II. AN INTERGENERATIONAL RATIONALE FOR FERTILITY ASSUMPTIONS IN LONG TERM WORLD POPULATION PROJECTIONS

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A. INTRODUCTION AND FRAMEWORK OF THE ANALYSIS

The analysis of fertility and generative behaviour usually concentrates on persons, couples, families, social communities, ethnic groups, nations or generations. These individuals and groups are examples for various decision making units with different objective functions and specific constraints for their generative behaviour. In this contribution, the elementary units for fertility decisions are generations. Generations may be regarded as the most natural units of making fertility decisions because the mere existence of any generation depends on the fertility decisions of the preceding ones.

There are many emotional, cultural, social and economic interactions between the behavioural units making fertility relevant decisions. This is especially true for generations which are very intensively connected by family ties, by kinship and by the societal and institutional regulations like the financial arrangements in the educational system, the health system and the pension system. Fertility theories differ very much according to the kind of the explanatory variables taken into account in the approaches of the various scientific disciplines like the economic theory of fertility, the anthropological-sociological fertility theories and the demographic theories, e.g. the transition theory and the biographic theory of fertility which tries to combine the explanatory power of different disciplines in an holistic approach.[1] Even more relevant than these distinctions, which emphasize the theoretical characteristics of the underlying scientific approaches, is the methodological question of whether the generations are treated as separate single units, which make their fertility decisions independently from each other by maximizing their objective functions separately or whether they are regarded as *one transgenerational* decisions making unit constituting a chain of consecutive generations.

In this contribution, the fertility analysis of generations is based on two concepts: The first is denoted as the *"chain of generations concept"* in which the successive generations are linked by intergenerational financial transfers. The corresponding chain of successive generations constitutes the decision making unit for fertility decisions. In the second concept, denoted as the *"single generation concept"*, fertility is analysed in the framework of a model including intergenerational transfers in the same way as in the chain of generations concept, but treats each generation as a separately acting unit which tries to maximize its objective function independently from the actions of the preceding and of the succeeding generations.

B. THE THEORETICAL CONCEPT[2]

Most models of optimal population growth and fertility are developed by economists in the framework of neoclassical economic theory. These models are based on the objective functions of optimum per capita output, consumption and on central economic variables like the interest rate and the rate of savings. Contrary to these neoclassical models, which are based on restrictive economic concepts like production and utility functions, the subsequent models use more general notions.[3] The assumptions made are as follows:

(1) Every generation in the sequence of generations is linked both to the preceding and to the succeeding one by way of intergenerational transfers. During childhood and youth, each generation starts out as a recipient of material support from it's parent's generation. During it's mid-phase, each generation provides material support to two

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other generations, i.e. to it's children and to it's parents who have now grown old. Finally, it enters the third phase during which it in turn is a net recipient of assistance from its children who have now entered the mid-phase. Special assumptions on the length of the three phases are not made. Furthermore, it is not necessary to make explicit assumptions on the life expectancy of the generations or on the relative length of the three phases (see figure 1).

(2) The relative sizes of the generations in demographic terms are significant for the balance between the support received and given during an entire life-course. This raises the question of how significant the size of a particular generation, as determined by the birth rate, will be for the ratio of assistance received to assistance given. Let the following be the notation used to analyse this relationship:

