

# The impact of taxation on upper and lower bounds of enterprise value\*

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## Abstract

This paper derives and draws on simple formulae for the upper and lower bounds to the value of a series of risky cash flows in order to provide some instructive insights in the impact of taxation on these bounds.

The formulae are based on no-arbitrage conditions in a setting that is a straightforward extension of the COX, ROSS, AND RUBINSTEIN [2] option-pricing model to an incomplete market model and look exactly like the popular GORDON growth formula.

Although based on stylized facts concerning the tax scheme the results promise to be a reliable guide for further research in this field.

**Keywords:** arbitrage theory, incomplete markets, taxes, enterprise value

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# 1 Introduction

It is really astonishing that so many financial analysts hang on to the practice of pinning down the value of a business sharply to one point. They should know for better that this practice requires undue restrictions as for example assuming complete markets or that the TOBIN-separation holds although it is an empirical fact that it does not.<sup>1</sup>

This paper avoids the above mentioned problem adding the least possible complexity by reference to an incomplete market model that is a straightforward extension of the COX, ROSS, AND RUBINSTEIN [2] option-pricing model. Within this setting the upper and lower bounds to the enterprise value implied by no-arbitrage conditions simply focus on the best-case and the worst-case scenario, respectively, and convexity becomes a crucial determinant for the impact of taxation. To be more precise: What counts is how the enterprise cash flows behave relative to the price of some exchange-traded reference asset in the best and in the worst case, respectively. If in principal it makes no difference as to this behavior whether you are in the best or the worst of all worlds, that is to say if the payoff characteristic is convex or concave in any of these worlds, the upper and lower bound always move to the same direction as long as you switch between symmetric<sup>2</sup> tax schemes. This does not hold however for asymmetric tax schemes.

## 2 The one-period case

Let  $\Omega = \{\omega_1, \omega_2, \omega_3, \omega_4\}$  be the probability set and

$$C_{t_1}(\omega) = \begin{cases} g_{t_0}^u \cdot C_{t_0} & \text{if } \omega = \omega_1 \\ b_{t_0}^u \cdot C_{t_0} & \text{if } \omega = \omega_2 \\ g_{t_0}^d \cdot C_{t_0} & \text{if } \omega = \omega_3 \\ b_{t_0}^d \cdot C_{t_0} & \text{if } \omega = \omega_4 \end{cases} \quad (1)$$

<sup>1</sup> This is really bad because you can miss the value to a specific investor by far if this investor's portfolio is significantly different from the reference portfolio your calculations are based on. As a remedy WILHELM [5] has proposed to value uncertain cash-flows as good as possible by replication with traded assets so that only the residuum remains susceptible to undue restrictions of investor-specific preferences and endowments.

<sup>2</sup> This means that short positions and long positions induce exactly the same tax payments in absolute terms.

with ('b' stands for 'bad' and 'g' stands for 'good' )

$$b_{t_0}^m \leq g_{t_0}^m \quad (m \in \{d, u\}) \quad (2)$$

be the payoff characteristic of the enterprise to be valued. Further let

$$S_{t_0}(\omega) = S_{t_0} \text{ for } \omega \in \Omega \quad \begin{array}{l} / \\ \backslash \end{array} \quad \begin{array}{l} S_{t_1}(\omega) = u_{t_0} \cdot S_{t_0} \text{ if } \omega \in \{\omega_1, \omega_2\} \\ \\ S_{t_1}(\omega) = d_{t_0} \cdot S_{t_0} \text{ if } \omega \in \{\omega_3, \omega_4\} \end{array}$$

be the price movement of some exchange traded risky asset which together with some money market fund certificate (MMF) with market prices

$$B_{t_0}, B_{t_1} := B_{t_0}(1 + r_{t_0}) \quad (3)$$

are the only exchange-traded instruments the enterprise value shall refer to.<sup>3</sup>

## 2.1 Lower Bound

From COX, ROSS, AND RUBINSTEIN [2] it is well known that portfolio

$$\begin{aligned} \mathbf{x}_{t_0} &:= \begin{pmatrix} x_{t_0}^S \\ x_{t_0}^B \end{pmatrix} \\ &= \begin{pmatrix} \frac{b_{t_0}^u - b_{t_0}^d}{u_{t_0} - d_{t_0}} \frac{C_{t_0}}{S_{t_0}} \\ \frac{-b_{t_0}^u d_{t_0} + b_{t_0}^d u_{t_0}}{u_{t_0} - d_{t_0}} \frac{C_{t_0}}{B_{t_1}} \end{pmatrix} \\ &= \begin{pmatrix} 1 & 0 \\ 0 & \frac{d_{t_0} u_{t_0}}{B_{t_0}^{-1} B_{t_1}} \end{pmatrix} \begin{pmatrix} \frac{b_{t_0}^u - b_{t_0}^d}{u_{t_0} - d_{t_0}} \frac{C_{t_0}}{S_{t_0}} \\ \frac{b_{t_0}^d}{d_{t_0}} - \frac{b_{t_0}^u}{u_{t_0}} \frac{C_{t_0}}{B_{t_0}} \end{pmatrix} \end{aligned}$$

<sup>3</sup> This confinement is for the sake of simplicity only.

comprising a number of  $x_{t_0}^S$  risky assets and  $x_{t_0}^B$  money market fund certificates (MMF) generates cash flows

$$P_{t_1}(\mathbf{x}_{t_0}) = \begin{cases} b_{t_0}^u \cdot C_{t_0} & \text{if } \omega \in \{\omega_1, \omega_2\} \\ b_{t_0}^d \cdot C_{t_0} & \text{if } \omega \in \{\omega_3, \omega_4\}. \end{cases} \quad (4)$$

