POLYAKOV LOOP CORRELATIONS IN LANDAU GAUGE AND THE HEAVY QUARK POTENTIAL

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We calculate Polyakov loop correlation functions in SU(3) gauge theory on a $12^3\times 4$ lattice. We determine from this the colour averaged heavy quark potential and compare it with the corresponding colour singlet potential in Landau gauge. A comparison with finite temperature perturbation theory shows that perturbative relations are at best recovered for very high temperatures.

Numerical studies of the QCD plasma phase have shown that quantities that are sensitive to short distance (large momentum) structure of the plasma are well described by finite temperature perturbation theory. In particular the energy density of a quark–gluon plasma closely resembles ideal gas behaviour above the deconfinement/chiral phase transition [#1]. Recent studies of long distance features like Debye screening of external colour charges [2,3] have, however, shown that even at rather high temperatures numerical results do not agree with perturbative results [4]. This may not be too surprising, as various non-perturbative modes may play a role in the long distance (low momentum) sector of the QCD plasma [1,5,6]. In fact it has been argued that the Debye mass itself cannot be defined perturbatively [7] and that the standard definition as the zero momentum limit of the zeroth component of the vacuum polarization tensor is meaningless. In any case this is a gauge invariant concept only in leading order perturbation theory.

Understanding Debye screening in non-abelian gauge theories is in itself of fundamental importance and plays a keyrole in the discussion of convergence of finite temperature perturbation theory [8]. In order to see whether contact can be made between non-perturbative calculations of the heavy quark potential and finite temperature perturbation theory we studied the colour averaged heavy quark potential as well as colour singlet and octet potentials in Landau gauge. We prefer this gauge over e.g. the axial gauge at finite temperature because of the rotational symmetry of the gauge condition. This allows us to check various relations among these potentials given in perturbation theory [4].

For temperatures above deconfinement the colour averaged heavy quark potential is defined in terms of Polyakov loop correlation functions on a lattice of size $N_\sigma^3\times N_\tau$,

$$\exp[-V(r)/T]=\langle \mathrm{Tr}\,L(R)\,\mathrm{Tr}\,L^\dagger(0)\rangle/\langle L\rangle^2\,, \tag{1}$$

with $\mathrm{Tr}\,L(R)=\mathrm{Tr}\prod_{i=1}^{N_\tau}U_{(\mathbf{R},i),0}$ denoting the Polyakov loop and $\langle L\rangle=\langle \mathrm{Tr}\,L(0)\rangle$. Temperature T and distance r are measured in units of the lattice spacing a and are given by

$$1/T=N_\tau a\,,\quad r=Ra\,,\quad R=1,2,\ldots,N_\sigma/2\,. \tag{2}$$

At high temperature, perturbation theory predicts for this colour averaged potential

$$V(r)/T=-\tfrac{1}{16}V_1(r)^2/T^2\,, \tag{3}$$

where $V_1(r)$ is the singlet term, which is expected to be of Debye screened Coulomb form

$$V_1(r)=[\alpha(T)/r]\exp[-\mu_D(T)r]\,, \tag{4a}$$

[#1] For a review see ref. [1].

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with

$$\alpha(T) = -g^2(T)(N^2-1)/8\pi N . \quad (4b)$$

Here $\mu_D(T) = \sqrt{\frac{1}{3}N}\, g(T)\, T$ is the Debye mass in the case of a pure SU(N) gauge theory. It should be stressed, however, that the Debye screening mass in the potential is not a perturbative result, but rather is based on the usual attempt to reproduce non-perturbative features in perturbative calculations by summing up certain infinite subsets of diagrams [9]. Thus we are actually discussing the incorporation of non-perturbative effects in the framework of perturbation theory when we compare eqs. (3) and (4) with Monte Carlo simulations. The simulations [2,3] so far seem to indicate that the colour averaged potential behaves like $V(r) \sim \exp(-cr)/r$ rather than showing the expected $1/r^2$ behaviour.