Gx = the size of generation x

Gx-1 = the size of generation x's parental generation

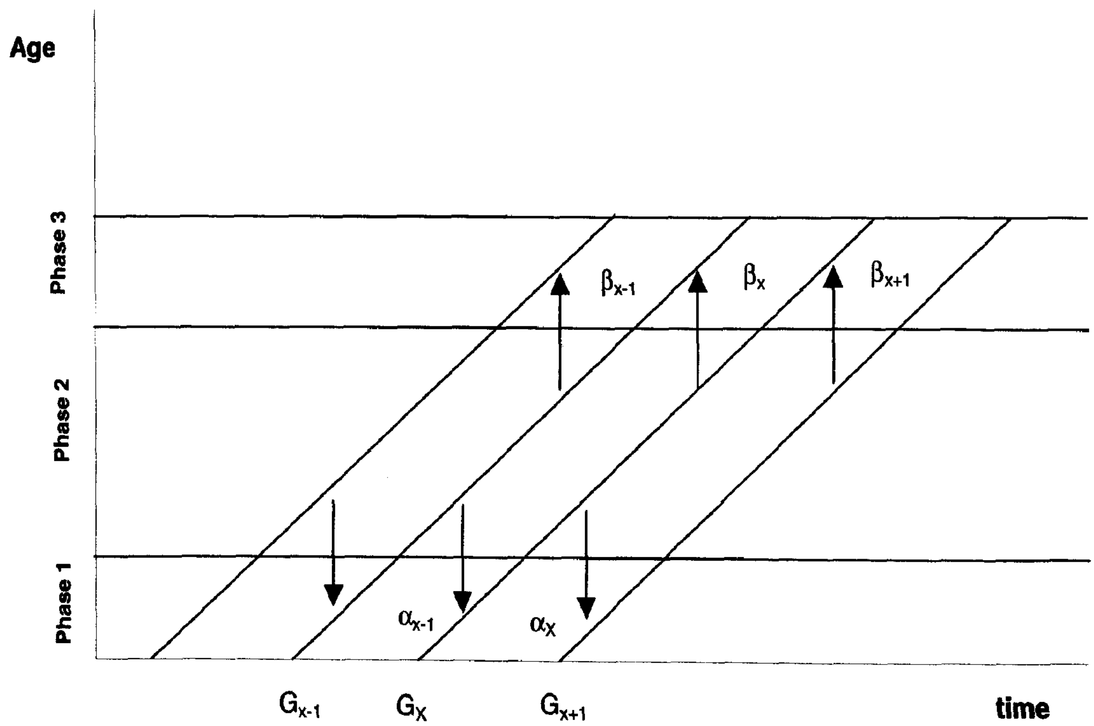
Gx+1 = the size of generation x's children's generation

αx = the services rendered and assistance given by generation x per head of its children's generation

βx = the services rendered and assistance given by generation x per head of its parents' generation

The value of the services rendered and assistance given by generation *x* to it's children's generation can be obtained by multiplying the size of its children's generation by the services per head of that generation, i.e. by the expression $_xG_{x+1}$. Likewise, the services and assistance rendered to the parental generation is $_xG_{x-1}$. That means that generation *x* provides a total amount of service and assistance

Figure I. Intergenerational transfers in a chain of generation



to other generations of

$$\alpha_x G_{x+1} + \beta_x G_{x-1} \qquad (1)$$

Correspondingly, generation x in turn will receive a total of

$$\alpha_{x-1} G_x + \beta_{x+1} G_x \qquad (2)$$

from its predecessor and successor generations.

The services/assistance given or received (or) have the index x appended to them because each generation can potentially have its own approach to these activities.

(3) The ratio of the services/assistance received and given to or by generation x is referred to as the "intergenerational transfer quotient Tx for generation x":

$$T_x = \frac{\alpha_x G_{x+1} + \beta_x G_{x-1}}{\alpha_{x-1} G_x + \beta_{x+1} G_x} \qquad (3)$$

The basic assumption made is that each generations objective function is to minimize its transfer quotient T_x.

C. Fertility in the "Chain of Generations Model"

The transfer quotient depends on the sizes of the three generations of G_{x-1}, G_x and G_{x+1}. To minimize the transfer quotient for generation x is not a trivial problem. For example, one important aspect of a favourable quotient for generation x is that the number of its children, i.e. G_{x+1} should not be too large. However, because the same argument applies to all generations, including the preceding one G_{x-1}, generation x's size when in the denominator of the transfer quotient would be all the smaller, making its transfer quotient less favourable, the more the parental generation G_{x-1} kept down the number of children it had for the sake of improving its own transfer quotient. In other words, this is a trans-generational, dynamic optimization problem.