Regarding the definitions

$$q_{t_0} := \frac{1 + r_{t_0} - d_{t_0}}{u_{t_0} - d_{t_0}} \quad (5)$$

$$\bar{b}_{t_0} := b_{t_0}^u \cdot q_{t_0} + b_{t_0}^d \cdot (1 - q_{t_0}) - 1 \quad (6)$$

and

$$\bar{g}_{t_0} := g_{t_0}^u \cdot q_{t_0} + g_{t_0}^d \cdot (1 - q_{t_0}) - 1 \quad (7)$$

the market price of this portfolio at  $t_0$  might be written as

$$\begin{aligned} P_{t_0}(\mathbf{x}_{t_0}) &= \left( \frac{b_{t_0}^u - b_{t_0}^d}{u_{t_0} - d_{t_0}} + \frac{1}{B_{t_0}^{-1} B_{t_1}} \frac{-b_{t_0}^u d_{t_0} + b_{t_0}^d u_{t_0}}{u_{t_0} - d_{t_0}} \right) C_{t_0} \\ &= \frac{1}{B_{t_0}^{-1} B_{t_1}} \left( b_{t_0}^u \frac{B_{t_0}^{-1} B_{t_1} - d_{t_0}}{u_{t_0} - d_{t_0}} + b_{t_0}^d \frac{u_{t_0} - B_{t_0}^{-1} B_{t_1}}{u_{t_0} - d_{t_0}} \right) C_{t_0} \\ &= \frac{1}{B_{t_0}^{-1} B_{t_1}} \left( b_{t_0}^u \cdot q_{t_0} + b_{t_0}^d \cdot (1 - q_{t_0}) \right) C_{t_0} \\ &= \frac{1 + \bar{b}_{t_0}}{1 + r_{t_0}} C_{t_0}. \end{aligned}$$

From (2) and (4) it follows that buying the enterprise i.e. cash flow  $C_{t_1}$  at price  $\Pi_{t_0}(C_{t_1})$  while at the same time short selling portfolio  $\mathbf{x}_{t_0}$  will payoff

$$C_{t_1}(\omega) + P_{t_1}(-\mathbf{x}_{t_0})(\omega) \geq 0 \text{ for each } \omega \in \Omega$$

at time  $t_1$  and thus would be an arbitrage opportunity if

$$\Pi_{t_0}(C_{t_1}) + P_{t_0}(-\mathbf{x}_{t_0}) \leq 0.$$

Hence in the absence of any impediment to trade such as transaction costs or taxes the enterprise must have a price

$$\Pi_{t_0}(C_{t_1}) > -P_{t_0}(-\mathbf{x}_{t_0}) = P_{t_0}(\mathbf{x}_{t_0})$$

in order to prevent arbitrage.

Nevertheless there might be other arbitrage opportunities. To preclude any such arbitrage opportunity  $\Pi_{t_0}(C_{t_1})$  must be higher than the *most expensive* portfolio with a cash flow that is weakly dominated by payoff characteristic (1). In what follows we will use linear programming to show that portfolio  $\mathbf{x}_{t_0}$  is the most expensive weakly dominated portfolio indeed, and thus the lower bound to the enterprise value is

$$\underline{V}_{t_0}(C_{t_1}) = P_{t_0}(\mathbf{x}_{t_0}) = \frac{1 + \bar{b}_{t_0}}{1 + r_{t_0}} C_{t_0}.$$

Taking the MMF certificates as a numeraire the objective is

$$z_P := x_{t_0}^B + x_{t_0}^S \cdot B_{t_0}^{-1} \cdot S_{t_0} \rightarrow \max_{x_{t_0}^B, x_{t_0}^S \in \mathbb{R}} !$$

and the constraints for this portfolio to be weakly dominated by (1) are

$$\begin{aligned} x_{t_0}^B + x_{t_0}^S \cdot u_{t_0} \cdot B_{t_1}^{-1} \cdot S_{t_0} &\leq g_{t_0}^u \cdot B_{t_1}^{-1} \cdot C_{t_0} \\ x_{t_0}^B + x_{t_0}^S \cdot u_{t_0} \cdot B_{t_1}^{-1} \cdot S_{t_0} &\leq b_{t_0}^u \cdot B_{t_1}^{-1} \cdot C_{t_0} \\ x_{t_0}^B + x_{t_0}^S \cdot d_{t_0} \cdot B_{t_1}^{-1} \cdot S_{t_0} &\leq g_{t_0}^d \cdot B_{t_1}^{-1} \cdot C_{t_0} \\ x_{t_0}^B + x_{t_0}^S \cdot d_{t_0} \cdot B_{t_1}^{-1} \cdot S_{t_0} &\leq b_{t_0}^d \cdot B_{t_1}^{-1} \cdot C_{t_0}. \end{aligned}$$

Duality theory says that if there is a solution to the above program there is also one for the following program

$$z_D := B_{t_1}^{-1} C_{t_0} \left( g_{t_0}^u \cdot q(\omega_1) + b_{t_0}^u \cdot q(\omega_2) + g_{t_0}^d \cdot q(\omega_3) + b_{t_0}^d \cdot q(\omega_4) \right) \rightarrow \min_{q(\omega_1), \dots, q(\omega_4) \geq 0} ! \quad (8)$$

s.t.

$$q(\omega_1) + q(\omega_2) + q(\omega_3) + q(\omega_4) = 1$$

$$B_{t_1}^{-1} \cdot [S_{t_0} \cdot u_{t_0} \cdot (q(\omega_1) + q(\omega_2)) + S_{t_0} \cdot d_{t_0} \cdot (q(\omega_3) + q(\omega_4))] = B_{t_0}^{-1} \cdot S_{t_0}$$

with exactly the same objective value. In combination with (3) and (5) the two constraints are equivalent to

$$q(\omega_1) + q(\omega_2) = q_{t_0}$$

$$q(\omega_3) + q(\omega_4) = 1 - q_{t_0}$$

so that (8) simplifies to

$$B_{t_1}^{-1} C_{t_0} \left( (g_{t_0}^u - b_{t_0}^u) \cdot q(\omega_1) + b_{t_0}^u \cdot q_{t_0} + (g_{t_0}^d - b_{t_0}^d) \cdot q(\omega_3) + b_{t_0}^d \cdot (1 - q_{t_0}) \right) \rightarrow \min_{q(\omega_1), q(\omega_3) \geq 0} !$$

