

THE SU(2) ADJOINT HIGGS MODEL AT POSITIVE TEMPERATURE: A MONTE CARLO STUDY

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We investigate the phase structure of the Georgi–Glashow model at positive temperature, realized by using an asymmetric $6^3 \times 3$ lattice. We find a clear signal for the deconfining phase transition at all values of the Higgs coupling constant. On the other hand, there is no unambiguous signal for a transition between a “Higgs” and a “symmetric” phase at moderate β ; very high values of β cannot be studied with the icosahedral subgroup used here, because of the freezing transition. We use some new observables as probes of the different behaviour of the system at different values of the Higgs coupling constant.

1. Introduction. Higgs fields in the adjoint representation of a unitary group play an essential part in grand unified theories [1]. Effective potential calculations [2] suggest the existence of a first order phase transition from a “symmetric” to a “Higgs” phase as the temperature is lowered; this is the basis of the so-called inflationary scenario for the early universe [3].

For this reason, we find it worthwhile to study the phase structure of the simplest model with adjoint Higgs fields, the Georgi–Glashow model [4] at positive temperature. This model has been studied before at zero temperature (at least, this was the interpretation used) by Lang et al. [5] and by Brower et al. [6].

The Georgi–Glashow model involves the coupling of a Higgs field ϕ in the adjoint representation of SU(2) to a standard SU(2) Yang–Mills field. We fix the length of the Higgs field at $|\phi| = 1$. The lattice action is then the following:

$$S = \frac{1}{2}\beta S_W + \frac{1}{2}\beta_H S_H. \quad (1)$$

Here S_W is the standard Wilson action for the gauge field $\{U_{n\mu}\}$

$$S_W = -\sum_p \text{tr} U_p, \quad (2)$$

where $U_p = \prod_{\partial p} U_{n\mu}$ is the ordered product of the SU(2) valued link variables $U_{n\mu}$ around the boundary of the plaquette p and the sum is over all plaquettes in a chosen orientation. Furthermore,

$$S_H = -\frac{1}{2} \sum_{n,\pm\mu} \phi_i(n) \phi_j(n+\mu) \text{tr} U_{n\mu} \sigma_i U_{n\mu}^\dagger \sigma_j, \quad (3)$$

where $\phi_i(n)$, $i = 1, 2, 3$ are the components of the Higgs field at site n obeying $\phi_1^2 + \phi_2^2 + \phi_3^2 = 1$ and σ_i are the Pauli matrices. Expectation values are to be calculated with the probability measure

$$d\mu = Z^{-1} \exp(-S) \prod_{n,\mu} dU_{n,\mu} \prod_n d\Omega(\phi(n)), \quad (4)$$

where $dU_{n\mu}$ is the Haar measure on $SU(2)$, $d\Omega$ the standard measure on the unit sphere in \mathbf{R}^3 and Z is chosen to make $\int d\mu = 1$.

For not too large β , it is convenient to eliminate the Higgs field degrees of freedom by going to the unitary gauge. We use the gauge freedom to rotate ϕ into the two-direction; the Higgs action becomes

$$S_H = -\sum_{n,\mu} \text{tr} U_{n\mu} \sigma_2 U_{n\mu}^\dagger \sigma_2, \quad (5)$$

and expectation values will be computed with the measure

$$d\mu = Z^{-1} \exp(-S) \prod_{n,\mu} dU_{n\mu}. \quad (6)$$

To simulate finite temperature field theory on an euclidean space-time lattice, we work on an asymmetric lattice of size $N_\sigma^3 \times N_\tau$, N_σ (N_τ) denoting the number of lattice points in space (time) direction. As we use equal couplings for space- and timelike links, the temperature $T = 1/N_\tau$ is fixed in units of the lattice spacing a . Varying the temporal lattice size N_τ allows us to relate the lattice spacing a with the couplings β and β_H using renormalization group considerations, which in turn allows us to define a continuum limit of the lattice model at fixed physical temperature. We will, however, not be concerned with this problem in the following; instead we will work at a fixed temporal lattice size N_τ and vary the couplings β and β_H . This should still allow us to explore the phase boundaries of the model.

