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Thermal Green's functions and transport coefficients on the lattice

F. Karsch
CERN, Theory Division, 1211 Geneva 23, Switzerland

H. W. Wyld
University of Illinois at Urbana—Champaign, Department of Physics, Loomis Laboratory, 1110 West Green Street, Urbana, Illinois 61801

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We discuss the formalism needed to extract transport coefficients from SU(3) gluon thermodynamics on Euclidean lattices. These can be obtained from analytic continuations of thermal correlation functions of the energy-momentum tensor. The feasibility of such a procedure is analyzed. A Monte Carlo simulation on an $8^3\times4$ lattice has been performed as a first attempt to study the behavior of thermal correlation functions of the energy-momentum tensor. The results are compared with analytic calculations of transport coefficients in the collision-time approximation.

I. INTRODUCTION

The analysis of QCD thermodynamics using nonperturbative lattice techniques up to now has mainly concentrated on equilibrium properties of the theory.[1,2] This has given important information about the existence of a phase transition to a quark-gluon plasma, the critical parameters of the transition as well as the equation of state. These properties will be studied in forthcoming heavy-ion experiments whose first generation has started at CERN.[3] In these experiments, however, hot QCD matter will not be produced in equilibrium; nonequilibrium effects will be significant.

Transport coefficients are important ingredients in the description of disspative effects in an expanding quark-gluon plasma using hydrodynamical equations of motion.[4–6] It has been suggested that within the linear-response approach transport coefficients could be calculated in the framework of lattice Monte Carlo (MC) simulations.[5,7] In general this will require the calculation of thermal correlation functions of the energy-momentum tensor, which then have to be analytically continued to real time. Besides the problem of analytic continuation, this procedure clearly is difficult due to the fact that the analytic structure of the thermal correlation functions has to be extracted from their long-distance behavior. Thus rather large lattices in temporal (imaginary time) direction will be necessary to obtain reliable results.

A less ambitious approach is to use the information obtained from Monte Carlo simulations at finite temperature to check the validity of approximate analytic calculations,[5,6] which have been performed in the collision-time approximation. Here data from lattices of moderate size may be meaningful and the data may be used as input for the analytic calculations.

The purpose of this paper is to study thermal correlation functions of the energy-momentum tensor at finite temperature. To this end we have performed a Monte Carlo simulation on a rather small $8^3\times4$ lattice for the SU(3) Yang-Mills systems. This will give first indications on the analytic structure of the thermal correlation functions and the validity of the collision-time approximation for temperatures close to the deconfinement transition temperature. The paper is organized as follows. In Sec. II we will discuss the method to calculate transport coefficients once the analytic structure of thermal Green's functions is known.[7,8] Section III is devoted to a discussion of the lattice version of the energy-momentum tensor and correlation functions suitable for the calculation of transport coefficients. In Sec. IV we discuss our MC results and their implications for transport coefficients. Finally in Sec. V we give our conclusions and discuss the prospect of performing these calculations on larger lattices.

II. KUBO FORMULAS FOR TRANSPORT COEFFICIENTS AND ANALYTIC CONTINUATION

The calculation of transport coefficients for the quark-gluon plasma has recently attracted much attention[4–6] since dissipative effects are expected to play an important role during the expansion of the plasma. In particular the rate of entropy production is directly related to shear viscosity η, bulk viscosity ζ, and thermal conductivity χ (Ref. 5). Analytic calculations have been performed to estimate these coefficients in the collision time approximation.[5,6] In general transport coefficients can be calculated, within the linear-response approach, from Kubo-type formulas as integrals over retarded Green's functions for the energy-momentum tensor.[7,8] Following the derivation of Zubarev[9] one obtains for shear viscosity η, bulk viscosity ζ, and heat conductivity χ,

$$\chi=-\frac{1}{T}\int d^3x'\int_{-\infty}^{t}dt_1 e^{\epsilon(t_1-t)}\int_{-\infty}^{t_1}dt'\langle T_{01}(x,t)T_{01}(x',t')\rangle_{\rm ret}\ , \tag{2.1a}$$

$$\eta=-\int d^3x'\int_{-\infty}^{t}dt_1 e^{\epsilon(t_1-t)}\int_{-\infty}^{t_1}dt'\langle T_{12}(x,t)T_{12}(x',t')\rangle_{\rm ret}\ , \tag{2.1b}$$

$$\tfrac{4}{3}\eta+\zeta=-\int d^3x'\int_{-\infty}^{t}dt_1 e^{\epsilon(t_1-t)}\int_{-\infty}^{t_1}dt'\langle T_{11}(x,t)T_{11}(x',t')\rangle_{\rm ret}\ . \tag{2.1c}$$

