Nuclear Physics B220[FS8] (1983) 223-236
© North-Holland Publishing Company

UNIVERSALITY IN FINITE TEMPERATURE LATTICE QCD

R.V. GAVAI*, F. KARSCH** and H. SATZ

Fakultät für Physik, Universität Bielefeld, Germany

Received 26 October 1982
(Revised 19 January 1983)

In the lattice regularization of QCD, physical results must be independent of the choice of lattice action. Finite temperature thermodynamics provides a very sensitive test of this universality, providing a functional comparison between the predictions of different actions. We study thermodynamics using Wilson's, Manton's and Villain's actions as well as the mixed fundamental-adjoint form of Bhanot and Creutz. Our results support universality in all cases, but indicate that in general the region of couplings in present lattice calculations requires the inclusion of higher order effects in the perturbative solution of the renormalization group equation.

1. Introduction

The lattice regularization of QCD is a very fruitful method both for confinement studies [1] and for finite temperature thermodynamics [2]. The same classical continuum theory is, however, obtained from a wide class of lattice actions, leading to a universality requirement for lattice evaluations: the resulting physical quantities must be independent of the choice of action. Thus e.g. the relation between mass gap and string tension, or between deconfinement temperature and string tension, must become the same in the continuum limit of sufficiently small bare coupling g^2, whatever action is used in the lattice formulation.

Numerical calculations in lattice QCD are, however, performed for finite g^2; in the scaling region of the theory, such calculations are expected to give us the correct continuum limit. Hence we must verify if universality holds within the scaling region of specific lattice formulations.

At $T=0$, the string tension σ has been calculated in the scaling region for different actions [3-6] x, each yielding σ/Λ_L^2, where Λ_L is the lattice scale parameter. Different actions lead to different values of σ/Λ_L^2, so that $\Lambda_L=\Lambda_L^x$ must depend on x. The ratios of Λ_L^x values for different x have been calculated in the weak coupling limit $g^2\to 0$ [7]. Comparing the ratios from the numerical evaluation with

* Alexander von Humboldt fellow.
** Present address: CERN, Geneva, Switzerland.

the weak coupling predictions can give us some indications about the validity of universality. It is not an unambiguous test, however, as the scaling region is generally larger than the region of validity of the lowest order weak coupling expansion; in other words, at finite g^2, higher order terms in the g^2 expansion may become important.

At $T \neq 0$, the colour deconfinement temperature T_c can be chosen as relevant physical quantity in place of σ; Monte Carlo studies now yield T_c/Λ_L^x for different actions [8, 9], and we may compare these results to weak coupling predictions.

In either case discrepancies between numerical results and weak coupling ratios may be due to higher order corrections in g^2, and for these so far only estimates exist [10]. To circumvent this difficulty, one can compare the ratios of physical quantities, e.g. σ/T_c^2, for different actions, in order to obtain a better test of universality.

One aim of the present paper is to carry out this test, as well as to study the role of higher order terms in the weak coupling expansion in this context.

Moreover, in finite temperature QCD one obtains physical quantities as functions of temperature, and this provides us with the possibility of a much more sensitive test of universality. Calculating an observable $O(T)$ on the lattice for different actions now requires functional agreement over a whole range, rather than the coincidence of two points only. The second aim of this paper is such a functional test of universality.

Finite temperature tests appear of particular interest in view of a recent study [6], showing at $T = 0$ considerable discrepancies between Monte Carlo results and lowest order weak coupling predictions, in the case of an action consisting of a mixture of fundamental and adjoint gauge group representations [11]. Do these discrepancies persist at $T \neq 0$, and do they also lead to functional discrepancies?

The plan of this paper is the following. In sect. 2, we present our results using Wilson's, Manton's and Villain's forms of the lattice action for SU(2) Yang-Mills theory. Besides T_c and σ, we compare the deconfinement order parameter [12, 13] $\langle |L| \rangle$ and the energy density ε as functions of temperature for the different actions. In sect. 3, we consider specifically the mixed fundamental-adjoint action of ref. [6], determining T_c as well as $\langle |L| \rangle (T)$. Sect. 4 summarizes the conclusions of our work.

