SU(2) STRING TENSION FROM LARGE WILSON LOOPS

F. KARSCH
CERN, Geneva, Switzerland

and

C.B. LANG
Institut für Theoretische Physik, Universität Graz, Austria

We determine expectation values of Wilson loops and correlations of Polyakov loops on lattices of size 10×16^3 and 8×16^3 at β values 2.25 and 2.375. Utilizing a recently proposed method to reduce the variance of loop expectation values, we are able to measure loops up to 6×6. We find $\Lambda_L = 0.0151 \pm 0.0006 \sqrt{\kappa}$ at $\beta = 2.375$.

Recent measurements of the string tension in SU(3) [1,2] have revealed that the so far accepted ratio $\Lambda_L/\sqrt{\kappa}$ = 0.006 ± 0.002 [3] has to be considered as a lower bound and presumably to be corrected upwards by a significant amount. It seems that one has underestimated two aspects: (a) the asymptotic domain for large Wilson loops of size $R \times T$, i.e. $T \to \infty$ and large R, may not have been reached in the calculations, and (b) finite size (N_s) and finite temperature (N_t) effects (on lattices of size $N_t \times N_s^3$) may be larger than considered.

On one hand it appears to be extremely hard to investigate Wilson loops large with regard to the correlation length in the accessible scaling region (e.g. 4×4 or 5×5 at values $4/g^2 \approx 2.25-2.4$ for SU(2)). On the other hand there is only sparse information on the finite temperature dependance of the string tension. Usually one assumes that determination of the dimensionless string tension $a^2\kappa$ from Wilson loops gives the result at zero temperature whereas the determination via correlations of thermal (Polyakov) loops gives the values for non-zero temperature. Polyakov loop calculations of the conventional type have been restricted to $N_t \leqslant 6$; the result is exponentially damped like $\exp(-N_t Ra^2\kappa)$ which so far prohibited to obtain reliable signals at larger values of N_t (cf. ref. [4]).

Recently Parisi et al. [1] suggested to measure these correlations with the help of modified thermal loop operators. The essence of this suggestion is to reduce the statistical noise by substituting the link operator by an expression giving the same mean values with less variance. The newly introduced operator contains the products of the other links along the six plaquettes bordering the original link. The technique has been applied to spin models some time ago [5].

We briefly recapitulate the formalism extending it to operators which contain both the usual link variables U and the so-called modified ones defined by

$$\bar{U}_{x,\mu} \equiv \left(\int dU_{x,\mu} \, U_{x,\mu} \exp(g^{-2} \, \mathrm{tr} \, U_{x,\mu} X_{x,\mu}^{\dagger} + \text{h.c.}) \right) \left(\int dV_{x,\mu} \exp(g^{-2} \, \mathrm{tr} \, V_{x,\mu} X_{x,\mu}^{\dagger} + \text{h.c.}) \right)^{-1} , \qquad (1)$$

where $X_{x,\mu}^{\dagger}$ denotes the sum of the ordered products of the other three links around the bordering plaquettes, i.e.

$$U_{x,\mu} X_{x,\mu}^{\dagger} = \sum_{P \supset \text{link}(x,\mu)} U_P . \qquad (2)$$

We call $\bar{U}$ a modified link variable and $X_{x,\mu}$ the neighbourhood of $U_{x,\mu}$; note that usually X is not an element of

the group. Let us discuss the expectation value of $A U_i B$ where A and B contain link variables other than U_i. Utilizing the identity

$$\int dU \, dV \, U \exp\{g^{-2} \, \text{tr} \, [(U+V)X^\dagger] + \text{h.c.}\} = \int dU \, dV \, V \exp\{g^{-2} \, \text{tr} \, [(U+V)X^\dagger] + \text{h.c.}\} , \tag{3}$$

one finds

$$\langle A \bar{U}_i B \rangle = \left[Z^{-1} \int \left(\prod_{j \neq i} dU_j \right) dU_i \, dV_i \, A V_i B \exp\{g^{-2} \, \text{tr} \, [(U+V)_i X_i^\dagger] + \text{h.c.} + \text{other terms}\} \right]$$

$$\times \left(\int dV_i \exp(g^{-2} \, \text{tr} \, V_i X_i^\dagger + \text{h.c.}) \right)^{-1}$$

