THE SU(3) β-FUNCTION AT LARGE β

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MCRG results are presented for the SU(3) β-function obtained from optimized blockspin transformations at β = 6.9 and 7 2. At both couplings for the shift $\Delta\beta$, corresponding to a change of length scale by a factor 2, $\Delta\beta$ = 0.51 is found. Although the error is large, deviations from asymptotic scaling seem to be larger than expected on the basis of earlier calculations.

Monte Carlo renormalization group (MCRG) methods have been used during the recent years to extract information about the β-function of lattice QCD from numerical simulations and to determine the regime of couplings where asymptotic scaling sets in, i.e. where physical observables scale according to the universal part of the SU(N) β-function

$$\beta(g) = -b_0 g^3 - b_1 g^5 + O(g^7), \tag{1}$$

with

$$b_0 = 11N/48\pi^2, \quad b_1 = \tfrac{34}{3}(N/16\pi^2)^2. \tag{2}$$

Previously we have used optimized blockspin transformations [1] and perturbatively improved Wilson loop ratio tests [2] to determine the β-function of pure SU(3) gauge theory for couplings up to β = 6.6. The results obtained with these different methods were consistent with each other and gave evidence for the presence of a pronounced dip in the β-function around β = 6.0 and a subsequent approach to the asymptotic value expected in the regime of validity of eq. (1)

$$\Delta\beta(\beta) = \beta(a) - \beta'(ba)$$
$$= (4Nb_0 + 8N^2 b_1/\beta)\log b + O(\beta^{-2}). \tag{3}$$

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Here b denotes the change in length scale per blocking step. Results obtained by us with a scale factor b = 2 [1,2] were broadly consistent with other MCRG calculations which used different scale factors [3] [‡1] and also with scaling behavior of physical observables like the deconfinement temperature [5] and string tension [6] (although the 0^+ glueball may be exceptional [7]) All these calculations seem to suggest that deviations from asymptotic scaling, as described by eq. (1), are small for couplings β somewhat larger than β = 6.0 [‡2].

The largest β value at which scaling has been tested by using $\sqrt{3}$ blockspin transformations has been β = 7.0 on a 9^4 lattice [3]. At this coupling agreement with asymptotic scaling has been found. Here we want to present our results obtained with b = 2 blockspin transformations using a 16^4 lattice at β = 6.9 and 7.2 as starting lattice. We seem to find larger deviations from asymptotic scaling than expected on the basis of earlier calculations. We will also discuss possible sources of

[‡1] Recently Petcher has tried to fit MCRG results for $\delta\beta$ to phenomenological ansatz for the β-function. He has found quite good agreement between fits to the b = 2 and b = $\sqrt{3}$ blockspin transformation data [4]

[‡2] A recent analysis of the scaling behavior of the heavy quark potential for couplings β in the interval (6.1, 6.7), however, finds still substantial deviations from asymptotic scaling [8].

systematic errors in our calculations.

Starting with a 16^4 lattice allows us to perform three subsequent block transformations. On each blocking level Wilson loops are measured and compared with corresponding loops measured on different blocking levels of 8^4 starting lattices at $\beta = 6.3, 6.4, 6.6$ and 6.7. The details of our blockspin transformation as well as the procedure followed to extract $\Delta\beta$ are discussed in ref. [1]. Our analysis at $\beta = 6.9$ and 7.2 follows the same criteria as used in earlier calculations and also the amount of statistics gathered at each value of β was the same (60 configurations on the 16^4 lattices and 96 configurations on the 8^4 lattices) To avoid the influence of correlations among the 60 configurations used in the analysis we separated them by 224 pseudo-heat-bath sweeps [9] at $\beta = 6.9$ and 112 at $\beta = 7.2$.

As discussed in ref. [1] we use a blockspin transformation suggested by Swendsen [10] which has an adjustable parameter P, i.e. gauge fields V_{AB} connecting sites A and B on the blocked lattice are selected according to the probability distribution

$$\mathrm{prob}(V_{AB}) \sim \exp[\tfrac{1}{2}P(\mathrm{Tr}\ V_{AB}X^{\dagger} + \mathrm{h.c.})], \tag{4}$$

with X denoting the sum over products of gauge fields along a selected set of paths connecting sites A and B [‡3]. In our previous analysis we found that on a given blocking level results for $\Delta\beta(\beta)$ depend linearly on $1/P$. Thus interpolation between different P-values could be used to determine the optimal P-value where best agreement between different observables could already be reached after a few blocking steps. In our present analysis we used $P = 25$ and 30 at $\beta = 6.9$ and $P = 27$ and 40 at $\beta = 7.2$. Results for $\Delta\beta(\beta)$ after two blocking steps are shown in figs. 1 and 2. The matching predictions for 4 selected loops at different blocking levels and P-values are summarized in tables 1 and 2. There we also show an estimate for the asymptotic $\Delta\beta^{(n=\infty)}$ value which would be expected if an infinite number of blocking steps were possible [1].