2. Wilson, Manton and Villain actions

In this section, we want to study the continuum limit of physical quantities calculated on euclidean space-time lattices, using the SU(2) form of the Wilson [14], Manton [15] and Villain [16] actions. These actions differ from each other at finite lattice spacing a, but lead to the same classical continuum limit for $a \to 0$. Their common feature is the dependence on a single dimensionless coupling g, which in the scaling region of the theory is related to the lattice spacing a through the

Callan-Symanzik equation

$$a\,\mathrm{d}g(a)/\mathrm{d}a = \beta(g). \tag{1}$$

Perturbation calculations yield for SU(2)

$$\beta(g) = \beta_0 g^3 + \beta_1 g^5 + \beta_2 g^7 + \mathrm{O}(g^9), \tag{2}$$

with

$$\beta_0 = 11/24\pi^2, \qquad \beta_1 = 17/96\pi^4. \tag{3}$$

The coefficient β_2 depends on the form of the action and has so far not been fully calculated for the cases considered here. Integrating eq. (1), (2) we obtain the relation

$$a\Lambda_{\mathrm{L}} = \left\{1 - \left(\frac{\beta_0\beta_2 - \beta_1^2}{2\beta_0^3}\right)g^2 + \mathrm{O}(g^4)\right\}\exp\left\{-\frac{1}{2\beta_0 g^2} - \frac{\beta_1}{2\beta_0^2}\ln(\beta_0 g^2)\right\}, \tag{4}$$

where the lattice scale parameter Λ_{L} is a regularization scheme dependent integration constant.

In Monte Carlo calculations, the scaling region of a given formulation is generally taken to be that range of couplings in which physical quantities of dimension a scale according to the leading term of eq. (4),

$$a\Lambda_{\mathrm{L}} = \exp\left\{-\frac{1}{2\beta_0 g^2} - \frac{\beta_1}{2\beta_0^2}\ln(\beta_0 g^2)\right\}. \tag{5}$$

An evaluation in this range then yields physical observables in units of Λ_{L}. By universality, these quantities have to be independent of the form of the action, and we can thus determine the ratios of the lattice scale parameters for different actions. These can then be compared with weak coupling limit predictions for $g^2 \to 0$. Before interpreting this as a test of universality, we have to make sure, however, that eq. (5) is indeed applicable to the data used. The higher order corrections terms in eq. (4) introduce (via β_2) an action-dependent correction

$$\Lambda_{\mathrm{L}} \to \Lambda_{\mathrm{L}} \Big/ \left(1 - \frac{\beta_2\beta_0 - \beta_1}{2\beta_0^3}g^2\right), \tag{6}$$

which, when observed over a small range of g^2 would appear simply as a change in the corresponding Λ_{L}.

We want to show in the following that finite temperature Monte Carlo studies provide a particularly sensitive test both of universality and of the role of higher order terms in eq. (4). We begin by defining the three actions to be considered in this section. On an isotropic lattice, i.e., one with equal lattice spacings in space and time directions, they have the common structure

$$S(U)=\sum_{(\mathrm{P})} S(U_\mathrm{P}), \tag{7}$$

where the sum is over all elementary plaquettes of the lattice; the plaquette action $S_\mathrm{P} \equiv S(U_\mathrm{P})$ is a function of the plaquette variables U_P and the dimensionless coupling g. The plaquette variable is given by

$$U_\mathrm{P} = U_{ij}U_{jk}U_{kl}U_{li}, \tag{8}$$

for a plaquette $P=(ijkl)$; the U_{ij} are SU(2) elements associated to the link joining the adjacent sites i and j. In terms of the angular variable θ_P and the Pauli matrices σ_i, U_P can be written as

$$U_\mathrm{P} = \mathbb{1}\cos\theta_\mathrm{P} + i\boldsymbol{\sigma}\cdot\boldsymbol{\eta}_\mathrm{P}\sin\theta_\mathrm{P}, \tag{9}$$

where $\boldsymbol{\eta}_\mathrm{P}$ is a three-dimensional unit vector specifying the remaining two Euler angles. Using this notation, we have

$$S_\mathrm{P}^\mathrm{W} = \frac{4}{g^2}(1-\cos\theta_\mathrm{P}), \tag{10}$$

for the Wilson action [14],

$$S_\mathrm{P}^\mathrm{M} = \frac{2}{g^2}\theta_\mathrm{P}^2, \tag{11}$$

for the Manton action [15], and

$$S_\mathrm{P}^\mathrm{V} = -\ln\left\{\frac{\sum_{l=0}^{\infty}(l+1)\sin^{-1}\theta_\mathrm{P}\sin[(l+1)\theta_\mathrm{P}]\exp[-\frac{1}{8}l(l+2)g^2]}{\sum_{l=0}^{\infty}(l+1)^2\exp[-\frac{1}{8}l(l+2)g^2]}\right\}, \tag{12}$$

for the Villain action [16].