The problem can best be expressed by asking what ratio between the generations G_{x-1}, G_x and G_{x+1} will yield the optimum, i.e. the lowest, transfer quotient. The numerical ratio between two consecutive generations is termed the net reproduction rate (NRR). Since any particular NRR always relates two generations to one another, the three generations involved in the transfer quotients can be represented by two net reproduction rates, as follows:

$$\frac{G_{x+1}}{G_x} = NRR_x \qquad (4a)$$

$$\frac{G_x}{G_{x-1}} = NRR_{x-1} \qquad (4b)$$

By substituting these expressions into the definitional equation (3) for the transfer quotient, we obtain:

$$T_x = \frac{\alpha_x NRR_x + \beta_x \frac{1}{NRR_{x-1}}}{\alpha_{x-1} + \beta_{x+1}} \qquad (5)$$

To begin with, let us seek to establish the optimum value of the transfer quotient when net reproduction rates and the "assistance output" rates and specific to the generations are equal, i.e. when

$$NRR_x = NRR_{x-1} = NRR \qquad (6a)$$

$$\alpha_x = \alpha_{x-1} = \alpha \qquad (6b)$$

$$\beta_x = \beta_{x+1} = \beta \qquad (6c)$$

In this case, instead of equation (5) we have the simplified expression

$$T = \frac{\alpha NRR + \beta \frac{1}{NRR}}{\alpha + \beta} \qquad (7)$$

The net reproduction rate yielding the optimum, i.e. lowest, value for the transfer quotient is found by setting the derivative of T with respect to NRR to zero. The result is:

$$NRR^{opt} = \sqrt{\frac{\beta}{\alpha}} \tag{8a}$$

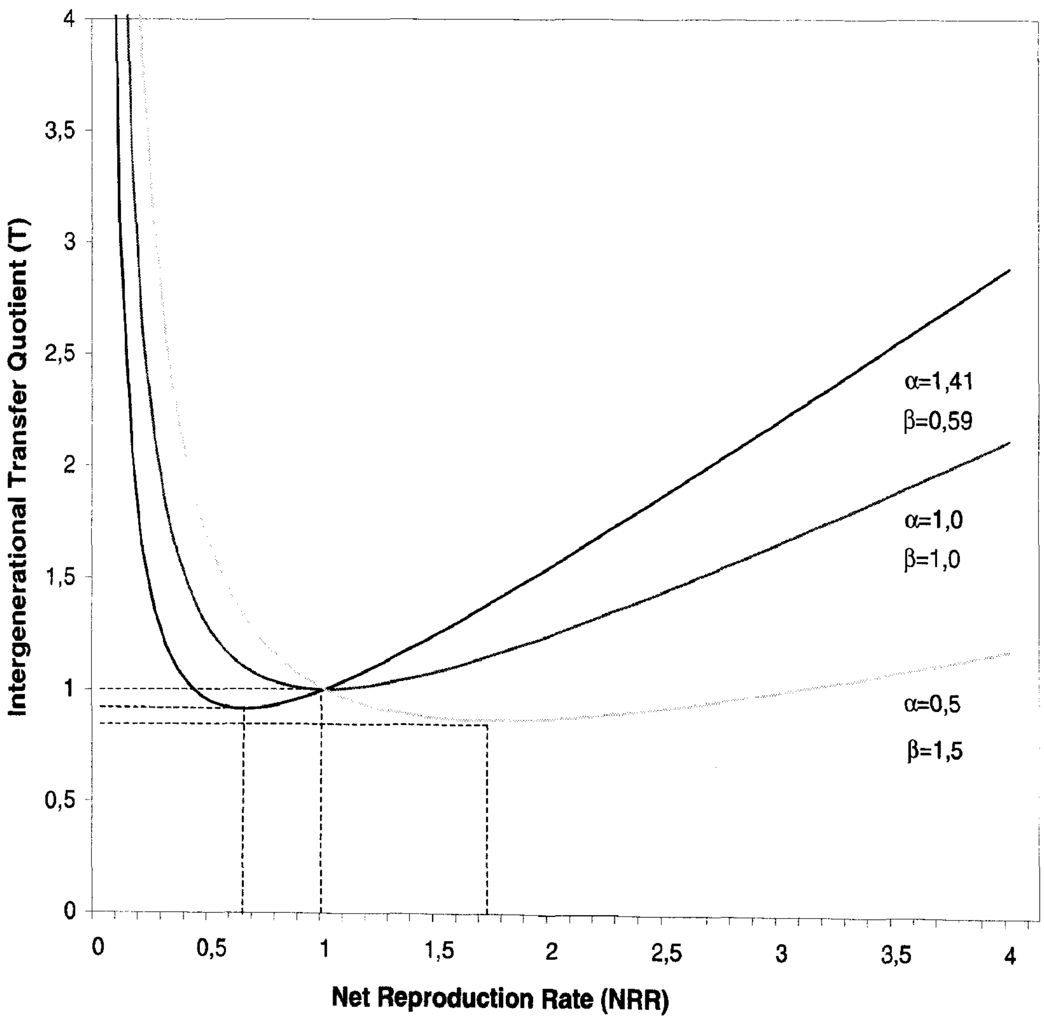
Substitution of NRR^{opt} from equation (8a) into equation (7) yields the optimal value of the transfer quotient:

