THE β-FUNCTION AND POTENTIAL AT $\beta = 6.0$ AND 6.3 IN SU(3) GAUGE THEORY

K.C. BOWLER[a], F. GUTBROD[b], P. HASENFRATZ[c], U. HELLER[d], F. KARSCH[e], R.D. KENWAY[a], I. MONTVAY[b], G S. PAWLEY[a], J. SMIT[f], and D J. WALLACE[a]

[a] *Physics Department, University of Edinburgh, Edinburgh EH9 3JZ, Scotland, UK*
[b] *DESY, Notkestrasse 85, D 2000 Hamburg 52, Germany*
[c] *Institut für Theoretische Physik, University of Bern, Sidlerstrasse 5, 3012 Bern, Switzerland*
[d] *CERN, 1211 Geneva 23, Switzerland*
[e] *Department of Physics, University of Illinois at Urbana-Champaign, 1110 West Green Street, Urbana, IL 61801, USA*
[f] *Institute of Theoretical Physics, University of Amsterdam, Valckenierstraat 65, 1018 XE Amsterdam, The Netherlands*

Received 13 June 1985, revised manuscript received 5 August 1985

Comparing optimized Wilson loop ratios on 16^4 and 8^4 lattices, matching predictions are obtained at $\beta = 6.0$ and 6.3 The static quark potential and string tension are also studied at these coupling constant values

An increasingly consistent picture of the continuum limit of SU(3) lattice gauge theory is emerging from recent Monte Carlo renormalization group studies [1–4]. These results suggest scaling behaviour for $\beta \gtrsim 5.7$ with a non-trivial, non-perturbative β-function. Asymptotic scaling seems to start only somewhat beyond $\beta = 6.0$. These predictions are in qualitative consistency with recent potential, string-tension [5–12] and critical-temperature [13,14] determinations.

It is relatively easy to give reliable statistical error estimates in these calculations. It is, however, much more difficult to keep control over the possible systematical errors. In a recent paper [2] the β-function [more precisely $\Delta\beta(\beta)$] was determined at $\beta = 6.0$, 6.3 and 6.6 using a non-perturbatively optimized block transformation in a MCRG study [15] [‡1]. The systematical errors were estimated by comparing the results of subsequent blocking steps and also by checking the effect of non-leading operators. In the present note we report on a calculation of $\Delta\beta(\beta)$ at $\beta = 6.0$ and 6.3 obtained by the optimized ratio method [1]. Since the analysis was performed on the same set of gauge configurations as in the previous study, the results give an independent estimate of the systematical errors involved. The ratio test [19] relies on a high-precision measurement of Wilson-loop expectation values which can be used at the same time to study the quark–antiquark potential and string tension. The results on these observables will be discussed also.

We analyzed 60 and 58 configurations at $\beta = 6.0$ and 6.3 respectively. These 16^4 configurations were separated by 112 pseudo heat-bath sweeps. They were created on DAP machines in Edinburgh. The loop measurements were done on a CYBER 205 computer in Amsterdam. In order to reduce the statistical errors the "multihit technique" of ref. [20] was applied. Neither time-correlation measurements, nor bunching the numbers indicated any correlations between

[‡1] A similar idea has been put forward and applied to the 3d Ising model recently by Swendsen [16] The basic idea of an MCRG analysis is suggested by Ma and Swendsen [17] Wilson [18] studied the method in gauge theories

0370-2693/85/$ 03.30 © Elsevier Science Publishers B.V. (North-Holland Physics Publishing Division)

subsequent configurations. Of course, we cannot exclude time correlations whose time scale exceeds O(5000) sweeps.

The Wilson-loop expectation values and errors at $\beta = 6.0$ and $\beta = 6.3$ are summarized in table 1. Comparison with other high-statistics results [6] shows that the multihit technique becomes quite effective for larger loops for which the statistical errors are reduced significantly The $\beta = 6.0$ data are in full consistency with those of ref. [9]

The ratio test requires Wilson-loop expectation values on 8^4 lattices data at $\beta = 5.6$, 5.7, 5.8 and 5.9 were used linearly interpolating to obtain expectation values at intermediate β values. The loops on the 8^4 lattices were calculated at CERN and the numbers were collected in ref. [21].

