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An Approach for Forecasting Life Expectancy and its Application in Germany*

Ein Ansatz für die Prognose der Lebenserwartung und seine Anwendung in Deutschland

Une approche en vue de pronostiquer l'espérance de vie et sa réalisation en Allemagne

Zusammenfassung

In den meisten Mortalitätsstudien wird die Dynamik in der Veränderung der Lebenserwartung auf der Basis der Querschnittsanalyse für eine bestimmten Periode untersucht. Die Längsschnittanalyse bzw. der Kohortenansatz sind methodologisch befriedigender, haben aber den Nachteil, dass die Mortalitätsangaben nur für diejenigen Kohorten komplett sind, die bis zu 100 Jahre zurückreichen; das bedeutet, dass dieser Ansatz für die Prognose nur geringen Nutzen hat.

Dieser Artikel stellt eine neue Methode vor, die die Perioden- und Kohortenanalyse kombiniert. Sie verbindet deshalb den Vorteil der Periodenanalyse (die Verwendung aktuellerer statistischer Daten) mit den methodologischen Vorzügen des Kohortenansatzes. Die Analyse basiert auf den deutschen Sterbetafeln zurückgehend bis zum Jahr 1871. Auf dieser empirischen Basis beruht der Ansatz für die Prognose der Lebenserwartung bis zum Jahr 2080.

Die mittlere Variante prognostiziert ein Anwachsen der Lebenserwartung für die Männer ausgehend von 73,3 Jahren in den Jahren 1994/96 auf 81 Jahre im Jahr 2080 und für die Frauen von 79,7 auf 87 Jahre. Die Zunahme der Lebenserwartung (d.h. der ferneren Lebenserwartung) ist für die höheren Altersgruppen beträchtlich größer als für die unter 50-Jährigen. Die Ergebnisse werden in Form von Survival-Funktionen dargestellt, die es ermöglichen, die altersspezifischen Sterbewahrscheinlichkeiten für beide Geschlechter und die fernere Lebenserwartung für jedes Jahr des Prognosezeitraumes abzuleiten.

1. Introduction

Investigations into mortality and life expectancy can be classified into one of these five main areas:

(1) Analysing and forecasting the life expectancy of populations
(2) Distinguishing between life expectancies within a population depending on the number of years spent in good health (the "active life expectancy") and those spent in poor health or with disabilities
(3) The maximum potential lifespan of a particular individual
(4) The average lifespan of a population composed of genetically different individuals with differing maximum lifespans

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(5) The "verified longest lived individual" of a particular species, which is used as an indicator of the maximum lifespan potential (MLSP).

The problems examined in this paper fall into the first of these broad categories, the objective being to analyse the dynamics of life expectancy in Germany since the late 19th century and, on that basis, to offer a long-term forecast of the likely growth in life expectancy during the 21st century. The paper introduces a new methodological approach which combines the classic cross-sectional (period) and longitudinal (cohort) analytical methods (Section 2). The approach is used to present a forecast of life expectancy in Germany up to the year 2080 (Section 3). These results are subsequently compared with those issued by other bodies (Section 4).

A key concern underlying this enquiry is the major impact of increasing life expectancy on the demographic ageing of society, along with the resulting financial problems for the pension and health care systems. A. Börsch-Supan and the author recently investigated possibilities of reforming the pension system by way of a partial transition from the present "pay-as-you-go" system, which is severely affected by demographic ageing, to a mixed system (Birg/Börsch-Supan 1999). They conducted that enquiry on behalf of the Association of German Insurance Companies (Gesamtverband der deutschen Versicherungswirtschaft).

## 2. An Approach for Forecasting Life Expectancy and Median Age

The method most often used to compute life expectancy is based on a period analysis of death probabilities. The life expectancy for a particular year is worked out from the death probabilities of each of the approximately 100 birth cohorts alive at that time. The result obtained will also apply specifically to one of these birth cohorts currently living, so this cohort can be referred to as the "representative cohort"; it is impossible to say beforehand what age the representative cohort will be. In recent decades death probabilities have been steadily falling, that means the age-specific death probabilities of the cohorts that are younger than the representative cohort in the year in question will be lower than that of the representative cohort itself. Conversely, the death probabilities of people born before those in the representative cohort will be relatively higher. Consequently, the overall life expectancy calculated in any particular calendar year will be relatively too low for people born later than the representative age cohort, and relatively too high for those born earlier.

Cohort analysis is a longitudinal method well suited to analyse mortality on a differential basis by birth cohort, and to calculate the differing increments in life expectancy from one birth cohort to the next (Bomsdorf 1993; Dinkel/Höhn/Scholz 1996). However, the methodological strong-points of cohort analysis are won only at the cost of having to use empirical data that do not reflect current conditions, as they refer to periods going some way back in time. For example, the youngest cohort today (in 1999) for which we can properly establish age-specific death probabilities stretching over an entire lifespan of approximately 100 years was born in 1899. Similarly, we shall not be able to refer to the cohort analysis of a full set of death probabilities for those born in 1960 until the year 2060.

This lack of current applicability in the empirical data used in cohort analysis is liable to be especially problematical if increases in life expectancy are concentrated on people's further life expectancy at an advanced age. For example, if the bulk of an improvement

Fig. 1: Lexis diagram showing the difference between survival functions obtained by period and cohort analysis

Abb. 1: Lexisdiagramm zu den Unterschieden zwischen Überlebensfunktionen nach der Perioden- und der Kohortenanalyse
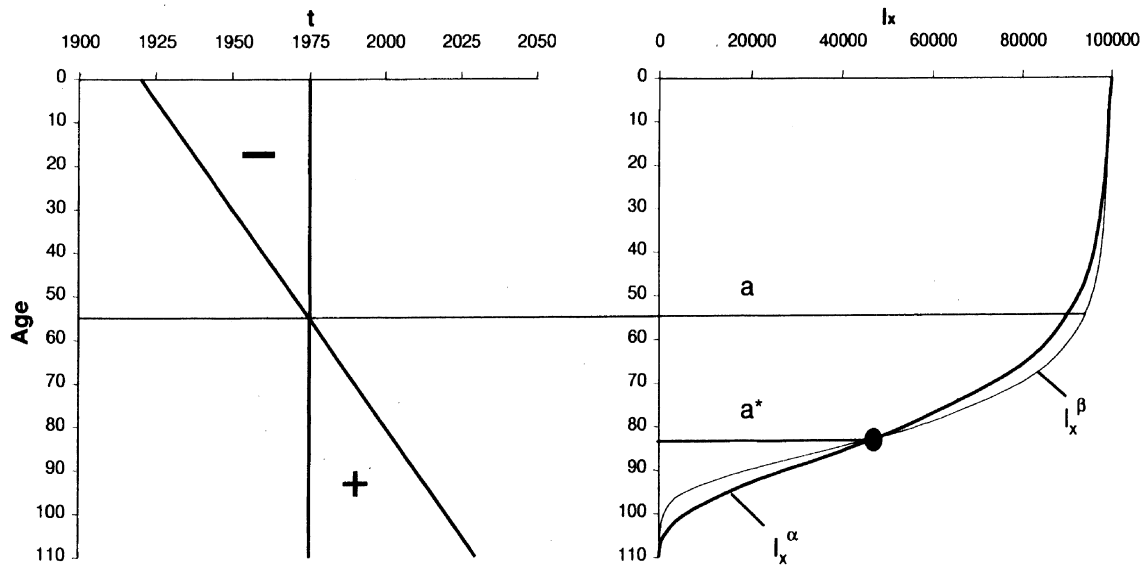
Source: H. Birg, University of Bielefeld, IBS, 1999.

in life expectancy were to affect the further expectancies of those aged 70 and upwards, it would not show up in the data until the death probabilities of this birth cohort (those born 1960) became apparent from the year 2030 onwards. This example demonstrates that, for all its methodological merits, cohort analysis also has some major disadvantages, particularly for making long-term forecasts of life expectancy.

Period analysis of mortality is an attempt to avoid the lack of current empirical data suffered by cohort analysis, by using the mortality figures for all 100 birth cohorts alive simultaneously in a particular year. However, the attempt to solve the problem in this way is all the more likely to generate methodological fuzziness if life expectancy is increasing or decreasing from one birth cohort to the next.

Figures 1 and 2 show the methodological differences between the period and the cohort methods of analysing mortality. The left-hand section of *Figure 1* is the Lexis diagram showing the life line of a particular cohort. If age-specific death probabilities are declining over time, these probabilities in the period t up to age a will now be lower than those of the cohort shown (the area marked with a minus sign in *Figure 1*), whereas they will be higher from age a upwards (shown by the plus sign). The consequence of this is that the cohort's survival function l_x^α will run below that of the period life table, l_x^β, up to a certain age a^*, and above it thereafter. Due to the cumulative effects of the death probabilities on the value of the survival function, $a^* > a$.

If mortality is decreasing over time, the survival functions of different age cohorts shown in *Figure 2* are ranged with the older cohorts to the left and the younger to the right. The survival function of the period life table intersects those of the cohorts in several places, such that the l_x^β function runs above the cohort's survival function l_x^α when it is to the left of the intersection, and below it when it is to its right. If mortality is increasing over time,

Fig. 2: Derivation of the period survival function l_x^β from the cohort survival functions l_x^α for increasing and decreasing life expectancies

Abb. 2: Ableitung der Perioden-Survivalfunktion l_x^β von den Kohorten-Survivalfunktionen l_x^α für steigende und abnehmende Lebenserwartung