$$T^{opt} = \frac{2\sqrt{\alpha\beta}}{\alpha + \beta} \tag{8b}$$

The dependence of the transfer quotient upon the net reproduction rate as expressed in equation (7) is portrayed graphically in figure 2. As the net reproduction rate increases, the transfer quotient initially falls, as the support provided to the older generation is spread among more people in the middle generation. However, because the effort they need to make for the young generation also increases as a result, the optimum value of the transfer quotient is reached with a net reproduction rate of exactly one. For all NRR figures above that, the transfer quotient increases in proportion.

The conclusions which can be drawn from this outcome are directly apparent from the equation (8a) showing the optimum NRR and from equation (8b) showing the optimal transfer quotient:[4]

Figure 2. Dependance of the International Transfer Quotient upon the Net Reproduction Rate



(a) A country's optimum net reproduction rate does *not* depend on the actual level of per capita assistance provided to the succeeding generation (α) or to the older generation (β), but on the ratio of the latter to the former (β/α) So if both forms of support are larger in country B than in country A by the same margin, there will be no affect on the optimum net reproduction rate.

(b) The larger the amount of assistance provided per capita to the younger generation (α) relative to that provided per capita to the older generation (β), the lower the optimum net reproduction rate will be, and vice versa.

(c) If the value of the assistance given to the younger and older generations is equal on a per capita basis (α=β), the optimum net reproduction rate = 1, regardless of the actual amount of support transferred from generation to generation: thus the population will remain constant without any need for immigration or emigration.

(d) If the assistance given to the young generation is greater than that given to the older one (α>β), the optimum net reproduction rate < 1, which means the population will decline if there is no net immigration.

(e) If the assistance given to the young generation is less than that given to the older one (α<β), the optimum net reproduction rate > 1, which means the population will grow if there is no net emigration.

(f) The social and economic power to determine the relative amounts of per capita assistance given to the young and the old normally lies with the generation in the middle which is active in the working world. It is to the advantage of this active generation if it keeps a damper on the amount of assistance given to the young per head while favouring assistance to the old, particularly since this generation has already left it's own phases of childhood and youth, in which it was a recipient, whereas its phase of old-age when it will again require support still lies ahead. Consequently, in any society, like numerous developing countries, which does not protect children from being exploited by the middle generation (e.g. by prohibiting child labour or passing laws for the benefit of children and young people, say on school provision), one would expect the balance of assistance provided to shift in favour of the older generation, making β>α, and the optimum net reproduction rate > 1, resulting in persisting population growth. Figure 2 outlines these links by examining three examples:

Example 1 (less developed countries):

α = 0.50
β = 1.50
NRRopt = 1.732

Numerous developing countries with high net reproduction rates correspond to example 1.

Example 2 (stationary populations and world population as a whole):

α = 1.00
β = 1.00
NRR_{opt} = 1.00

This example represents an ideal case in which a population remains constant when viewed net of migratory flows. It's net reproduction rate is 1.00 (approx. 2 children eventually reaching adult age and reproducing themselves per woman).