Let us now discuss the various limiting cases of our lattice model. For $\beta_H = 0$, we have the pure $SU(2)$ lattice gauge model which is known to undergo a deconfining phase transition [7-9] at some $\beta = \beta_c(N_\tau)$ for all N_τ . Monte Carlo simulations support the continuous nature of this transition and indicate [9] that the transition will also persist in the continuum limit of the theory as the critical coupling scales according to the $SU(2)$ renormalization group equation.

In the limit $\beta_H \rightarrow \infty$, the model reduces to the pure $U(1)$ lattice gauge model that also shows a second order [10] deconfining phase transition. It is known – a fact which is also borne out by our data – that

$$\beta_c(\beta_H = \infty, N_\tau) < \beta_c(\beta_H = 0, N_\tau). \quad (7)$$

For $\beta = \infty$, the gauge field becomes a pure gauge, at least locally; because of the periodic boundary conditions, the gauge field cannot, in fact, be gauged away completely, since $U_p = \mathbf{1}$ for all plaquettes still allows non-trivial Polyakov (thermal Wilson) loops. A little thought shows, however, that these Polyakov loops will essentially (in the infinite volume limit exactly) be frozen to values in the centre Z_2 of $SU(2)$, so the Higgs fields do not feel them and the form (3) of the Higgs action reduces to the action of the classical $O(3)$ Heisenberg model. This is a little harder to see in the unitary gauge action (5) (cf. ref. [5]). Also, the usual Monte Carlo algorithms have difficulties in the unitary gauge for large β .

The critical temperature of the Heisenberg $O(N)$ models is known to be well approximated (to about 10%) by the so-called infrared bound [11], which for our case states:

$$\beta_{H,c} \leq NI(d, N_\tau), \quad (8)$$

where

$$I(d, N_\tau) \equiv (2\pi)^{1-d} \sum_{n=1}^{N_\tau} N_\tau^{-1} \int_{|p_\mu| \leq \pi} d^{d-1} p \times \left(\sum_{i=1}^{d-1} (1 - \cos p_i) + 1 - \cos(2\pi n/N_\tau) \right)^{-1}, \quad (9)$$

and d is the space-time dimension, i.e., 4 in our case. We have in general:

$$I(d+1, 1) = I(d, \infty) \equiv I(d) = (2\pi)^{-d} \int d^d p \left(\sum_{i=1}^d (1 - \cos p_i) \right)^{-1}. \quad (10)$$

So we see that by varying N_τ we interpolate between three and four dimensions. We have computed $I(4, N_\tau)$ for various N_τ and used the bound (9) to estimate $\beta_{H,c}$. The results are shown in table 1. It can be seen from this table that at $\beta = \infty$ the finiteness of N_τ , i.e., the non-zero temperature, has only a very small effect on the location of the phase transition.

Finally, for $\beta = 0$ our lattice model with frozen $|\phi| = 1$ reduces to a trivial one-link model without any phase transitions (see,

Table 1

The infrared bound on the critical β_H for the finite temperature ($\sim 1/N_T$) transition of the four-dimensional O(3) Heisenberg model.

N_T	$\frac{3}{2}I(4, N_T)$
1	1.5
2	1.08
3	1.0
4	0.96
5	0.96
6	0.95
≥ 7	0.94

however, ref. [12] for variable radial components). Thus, from the discussion of the boundaries of the phase diagram, we might expect the phase diagram displayed in fig. 1, which also presents our results for the $6^3 \times 3$ lattice used (the discussion of the results is relegated to section 3). Indeed, the existence of the deconfining transition for the pure SU(2) model [8] works just as well for $\beta_H > 0$. Thus there will be deconfinement for $\beta > \beta_c(\beta_H, N_T)$ for all β_H , as the critical points on the U(1) and SU(2) boundaries enter the interior of the phase diagram. Our data show this phenomenon clearly. On the other hand, whether the critical point on the $\beta = \infty$ boundary will really enter the interior of the phase diagram, as indicated by the dashed horizontal line in fig. 1, is unclear.