l_x^β-function for decreasing life expectancy

l_x^β-function for increasing life expectancy

l_x^α-functions

Survival

Age
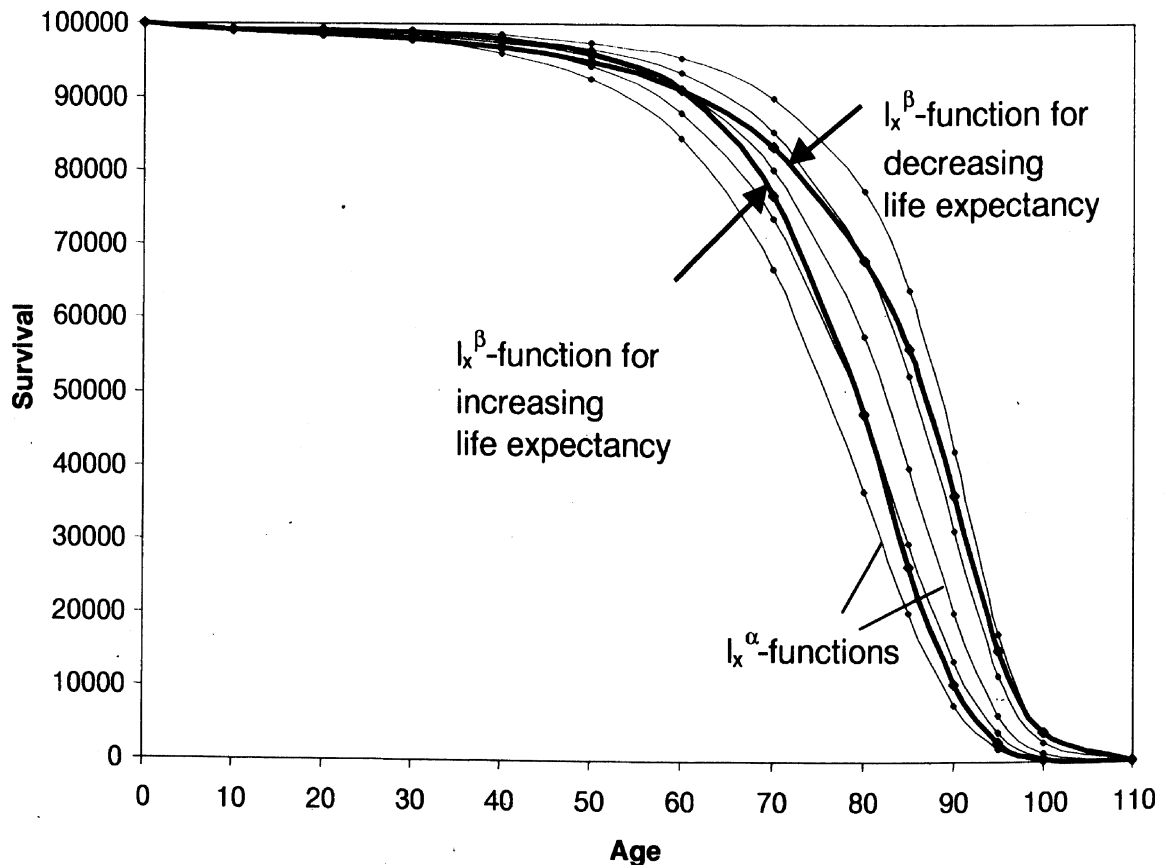
Source: H. Birg, University of Bielefeld, IBS 1999

i.e. from one cohort to the next, the survival function obtained from the period analysis runs below the cohort-analysis survival function to the left of the intersection, and above it when it is to the right of the intersection.

To utilize the benefits of longitudinal, i.e. cohort, analysis for computing scenarios to develop long-term forecasts of life expectancy without having to be bound by its disadvantage of only being able to provide information referring largely to the past, the method demonstrated in this paper combines the cohort and period types of analysis. The methodological approach is shown in schematic form in *Table 1*.

The approach starts out from the age and gender-specific death probabilities shown by the classic period life tables. Methodologically satisfactory life tables have been available in Germany since the 1871/78 period. When the death probabilities are compiled for all the calendar years in which life tables have been computed it gives us the matrix of age-specific death probabilities shown in the first quadrant of *Table 1*. The death probabilities of a particular cohort (q_x) are aligned diagonally in the matrix, and those of a particular calendar year are aligned vertically.

If the q_x values along a diagonal are linked to obtain the survival function for a particular birth cohort, this gives us the diagonally arranged values shown in the second quadrant of *Table 1*. If the q_x values are linked vertically instead of diagonally, this gives us the

Tab. 1: Cohort and period analysis of life expectancy, and the combination of the two forms (the "γ approach")
Kohorten- und Periodenanalyse der Lebenserwartung und die Verbindung der beiden Formen (der „γ-Ansatz")

Matrix of death probabilities ($q_{x,t}$) for both sexes, classified by age (x) and period (t)

$q_{0,t}$	$q_{0,t+1}$	$q_{0,t+2}$	$\cdots$	$q_{0,t+n}$
$q_{1,t}$	$q_{1,t+1}$	$q_{1,t+2}$	$\cdots$	$q_{1,t+n}$
$q_{2,t}$	$q_{2,t+1}$	$q_{2,t+2}$	$\cdots$	$q_{2,t+n}$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$q_{n,t}$	$q_{n,t+1}$	$q_{n,t+2}$	$\cdots$	$q_{n,t+n}$

1. Cohort analysis
Death probabilities are linked diagonally to make up l_x^a trajectories for individual cohorts

$l_{0,t}^a$	$l_{0,t+1}^a$	$l_{0,t+2}^a$	$\cdots$	$l_{0,t+n}^a$
$l_{1,t}^a$	$l_{1,t+1}^a$	$l_{1,t+2}^a$	$\cdots$	$l_{1,t+n}^a$
$l_{2,t}^a$	$l_{2,t+1}^a$	$l_{2,t+2}^a$	$\cdots$	$l_{2,t+n}^a$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$l_{n,t}^a$	$l_{n,t+1}^a$	$l_{n,t+2}^a$	$\cdots$	$l_{n,t+n}^a$

2. Period analysis
Death probabilities are linked vertically to make up l_x^β trajectories for individual periods

| $l_{0,t}^\beta$	$l_{0,t+1}^\beta$	$l_{0,t+2}^\beta$	$\cdots$	$l_{0,t+n}^\beta$
$l_{1,t}^\beta$	$l_{1,t+1}^\beta$	$l_{1,t+2}^\beta$	$\cdots$	$l_{1,t+n}^\beta$
$l_{2,t}^\beta$	$l_{2,t+1}^\beta$	$l_{2,t+2}^\beta$	$\cdots$	$l_{2,t+n}^\beta$
$\cdots$	$\cdots$	$\cdots$	$\cdots$	$\cdots$
$l_{n,t}^\beta$	$l_{n,t+1}^\beta$	$l_{n,t+2}^\beta$	$\cdots$	$l_{n,t+n}^\beta$

3. Combined cohort and period analysis
l_x^β values are linked diagonally to make up diagonal l_x^γ trajectories, where $l_{ij}^\beta = l_{ij}^\gamma$

| $l_{0,t}^\gamma$				
	$l_{1,t+1}^\gamma$			
		$l_{2,t+2}^\gamma$		
			$\cdots$	
				$l_{n,t+n}^\gamma$

survival function provided by a period life table (third quadrant). In line with standard practice, the symbol l_x is used for a survival function (where x is the age in years). To distinguish between the survival functions obtained by cohort and period analysis, the

Fig. 3: γ trajectories for men based on the German life tables from 1959/61 to 1994/96

Abb. 3: γ-Trajektoren für Männer gemäß den deutschen Sterbetafeln von 1959/61 bis 1994/96
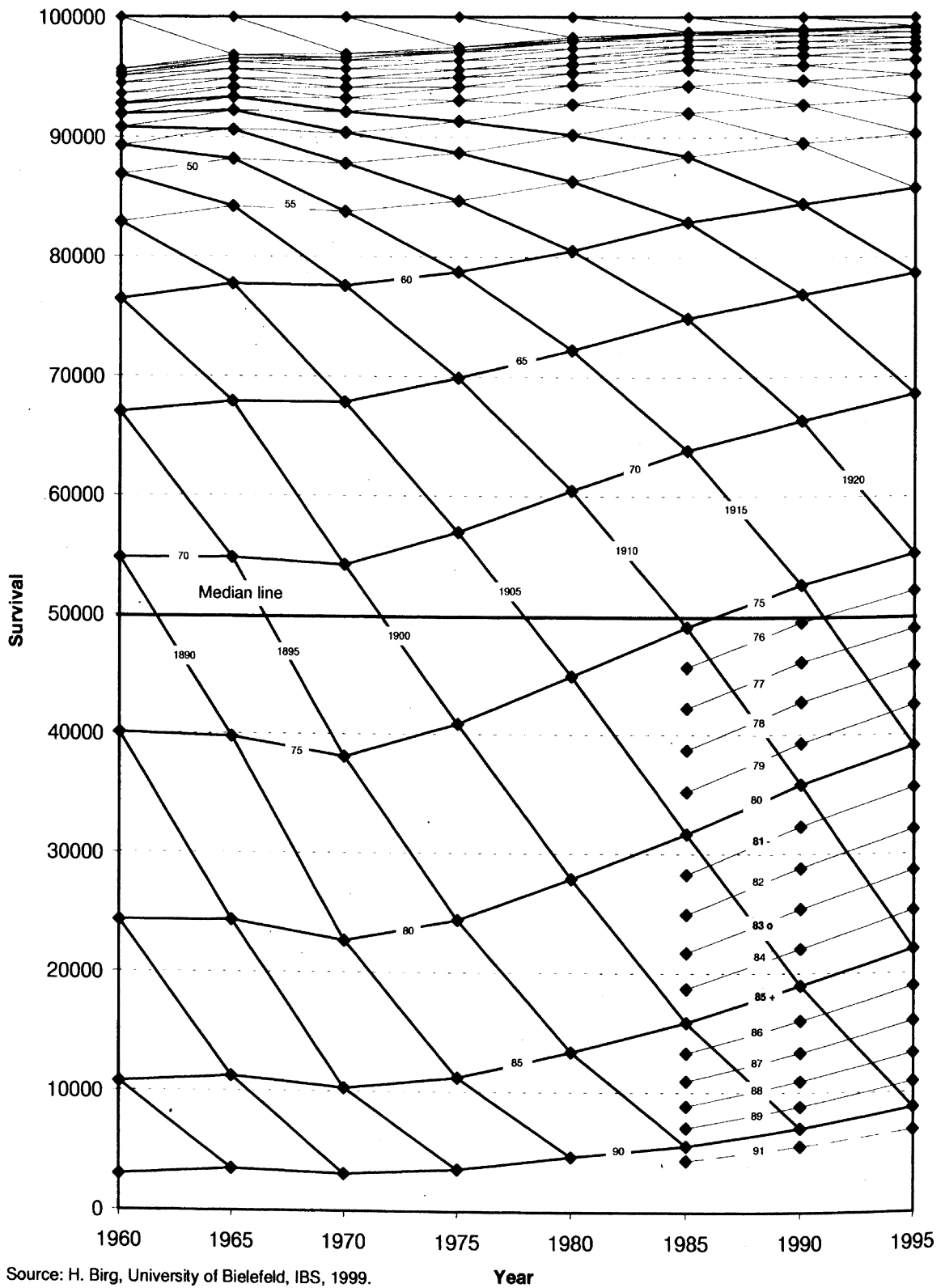
Source: H. Birg, University of Bielefeld, IBS, 1999.
Data: life tables, Federal Statistical Office Germany.