Here the limit $\epsilon\to+0$ has to be taken, and $T_{\mu\nu}$ denotes the energy-momentum tensor, which in terms of the field strength tensor $F_{\mu\nu}$ is given by

$$T_{\mu\nu}=2\,\mathrm{Tr}(F_{\mu\sigma}F_{\nu\sigma}-\tfrac{1}{4}\delta_{\mu\nu}F_{\rho\sigma}F_{\rho\sigma})\ . \tag{2.2}$$

The retarded Green's function $\langle A(x,t)B(x',t')\rangle_{\rm ret}$ is defined as

$$\langle A(x,t)B(x',t')\rangle_{\rm ret}=-i\theta(t'-t)\langle[A(x,t),B(x',t')]\rangle_0 \tag{2.3}$$

with $\langle\ldots\rangle_0$ denoting the usual thermal expectation value of the correlation function with respect to the thermal-equilibrium distribution. The correlation functions appearing in Eq. (2.1) represent correlations in real time. In a nonperturbative lattice calculation on a Euclidean lattice we obtain instead thermal Green's functions. The determination of the retarded Green's functions in Eq. (2.1) thus requires an analytic continuation of the thermal Green's functions:

$$G_\beta(x,t)=-i\langle A(x,\tau)B(0,0)\rangle_0,\quad 0<\tau<\beta=1/T\ . \tag{2.4}$$

The formalism to do this has recently been discussed by Hosoya *et al.*[8] (see also Ref. 10).

Knowing the thermal Green's function $G_\beta(x,\tau)$, we can determine the Fourier transform $G^\beta(p,\omega_n)$ at the discrete set of frequencies $\omega_n=2\pi n/\beta$ with $n=0,\pm1,\ldots$. In terms of the spectral function $\rho(p,\omega)$, G^β is given as

$$G^\beta(p,\omega)=\int d\omega\frac{\rho(p,\omega)}{i\omega_n-\omega}\ . \tag{2.5}$$

Once the spectral function ρ is known, the analytic continuation is immediate as the retarded Green's function has the same spectral representation as G^β with $i\omega_n$ replaced by $p_0+i\epsilon$.

Thus the aim of a nonperturbative MC calculation of transport coefficients should be to extract from thermal correlation functions of suitable components of the energy-momentum tensor the spectral function $\rho(p,\omega)$ and use it as the input for an analytic continuation. In practice, however, we are far from this ideal possibility. On lattices with a finite number of sites in the temporal direction the thermal correlation functions can only be determined for a finite number of points, so that even with precise data the analytic continuation is in principle impossible. The best we can do is make a reasonable ansatz for the spectral density which contains a few parameters. Using the Fourier transform of Eq. (2.5) gives then an ansatz for the thermal correlation function $G_\beta(x,\tau)$ which can be used to fit the Monte Carlo data.

It is expected and supported by existing MC data[1,2] that the SU(3) Yang-Mills system in the limit of low (high) temperatures is well approximated by an ideal glueball (gluon) gas. Thus it is presumably a good starting point to use the spectral function of a free theory and modify this to take into account the influence of weak interactions.[7] Of course, a more refined ansatz will be necessary in the transition region. The zero-momentum spectral density function for a free theory is given by

$$\rho(\omega)=A(1-e^{-\beta m})[\delta(m-\omega)-\delta(m+\omega)]\ . \tag{2.6}$$

Here the coefficients A,m determine the amplitude and exponential decay of the thermal Green's function. Using Eq. (2.5) and performing the Fourier transform one finds, for $G_\beta(\tau)$,

$$G_\beta(\tau)=-iA(e^{-m\tau}+e^{-m(\beta-\tau)}),\quad 0\le\tau\le\beta\ . \tag{2.7}$$