Example 3 (more developed countries):

α = 1.41
β = 0.59
NRR_{opt} = 0.65

The figures in example 3 were chosen so as to reflect roughly the circumstances in a developed country like Germany today. In this case, the net reproduction rate is 0.65. Please note that the optimum net reproduction rate depends solely on the *ratio* of α to β, and not on the absolute value of either parameter. That being the case, the figures shown in the example for α and β do not actually have to agree with the *absolute* parameters existing in the real world but only with their relative values. One may conclude from this example that the objective of maintaining a constant population, with a net reproduction rate of 1.0, without the need for net immigration will be unattainable as long as the assistance provided to the younger generation (per head of that generation) is greater than that given, per head, to the older generation. Is the per capita inter-generational transfer in favour of the younger generation lower or higher than that in favour of the older generation? The answer cannot be found simply by examining statistics on family or household income and expenditure, for these figures are, firstly, themselves influenced by government policy on families, and secondly, they fail to take into account any of the government services and infrastructure provided to the younger or to the older generation. What is needed is an assessment which takes in all real payments or transfers of assistance, so that would have to include such items as expenditure on the educational system etc. The same naturally applies to the support given to the older generation. Many of the real provisions made by the state are economic quantities which cannot be directly captured in statistical information, but they can certainly be empirically estimated using statistics as a basis, though the necessary research input is high.

Another issue to be addressed in this theoretical treatment is that of what effects can be expected to be generated if an ever-greater proportion of the transfers per head of the younger or older generation are no longer made by individuals or families, but by society as a whole or by government bodies. Suppose the sum of individual services (α^i) and societal services (α^s) per head of the younger generation is constant, and likewise for the services to the older generation:

$$\bar{\alpha} = \alpha^i + \alpha^s \qquad \alpha^i = \bar{\alpha} - \alpha^s \tag{9}$$

$$\bar{\beta} = \beta^i + \beta^s \qquad \beta^i = \bar{\beta} - \beta^s \tag{10}$$

Let us further assume that the members of the middle generation only bear their individual share of the services given, although they have been, or will later be, recipients both of the individual and of the societal assistance given to younger and older people. Based on these assumptions, the numerator of the transfer quotient, showing the services or assistance given out, will only contain the individual items, whereas the denominator will show both the individual and societal transfers received by the same generation during its lifetime:

$$T_x = \frac{\alpha^i NRR_x + \beta^i \dfrac{1}{NRR_{x-1}}}{\left(\alpha^i + \alpha^s\right) + \left(\beta^i + \beta^s\right)} \tag{11}$$

Here too, let us begin by assuming a net reproduction rate which remains constant from generation to generation (NRRx-1 = NRRx = NRR), yields the following optimum net reproduction rate, where the transfer quotient is at a minimum:

$$NRR^{opt} = \sqrt{\frac{\bar{\beta} - \beta^s}{\bar{\alpha} - \alpha^s}} \tag{12}$$

The derivation of the optimum net reproduction rate is based on legally and culturally defined standards for the assistance provided, per capita, to the younger or older generation. If these findings are applied to the situation, say, in Germany, the following statements can be made:

Summary (with reference to a more developed country like Germany):

(I) The greater the proportion of the assistance provided (per capita) to people in the late old-age phase of the life-cycle which is borne by society at large or by the state, the lower the optimum net reproduction rate will be, all other factors being equal. In Germany, for example, the birth rate began to decline at the time a collective insurance programme for old-age pensions was introduced (in the Bismarckian social reforms of the 1890s), thus backing up this finding. Of course, one should not take that to mean that the introduction of a state social insurance scheme was the only factor behind the fall in the birth rate in the 20th century.

(II) The greater the proportion of the assistance provided (per capita) to children and young people which is borne by society at large or by the state, the higher the optimum net reproduction rate will be. It is this functional relationship which nurtures the hope in industrial countries that it will be possible to raise the net reproduction rate substantially with the help of government policy towards families.

(III) Whether the net reproduction rate is greater than, equal to or less then unity, or in other words whether the population net of migration will grow, remain constant or shrink in the long term, depends on the ratio between the portion of per capita assistance provided by society at large to the older generation and the portion of per capita assistance it provides to the younger generation.