Year

Fig. 4: γ trajectories for women based on the German life tables from 1959/61 to 1994/96
Abb. 4: γ-Trajektoren für Frauen gemäß den deutschen Sterbetafeln von 1959/61 bis 1994/96
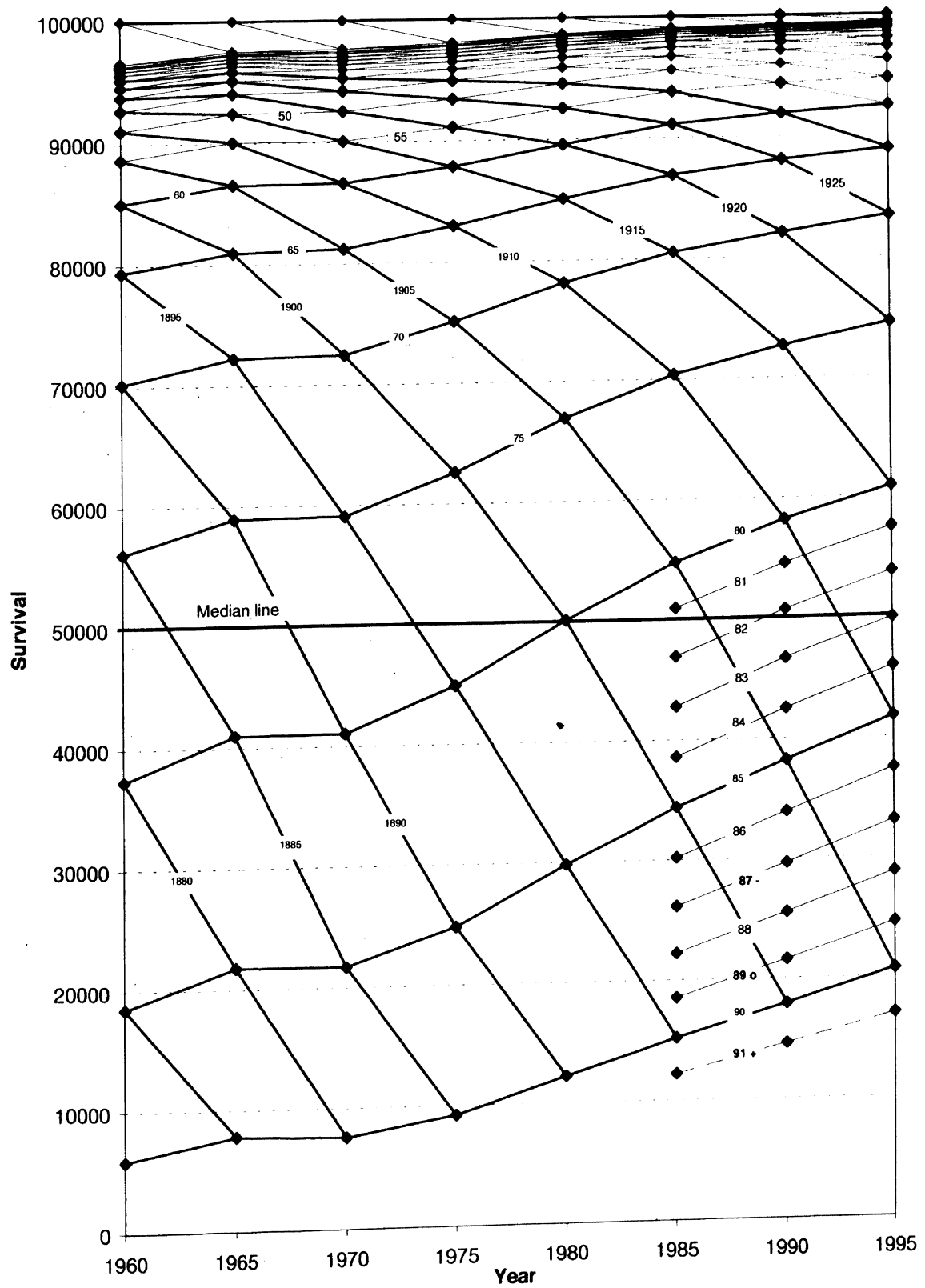
Source: H. Birg, University of Bielefeld, IBS, 1999.
Data: Life tables, Federal Statistical Office Germany.

former is given the superscript α and the latter the superscript β. Values in a survival function belonging to the same cohort are described as an l_x^α trajectory, while those in a function belonging to the same period form an l_x^β trajectory.

The combination of the period and cohort analytical methods is achieved by linking up the l_x^β values in the period-analysis survival function *diagonally* instead of vertically, to constitute a new form of *longitudinal trajectory*, l_x^γ (fourth quadrant of *Table 1*). Picking up on the use of the γ symbol, this method will be referred to in short as the "γ *approach*", to distinguish it from the α (cohort) and β (period) approaches.

The empirical values on the γ trajectories are shown separately for males and females in *Figures 3 and 4*. The γ trajectories, plotted from top left to bottom right, are similar in form to the survival functions of cohorts – a visual impression reinforced by the fact that the trajectories are labelled with different years, starting in 1890. In reality, though, the curves reflect a substantially more complex set of information, as each single l_x^γ value for the period t takes in the death probabilities of several birth cohorts or, to be precise, all cohorts of the ages 0, 1, ..., x at time t.

The γ approach is not just a new variant of the classic cohort analysis of survival functions, and this distinction can also be illustrated empirically. Classic cohort analysis of a survival function will always produce a curve that declines with increasing age. Although the γ trajectories also decline from age 10 onwards, they rise in some instances in the 0–10 age range. This is due to a pronounced decrease in infant and child mortality and, in

Fig. 5: Percentage of people surviving to age 70, 75, 80, 85 and 90 according to German life tables from 1871/81 to 1994/96

Abb. 5: Anteil der bis zum Alter von 70, 75, 80, 85 und 90 Jahren überlebenden Personen gemäß deutschen Sterbetafeln von 1871/81 bis 1994/96
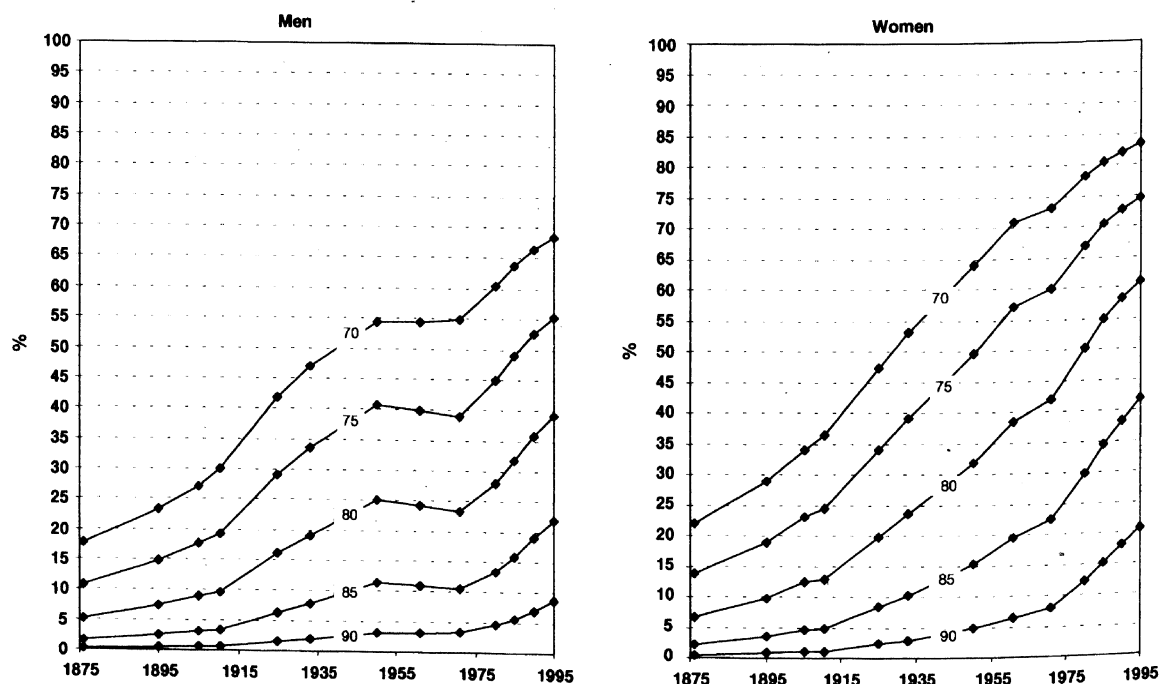
Source: H. Birg, University of Bielefeld, IBS, 1999.
Data: Federal Statistical Office Germany.

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methodological terms, results from the intersection of the cohort and period survival function curves shown in *Figure 2*.

Figures 3 and 4 include another type of curve – referred to here as *iso-age lines* – and these run from bottom left to top right. They are labelled with particular ages. The "80" curve, for example, shows the proportion (out of 100,000 live-born people) reaching age 80. The l_{80}^{β} values are obtained from the life tables prepared in the calendar years shown on the horizontal axis (*Figure 5*).

Figures 3 and 4 provide the following important information and indications relevant to forecasting life expectancies

● the horizontal median line is intersected by the iso-age lines at steadily increasing ages
● the cross-hatched pattern of lines and the shifts in them is very regular: a quality that can be utilized when making forecasts.

The basic principle underlying the forecasting method is to begin by determining median age, by forecasting the years in which the iso-age lines will intersect the median line, before going on to derive a life expectancy forecast from the median age in a second step. When making this derivation, reference will be made to the empirical connection between life expectancy and median age illustrated in *Figure 6*.

The individual steps involved in forecasting life expectancies are detailed in *Figure 7*. The starting point comprises the death probabilities (q_x) specified for particular ages and for the two sexes in the life tables published from 1871/81 to 1994/96. On that basis, **Step 1**
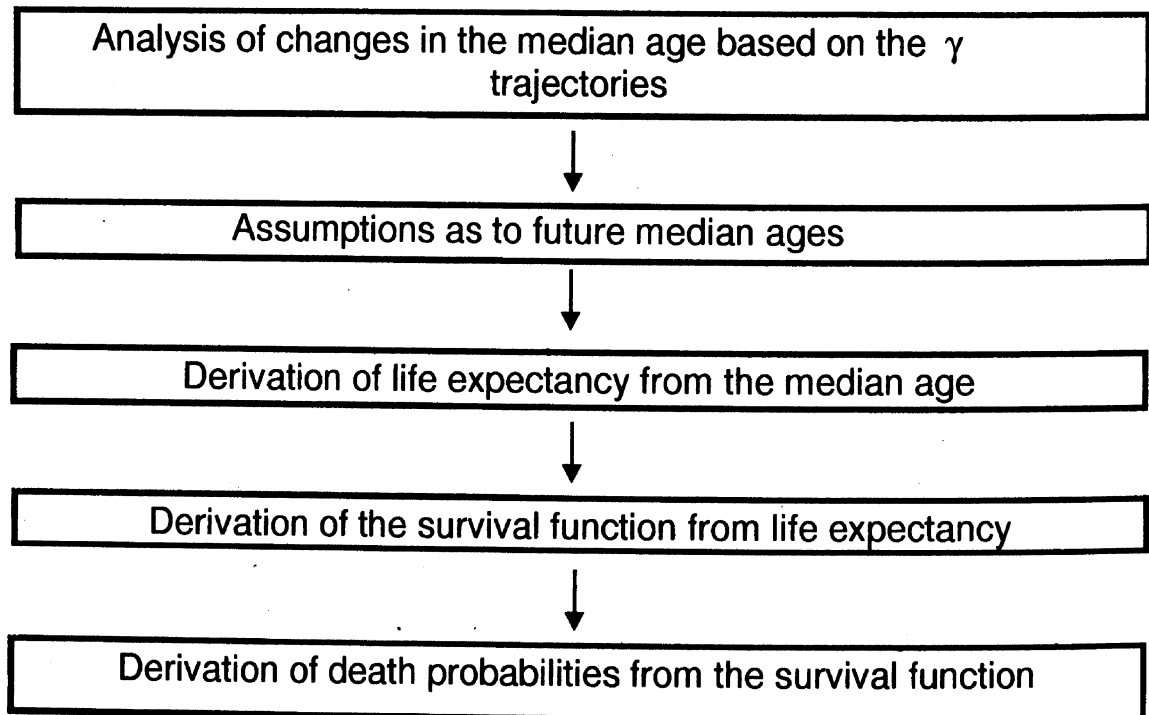
Fig. 6:   Correlation between life expectancy and mean age for the past and projections for the future
Abb. 6:   Korrelation zwischen Lebenserwartung und mittlerem Alter in der Vergangenheit und für die Zukunft