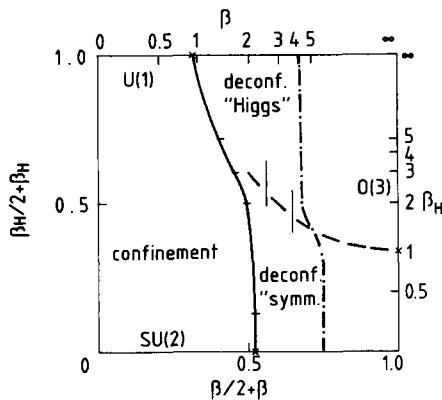


Fig. 1. Tentative phase diagram for the lattice $6^3 \times 3$. The dashed-dotted line indicates the icosahedron freezing transition as given in ref. [5].

2. Monte Carlo simulations. To clarify the structure of the phase diagram discussed in the previous section, we have performed Monte Carlo simulations on a $6^3 \times 3$ lattice using the icosahedral subgroup of SU(2). Let us first discuss the deconfining transition as a function of β and β_H . As we are dealing with an SU(2) Higgs model with the Higgs fields in the adjoint representation of SU(2), the lattice action eq. (1) is still invariant under global Z(2) transformations of the form:

$$U_{x,0} \rightarrow z U_{x,0}, \quad z \in Z(2), \quad x \equiv (x_0, \mathbf{x}), \quad x_0 \text{ fixed.} \quad (11)$$

Thus the trace of the Polyakov loop

$$\bar{L} \equiv N_\sigma^{-3} \sum_x \text{tr} L_x, \quad L_x \equiv \prod_{x_0=1}^{N_T} U_{(x_0, \mathbf{x}), 0}, \quad (12)$$

is still an order parameter for the deconfining transition.

We have measured $\langle \bar{L}^2 \rangle$ as a function of β for several values of β_H , using the unitary gauge [see eq. (5)]. The results are presented in fig. 2. They show clearly the deconfining transition characterized by the appearance of a non-zero value for $\langle \bar{L}^2 \rangle$. It is also clearly visible that this transition moves towards smaller β as β_H increases, as announced. We remark that at high

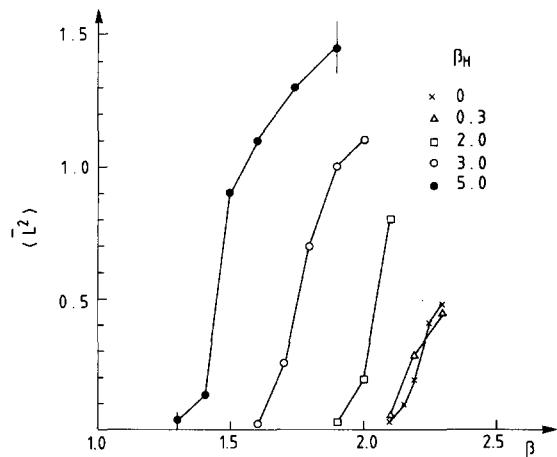


Fig. 2. The order parameter $\langle \bar{L}^2 \rangle$ for the deconfining transition as a function of β for $\beta_H = 0.0, 0.3, 2.0$ and 3.0 . For $\beta_H = 5.0$, the quantity $\langle \bar{L}^2 \rangle + \langle (\sigma_2 \bar{L})^2 \rangle$ is plotted instead. Typical error bars are also indicated.

β_H , the adequate quantity to use is $\langle \bar{L}^2 \rangle + \langle (\sigma_2 \bar{L})^2 \rangle$, which is constant while the terms of the sum oscillate. Here

$$\overline{\sigma_2 \bar{L}} \equiv N_\sigma^{-3} \sum_x \text{tr}(\sigma_2 L_x), \quad (13)$$

see the discussion further below.

