Nuclear Physics A418 (1984) 467c-476c
North-Holland, Amsterdam

THE INFLUENCE OF QUARKS ON THE SU(3) DECONFINEMENT PHASE TRANSITION

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We discuss the influence of quarks on the first order deconfinement phase transition present in pure SU(3) lattice gauge theory. Strong coupling considerations as well as Monte Carlo simulations on a $8^3 \times 2$ lattice indicate that this transition weakens rapidly with decreasing quark mass and disappears below a critical mass of the order of GeV.

1. INTRODUCTION

The existence of a finite temperature deconfining phase transition in a gauge theory is a long standing conjecture. For pure lattice gauge theories its existence has been proved analytically[1,2] and Monte Carlo calculations with SU(2) and SU(3) gauge groups have demonstrated convincingly that this phase transition survives the continuum limit[3-6]. These Monte Carlo results show a clear first order phase transition for pure SU(3) gauge theories[7,8] in accordance with theoretical expectations[9,10]. They also yield quantitative estimates for the critical temperature and latent heat[7,8,11,12] of the transition which are important parameters in judging the feasibility of producing gluon matter in laboratory experiments.

Of course (assuming that QCD is the correct theory), these experiments would deal with the complete theory: gauge and quark fields in interaction. While in many spectroscopical problems the effect of (virtual) quarks is believed to be small, in the case of the deconfinement phase transition it is expected to be relevant and qualitative. Indeed there are several indications that fundamental matter fields may destroy the phase transition present in the pure gauge sector of lattice gauge theories[13-17]: in pure SU(N) lattice gauge theories the deconfinement phase transition is related to the spontaneous breakdown of a global Z(N) symmetry of the pure gauge action. The nature of this phase transition can be analyzed by relating the finite temperature SU(N) theories to effective three-dimensional spin models with a global Z(N) symmetry[9]. In particular, in the case of SU(3) this led to a successful prediction of a first order phase transition. Similar considerations indicate that SU(N) gauge theories in the presence of fundamental matter fields are closely related to Z(N) spin models in the presence of an external field[14]. This suggests the disappearance of the deconfinement transition in the presence of

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arbitrary heavy quarks, if the transition was second order originally, or below a critical quark mass, or if the transition was first order in the pure gauge theory.

Thus in the absence of a global symmetry which allows to distinguish between different states of matter, the existence of a deconfinement phase transition becomes a quantitative rather than a qualitative problem governed by the relevant number of flavours and quark masses entering the QCD Lagrangian.

To clarify the fate of the first order deconfining phase transition present in pure SU(3) gauge theories, an explicit calculation in the full theory is needed. In the following section we will discuss a first quantitative analysis of this problem both in the strong coupling limit of lattice QCD[17] and by performing a Monte Carlo simulation on a $8^3 \times 2$ lattice[16]. Section 3 contains our conclusions.

2. DECONFINEMENT IN THE PRESENCE OF VIRTUAL QUARKS

2.1 General considerations

The inclusion of fermions in the Euclidean lattice formulation of gauge field thermodynamics[18,19] follows closely the zero temperature formalism. On a lattice of size $N_\sigma^3 \times N_\beta$ with lattice spacing a temperature T and volume V are given by $1/T = N_\beta a$ and $V = (N_\sigma a)^3$. The Euclidean partition function Z_E can be expressed in terms of bosonic link variables $U_{x,\mu} \in SU(N)$ alone, which obey periodic boundary conditions. In the case of n_f quark flavours we have

$$Z_E = \int \prod_{\substack{\text{links} \\ x,\mu}} dU_{x,\mu} \exp\{S_{eff}\}, \tag{1}$$

where the effective action

$$S_{eff} = S_G + S_F \tag{2}$$

contains the gluonic contribution

$$S_G = \frac{1}{g^2} \sum_{\text{plaquettes}} (\mathrm{Tr}\, U_{x,\mu} U_{x+\mu,\nu} U^+_{x+\nu,\mu} U^+_{x,\nu} + cc) \tag{3}$$

and a fermionic part resulting from the integration over the fermionic field variables