A quantity of essential interest for the thermodynamics of Yang-Mills systems is the deconfinement temperature T_c, above which colour screening deconfines the gluonium colour singlets. The thermodynamics of the phase transition at this temperature can be studied in two ways. On the one hand, one can evaluate the expectation value of a thermal Wilson loop [12, 13],

$$\langle|L|\rangle = \mathrm{e}^{-F_\mathrm{q}/T}\begin{cases} =0 & T\leqslant T_\mathrm{c} \\ \neq 0 & T>T_\mathrm{c}\end{cases}, \tag{13}$$

which is related to the free energy F_q of a static quark placed into a gluonic system of temperature T. On the other hand, one can calculate the energy density of the gluonic system as such [2, 8], and then determine the critical temperature from the singularity in the specific heat. On finite lattices, the latter appears to provide a more precise determination; it yields [8, 9] for the three forms (10–12), respectively

$$T_c = \begin{cases} 42.8\Lambda_L^W \\ 10.5\Lambda_L^M \\ 27.3\Lambda_L^V \end{cases} . \tag{14}$$

From these results and the requirement of a universal T_c one obtains the lattice scale ratios shown in table 1.

At $T = 0$, analogous results are obtained for the string tension σ [3, 4, 5]

$$\sqrt{\sigma} = \begin{cases} (83.3 \pm 13.9)\Lambda_L^W \text{ [3, 4]} \\ (16.2 \pm 0.5)\Lambda_L^M \text{ [5]} \\ (48.5 \pm 2.6)\Lambda_L^V \text{ [5]} \end{cases} . \tag{15}$$

The corresponding Λ ratios are also shown in table 1.

Both the T_c and the $\sqrt{\sigma}$ ratios are compared in table 1 with the weak coupling limit prediction for $g^2 \to 0$ [7]. In both cases, we have order of magnitude agreement, but also clear discrepancies. To see whether these might be due to higher order terms in eq. (4), we consider the dimensionless ratio

$$R \equiv \sqrt{\sigma}/T_c , \tag{16}$$

for the three actions. From eq. (14) and (15) we obtain

$$R_W = 1.94 \pm 0.33 , \tag{17a}$$

$$R_M = 1.54 \pm 0.05 , \tag{17b}$$

$$R_V = 1.78 \pm 0.10 . \tag{17c}$$

TABLE 1

	From σ	From T_c	From theory*	From theory†
Λ_L^M/Λ_L^W	5.14 ± 0.87	4.08	3.07	3.33
Λ_L^M/Λ_L^V	2.99 ± 0.19	2.60	2.45	2.92

*With $g^2 = 0$, from ref. [7].
†With higher order corrections, from ref. [10].

Since these results agree, as also noted in ref. [9], within the quoted errors, we conclude that universality in fact holds and that the observed deviations from the weak coupling limit are indeed finite-g effects.

So far, our test of universality consisted in the comparison of numerical ratios. Finite temperature thermodynamics, as already mentioned, gives us a further and much more sensitive test: using different actions, we can compare corresponding physical quantities over an entire range of temperatures. We have, therefore, studied, for each action, both the thermal Wilson loop $\langle |L| \rangle$ and the energy density ε of the gluon system, on a $10^3 \times 3$ lattice for g^2 values leading to temperatures around T_c.

In fig. 1, we show $\langle |L| \rangle$ for the three actions as a function of T/T_c, with T_c given by eq. (14). The data points seem to fall on a universal curve over the whole temperature range considered. In some sense this observed action-independence of $\langle |L| \rangle$ is rather astonishing. For, strictly speaking, $\langle |L| \rangle$ by itself is not a physical quantity although it has been widely used [12, 13] in finite temperature studies as a deconfinement order parameter. As mentioned in eq. (13), it describes the free energy of a static quark and as such still contains the divergent self-energy term

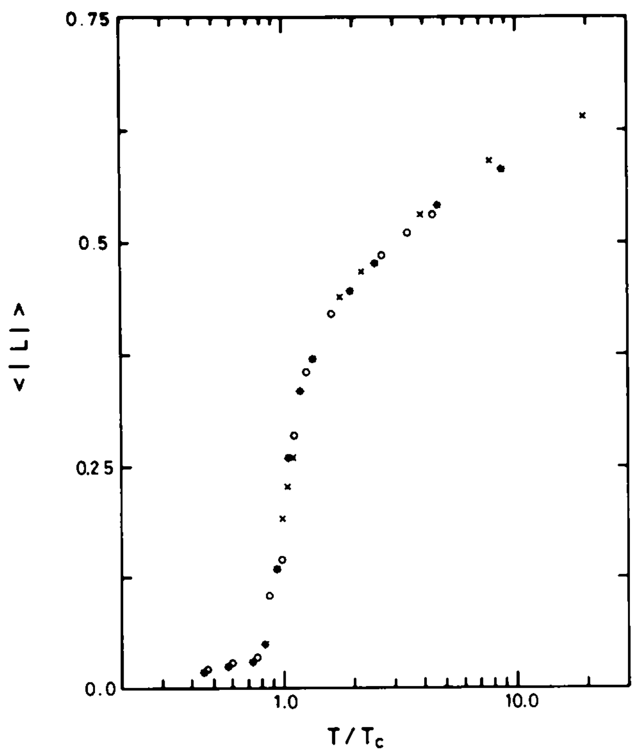

Fig. 1. Thermal Wilson loop as a function of T/T_c, calculated on a 3×10^3 lattice for Wilson action ($\times$), Manton action ($*$) and Villain action ($\bigcirc$). Here T is the temperature of the SU(2) gluon matter while T_c is the deconfinement temperature.