$$= \left[Z^{-1} \int \left(\prod_{j \neq i} dU_j \right) dU_i \, dV_i \, A U_i B \exp\{g^{-2} \, \text{tr} \, [(U+V)_i X_i^\dagger] + \text{h.c.} + \text{other terms}\} \right]$$

$$\times \left(\int dV_i \exp(g^{-2} \, \text{tr} \, V_i X_i^\dagger + \text{h.c.}) \right)^{-1}$$

$$= Z^{-1} \int \left(\prod_j dU_j \right) A U_i B \exp(g^{-2} \, \text{tr} \, U X_i^\dagger + \text{h.c.} + \text{other terms}) = \langle A U_i B \rangle , \tag{4}$$

provided neither A nor B contain U_i or modified link variables whose neighbourhood contains U_i (any link variable may be integrated only once!).

This possibility to substitute U by $\bar{U}$ generalizes obviously to characters and products of characters of operators and has therefore a wide range of possible applications. Since $\bar{U}$ is already the result of an averaging process one expects a significant decrease in the variance of the operator expectation values at least as long as the correlation length is of the order of the lattice spacing. Parisi et al. [1] applied this method to the measurement of correlations of thermal loops with spatial distance 2 and above. In the present work we exploit the possibility to mix link variables with modified link variables in the operators to determine both correlations of Polyakov loops and rectangular Wilson loops up to size 6×6 (see fig. 1). In parallel we also measure the expectation values of the old operators for comparison of the efficiency.

The gauge group is SU(2). For practical reasons we approximate it by its 120-element icosahedral subgroup $\tilde{Y}$. It is well known that this approximation is reliable for $4/g^2 \equiv \beta < 6$ [6]. For SU(2) one may evaluate (1) explicitely to obtain

$$\bar{U} = X I_2(\beta\lambda)/\lambda I_1(\beta\lambda) , \qquad \lambda \equiv (\det X)^{1/2} , \tag{5}$$

where I_1, I_2 are the modified Bessel functions. For given β this ratio is tabulated to be used in the subsequent measurements. We work with lattices of size 10×16^3 and 8×16^3 at $\beta = 2.25$ and 10×16^3 at $\beta = 2.375$. Some of our results may be compared with those of ref. [4] for lattice size 6×16^3.

Equilibrium configurations are obtained by preparing first $N_t \times 8^3$ configurations in equilibrium after 500 MC iterations, copying them to $N_t \times 16^3$ and performing another 400 iterations. Then we measure during the next 400 iterations: correlations of thermal loops for each configuration, Wilson loops in the old way every 5th configuration (i.e. on a total of 80), Wilson loops with the new method every 25th configuration (i.e. a total of 16). The
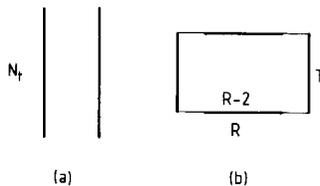

Fig. 1. (a) Thermal loops constructed from modified link variables (bold lines), (b) a Wilson loop of size $R \times T$ constructed from 4 link variables and $2R + 2T - 4$ modified link variables.

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program is a multispin coded version of that given in ref. [7].

Before we discuss the results let us compare the time requirements of the new method and its efficiency with regard to the decrease of variance with that of the classical method. For lattice size 10×16^3 the measurement of all rectangular Wilson loops with $1 \leqslant R, T \leqslant 5$ took 18.25 s (all values are for a CDC 7600) with the standard method, roughly a factor 5 less than the time required for the new method, that is 80 s for the determination of loops with $4 \leqslant R, T \leqslant 6$. For Polyakov loops the corresponding numbers are 0.03 and 2.15 respectively. The time for one MC update was 7.6 s (46 μs per link). Comparison of results, where they have been determined both ways, show agreement within the errors with standard deviations via the new method typically smaller by factors 5–10 for the rectangular loops and 10–20 for the thermal loop correlations. Since these statistical errors depend on the CP time by square-root law the new method wins by a factor 5–15 (in CP time) in the case of large fluctuations.

The free energy of a static charge–anticharge pair may be obtained from the correlation of thermal loops

$$L(x) = \text{tr} \prod_{i=0}^{N_t-1} U_{x+i\mu,\mu} \,, \tag{6}$$

i.e. the static potential

$$V_{N_t}(R) = -N_t^{-1} \log (\langle L(0)L(R)\rangle - \langle L(0)\rangle^2) \,. \tag{7}$$

For a critical temperature $T_c = 43\Lambda_L$ [8] the values $N_t = 10, 8, (6)$ correspond to temperatures $0.35\,T_c, 0.43\,T_c$, $(0.58\,T_c)$ at $\beta = 2.25$ and $0.48\,T_c, 0.59\,T_c, (0.79\,T_c)$ at $\beta = 2.375$.