$$S_F = \sum_{f=1}^{n_f} \mathrm{Tr}\, \ell n(\mathbf{1} - K_f M). \tag{4}$$

In the case of Wilson fermions the fermionic matrix M is given by

$$M_{\mu,xy} = (1-\gamma_\mu)\tilde{U}_{x,\mu}\,\delta_{x,y-\mu} + (1+\gamma_\mu)\tilde{U}^{+}_{y,\mu}\,\delta_{x,y+\mu} \tag{5}$$

where $\tilde{U}_{x,\mu} \equiv \tilde{U}_{(\vec{x},x_4),\mu} = (1 - 2\delta_{x_4,N_\beta}\delta_{\mu,4})U_{x,\mu}$ due to the antiperiodic boundary conditions for fermions in the temperature direction. The quark masses m_f enter the partition function through the hopping parameter K_f, which in the naive continuum limit is given by $K_f = (8 + 2m_f a)^{-1}$.

The relevance of quarks for the problem of the existence of a finite temperature deconfining phase transition in QCD becomes obvious from the close relation between deconfinement and the spontaneous breaking of a global Z(N) symmetry realized in the gluonic part of the effective action. The gluonic action S_G is invariant under the global Z(N) transformation

$$U_{(\vec{x},x_4),4} \rightarrow zU_{(\vec{x},x_4),4} \quad , \quad x_4 \text{ fixed, } z \in Z(N). \tag{6}$$

However, Wilson loops which are closed due to the periodicity of the Euclidean lattice in the temperature like the thermal Wilson line

$$L_{\vec{x}} = \prod_{x_4=1}^{N_\beta} U_{(\vec{x},x_4),4} \tag{7}$$

transform non-trivially under the above transformation. As these kind of loops contribute to the fermionic part S_F of the effective action, the global Z(N) symmetry gets explicitly broken due to the presence of quarks. Only in the pure gauge theory the expectation value of the thermal Wilson line $\langle \mathrm{Tr}\, L_{\vec{x}} \rangle$ is an order parameter which allows us to distinguish between a Z(N) symmetric and a spontaneously broken phase. As $\langle \mathrm{Tr}\, L_{\vec{x}} \rangle$ also describes the free energy of a static quark in a gluonic heat bath

$$\langle \mathrm{Tr}\, L_{\vec{x}} \rangle \sim e^{-F_q/T} \tag{8}$$

the appearance of spontaneous symmetry breaking clearly corresponds to the deconfinement phase transition. In the presence of quarks $\langle \mathrm{Tr}\, L_{\vec{x}} \rangle$ is no longer an order parameter, i.e., $\langle \mathrm{Tr}\, L_{\vec{x}} \rangle \neq 0$ for all temperatures. Nonetheless a singularity in $\langle \mathrm{Tr}\, L_{\vec{x}} \rangle$ may still signal a phase transition. Probably the most direct way to search for a non-analyticity of the partition function is to look for a discontinuity in the plaquette expectation values

$$\langle P \rangle = \langle 1 - \frac{1}{N}\mathrm{Re}\,\mathrm{Tr}\, U_{x,\mu}U_{x+\mu,\nu}U^{+}_{x+\nu,\mu}U^{+}_{x,\nu} \rangle \tag{9}$$

or one of its derivatives with respect to g^{-2}. The expectation value $\langle P \rangle$ is directly related to the latent heat[12] and the free energy density[18]. A jump in $\langle P \rangle$ like in $\langle \mathrm{Tr}\, L_{\vec{x}} \rangle$ would therefore signal a first order phase transition.

To study the influence of quarks on the first order phase transition observed in pure SU(3) lattice gauge theory we have to deal with the contribution of S_F to the effective action. Unfortunately, an expansion of S_F in terms of the hopping parameter K_f leads to a highly non-local form of the effective action as Wilson loops of arbitrary length contribute. As the contribution of the long path becomes increasingly important for larger K_f, this approach seems to be extremely difficult for light quarks. In the case of heavy quarks (small K_f), however, the expansion can be truncated after a few terms[20] and the problem of performing a trustworthy calculation in the presence of heavy quarks becomes manageable. Unfortunately, it also becomes irrelevant for physics; the hadronic world contains three light quark species.