Regarding that (2) implies

$$0 \leq g_{t_0}^m - b_{t_0}^m \quad (m \in \{d, u\}) \quad (9)$$

it becomes obvious that both objective functions have optimal value

$$z_P^* = z_D^* = (1 + \bar{b}_{t_0}) B_{t_1}^{-1} C_{t_0} = B_{t_0}^{-1} P_{t_0}(\mathbf{x}_{t_0})$$

and that  $\mathbf{x}_{t_0}$  is the most expensive weakly dominated portfolio indeed. Moreover it shows that  $\bar{b}_{t_0}$  and  $\bar{g}_{t_0}$  might be interpreted as expected worst-case and best-case growth rates under measure  $\mathbb{S}$  designed to make the price of the exchange-traded risky asset measured in units of the numeraire a martingale. They will be referred to as *pseudo growth rates*

$$\bar{c}_{t_0} := E_{\mathbb{S}} (C_{t_0}^{-1} C_{t_1}^c | \mathcal{F}_{t_0}) - 1 \quad (c \in \{b, g\}).$$

in what follows.<sup>4</sup>

## 2.2 Upper Bound

The line of reasoning that leads to the upper bound rests on difference arbitrage: If portfolio  $\mathbf{y}_{t_0}$  with payoff  $P_{t_1}(\mathbf{y}_{t_0})$  at time  $t_1$  weakly dominates the cash flow  $C_{t_1}$  generated by the enterprise at time  $t_1$  that is if

$$P_{t_1}(\mathbf{y}_{t_0})(\omega) \geq C_{t_1}(\omega) \text{ for each } \omega \in \Omega,$$

a potential buyer would rather buy this portfolio than the enterprise if at time  $t_0$  it would cost no more than the enterprise that is if

$$P_{t_0}(\mathbf{y}_{t_0}) \leq \Pi_{t_0}(C_{t_1}).$$

Assume that  $\mathbf{y}_{t_0}$  is the *cheapest* dominating portfolio then its price  $P_{t_0}(\mathbf{y}_{t_0})$  marks the upper bound  $\bar{V}_{t_0}(C_{t_1})$  to the price of a business with payoff  $C_{t_1}$ .

Proceeding exactly as above leads to the conclusion that  $\mathbf{y}_{t_0}$  is the portfolio with cash flows

$$C_{t_1}(\omega) = \begin{cases} g_{t_0}^u \cdot C_{t_0} & \text{if } \omega \in \{\omega_1, \omega_2\} \\ g_{t_0}^d \cdot C_{t_0} & \text{if } \omega \in \{\omega_3, \omega_4\} \end{cases} \quad (10)$$

<sup>4</sup> One obvious implication of our central result is that if the lower of the two possible cash flows generated by the enterprise contingent on the asset price and the asset price itself are perfectly correlated, that is if

$$\frac{b_{t_0}^u}{u_{t_0}} = \frac{b_{t_0}^d}{d_{t_0}},$$

we get

$$\mathbf{x}_{t_0} = \begin{pmatrix} \frac{b_{t_0}^d C_{t_0}}{d_{t_0} S_{t_0}} \\ 0 \end{pmatrix}$$

and the lower bounds simplifies to

$$\underline{V}_{t_0}(C_{t_1}) = \frac{b_{t_0}^d C_{t_0}}{d_{t_0} S_{t_0}} S_{t_0} = \frac{b_{t_0}^d}{d_{t_0}} C_{t_0}.$$

and composition

$$\begin{aligned}
\mathbf{y}_{t_0} &:= \begin{pmatrix} y_{t_0}^S \\ y_{t_0}^B \end{pmatrix} \\
&= \begin{pmatrix} \frac{g_{t_0}^u - g_{t_0}^d}{u_{t_0} - d_{t_0}} \frac{C_{t_0}}{S_{t_0}} \\ \frac{-g_{t_0}^u d_{t_0} + g_{t_0}^d u_{t_0}}{u_{t_0} - d_{t_0}} \frac{C_{t_0}}{B_{t_1}} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ 0 & \frac{d_{t_0} u_{t_0}}{B_{t_0}^{-1} B_{t_1}} \end{pmatrix} \begin{pmatrix} \frac{g_{t_0}^u - g_{t_0}^d}{u_{t_0} - d_{t_0}} \frac{C_{t_0}}{S_{t_0}} \\ \frac{g_{t_0}^d}{d_{t_0}} - \frac{g_{t_0}^u}{u_{t_0}} \frac{C_{t_0}}{B_{t_0}} \end{pmatrix}.
\end{aligned}$$

From this we get

$$\bar{V}_{t_0}(C_{t_1}) = P_{t_0}(\mathbf{y}_{t_0}) = \frac{1 + \bar{g}_{t_0}}{1 + r_{t_0}} C_{t_0}$$

for the upper bound to the enterprise value.

**Summary 2.1** *The upper and lower bounds may be derived by referring to the best- and worst-case scenarios and valuing them as if the market were complete.*

### 3 The multi-period case

The  $n$ -period case is a straightforward extension of the one-period case with

$\omega \in \Omega := \{\omega_1, \omega_2, \omega_3, \omega_4\}^n$  and the portfolios  $\mathbf{x}_{t_0}, \mathbf{y}_{t_0}$  being replaced by trading strategies  $(\mathbf{x}_{t_i})_{i=0}^{n-1}, (\mathbf{y}_{t_i})_{i=0}^{n-1}$  that have to be determined recursively as follows: Let

$$\begin{aligned}
P_{t_i}^{b,m} &:= P_{t_i}(\mathbf{x}_{t_i}(\omega))(\omega) \\
&= x_{t_i}^S(\omega) \cdot S_{t_i}(\omega) + x_{t_i}^B(\omega) \cdot B_{t_i} \text{ for } \omega \in S_{t_i}^{-1}(m \cdot S_{t_{i-1}}) := \{\omega \mid S_{t_i}(\omega) = m \cdot S_{t_{i-1}}\}
\end{aligned}$$

be a short cut for the price of portfolio  $\mathbf{x}_{t_i}$  you need at time  $t_i$  in order to replicate  $C_{t_n}^b(\omega)$  at time  $t_n$  depending on the price move  $m$  of the exchange-traded risky asset from time  $t_{i-1}$  to