We observe that the P dependence of $\Delta\beta(\beta)$ becomes smaller with increasing number of blocking steps. This is expected, as ultimately $\Delta\beta(\beta)$ would be P independent if enough blocking steps could be performed to reach the renormalized trajectory. A comparison of the present results with corresponding data at $\beta = 6.6$

‡3 We use the set of 7 paths defined as scheme 1 in ref. [1].

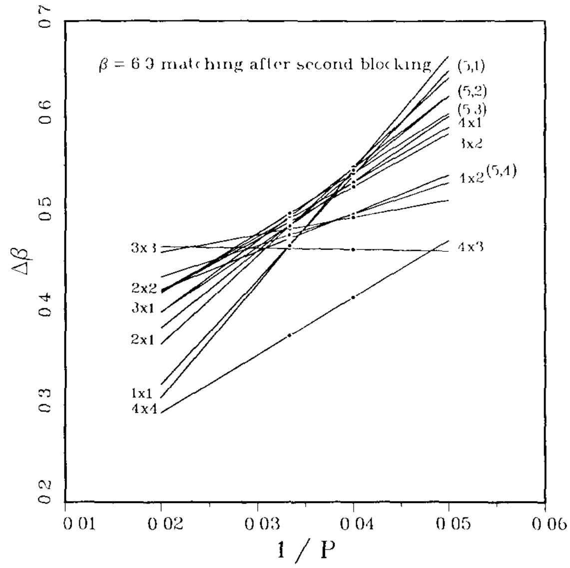

Fig. 1. The matching predictions obtained from various block loops after the second blocking step at $\beta = 6.9$. Dots show the measurements at $P = 25$ and 27. Errors have been omitted for clarity. They range from 0.005 for the 1×1 loop to 0.018 for the 4×4 loop. Results labeled with $(5, t)$ are obtained from non-planar loops of length t in time direction and $\sqrt{2}$ in the $x - y$ plane (see ref; [1] for details of the notation).

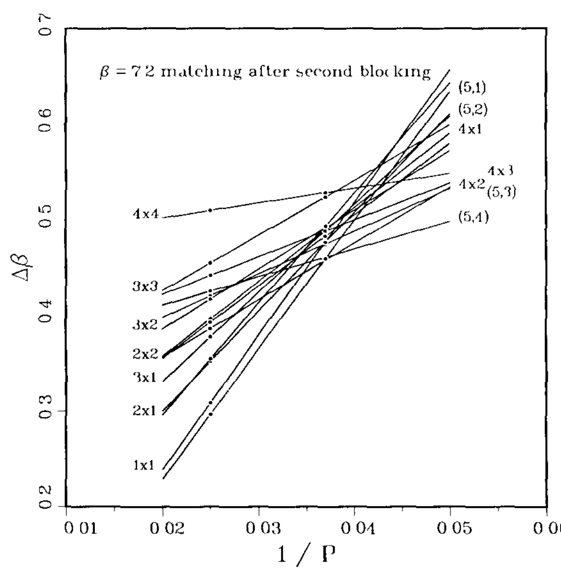

Fig. 2. The matching predictions obtained from various block loops after the second blocking step at $\beta = 7.2$. Dots show the measurements at $P = 25$ and 27. Errors have been omitted for clarity. They range from 0.017 for the 1×1 loop to 0.055 for the $(5,4)$ loop.

Table 1
The matching predictions $\Delta\beta(\beta = 6.9)$ are summarized for 4 different block loops at different blocking levels and different values of P. Also given are estimates for $\Delta\beta^{(n=\infty)}$ as discussed in ref [1]