In order to analyze this discrepancy further and see in which sense the perturbative limit can be reached, we have studied the potential in more detail for the pure SU(3) gauge theory. In addition to the colour averaged potential defined by eq. (1) we also study the singlet and octet potentials defined as [4,10]

$$\exp[-V_1(r)/T] = 3\langle \mathrm{Tr}\, L(R)\, L^\dagger(0)\rangle / \langle L\rangle^2 , \quad (5)$$

$$\begin{aligned}\exp[-V_8(r)/T] &= \tfrac{9}{8}\langle \mathrm{Tr}\, L(R)\, \mathrm{Tr}\, L^\dagger(0)\rangle / \langle L\rangle^2 \\ &\quad - \tfrac{3}{8}\langle \mathrm{Tr}\, L(R)\, L^\dagger(0)\rangle / \langle L\rangle^2 \\ &= \tfrac{9}{8}\exp[-V(r)/T] - \tfrac{1}{8}\exp[-V_1(r)/T] . \end{aligned} \quad (6)$$

As these quantities are not gauge invariant we study them in Landau gauge. In perturbation theory the singlet and octet potentials are related by

$$V_1(r)/V_8(r) = -8 + O(g^4) . \quad (7a)$$

We note that in this relation the $O(g^2)$ corrections cancel, while this is not the case for similar relations deduced from eq. (3):

$$V_1(r)^2/TV(r) = -16 + O(g^2) . \quad (7b)$$

To study these potentials we performed Monte Carlo simulations on a $12^3 \times 4$ lattice at three couplings $\beta = 5.75$, 6.10 and 8.00. As the deconfinement phase transition on this sized lattices occurs at $\beta_c \simeq 5.69$ [11], the above β values correspond to $T/T_c = 1.16$, 2.3 and 13.7 [#2]. In units of the deconfinement temperature T_c. At each β value we have performed 42 000 iterations. Measurements have been performed every tenth iteration. On these configurations we also fixed the Landau gauge following the procedure outlined in ref. [13], i.e. we first fixed the axial gauge and then iteratively fixed the Landau gauge by maximizing the quantity

$$\mathrm{Re}\,\mathrm{Tr} \sum_{\mu=0}^{3} (U_{x,\mu} + U^\dagger_{x-\mu,\mu}) \quad (8)$$

on each site. The iterative process has been performed 100 times. We checked rotational symmetry by comparing $\langle \mathrm{Tr}\, U_{x,\mu}\rangle$ in the four directions. After 100 iterations the differences between these values were found to be less than 2%.

The error analysis has been performed by subdividing the total data sample in 8 blocks of 512 measurements each. On each of these blocks we determined the potentials according to eqs. (1), (5) and (6). Errors are then calulated as statistical errors of these eight measurements of the potentials. In fig. 1 we show results for the colour averaged (fig. 1a) and singlet and octet (fig. 1b) potentials at the three β values studied.

Our results are summerized in table 1 and shown in fig. 1. From this we see that indeed the octet potential is repulsive, while the singlet potential is attractive. In fig. 2 we show the ratio between singlet and octet potentials (fig. 2a) as well as $-V_1(r)^2/V(r)T$ (fig. 2b) as functions of R. We note that for lower temperatures the ratio $-V_1(r)/V_8(r)$ rises fast, indicating that the octet potential drops fast. At large distances the colour singlet potential thus gives the dominant contribution to the colour averaged potential for temperatures close to T_c. In the perturbative regime we would expect the relations (7a) and (7b) to hold. For the largest β value we indeed find the expected ratio between singlet and octet potentials, while the relation between colour averaged and singlet potential is in general not yet fulfilled. This may reflect the different order in g^2 neglected in (7a) respectively (7b). Apparently we find the best agreement with perturbation theory at short distances. At large distances, the "non-perturbative" screening

[#2] Here we have taken into account the observed scaling violations for T_c/Λ_L. For further details see ref. [12].

