PHASE STRUCTURE OF O(N)-SYMMETRIC ϕ_3^6 MODELS AT SMALL AND INTERMEDIATE N

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To test on the occurrence of the non-trivial BMB fixed point we study the phase diagram of O(N)-symmetric ϕ_3^6 models in the limit of infinite ϕ^6-coupling. Our Monte Carlo results for $N=5$, 10 and 30 confirm that the so-called BMB phenomenon could be a large-N effect.

An O(N)-symmetric $(\phi^2)^3$ theory in three dimensions with N scalar fields has several interesting features in the large-N limit [1–4]. For infinite N and in the limit of vanishing couplings μ_R and λ_R – the renormalized couplings of the quadratic and quartic terms in the action – scale invariance is an exact symmetry, the perturbative β-function of the $(\phi^2)^3$ coupling η_R vanishes. For a certain value of η_R, η_C, scale invariance is spontaneously broken, a mass is generated through dimensional transmutation. The former dimensionless free coupling parameter η_R is now fixed to η_C, while the corresponding degree of freedom is transmuted to the mass scale m. The fixed point at $\eta_R = \eta_C = 16 \pi^2$ (the so-called BMB fixed point) is non-trivial ($\eta_C = 0$) and UV-stable. This is to be distinguished from the fixed point at $\eta_R = \eta_P = 192$ which has been determined as a zero of the perturbative part of the β-function when $1/N$ corrections are taken into account [5]. However, for $\eta_R > \eta_C$, the non-perturbative contribution to the β-function dominates the perturbative one [1], at least for infinite N, where the perturbative part vanishes. Therefore in this limit the flow of couplings close to the η_R-axis is governed only by the BMB fixed point. These features are summarized as the BMB phenomenon [2].

For $N<\infty$ but still very large, one has to determine which of the following alternatives is realized. Either the non-perturbative part of the β-function still dominates the perturbative one, which is now dif-

ferent from zero and has the fixed point of ref. [5], or the non-perturbative contribution disappears (since it is a mere $N=\infty$ effect). In the first case the flux of couplings (close to the η_R-axis) would no longer exclusively, but still mainly, be governed by the BMB fixed point. Also the phase structure should exhibit the characteristic features due to the BMB phenomenon in this case [1,2].

To decide which of these features survives to finite N we have performed a Monte Carlo simulation for the ϕ_3^6 model in the limit of infinite ϕ^6-coupling. Since we use the lattice regularization we will not discuss any question related to the remnant of scale invariance. We concentrate on the existence of the BMB fixed point. A lattice theory defined at (or close to) this point had the chance of having an interacting theory as continuum limit which is stable, ultra-violet non-trivial and asymptotically not free.

There was already an extensive search for a non-trivial fixed point in ϕ^4 in four dimensions. Among other methods, Monte Carlo renormalization group calculations were used [6]. Starting at a large bare ϕ^4-coupling, a non-trivial fixed point could stop the block-renormalized quartic (and higher order) couplings from running to zero, i.e. it could prevent the triviality of ϕ^4 (although the mere existence of such a point wase not sufficient to prove the non-triviality). However, all results obtained up to now are negative in the sense that the observed flow of couplings is compatible with a free ϕ_4^4 continuum limit.

To find an indication for the BMB fixed point we could do the same type of (Monte Carlo) renormalization group calculations in ϕ_3^6 measuring the flow of couplings in a truncated space of three couplings for example. Such a kind of analysis was started also in ref. [7]. Instead of that we follow a proposal of David et al. [3] who considered the limit of $\eta_L \to \infty$ (η_L is the bare $(\phi^2)^3$ coupling on the lattice). In this limit they formulated an almost necessary condition for the occurrence of the BMB phenomenon. It is this condition we are going to check within the Monte Carlo method.

