is relevant for categorizing frequently yawning persons as sleepy heads. Evidently, the introduction of new dimensions and new characteristic properties results in a reduction of the extensional meaning again. The meaning created by the modification process will be vague as well of course and it may itself be the starting point of changing processes, if it has become socially stable. As far as our example "Schlafmütze" is concerned, we have to take into account different versions of derived readings. Presumably a first version of the kind "lie-abed, sleeping much" can be regarded as the basic one (judged by the intuitive order of derivability stages); besides this there are more metaphoric versions such as "boring", "inattentive" or something like that. In the sense of my vagueness definition the latter versions are not obtainable from the basic version only be exhausting its vagueness domain. But in general, without serious empirical investigations we can hardly determine the exact limit between vagueness and the creation of new meanings. Perhaps, the notion of vagueness should also be extended in order to cover all the phenomena of dynamic processes of meaning-modification.

In my opinion the outlined scheme for modelling meaning-modification processes defines a frame within which the research of empirical semantics should be intensified in the future.I want to point out two important tasks finally. First, we have to investigate much more systematically which meaning-modification strategies communicators apply. For this task we must discuss among other things, toplogical facts to a considerable amount. For on the one hand the above sketched procedures of varying dimensions must be topologically described; a particular, very interesting problem in this area is the fact (overlooked up to now) that the property of topological connectedness plays an important role for the definition and selection of extensional meanings. On the other hand, as I could only vaguely indicate in section 2, I assume that the change of extensional meanings can be modelled as a continuous process. In other words, for describing such processes the topological concepts of convergence and continuity can be applied.

Secondly, it would be necessary to intensify the empirical research on socially valid standards fixing the extent to which newly created meanings may deviate from standard meanings. Of course such standards are not absolutely valid but depend on situational parameters like, e.g., the communicative competence presupposed. For instance, if adults converse with children, they are easily prepared to accept strongly deviant meanings. Another important parameter is given by the situative accessibility of non-standardized meanings. The degree of accessibility can be raised, e.g. by additional communication on the level of nonverbal

behaviour. Theoretically however, the concept of accessibility should as well be explicated by topological means since it is based on the notion of proximity too.

FOOTNOTES

*I want to express my thanks to my friend Hannes Rieser for many discussions and helping with the English. For helpful comments I am also indebted to Hans-Jürgen Eikmeyer and Manfred Pinkal.

[1]One of the merits of Eikmeyer/Rieser 1978 was it to discuss the links between vagueness and precisification in a way more specific than usual. Both concepts are made dependent upon context changes and speaker groups.

[2]The reader who is not interested in details of mathematical modelling may go on to section 3.

[3]The reader who is not interested in mathematical details may skip over the following discussion.

[4]If it is possible one should choose any z in the middle of $Y^1_{\gamma,n}$, i.e. the resulting two parts of $Y^1_{\gamma,n}$ (left to z and right to z) should be equipollent.

[5]Moreover, this example shows that the choice of the grading relation is often correlated with the choice of a correspondingly fine screen of verbal categories.

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