typical of a point charge [17]. In order to extract the true order parameter from it, and thus test universality, one has to compute this divergent contribution and subtract it from F_q. For our purpose, however, it suffices to argue that the divergent contribution is independent of action. Unfortunately, we do not have a rigorous argument to show this, but intuitively it is expected to be so. The divergence is related to the vanishing size of the point charge, and hence the divergent contribution is expected to be independent of temperature. If this be so, one can subtract [18] the free energy F_q at a fixed temperature T_0 from F_q at all T to get rid of the self-energy contribution. Thus

$$F_q(T_0) = -T_0 \ln \langle |L| \rangle|_{T_0},$$

$$F_q^{\text{phys}} = F_q - F_q(T_0),$$

so that

$$\langle |L| \rangle / \left(\langle |L| \rangle|_{T=T_0} \right)^{T_0/T} = e^{-F_q^{\text{phys}}/T},$$

is the true order parameter. Since our $\langle |L| \rangle$ data themselves fall on a universal curve, it follows that also the order parameter is universal.

However, in view of the important rôle played by $\langle |L| \rangle$ as deconfinement order parameter in finite temperature QCD, the subtraction scheme for the self-energy term should be studied in more detail; work in this direction is in progress and will be reported elsewhere [19].

In fig. 2, we show the energy density ε of the SU(2) gluon matter as a function of T/T_c. The data points shown here are obtained by subtracting space-like and time-like plaquette averages [2, 8, 9]; higher order corrections, proportional to the derivatives of g^2, are not included, since they have not yet been calculated for Manton and Villain actions. Here, too, one sees that the data fall on a universal curve for all three actions.

From the functional universality of $\langle |L| \rangle$ and ε we can conclude even more strongly that the deviation of lattice parameter ratios from their weak coupling limit must be due to higher order corrections of the form (6).

The influence of the action dependent term β_2 has been estimated [10]; for our lattice parameter ratios one finds

$$\Lambda_L^M / \Lambda_L^W = 3.07(1 + 0.047 g^2), \tag{18a}$$

$$\Lambda_L^M / \Lambda_L^V = 2.45(1 + 0.077 g^2). \tag{18b}$$

At $g^2 \approx 2$, which corresponds both to the temperature and σ values in question, this leads to the corrected values included in table 1. The corrections are clearly seen to reduce the discrepancies, as expected.

We conclude: for Wilson, Manton and Villain actions, universality is well supported by finite temperature thermodynamics. Present lattice sizes, which require g^2 to be in the "cross-over region" of $T=0$ studies, lead to small but noticeable effects from higher order terms in the scaling relation (4).

3. The fundamental adjoint action

In this section, we want to consider the action obtained by generalizing Wilson's form to include a term corresponding to the adjoint representation of the SU(N) colour group [11]. For SU(2) it is defined as

$$S_A = \beta \sum_{\langle P \rangle} (1 - \cos\theta_P) + \beta_A \sum_{\langle P \rangle} \sin^2\theta_P, \tag{19}$$

where we have used the conventional notation β, β_A for the lattice couplings; in the continuum limit

$$g^{-2} = \left(\tfrac{1}{4}\beta + \tfrac{1}{2}\beta_A\right). \tag{20}$$

This action has a considerably richer phase structure [11] than the pure Wilson action, as shown in fig. 3. It is therefore of great interest to see if universality also holds in this more general case.

