THE SU(3) DECONFINEMENT PHASE TRANSITION IN THE PRESENCE OF QUARKS

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The fate of the deconfinement phase transition is studied as a function of the quark mass on a $8^3 \times 2$ lattice. The first order phase transition present in the pure SU(3) lattice gauge theory weakens rapidly as the quark mass is decreased and no such transition is observed below a critical mass value. This critical mass appears to be large, of the order of GeV.

The presence of a deconfining phase transition in SU(2) and SU(3) lattice gauge theories has been convincingly demonstrated by Monte Carlo studies [1–4]. The results show a clean first order phase transition for pure SU(3) gauge theories [5,6] in accordance with the theoretical expectations [7] [‡1]. There are quantitative estimates for the critical temperature and latent heat [5,6,9] which are important parameters in judging the feasibility of producing gluon matter in laboratory experiments.

Of course (assuming that QCD is the correct theory), these experiments would deal with the complete theory: gauge and quark fields in interaction. While in many spectroscopical problems the effect of (virtual) quarks is believed to be small, in the case of the deconfinement phase transition it is expected to be relevant and qualitative. Really, the same kind of theoretical arguments which predict the universality classes of the phase transitions in different pure gauge theories [7], suggest the disappearance of these phase transitions in the presence of fundamental matter fields – at least if the transition was second order originally [10]. Of course, some rapid change – like a spectacular bump in the specific heat, for instance – is not excluded by these general considerations. This is a difficult quantitative problem, just like the question of what happens with the first order phase transition present in pure SU(3) gauge theory. In this case, an explicit calculation in the full theory which takes into account the influence of virtual quarks, is needed [‡2]

Including virtual quarks in a lattice calculation is notoriously difficult: the resulting effective action is highly non-local, it is composed of all kinds of Wilson loops describing the space–time propagation of virtual quarks. One might, however, consider a hypothetical world, where the quarks are very heavy – in this case the long paths are suppressed, the effective action can be truncated, and the problem becomes manageable [‡3]. Unfortunately, it also becomes irrelevant for physics; the hadronic world contains three light quark species.

There is, however, an exception to the conclusion above. It might happen that all the important changes in the nature of the deconfining phase transition occur already for heavy quarks, that the first order phase transition is destroyed and smoothed out already at a

[‡1] The relevance of the underlying $Z(N)$ symmetry has been pointed out also in ref. [8].

[‡2] Some preliminary results on including virtual quark loops in a finite temperature study of SU(2) have been presented by Kuti and Polónyi [11].

[‡3] The problem of using a truncated effective action in a Monte Carlo simulation is discussed in ref. [12].

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mass value which is large. The calculations described in this paper show that on an $8^3 \times 2$ lattice indeed, this is the case. Due to the small size of the lattice along the temperature direction this result can be taken only as an indication concerning the expected behaviour of continuum SU(3) QCD.

Before describing the results obtained in a Monte Carlo calculation, let us analyze briefly a simplified model which exhibits all the essential phenomena ‡4. The model describes the thermodynamics of Z(3) flux tubes and Z(3) charges ("quarks") living on the links and sites of a three-dimensional lattice. A configuration is characterized by the integer valued link $\{n_{i,\mu}\}$ and site $\{k_i\}$ variables (taking the values $0, \pm 1$). At every lattice point, the Z(3) flux is conserved. The energy of a configuration is given by

$$E(\{n_{i,\hat{\mu}}\}, \{k_i\}) = \sum_{\text{links}\, i,\hat{\mu}} \sigma n_{i,\hat{\mu}}^2 + \sum_{\text{sites}\, i} m k_i^2 , \tag{1}$$

and the partition function is defined as

$$Z = \Bigg(\prod_{\text{links} i,\hat{\mu}} \sum_{n_{i,\hat{\mu}}} \Bigg) \Bigg(\prod_{\text{sites}\, i} \sum_{k_i} \Bigg) \exp\left[-\beta E(\{n_{i,\hat{\mu}}\},\{k_i\})\right]$$

$$\times \prod_i \delta_{\Delta_{\hat{\mu}} n_{i-\hat{\mu},\hat{\mu}},\, k_i (\mathrm{mod}\, 3)} , \tag{2}$$

where the Kronecker δ ensures the flux conservation at every lattice site, and β is the inverse temperature, $\beta = 1/T$. The δ constraint can be resolved by introducing a site variable $z_i \in Z(3)$:

$$\prod_i \delta_{\Delta_{\hat{\mu}} n_{i-\hat{\mu},\hat{\mu}},\, k_i (\mathrm{mod}\, 3)}$$

$$= \prod_i \Bigg(\frac{1}{3} \sum_{z_i \in Z(3)} z_i^{\Delta_{\hat{\mu}} n_{i-\hat{\mu},\hat{\mu}} - k_i} \Bigg) . \tag{3}$$