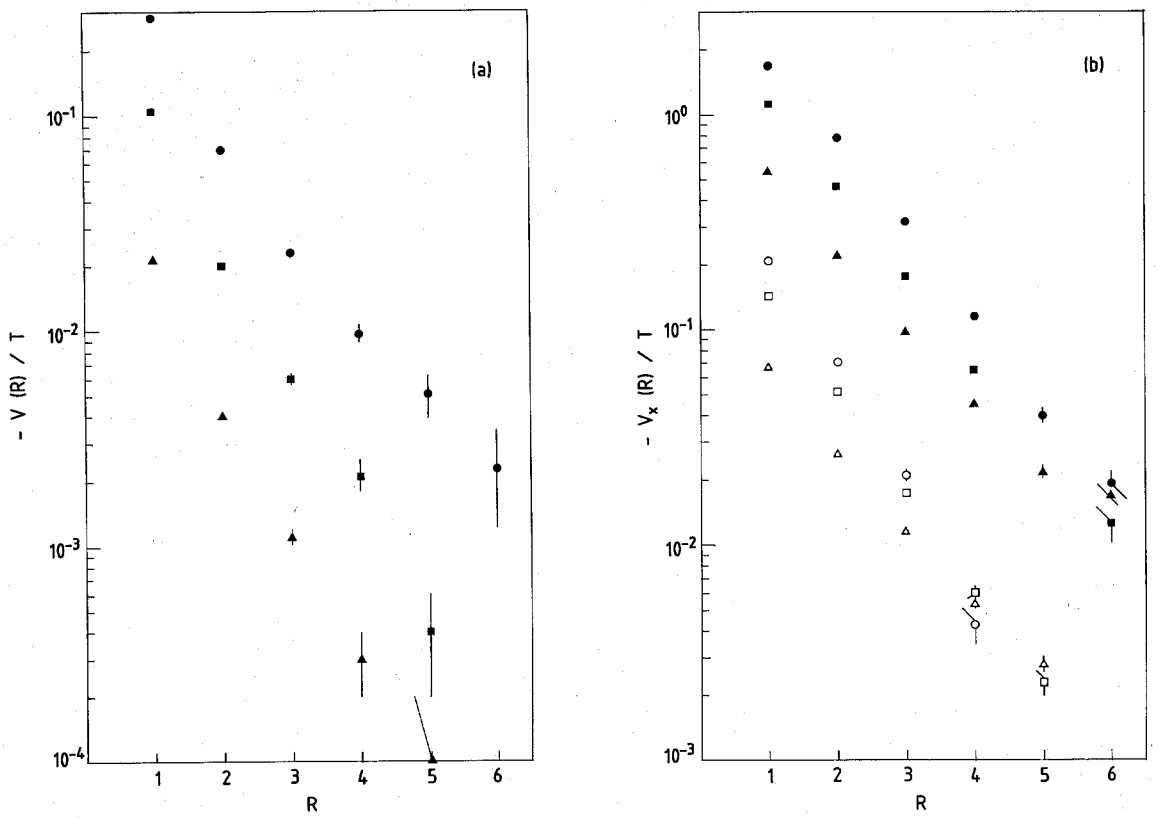

Fig. 1. (a) The function $-V(R)/T$ versus lattice distance R on a $12^3\times 4$ lattice at $\beta=5.75$ (circles), 6.10 (squares) and 8.00 (triangles), where $V(R)$ is the colour averaged potential. (b) The same as in (a), with filled symbols for the singlet potential $-V_1(R)/T$ and open symbols for the octet potential $V_8(R)/T$.

Table 1
Monte Carlo data for the colour averaged potential $V(R)$, singlet potential $V_1(R)$ and octet potential $V_8(R)$ in units of the temperature T at three β values $\beta=5.75$, 6.10 and 8.00. Details about the data sample and error analysis are discussed in the text.

