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A lower bound on T_{SR}/m_H in the O(4) model on anisotropic lattices

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Results of an investigation of the O(4) spin model at finite temperature using anisotropic lattices are presented. In both the large N approximation and numerical simulations using the Wolff cluster algorithm we find that the ratio of the symmetry restoration temperature T_{SR} to the Higgs mass m_H is independent of the anisotropy ξ. From the numerical simulations we obtain a lower bound of $T_{SR}/m_H \simeq 0.58 \pm 0.02$ at a value for the Higgs mass $m_H a_s \simeq 0.5$, which is lowered further by about 10% at $m_H a_s \simeq 1$. Requiring certain timelike correlation functions to coincide with their spacelike counterparts, quantum and scaling corrections to the anisotropy are determined and are found to be small i.e., the anisotropy is found to be close to the ratio of spacelike and timelike lattice spacings.

1. Introduction

The fate of a spontaneously broken gauge theory at finite temperatures of the order of the symmetry breaking scale has attracted attention for a considerable period now. Such investigations are of importance to the physics of the very early universe. Two prime examples are the inflationary universe and the generation of the baryon asymmetry. It has been argued [1] that all the baryon asymmetry generated at the GUT scale is washed out by non-perturbative effects near the electroweak phase transition. Whether any extra mechanism exists to create a fresh baryon asymmetry [2] near this phase transition remains unclear. Although symmetry restoring phase transitions in spontaneously broken gauge theories are crucial for these areas, our knowledge about them comes chiefly from perturbation theory [3] which can be expected to be rather inadequate for dealing with the anticipated presence of certain intrinsic nonperturbative effects near such phase transitions [4].

Motivated by the desire to learn more about the non-perturbative aspects of the symmetry restoration transition, exploratory lattice investigations of SU(2) Higgs-gauge models and nonlinear O(N) models at finite temperature have been made [5-9]. These models are expected to be trivial, giving rise to an upper bound on the Higgs mass of about 650 GeV at values of the (lattice) Higgs mass close to the lattice cutoff $m_H a_s \simeq 1$. Correspondingly at finite temperature one expects a lower bound on the symmetry restoration temperature T_{SR} in units of the inverse Higgs mass m_H^{-1}. For the standard model the gauge couplings at the weak symmetry breaking scale and the Yukawa couplings, with the possible exception of that of the top quark, are small. Neglecting them as a first approximation, one arrives at an O(4) symmetric scalar model. As in the case of the bound on the Higgs mass, one can then hope that the lower bound on T_{SR}/m_H can be obtained by studying the O(4) model, rather than the more complicated SU(2) fermion-Higgs model.

In this note we study the O(4) model at finite temperature using numerical simulation of the euclidian path integral on lattices with anisotropic spacings in time (temperature) and spatial directions. We also compare our results with analytic results obtained in the lowest order large N expansion. Anisotropic couplings allow, at least in principle, a continuous tuning of the temperature, while the Higgs mass in units of the lattice spacing can stay fixed at values $m_H a_s \simeq 1$. Consequently a study of temperature effects of the

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theory at a correlation length of the Higgs particle of order unity becomes feasible without changing the lattice size or losing the resolution in the temperature direction and the relevant information about the lower bound on T_{SR}/m_H can be extracted. In addition, anisotropic lattices allow us to distinguish the finite temperature effects, which in the euclidian formulation that we employ could be regarded as a special type of finite size effects, from other finite size effects since the finite temperature effects have to be independent of the anisotropy in the scaling region.

The plan of this paper is as follows: In the next section we define the model and give details of our methods to study it in the large N limit and, for $N=4$, using numerical simulations. The procedure to obtain the ratio T_{SR}/m_H is described here. Section 3 is devoted to the discussion of our results and conclusions are presented in the final section. Some of our results have already been presented in a preliminary form in ref. [10].