(IV) For example, the introduction of nursing-care insurance in Germany in 1995 has raised the proportion of per capita services to the older sections of the population which is borne by society or at least collectively, the effect of which is to lower the optimum net reproduction rate. So in a population like Germany's which is shrinking since 1972 without net immigration, the introduction of nursing-care insurance for senior citizens will mean that net immigration needs to be even higher than it already was in order to maintain a constant population. (In the early 1990tees the number of refugees asylum seekers and other immigrants was above one million per annum and above the number of births so that Germany's population grew despite of the birth deficit.)

Nursing-care insurance thus intensifies the cause of the low birth rate and of the aging of Germany's population, which is the actual reason for introducing the insurance scheme in the first place. From the purely demographic point of view, the measure is counter-productive, apart from which it breaches the principles laid down in the Federal Constitution Court's much-publicized judgement of July 7, 1992 (on pension rights for the women who had worked to clear the rubble in Germany's cities after the World-War-II bombings), because it increases still more the transfer payments made by families with several children to pensioners with few or no children, instead of reducing this "inverse solidarity".

D. Fertility in the "single generation model"[5]

So far, we have set out to establish the optimum net reproduction rate on the basis of the functional relationship between the NRR and the intergenerational transfer quotient, while assuming that the net reproduction rate sought or obtained will be equal in all generations. In other words, we imagined that what might be termed a "chain of generations" existed as the focus of people's actions, linking the different sections of the population together in a community of consecutive generations giving assistance and reciprocating it.

Let us now drop this rather idealistic conception in favour of a more realistic view, enquiring what the optimum number of children per woman will be if the focus of action is not the community of generations but a single one, generation x. So the new question posed is: What are the optimum patterns of reproductive behaviour and family structure in terms of the transfer quotient for the generation under examination if it seeks solely to optimize the benefits to itself?

Taking generation x's point of view in isolation, the outcome of this seems to be directly apparent from equation (5). The only quantities which generation x can influence in a bid to minimize its transfer quotient are the number of children it has, the amount of assistance it provides per head of its children (αx) and the amount of assistance it provides per head to its parent's generation (βx) The generation x's transfer quotient will be at an optimum when its net reproduction rate NRRx, the amount of assistance αx provided per child and the amount of assistance per head provided to the parental generation βx are all at their lowest. In contrast to the outcome of the trans-generational optimization problem considered prior to this one, the transfer quotient seems now to be at its lowest, when the number of children per woman is zero. But this simple outcome is only valid for a rather unrealistic condition. In the following it will be demonstrated that the result is more complicated.

The central assumptions made in this argument are that generation x's transfer quotient is independent of the values of α and β and independent of the net reproduction rate of the preceding and succeeding generations. Which net reproduction rate is optimal, if these assumptions do not hold? To answer this question four cases will be distinguished:

Case 1: Equal values of α for all generations and generation specific values of β_x and NRR_x,

Case 2: equal vaues of β for all generations and generation specific values of α_x and NRR_x,

Case 3: equal values of α, equal values of β and generation specific net reproduction rates NRR_x, and

Case 4: equal net reproduction rates for all generations and generation specific values of α_x and β_x.

In the following it will be shown that in these four cases the problem of the single generation model equals the problem treated in game theory: There is no way for a single generation x to optimize its transfer quotient independently from the preceding and succeeding generations.

Case 1

In case (1) the central assumption and the equation for the transfer quotient are given by the equations:

$$\alpha_x = \alpha_{x-1} = \alpha \tag{13}$$

$$T_x = \frac{\alpha NRR_x + \beta_x \dfrac{1}{NRR_{x-1}}}{\alpha + \beta_{x+1}} \tag{14}$$

The optimal value of T_x is found setting the partial derivative of T_x with respect to α to zero:

$$\frac{\delta T_x}{\delta \alpha} = \frac{NRR_x(\alpha + \beta_{x+1}) - (\alpha NRR_x + \beta_x / NRR_{x-1})}{(\alpha + \beta_{x+1})^2} = 0 \tag{15}$$

The condition for the minimum of T_x derived from (15) is:

$$NRR_x \cdot NRR_{x-1} = \frac{\beta_x}{\beta_{x+1}} \tag{16}$$

Substitution of NRR_x from equation (16) into equation (14) yields the optimal value of the transfer quotient:

$$T_x^{opt} = \frac{\alpha NRR_x + NRR_x \cdot \beta_{x+1}}{\alpha + \beta_{x+1}} = NRR_x \tag{17}$$

The interpretation of equations (16) and (17) yields: It is not possible for generation x to minimize T_x by a low value for its net reproduction rate independently from the value of the net reproduction rate of generation x+1 since according to equation (16) NRR_x and NRR_{x+1} are not independent. If in equation (16) NRR_x is decreased, NRR_{x-1} has to be increased so that the minimum of T_x can not be reached simply by a decrease of NRR_x.