In the $\beta_H = 0$ case, it has been shown [13] that the deconfining transition leads to an abrupt change in the difference of the spatial and temporal plaquettes, i.e., plaquettes with only spacelike and with space- and timelike links respectively. In the pure SU(2) Yang-Mills model, this difference gives the dominant contribution for large β [13] to the gauge field physical energy density:

$$\epsilon_G a^4 = 3\beta (\langle \text{tr } U_p \rangle|_{\text{pspatial}} - \langle \text{tr } U_p \rangle|_{\text{ptemporal}}). \quad (14)$$

Furthermore, $\epsilon_G a^4$ approaches its limiting ideal gas value,

$$\epsilon_{SB} a^4 = f c(N_\sigma, N_\tau) \frac{1}{15} \pi^2 N_\tau^{-4}, \quad (15)$$

already for β values quite close to the critical point. In eq. (15), f counts the degrees of freedom besides the spin degrees of freedom, i.e., $f = 3$ for SU(2), and $c(N_\sigma, N_\tau)$ includes finite size corrections [14]. In fig. 3, we present data for $\epsilon_G a^4$ at $\beta_H = 5$. They show a sudden increase around $\beta = 1.3$, like the corresponding Polyakov loop in fig. 2. Above this value, $\epsilon_G a^4$ approaches the asymptotic value $\epsilon_{SB} a^4$ with $f = 1$ indicating that, for $\beta_H = 5$, only one massless gauge boson remains participating in the thermal equilibrium (black body radiation); i.e., two of the three

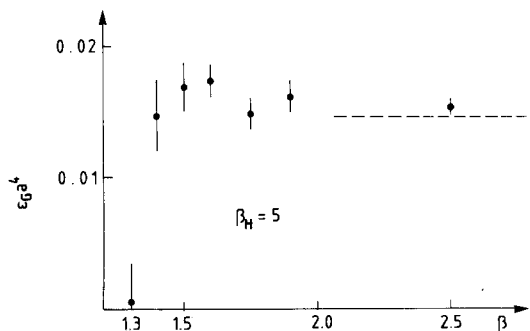


Fig. 3. The Yang-Mills part of the energy density, $\epsilon_G a^4$, as a function of β at $\beta_H = 5.0$. The dashed line corresponds to an ideal gas with one degree of freedom ($f = 1$).

gluons present at $\beta_H = 0$ become heavy and we are left with only one massless gauge boson, “photon” (compare with fig. 11 in ref. [13] for pure SU(2), where all three degrees of freedom are visible to the right of the phase transition).

Thus the deconfining phase transition is well established by our measurements. Moreover, a clear difference shows up between the upper- and lower-right-hand part of the phase diagram fig. 1. Much more difficult, however, is the identification of a phase boundary between a “Higgs” phase (β_H large) and a “symmetric” phase (β_H small). In fact, at the moderate β values for which we can take reliable measurements with the icosahedral subgroup of SU(2), no clear signal of such a transition is seen. Nevertheless, it is interesting to study how the system changes with β_H .

At $\beta_H = \infty$, the Polyakov loops are restricted to the U(1) subgroup $H = \{\exp(i\tau\sigma_2)\}$ of SU(2), and behave like the spins in an xy model, showing a global U(1) symmetry. For $\beta_H < \infty$, this U(1) symmetry is broken down to a Z(2) symmetry; the Polyakov loop variable L_x is no longer forced to lie in H, but will fluctuate around it; on the other hand, the neighbourhood of the centre Z(2) = {1, -1} is favoured energetically. This effect becomes more pronounced for $\beta_H \rightarrow 0$, whereas for large β_H , L_x will still be distributed almost uniformly on a strip surrounding H in SU(2).

To study these phenomena, we measured both \bar{L} and $\sigma_2 \bar{L}$ [eqs. (12) and (13)] using the action eq. (5). Notice that $\sigma_2 \bar{L}$ is invariant under the residual gauge symmetry but not under the full SU(2) symmetry. Therefore, it is meaningful only in the unitary gauge. Fig. 4 shows the “time evolution” of these quantities. They are clearly correlated and therefore, if taken as coordinate in a plane, move on a circle. At small β_H , however, they seem to have come to a standstill and execute at best small oscillations around $|\bar{L}| = 1$ and $\overline{\sigma_2 \bar{L}} = 0$ (see fig. 4a). On the other hand, the deconfining transition shows up clearly as the collapse of the radius of the circle for small β (see fig. 4b). The resulting averages $\langle \bar{L}^2 \rangle$ and $\langle (\sigma_2 \bar{L})^2 \rangle$ over a total of ~ 2000 iterations are shown in fig. 5. Again, the change in