An obvious way to introduce the effect of interactions in the system is to replace the δ functions in Eq. (2.6) by smeared δ functions:

$$\rho(\omega)=\frac{A(1-e^{-\beta m})}{\pi}\left[\frac{\gamma}{(m-\omega)^2+\gamma^2}-\frac{\gamma}{(m+\omega)^2+\gamma^2}\right]. \tag{2.8}$$

This introduces a third parameter γ, which has to be determined from fits of the thermal correlation functions. Using Eq. (2.8) we find

$$G_{\rm ret}(t)=-2A(1-e^{-\beta m})\theta(t)e^{-\gamma t}\sin(mt) \tag{2.9}$$

but obtain $G_\beta(\tau)$ only in terms of an integral over the spectral density

$$G_\beta(\tau)=-i\int d\omega\frac{e^{\beta\omega}}{e^{\beta\omega}-1}e^{-\omega\tau}\rho(\omega)\ . \tag{2.10}$$

To show the influence of the interaction parameter γ on the thermal correlation functions we give in Fig. 1 an example of the behavior of $G_\beta(\tau)$ for $m\beta=12.0$ and $\gamma/m=0$, 0.05, 0.1. These parameters reflect typical values used in fits of the MC data we will discuss in Sec. IV. As can be seen, a nonvanishing value of γ results in a flattening of the correlation functions in the central region. Using this ansatz for the analytic structure of the thermal Green's functions the transport coefficients can be calculated. From Eq. (2.1) we find

$$\alpha=2A(1-e^{-\beta m})\frac{2\gamma m}{(\gamma^2+m^2)^2} \tag{2.11}$$

with $\alpha=\eta$, $\frac{4}{3}\eta+\zeta$, or χT.

This simple ansatz already involves three parameters which have to be determined from the Monte Carlo data. This is the maximum number of parameters one can possibly determine from the MC simulation on an $8^3\times4$ lattice which we are going to discuss in Sec. IV. However, it should be emphasized that our ansatz, Eq. (2.8), may

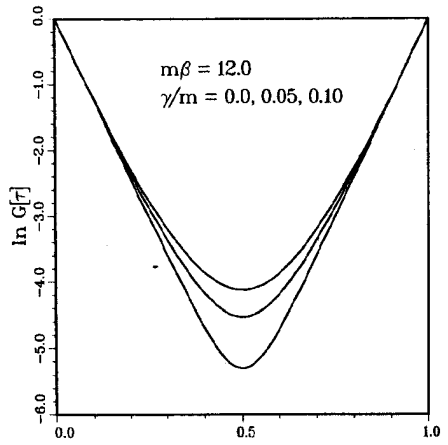


FIG. 1. Thermal Green's function $G_\beta(\tau)$ vs τ for fixed $m\beta=12.0$ and various values of the interaction parameter $\gamma/m=0.0$, 0.05, and 0.10.

neglect important features of the spectral density function which should become visible if more parameters could be determined. Thus, as discussed, for example, in Sec. III of Ref. 4, the hydrodynamical equations predict the existence of poles in the Green's function at $\omega=-ik^2D_T$ and $\omega=c_s k-ik^2\Gamma$, where D_T is the thermal diffusivity, c_s is the sound speed, and Γ is the sound wave damping constant. Furthermore, the residues of these poles are constrained by analyticity requirements and the f sum rule. It is clear that our present data is inadequate to determine the parameters necessary to describe these modes, so we shall do the best we can with the simple ansatz (2.8).

III. THE ENERGY-MOMENTUM TENSOR ON THE LATTICE

In Wilson's lattice approach[11] to continuum QCD components of the field strength tensor $F_{\mu\nu}$ are expressed in terms of plaquette variables $U_{x,\mu\nu}$

$$U_{x,\mu\nu}=U_{x,\mu}U_{x+\mu,\nu}U^{\dagger}_{x+\mu+\nu,\mu}U^{\dagger}_{x,\nu} \tag{3.1}$$

with $U_{x,\mu}$ an element of SU(3) defined on links $(x,x+\mu)$ of a four-dimensional hypercubic lattice. In the naive continuum limit we find