Starting from the bare continuum action

$$S = -\int d^3x \left[\tfrac{1}{2}(\partial_\mu \phi)^2 + \tfrac{1}{2}\tilde{\mu}_0^2 \phi^2 \right.$$

$$\left. + \tfrac{1}{4}\tilde{\lambda}_0(\phi^2)^2 + \tfrac{1}{6}\tilde{\eta}_0(\phi^2)^3 \right], \tag{1}$$

where $\phi(x)$ is an N-component scalar field, the phase diagram in the space of renormalized couplings (μ_R, λ_R, η_R) was derived from an effective potential calculation at infinite N [2]. Here we summarize only those features which are of interest for our following discussion. Points (μ_R, λ_R, η_R) with $\mu_R = 0 = \lambda_R$ and $\eta_R < \eta_C$ are tricritical, since the phase transition related to the spontaneous breaking of the $O(N)$ symmetry changes from first to second order at these points. At ($\mu_R = 0$, $\lambda_R = 0$, $\eta_R = \eta_C$) we have the BMB fixed point. For points (μ_R, λ_R, η_R) with $\eta_R > \eta_C$ a surface of first-order transitions continues into the $O(N)$ symmetric phase [1]. The order and type of phase transitions depend then on the value of λ_R. There exists a certain range of λ_R where for fixed η_R, λ_R and increasing μ_R^2, we first cross a second-order symmetry breaking transition and later on a first-order transition lying in the symmetric phase. We call it "situation I" if we observe the phase structure as it is seen crossing the η_R-axis for values $\eta_R < \eta_C$, and "situation II" if the phase structure is like that for $\eta_R > \eta_C$ and λ_R our of the range mentioned above.

Which situation occurs at $\eta_R \to \infty$ at finite N? For $N < \infty$, the assumption is that the qualitative distinction between I and II remains the same at finite N. As was argued in ref. [3], if the tricritical line extends to infinite η_R (situation I), the BMB fixed point can almost be excluded. In this case a simple meeting of first- and second-order transition lines in a tricritical point would indicate that no BMB fixed

point exists at finite η_R. Otherwise it would lead to a complicated phase structure at finite η_R corresponding to situation II. [Since the corresponding structure in the space of three couplings would be only a (almost) necessary condition, further investigations would be necessary to prove its existence.]

The $\eta_R \to \infty$ limit of the ϕ_3^6 model is defined in the following steps [3]. Expand (2) [a lattice version of eq. (1)] in powers of $1/\eta_L$,

$$S_L = -\sum_x \left(\frac{1}{2} \sum_{\mu=1}^3 [\phi(x+\mu) - \phi(x)]^2 \right.$$

$$+ \tfrac{1}{2}\mu_L^2 \phi^2(x) + (1/4N)\lambda_L[\phi^2(x)]^2$$

$$\left. + (1/6N^2)\eta_L[\phi^2(x)]^3 \right), \tag{2}$$

tune μ_L and λ_L such that a non-trivial limit-action results, finally redefine the field variables and couplings in a suitable way. Then the resulting action up to terms of order $1/\eta_L$ is given by

$$S_{L\eta} = N \sum_x \left(\mu \rho(x) \right.$$

$$\left. + \beta \sum_{\mu=1}^3 \rho(x)\rho(x+\mu)\Omega(x)\Omega(x+\mu) \right). \tag{3}$$

$\Omega(x)$ is an $O(N)$-vector normalized to one, $\rho(x) \in \{0, 1\}$ play the role of vacancy variables, β and μ are the remaining two coupling parameters (μ should not be confused with μ_L or $\tilde{\mu}_0$).