Case 2

In case (2) the basic assumption and the corresponding transfer quotient are given by equations (18) and (19):

$$\beta_x = \beta_{x-1} = \beta \qquad (18)$$

$$T_x = \frac{\alpha_x NRR_x + \beta \dfrac{1}{NRR_{x-1}}}{\alpha_{x-1} + \beta} \qquad (19)$$

Setting the partial derivative of Tx with respect to β to zero

$$\frac{\delta T_x}{\delta \beta} = \frac{(\alpha_{x-1} + \beta)/NRR_{x-1} - (\alpha_x NRR_x + \beta / NRR_{x-1})}{(\alpha_{x-1} + \beta)^2} = 0 \qquad (20)$$

The condition for the minimum of Tx derived from (20) is:

$$\alpha_x NRR_x = \frac{\alpha_{x-1}}{NRR_{x-1}} \qquad (21)$$

Substitution of NRRx from equation (21) into equation (19) yields the optimal transfer quotient:

$$T_x^{opt} = \frac{\dfrac{\alpha_{x-1}}{NRR_{x-1}} + \dfrac{\beta}{NRR_{x-1}}}{\alpha_{x-1} + \beta} = \frac{1}{NRR_{x-1}} = \frac{\alpha_x}{\alpha_{x-1}} NRR_x \qquad (22)$$

Interpreting these equations the result is the following. The transfer quotient of generation x is low, if NRR_x is low, but according to condition (21) a decrease of NRR_x is not possible without an increase of NRR_{x-1}. If generation x-1 minimizes its own transfer quotient by a low NRR_{x-1} the net reproduction rate of generation x-2 has to be increased and so on. As in case (1) the conclusion of case (2) is that there is no way for a single generation to achieve its optimal transfer quotient independently from the preceding and the succeeding generations.

Case 3

The following assumptions are made:

$$\alpha_x = \alpha_{x-1} = \alpha \tag{23a}$$

$$\beta_x = \beta_{x-1} = \beta \tag{23b}$$

Using the definition

$$\gamma = \alpha + \beta \tag{24}$$

the transfer quotient is

$$T_x = \frac{(\gamma - \beta) NRR_x + \dfrac{\beta}{NRR_{x-1}}}{\gamma} = \left(1 - \frac{\beta}{\gamma}\right) NRR_x + \frac{\dfrac{\beta}{\gamma}}{NRR_{x-1}} \tag{25}$$

Setting the partial derivative of Tx with respect to γ equal to zero

$$\frac{\delta T_x}{\delta \gamma} = -NRR_x + \frac{1}{NRR_{x-1}} = 0 \tag{26}$$

we obtain the condition for the minimum of Tx:

$$NRR_x \cdot NRR_{x-1} = 1 \tag{27}$$

Substitution of NRRx-1 from equation (27) into equation (25) yields:

$$T_x^{opt} = NRR_x \tag{28}$$

The interpretation of equations (27) and (28) results in the same conclusions as in the cases (2) and (3).

Case 4

In this case the basic assumption is that the net reproduction rates of the generations are equal but the values of the α's and β's are different:

$$NRR_x = NRR_{x-1} = NRR \tag{29}$$

$$T_x = \frac{\alpha_x NRR + \beta_x \frac{1}{NRR}}{(\alpha_{x-1} + \beta_{x+1})} \tag{30}$$

Setting the partial derivative of T_x with respect to NRR equal to zero:

$$\frac{\delta T_x}{\delta NRR} = \frac{\alpha_x - \beta_x \frac{1}{NRR^2}}{\alpha_{x-1} + \beta_{x+1}} = 0 \tag{31}$$

yields the optimal net reproduction rate

$$NRR^{opt} = \sqrt{\frac{\beta_x}{\alpha_x}} \tag{32}$$

and the optimal transfer quotient

$$T_x^{opt} = \frac{2\sqrt{\alpha_x \cdot \beta_x}}{\alpha_{x-1} + \beta_{x+1}} \tag{33}$$

The interpretation of these equations is: Generation x cannot minimize its transfer quotient simply by decreasing the values αx and βx independently from the preceding and succeeding generations, because a corresponding decrease of αx-1 and βx+1 in equation (33) would cause a rise of Tx.