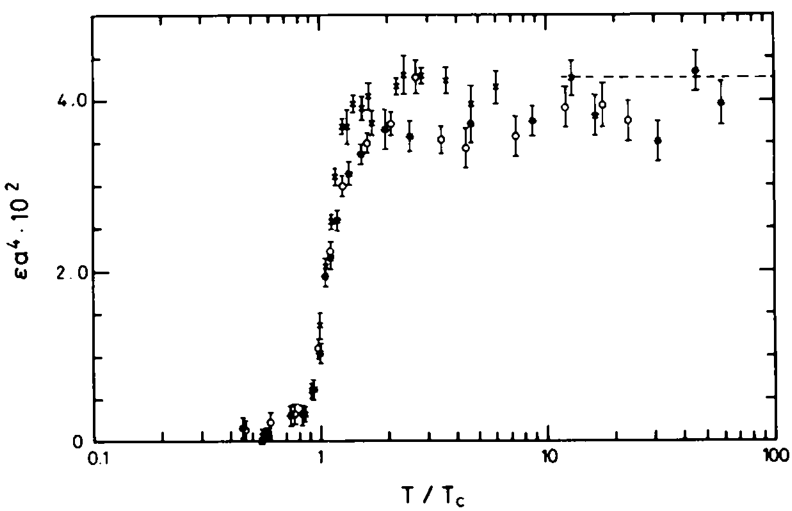

Fig. 2. The energy density of the SU(2) gluon matter versus T/T_c, calculated on a 3×10^3 lattice for Wilson action (×), Manton action (•) and Villain action (○). T, T_c are the same as in fig. 1. The dotted lines shows the Stefan-Boltzmann limit for the energy density: $\varepsilon = \frac{1}{5}\pi^2 T^4$.

Let us denote the lattice scale parameter for the action (19) by Λ_L^A; it has been studied at $T = 0$ by Bhanot and Dashen [6]. In the weak coupling limit, Λ_L^A is related to the pure Wilson action scale Λ_L^W by

$$\Lambda_L^W/\Lambda_L^A = \exp\{15\pi^2\beta_A/[22(\beta + 2\beta_A)]\}. \qquad (21)$$

In the scaling region of eq. (19) one can thus obtain a prediction for the scale parameter of the pure Wilson action

$$\Lambda_{\beta_A} \equiv \Lambda_L^A \exp\{15\pi^2/[22(2 + \beta/\beta_A)]\}, \qquad (22)$$

provided the weak coupling form (21) is applicable in the region of couplings considered. If this is the case,

$$\Lambda_{\beta_A} = \Lambda_{\beta_A = 0} \equiv \Lambda_L^W, \qquad (23)$$

should hold as consequence of universality.

In ref. [6] it is found, however, that

$$\Lambda_{\beta_A = 1.21} = (3.0 \pm 0.3) \times 10^{-3}\sqrt{\sigma}, \qquad (24)$$

which is a factor four lower than the value of Λ_L^W obtained in refs. [3,4] and shown in eq. (15). Furthermore, $\Lambda_0/\Lambda_{\beta_A}$ was seen to decrease parabolically towards unity as

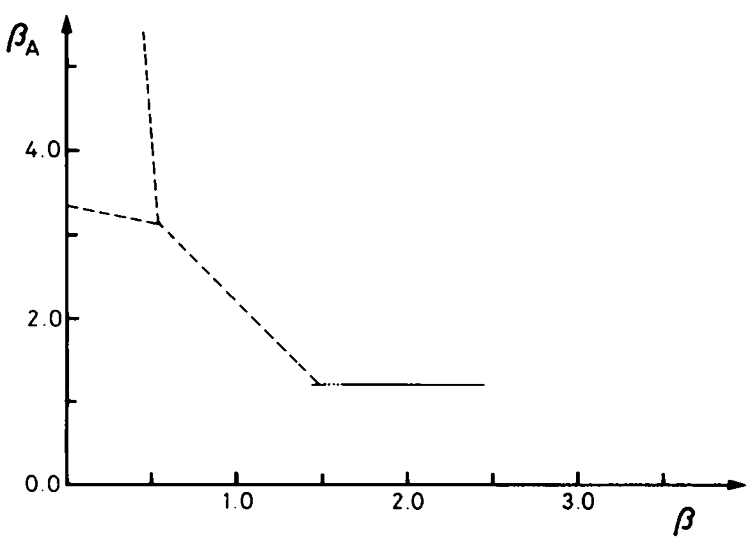

Fig. 3. The phase diagram of the action defined by eq. (19) (from ref. [11]). The dashed lines are first order transition lines. The dotted line shows the region of calculation of ref. [6], while our calculations span the region covered by both the solid and dotted lines.

$\beta \to 0$. The discrepancy shown by eq. (24) and the functional dependence of Λ_{β_A} on β_A led Bhanot and Dashen to question the validity of universality in the cross-over region of the coupling (g^2 around two), where these results were obtained.

To see if these effects persist at finite temperature, we have calculated the order parameter $\langle |L| \rangle$ as function of temperature, choosing $\beta_A = 1.21$ as in ref. [6], and varying β from 1.45 to 2.45; in ref. [6], $1.52 \leqslant \beta \leqslant 1.6$, so that the β range we consider here is quite a bit larger (see fig. 3). The physical temperature in units of $\Lambda_{\beta_A = 1.21}$ is obtained in terms of β, β_A by use of the renormalization group relation (5), eq. (20) and eq. (21).