There is, however, an exception to the conclusion above. It might happen that all important changes in the nature of the deconfinement phase transition occur already for heavy quarks, that the first order phase transition is destroyed and smoothed out already at a mass value which is large.

To lowest order in the hopping parameter (for $N_\beta \leq 3$) the effective action has the form

$$S_{eff} = S_G + 2n_f(2K)^{N_\beta} \sum_{\vec{x}} (\mathrm{Tr}\, L_{\vec{x}} + \mathrm{Tr}\, L^{+}_{\vec{x}}) \tag{10}$$

where we have taken all fermion masses to be equal, $K_f \equiv K$. The thermal Wilson line thus provides an explicit Z(N) symmetry breaking term in the effective action similar to an external magnetic field in spin systems with a field strength $H = 2n_f(2K)^{N_\beta}$. Before we analyze the influence of this symmetry breaking term on the deconfinement phase transition in a Monte Carlo calculation, let us discuss what one obtains in the strong coupling limit.

2.2 Strong coupling limit

It is generally expected that in the pure gauge sector, the deconfinement phase transition is due to long range fluctuations in the thermal Wilson line while spatial degrees of freedom do not show critical behaviour in the transition region[9,10]. It is therefore suggestive to rewrite the partition function in terms of this variable and try to integrate out those variables which do not lead to long range fluctuations. In a strong coupling expansion this can be done systematically[9,10,21] and leads in lowest order to the following approximation to the SU(3) partition function in the presence of quarks

$$Z_E \simeq \int \prod_{\vec{x}} dL_{\vec{x}} \exp \{\beta' \sum_{\vec{x},\ell} (\mathrm{Tr}\, L_{\vec{x}} \cdot \mathrm{Tr}\, L_{\vec{x}+\ell} + cc) + H \sum_{\vec{x}} (\mathrm{Tr}\, L_{\vec{x}} + \mathrm{Tr}\, L_{\vec{x}}^{+})\} \qquad (11a)$$

with

$$\beta' = \left(\frac{1}{3g^2}\right)^{N_\beta} , \qquad (11b)$$

$$H = 2n_f (2K)^{N_\beta} . \qquad (11c)$$

This is a three-dimensional spin model for the thermal Wilson line in an external field H. The phase diagram of this strong coupling approximation can be studied in a meanfield approach[17]. Substituting $\mathrm{Tr}\, L_{\vec{x}}$, $\mathrm{Tr}\, L_{\vec{x}}^{+}$ by the mean value

$$M = \langle \mathrm{Tr}\, L_{\vec{x}} \rangle = \langle \mathrm{Tr}\, L_{\vec{x}}^{+} \rangle, \qquad (12)$$

which can be taken to be real, leads to the mean field free energy

$$F_{MF} = -\ln Z_{MF} - 6\beta' \langle \mathrm{Tr}\, L_{\vec{x}} \rangle_M^2 + (12\beta' M - 2H) \langle \mathrm{Tr}\, L_{\vec{x}} \rangle_M \qquad (13)$$

with

$$Z_{MF} = \int dU \exp\{-6\beta' M (\mathrm{Tr}\, U + \mathrm{Tr}\, U^{+})\} \qquad (14)$$

and

$$\langle \mathrm{Tr}\, U \rangle_M = \frac{1}{Z_{MF}} \int dU\, \mathrm{Tr}\, U \exp\{-6\beta' M (\mathrm{Tr}\, U + \mathrm{Tr}\, U^{+})\}. \qquad (15)$$

The expectation value of the thermal Wilson line M is determined self-consistently by minimizing F_{MF}. For H = 0 this leads indeed to a discontinuity in M as a function of β' thus indicating a first order phase transition in the pure SU(3) gauge theory as expected[22]. However, as can be seen in Fig. 1, this transition gets weakened rapidly in the presence of an external field H. The jump in the mean value M decreases rapidly with increasing H and disappears already for a critical field

$$H_c \simeq 0.059. \qquad (16)$$

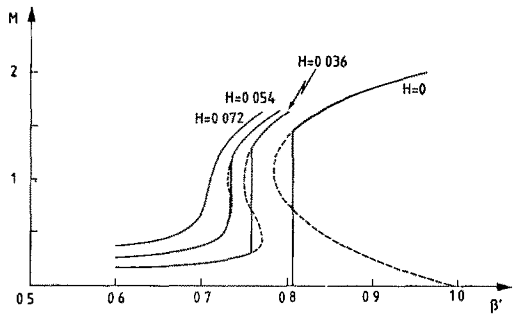

FIGURE 1
M versus the coupling β' for various values of the external field H. The solid (broken) lines correspond to stable (metastable) solutions of the mean field equation.