2. The anisotropic O(N) model

The anisotropic $O(N)$ symmetric spin model on lattices with spatial extension N_s and temporal extension N_t is defined by the action

$$S=-N\beta\left(\gamma\sum_x S_x\cdot S_{x+\hat{0}}+\frac{1}{\gamma}\sum_{x,j} S_x\cdot S_{x+\hat{j}}\right), \qquad (1)$$

or alternatively

$$S=-2\kappa\left(\gamma\sum_x S_x\cdot S_{x+\hat{0}}+\frac{1}{\gamma}\sum_{x,j} S_x\cdot S_{x+\hat{j}}\right). \qquad (2)$$

Here the spins S_x are unit vectors in $O(N)$, γ is the anisotropy coupling and β or κ denote the hopping parameter. Isotropic lattices are defined by $\gamma=1$. For the study of the large N limit we take the first form of the action, eq. (1), keeping β finite, while for the O(4) model we use the more conventional second form, eq. (2).

Denoting the lattice spacing in spatial directions a_s and in the temporal direction a_t, the anisotropy parameter ξ is the ratio of spacelike to timelike lattice spacings:

$$\xi=\frac{a_s}{a_t}. \qquad (3)$$

In the naive continuum limit and for noninteracting theories $\xi=\gamma$. However, quantum and scaling corrections can modify this relation [11]. For a given anisotropy coupling γ, ξ can be determined by the requirement that physics, e.g. the fall-off of correlation functions, is the same in the temporal and spatial directions. As the O(4) model is weakly interacting we expect only a small renormalization of ξ with respect to the bare coupling γ. We also expect at the critical point $\xi=\gamma$, as the renormalized coupling of the O(4) model vanishes there. The anisotropy ξ is easily calculable in the large N limit for the symmetric phase of the model. There we found that the relevant contributions to ξ are of order $O(a^2)$, i.e. a scaling violation effect. The same conclusion can be inferred for the broken phase. We also looked at contributions of ξ in renormalized perturbation theory of the $\lambda\phi^4$ model. Up to two loops we again found $O(a^2)$ effects, in contrast to theories involving gauge fields where quantum corrections of order $O(g^2)$ occur [11]. We conjecture that for our model the difference of ξ from γ is order $O(a^2)$ to all orders in perturbation theory.

On the anisotropic lattice the physical three-dimensional spatial volume and the temperature are respectively given by $V_3=N_s^3a_s^3$ and $T=1/N_ta_t=\xi/N_ta_s$. It is therefore possible to vary ξ and N_t simultaneously at fixed ratio ξ/N_t, without changing the temperature in units a_s^{-1}, or changing the spatial volume. This amounts effectively to a change in resolution in the temporal direction: a_t is changed while N_ta_t is kept fixed. At least in the scaling region physical results should then be independent of the anisotropy ξ. A verification of this property will provide a valuable consistency check to our analysis.

Earlier studies [6,7] of the symmetry restoration phase transition in the O(4) symmetric spin model on isotropic lattices revealed that it was only possible to determine the symmetry restoration temperature T_{SR} for values of the Higgs mass which barely exceeded a value of $m_Ha_s\simeq 0.4$ on reasonable lattice sizes. Furthermore, increasing the Higgs mass in units of the lattice spacing a_s, one expects a logarithmically slow decrease of T_{SR}/m_H, driving numerical simulations on isotropic lattices to larger temperatures and smaller N_t values, therefore loosing resolution in the time direction. However, choosing the anisotropy coupling $\gamma>1$ it is possible to explore the model at values $m_H a_s \simeq 1$ and at larger values of the temperature $1/N_t a_t$ without giving up a reasonable discretization in the time direction, i.e. in our case it was possible to simulate the region $m_H a_s \simeq 1$ on an $N_t = 4$ lattice. In this way it will be possible for the first time to explore regions of the theory where the Higgs mass takes values of the order of the cutoff and a numerical determination of the lower bound on T_{SR} becomes feasible.

