SU(4) deconfining transition at strong coupling: A Monte Carlo study

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The effective theory describing SU(4) lattice gauge theory at finite temperature and strong coupling is investigated via Monte Carlo methods. A first-order deconfining transition is found. Introducing dynamical quarks leads to the disappearance of the transition at a large value of the quark mass, in agreement with recent mean-field calculations.

The behavior of SU(N) lattice gauge theories at finite temperature is an important theoretical question.[1–3] The $N=2$ and 3 deconfining transitions are well understood, but there has been some controversy as to the nature of the deconfining transitions for $N>3$.[3–5] In a recent paper[5] we found that such transitions were first order in the pure gauge sector at strong coupling, and were destroyed by the introduction of quarks of quite large mass. These conclusions were based on mean-field theory, which is not always reliable. It is thus desirable to test them via the Monte Carlo method. In this paper we present numerical simulations for the SU(4) case.[6] The $N>4$ cases should have a similar behavior.

Rather than treat the full SU(4) gauge theory, we start by rederiving an effective theory of Wilson line variables at strong coupling.[3–5,7] This is accomplished by neglecting spacelike plaquette variables (which is justified at strong coupling) and then performing the group integrations over the spacelike links. Thus although the original theory is defined on an $N_t \times N_s^3$ lattice, the effective theory is defined on an $N_s \times N_s \times N_s$ lattice (N_s = spatial extent, N_t = "time" or temperature extent). The effective partition function is given by[5]

$$Z(\beta')=\int \prod_{\vec{x}} dW_{\vec{x}} \exp\left[\beta' \sum_{\vec{x},\hat{i}} \operatorname{tr} W_{\vec{x}} \operatorname{tr} W^{\dagger}_{\vec{x}+\hat{i}} + \text{c.c.}\right] , \qquad (1)$$

where the integral is over the invariant group measure on each site of the three-dimensional lattice, the $W_{\vec{x}}$'s are the Wilson line variables defined on each site $\vec{x}$, the sum runs over the links of the spatial lattice, and $\beta' = z_{1;0}^{N_t}$ ($z_{1;0}$ is the character coefficient of the fundamental representation in a character expansion of the original lattice action). The simulation is simplified by noting that the $W_{\vec{x}}$'s can be diagonalized (only traces appear). Since $W \in \text{SU}(4)$, its eigenvalues can be written in the form $\exp(i\theta_j)$, $j=1,\ldots,4$ with $\theta_4=-\theta_1-\theta_2-\theta_3$. In terms of the θ's, Eq. (1) can be written as[4]

$$Z(\beta')=\int \prod_{\vec{x}} \left[\prod_i d\theta_i(x) M(\theta(\vec{x}))\right] \exp\left[2\beta' \sum_{\vec{x},\hat{i}} \sum_{i,j} \cos[\theta_i(\vec{x})-\theta_j(\vec{x}+\hat{i})]\right] . \qquad (2)$$

The group-measure factor M appearing in Eq. (2) is

$$M(\theta)=\prod_{i<j} \sin^2\left[\frac{\theta_i-\theta_j}{2}\right] . \qquad (3)$$

We include dynamical heavy quarks using the leading-order term in a hopping-parameter expansion.[5,8] This has the effect of adding a term

$$h \sum_{\vec{x}} \operatorname{tr} W_{\vec{x}} + \text{c.c.} = 2h \sum_{\vec{x},i} \cos[\theta_i(\vec{x})] \qquad (4)$$

to the effective action, where

$$h = 2n_f(2K)^{N_t} , \qquad (5)$$

K being the Wilson hopping parameter, and n_f the number of flavors. Equation (4) breaks the $Z(4)$ symmetry of Eq. (2), which, as is well known, characterizes deconfinement.[3] h acts as a symmetry-breaking external field.

A simple Metropolis algorithm was used, generating θ configurations from the effective partition function Eq. (2), with 10 hits per site and, for most of our data, $N_s=7$. In Fig. 1 the average action $\langle S \rangle$ (normalized to 1 for $\beta' \to \infty$) for the pure gauge theory ($h=0$) is plotted versus β' in a thermal cycle. The program was checked using strong- and weak-coupling expansions for $\langle S \rangle$. There is clear hysteresis between $\beta'=0.146$ and 0.155, indicating the presence of a first-order transition.