Although these strong coupling results may only be indicative for the continuum limit of lattice QCD, it is tempting to relate the critical field H_c found in this strong coupling calculation to actual physical quantities. An appropriate relation between the hopping parameter K and the quark mass m_q for small values of K is $2K \approx \exp(m_q q)$[15]. This allows us to determine the critical quark mass below which the deconfinement phase transition disappears in units of the critical temperature $T_c = (N_\beta a)^{-1}$

$$m_q/T_c = \ln (H_c/2n_f). \tag{17}$$

In the case of three quark flavours, which is most relevant for QCD, this leads to a large critical quark mass

$$m_q = 4.6\ T_c. \tag{18}$$

2.3 Monte Carlo calculations

Let us now turn to the Monte Carlo results obtained for a SU(3) theory with quarks using the effective action Eq. (10)[16]. The lattice size was taken to be $8^3 \times 2$. On this lattice the pure SU(3) gauge theory ($H \equiv 0$) exhibits a first order transition at $\beta(\equiv 6/g^2) = 5.11$ [7,8] with large discontinuities

$$\Delta L = 1.01 \pm 0.05 \tag{19a}$$

$$\Delta p = 0.034 \pm 0.005 \tag{19b}$$

in the thermal Wilson line and plaquette expectation values[7,8,12,16]. Our aim was to determine the critical line of first order transitions in the (H,β) plane, which starts at (0.0,5.11). To achieve this a value of β (<5.11) was fixed then H was tuned in trying to locate two metastable co-existing states, signaling a first order transition. At every point (H,β) two runs were made starting from ordered or random configurations and performing 2500-4000 iterations. In the critical region we observed two metastable states with occasional phase flips between them. Figure 2 shows the results for the thermal loop expectation value and the average plaquette at $\beta = 5.0$ as a function of $\sqrt{H}$.

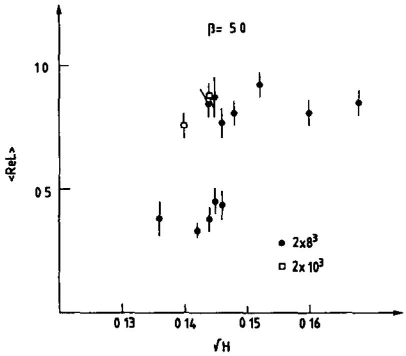

Fig. 2a

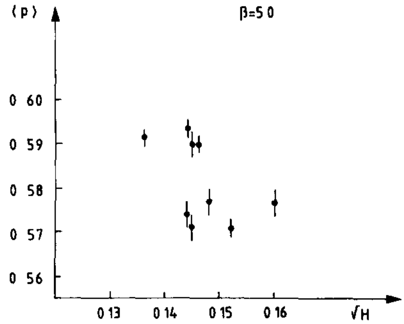

Fig. 2b

FIGURE 2

The thermal loop expectation value (a) and the average plaquette value (b) as a function of $\sqrt{H}$ at $\beta = 5.0$.

A clear signal for a first order phase transition can still be seen at $H \simeq 0.021$. However, compared to the corresponding quantities at $H = 0$ [Eq. (19)], the jump is already reduced by a factor of 2. Figure 3 shows the same quantitites at $\beta = 4.9$ where the discontinuities become even smaller.