In both the large N calculation and the numerical simulations our procedure to investigate finite temperature effects consists of two steps. First we determine, at a given value of the anisotropy coupling γ, the critical coupling on $N_s^3 \times N_t$ lattices. Studying the large N limit in leading order, β_c is obtained by solving numerically the saddle point equation

$$\beta_c(N_t) = \frac{\gamma}{N_t N_s^3} \sum_p' \frac{1}{D(p)} \tag{4}$$

for $N_s \to \infty$, where $D(p)$ is given by

$$D(p) = 4\gamma^2 \sin^2(\tfrac{1}{2}p_0) + 4 \sum_j \sin^2(\tfrac{1}{2}p_j) , \tag{5}$$

with the momenta p_μ given by $p_\mu = 2\pi n_\mu / N_\mu$, $n_\mu = 0, ..., N_\mu - 1$, where $N_0 = N_t$ and $N_j = N_s$. The prime on the sum in eq. (4) indicates that the zero mode $p=0$ is being left out. In Monte Carlo (MC) simulations the unique crossing point of the Binder cumulant $g_R = \langle M^4 \rangle / \langle M^2 \rangle^2$ for various volumes N_s^3 and at given values of the anisotropy coupling γ yields $\kappa_c(\infty, N_t)$. Here M is the order parameter, defined by $M = \langle (M^\alpha M^\alpha)^{0.5} \rangle$, where M^α is given by

$$M^\alpha = \frac{1}{N_s^3 N_t} \sum_x S_x^\alpha \tag{6}$$

and α denotes the O(N) index. Alternatively, one may use the peak position of the susceptibility $\chi = N_t N_s^3 (\langle M^2 \rangle - \langle M \rangle^2)$, to define $\kappa_c(N_s, N_t)$. Using the critical exponents of the O(4) model in three dimensions, $\kappa_c(\infty, N_t)$ can then be obtained using the finite size scaling theory. We employed both methods and checked that they yield consistent results.

Secondly the Higgs mass and the renormalized field expectation value were then determined at zero temperature at the coupling $\kappa_c(\infty, N_t)$ on $N_s^3 \times \gamma N_s$ lattices. For the determination of the renormalized field expectation v_R in units of a_t^{-1} we proceed in case of our Monte Carlo simulation as follows: The dimensionless quantity $v_R a_t$ is given by an estimator for the field expectation value Σ, which is properly normalized by its corresponding wave function renormalization constant Z: $v_R a_t = \Sigma / \sqrt{Z}$. Note here that neither quantity Σ nor Z are fixed numbers in the theory. It is possible to redefine Σ and Z by overall multiplicative factors, such that the physical quantity $v_R a_t$ stays fixed. In our case we chose the expectation value of the mean field multiplied with a convenient factor $(\sqrt{2\kappa/\gamma}) \langle M \rangle$ as an estimator for the field expectation value Σ. The corresponding wave function renormalization constant can then be derived from the behavior of the O(4) symmetric zero momentum correlation function

$$G(n) = \frac{2\kappa}{4N_s^3 \gamma^2} \sum_x \langle S_0^\alpha S_{x,ne_t}^\alpha \rangle , \tag{7}$$

which is defined in the temporal direction of the lattice. Using chiral perturbation theory one finds for large values of n on a periodic symmetric box, that $G(n)$ has the shape of a parabola. This is due to the presence of massless Goldstone bosons in the theory:

$$G(n) = Z \frac{3}{2V} (n - \tfrac{1}{2}N_t)^2 + \text{const} . \tag{8}$$

Expressing the volume V in units of a_t, $V = \xi^3 N_s^3 N_t$, the desired wave function renormalization constant Z can in principal be determined. In our actual data analysis we have also considered the contribution of the scalar particle to eq. (7), for a detailed description of the procedure see ref. [12].

For the determination of the Higgs mass we project the scalar fields S_x^α individually in each configuration onto the direction of the mean field $M^\alpha / |M|$ and we obtain a field operator which has a good overlap with the Higgs particle:

$$S_{\sigma,x} = \frac{S_x^\alpha M^\alpha}{|M|} . \tag{9}$$

The Higgs mass $m_H a_t$ can then be extracted from the exponential decay of the zero (spatial) momentum correlation functions of the operator $S_{\sigma,x}$.