Men   Women

Median age   Median age
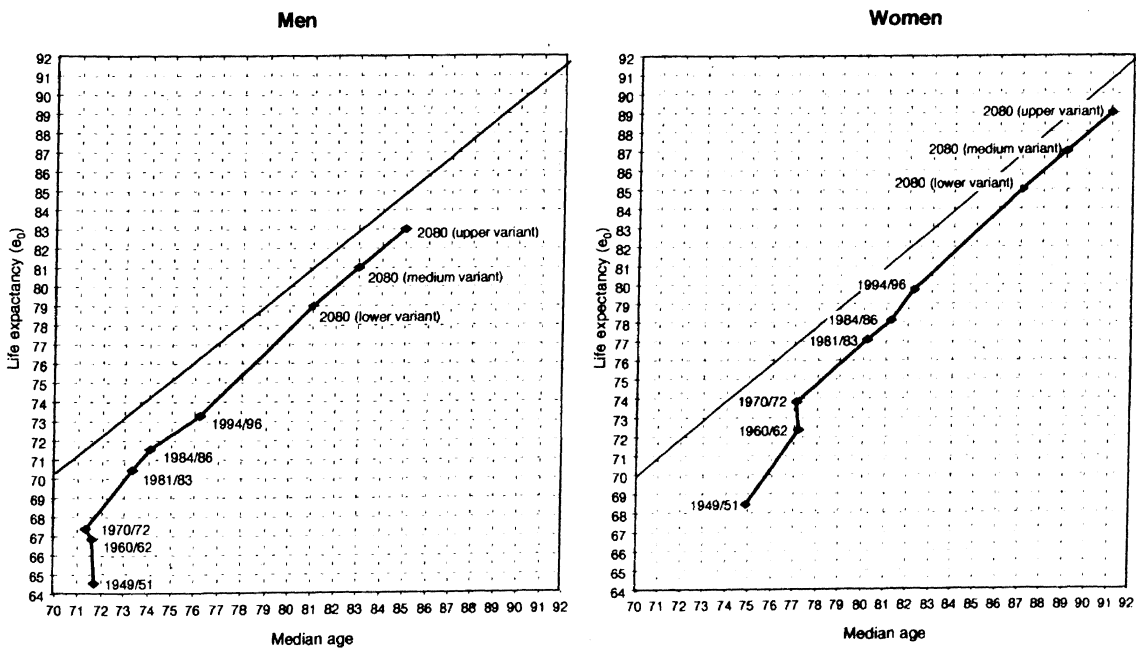
Source: H. Birg, University of Bielefeld, IBS, 1999.
Data for the past: Federal Statistical Office Germany.

Fig. 7: Steps in forecasting life expectancy based on the γ approach
Abb. 7: Schritte zur Prognose der Lebenserwartung auf der Basis des γ-Ansatzes

Analysis of changes in the median age based on the γ trajectories

$\downarrow$

Assumptions as to future median ages

$\downarrow$

Derivation of life expectancy from the median age

$\downarrow$

Derivation of the survival function from life expectancy

$\downarrow$

Derivation of death probabilities from the survival function

is to analyse the change over time in median age and the percentage of people surviving to the ages of 70, 75, ..., 90 (*Figures 3 and 4*).

In **Step 2**, scenarios for the change of median age in future years are worked out by referring back to the changes in the past decades. On this basis, assumptions are specified on future target values for median age, in turn assuming a gradual, non-linear transition to a stable value in the year 2080. From that year on, median age is assumed to remain constant, as no estimates beyond this point are required for the purposes of the current enquiry.

Step 3 computes life expectancy from the empirical relationship between life expectancy and median age (*Figure 6*).

In **Step 4**, the life expectancy $e_0(t)$ for a forecast year t has a survival function $l_x(t)$ accorded to it, initially using a function derived from past experience, but taking due account of the fact that the increases in further life expectancy occurring in recent decades have been especially marked in the high age groups. The transformation function used for this purpose includes a parameter $u(t,x)$ chosen to ensure that the life expectancy already found to which $l_x(t)$ belongs will be reached exactly in the target year[1] (Fig. 8):
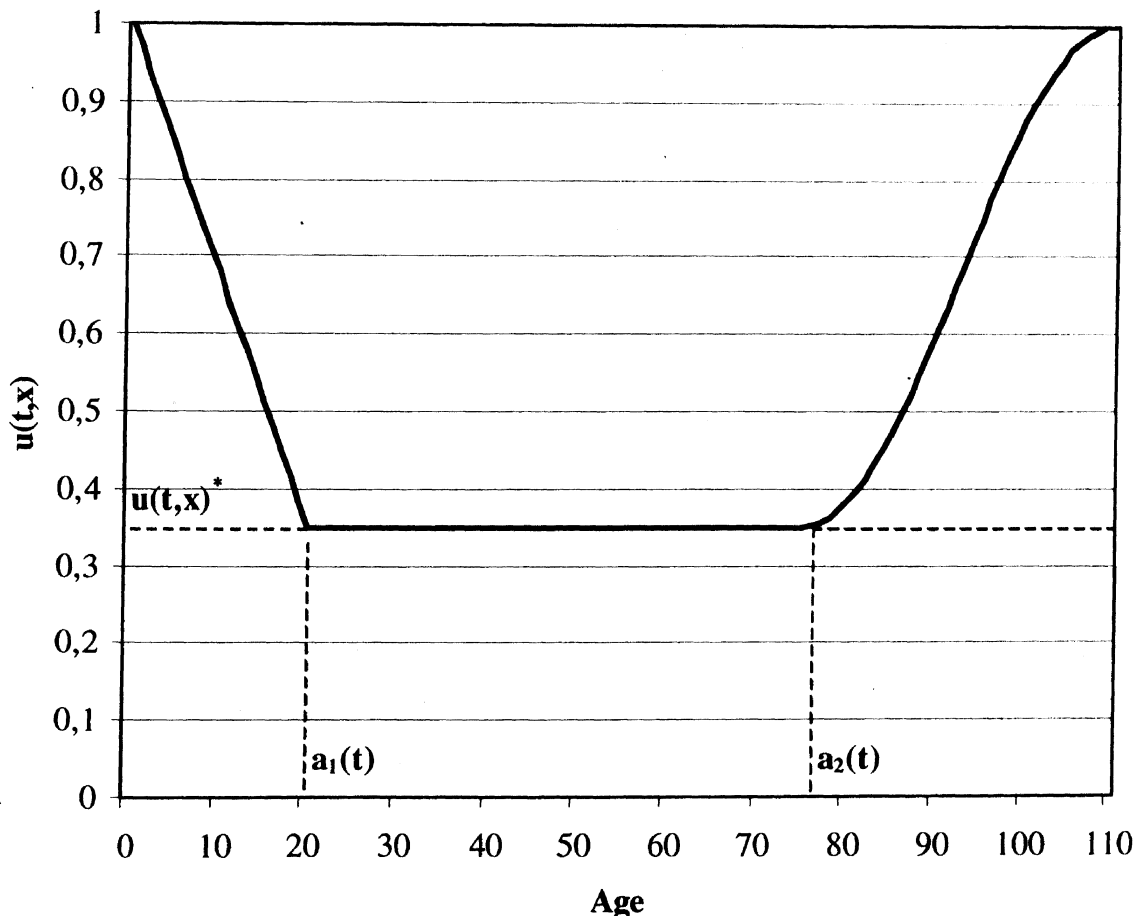
$$l_x(t) = (l_x(94/96))^{u(t,x)}$$
$$x = 0, 1, ..., 110$$
where $t = 2000, 2010, ..., 2080$
$$0 < u(t,x) \leq 1$$
$$0 \leq l_x(t) \leq 1$$

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Fig. 8: Specification of the function u(t,x)
**Abb. 8:** Spezifizierung der Funktion u(t,x)

The smaller $u(t,x)$, the higher are the values in the survival function $l_x(t)$, and the longer the corresponding life expectancy $e_0(t)$. The parameter $u(t,x)$ is not given the same value for all ages: the value first declines with age until it reaches a floor at $u(t,x)^*$ from which it later increases again. This choice of parameter ensures that the growth in life expectancy will be greater at older ages. Thus, the $u(t,x)$ function is divided into three sections. In the first section, covering the range $x = 0$ to $x = a_1(t)$, the function declines on a linear basis. Then, over the range $x = a_1(t)$ to $x = a_2(t)$, it is constant at $u(t,x)^*$.

Finally, over the range $x > a_2(t)$ the $u(t,x)$ function is an s-curve rising to unity. An s-curve was chosen in preference to a linear function in the final section to achieve a gradual change of the curvature in the area around $a_2(t)$. Around $x = a_1(t)$, this gradual transition occurs even with a linear function, as the change in the survival function is much less marked for people in the younger and middle age groups than it is for the older ones.

The choice of the two age thresholds $a_1(t)$ and $a_2(t)$, and of the parameter floor $u(t,x)^*$ makes it possible to fine tune the transformation of the survival function so closely that the desired life expectancy in the target year can be precisely attained[2].

In the final stage (Step 5), the death probabilities are derived from the survival functions for the years 1996, 1997, ..., 2080. Survival functions for the intervening years between the fixed coordinates in 2000, 2010, ..., 2080 are obtained by interpolation.

The key distinction between this approach and other methods of forecasting life expectancies[3] is that, in most other cases, death probabilities function as an input to the forecasting procedure and survival functions are the outcome, whereas in this approach it is the other way round: survival functions are a point of departure, and death probabilities are computed as an outcome. The approach chosen here has three favourable features when it comes to long-term forecasting of life expectancy:

(1) Changes in survival functions that have occurred in the past are much more stable in direction over time than the pronounced fluctuations in death probabilities. This stability is beneficial for forecasting purposes.

(2) Interdependencies exist between specific death probabilities, and these give rise to potential problems if the death probabilities for distinct ages are extrapolated separately from one another, as is the case in most mortality forecasts. If survival functions are chosen as the central empirical basis, these interdependencies are automatically taken into account.

(3) The approach chosen allows the specific benefits of period analysis – its more up-to-date empirical data than those obtained from cohorts on completion of a full life-span – to be combined with the methodological advantages of cohort analysis.