Another way to estimate the potential is from ratios of elongated $(T \gg R)$ Wilson loops [9]

$$V^{T,T-1}(R) = -\log [W(T,R)/W(T-1,R)] \,. \tag{8}$$

Table 1
Values for the static potential obtained from elongated Wilson loops and correlations of thermal loops. The data for lattice size 6×16^3 are from ref. [4].

β	n_t	n_s	10×16^3		8×16^3		6×16^3
			$V^{n_{t_1}n_t-1}(n_s)$	$V_{10}(n_s)$	$V^{n_{t_1}n_t-1}(n_s)$	$V_8(n_s)$	$V_6(n_s)$ [a]
2.25	5	1	0.460 ± 0.003	0.462 ± 0.005	0.463 ± 0.004	0.464 ± 0.002	0.459 ± 0.002
	5	2	0.764 ± 0.010	0.768 ± 0.003	0.766 ± 0.012	0.766 ± 0.002	0.748 ± 0.010
	5	3	0.988 ± 0.038	$1.073^{+0.103}_{-0.050}$	0.986 ± 0.041	1.033 ± 0.014	0.962 ± 0.023
	6	4	$1.218^{+0.129}_{-0.123}$	$1.046^{+0.068}_{-0.040}$	–	–	1.132 ± 0.078
2.375	5	1	0.368 ± 0.002	0.389 ± 0.001	–	–	0.382 ± 0.002
	5	2	0.589 ± 0.007	0.590 ± 0.001	–	–	0.563 ± 0.002
	5	3	0.720 ± 0.013	0.730 ± 0.010	–	–	0.668 ± 0.010
	6	4	0.832 ± 0.036	0.845 ± 0.010	–	–	0.738 ± 0.014
	6	5	0.920 ± 0.061	0.918 ± 0.020	–	–	–
	6	6	0.968 ± 0.099	$0.939^{+0.028}_{-0.022}$	–	–	–
		7	–	$0.953^{+0.033}_{-0.025}$	–	–	–
		8	–	$1.018^{+0.076}_{-0.043}$	–	–	–

[a] Data from ref. [4].

In table 1 we compare the values for the potential defined in (7) and (8). Let us mention that without the variance reducing method of ref. [1] it would have been hardly possible at all to measure the Polyakov loop correlations at lattices with $N_t = 10$. We find surprisingly good agreement between the values obtained from the two different definitions of V. For $\beta = 2.25$, where we have values for $N_t = 10$ and $N_t = 8$ we find no significant difference between them, the first indication of temperature dependance may be seen from the comparison with results from ref. [4] for $N_t = 6$.

Estimating the string tension from the asymptotic slope at $\beta = 2.25$ we average over the four (from 10×16^3 and 8×16^3) values $V(3) - V(2)$ and obtain $a^2\kappa = 0.254 \pm 0.040$, at $\beta = 2.375$ a linear fit to $V(3)$, $V(4)$ and $V(5)$ gives $a^2\kappa = 0.104 \pm 0.009$. The potential for $\beta = 2.375$ exhibits for $R \geqslant 6$ a tendency to flatten faster than one naively expects from finite size periodicity; a fit with a sum of exponentials accounting for the periodicity is not satisfactory. A possible explanation may be an underestimation of the errors: at larger distances the correlation might need a very high number of iterations in order to build up correctly. Much longer runs could clarify the situation.

We compare the values for the string tension with the Creutz-type estimates from ratios of almost quadratic Wilson loops,

$$\chi(R, T) = -\log\left[W(R, T)W(R-1, T-1)/W(R, T-1)W(R-1, T)\right] . \tag{9}$$

The new method allows to obtain reliable values at $\beta = 2.25$ up to $\chi(4, 4)$, i.e. including 4×4 loops and at $\beta = 2.375$ up to $\chi(5,5)$. The values for the next larger loop size are already of the order of the errors. In fig. 2 we show the dependance on the loop size and find a clear signal that most of the prior determinations have not obtained the asymptotic regime. We see from the figure that the estimate for $a^2\kappa$ for large R is more reliable for $\beta = 2.375$ than for $\beta = 2.25$, where no asymptotic saturation is observed.