Introducing O(N) invariant correlation functions defined on the main axis of the lattice in time direction

$$C_t(n) = \frac{1}{N_s^3 N_t} \sum_x S_x^\alpha S_{x+ne_t}^\alpha , \tag{10}$$

and in space direction

$$C_s(n) = \frac{1}{N_s^3 N_t} \sum_x S_x^\alpha S_{x+ne_s}^\alpha \tag{11}$$

we demand invariance with respect to an interchange of the spatial and temporal directions. We match the correlation functions in temporal and spatial directions at equal distance n, by scaling the temporal direction by a factor ξ, which determines the anisotropy. As we shall see below, the difference of ξ from γ was found to be rather small, being of the order of at most 3% for all γ values we studied.

3. Results

The numerical computations have been performed using the nonlocal Wolff cluster algorithm. The employed statistics was about 10^5 sweeps for each simulated lattice size and set of couplings. At finite temperature we simulated N_t and γ values as given in table 1. In each case we performed simulations with $N_s = 18$ and $N_s = 24$ at few values of the hopping parameter κ. We employed the spectral density method in order to determine the maximum of the susceptibility and the crossing point of the Binder cumulant. At zero temperature, with $\kappa = \kappa_c(\infty, N_t)$, we performed simulations on $18^3 \times \gamma 18$ lattices with γ equal to the cited values.

Fig. 1 exhibits our results for both g_R and χ on $18^3 \times 6$ and $24^3 \times 6$ lattices for $\gamma = 1.5$. We used the spectral density method to obtain the smooth curves shown from our data, shown by crosses. Similar results have also been obtained for all other values of γ and N_t. In each case we obtained $\kappa_c(\infty, N_t)$ by using

Table 1
Critical hopping parameters at given N and γ for the $N_s \to \infty$ limit. The third column denotes our result from numerical simulations while the last column ($\hat{\kappa}$) denotes results from the large N expansion.

N_t	γ	$\kappa_c(\infty, N_t)$	$\hat{\kappa}_c(\infty, N_t)$
6	1.0	0.3060(3)	0.314594
4	1.0	0.3103(3)	0.320871
6	1.5	0.3645(3)	0.377673
8	2.0	0.3912(3)	0.408333
3	1.5	0.3913(3)	0.415998
4	2.0	0.4171(3)	0.446373

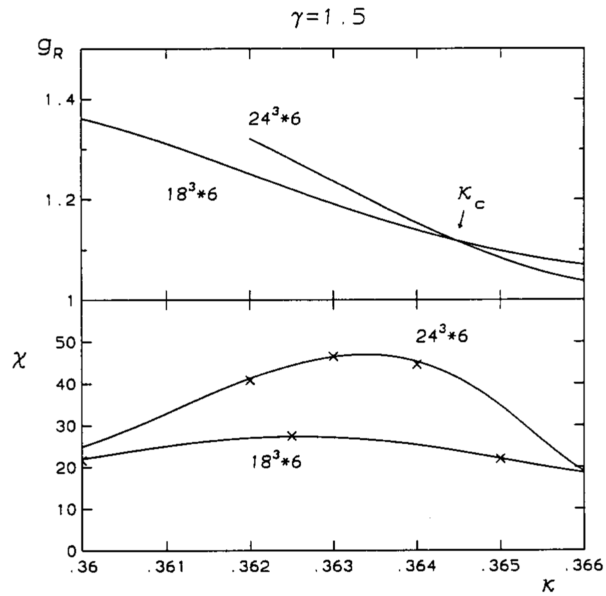

Fig. 1. Results for thermodynamic quantities at $\gamma = 1.5$.

both the crossing point of g_R and the finite size scaling of the peak position of the susceptibility. Both estimates were always found to be consistent, although we preferred to use the former for determining m_H. Table 1 contains our results for κ_c as a function of N_t and γ from the numerical simulations, along with the corresponding results from the large N expansion. One finds a sizable but less than ~7% difference between the two estimates, which is of the same order as the discrepancy observed by comparing the zero temperature critical hopping parameter from the large N expansion with high precision numerical simulations.