β	$\langle L\rangle$	R	$-V(R)/T$	$-V_1(R)/T$	$V_8(R)/T$
5.75	0.53753 (191)	1	0.2790 (26)	1.6946 (63)	0.2152 (13)
		2	0.0687 (17)	0.7847 (54)	0.0715 (11)
		3	0.0231 (9)	0.3199 (28)	0.0211 (10)
		4	0.0096 (9)	0.1138 (30)	0.0043 (8)
		5	0.0051 (11)	0.0403 (28)	−0.0005 (11)
		6	0.0024 (11)	0.0194 (25)	−0.0002 (10)
6.10	0.81948 (98)	1	0.10422 (48)	1.1156 (25)	0.1426 (5)
		2	0.02005 (31)	0.4596 (20)	0.0515 (5)
		3	0.00599 (14)	0.1766 (12)	0.0176 (3)
		4	0.00209 (27)	0.0642 (13)	0.0060 (3)
		5	0.00041 (19)	0.0222 (14)	0.0023 (3)
		6	0.00066 (46)	0.0127 (24)	0.0009 (5)
8.00	1.44292 (119)	1	0.02148 (14)	0.5445 (17)	0.0683 (3)
		2	0.00402 (9)	0.2210 (18)	0.0267 (3)
		3	0.00110 (7)	0.0969 (19)	0.0116 (2)
		4	0.00032 (6)	0.0448 (18)	0.0054 (2)
		5	0.00012 (6)	0.0232 (16)	0.0028 (2)
		6	0.00015 (8)	0.0172 (16)	0.0020 (2)

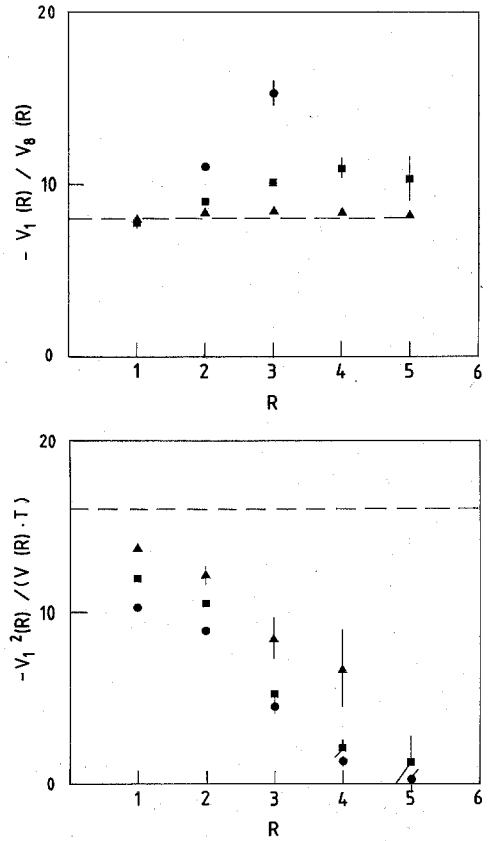

Fig. 2. The ratios $-V_1(R)/V_8(R)$ (a) and $-V_1(R)^2/V(R)T$ (b) versus R for three values of β. Symbols are as in fig. 1. The dashed line corresponds to the perturbative result.

factor $\exp[-\mu_D(T)r]$ becomes important, and we see in particular in fig. 2b that even at high temperature relation (3) is not well satisfied. On the other hand, the improved agreement at short distances shows that the "truly" perturbative aspects are better and better reproduced as T increases.

Our present analysis of the heavy quark potential is limited to distances $0.25 \leqslant rT \leqslant 1.5$, whereas the simple Debye screening form of the potential can become valid only in the limit $rT \to \infty$. At intermediate distances the structure of the potential may be more complicated [14]. To study the functional behaviour in more detail we parametrize the potentials in the general form

$$V(r)/T = [c(T)/(rT)^d] \exp(-\mu r) . \qquad (9)$$

We thus allow for an arbitrary power d in the "Coulomb" factor and assume a simple exponential decay governed by a screening mass μ. Note that this parametrization gives an effective screening mass describing the behaviour of the potential at intermediate distances. To study the asymptotic behaviour at large distances one could also use an ansatz including higher excited states, i.e. additional exponentials in eq. (9). We prefer here to define an effective screening mass $\mu_{R,d}$ at distance R through

$$\begin{aligned} V(R)/V(R+1) = \{ & R^{-d} \exp(-\mu_{R,d} R) \\ & + (12-R)^{-d} \exp[-\mu_{R,d}(12-R)]\} \\ & \times \{(R+1)^{-d} \exp[-\mu_{R,d}(R+1)] \\ & + (11-R)^{-d} \exp[-\mu_{R,d}(11-R)]\}^{-1} . \end{aligned} \qquad (10)$$