Fig. 9: Historical life expectancy curve for men in Germany and the forecast for male German nationals in the former West German states, to 2100

Abb. 9: Historische Lebenserwartung der Männer in Deutschland und Prognose bis 2100 für deutsche Männer in den alten Bundesländern
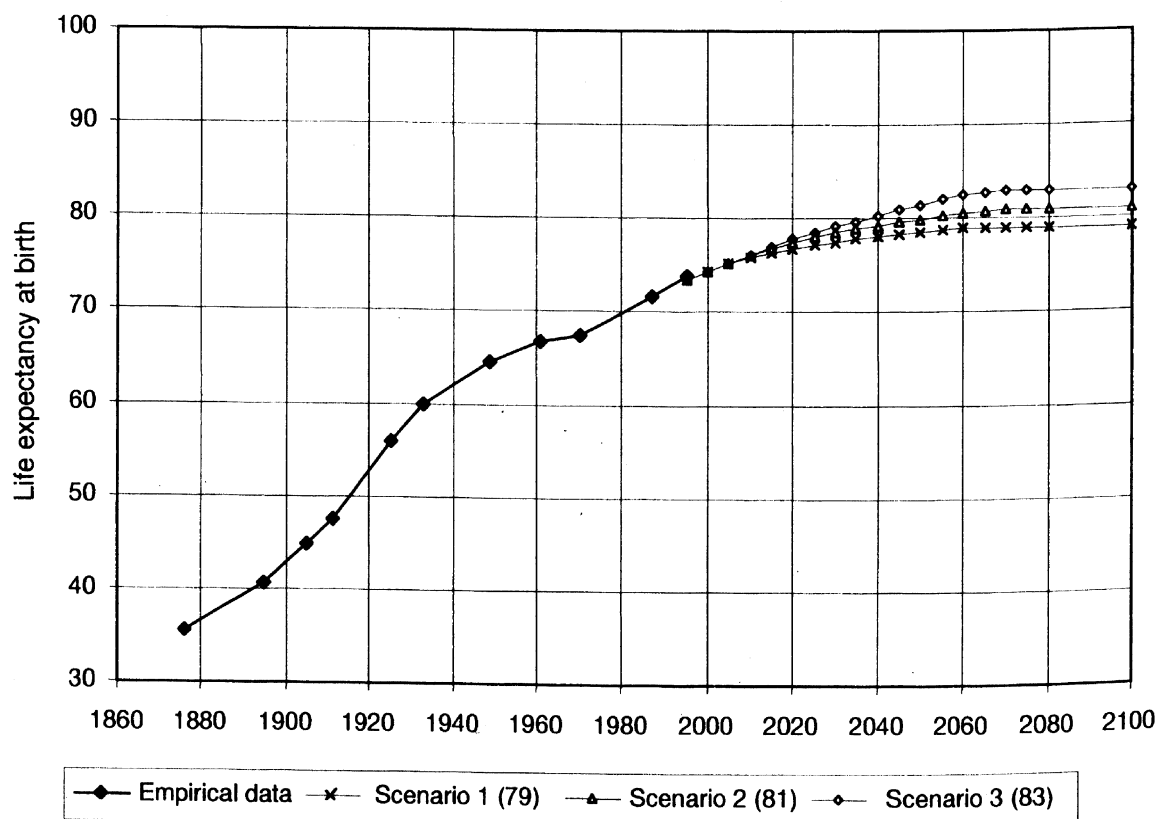
Data up to 1996:
Statistical Office Germany, Fachserie 1, Reihe 1, Gebiet und Bevölkerung 1996
Birg/Flöthmann, IBS, Univ. Bielefeld, 1999

3. Results obtained by applying the approach to forecasting life expectancy in Germany up to 2080

The forecast results set out in *Table 2* and *Figures 9 and 10* show the median age and life expectancy as they develop from the 1994/96 life table to the target year of 2080, and

Tab. 2: Scenarios for median age at death and life expectancy
Szenarien für mittleres Sterbealter und Lebenserwartung

	Median age at death		Life expectancy	
	1994/96	2080	1994/96	2080
		– Men –		
Mortality scenario 1	76,2	81	73,3	79
Mortality scenario 2	76,2	83	73,3	81
Mortality scenario 3	76,2	85	73,3	83
		– Women –		
Mortality scenario 1	82,2	87	79,7	85
Mortality scenario 2	82,2	89	79,7	87
Mortality scenario 3	82,2	91	79,7	89

Fig. 10: Historical life expectancy curve for women in Germany and the forecast for female German nationals in the former West German states, to 2100

Abb. 10: Historische Lebenserwartung der Frauen in Deutschland und Prognose bis 2100 für deutsche Frauen in den alten Bundesländern
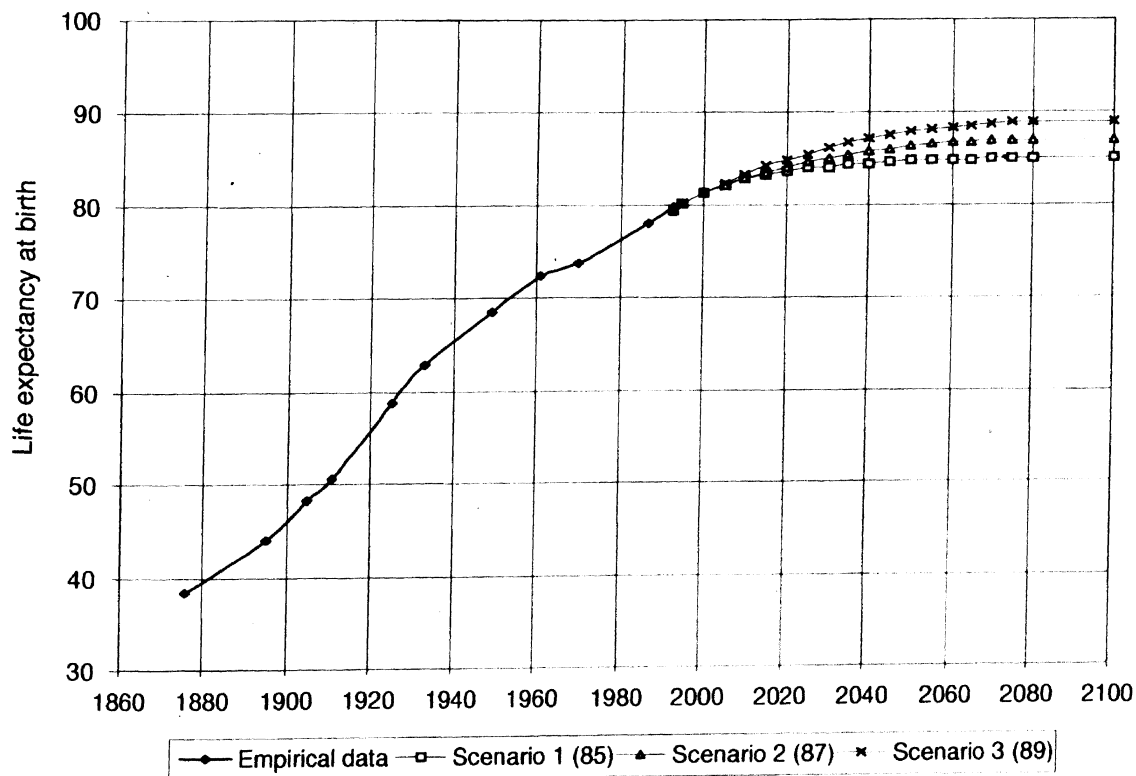
—●— Empirical data —□— Scenario 1 (85) —▲— Scenario 2 (87) ✕ Scenario 3 (89)

Data up to 1996:
Statistical Office Germany, Fachserie 1, Reihe 1, Gebiet und Bevölkerung 1996
Birg/Flöthmann, IBS, Univ. Bielefeld, 1999

Tab. 3: Mortality scenarios differentiated by population groups and gender[1]
**Mortalitätsszenarien unterschieden nach Bevölkerungsgruppen und Geschlecht**

	Scenario 1			Scenario 2			Scenario 3		
	1993/95[1]	2035	2080	1993/95[1]	2035	2080	1993/95[1]	2035	2080
– Men –									
German nationals/Former West Germany	73	77.9	79	73	79	81	73	79.8	83
German nationals/Former East Germany	70.3	77.9	79	70.3	79	81	70.3	79.8	83
Immigrants	–	–	–	80.3	80.6	81	–	–	–
– Women –									
German nationals/Former West Germany	79.5	84.4	85	79.5	85.4	87	79.2	86.8	89
German nationals/Former East Germany	77.9	84.4	85	77.9	85.4	87	77.9	86.8	89
Immigrants	–	–	–	85.2	86.3	87	–	–	–

[1] This table is not based on the 1994/96 life table, but on the unpublished life table compiled by the Federal Statistical Office, differentiating between German nationals (long-term residents) and immigrants.

are divided into a lower, middle and upper variant. In the middle variant, average male life expectancy increases from 73.3 years in 1994/96 to 81 years by 2080, and the female life expectancy from 79.7 to 87 years. Over the same period, the median age at death is forecast to increase from 76.2 to 83 years for men and from 82.2 to 89 years for women.

The figures for German citizens have also been broken down to cover the former West and East German states. This was not possible for citizens of other countries or for German nationals who are recent immigrants, as the life tables covering these sections of the population contain a number of sources of imprecision. There is a mortality differential of about seven years between resident Germans and immigrants, but the gap is partly explicable by grey areas in the statistical data. Because some of the foreign nationals who leave Germany again do so without reporting their departure to the registry of inhabitants, the number of foreign nationals currently resident in the country tends to be estimated too high, so the age-specific death probabilities are underestimated as a result. But although it is clear that life expectancy in these population groups is lower than the life tables suggest, the actual magnitude of the error is not known. On the one hand, migration does have a positive selection effect on life expectancy (people who are very ill are also less mobile, and hence are less likely to migrate than healthier people), but how many years this adds on to actual life expectancies of immigrants in Germany is open to conjecture[4]. All three mortality scenarios assume that the life expectancy of foreign nationals or immi-