$$U_{x,\mu\nu}=1+ia^2gF^b_{\mu\nu}T^b-\tfrac{1}{2}a^4g^2F^b_{\mu\nu}F^c_{\mu\nu}T^bT^c \tag{3.2}$$

with a denoting the lattice spacing and g being the bare coupling constant. The generators $T^b, b=1,2,\ldots,8$ of the SU(3) group are traceless and satisfy $\mathrm{Tr}T^bT^c=\frac{1}{2}\delta_{bc}$. Thus we see that diagonal components of the energy-momentum tensor can be expressed in terms of plaquette variables

$$T_{\mu\mu}(x)=\frac{2}{g^2}\left[-\sum_{\nu\neq\mu}\mathrm{Tr}U_{x,\mu\nu}+\sum_{\sigma,\nu\neq\mu,\sigma>\nu}\mathrm{Tr}U_{x,\sigma\nu}\right]. \tag{3.3}$$

In the definition of diagonal components of $T_{\mu\nu}$ we have ignored $O(g^2)$ terms which appear in the definition of energy (T_{00}) and pressure (T_{ii}) as derivatives of the partition function[12] and give rise to the trace anomaly of $T_{\mu\nu}$ (Ref. 13). Thus in our approximation $T_{\mu\nu}$ is traceless. As in calculations of T_{00} we expect the influence of the neglected terms to be small in correlation functions.

For the off-diagonal components it is necessary to combine two plaquettes in different hyperplanes

$$T_{\mu\nu}(x)=\frac{2}{g^2}\mathrm{Tr}\tilde{U}_{x,\mu\sigma}\tilde{U}_{x,\nu\sigma}\,, \tag{3.4}$$

where we have defined the traceless antisymmetrized plaquette variable $\tilde{U}_{x,\mu\nu}$ as

$$\tilde{U}_{x,\mu\nu}=-\frac{i}{2}(U_{x,\mu\nu}-U_{x,\nu\mu})_{\text{traceless part}}\,. \tag{3.5}$$

Using these definitions for the energy-momentum tensor we can calculate thermal correlation functions of its components. In the evaluation of those correlation functions relevant for the calculation of transport coefficients we will take advantage of the cubic symmetry of the space components of $T_{\mu\nu}$ which implies that the general tensor structure of spacelike correlation functions is given by

$$\langle T_{ij}(x)T_{kl}(y)\rangle=A(x,y)(\delta_{ik}\delta_{jl}+\delta_{il}\delta_{jk})+B(x,y)\delta_{ij}\delta_{kl}\,,$$
$$i,j,k,l=1,2,3\,. \tag{3.6}$$

Equation (3.6) implies that correlations of off-diagonal space-space components of $T_{\mu\nu}$ can be expressed in terms of diagonal elements. In particular, we have

$$\langle T_{12}(x)T_{12}(y)\rangle=\tfrac{1}{2}[\langle T_{11}(x)T_{11}(y)\rangle-\langle T_{11}(x)T_{22}(y)\rangle]\,. \tag{3.7}$$

This allows us to express the shear viscosity η in terms of diagonal elements of $T_{\mu\nu}$, which will be an advantage in practical Monte Carlo simulations. Moreover this representation makes manifest that expectation values of off-diagonal components of T_{ij} vanish in thermodynamic equilibrium. Using Eq. (3.7) and summing over all possible space directions makes this statement true on each individual gauge configuration. This reduces the statistical error and makes η the most accessible coefficient in MC simulations.

Before discussing the MC data let us give the analogue of Eq. (2.11) in terms of the corresponding lattice parameters. On the lattice the zero-momentum thermal correlation functions of $T_{\mu\nu}$ have dimension a^{-5}. A fit thus gives us transport coefficients in terms of the dimensionless parameters $A_l=Aa^5$, $m_l=ma$, and γ/m. With the temperature given by $N_\tau a=1/T$, Eq. (2.11) becomes

$$\frac{\alpha}{T^3}=N_\tau{}^3\frac{2A_l}{m_l{}^2(1-e^{-N_\tau m_l})}\frac{2\gamma/m}{[1+(\gamma/m)^2]^2}\,. \tag{3.8}$$

IV. MONTE CARLO RESULTS

We have performed a Monte Carlo simulation for the SU(3) Yang-Mills system on an $8^3\times4$ lattice to measure various thermal correlation functions of the energy-momentum tensor which are relevant for the calculation of transport coefficients. In particular we have measured the following three zero-momentum, thermal correlation functions:

TABLE I. Thermal Green's function $C_\alpha(\tau)$ for $\tau=0,1,2$ at $6/g^2=4.8$, 5.2, 5.6, and 5.8 based on 12 000, 28 000, 36 000, and 36 000 measurements, respectively.