Data were obtained for the mixed action (19) and the pure Wilson action (10) on a $12^3 \times 6$ lattice. Each data point is an average over more than 1000 iterations after attaining thermal equilibrium. The rather large lattice size was necessary to cover near T_c the β interval of ref. [6] and to reduce finite size effects. In fig. 4a, we compare $\langle |L| \rangle$ for the Wilson action with $\langle |L| \rangle$ for the mixed action, plotting both as a function of T/Λ_L^W, where we have used the universality relation (23). Except possibly at high T, the two curves disagree both pointwise and in functional behaviour. Using the empirical value (24) of ref. [6] rather than the universality limit (23) yields fig. 4b. The curves are now closer to each other in the region $T/\Lambda_L^W \simeq$ 50-100 (where β lies in the range used in ref. [6]), but the functional disagreement persists.

Finite temperature calculations thus confirm the shift of $\Lambda_{\beta_A = 1.21}$ by a factor $\frac{1}{4}$ with respect to the weak coupling value and in addition show functional differences between the two actions. What is the reason for these discrepancies?

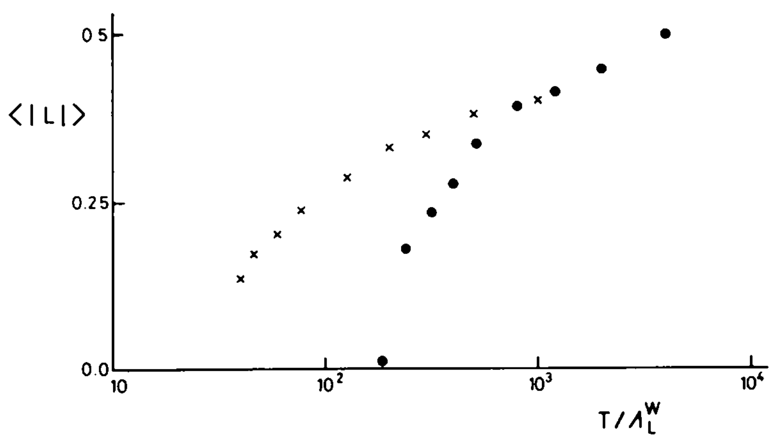

Fig. 4a. Thermal Wilson loop $\langle |L| \rangle$ as a function of T/Λ_L^W, calculated on a 6×12^3 lattice for Wilson action (×) and the mixed fundamental-adjoint action (•) at $\beta_A = 1.21$. For the latter, eq. (23) has been used to obtain T in the units of Λ_L^W.

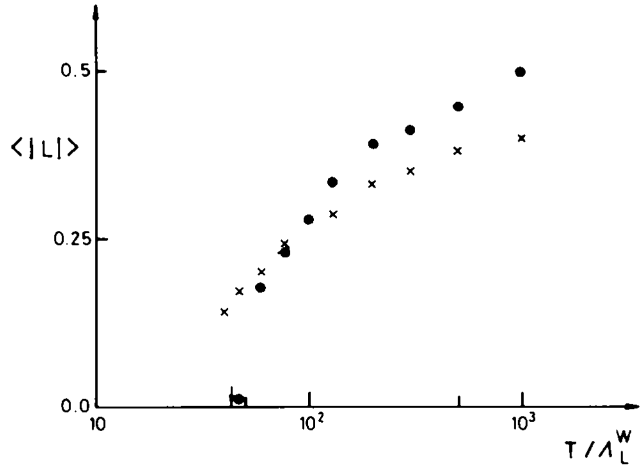

Fig. 4b. Same as fig. 4a, but using eq. (24) to obtain T in the units of $\Lambda_{\mathrm{L}}^{\mathrm{W}}$ for the mixed action.

As in sect. 2, in the comparison of lattice scale ratios at finite g^2 with their weak coupling limits, we have here in eqs. (5/20) neglected higher order corrections in g^2. For tests of universality in sect. 2, we could avoid this problem either by comparing T_c and $\sqrt{\sigma}$ directly, or by rescaling the Λ_{L} values over a restricted region of finite g^2 by a constant factor, in order to approximate a correction of form (6). In the present section, the situation is more complex, since any corrections can depend on the two variables β and β_{A}. The β_{A} dependence of $\Lambda_{\beta_{\mathrm{A}}}$ observed in ref. [6] and the functional discrepancies for the $\langle |L| \rangle$ curves at fixed β_{A}, shown in fig. 4, indeed suggest such a behaviour.

We shall here pursue two alternatives: we shall consider the effect of higher order corrections recently estimated [10] for eq. (21), and we shall study if $\Lambda_0/\Lambda_{\beta_{\mathrm{A}}}$ at fixed β_{A} with decreasing g^2 approaches unity, which it should if deviations are finite g^2 effects.