Fig. 11: **Mortality assumptions for men up to 2100, differentiating between German nationals and immigrants, in old West and former East German states**
Abb. 11: **Mortalitätsannahmen für Männer bis 2100, unterschieden nach Deutschen und Ausländern in den alten und neuen Bundesländern**
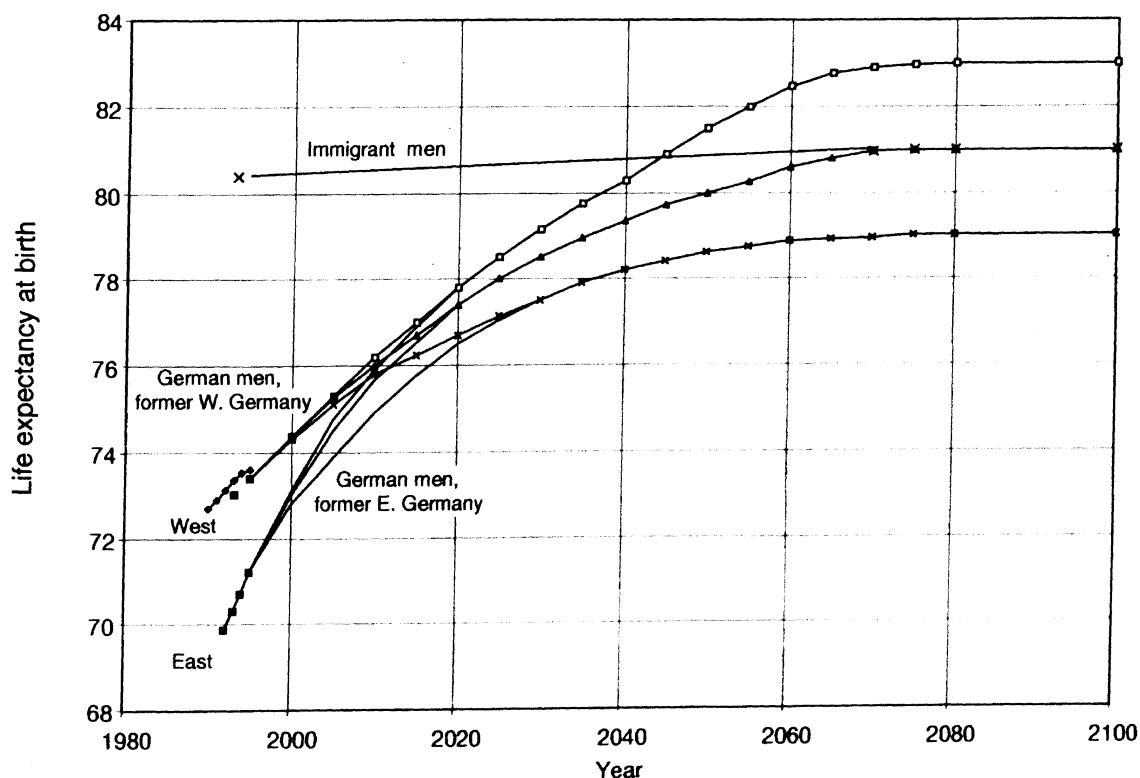
Data 1991-1996: Federal Statistical Office Germany
Birg/Flöthmann, IBS, Univ. Bielefeld, 1999

Fig. 12: Mortality assumptions for women up to 2100, differentiating between German nationals and immigrants, in old West and former East German states

Abb. 12: Mortalitätsannahmen für Frauen bis 2100, unterschieden nach Deutschen und Ausländern in den alten und neuen Bundesländern
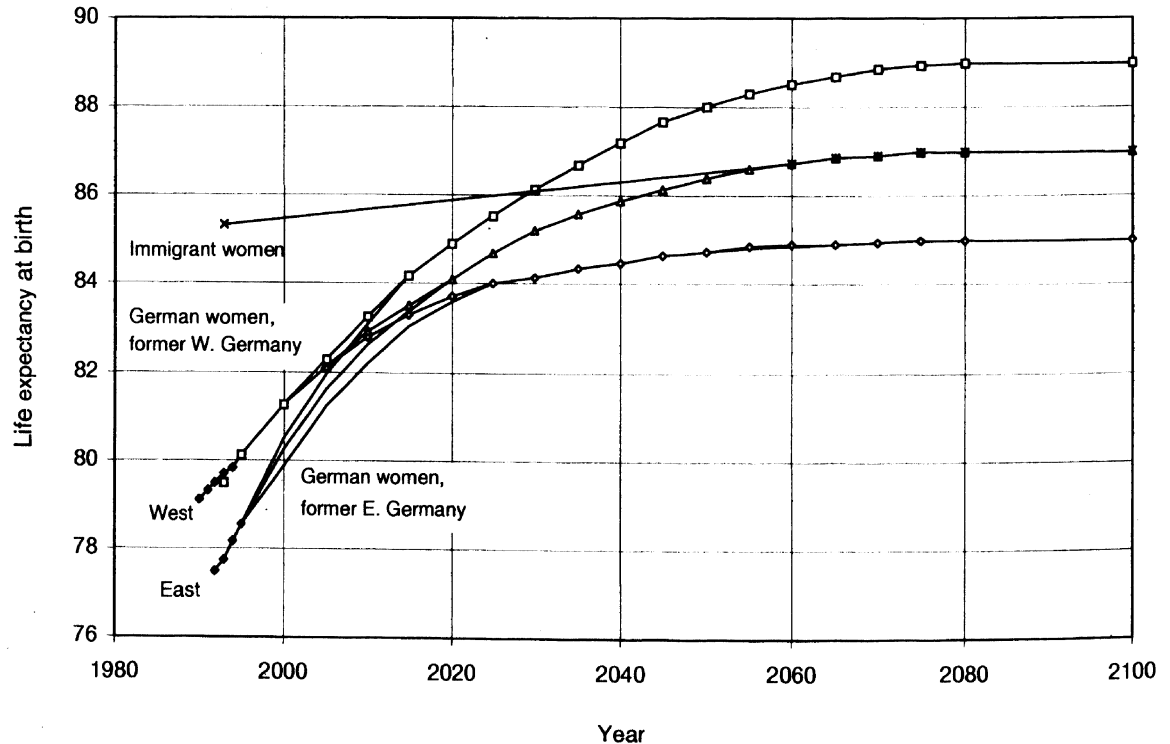
Data 1991-1996: Federal Statistical Office Germany
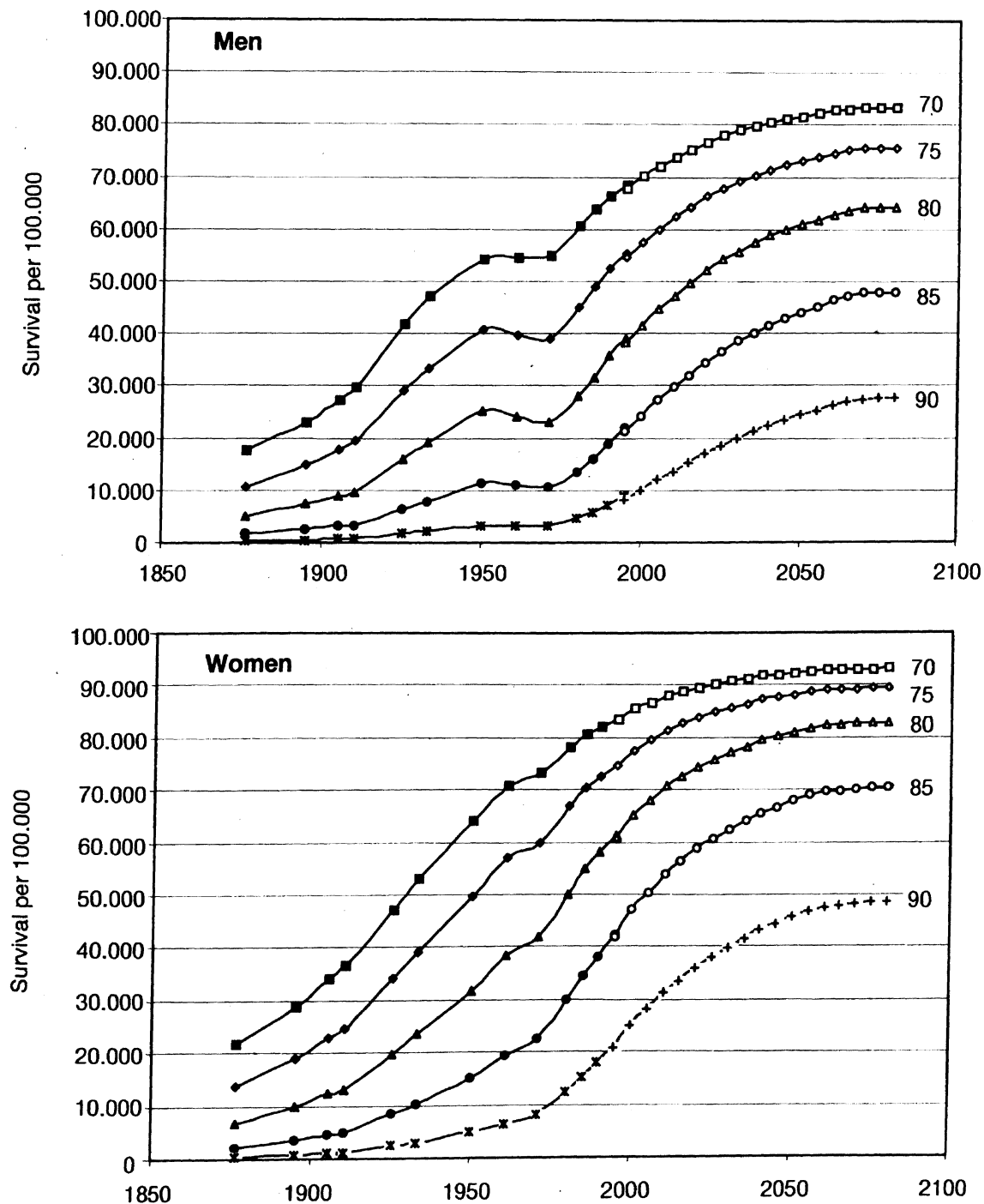Birg/Flöthmann, IBS, Univ. Bielefeld, 1999

grants resident in Germany will approach that of the German nationals over time. Given that the latter currently have a lower statistical life expectancy, this means in effect that the life expectancies of foreign nationals and immigrants are assumed to increase only slightly during the forecast period (*Table 3*). How the curves approach one another is illustrated in all the scenarios in *Figures 11* (for men) and *12* (for women).

One way of performing a strict formal test of the quality of a forecasting approach is to take the curves showing the number of people surviving up to the ages of 70, 75, ..., 90 in the past (*Figure 5*) and to extrapolate them into the future using the survival functions obtained in the forecasting approach. The past curves run perfectly smoothly into the curves for the period up to 2080 (*Figure 13*). The only slight differences between the last empirically established value and the first forecast value are attributable to the fact that the past data apply to Germany as a whole, whereas the forecast covers only the former West German states. Thus, the forecasting approach passed this strict test very well indeed.