The most interesting quantity is the Wilson line order parameter characterizing quark confinement,[9]

$$\frac{1}{4}\frac{1}{N_s^3}\left\langle \left| \sum_{\vec{x}} \operatorname{tr} W_{\vec{x}} \right| \right\rangle = \frac{1}{4}\frac{1}{N_s^3}\left\langle \left[\left(\sum_{\vec{x}} \operatorname{tr} W_{\vec{x}} \right) \left(\sum_{\vec{x}} \operatorname{tr} W^{\dagger}_{\vec{x}} \right) \right]^{1/2} \right\rangle , \qquad (6)$$

which is related to the free energy of an isolated quark. This is plotted in a thermal cycle in Fig. 2, showing a similar hysteresis to $\langle S \rangle$. The dots are the result of mixed starts at various values of β', and show good agreement with the thermal cycle. The coexistence of two phases at $\beta'=0.155$ is quite evident from hot and cold starts (the points × and ○ in Fig. 2). The behavior of the order parameter as a function of the number of iterations on a 16^3 lattice (Fig. 3) shows that these two phases are quite stable at $\beta_c'=0.155$. This value for β_c' is in excellent agreement with mean-field

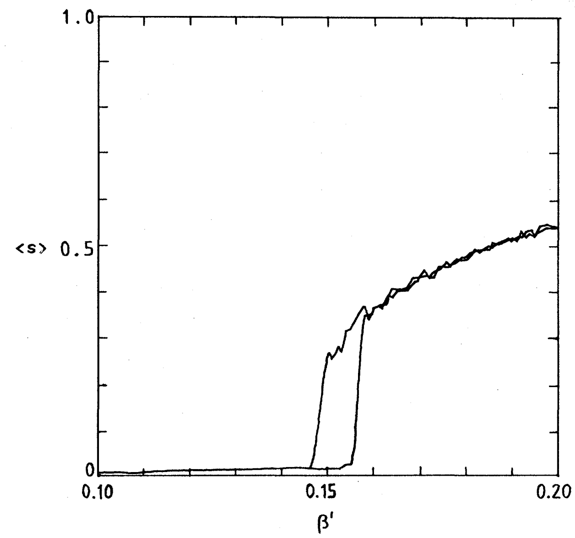

FIG. 1. Average action, $\langle s \rangle \equiv \frac{1}{32}\langle \mathrm{tr}\, W_{\vec{x}}\, \mathrm{tr}\, W^{\dagger}_{\vec{x}+\hat{i}} + \text{c.c.}\rangle$, plotted vs β'. After an initial 200 sweeps at 0.1, β' was increased from 0.1 to 0.2, and then decreased back to 0.1, in steps of $\Delta\beta' = 0.001$, with 50 iterations per step. The small-β' and large-β' tails of the curve are in good agreement with small-β' and large-β' expansions for $\langle S \rangle$. This is for a 7^3 lattice with 10 hits/site, for which the acceptance rate was typically 20%.

theory (which gives $\beta_c' = 0.16$, Ref. 5). The effective theory proved to be a very good approximation for $N = 3$ with $N_t = 1$.[5] We therefore predict, using the relation between $z_{1;0}$ and β $[= 1/(4g^2)]$, that the full SU(4) pure gauge theory has a first-order deconfining transition at $\beta_c = 0.148$ for $N_t = 1$. The smallness of β_c is consistent with our strong-coupling approximation.

Of course one is ultimately interested in taking $N_t \rightarrow \infty$

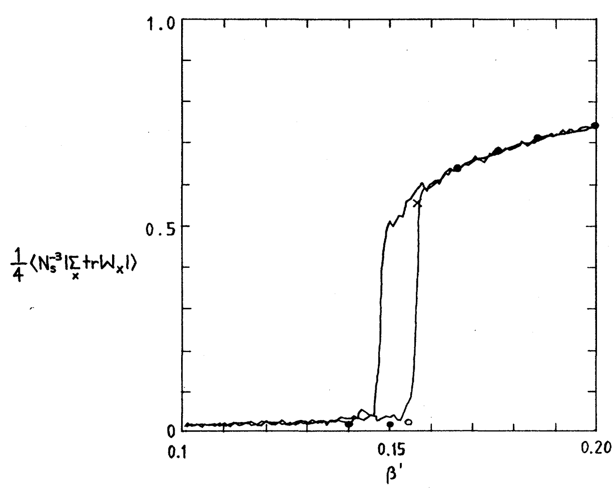

FIG. 2. Thermal cycle for the order parameter $\frac{1}{4}N_s^{-3} \times \langle | \sum_{\vec{x}} \mathrm{tr}\, W_{\vec{x}} | \rangle$, obtained in exactly the same manner as for the average action. The dots represent mixed starts, in which half of the lattice was placed in the high-temperature phase and half in the low-temperature phase. The points × and ○, both at $\beta_c' = 0.155$, are obtained from hot and cold starts, respectively, on a 16^3 lattice.