In ref. [10], the dominant next order corrections to eq. (21) are estimated to yield

$$\Lambda_0/\Lambda_{\beta_{\mathrm{A}}} = \left[1 + (0.107r + 20.188r^2)g^2\right],$$

$$r \equiv -\beta_{\mathrm{A}}/\left[2(\beta + 2\beta_{\mathrm{A}})\right]. \tag{25}$$

In ref. [6], the ratio $\Lambda_0/\Lambda_{\beta_{\mathrm{A}}}$ was calculated at a fixed value of the string tension $\sigma(\beta, \beta_{\mathrm{A}})$; we have therefore calculated this ratio in the same way, using eq. (25). The result is shown in fig. 5, together with the curve for $a^2\sigma = 0.14$ from ref. [6]. We see that the higher order corrections of eq. (25) account for all deviations at $\beta_{\mathrm{A}} < 0$ and for 50% or more of the observed discrepancies at $\beta_{\mathrm{A}} > 0$.

We note that the higher order corrections in eq. (25) are quite large, in contrast to the small modifications in eqs. (18). The deviations here are thus expected to be larger than for the actions considered in sect. 2.

To test whether the deviations of $\Lambda_0/\Lambda_{\beta_A}$ from unity at finite β_A are a consequence of finite coupling, we study the behaviour of this ratio for increasing temperature, i.e., for decreasing g^2. If our conjecture is right, we expect the ratio to approach unity with $g^2 \to 0$. As our calculations of $\langle |L| \rangle$ cover an extensive range of β values, we can carry out such a test: we calculate the ratio $\Lambda_0/\Lambda_{\beta_A=1.21}$ for which a given value of $\langle |L| \rangle$ using the mixed action coincides with the same value using the Wilson action. (For example, the points for $\langle |L| \rangle = 0.25$ coincide when the ratio $\Lambda_0/\Lambda_{\beta_A=1.21}$ is approximately four, see fig. 4.) The result, plotted as a function of $4/g^2$, is shown in fig. 6. Here $4/g^2$ corresponds to the temperature for the Wilson action; the conclusions remain unchanged, however, if we use the mixed action instead. We thus observe that Wilson and mixed actions at sufficiently large T indeed lead to the same physical results.

These two independent checks lead us to conclude that the discrepancies between the results from the mixed and the pure Wilson action are in fact also due to the neglect of higher order terms in the scaling relations (5/22) and not to a violation of universality.

In closing this section, we comment briefly on two other alternatives to account for the results of ref. [6]. It has been suggested [20], that instead of including higher order terms in g^2, one should resum the perturbation series with $1/N$ as expansion parameter. While this does seem to provide the observed decrease of $\Lambda_{\beta_A=1.21}$ in

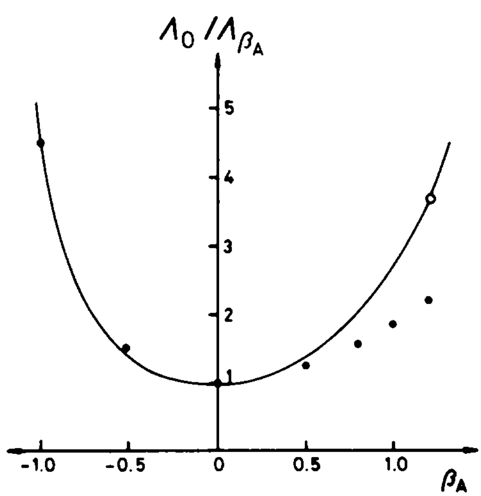

Fig. 5. $\Lambda_0/\Lambda_{\beta_A}$ as a function of β_A. The open circle and the curve show the results of ref. [6], while the full points exhibit the predictions of weak coupling expansion (eq. (25)).

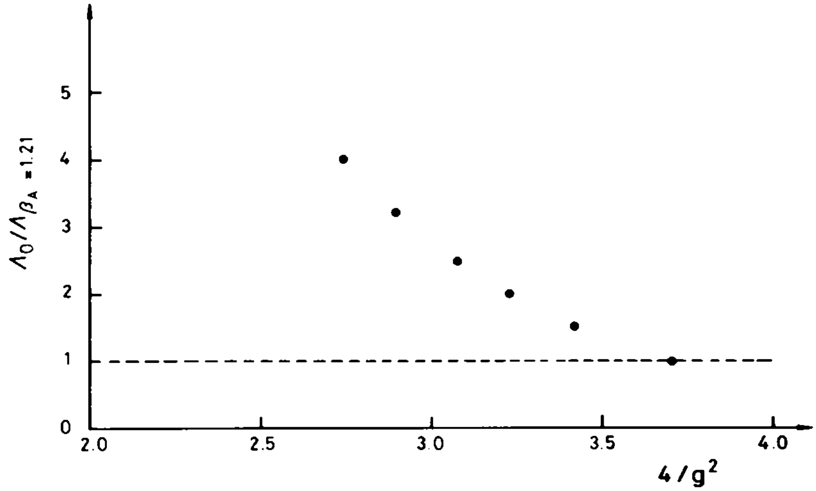

Fig. 6. $\Lambda_0/\Lambda_{\beta_A = 1.21}$ as a function of $4/g^2$. The dashed line is the prediction of the lowest order of the weak coupling expansion.