The starting point of the ratio test is the observation [19] that appropriate ratios of Wilson-loop expectation values are free of self mass and corner singularities, and, therefore, satisfy a homogeneous renormalization group equation if the loops involved are large enough. In case of loops of relatively small size, the remaining lattice artifacts can be cancelled order by order in perturbation theory [1,22]. By comparing ratios obtained on an 8^4 lattice with those composed of twice as large loops on a 16^4 lattice, an estimate of $\Delta\beta(\beta)$ is obtained. The function $\Delta\beta(\beta)$ gives the change of the coupling β which corresponds to a change of the lattice unit by a factor of 2.

The results on $\Delta\beta(\beta)$ obtained by using one-loop improved mixed ratios are summarized in table 2. The spread in the matching predictions (characterised by $\Delta\beta_{max} - \Delta\beta_{min}$) is small. On the basis of table 2 we conclude that

$$\Delta\beta(6.0) = 0.36 \pm 0\ 03,$$
$$\Delta\beta(6.3) = 0.45 \pm 0.03 \qquad (1)$$

Comparing these predictions with those obtained by MCRG blocking [2] [$\Delta\beta(6.0) = 0.35 \pm 0.02$ and 0.34 ± 0.02, $\Delta\beta(6.3) = 0.43 \pm 0.03$] a new estimate of the systematical errors is obtained.

The expectation values quoted in table 1 can be used to determine the potential energy of a pair of static quark–antiquark sources. The function

$$V_T(R) = -\ln\left[\langle W(R,T)\rangle / \langle W(R,T-1)\rangle\right] \qquad (2)$$

Table 1
Wilson-loop expectation values at $\beta = 6\ 0$ and $6\ 3$

Loops		Wilson-loop expectation values	
I	J	$\beta = 6\ 0$	$\beta = 6\ 3$
1	1	0 593617(94)	0 622487(52)
1	2	0 383437(147)	0 421953(86)
1	3	0 252418(158)	0 291501(109)
1	4	0 166949(145)	0 202325(111)
1	5	0 110532(125)	0 140607(101)
1	6	0 073194(101)	0 097755(84)
1	7	0 048475(81)	0 067983(66)
1	8	0 032090(65)	0 047289(57)
2	2	0 189867(152)	0 229388(103)
2	3	0 101181(128)	0 133662(100)
2	4	0 055074(98)	0 079470(76)
2	5	0 030188(69)	0 047575(58)
2	6	0 016592(44)	0 028551(42)
2	7	0 009120(27)	0 017153(30)
2	8	0 005010(18)	0 010312(21)
3	3	0 046832(101)	0 070118(87)
3	4	0 022677(71)	0 038354(67)
3	5	0 011151(43)	0 021284(48)
3	6	0 005518(28)	0 011882(34)
3	7	0 002733(16)	0.006648(22)
3	8	0 001353(9)	0 003733(14)
4	4	0 010031(52)	0 019758(51)
4	5	0 004547(28)	0 010413(38)
4	6	0 002084(18)	0 005546(24)
4	7	0 000961(10)	0 002970(15)
4	8	0 000444(5)	0 001593(9)
5	5	0 001926(15)	0 005253(29)
5	6	0 000825(9)	0 002692(19)
5	7	0 000356(5)	0 001390(10)
5	8	0 000154(3)	0 000720(6)
6	6	0 000334(7)	0 001334(13)
6	7	0 000136(4)	0 000667(7)
6	8	0 000055(2)	0 000336(5)
7	7	0 000052(2)	0 000326(5)
7	8	0 000020(1)	0 000158(3)
8	8	0 000007(1)	0 000074(2)

gives an upper bound for the true potential energy $V(R) = \lim_{T\to\infty} V_T(R)$.

The corrections which are due to excited states lying above the q–$\bar{\text{q}}$ ground state, are expected to decay exponentially in T. The leading correction can be parametrized as

$$V_T(R) = V(R) + \alpha\, e^{-\delta T}. \qquad (3)$$

Taking three subsequent values $V_{T_0-2}(R)$, $V_{T_0-1}(R)$ and $V_{T_0}(R)$, the three unknown parameters of eq. (3) are determined. By increasing T_0 up

Table 2
One-loop improved ratio results Ratios, whose $\Delta\beta$ prediction contains a statistical error exceeding the error cut are excluded from the analysis $\Delta\beta_{min}(\Delta\beta_{max})$ gives the smallest (largest) matching value in the set surviving the statistical error cut