$t_i$ . By analogy to the one-period case we know that

$$\begin{aligned}
\mathbf{x}_{t_{n-1}} &:= \begin{pmatrix} x_{t_{n-1}}^S \\ x_{t_{n-1}}^B \end{pmatrix} \\
&= \begin{pmatrix} \frac{b_{t_{n-1}}^u - b_{t_{n-1}}^d}{u_{t_{n-1}} - d_{t_{n-1}}} \frac{C_{t_{n-1}}^b}{S_{t_{n-1}}} \\ \frac{-b_{t_{n-1}}^u d_{t_{n-1}} + b_{t_{n-1}}^d u_{t_{n-1}}}{u_{t_{n-1}} - d_{t_{n-1}}} \frac{C_{t_{n-1}}^b}{B_{t_{n-1}}} \end{pmatrix} \\
&= \begin{pmatrix} 1 & 0 \\ 0 & \frac{d_{t_{n-1}} u_{t_{n-1}}}{B_{t_{n-1}}^{-1} B_{t_{n-1}}} \end{pmatrix} \begin{pmatrix} \frac{b_{t_{n-1}}^u - b_{t_{n-1}}^d}{u_{t_{n-1}} - d_{t_{n-1}}} \frac{C_{t_{n-1}}^b}{S_{t_{n-1}}} \\ \frac{b_{t_{n-1}}^d - b_{t_{n-1}}^u}{d_{t_{n-1}} - u_{t_{n-1}}} \frac{C_{t_{n-1}}^b}{B_{t_{n-1}}} \end{pmatrix} \quad (11)
\end{aligned}$$

has market price

$$\begin{aligned}
P_{t_{n-1}}(\mathbf{x}_{t_{n-1}}) &= \left( \frac{b_{t_{n-1}}^u - b_{t_{n-1}}^d}{u_{t_{n-1}} - d_{t_{n-1}}} + \frac{1}{B_{t_{n-1}}^{-1} B_{t_{n-1}}} \frac{-b_{t_{n-1}}^u d_{t_{n-1}} + b_{t_{n-1}}^d u_{t_{n-1}}}{u_{t_{n-1}} - d_{t_{n-1}}} \right) C_{t_{n-1}}^b \\
&= \frac{1 + \bar{b}_{t_{n-1}}}{1 + r_{t_{n-1}}} C_{t_{n-1}}^b
\end{aligned}$$

at time  $t_{n-1}$ . Hence, if the short term interest rate evolves deterministically the numbers of risky assets and MMF certificates needed at time  $t_{n-2}$  are

$$\begin{aligned}
x_{t_{n-2}}^S &= \frac{P_{t_{n-1}}^{b,u} - P_{t_{n-1}}^{b,d}}{(u_{t_{n-2}} - d_{t_{n-2}}) \cdot S_{t_{n-2}}} \\
&= \frac{1 + \bar{b}_{t_{n-1}}}{1 + r_{t_{n-1}}} \frac{b_{t_{n-2}}^u - b_{t_{n-2}}^d}{u_{t_{n-2}} - d_{t_{n-2}}} \frac{C_{t_{n-2}}^b}{S_{t_{n-2}}}
\end{aligned}$$

and

$$\begin{aligned}
x_{t_{n-2}}^B &= \frac{P_{t_{n-1}}^{b,m} - x_{t_{n-2}}^S \cdot m \cdot S_{t_{n-2}}}{B_{t_{n-1}}} \\
&= \frac{1 + \bar{b}_{t_{n-1}}}{1 + r_{t_{n-1}}} \frac{C_{t_{n-2}}^b}{B_{t_{n-1}}} \left( b_{t_{n-2}}^m - m \cdot \frac{b_{t_{n-2}}^u - b_{t_{n-2}}^d}{u_{t_{n-2}} - d_{t_{n-2}}} \right) \\
&= \frac{1 + \bar{b}_{t_{n-1}}}{1 + r_{t_{n-1}}} \frac{d_{t_{n-2}} u_{t_{n-2}}}{B_{t_{n-2}}^{-1} B_{t_{n-1}}} \frac{b_{t_{n-2}}^d - b_{t_{n-2}}^u}{d_{t_{n-2}} - u_{t_{n-2}}} \frac{C_{t_{n-2}}^b}{B_{t_{n-2}}} \quad (m \in \{d, u\})
\end{aligned}$$

or equivalently

$$\mathbf{x}_{t_{n-2}} = \frac{1 + \bar{b}_{t_{n-1}}}{1 + r_{t_{n-1}}} \begin{pmatrix} \frac{b_{t_{n-2}}^u - b_{t_{n-2}}^d}{u_{t_{n-2}} - d_{t_{n-2}}} \frac{C_{t_{n-2}}^b}{S_{t_{n-2}}} \\ \frac{d_{t_{n-2}} u_{t_{n-2}}}{1 + r_{t_{n-2}}} \frac{b_{t_{n-2}}^d - b_{t_{n-2}}^u}{u_{t_{n-2}} - d_{t_{n-2}}} \frac{C_{t_{n-2}}^b}{B_{t_{n-2}}} \end{pmatrix}.$$