Now the summation over $n_{i,\hat{\mu}}$ and k_i can be done and we end up with a Z(3) spin model in an external magnetic field

$$Z \sim \prod_i \sum_{z_i \in Z(3)} \exp \Bigg(\beta^* \sum_{\text{links} i,\hat{\mu}} \tfrac{1}{2} (z_i z_{i+\hat{\mu}}^{-1} + \text{c.c.})$$

$$+ H \sum_{\text{sites}\, i} \tfrac{1}{2} (z_i + \text{c.c.}) \Bigg) , \tag{4}$$

where

$$\beta^* = \tfrac{2}{3} \ln\{[1 + 2\exp(-\beta\sigma)]/[1 - \exp(-\beta\sigma)]\},$$

$$H = \tfrac{2}{3} \ln\{[1 + 2\exp(-\beta m)]/[1 - \exp(-\beta m)]\} . \tag{5}$$

For $m = \infty$ (no matter), the magnetic field H is zero and the spin model has a first order phase transition separating the disordered and magnetized phases. It is easy to see (by calculating the free energy of static sources, for instance) that this corresponds to the deconfining phase transition in terms of the original variables.

For $m = 0$ ($H = \infty$) the model becomes trivial and no phase transition occurs. The magnetization $M = \langle \mathrm{Re}\, z \rangle$ as a function of β^*, and the phase transition points are shown in fig. 1 for different values of H, as it is predicted by mean field theory. The jump in M (and similarly in the latent heat) decreases rapidly as H is turned away from zero (the mass is decreased), and at $H \approx 0.02$ the phase transition becomes second order. Below a critical mass value ($m_{\min} \approx 4.4\, T_c$) the deconfining phase transition disappears and is smoothed out completely as the mass is decreased further, the bump in the specific heat becomes broad and small and finally disappears.

Let us now turn to the Monte Carlo results obtained in an SU(3) theory with quarks. The lattice size was taken to be $8^3 \times 2$. On this lattice, the pure gauge theory exhibits a first order phase transition at β ($\equiv 6/g^2$) = 5.11 [5] which is still in the strong coupling regime of the pure gauge theory. However, the value of the critical temperature of the pure gauge theory deduced from these lattices agrees well with those obtained from larger lattices which allow to go to values of β deeper in the continuum regime. We introduce n_f quark species having a hopping parameter $K(\beta)$ ‡5. Our aim is to find the critical line in the (K^2, β) plane. This critical line starts at the point (0.0, 5.11), and it can be followed in the vicinity of this starting point by simulating a truncated effective action applicable for small K (heavy quarks). The effective action on an $N_s^3 \times 2$ lattice has the form [12,15]

‡4 Many of the points of the following discussion are well known [13], see also refs. [7,8,10]. It prepares the analysis of the subsequent Monte Carlo results only.

‡5 We use Wilson fermions [14].

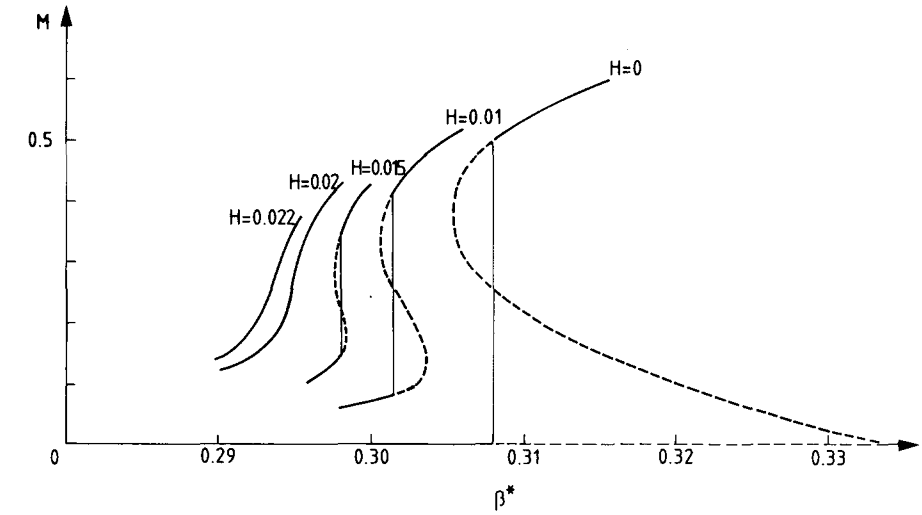

Fig. 1. The magnetization M of a Z(3) spin model versus the coupling β^* for various values of the external field H. The solid (broken) linse corresponds to stable (metastable) solutions of the mean field equation.