Here we take into account the periodicity of our $12^3 \times 4$ lattice. For any value of d these effective masses approach the same limiting value

$$\mu = \lim_{R \to \infty} \mu_{R,d} . \qquad (11)$$

If the potential drops according to a power law $1/r^{d_0}$, the approximants $\mu_{R,d}$ approach μ from above (below) for $d < d_0$ ($d > d_0$). The "best" choice for the exponent d obviously is obtained, if $\mu_{R,d}$ is roughly R independent already at short distances. In fig. 3 we show $\mu_{R,d}$ for the colour averaged potential and three different values of d. We see that apparently we need $1 \leqslant d \leqslant 2$ to get $\mu_{R,d}$ approximately R independent. In fig. 4 we show the behaviour of $\mu_{R,d}$ for the singlet potential. Here we seem to need $d \simeq 0$ [#3]. Detailed results for d and μ, obtained from a two-parameter fit of the ratios $V(R-1)/V(R)$, are given in table 2. Let us compare these results in some detail with perturbative calculations.

We noticed already in connection with fig. 2 that at $\beta = 5.75$ and 6.10 we do not see any indications for perturbative behaviour. This also follows from the analysis of the functional form of the potentials. At $\beta = 8.00$ the powerlike behaviour of the singlet potential is best described by $d = 0.24$ in the range

[#3] Note that for $d = 0$ the effective masses rise with increasing R. Similar behaviour has been found for the gluon propagator in Landau gauge [15]. There the increase of the effective mass has been taken as evidence for the non-positivity of the transfer matrix in this gauge.

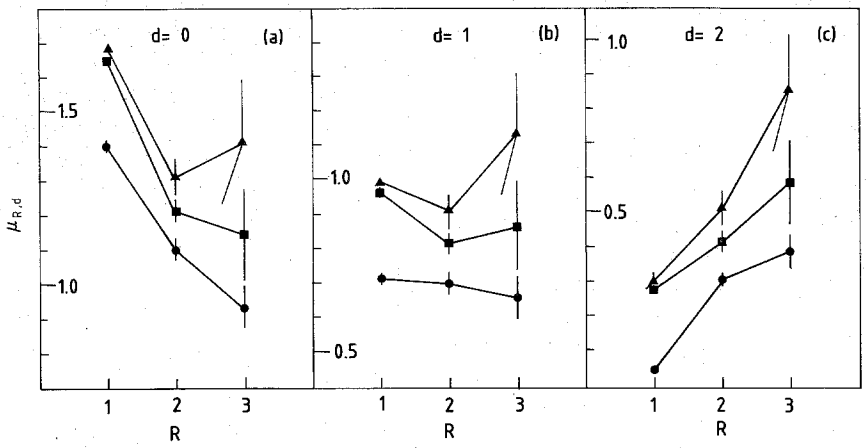

Fig. 3. Effective masses $\mu_{R,d}$ in inverse lattice spacings for the colour averaged potential versus R and $d=0$ (a), $d=1$ (b) and $d=2$ (c). Results are shown for the three values of β (symbols as in fig. 1). Lines are drawn to guide the eye.