Let us discuss scaling by comparing ratios $a(2.25)/a(2.375)$ obtained in several ways.

(i) If one tries to change the scale of a for the potential values $V(2)$ to $V(5)$ at $\beta = 2.375$ such that they agree with the shape of the potential at $\beta = 2.25$ we obtain rough agreement for

$$a(2.25)/a(2.375) = 1.65 \pm 0.15 . \tag{10}$$

(ii) Comparison of the "asymptotic" slopes gives

$$a(2.25)/a(2.375) = 1.56 \pm 0.25 . \tag{11}$$

This value might well be smaller due to the discussed uncertainty of the string tension at $\beta = 2.25$. Asymptotic scaling as predicted from lowest-order RG gives
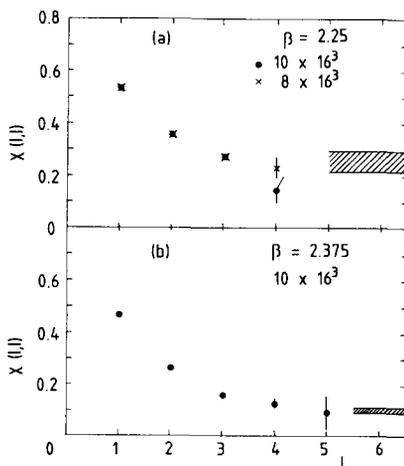

Fig. 2. Values of $\chi(l, l)$ for different values of l compared to the estimates of the asymptotic values from the potential slopes (shaded region): (a) $\beta = 2.25$, (b) $\beta = 2.375$.

$$a(2.25)/a(2.375) = 1.37\,[1 + O(g^2)]\,. \tag{12}$$

Although this might imply a slight violation of asymptotic scaling we think that one also could blame the questionable value of $a^2\kappa$ at the smaller β value. From the reliable value of $a^2\kappa$ at $\beta = 2.375$ we obtain

$$\Lambda_L = (0.0151 \pm 0.0006)\sqrt{\kappa}\,, \tag{13}$$

larger than previously published values [+1].

The method proposed by Parisi et al. [1] to reduce the variance in measurements of thermal loops is well apt for rectangular Wilson loops too. The improvement is roughly a factor 5—15 in overall gain (2—4 in the statistical errors) when the correlation length is of the order of the lattice spacing. Correlations of thermal loops may in this way be measured even for lattices with $N_t = 10$, and Wilson loops for R, T larger than usual up to now. Earlier determinations of the string tension at the onset of the scaling regime might not have been extracting the correct asymptotic slope.

[+1] Having completed this paper we became aware of ref. [10] where an even larger value is obtained with the conventional method, and of ref. [11] where correlations of thermal loops for SU(2) are determined with the new method too.

References

[1] G. Parisi, R. Petronzio and F. Rapuano, Phys. Lett. 128B (1983) 418.
[2] F. Gutbrod, P. Hasenfratz, Z. Kunszt and I. Montvay, Phys. Lett. 128B (1983) 415.
[3] M. Creutz, Phys. Rev. Lett. 45 (1980) 313;
    E. Pietarinen, Nucl. Phys. B180 (1981) 349;
    M. Creutz and K.J.M. Moriarty, Phys. Rev. D26 (1982) 2166.
[4] C.B. Lang and C. Rebbi, Phys. Lett. 115B (1982) 137.
[5] H.B. Callen, Phys. Lett. 4B (1963) 161;
    R.L. Dobrushin, Theory Prob. Appl. 13 (1969) 387;
    O.E. Lanford III and D. Ruelle, Commun. Math. Phys. 13 (1969) 194.
[6] G. Bhanot and C. Rebbi, Nucl. Phys. B180 (1981) 469;
    D. Petcher and D. Weingarten, Phys. Rev. D22 (1980) 2465.
[7] G. Bhanot, C.B. Lang and C. Rebbi, Comput. Phys. Commun. 25 (1982) 57.
[8] J. Engels, F. Karsch, I. Montvay and H. Satz, Nucl. Phys. B205 (1982) 545.
[9] J.D. Stack, Phys. Rev. D27 (1983) 412.
[10] F. Gutbrod and I. Montvay, DESY preprint 82-112 (November, 1983).
[11] A. Billoire and E. Marinari, CEN Saclay preprint (December, 1983).