$$S_{\mathrm{eff}} = \frac{\beta}{6} \sum_{\text{plaquettes}} (\mathrm{tr}\, UUUU + \text{c.c})$$

$$+ n_{\mathrm{f}} \Bigg[8K^2 \sum_{\text{sites}} (\bigcirc + \text{c.c.})$$

$$+ 2K^4 \sum_{\text{sites}} \Big(\quad + \quad - 4 \,\bigcirc\!\!\bigcirc + \text{c.c.} \Big)$$

$$+ \mathrm{O}(K^6) \Bigg] , \qquad (6)$$

using an obvious notation. Already in lowest order, quark loops closed by the antiperiodic boundary conditions in the temperature direction contribute to the effective action and destroy the global Z(3) symmetry present in the SU(3) Yang–Mills action. For very small K^2, only the leading order term, proportional to $H \equiv 8n_{\mathrm{f}}K^2$, is kept.

The gauge field configurations were generated with this truncated effective action. In the procedure, a value of β (< 5.11) was fixed, then H was tuned in trying to locate two metastable co-existing states, signaling a first order phase transition.

We used a Metropolis algorithm with four hits/link and performed 2500–4000 iterations at a given (H, β) value. At every (H, β) points two runs were made starting from an ordered and a random configuration. Outside the critical region both runs ended up in the stable phase after a transient period of 500–700 iterations. In the critical region we observed two metastable states with occasional phase flips between them after many iterations. A typical example is shown in fig. 2, where the thermal loop value is plotted against the number of iterations in a single run. Between the 750th and the 2100th iterations the system is in the confined phase, then, after a short, sharp transition region it stays in the deconfined phase for more than 1600 iterations.

Our data analysis was intentionally biased towards weakening the effect of quarks: that is, in case of ambiguities (for instance, sometimes it is not clear whether certain data points lie in a transition region or are part of one of the metastable phases) we chose that interpretation which increased the strength of the first order transition.

Figs. 3 and 4 give the thermal loop expectation value as the function of $\sqrt{H}$ for $\beta = 5.0$ and 4.9, respectively. Our search for the critical region was helped by the observation that, for small H, the difference $\beta_{\mathrm{c}}(H=0) - \beta_{\mathrm{c}}(H)$ can be predicted in terms of pure gauge theory quantities. The free energy density $F = -N_s^{-3} \ln Z$ can be expanded around $H = 0$ slightly above and below the phase transition point. Using the

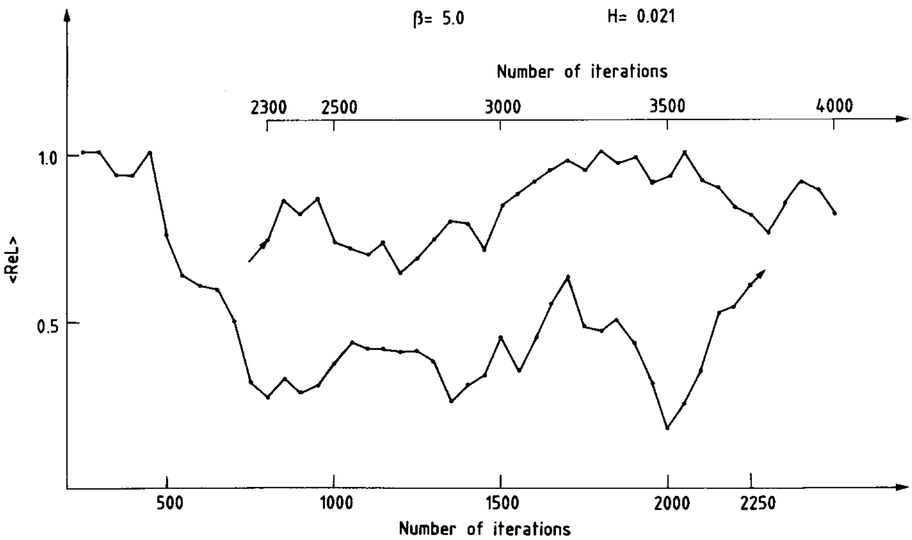

Fig. 2. Expectation value of the thermal Wilson loop at $\beta = 5.0$ and $H = 0.021$ versus the number of iterations. The dots are averages over 50 iterations. After ~2100 iterations there is a phase flip into the deconfining phase. (The iteration numbers between 2300 and 4000 are given above the curves. The part of the curve after the flip is shifted to the left to show the effect clearly.)