Further recursion leads to

$$\mathbf{x}_{t_{n-i}} = \left( \prod_{j=n-i+1}^{n-1} \frac{1 + \bar{b}_{t_j}}{1 + r_{t_j}} \right) \begin{pmatrix} \frac{b_{t_{n-i}}^u - b_{t_{n-i}}^d}{u_{t_{n-i}} - d_{t_{n-i}}} \frac{C_{t_{n-i}}^b}{S_{t_{n-i}}} \\ \frac{d_{t_{n-i}} u_{t_{n-i}}}{1 + r_{t_{n-i}}} \frac{b_{t_{n-i}}^d - b_{t_{n-i}}^u}{u_{t_{n-i}} - d_{t_{n-i}}} \frac{C_{t_{n-i}}^b}{B_{t_{n-i}}} \end{pmatrix} \quad (12)$$

and

$$\begin{aligned} P_{t_{n-i}}(\mathbf{x}_{t_{n-i}}) &= \prod_{j=n-i+1}^{n-1} \frac{1 + \bar{b}_{t_j}}{1 + r_{t_j}} \left( \frac{b_{t_{n-i}}^u - b_{t_{n-i}}^d}{u_{t_{n-i}} - d_{t_{n-i}}} + \frac{1}{B_{t_{n-i}}^{-1} B_{t_{n-i+1}}} \frac{-b_{t_{n-i}}^u d_{t_{n-i}} + b_{t_{n-i}}^d u_{t_{n-i}}}{u_{t_{n-i}} - d_{t_{n-i}}} \right) C_{t_{n-i}}^b \\ &= \prod_{j=n-i}^{n-1} \frac{1 + \bar{b}_{t_j}}{1 + r_{t_j}} C_{t_{n-i}}^b. \end{aligned}$$

Rearranging terms according to

$$\begin{aligned} B_{t_{n-i}}^{-1} P_{t_{n-i}}(\mathbf{x}_{t_{n-i}}) &= \frac{B_{t_{n-i}}^{-1}}{B_{t_{n-i}}^{-1} B_{t_{n-i+1}}} \prod_{j=n-i+1}^{n-1} \frac{1 + \bar{b}_{t_j}}{B_{t_j}^{-1} B_{t_{j+1}}} \left( b_{t_{n-i}}^u \cdot q_{t_{n-i}} + b_{t_{n-i}}^d \cdot (1 - q_{t_{n-i}}) \right) C_{t_{n-i}} \\ &= B_{t_{n-i+1}}^{-1} \left( P_{t_{n-i+1}}^{b,u} \cdot q_{t_{n-i}} + P_{t_{n-i+1}}^{b,d} \cdot (1 - q_{t_{n-i}}) \right) \end{aligned}$$

reveals that the market price of portfolio  $\mathbf{x}_{t_{n-i}}$  is a martingale if measured in units of MMF certificates. Thus, rebalancing this portfolio from time to time must be self financing.<sup>5</sup> Hence, regarding the identity

$$C_{t_0}^b \equiv C_{t_0}$$

we finally arrive at

$$V_{t_0}(C_{t_k}) = \prod_{j=0}^{k-1} \frac{1 + \bar{b}_{t_j}}{1 + f_{t_j}} C_{t_0}$$

<sup>5</sup> Cf. HARRISON AND KREPS [4].

and

$$\bar{V}_{t_0}(C_{t_k}) = \prod_{j=0}^{k-1} \frac{1 + \bar{g}_{t_j}}{1 + f_{t_j}} C_{t_0},$$

respectively, by substituting the short rates for future periods with the respective forward rates

$$f_{t_i} := f(t_0, t_i, t_{i+1}) = r(t_i, t_{i+1}) =: r_{t_i}$$

implied by the term structure of interest rate in  $t_0$ .<sup>6</sup>

If predictability beyond time  $t_i$  is limited so that it does not make sense to be too specific about the scenario from that time on the following assumptions seem adequate

$$b_{t_j}^m = b^m \text{ for all } j \geq i$$

and

$$u_{t_j} \cdot d_{t_j} = 1 \text{ for all } j \geq i.$$

The growth factors  $u_{t_j}$  and  $d_{t_j}$  then can be traced back to the implied volatility  $\sigma$  of the exchange-traded asset according to

$$u_{t_j} = e^{\sigma} \text{ for all } j \geq i.$$

Adding the assumption of a constant spot rate  $r$  that is

$$B_{t_j}^{-1} B_{t_{j+1}} = 1 + r \text{ for all } j \geq i$$

<sup>6</sup> It is well known since COX, INGERSOLL, AND ROSS [1] that future spot rates must be equal to implied forward rates to preclude arbitrage if the interest rate evolves deterministically.

we get

$$\begin{aligned}\bar{b}_{t_j} = \bar{b} &:= b^d + (b^u - b^d) \cdot q - 1 \\ &= b^d - 1 + (b^u - b^d) \cdot \frac{(1+r) \cdot e^\sigma - 1}{e^{2\sigma} - 1} \quad \text{for all } j \geq i+1\end{aligned}$$

and thus for any given  $i$ ,  $1 \leq i \leq k$

$$\underline{V}_{t_0}(C_{t_k}) = \prod_{h=0}^{i-1} \frac{1 + \bar{b}_{t_h}}{1 + f_{t_h}} \left( \frac{1 + \bar{b}}{1 + r} \right)^{k-i} C_{t_0}$$

and

$$\bar{V}_{t_0}(C_{t_k}) = \prod_{h=0}^{i-1} \frac{1 + \bar{g}_{t_h}}{1 + f_{t_h}} \left( \frac{1 + \bar{g}}{1 + r} \right)^{k-i} C_{t_0}.$$