$6/g^2$	τ	C_η	C_χ	$C_{4\eta/3+\zeta}$
	0	6.543±0.030	1.937±0.024	9.987±0.041
4.8	1	0.232±0.022	−0.001±0.024	0.258±0.030
	2	0.044±0.031	0.008±0.024	0.048±0.042
	0	6.878±0.021	2.364±0.037	10.614±0.028
5.2	1	0.277±0.014	0.015±0.037	0.288±0.020
	2	−0.009+0.020	0.001±0.037	−0.029±0.028
	0	6.005±0.016	2.768±0.160	9.362±0.022
5.6	1	0.347±0.011	0.090±0.160	0.360±0.016
	2	0.050±0.016	0.064±0.160	0.023±0.022
	0	5.325±0.014	2.691±0.091	8.292±0.019
5.8	1	0.334±0.010	0.001±0.091	0.368±0.014
	2	0.045±0.014	0.014±0.091	0.046±0.019

$$C_\chi=\sum_x \langle T_{01}(0,0)T_{01}(x,\tau)\rangle\ , \tag{4.1a}$$

$$C_\eta=\sum_x \langle T_{12}(0,0)T_{12}(x,\tau)\rangle\ , \tag{4.1b}$$

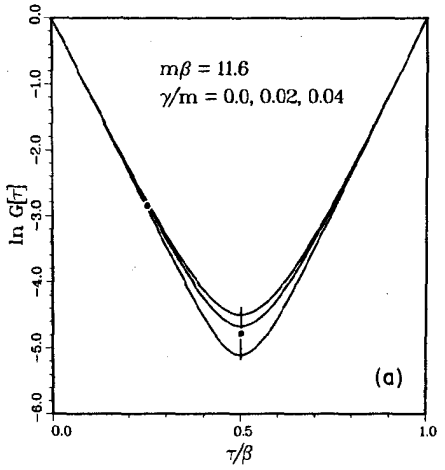
$$C_{4\eta/3+\zeta}=\sum_x \langle T_{11}(0,0)T_{11}(x,\tau)\rangle\ , \tag{4.1c}$$

with $\tau=0,1,2$. In the actual calculation we averaged over all equivalent space directions and used Eq. (3.7) to evaluate C_η.

At best we can only measure correlations up to distance 2 on a lattice with only four sites in the temporal direction. Thus we clearly cannot get much information about the analytic structure of these correlation functions on lattices of sizes $8^3\times 4$. Already the simple ansatz discussed in Sec. II requires the determination of three parameters, which, moreover, should be extracted from the long-distance part of the correlation functions. Actually it turned out that the correlation functions drop quite rapidly as functions of τ, making even a measurement at distance 2 quite difficult. We will thus first restrict ourselves to extracting information about the mass parameter m controlling the exponential decay of the correlation functions. In Table I we summarize our results obtained at $6/g^2=4.8$, 5.2, 5.6, and 5.8. The data are based on runs with up to 180 000 sweeps with measurements taken every fifth sweep. We have checked that these measurements were statistically independent by monitoring their time correlations. As can be seen only C_η could be measured at distance 2. This is partly due to the fact that the disconnected part of the correlation function, Eq. (4.1b), is strictly zero on each individual gauge configuration as we have discussed in the previous section. The correlations of off-diagonal matrix elements of $T_{\mu\nu}$ entering the heat conductivity could not be measured. This is due to the circumstance that complicated six link operators are involved in the definition of T_{0i}. In the following we thus will restrict our discussion to the behavior of C_η. The measurements at $6/g^2=5.6$ and 5.8 are performed close to the critical coupling for the deconfinement transition, which one this size lattice occurs at $6/g^2=5.68$ (Ref. 14). Comparing the behavior of C_η below and above the phase transition, we see that it behaves in much the same way in both cases, dropping rapidly, indicating a large effective mass for the particles in the system. Thus even above T_c we observe the influence of massive excitations, which shows that close to T_c the plasma cannot be described by a free gluon gas. This observation is in agreement with recent results of Ref. 15. The effective masses we obtain from ratios of successive measurements of C_η by fitting these with the periodic ansatz, Eq. (2.7), are given in Table II. We see that at low temperature these effective masses approach the lowest glueball mass, as expected.[7] Indeed, for the smaller values of $6/g^2$ our results are in agreement with glueball masses extracted from the short-distance part of plaquette correlation functions.[16] In the vicinity of the deconfinement transition, however, the masses in lattice units stay more or less constant, in contrast with the glueball case where they drop significantly over the range of $6/g^2$ considered here.[16] In units of the