equation $\delta F/\delta H\,(\beta, H=0) = -2\langle L\rangle$, we obtain

$$F(\beta,H) = F(\beta,0), \quad \beta < \beta_c(H=0) - \epsilon\,,$$

$$F(\beta,H) = F(\beta,0) - 2\Delta(\langle L\rangle)\cdot H, \quad \beta > \beta_c(H=0) + \epsilon\,, \qquad (7)$$

where $\Delta(\langle L\rangle)$ is the jump in $\langle L\rangle$ at $\beta_c(H=0)$. Then it is easy to see that the new break point of the free energy density is determined by the equation

$$\beta_c(H) = \beta_c(0) + [2\Delta(\langle L\rangle)/\Delta(\partial F/\partial\beta)_{H=0}]\cdot H\,, \qquad (8)$$

where $(\delta F/\delta\beta)_{H=0}$ can be expressed in terms of the jump in the plaquette expectation values at $\beta_c(H=0)$. Monte Carlo data for the pure gauge theory give then

$$\beta_c(H) = \beta_c(0) - (4.94 \pm 0.75)\cdot H\,. \qquad (9)$$

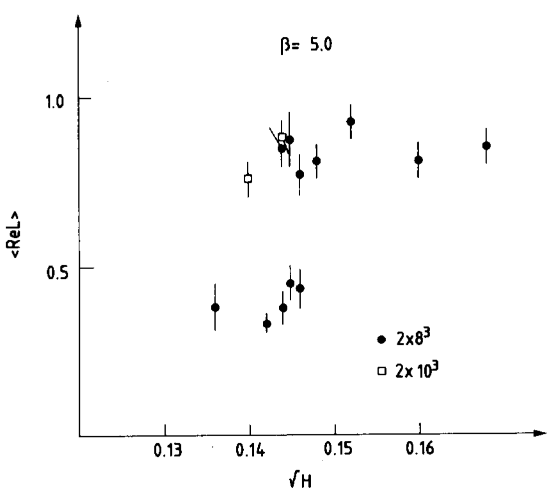

Fig. 3. The thermal loop expectation value as the function of $\sqrt{H}$ at fixed $\beta = 5.0$.

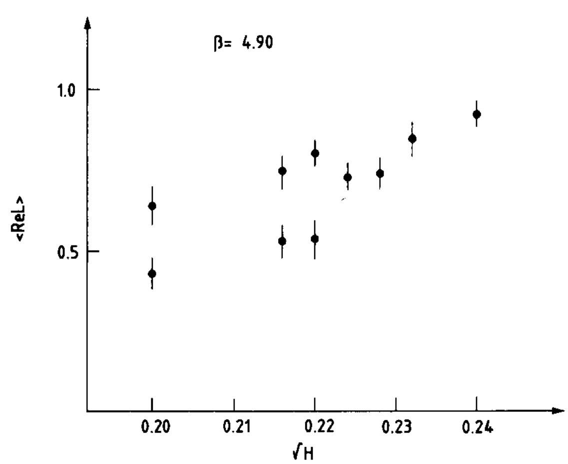

Fig. 4. The same as fig. 3, but at $\beta = 4.9$.