Given the definition

$$\theta(x, y) := \frac{1+x}{1+y}$$

and  $\bar{b} \neq r$  the upper and lower bounds to the value of a *series* of cash flows  $(C_{t_i})_{i=1}^k$  at times  $t_1, \dots, t_k$  are<sup>7</sup>

$$\underline{V}_{t_0} \left( (C_{t_i})_{i=1}^k \right) = \left( \sum_{h=0}^{i-1} \prod_{g=0}^h \theta(\bar{b}_{t_g}, f_{t_g}) + \prod_{h=0}^{i-1} \theta(\bar{b}_{t_h}, f_{t_h}) \frac{1 + \bar{b}}{r - \bar{b}} \left( 1 - \theta(\bar{b}, r)^{k-i} \right) \right) C_{t_0}$$

and

$$\bar{V}_{t_0} \left( (C_{t_i})_{i=1}^k \right) = \left( \sum_{h=0}^{i-1} \prod_{g=0}^h \theta(\bar{g}_{t_g}, f_{t_g}) + \prod_{h=0}^{i-1} \theta(\bar{g}_{t_h}, f_{t_h}) \frac{1 + \bar{g}}{r - \bar{g}} \left( 1 - \theta(\bar{g}, r)^{k-i} \right) \right) C_{t_0}.$$

<sup>7</sup> It is well known that

$$C_{t_0} \cdot \sum_{i=1}^n \theta^i = C_{t_0} \cdot \frac{\theta^{-1} - 1}{\theta^{-1} - 1} \cdot \sum_{i=1}^n \theta^i = C_{t_0} \cdot \frac{\theta^{-1} \cdot \sum_{i=1}^n \theta^i - \sum_{i=1}^n \theta^i}{\theta^{-1} - 1} = C_{t_0} \cdot \frac{\sum_{i=0}^{n-1} \theta^i - \sum_{i=1}^n \theta^i}{\theta^{-1} - 1} = C_{t_0} \cdot \frac{1 - \theta^n}{\theta^{-1} - 1}.$$

For  $f_{t_g} = r$  and  $\bar{c}_{t_g} = \bar{c}$  ( $g = 0, \dots, k-1$ ;  $c \in \{b, g\}$ ) the above formulae simplify to

$$\underline{V}_{t_0} \left( (C_{t_i})_{i=1}^k \right) = \frac{1 + \bar{b}}{r - \bar{b}} \left( 1 - \theta(\bar{b}, r)^k \right) C_{t_0}$$

and

$$\bar{V}_{t_0} \left( (C_{t_i})_{i=1}^k \right) = \frac{1 + \bar{g}}{r - \bar{g}} \left( 1 - \theta(\bar{g}, r)^k \right) C_{t_0}.$$

For

$$\bar{c} < r \Leftrightarrow \frac{c^u - c^d}{u - d} < \frac{1 + r - c^d}{1 + r - d} \quad (c \in \{b, g\}) \quad (13)$$

the limits for an infinite series of cash flows are

$$\begin{aligned} \lim_{k \rightarrow \infty} \underline{V}_{t_0} \left( (C_{t_i})_{i=1}^k \right) &= \frac{1 + \bar{b}}{r - \bar{b}} \left( 1 - \lim_{k \rightarrow \infty} \theta(\bar{b}, r)^k \right) C_{t_0} \\ &= \frac{1 + \bar{b}}{r - \bar{b}} C_{t_0} \end{aligned}$$

and

$$\lim_{k \rightarrow \infty} \bar{V}_{t_0} \left( (C_{t_i})_{i=1}^k \right) = \frac{1 + \bar{g}}{r - \bar{g}} C_{t_0}.$$

The right hand sides of the above formulae perfectly resemble the structure of the so called GORDON<sup>8</sup> growth formula.

## 4 The impact of taxes

We will analyze the impact of three tax schemes  $TS \in \{GI, I, I^+\}$  using stylized facts to gain some useful insights without adding too much complexity. The tax rates  $t_i$  applicable at time  $t_i$  are the same for any kind of income.<sup>9</sup> Tax schemes *GI* and *I* require taxes on realized and unrealized gains with MMF certificates to be paid immediately and grant immediate

<sup>8</sup> Cf. GORDON [3].

<sup>9</sup> So they also apply to the cash flows  $C_{t_i}$

payments on realized and unrealized losses with MMF certificates ('I' stands for '*Interest*'). Tax scheme *GI* treats realized and unrealized gains and losses with stocks ('G' stands for '*Capital Gain*') the same way as tax schemes *GI* and *I* treat gains and losses with MMF certificates. Tax schemes *GI* and *I* are symmetric in the sense that short positions and long positions induce exactly the same tax payments in absolute terms. Thus, given the definition

$$\mathbf{x}_{t_i}^{TS} := \begin{pmatrix} x_{t_i}^{S,TS} \\ x_{t_i}^{B,TS} \end{pmatrix}$$

the fundamental equation

$$\underline{V}_{t_0}^{TS} = -P_{t_0}(-\mathbf{x}_{t_0}^{TS}) = P_{t_0}(\mathbf{x}_{t_0}^{TS}) \quad (TS \in \{GI, I\})$$

still holds. This is not true for tax scheme  $I^+$  that is inspired by non deductability of interest paid and therefore subjects only long positions in MMF certificates to taxation. Given the definition

$$\begin{aligned} r_{t_i}^{tax} &:= (1 - t_{i+1})r_{t_i} \\ &= (1 - t_{i+1})f_{t_i} =: f_{t_i}^{tax} \end{aligned}$$

by analogy to a world without taxes we get

$$\begin{aligned} x_{t_{n-1}}^{S,GI} &= \frac{(1 - t_n)(b_{t_{n-1}}^u - b_{t_{n-1}}^d)}{u_{t_{n-1}} - (u_{t_{n-1}} - 1)t_n - (d_{t_{n-1}} - (d_{t_{n-1}} - 1)t_n)} \frac{C_{t_{n-1}}^b}{S_{t_{n-1}}} \\ &= \frac{b_{t_{n-1}}^u - b_{t_{n-1}}^d}{u_{t_{n-1}} - d_{t_{n-1}}} \frac{C_{t_{n-1}}^b}{S_{t_{n-1}}} \\ &= x_{t_{n-1}}^S \end{aligned} \tag{14}$$

and

$$\begin{aligned} x_{t_{n-1}}^{S,I} &= \frac{(1 - t_n)(b_{t_{n-1}}^u - b_{t_{n-1}}^d)}{u_{t_{n-1}} - d_{t_{n-1}}} \frac{C_{t_{n-1}}^b}{S_{t_{n-1}}} \\ &= (1 - t_n)x_{t_{n-1}}^S \end{aligned} \tag{15}$$