Fig. 2 compares the spacelike correlation function $C_s(n)$ on an $18^3 \times 36$ lattice at $(\kappa, \gamma) = (0.3912, 2.0)$ with the corresponding timelike correlation function $C_t(n/\xi)$ at scaled distance n/ξ. One sees that the two are in nice agreement with each other. Table 2 contains, together with other quantities, the measured anisotropy ξ. The deviations of ξ from γ are small, on the few percent level, which is in accord with the expectations near a gaussian fixed point.

The Higgs mass $m_H a_t$ was then obtained from an exponential fit to the connected zero momentum correlation functions of the operator (9). These values of the Higgs mass are listed together with $m_H a_s$ in table 2. Using these results, the ratio $T_{SR}/m_H =$

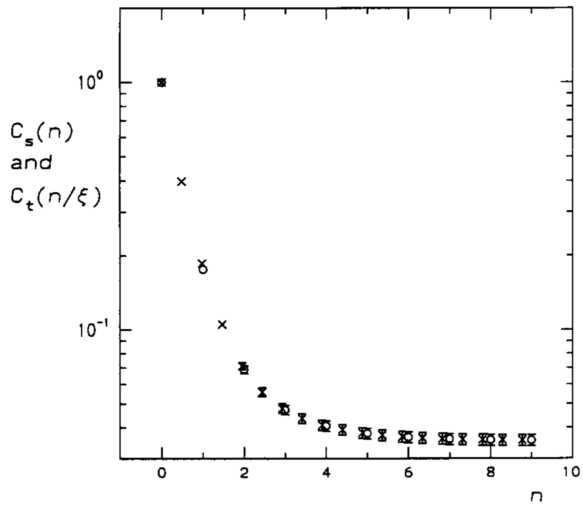

Fig. 2. $C_s(n)$ and $C_t(n/\xi)$ at $(\kappa, \gamma)=(0.3912, 2.0)$ on a $18^3\times 36$ lattice.

$1/N_t m_H a_t$ shown in the table was obtained for various γ and N_t. As expected, we observe the ξ-independence of the ratio at fixed values of $m_H a_s$, demonstrating the internal consistency of our finite temperature formulation of the theory on anisotropic lattices. Considering fluctuations around the saddle point in the large N limit, one can obtain the Higgs mass m_H at $\beta_c(\infty, N_t)$ at given γ. Fig. 3 shows these large N results for T_{SR}/m_H. They are also seen to be almost independent of the anisotropy ξ.

We have also collected in table 2 our results for the expectation value of the mean field $\langle M\rangle$, the wave function renormalization constant Z, $v_R a_t$ and finally the ratio T_{SR}/v_R, which have been determined from the Monte Carlo data by the methods described above. Once again the ξ independence of the ratio at fixed $m_H a_s$ in nicely born out. Large N results for the ratio T_{SR}/v_R are also shown in fig. 3. The renormalized vacuum expectation value of the field in the large N calculation is given by $v_R^2 = N(\beta_c(\xi L_t)-\beta_c(\infty))$ and we have set $N=4$ [#1]. Again, the ratio T_{SR}/v_R is almost independent of the anisotropy ξ. The large N results, shown in fig. 3, agree quite well with the numerical results at $N=4$ of table 2.