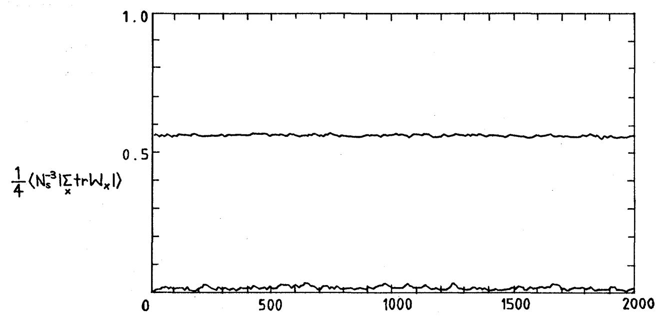

FIG. 3. Order parameter of Fig. 2 vs iterations on a 16^3 lattice. The upper line is a hot start, the lower a cold start.

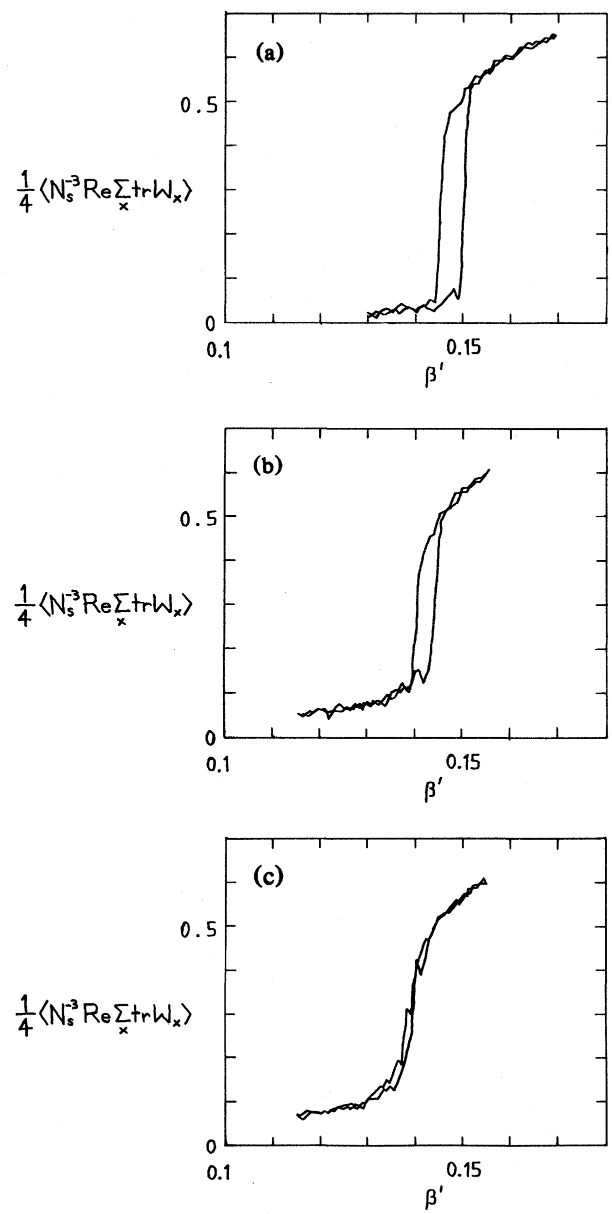

FIG. 4. Thermal cycle for the order parameter $\frac{1}{4}\mathrm{Re}\langle \mathrm{tr}\, W \rangle$ with the external field (a) $h = 0.02$, (b) $h = 0.06$, and (c) $h = 0.08$.

and $\beta_c \rightarrow 0$ in accordance with asymptotic freedom. The strong-coupling approximation ($N_t = 1$) is far from that limit. We can therefore draw no definite conclusion as to the nature of the transition in the physical, continuum limit. However, it is possible that our overall picture remains at least qualitatively correct.

Clearly the strong-coupling effective theory is not adequately described by a four-state clock model.[4] This model is known to have a second-order phase transition.[3,4,10]

The effect of dynamical heavy quarks is to increase h from zero. This causes the order parameter to be always nonzero, although it may still have a discontinuity. Thermal cycles for the order parameter are shown in Figs. 4(a), 4(b), and 4(c) for $h = 0.02$, 0.06, and 0.08, respectively. The critical temperature clearly decreases, since at $h = 0.02$, $\beta_c' = 0.15$, and at $h = 0.06$, $\beta_c' = 0.143$. There is virtually no hint of any hysteresis at $h = 0.08$, so for $h > h_c = 0.08$, the first-order transition disappears. [The value for h_c found in Ref. 5 (0.047) is probably too small due to the shortness of the series used to obtain it. A short expansion is not enough to display a large latent heat.] Using the crude relation between h and a quark mass as given in Ref. 5, the critical quark mass below which there is no longer a transition is (for $n_f = 3$) $m_c = 4.3 T_c$, where T_c is the deconfining temperature at h_c. Thus the critical mass is quite large. However, at small masses[11,12] there might still be critical behavior due to a chiral transition.

When this work was completed we received a preprint by M. Gross and J. F. Wheater (Oxford University) containing similar results for the pure gauge sector.

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