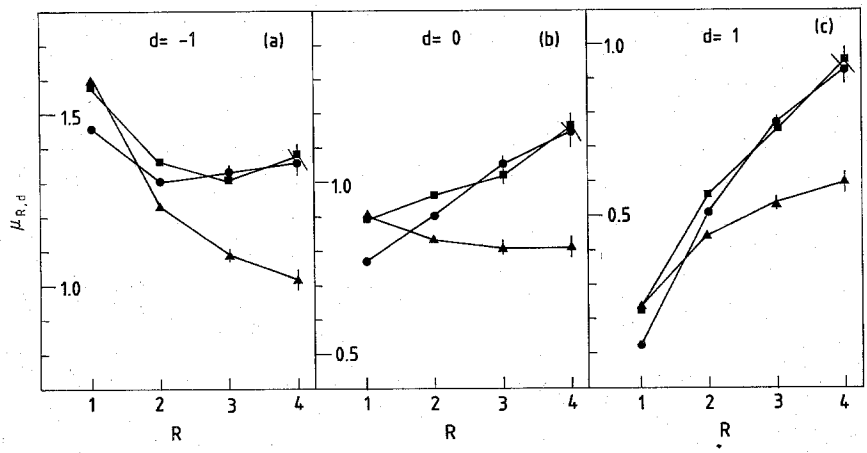

Fig. 4. The same as fig. 3, now for the colour singlet potential and $d=-1$ (a), $d=0$ (b) and $d=1$ (c).

Table 2
The power d and Debye mass μ for the colour averaged and singlet potentials obtained from a χ^2 fit of the ratios $V(R-1)/V(R)$ as discussed in the text.

β	Singlet		Colour averaged	
	μ	d	μ	d
5.75	1.11 (1)	−0.49 (1)	0.61 (7)	1.15 (10)
6.10	1.07 (1)	−0.26 (1)	0.55 (7)	1.59 (11)
8.00	0.74 (2)	0.24 (3)	0.58 (6)	1.59 (9)

$0.25<rT<1.5$. Besides this we see from figs. 4b and 4c that the effective masses extracted for $d=0$ and $d=1$ seem to give quite accurate upper and lower bounds for the asymptotic value of the screening mass at $\beta=8.00$:

$$2.4<\mu/T<3.2\,. \qquad (12)$$

This is to be compared with the perturbative result, $\mu_D/T=g(T)$. The large value found for μ/T would thus require a rather large value for the temperature-

dependent running coupling constant. To explain it by lowest order perturbation theory would demand an unreasonably high Λ value in the running coupling constant

$$g^2(T)=24\pi^2/33\ln(T/\Lambda)\,, \qquad (13)$$

namely $\Lambda \gtrsim 3T_c$. This in turn makes it likely to expect higher order terms to be relevant in the perturbative analysis. Unlike the singlet potential, which could be measured up to distances $rT=1.5$, the colour averaged potential could be determined by us only up to $rT=1$. This makes the bounds on the asymptotic values of the screening mass less stringent. At $\beta=8.00$ we find

$$2.0 \leqslant \mu/T \leqslant 4.0\,. \qquad (14)$$

This makes it difficult to judge whether asymptotically the relation $\mu_{\text{average}}=2\mu_{\text{singlet}}$ holds. To investigate the effects at finite lattice spacing, we have fitted our data to a lattice version of eq. (9), defined from a discretized Fourier transform. It turns out, however, that the mass value obtained from this fit differs very little from the one obtained by using eq. (9) and including the lattice periodicity.

At finite distances $rT \gtrsim 1$ we start feeling the finite size of the lattice used at present and we have to worry about the influence of these finite size effects. Work in this direction is in progress [16].

In conclusion, we find that the heavy quark potential exhibits a complicated structure at short and intermediate distances studied by us, i.e. $0.25 \leqslant rT \leqslant 1.5$. We see that for temperatures close to T_c, $1.0 < T/T_c \lesssim 3.0$, perturbative relations fail to describe the potential and no indications are found that they could describe the large distance behaviour of the potential. At large temperatures, $T \sim 10T_c$, we find some indications for the validity of perturbative relations like eqs. (3) and (7). However, even here the screening masses turn out to be rather large. Presumably even higher temperatures are needed to find complete agreement with perturbation theory.

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