At finite N we expect a second-order symmetry breaking transition for $\mu \to \infty$, where $\langle \Omega \rangle$, the mean magnetization density, gets a non-vanishing expectation value, while $\langle \rho \rangle$, the mean vacancy density, equals one for all values of β (for $N = 1$, this is the second-order phase transition of the $d = 3$ Ising model). Decreasing μ dilutes the $O(N)$ spin system, $\langle \rho \rangle < 1$. It becomes more difficult to order the system, the inverse critical temperature β_c increases. For μ sufficiently small a second-order transition is now observed in both quantities $\langle \rho \rangle$ and $\langle \Omega \rangle$. At $\beta \to \infty$ and $\mu \to -3\beta$, we expect a first-order symmetry breaking transition, across which both $\langle \rho \rangle$ and $\langle \Omega \rangle$ are expected to jump (the first order is easily understood, since, depending on $\langle \rho \rangle$, the two situations
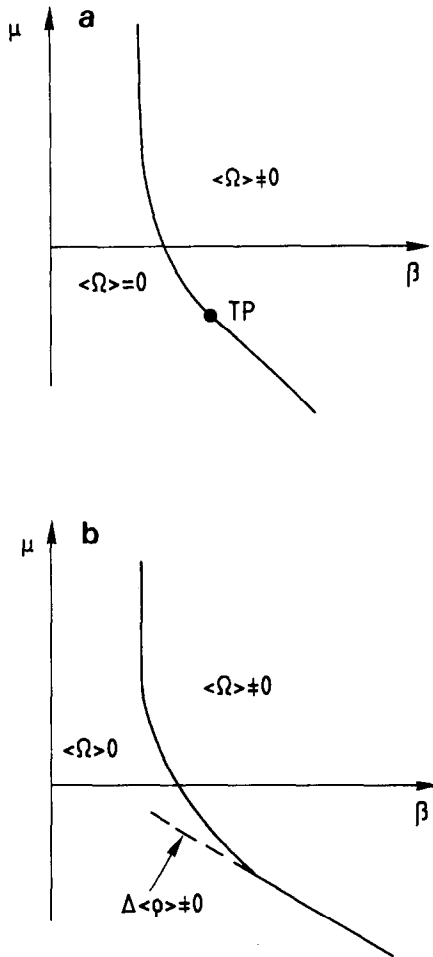
Fig. 1. Possible phase structure of $\eta_R \to \infty$ ϕ_3^6-model. One shows the case where second-order and first-order lines meet in a tricritical point (TP) (a) and the more complicated line meeting which would favour the existence of a BMB fixed point (b). See text for more explanations.

where all Ω's are aligned or where they are completely random are strongly favoured by the action).

As we have indicated above, the interesting part of the phase diagram is where the first- and second-order transition lines meet. In the space of the two couplings μ and β, the distinction between situations I and II is reflected as shown in figs. 1a and 1b. Either the second-order line goes over into the first-order line leading to the tricritical point TP or it ends on the first-order line such that the first-order line continues into the $O(N)$-symmetric phase. Symmetry breaking occurs along the full lines $\langle \rho \rangle$ and $\langle \Omega \rangle$

change according to the order of the transition as outlined above. Along the dashed line (fig. 1b) only $\langle \rho \rangle$ is expected to jump, while $\langle \Omega \rangle$ vanishes on both sides of this transition line.

On the basis of a mean-field calculation, David et al. have estimated which line meeting they expect depending on N. Integration of the Ω-fluctuations leads them to an effective vacancy action, from which they conclude that case I occurs for $N \leqslant 28$, while case II is possible only if $N > 28$. Therefore $N = 29$ is a lower bound for the occurrence of the BMB phenomenon in their analysis.

We have analyzed the type of line meeting more directly using the Monte Carlo method in the relevant part of the phase diagram. On an 8^3-lattice we measured $\langle \rho \rangle$ and $\langle \Omega \rangle$ as a function of μ and β. $\langle \Omega \rangle$ was defined as

$$\langle \Omega \rangle := \frac{1}{\text{Iter}} \sum_{\text{Iter}} \left| \frac{1}{\text{Vol}} \sum_x \Omega_x \right|. \tag{4}$$

The sums run over all lattice sites x and all equilibrium configurations Iter. For the upgrading procedure we used the Metropolis algorithm. To achieve an acceptance rate between 30% and 50%, we had to tune the "distance" of the rotation matrices R from the identity, the R's were used for the upgrading of Ω. At each point (μ, β) we have performed 10000 iterations for $N = 5$, 10 and 6000 for $N = 30$. The first 2000 have been discarded for equilibration. Errors have been calculated by grouping the data in blocks of 1000 for $N = 5$, 10 and blocks of 500 for $N = 30$ and determining the statistical fluctuations of the block-averages.