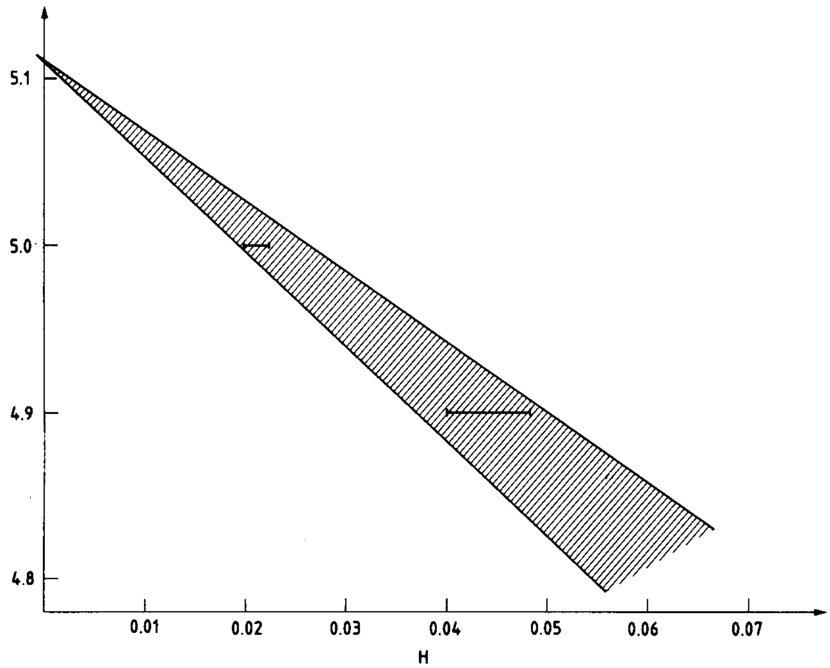

Fig. 5. The critical coupling β_c versus H. The dashed region indicates the prediction of eq. (9) for small external fields H. The dots are Monte Carlo data obtained by including terms of order K^2 in the effective action, eq. (6).

This prediction agrees well with the direct Monte Carlo measurements using the truncated effective action (fig. 5).

As H is increased (the quark mass is decreased), the critical β coupling is decreasing and the corresponding jump $\Delta\langle L\rangle$ of the first order transition is decreasing rapidly as can be seen in fig. 6.

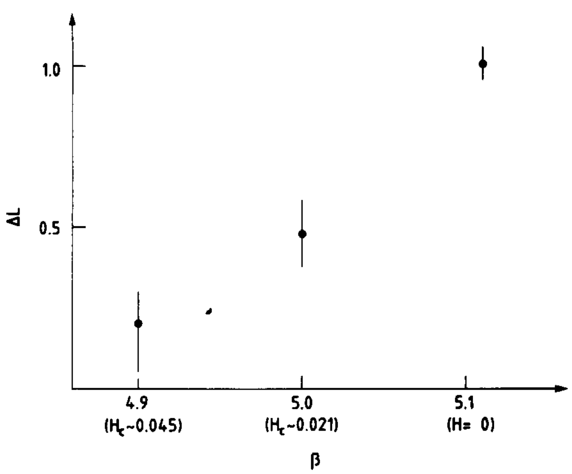

Fig. 6. The jump $\Delta\langle L\rangle$ in the expectation value of the thermal Wilson loop as a function of β_c.

It is interesting to note that the same phenomenon can be observed even on simpler quantities which are not related to the Z(3) symmetry like the plaquette expectation value. Although $\langle p\rangle$ has no direct thermodynamical interpretation [‡6], the first order deconfining phase transition creates a jump in $\langle p\rangle$, and this jump decreases (as the quark mass is decreased) qualitatively the same way as $\Delta\langle L\rangle$.

Fig. 6 suggests that the first order jump becomes zero somewhere around $\beta \approx 4.85$, which, according to fig. 5, corresponds to $H \approx 0.055$. Therefore even at the endpoint of the critical line of first order transitions H is small, describing heavy quarks. The u, d and s quarks can be thought degenerated on this scale, $n_f = 3$ can be taken, which gives $K \approx 0.05$. The K value corresponding to light or massless quarks is

‡6 Actually, the jump of $\langle p\rangle$ is related to the latent heat [17] while the difference between the space- and time-like plaquette expectation values is connected to the energy density of the glue sector.

around $K_c \approx 0.2$ in this coupling constant region [16]. [As a crude estimate, we might use the relation $\frac{1}{2}(1/K - 1/K_c) = \exp(m_q a) - 1$, which gives $m_q/T_c = m_q a/T_c a = 2.1/0.5 = 4.2$ [‡7].] Therefore the endpoint of the first order phase transitions is far away from the physical region. It is not excluded that the system remembers the phase transition even at such a distance (via some spectacular bump, for instance) but it is not plausible a priori.

The question of the existence of the deconfining phase transition in QCD is basically important in thinking about the relevant heavy ion experiments. In order to find a convincing answer one should go deeper into the continuum limit by increasing the temperature size of the lattice. On the basis of the present work we believe the problem is not hopelessly difficult.

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Note added in proof. The simplified model in eq. (4) is discussed in a recent paper [18] by DeGrand and DeTar. The conclusion is the same as obtained here.

‡7 In leading order S_{eff} can be also interpreted as coming from the naive fermion formulation. Using this interpretation H is related to the quark mass via the relation $H = (4 \cdot n_f/16) \times (1/2m_q a)^2$, giving $m_q/T_c \sim 3.7$ in good consistency with the above numbers.

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