TABLE II. Effective masses extracted from the exponential decay of the thermal Green's functions at various values of $6/g^2$. $m_\alpha(\tau)$ denotes masses extracted from ratios of the correlation function C_α at distance τ and $\tau-1$. It was not possible to obtain masses corresponding to the heat conductivity Green's function.

$6/g^2$	τ	m_η	$m_{4\eta/3+\zeta}$
4.8	1	3.34±0.10	3.66±0.13
	2	2.35±1.30	2.36±2.08
5.2	1	3.21±0.06	3.61±0.07
	2		
5.6	1	2.85±0.04	3.26±0.05
	2	2.62±0.42	3.45±3.31
5.8	1	2.77±0.03	3.12±0.04
	2	2.70±0.40	2.77±0.58



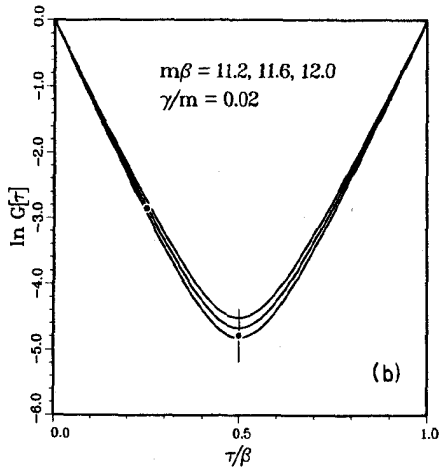


FIG. 2. (a) Thermal Green's function $G_\beta(\tau)$ vs τ for $m\beta=11.6$ and various values of the interaction parameter $\gamma/m=0.0$, 0.02, and 0.04. Also shown are Monte Carlo data from simulations at $6/g^2=5.6$ for the shear viscosity correlation function C_η. (b) Thermal Green's function $G_\beta(\tau)$ vs τ for various values of $m\beta=11.2$, 11.6, and 12.0 and the interaction parameter $\gamma/m=0.02$. Also shown are Monte Carlo data from simulations at $6/g^2=5.6$ for the shear viscosity correlation function C_η.

temperature we thus find

$$\frac{m}{T}\simeq 8-12 \tag{4.2}$$

in the temperature range close to T_c. One should, however, be aware that in most cases we got information on the masses only from distance 0 and 1 correlation functions. In the cases where we could measure up to distance 2 the masses drop by about 10%. We thus expect the asymptotic values to be within the lower bound given in Eq. (4.2).

Let us finally attempt to draw some conclusions about the analytic structure of the thermal correlation functions. As we have pointed out, we should extract the parameters A,m,γ entering the analytic ansatz, Eqs. (2.10) and (2.11), from the long-distance part of the correlation function. On our $8^3\times4$ lattice we have seen that a pure exponential fit already gives reasonable results. This would suggest that the parameter γ, which leads to a flattening of the correlation functions in the central region is small. We can obtain an estimate for γ by using the ansatz (2.10) and (2.11) to fit the whole correlation function. As the exponential decay then basically is fixed by the distance 0 and 1 results for the correlation function we get an upper bound for γ. Such a fit is shown in Fig. 2. From this we conclude that

$$\frac{\gamma}{m}<0.05\ . \tag{4.3}$$

Using this upper bound for γ and $A=C_\eta(0)$ we get from Eq. (3.8) for the shear viscosity

$$\frac{\eta}{T^3}<9.5 \tag{4.4}$$

close to T_c in the confined as well as in the deconfined phase. This is in reasonable agreement with the analytic estimates of Ref. 5 from the collision time approximation.