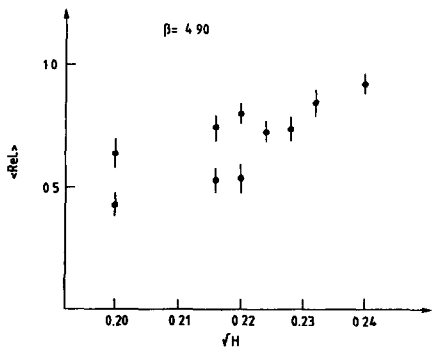

Fig. 3a

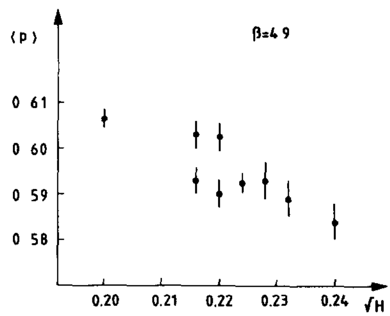

Fig. 3b

FIGURE 3

The same as Fig. 2 but at $\beta = 4.9$

In Fig. 4 we show the jump ΔL and Δp as a function of β and H which suggests that the line of first order transitions ends somewhere around

$$(H_c,\beta_c) \simeq (0.055, 4.85) \ . \tag{20}$$

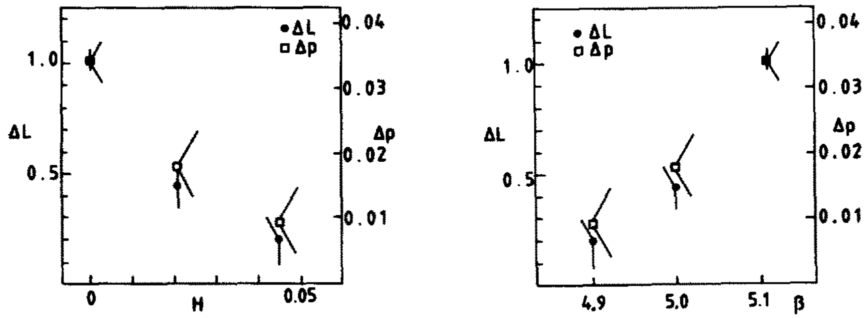

Fig. 4a Fig. 4b

FIGURE 4
The jump in the expectation value of the thermal Wilson line (ΔL) and the average plaquette value (Δp) as a function of H_c(a) and β_c(b).

It is remarkable how well the critical field H_c found in the Monte Carlo calculations agrees with the strong coupling result, Eq. (16). This might indicate that our Monte Carlo simulations still reflect strong coupling rather continuum behaviour. However, recent results obtained on a $8^3 \times 3$ lattice[23] seem to lead to a similar value for H_c. Let us therefore assume that our results resemble continuum physics and use H_c to determine a critical quark mass below which the first order deconfinement phase transition disappears. Our value for H_c is extremely small describing heavy quarks. The u,d and s quarks can be thought to be degenerate on this scale. Thus, with $n_f = 3$ we get $K \simeq 0.05$ for the critical hopping parameter. The value for massless quarks is around $K_c \simeq 0.2$ in this region of coupling constants[24]. Using as a crude estimate for the quark mass

$$\tfrac{1}{2}\left(\frac{1}{K} - \frac{1}{K_c}\right) = e^{m_q a} - 1 \tag{21}$$

we find

$$m_q/T_c = 4.2 \ . \tag{22}$$

This shows that the critical quark mass below which the deconfinement transition disappears seems to be large*.

3. CONCLUSIONS

We have studied the fate of the first order deconfinement phase transition of SU(3) lattice gauge theory in the presence of quarks. Strong coupling calculations, as well as Monte Carlo simulations, indicate that this transition disappears already for quite heavy quarks. Of course, these results do not exclude the possibility that in the physically interesting case of three light quark flavours, the system remembers the phase transition in the heavy quark sector by showing a large peak in the specific heat for instance. However, to study these effects considerably more work is necessary as long loops in the fermion determinant will contribute to the effective action.

ACKNOWLEDGEMENTS

It is a pleasure to thank my collaborators F. Green, P. Hasenfratz and I.O. Stamatescu for their support and many helpful discussions.

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*In leading order S_{eff} can be also interpreted as coming from the naïve fermion formulation. Using this interpretation, H is related to the quark mass via the relation $H = (4\ n_f/16)(1/2m_q a)^2$, giving $m_q/T_c \sim 3.7$ in good consistency with the above numbers.

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