E. CONCLUSION FOR THE WORLD POPULATION AS A CHAIN OF GENERATIONS

The general result of the interpretations of the four cases based on the single generation model is: For a single generation x, it is not possible to optimize its own benefits without any regard for what would happen if other generations acted in the same way. A specific generation can only reach its optimal fertility if the preceding and the succeeding generations also practice optimal fertility rates. A more general conclusion is: If the different generations chose to violate the universal ethical principle laid down in Immanuel Kant's *categoric imperative* - they cannot achieve their optimal transfer quotient and optimal fertility. If they acknowledge this principle they would act like a *community* or *chain of generations*, all of which would experience the optimum successively. But even if this principle would cause an in-built tendency to an optimal level of the net reproduction rate, the value of the reproduction rate can be less than one or more than one. *The level of the net reproductions rate would be one only in the ideal case that the amount of support given by a generation per head of its children's generation equals exactly the amount of support given per head of it's parent's generation.* This result can be interpreted as a rationale for fertility assumptions in long term world population projections if the world population is regarded as a chain of generations which tries to achieve an optimal solution of its dynamic intergenerational optimization problem.

NOTES

[1] Typical examples: Becker, G.S., *A treatise of the family*, Cambridge: Harvard University Press, 1981; Caldwell, J.C., *Toward a restatement of demographic transition theory*. In: Population and Development Review, Vol. 20, No. 4, Sept./Dec. 1976; Mackenroth, G., *Bevölkerungslehre*, Berlin 1953; Chesnais, J.C., *The demographic transition - stages, patterns and economic implications*, Oxford 1992; Birg, H., Flöthmann, E.-J., Reiter, I., *Biographic analysis of the demographic characteristics of the life histories of men and women in regional labour market cohorts as clusters of birth cohorts*. In: Becker, H.A. (Ed.), Life histories and generations, Vol. I, University of Utrecht, Faculty of Social Sciences, Utrecht 1991.

[2] This part is an extended version of the model published in Birg, H., *World Population Projections for the 21st Century - Theoretical Interpretations and Quantitative Simulations"*, Frankfurt/: Campus and New York: St. Martin's Press, 1995, Chapter 3.3, p. 67f.

[3] Examples for the type of models with non overlapping or overlapping generations all based on the philosophy of economic utility functions on which this paper does not build on are: Samuelson, P.A., *The optimum growth rate for population. And: Optimal social security in a life-cycle growth model.* Both contributions in: International Economic Review, Vol. 16, No. 3, 1975, p. 531-544. Arthur, B. and McNicoll, G., Samuelson, Population and Intergenerational Transfers. In: International Economic Review 19, p. 241-246. Lee, R.D., Fertility, Mortality and Intergenerational Transfers. In: Ermisch and Ogawa (Eds.), *The Family, the Market, and the State in Aging Societies*, Oxford University Press, 1994.

[4] The results are similar to those of the model of J. Bourgeois-Pichat which is based on more restrictive assumptions of the theory of stable population. See: *"Charge de la population active"*. In: Journal de la societé de statistique de Paris, Paris, Année 91, 1950, p. 94f. An extended version of the model of Bourgeois-Pichat has been developed by Höhne, H.-G.: *"Optimale Bevölkerungs-wachstumsrate - Eine Modifikation der Approximation von Bourgeois-Pichat"*. In: G. Buttler, H.-J. Hoffmann-Novotny and G. Schmitt-Rink (Eds): Acta Demographica, Heidelberg, 1991, p. 15-38.

[5] I am grateful for the fruitful comments made by Jürgen Schott, Technical University of Dresden, on the general model which encouraged me to extend the original version in this contribution.