for the number of risky assets needed at time  $t_{n-1}$ . Hence, again by analogy the required numbers of MMF certificates may be derived from the residual according to

$$\begin{aligned}
x_{t_{n-1}}^{B,GI} &= \frac{(1-t_n)b_{t_{n-1}}^m C_{t_{n-1}}^b - x_{t_{n-1}}^S \cdot (m - (m-1)t_n) \cdot S_{t_{n-1}}}{B_{t_n} - (B_{t_n} - B_{t_{n-1}})t_n} \\
&= \frac{(1-t_n)B_{t_n}}{(1-t_n)B_{t_n} + t_n B_{t_{n-1}}} x_{t_{n-1}}^B - \frac{t_n}{(1-t_n)B_{t_n} + t_n B_{t_{n-1}}} x_{t_{n-1}}^S S_{t_{n-1}} \\
&= x_{t_{n-1}}^B - \frac{t_n}{(1-t_n)B_{t_n} + t_n B_{t_{n-1}}} (x_{t_{n-1}}^B B_{t_{n-1}} + x_{t_{n-1}}^S S_{t_{n-1}})
\end{aligned} \tag{16}$$

and

$$\begin{aligned}
x_{t_{n-1}}^{B,I} &= \frac{(1-t_n)b_{t_{n-1}}^m C_{t_{n-1}}^b - (1-t_n)x_{t_{n-1}}^S \cdot m \cdot S_{t_{n-1}}}{B_{t_n} - (B_{t_n} - B_{t_{n-1}})t_n} \\
&= \frac{B_{t_n}}{(1-t_n)B_{t_n} + t_n B_{t_{n-1}}} (1-t_n)x_{t_{n-1}}^B \\
&= \frac{B_{t_{n-1}}^{-1} B_{t_n}}{(1-t_n)B_{t_{n-1}}^{-1} B_{t_n} + t_n} (1-t_n)x_{t_{n-1}}^B \\
&= \frac{1+r_{t_{n-1}}}{1+r_{t_{n-1}}^{tax}} (1-t_n)x_{t_{n-1}}^B.
\end{aligned} \tag{17}$$

Given the definitions

$$\begin{aligned}
q_{t_i}^{tax} &:= \frac{1+r_{t_i}^{tax} - d_{t_i}}{u_{t_i} - d_{t_i}} \\
\bar{b}_{t_i}^{tax} &:= b_{t_i}^d + q_{t_i}^{tax}(b_{t_i}^u - b_{t_i}^d) - 1
\end{aligned}$$

the market prices of portfolios  $\mathbf{x}_{t_{n-1}}^{GI}$  and  $\mathbf{x}_{t_{n-1}}^I$  at time  $t_{n-1}$  are

$$\begin{aligned}
P_{t_{n-1}}(\mathbf{x}_{t_{n-1}}^{GI}) &= P_{t_{n-1}}(\mathbf{x}_{t_{n-1}}) \left( 1 - \frac{t_n B_{t_{n-1}}}{(1-t_n)B_{t_n} + t_n B_{t_{n-1}}} \right) \\
&= P_{t_{n-1}}(\mathbf{x}_{t_{n-1}}) \left( \frac{(1-t_n)B_{t_{n-1}}^{-1} B_{t_n}}{(1-t_n)B_{t_{n-1}}^{-1} B_{t_n} + t_n} \right) \\
&= \frac{1+r_{t_{n-1}}}{1+r_{t_{n-1}}^{tax}} (1-t_n) P_{t_{n-1}}(\mathbf{x}_{t_{n-1}})
\end{aligned} \tag{18}$$

and

$$\begin{aligned}
P_{t_{n-1}}(\mathbf{x}_{t_{n-1}}^I) &= (1 - t_{t_n}) \left( \frac{b_{t_{n-1}}^u - b_{t_{n-1}}^d}{u_{t_{n-1}} - d_{t_{n-1}}} + \frac{1}{1 + r_{t_{n-1}}^{tax}} \frac{-b_{t_{n-1}}^u d_{t_{n-1}} + b_{t_{n-1}}^d u_{t_{n-1}}}{u_{t_{n-1}} - d_{t_{n-1}}} \right) C_{t_{n-1}}^b \\
&= \frac{1 + \bar{b}_{t_{n-1}}^{tax}}{1 + \bar{b}_{t_{n-1}}} \frac{1 + r_{t_{n-1}}}{1 + r_{t_{n-1}}^{tax}} (1 - t_{t_n}) P_{t_{n-1}}(\mathbf{x}_{t_{n-1}}) \\
&= \frac{1 + \bar{b}_{t_{n-1}}^{tax}}{1 + \bar{b}_{t_{n-1}}} P_{t_{n-1}}(\mathbf{x}_{t_{n-1}}^{GI}). \tag{19}
\end{aligned}$$

Calculating backwards in the same manner as in the absence of taxes in combination with the definitions

$$\begin{aligned}
C_{t_i}^{tax} &:= (1 - t_{t_i}) C_{t_i} \\
\alpha_j &:= \frac{1 + f_{t_j}}{1 + f_{t_j}^{tax}} \\
\beta_j &:= \frac{1 + \bar{b}_{t_j}^{tax}}{1 + \bar{b}_{t_j}} \\
\gamma_j &:= \frac{1 + \bar{g}_{t_j}^{tax}}{1 + \bar{g}_{t_j}}
\end{aligned}$$

finally leads to the following expressions for the lower and upper bounds in the multi-period case

$$\begin{aligned}
\underline{V}_{t_0}^I(C_{t_k}^{tax}) &= \prod_{j=0}^{k-1} \beta_j \cdot \underline{V}_{t_0}^{GI}(C_{t_k}^{tax}) = \prod_{j=0}^{k-1} \alpha_j \beta_j \cdot \underline{V}_{t_0}(C_{t_k}^{tax}) \\
\bar{V}_{t_0}^I(C_{t_k}^{tax}) &= \prod_{j=0}^{k-1} \gamma_j \cdot \bar{V}_{t_0}^{GI}(C_{t_k}^{tax}) = \prod_{j=0}^{k-1} \alpha_j \gamma_j \cdot \underline{V}_{t_0}(C_{t_k}^{tax}).
\end{aligned}$$