Another strict test involves comparing the increase in life expectancy from one age-group to another. In recent decades, the increase in further life expectancy for 80-year-olds has been greater than for 70-year-olds, and the increase for 70-year-olds greater than for 60-year-olds. To be realistic, it is important that a forecasting method should be able to reproduce these relative trends when it looks ahead. As *Figures 14 and 15* demonstrate, our approach also passed this important test: the increase in further life expectancy grows

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Fig. 13: Empirically established and forecasted figures for persons surviving to ages 70–90 (per 100,000) – German nationals in former West German states, middle variant (e_0 rising to 81 years by 2100)

Abb. 13: Empirisch gesicherte und prognostizierte Daten für die Zahl der Überlebenden bis zum Alter 70 bis 90 Jahren (je 100 000) – Deutsche in den alten Bundesländern, mittlere Variante (e_0 erreicht 81 bzw. 87 Jahre bis 2100)
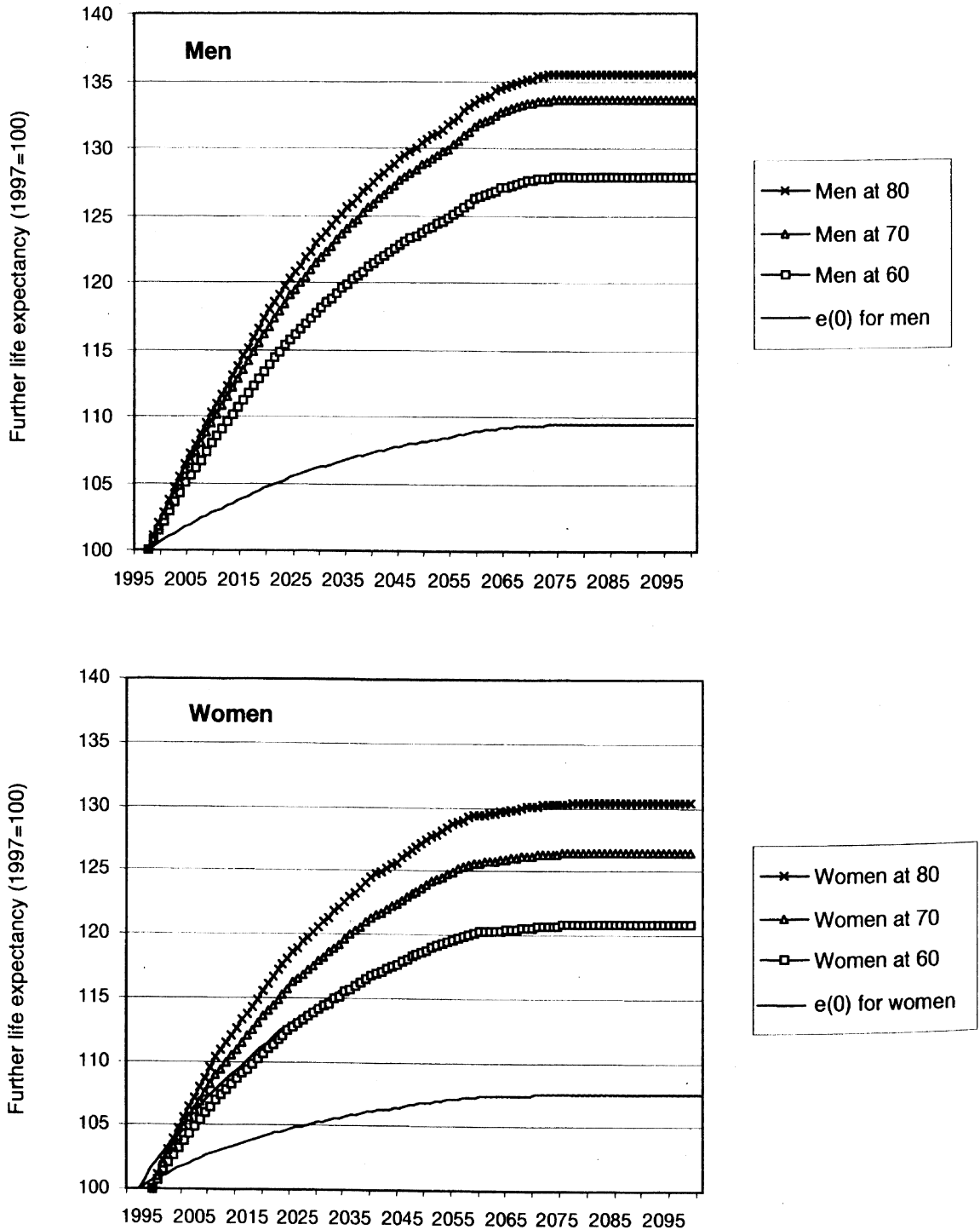

Data 1876 - 1996: Federal Statistical Office Germany.

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Fig. 14: Relative increase in the forecast of further life expectancies in the former West German states (middle variant)

Abb. 14: Relative Steigerung in der Prognose für die fernere Lebenserwartung in den alten Bundesländern (mittlere Variante)

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Fig. 15: Forecast development of further life expectancies for German nationals in the former West German states (variant 02)

Abb. 15: Prognostizierte Entwicklung der ferneren Lebenserwartung von Deutschen in den alten Bundesländern (Variante 02)

△ Women at 60
▲ Men at 60
◇ Women at 70
◆ Men at 70
□ Women at 80
■ Men at 80
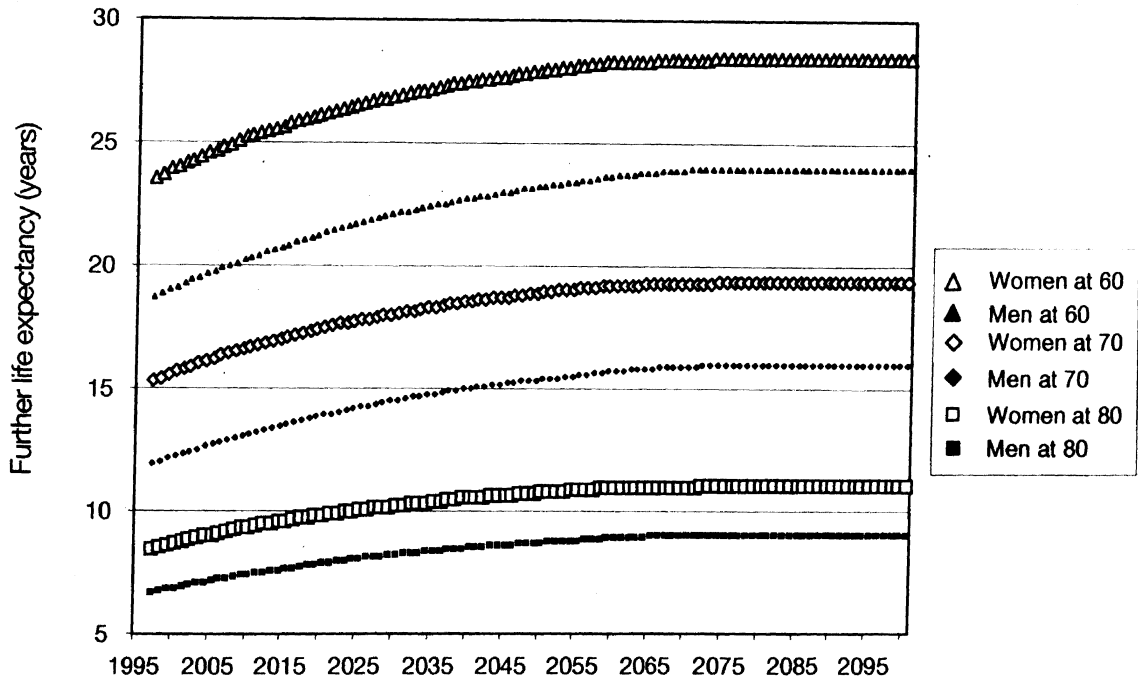
Birg/Flöthmann, IBS, Univ. Bielefeld 1999

Fig. 16: Shift of the I_x functions of the male population in the former West German states, based on an increase in life expectancy from 73 to 83 years by 2080

Abb. 16: Veränderung der I_x-Funktionen der männlichen Bevölkerung in den alten Bundesländern, basierend auf einer Steigerung der Lebenserwartung von 73 auf 83 Jahre bis zum Jahr 2080

Age

—□—1996 —▲—2000 —×—2010 —■—2020 —○—2030 —+—2040 —●—2050 —●—2060 —◇—2080
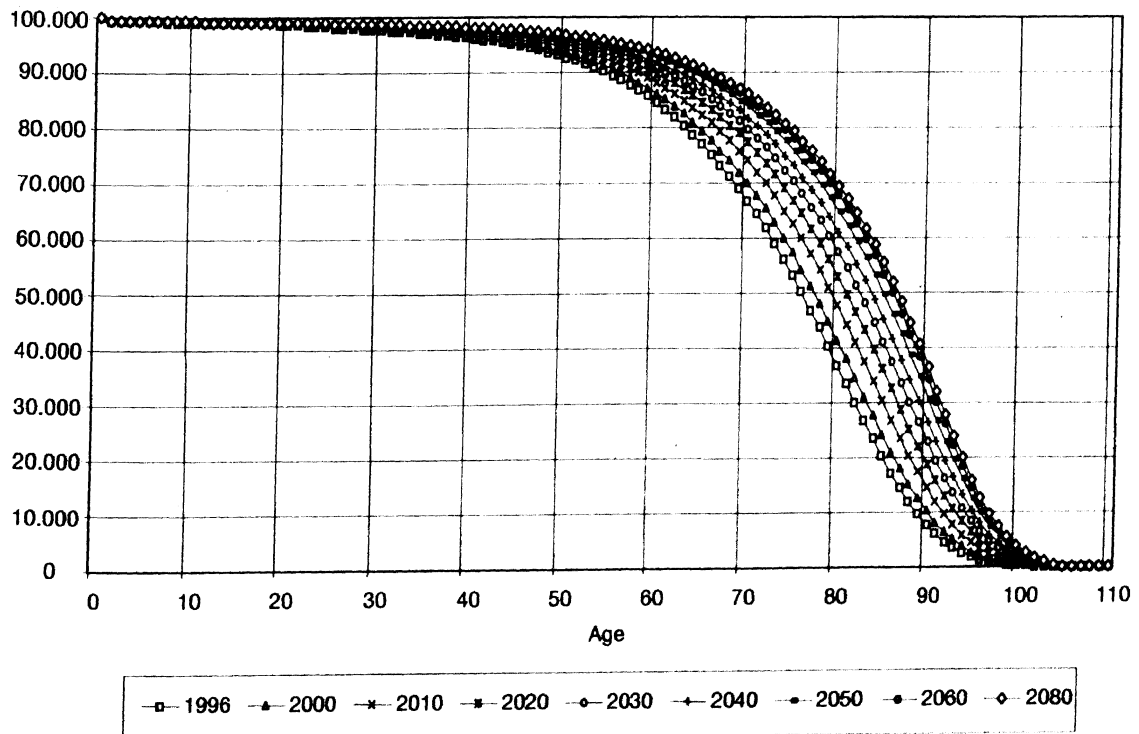
Birg/Flöthmann, IBS, Univ. Bielefeld 1999

in proportion to the age from which it applies. The same effect is apparent in the forecast survival functions shown in *Figure 16*: the ordinate value increases by a factor of 1.2 for 65-year-olds, of 1.8 for 80-year-olds, but of 4 times for 90-year-olds.