V. CONCLUSIONS

We have discussed the framework for a calculation of transport coefficients in the context of lattice gauge theory. This study made the limitations of such a program clear. From a first exploratory analysis of thermal Green's functions for the energy-momentum tensor on an $8^3\times4$ lattice we have seen that it will be quite difficult to measure these at large temporal distances, which would be necessary to get information about their analytic structure. The best results have been obtained for the correlation functions entering the determination of the shear viscosity, while space-time components of $T_{\mu\nu}$, which determine the thermal conductivity, were most difficult to measure.

From the exponential decay of the Green's functions we saw that in the confined region as well as in the deconfined region close to T_c the exponential decay of the correlation functions indicates the existence of excitations with masses of the order of the lowest glueball states. While this was expected in the confined region it shows that above T_c we do not just find a massless gluon gas; massive modes are still relevant.

Probably it will be rather difficult to extract a good value for η from simulations of the sort studied here for a while, but by analyzing the deviations from a pure exponential decay on larger lattices it may be possible to establish upper bounds on η, as we attempted to do on the $8^3\times4$ lattice. To get better results for correlation functions at larger distances one should investigate the possibility of using source methods for the energy-momentum tensor. The small number of temporal values accessible in a Monte Carlo simulation on a Euclidean lattice presents a severe restriction for the analysis of the analytic structure of the spectral density functions. It thus may be useful to introduce anisotropic lattices, i.e., lattices with dif-

ferent lattice spacings in the space and time directions.[12,17] In this way more lattice sites in the temporal direction could be introduced without the need of going to too large couplings $6/g^2$, where one would lose the signal for the correlation functions. In this way it might be possible to determine more than three parameters characterizing an ansatz for the spectral density function.

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[1]F. Karsch, in *Lattice Gauge Theory: A Challenge in Large-Scale Computing*, proceedings of the Wuppertal Workshop, 1985, edited by B. Bunk, K. H. Mutter, and K. Schilling (NATO Advanced Study Institute, Series B: Physics, Vol. 140) (Plenum, New York, 1986), p. 1.

[2]J. Cleymans, R. V. Gavai, and E. Suhonen, Phys. Rep. **130**, 219 (1986).

[3]M. A. Faessler, CERN Report No. CERN-EP/86-102, 1986 (unpublished).

[4]L. P. Kadanoff and P. C. Martin, Ann. Phys. (N.Y.) **24**, 419 (1963).

[5]A. Hosoya and K. Kajantie, Nucl. Phys. **B250**, 666 (1985).

[6]S. Gavin, Nucl. Phys. **A435**, 826 (1985).

[7]R. Horsley and W. Schoenmaker, Phys. Rev. Lett. **57**, 2894 (1986).

[8]A. Hosoya, M. Sakayami, and M. Takao, Ann. Phys. (N.Y.) **154**, 229 (1984).

[9]D. N. Zubarev, *Non-Equilibrium Statistical Thermodynamics* (Plenum, New York, 1974).

[10]G. Baym and N. D. Mermin, J. Math. Phys. **2**, 232 (1961).

[11]K. Wilson, Phys. Rev. D **10**, 2445 (1974).

[12]J. Engels, F. Karsch, I. Montvay, and H. Satz, Nucl. Phys. **B205**, 545 (1982).

[13]F. Fucito and B. Svetitsky, Phys. Lett. **131B**, 165 (1983).

[14]A. D. Kennedy, J. Kuti, S. Meyer, and B. J. Pendleton, Phys. Rev. Lett. **54**, 87 (1985).

[15]T. A. DeGrand and C. E. DeTar, Phys. Rev. D **34**, 2469 (1986).

[16]B. Berg and A. Billoire, Nucl. Phys. **B221**, 109 (1983).

[17]J. Engels, F. Karsch, and H. Satz, Nucl. Phys. **B205**, 239 (1982).