β	statistical error cut	number of contributing ratios	average $\Delta\beta$	$\Delta\beta_{min}$	$\Delta\beta_{max}$
6 0	0 05	4455	0 356	0 334	0 389
	0 03	2253	0 356	0 343	0 377
6 3	0 05	4113	0 447	0 434	0 463
	0 03	2201	0 449	0 435	0 463

to $T_{0\,max} = 7$ ($T = 8$ is strongly contaminated by finite size errors) the consistency of the fit can be checked. We found greater consistency this way than with other methods, e.g. fitting $V_T(R)$ to a form linear in $1/T(T-1)$. The method used to extrapolate in T was discussed and advocated in ref. [23]. Our prediction of the potential is plotted in figs. 1 and 2 at $\beta = 6.0$ and 6.3 respectively. The systematical errors from the T_0 dependence are within the statistical errors shown.

The larger-distance part of the potential in figs. 1 and 2 is not linear in R. Its behaviour is consistent with the form $cR + c'/R$ + const. for large R, as is expected for a fluctuating flux tube [24]. The errors are too large, however, to allow a sensible three-parameter χ^2 fit.

Assuming that the long-distance part of the potential contains a $-\frac{1}{12}\pi/R$ term [24] and subtracting this piece from the potential, the points $R \geqslant 2$ are consistent with a linear fit giving

$$\sigma^{1/2}a = 0.22 \pm 0.02, \quad \beta = 6.0,$$
$$\sigma^{1/2}a = 0.15 \pm 0.02, \quad \beta = 6.3. \tag{4}$$

The numbers in eq. (4) give (taking $\sigma^{1/2} \approx 0.42$ GeV) $a(\beta = 6.0) = 0.10 \pm 0.01$ fm and $a(\beta = 6.3) = 0.07 \pm 0.01$ fm. Our 16^4 lattice corresponds to a periodic box of size $\sim (1.6\text{ fm})^4$ and $\sim (1.0\text{ fm})^4$ at $\beta = 6.0$ and 6.3 respectively. Asymptotic scaling predicts $a(6.3)/a(6.0) = 0.71$, which is consistent with eq. (4). Using the one-loop formula for Λ^{latt} we obtain

$$\sigma^{1/2} = (94 \pm 9)\Lambda^{latt}, \quad \beta = 6.0,$$
$$\sigma^{1/2} = (90 \pm 12)\Lambda^{latt}, \quad \beta = 6.3. \tag{5}$$

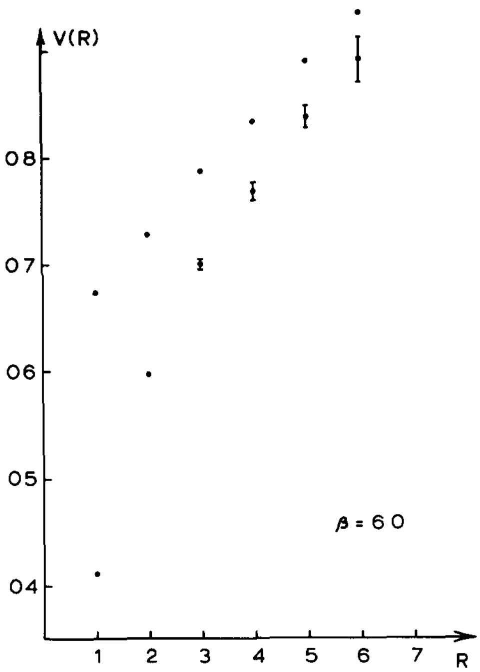

Fig 1 The potential at $\beta = 6\,0$ The upper points represent $V(R)$ shifted by $\pi/12R$, with error bars suppressed