4. The results compared with those of other studies

In an ideal world one would want to compare the findings of this study with those of others not just in terms of life expectancies at birth, but also in terms of the forecast of further life expectancies at more advanced ages. Unfortunately, the other four studies discussed here either do not permit this more subtle comparison at all or, as in the case of the

Tab. 4: **The assumptions on increased life expectancy in 21st century Germany made by various research institutes**
Annahmen zur steigenden Lebenserwartung in Deutschland für das 21. Jahrhundert, ermittelt von verschiedenen Forschungsinstituten

Institute	Increase in life expectancy at birth Starting from the 1994/1996 life table: 73 years (men), 80 years (women)		
	2035	2050	2080
Prognos (1998)[1]	3	0	0
UNO (Population Division, 1998)[2]	3	5	–
German Statistical Office ("8th coordinated projection")[3]	0*	–	–
German Statistical Office ("9th coordinated projection")[4]	4	5	–
Institute of Population Research and Social Policy (IBS), University of Bielefeld[5]			
Variant A	5	5	6
Variant B } Resident German nationals	6	7	8
Variant C	7	8	9
Variant D	5	7	8
Variant E } Total population	7	9.	9
Variant F	9	11	14

[1] Prognos (ed.), Prognos-Gutachten 1998, Frankfurt/M. 1998.
[2] UN (ed.), World Population Prospects – The 1998 Revision, New York 1999 (middle variant).
[3] *B. Sommer,* „Entwicklung der Bevölkerung bis 2040", in: Wirtschaft und Statistik, 1994, pp. 497–504.
[4] Statistisches Bundesamt, 9. Koordinierte Bevölkerungsvorausberechnung (probable assumtions as currently planed according to information supplied by the Fed. Statistical Office).
[5] Variants A–C are discussed in this contribution and in: *H. Birg and A. Börsch-Supan,* „Für eine neue Aufgabenteilung zwischen gesetzlicher und privater Altersversorgung", Gutachten im Auftrag des Gesamtverbandes der deutschen Versicherungswirtschaft, Berlin, November 1999. Variants D–F: See *H. Birg, E.-J. Flöthmann, T. Frein and K. Ströker,* Simulationsrechnungen zur Bevölkerungsentwicklung in den alten und neuen Bundesländern im 21. Jahrhundert, Materials published by the Institut für Bevölkerungsforschung und Sozialpolitik (University of Bielefeld), vol. 45, Bielefeld 1998.
* Assumes no further increase in life expectancy from the year 2000 onwards in former West Germany, while that in former East Germany continues rising until it has matched the western figure.

Prognos Institute forecast, only to a limited extent. They generally present figures on life expectancy at birth, and details of further life expectancy at advanced ages are largely absent. Likewise, the comparative studies do not provide median age figures, so another potential basis of comparison is lost.

Germany's *Federal Statistical Office* based its last published projections up to 2040, the *"8th coordinated population projection"* which came out in 1994, on the assumption that life expectancy would cease increasing in former West Germany after the year 2000: "Our assumption for mortality has been that, in line with the trend to date, it will decline further, meaning that the life expectancy of newly born people in former West Germany – currently (in 1992) 73.2 years for males and 79.6 years for females – will increase by another 1.5 years by the year 2000 and remain constant thereafter." (*Sommer* 1994: 497). In its next projections (the "9th coordinated population projection") due for publication in late 1999, the Federal Statistical Office will presumably substitute a more realistic assumption for its earlier view that life expectancies at birth would remain constant from 2000 onwards. If this as yet unpublished work is also included in the comparison, the assumed gain in life expectancy between the 1994/96 life table and the year 2035 comes out at between 3 and 9 years (*Table 4*). Looking ahead to the period after 2035, the *Prognos Institute* of Basle is the only one to assume constant life expectancy, while others assume that it will continue to increase up to 2050 or 2080. The author's mortality scenarios presented in this article have been labelled as variants A, B and C in *Table 4*.

How likely is it that the uppermost scenarios in *Table 4*, assuming a continuing growth in life expectancy of anything up to 14 years up to the end of the 21st Century, will become reality? Life expectancy would indeed increase by 10–14 years by 2080 if it were to maintain the rate of increase achieved in the past few decades, of approximately 1.8 to more than 2.0 years per decade. It is quite possible that this could happen, for even if there is only modest growth in economic output the increases in gross domestic product will mount up in coming decades, making more financial resources available to the health-care system. That should mean that the advances in medicine already achieved and those now foreseeable will indeed be implemented, introducing more effective methods of combating the most important causes of death (cardiovascular diseases and malignant cancer) and bringing other improvements in care, regardless of how the health system as we know it today is reformed. Other factors promoting longevity include the usually positive changes in health-consciousness and a tendency for members of the public to lead healthier lives, while health risks at the workplace are also likely to grow smaller as working hours also decrease, the potential for preventive health care is being better utilized by promoting awareness and improving public information. Finally, there are the selection effects stemming from immigration, which will become more significant if the immigrant population grows proportionately greater in years to come[5].

Some further potential for growth in life expectancy arises from the fact that the after-effects of the two world wars which weakened the long-term trend in increasing life expectancy for men (but not for women) will at last have been overcome in the 21st century. Just how strong these effects were, or indeed to some extent still are, is immediately evident if the proportion of people surviving to the ages of 70, 80, 85 and 90 are compared for men and women. *Figure 5* shows that the life tables from 1959/60 to 1974/76 bring the rising curves for men to a halt. The flat or dipping sections of the curves reflect those who died in or as a result of World War I, and the effect of post-war food shortages. For

example, the curve for people surviving to age 75–80 dips between 1955 and 1975[6]. The curves for women, by way of contrast, continue upwards throughout.

Anmerkungen

[1] By virtue of its simplicity coupled with the amenability of its parameters to substantial interpretation, the transformation function used here is better suited to the task in hand than the complex function used by *Heligman-Pollard* ("The age pattern of mortality", in: *Journal of the Institute of Actuaries*, vol. 107, pp. 49–80), or the survival function proposed by *A. Di Pino* and *P. Pirri* ("Analysis of survival functions by a logistic derivation model: the generalized moivre function", in: *Genus*, vol. LIV, nos. 3–4, pp. 35–54, 1998).

[2] The median age of the $l_x(t)$ survival function thus attained, though not a perfect match, comes sufficiently close to median age originally determined. A perfect match can be achieved by choosing appropriate values for the three parameters $a_1(t)$, $a_2(t)$ and $u(t)^*$, but in this instance, it was not felt necessary to include this additional iteration.

[3] A typical procedure is that described by *B. Schmithals* and *E. U. Schütz*: „Herleitung der DAV-Sterbetafel 1994R für Rentenversicherungen" in: The society bulletin of the Deutsche Gesellschaft für Versicherungsmathematik, vol. XXII (1995), no. 1, pp. 29–69. A general review of the various approaches in use is offered by *S. J. Olshansky*, "On Forecasting Mortality", in: *The Milbank Quarterly*, Vol. 66, no. 3, 1988. See also: *S. J. Olshansky, B. A. Carnes* and *C. Cassel*, "In Search of Methusalem: Estimating the upper limits of human longevity", in: Science, Vol. 250, Nov. 1990, pp. 634–640.

[4] For the city of Berlin a difference of more than 10 years was computed by *R. D. Scholz* and *H. Thoelke*, Lebenserwartung in Berlin 1986–1994 – Trends und regionale Unterschiede. Discussion paper no. 29, Senatsverwaltung für Gesundheit und Soziales, Berlin, 1997, Table 11, p. 28.

[5] An overview of the literature on mortality modelling and the potential gains in life expectancy is provided by *A. I. Yashin, I. A. Iachine* and *A. S. Begun*, "Mortality modelling in the population studies of ageing". Paper presented at the European Population Conference, The Hague, 1999.

[6] The fact that the 70–85 year survivor age-group was the most severely affected still only provides limited information as to exactly which age cohorts suffered the largest number of earlier mortalities, because the life tables from which the data were obtained are period tables. That means that a decline in, say, the number of men surviving to age 70 such as that occurring in the 1959/60 life table is not wholly attributable to the cohort born in 1890. In a period table, the probability of surviving to a certain age (e.g., 70) depends on the mortality in all birth cohorts who were still living alongside each other in the year when the life table was compiled. In other words, the probability of surviving to age 70 was also influenced by the cohorts who, in 1959/60, were younger than their counterparts born in 1890. The younger cohorts also included men who died as a result of World War II. Thus the dipping curves in *Figure 5* are probably caused by the Second as well as the First World War, also taking in the periods of post-war deprivation and the economic depression of 1932. On mortality in particular male age cohorts, see *H. Birg* and *H. Koch, Der Bevölkerungsrückgang in der Bundesrepublik Deutschland*, Frankfurt and New York 1987, Figure SM1, p. 119. See also *R. H. Dinkel, C. Höhn* and *R. D. Scholz* (eds.), *Sterblichkeitsentwicklung unter besonderer Berücksichtigung des Kohortenansatzes*, Munich 1996; *E. Bomsdorf, Generationensterbetafeln*, Cologne 1993; *K.-A. Schäffer*, "Analyse der Männersterblichkeit in der Bundesrepublik Deutschland (alte Länder)", in: *Allgemeines Statistisches Archiv 1980*, pp. 411–432.

Summary

In most mortality studies, the dynamism of changes in life expectancy is based on period analysis using cross-sectional data referring to a specific period. The cohort approach is methodologically more satisfactory, but it has the disadvantage that, by the time the mortality figures for a particular cohort are complete, they reach back up to 100 years in time, which means the approach has little value as a basis for forecasting.

This paper presents a new approach which combines period and cohort analysis. It therefore offers the advantage of period analysis (the use of more up-to-date statistical data) coupled with the methodological benefits of the cohort approach. The analysis is

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based on German life tables going back as far as 1871. On this empirical basis, the approach is used to issue a forecast of life expectancies up to the year 2080.

The middle variant forecasts a growth in male life expectancy from 73.3 years in 1994/96 to 81 years in 2080, and in female life expectancy from 79.7 to 87 years. The gain in life expectancy (i.e., in further life expectancy) is considerably greater for the older age-groups than it is at ages below 50 years. The results are presented in the form of survival functions from which age-specific death probabilities for both sexes and further life expectancies can be derived for each year during the forecast period.

Résumé

Dans la plupart des études de mortalité, la dynamique du changement de l'espérance de vie est basée sur une analyse périodique de données de coupe transversale d'une période déterminée. L'approche de cohortes s'y prête mieux du point de vue méthodologique, mais elle a l'inconvénient que les données de mortalité ne sont complètes que pour les cohortes remontant à 100 ans dont résulte que cette approche n'a qu'une valeur minime pour le pronostic.

Cet article présente une approche nouvelle combinant l'analyse périodique avec l'analyse de cohortes pour associer ainsi l'avantage de la première (l'utilisation de données statistiques plus actuelles) aux avantages méthodologiques de l'approche de cohortes. L'analyse est basée sur les tables de mortalité allemandes datant en arrière jusqu'à l'an 1871. Sur cette base est fondée l'approche pour le pronostic de l'espérance de vie jusqu'à l'an 2080.

La variante moyenne prognostique une augmentation de l'espérance de vie pour les hommes à 81 ans en 2080, partant de 73,3 ans des années 1994/96, et pour les femmes de 79,7 à 87 ans. L'accroissement de l'espérance de vie est bien plus important pour les groupes d'âge élevé que pour les personnes âgées de moins de 50 ans. Les résultats sont présentés sous forme de fonctions de survie permettant d'en dériver les quotients de mortalité par âge pour les deux sexes et l'espérance de vie additionnelle pour chaque année de la période considérée.

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