In figs. 2a–2d we show the behaviour of $\langle \rho \rangle$ and $\langle \Omega \rangle$ along lines of fixed μ close to the change from first- to second-order behaviour. In figs. 2a, 2b we see the second-order transition line of fig. 1 in $\langle \rho \rangle$ (fig. 2a) and $\langle \Omega \rangle$ (fig. 2b) for $\mu = 0$. The same quantities are plotted in figs. 2c, 2d for a smaller value of μ, $\mu = -0.2$, which cuts the phase diagram already in the first-order region. These figures clearly show that for $N = 5$ and 10, the first-order transition behaviour of $\langle \rho \rangle$ and $\langle \Omega \rangle$ is strongly correlated, i.e., we did not find any indication for an additional first-order transition in $\langle \rho \rangle$ which happens inside the phase where $\langle \Omega \rangle \simeq 0$. For $N = 30$, we analyzed the behaviour of $\langle \rho \rangle$ and $\langle \Omega \rangle$ close to the tricritical point in more detail. Result for runs at $\mu = 0.0$, -0.05
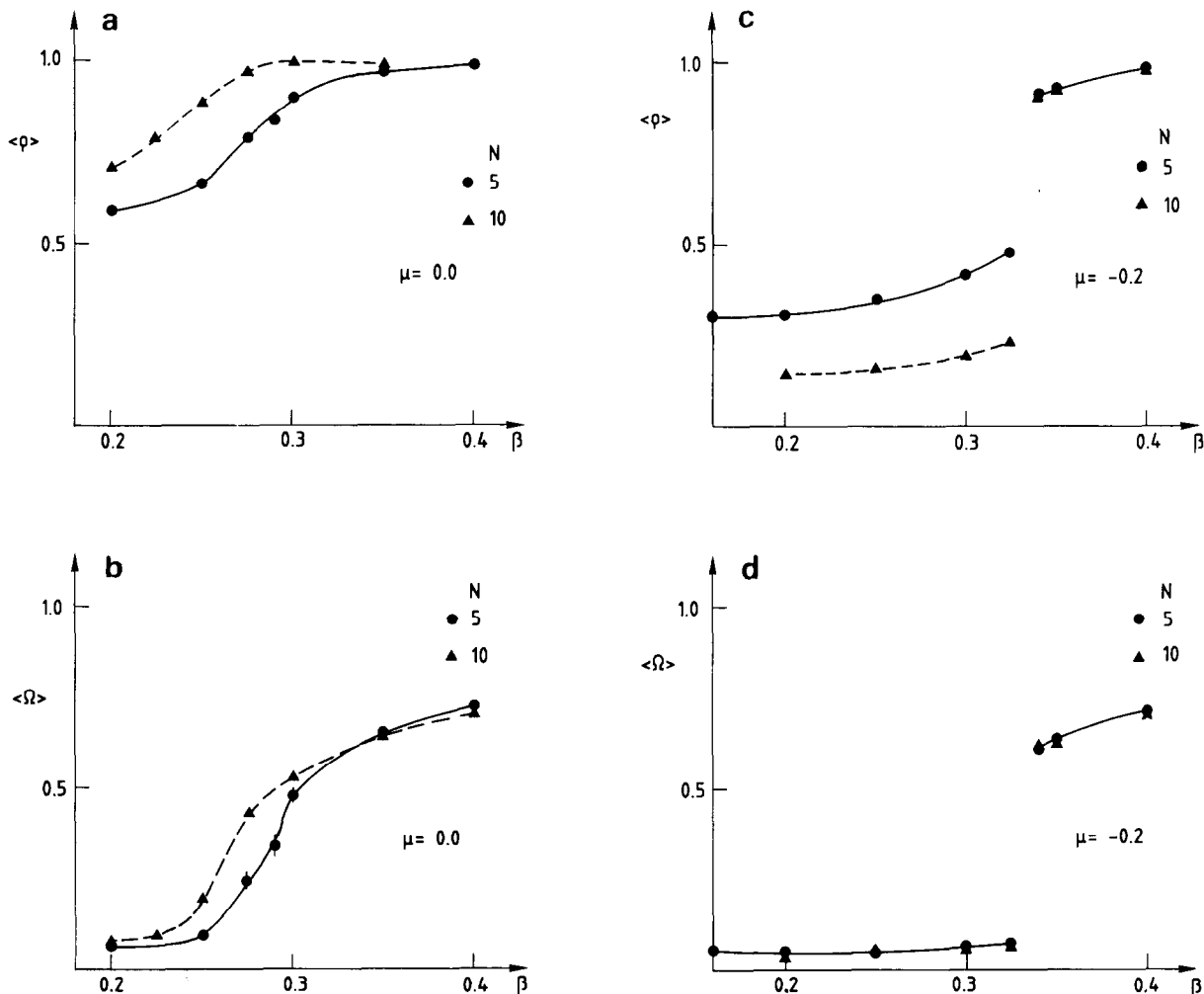
Fig. 2. Expectation values $\langle \rho \rangle$ and $\langle \Omega \rangle$ on an 8^3 lattice for $N = 5$ and 10 as a function of β. Shown are runs for $\mu = 0.0$ (a), (b), which yield second-order phase transitions and $\mu = -0.2$ (c), (d) where the transition is first order. Lines are drawn to guide the eye.