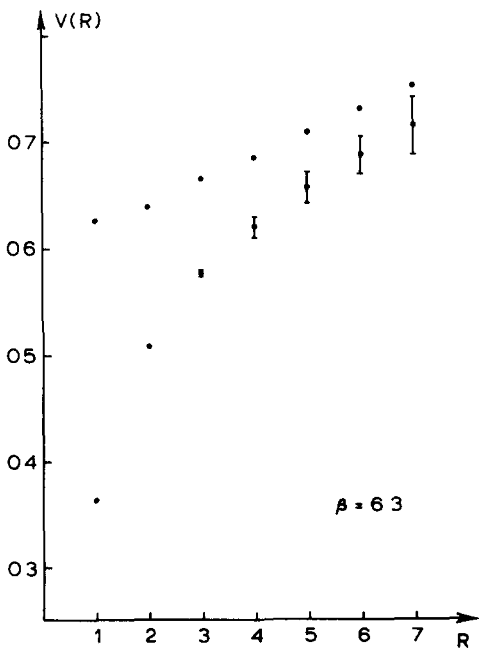

Fig 2 The potential at $\beta = 6\,3$

We should emphasize again that these predictions were obtained by assuming that the long-distance part of the potential contains a $-\frac{1}{12}\pi/R$ term Our data is consistent with this assumption but cannot really confirm it. Indeed it should be noted that the alternative interpretation of a $1/R$ term as a perturbative Coulomb effect [6] is also consistent and leads to a numerically similar coefficient.

We thank the NIKHEF-H for providing time on the Cyber 205 computer in Amsterdam and W.M. van Leeuwen for his help with this machine This work was supported in part by SERC grants NG 11849 and NG 14703.

References

[1] A Hasenfratz, P Hasenfratz, U Heller and F Karsch, Phys Lett 143B (1984) 193

[2] K C Bowler, A Hasenfratz, P Hasenfratz, U Heller, F Karsch, R D Kenway, H Meyer-Ortmanns, I Montvay, G S Pawley and D J Wallace, CERN preprint TH 3952 (1984), Nucl Phys B257 [FS14] (1985) 155

[3] R Gupta and A Patel, Caltech, preprint CALT-68-1142 (1984),
R Gupta, G Guralnik, A Patel, T Warnock and C Zemach, Caltech preprint CALT-68-1143 (1984)

[4] A D Kennedy, J Kuti, S Meyer and B J Pendleton, Phys Rev Lett 54 (1985) 87, Santa Barbara preprint NSF-ITP-85-11 (1985)

[5] F Gutbrod, P Hasenfratz, Z Kunszt and I Montvay, Phys Lett 128B (1983) 415,
A Hasenfratz, P Hasenfratz, U Heller and F Karsch, Z Phys C25 (1984) 191

[6] D Barkai, K J M Moriarty and C Rebbi, Phys Rev D30 (1984) 1293

[7] J Stack, Phys Rev D29 (1984) 1213

[8] S Otto and J D Stack, Phys Rev Lett 52 (1984) 2328, 53 (1984) 1028 (E)

[9] Ph de Forcrand, unpublished,
see also M Flensburg and C Peterson, Phys Lett 153B (1985) 412

[10] Ph de Forcrand and C Roisnel Phys Lett 137B (1984) 213

[11] Ph de Forcrand, G Schierholz, H Schneider and M Teper, DESY preprint 84-116 (1984)

[12] A Sommer and K Schilling, Z Phys C, to be published

[13] T Celik, J Engels and H Satz, Phys Lett 129B (1983) 323,
J Kogut, M Stone, H W Wyld, W R Gibbs, J Shigemitsu, S H Shenker and D K Sinclair, Phys Rev Lett 5D (1983) 393,
F Karsch and R Petronzio, Phys Lett 139B (1984) 403

[14] A D Kennedy, J Kuti, S Meyer and B J Pendleton, Phys Rev Lett 54 (1985) 87

[15] A Hasenfratz, P Hasenfratz, U Heller and F Karsch, Phys Lett 140B (1984) 76

[16] R H Swendsen, Phys Rev Lett 52 (1984) 2321

[17] S K Ma, Phys Rev Lett 37 (1976) 461,
R H Swendsen, Phys Rev Lett 42 (1979) 859

[18] K G Wilson, in Recent developments in gauge theories, eds G 't Hooft et al (Plenum, New York, 1980)

[19] M Creutz, Phys Rev D23 (1981) 1815,
R W B Ardill, M Creutz and K J M Moriarty, Phys Rev D27 (1983) 1956

[20] G Parisi, R Petronzio and F Rapuano, Phys Lett 128B (1983) 418

[21] See A Hasenfratz and P Hasenfratz, comps, Wilson loop expectation values in SU(3) (1984)

[22] F Karsch and U Heller, Nucl Phys B251 [FS13] (1985) 254

[23] N A Campbell, C Michael and P E L Rakow, Phys Lett 139B (1984) 288

[24] M Lüscher Nucl Phys B180 (1981) 317