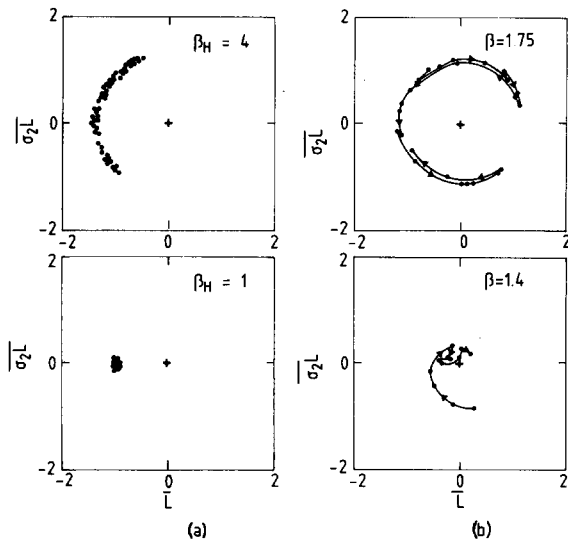


Fig. 4. Plots $\overline{\sigma_2 L}$ versus \overline{L} ; the points correspond to averages over 50 or 100 sweeps. (a) $\beta = 2.5$, various β_H ; (b) $\beta_H = 5.0$, various β .

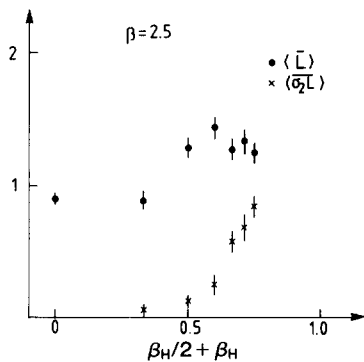


Fig. 5. $\langle \overline{L} \rangle$ (points) and $\langle (\overline{\sigma_2 L})^2 \rangle$ (crosses) for $\beta_H = 0.0, 1.0, 2.0, 3.0, 4.0, 5.0$ and 6.0 at $\beta = 2.5$.

behaviour taking place between $\beta_H = 1$ and $\beta_H = 5$ is clearly visible in these quantities. This change, however, cannot be taken as a signal for the Higgs transition: for $\beta = \infty$ the Higgs transition becomes the transition of the Heisenberg model, but in this limit, the Polyakov loop L_x will be frozen to the centre $Z(2)$, as discussed before, and no such changes as seen in figs. 4 and 5 will occur.

This discussion shows that the basic problem in the analysis of the Higgs transition is the lack of a suitable order parameter. There have been some proposals for an order parameter for the

Higgs transition [15]. However, they involve obtaining the asymptotic behaviour of some kinds of correlations for which large lattices are necessary. Nonetheless, even without an order parameter, a phase transition should also show up in the usual thermodynamic quantities; i.e., the difference of space-space and space-time plaquette expectation values [eq. (12)] which contributes to the energy density of the system. This quantity is shown in fig. 6 for $\beta = 2.5$ and various values of β_H . One sees a rapid change from the asymptotic SU(2) behaviour ($\beta_H = 0$; three gauge bosons) to the U(1) limit ($\beta_H = \infty$; one gauge boson) for $\beta_H \approx 1.0-4.0$, which, however, is difficult to interpret as a signal for a first order phase transition.