A remark concerning the error determination for the quantities as cited in table 2 and a comment on further possible systematic errors may be appropriate here. As can be noted, the ratios T_{SR}/m_H and T_{SR}/v_R exhibit sizable errors, as compared to the relatively small and purely statistical errors quoted for all the other quantities. These errors are mainly caused by the uncertainty of the finite temperature critical κ values (table 1), which is turn lead to relatively large errors for the zero temperature values of $m_H a_t$ and $v_R a_t$ to be used in the ratios. Also we have to expect zero temperature finite volume corrections to the quantities $m_H a_t$ and $v_R a_t$ used to construct the ratios as quoted in table 2. As we anticipate the finite volume corrections to the quantities T_{SR}/m_H and T_{SR}/v_R, quoted in table 2, to be significantly smaller on our lattices than the errors induced by the uncertainty of the critical points, we refrained from a detailed zero temperature finite size scaling analysis. Future simulations yielding more precise κ_c values will have to incorporate them.

Our data for T_{SR}/m_H as depicted in table 2 decrease, as expected, very slowly as the Higgs mass $m_H a_s$ in units of a_s is increased. Thus, depending on the choice of value of the correlation length up to

[#1] The normalization of v_R in ref. [10] differs by a factor $\sqrt{N}$ ($=2$ for $N=4$) from the one used here. This causes a difference of a factor 2 in the scale of fig. 2 there as compared to fig. 3 in this paper.

Table 2
Main results from the numerical simulation of the finite temperature O(4) model on anisotropic lattices.

N_s	N_t	γ	κ	ξ	$\langle M\rangle$	$m_H a_t$	Z	$m_H a_s$	$v_R a_t$	T_{SR}/m_H	T_{SR}/v_R
18	18	1.0	0.3060	1.00(1)	0.1305(3)	0.280(3)	0.96(02)	0.280(04)	0.1040(14)	0.593(39)	1.60(10)
18	18	1.0	0.3103	1.00(2)	0.2082(2)	0.428(3)	0.95(04)	0.428(07)	0.1682(37)	0.583(18)	1.485(64)
18	27	1.5	0.3645	1.51(2)	0.1913(1)	0.285(5)	0.96(03)	0.433(09)	0.1110(20)	0.583(30)	1.500(76)
18	36	2.0	0.3912	2.05(2)	0.1879(1)	0.212(1)	0.99(04)	0.436(06)	0.0832(15)	0.587(31)	1.501(91)
18	27	1.5	0.3913	1.51(3)	0.3838(1)	0.615(3)	0.97(05)	0.934(18)	0.2292(65)	0.541(07)	1.454(52)
18	36	2.0	0.4171	2.05(5)	0.3716(1)	0.457(5)	0.97(06)	0.937(25)	0.1719(61)	0.547(12)	1.453(66)

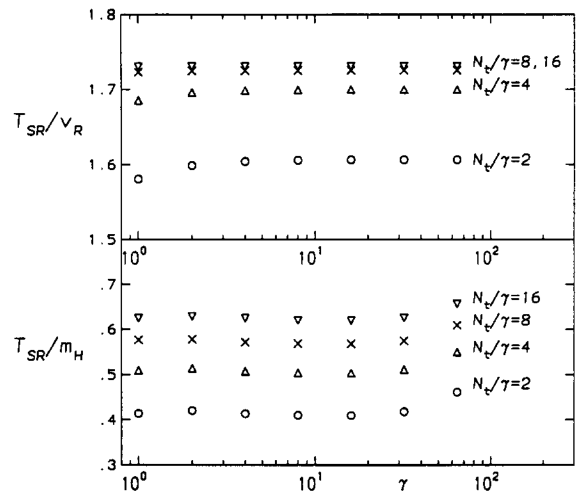

Fig. 3. Large N results for T_{SR}/v_R and T_{SR}/m_H and function of the anisotropy parameter γ.

which an effective theory can be defined, one obtains a lower bound on the ratio T_{SR}/m_H. Just as in the case of the determination of the upper bound to the Higgs mass it is expected, that this lower bound saturates for the theory under study, i.e. the O(4) model at infinite bare quartic coupling. From table 2 we estimate this bound to be 0.58 ± 0.02 for a correlation length of ~ 2, which further decreased by about 10% for a value of $m_H a_s \simeq 1$. Our data for T_{SR}/v_R show an approximate constant behavior as $m_H a_s$ is varied. The actual value is within the errors consistent with the value $\sqrt{2}$, which is the prediction of one-loop renormalized perturbation theory, though the data show some tendency to lie slightly above the perturbative value.