P	blocking step	□	▭		
25	1	0.568(6)	0.613(7)	0.518(5)	0.490(5)
	2	0.550(12)	0.551(12)	0.543(12)	0.538(13)
	3	0.517(53)	0.498(54)	0.515(61)	0.513(60)
	$4\Delta\beta^{(n=2)}/3 - \Delta\beta^{(n=1)}/3$	0.544(19)	0.531(19)	0.551(18)	0.554(18)
	$4\Delta\beta^{(n=3)}/3 - \Delta\beta^{(n=2)}/3$	0.51(7)	0.48(7)	0.51(8)	0.50(8)
30	1	0.386(2)	0.475(6)	0.360(2)	0.351(3)
	2	0.479(13)	0.496(14)	0.476(12)	0.472(13)
	3	0.486(36)	0.482(35)	0.494(37)	0.493(44)
	$4\Delta\beta^{(n=2)}/3 - \Delta\beta^{(n=1)}/3$	0.510(18)	0.503(20)	0.514(17)	0.512(18)
	$4\Delta\beta^{(n=3)}/3 - \Delta\beta^{(n=2)}/3$	0.49(5)	0.48(5)	0.50(5)	0.50(6)

Table 2
The matching predictions $\Delta\beta(\beta = 7.2)$ are summarized for 4 different block loops at different blocking levels and different values of P. Also given are estimates for $\Delta\beta^{(n=\infty)}$ as discussed in ref. [1].

P	blocking step	□	▭		
27	1	0.428(9)	0.506(7)	0.388(9)	0.373(9)
	2	0.475(17)	0.486(14)	0.467(14)	0.463(16)
	3	0.490(30)	0.517(36)	0.484(32)	0.470(50)
	$4\Delta\beta^{(n=2)}/3 - \Delta\beta^{(n=1)}/3$	0.491(26)	0.480(21)	0.493(21)	0.493(24)
	$4\Delta\beta^{(n=3)}/3 - \Delta\beta^{(n=2)}/3$	0.49(5)	0.53(5)	0.49(5)	0.47(7)
40	1	0.001(11)	0.171(10)	0.013(11)	0.049(11)
	2	0.327(9)	0.364(15)	0.329(12)	0.326(15)
	3	0.402(31)	0.454(34)	0.379(36)	0.339(43)
	$4\Delta\beta^{(n=2)}/3 - \Delta\beta^{(n=1)}/3$	0.435(16)	0.428(23)	0.434(19)	0.419(24)
	$4\Delta\beta^{(n=3)}/3 - \Delta\beta^{(n=2)}/3$	0.44(4)	0.48(5)	0.40(5)	0.34(6)

of ref. [1] shows, however, that matching predictions on a given blocking level depend more strongly on P as β increases. The strong P dependence still present after the second and third blocking step reflects itself in the poor agreement of the estimates for $\Delta\beta^{(\infty)}$ for different P-values as shown in tables 1 and 2. Although a 16^4 lattice at $\beta = 6.9$ is presumably already in the deconfined region [5] this should not invalidate the MCRG method in principal, since the universal perturbative β-function is well defined and calculable even in a finite box. However, in practice the transient effects may become more severe at larger β. It may well be that at this value of the coupling constant the renormalized trajectory cannot be brought close enough to the Wilson axis by our one parameter blockspin transformation so that it can be reached within the 3 blocking steps that we can perform. Our data seem to indicate that $\Delta\beta(\beta)$ is systematically lower than what has been found at $\beta = 6.6$.

For the optimal P-value, where the best consistent matching of all different Wilson loops after the second blocking step has been found, we deduce from figs. 1 and 2

$$P^{\text{opt}} = 27^{+6}_{-3}, \quad \beta = 6.9, \tag{5a}$$

$$P^{\text{opt}} = 25^{+5}_{-3}, \quad \beta = 7.2. \tag{5b}$$

These values are in agreement with those found at β = 6.3 and 6.6 [1]. They still seem to decrease somewhat with increasing β, although on the basis of a perturbative analysis it is expected that asymptotically, for large $\beta, P \sim \beta$ [11]. This is necessary to ensure that the blockspin transformation will not randomize the initial configuration. For the range of P-values given in eq. (5) we find as matching values

$$\Delta\beta(\beta = 6.9) = 0.51 \pm 0.06, \tag{6a}$$

$$\Delta\beta(\beta = 7.2) = 0.51 \pm 0.07. \tag{6b}$$

Thus although the matching values at β = 6.9 and 7.2 agree within errors with earlier results at β = 6.6 [1] they are substantially smaller than expected if asymptotic scaling were to hold. These results are also in conflict with those presented in ref. [3] at β = 7.0 for $\sqrt{3}$ blockspin transformations. However, in all these calculations the error on $\Delta\beta(\beta)$ is large. Much higher statistics would be necessary to clarify the behavior of the β-function at these large coupling values. An analysis of larger lattices would be necessary to get control over the influence of transient effects by performing more blocking steps.

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