and -0.1 are shown in fig. 3. Again this gives no indication for discontinuity in $\langle \rho \rangle$ other than that related to the $O(N)$-symmetry breaking transition. Other runs at fixed β led to the same conclusion. The resulting phase diagram is shown in fig. 4. This seems to exclude the existence of the BMB fixed point for $N \leqslant 30$. The interval for which we cannot exclude a branching of first- and second-order transition lines for $N = 30$ was reduced to $-0.1 \leqslant \mu \leqslant -0.05$. The tricritical point has been localized in the interval $\beta_c \in (0.22, 0.27)$, $\mu_c \in (-0.1, -0.05)$.

Let us briefly comment on the N-dependence of

our results. Increasing N for fixed μ and β induces two competitive effects. Increasing N leads to an increase in the entropy due to the increasing degrees of freedom of the Ω's. This tends to lead to a larger critical coupling β_c with increasing N. On the other hand, an increase in N for $\mu > 0$ ($\mu < 0$) leads to an increase (decrease) in the average occupation number $\langle \rho \rangle$. Thus for $\mu > 0$ this tends to shift β_c to smaller values. As long as $\langle \rho \rangle < 1$ this effect dominates for
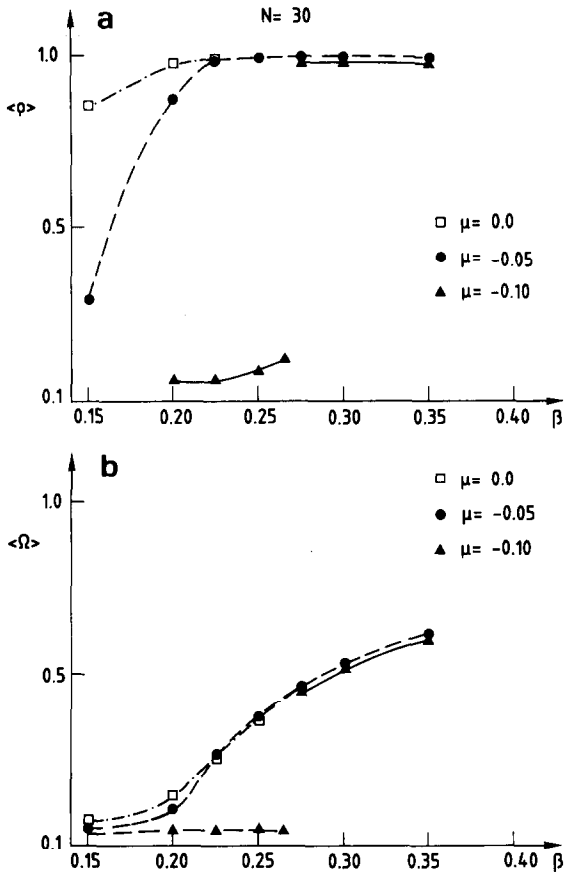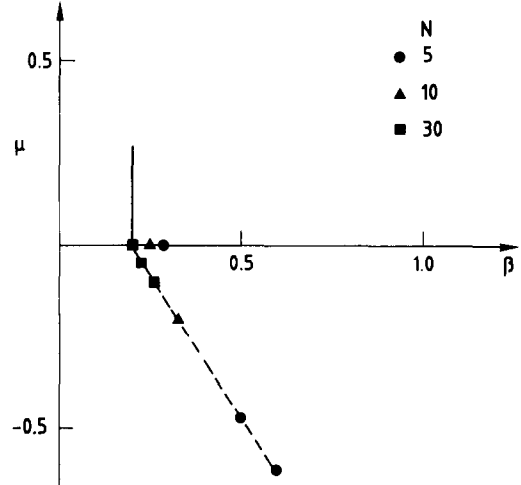
Fig. 4. Phase diagram of the $\eta_R \to \infty$ ϕ_3^6-model obtained from Monte Carlo simulations on an 8^3 lattice. The dashed line corresponds to first-order transitions while the solid line indicates second-order O(N)-symmetry breaking transitions.

Fig. 3. $\langle \rho \rangle$ and $\langle \Omega \rangle$ versus β for $N=30$ and various values of μ. Lines are drawn to guide the eye.