The question whether such a transition exists at higher β , ending up in the Heisenberg transition, is delicate. With a discrete subgroup we

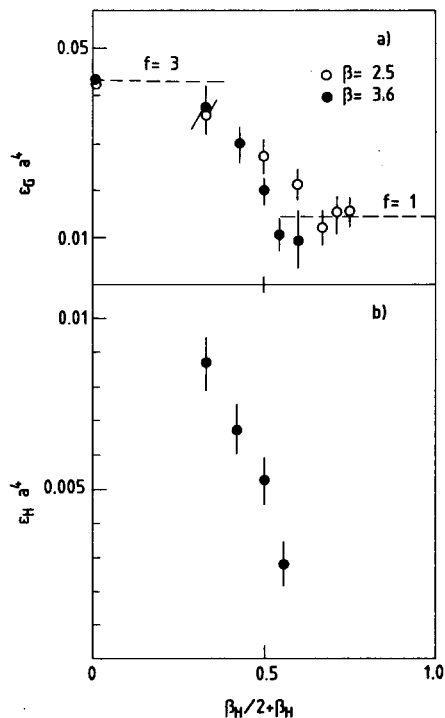


Fig. 6. (a) $\epsilon_G a^4$ for $\beta_H = 0.0, 1.0, 2.0, 3.0, 4.0, 5.0$ and 6.0 at $\beta = 2.5$, and for $\beta_H = 0.0, 1.0, 1.5, 2.0, 2.5$ and 3.0 at $\beta = 3.6$. The dashed lines correspond to an ideal gas with $f = 1$, respectively $f = 3$ degrees of freedom. (b) $\epsilon_H a^4$ for $\beta_H = 1.0, 1.5, 2.0$ and 2.5 at $\beta = 3.6$.

cannot actually approach this region because of the freezing transition. We ask, however, whether we might see something to the left of the latter. For this, we did a run at $\beta = 3.6$, which should be quite near the freezing transition as given in ref. [5]. To make sure that no metastabilities would be introduced through the use of the unitary gauge at this high β , we relaxed the gauge fixing for this run. We thus used eq. (3) and took continuous Higgs fields $\phi \in S_3$. The results are presented in fig. 6.

A still steeper behaviour is to be noticed in the region $\beta_H \approx 1.0-3.0$, especially for the Higgs contribution to the energy density:

$$\epsilon_H a^4 \equiv \beta_H (\langle \phi_i(n) \phi_j(n+\mu) \rangle \times \text{tr}[U_{n\mu} \sigma_i U_{n\mu}^\dagger \sigma_j])|_{\mu: \text{spacelike}} - \langle \phi_i(n) \phi_j(n+\mu) \rangle \text{tr}[U_{n\mu} \sigma_i U_{n\mu}^\dagger \sigma_j]|_{\mu=0}. \quad (16)$$

However, the general picture is similar to the one seen at $\beta = 2.5$.

The β_H dependence of the plaquette and the links is, in general, smoother than that of the physical energy density. We have also performed a heating-cooling cycle at $\beta = 3.6$, which gave no indication of a first order transition.

3. Discussion. Our results can be summarized in the tentative phase diagram given in fig. 1. The continuous line is well established, while the dashed horizontal line is not very well established, due certainly also to the softness of the presumed transition. It seems, however, that our data rule out a first order "Higgs" phase transition for $\beta \leq 3.6$. To clarify the situation, it might be worthwhile to measure at larger β , closer to the transition of the classical Heisenberg model. As we have stated earlier, this will require working with the full SU(2) group.

It should be noted that at finite temperature both the upper ("Higgs") and the lower ("symmetric") phases will show exponential clustering (Debye screening) and behave like a plasma, so it is not clear what might distinguish them. On the other hand, at $\beta = \infty$, the magnetizing transition of the Heisenberg model is certainly present and there is no doubt that nearby (i.e., at large β) one should observe at least a rapid variation of some quantities. The plaquette (and links) difference seems to be a

suitable quantity, as one sees in fig. 6. The change-over clearly becomes steeper by going from $\beta = 2.5$ to $\beta = 3.6$. It is possible that a transition would be more easily visible when varying the temperature by using asymmetric couplings, thus cutting the phase boundary at another angle.

Finally, we think that for a definitive answer, one should also consider the effect of the radial degree of freedom in the Higgs action with the full Higgs potential.

We should like to close with the remark that our results are certainly consistent with those of refs. [5] and [6]. We would, however, tend to interpret their data as finite temperature results. Certainly, the finite temperature deconfining transition should be visible in the phase diagrams given (on a 4^4 lattice it occurs at about $\beta \approx 2.3$ for $\beta_H = 0$).

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