comparison to Λ_0 for $\beta \simeq 1.6$, it does not yield the difference in the functional behaviour of $\langle |L| \rangle$. In a second proposal [21], it is suggested that weak coupling results of any kind are not applicable near the end point of the transition line in fig. 3, where the calculations of ref. [6] are carried out. There instead, the Monte Carlo data are to be described in terms of a higher order strong coupling expansion. We cannot exclude this alternative; note, however, that the pure Wilson action appears to be in accord with scaling in the g^2 region considered, and the ratio $\Lambda_0/\Lambda_{\beta_A}$ as seen in fig. 6, seems to approach unity rather smoothly. This does not suggest a change of regimes from strong to weak coupling in the g^2 range considered by us.

4. Conclusions

Comparing the finite temperature thermodynamics obtained with Wilson's, Manton's and Villain's actions for the SU(2) Yang-Mills system, we have found universal behaviour, both point-wise, for $\sqrt{\sigma}/T_c$, and functionally, for $\langle |L| \rangle$ and ε in their dependence on T/T_c. In the g^2 region considered, there are deviations, however, to the validity of lowest order perturbative solutions to the renormalization group equation.

A similar situation appears to arise for the mixed fundamental-adjoint action. We reproduce at finite temperature the discrepancies with respect to the Wilson action, as observed by Bhanot and Dashen [6]; moreover, the two actions are found to yield different behaviour as function of temperature. These discrepancies are, however, removed to 50% or more already by a partial inclusion of the next order in g^2; moreover, for decreasing g^2 at fixed β_A, they are found to disappear.

We thus conclude that it is indeed meaningful to evaluate physical quantities even in the cross-over region of the coupling. The neglect of terms beyond g^5 in the perturbation solution of the renormalization group equation may, however, not be generally possible in such regions.

References

[1] See, e.g., C. Rebbi, ICTP preprint IC/81/151; Brookhaven preprint BNL 31234
[2] See, e.g., H. Satz, Phys. Reports 88 (1982) 349
[3] M. Creutz, Phys. Rev. D21 (1980) 2308; Phys. Rev. Lett. 45 (1980) 313
[4] G. Bhanot and C. Rebbi, Nucl. Phys. B180[FS2] (1981) 469
[5] C.B. Lang, C. Rebbi, P. Salomonson and B.S. Skagerstam, Phys. Lett. B101 (1981) 173
[6] G. Bhanot and R. Dashen, Phys. Lett. 113B (1982) 299
[7] A. Gonzalez-Arroyo and C.P. Korthals-Altes, Nucl. Phys. B205[FS5] (1982) 46
[8] J. Engels, F. Karsch, H. Satz and I. Montvay, Nucl. Phys. B205[FS5] (1982) 545
[9] R.V. Gavai, Nucl. Phys. B215[FS7] (1983) 458
[10] H. Sharatchandra and P. Weisz, DESY preprint, DESY 81-083 (1981)
[11] G. Bhanot and M. Creutz, Phys. Rev. D24 (1981) 3212
[12] L. McLerran and B. Svetitsky, Phys. Lett. 98B (1981) 195; Phys. Rev. D24 (1981) 450
[13] J. Kuti, J. Polónyi and K. Szlachányi, Phys. Lett. 98B (1981) 199
[14] K.G. Wilson, Phys. Rev. D10 (1974) 2445
[15] N.S. Manton, Phys. Lett. 96B (1980) 328
[16] J. Villain, J. Phys. 36 (1975) 581;
J.M. Drouffe, Phys. Rev. D18 (1978) 1174
[17] J.D. Jackson, Classical electrodynamics (Wiley, 1962) 593
[18] L. McLerran, private communication
[19] R.V. Gavai and F. Karsch, in preparation
[20] Yu.M. Makeenko and M.I. Polikarpov, Nucl. Phys. B205[FS5] (1982) 386
B. Grossmann and S. Samuel, Columbia University preprint CU-TP-230 (1981)
[21] A. Gonzalez-Arroyo, C.P. Korthals-Altes, J. Peiro and M. Perrottet, Marseille preprint CPT-82/P.1400