$\mu \geqslant 0$ as can be seen in figs. 2a, 2b [#1]. Also the uncertainty in the determination of β_c grows with N. For $\mu = 0.0$ we find $\beta_c = 0.285 \pm 0.01$ ($N=5$), $\beta_c = 0.26 \pm 0.01$ ($N=10$) and $\beta_c = 0.2 \pm 0.015$ ($N=30$). We take this as an indication for stronger finite-size effects. They should increase with increasing N as long as the correlations among the Ω's increase. For $\mu < 0.0$ and in the region of first-order transitions β_c is basically independent of N for $N \geqslant 5$. For fixed μ the gap in $\langle \rho \rangle$ increases as it is to be expected from absorbing the change in N in a redefinition of μ, while $\Delta \langle \Omega \rangle$ is N independent at β_c. For $N=30$ we find that $\langle \rho \rangle$ practically jumps from

0 to 1 at β_c ($\Delta \langle \rho \rangle \geqslant 0.9$ for $\mu \leqslant -0.1$). Thus we expect that in this region $N=30$ is in fact a good approximation of the $N \to \infty$ limit.

In summary, we have found at finite N a situation which corresponds to case I at infinite N. The result confirms the expectation of David et al. for $N \leqslant 28$. Moreover, even for $N=30$ (just above the lower bound of ref. [3]), there was no indication for a qualitative change in the phase structure. Therefore, in the space of three couplings, the tricritical line seems to extend to infinite η_R, at least up to values of $N \leqslant 30$.

Of course the present analysis does not rule out the existence of any other non-trivial fixed point that is not related to the occurrence of the BMB phenomenon at infinite N.

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Note added. After completing the paper we became aware of ref. [8], where it was shown within a variational Ansatz that at $N=\infty$ a nearest neighbor lattice action does not provide a suitable regularization to show the BMB phenomenon.

[#1] For $\langle \rho \rangle \sim 1$ an effective shift to larger β_c is expected. The phenomenological N-dependence of β_c is then given by $\beta_c \sim 0.253 - 0.032/N$ [3].

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