It is interesting to compare our results for T_{SR}/m_H with the one-loop result as obtained in renormalized perturbation theory in the O(4) model. To this order the symmetry restoration temperature is given by [7]

$$\frac{T_{SR}}{m_H} = \left(\frac{6}{g_R}\right)^{1/2}, \tag{12}$$

where g_R is the renormalized quartic coupling of the model. Using previous high precision numerical determinations of g_R [13] as an input we draw in fig. 4 our numerical results for T_{SR}/m_H (crosses) together with the one-loop prediction as indicated by the curve and by the triangles. Here we observe sizable deviations when $m_H a_s$ takes values ≈ 1, indicating that higher order corrections are large at finite temperatures in a region of the model where the scalar correlation length is close to 1, see also ref. [14]. Including also results from the large N expansion in fig. 4, one also notices sizable deviations of the large N results from our data, though the overall trend is reproduced.

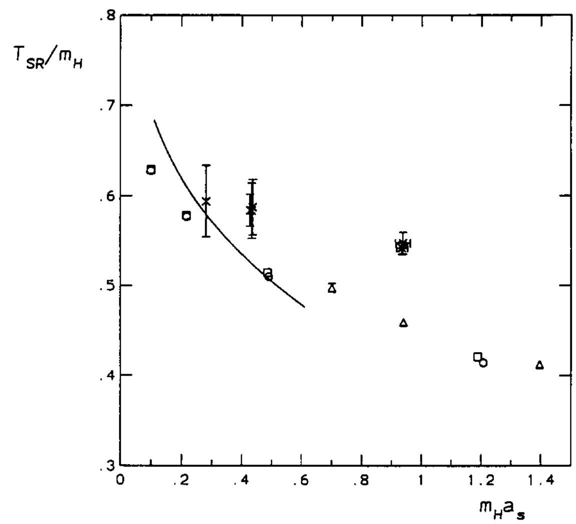

Fig. 4. T_{SR}/m_H as a function of $m_H a_s$. The crosses denote our numerical results, circles and boxes come from the large N expansion at various values of N_t with anisotropy parameter $\gamma = 1$ (circles) and $\gamma = 2$ (boxes), while the curve and triangles correspond to one-loop renormalized perturbation theory, eq. (12).

4. Conclusions

Using anisotropic lattices we have studied the finite temperature behavior of the O(4) theory in regions of the parameter space where the correlation length of a scalar particle is as low as ≈ 1. Depending on the maximal value of $m_H a_s$, one is willing to admit for a sensible definition of the effective theory, a lower bound on T_{SR}/m_H is derived. E.g. for a heavy Higgs particle which at a value of the cutoff $m_H a_s \simeq 0.5$ has a mass close to its triviality bound of about 650 GeV, we find $T_{SR} = 370$ GeV. This value is close to the value predicted by renormalized perturbation theory $T_{SR} = \sqrt{2}\, v_{weak}$ with $v_{weak} \simeq 250$ GeV and consistent with our finding that the ratio T_{SR}/v_R follows the perturbative answer in the whole considered correlation length interval. However, at correlation length 1 we start finding large deviations from one-loop perturbation theory for the quantity T_{SR}/m_H. Qualitatively, the lowest order large N expansion seems to reproduce all the features of the Monte Carlo (MC) data well. Even quantitatively the results are consistent with the naive expectation that they should be accurate to $O(1/N)$. In the large N expansion we were able to explore ξ-independence of T_{SR}/m_H and T_{SR}/v_R over larger ranges of ξ and for more values of N_t. This supports our belief that the early scaling evidence in the MC data even for $N_t=3$ and 4 lattices is no fluke. But it would be interesting to check this by simulating the theory at more ξ and N_t values.

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