Notice that  $f_{t_j}^{tax} < f_{t_j}$  has two instructive implications: Firstly, it implies  $q_{t_i}^{tax} < q_{t_i}$  which again implies the equivalence

$$\bar{c}_{t_i}^{tax} \leq \bar{c}_{t_i} \Leftrightarrow c_{t_i}^d \leq c_{t_i}^u \quad (c \in \{b, g\})$$



and thus

$$\beta_j \leq 1 \Leftrightarrow b_{t_j}^d \leq b_{t_j}^u$$

$$\gamma_j \leq 1 \Leftrightarrow g_{t_j}^d \leq g_{t_j}^u.$$

Secondly, as can be seen from

$$\begin{aligned} \alpha_j \beta_j &= \frac{\frac{1+\bar{b}_{t_j}^{tax}}{1+f_{t_j}^{tax}}}{\frac{1+\bar{b}_{t_j}}{1+f_{t_j}}} \\ &= \frac{\frac{b_{t_j}^u - b_{t_j}^d}{u_{t_j} - d_{t_j}} + \frac{1}{1+f_{t_j}^{tax}} \frac{-b_{t_j}^u d_{t_j} + b_{t_j}^d u_{t_j}}{u_{t_j} - d_{t_j}}}{\frac{b_{t_j}^u - b_{t_j}^d}{u_{t_j} - d_{t_j}} + \frac{1}{1+f_{t_j}} \frac{-b_{t_j}^u d_{t_j} + b_{t_j}^d u_{t_j}}{u_{t_j} - d_{t_j}}} \\ &= \frac{\frac{b_{t_j}^u - b_{t_j}^d}{u_{t_j} - d_{t_j}} + \frac{u_{t_j} d_{t_j}}{1+f_{t_j}^{tax}} \frac{\frac{b_{t_j}^d}{d_{t_j}} - \frac{b_{t_j}^u}{u_{t_j}}}{u_{t_j} - d_{t_j}}}{\frac{b_{t_j}^u - b_{t_j}^d}{u_{t_j} - d_{t_j}} + \frac{u_{t_j} d_{t_j}}{1+f_{t_j}} \frac{\frac{b_{t_j}^d}{d_{t_j}} - \frac{b_{t_j}^u}{u_{t_j}}}{u_{t_j} - d_{t_j}}} \end{aligned} \quad (20)$$

in combination with the fact that neither the nominator nor the denominator of (20) can get negative, it implies

$$\alpha_j \beta_j \leq 1 \Leftrightarrow \frac{b_{t_j}^d}{d_{t_j}} \leq \frac{b_{t_j}^u}{u_{t_j}}$$

and

$$\alpha_j \gamma_j \leq 1 \Leftrightarrow \frac{g_{t_j}^d}{d_{t_j}} \leq \frac{g_{t_j}^u}{u_{t_j}}.$$

Thus, if the payoff characteristic is convex for any trading interval up to  $t_k$  that is if

$$\frac{c_{t_i}^d}{d_{t_i}} < \frac{c_{t_i}^u}{u_{t_i}} \text{ for each } i \in \{0, \dots, k-1\}, c \in \{b, g\}$$

we get

$$\underline{V}_{t_0}^I (C_{t_k}^{tax}) < \underline{V}_{t_0} (C_{t_k}^{tax}) < \underline{V}_{t_0}^{GI} (C_{t_k}^{tax}) \quad (21)$$

and

$$\bar{V}_{t_0}^I (C_{t_k}^{tax}) < \bar{V}_{t_0} (C_{t_k}^{tax}) < \bar{V}_{t_0}^{GI} (C_{t_k}^{tax}) . \quad (22)$$

The first relation of these orders derives from the fact that strategies  $(\mathbf{x}_{t_i})_{i=0}^{k-1}$  and  $(\mathbf{y}_{t_i})_{i=0}^{k-1}$  require long positions in stocks and short positions in MMF certificates at all times  $t_i$  if the payoff characteristic is convex for all  $t_i$ . So tax scheme  $I$  reduces the financing cost of the long position in stocks compared to the absence of any taxation of exchange-traded assets. The second relation of these orders results from the comparably larger number of stocks necessary under tax scheme  $GI$  that overrules the effect of reduced financing costs.

If the payoff characteristic were concave constantly over time we would get

$$\underline{V}_{t_0} (C_{t_k}^{tax}) < \underline{V}_{t_0}^I (C_{t_k}^{tax}) < \underline{V}_{t_0}^{GI} (C_{t_k}^{tax}) \quad (23)$$

$$\bar{V}_{t_0} (C_{t_k}^{tax}) < \bar{V}_{t_0}^I (C_{t_k}^{tax}) < \bar{V}_{t_0}^{GI} (C_{t_k}^{tax}) \quad (24)$$

if

$$c_{t_i}^u \frac{d_{t_i}}{u_{t_i}} < c_{t_i}^d < bc_{t_i}^u \text{ for each } i \in \{0, \dots, k-1\}, c \in \{b, g\} \quad (25)$$

and

$$\underline{V}_{t_0} (C_{t_k}^{tax}) < \underline{V}_{t_0}^{GI} (C_{t_k}^{tax}) < \underline{V}_{t_0}^I (C_{t_k}^{tax}) \quad (26)$$

$$\bar{V}_{t_0} (C_{t_k}^{tax}) < \bar{V}_{t_0}^{GI} (C_{t_k}^{tax}) < \bar{V}_{t_0}^I (C_{t_k}^{tax}) \quad (27)$$

if

$$c_{t_i}^u \leq c_{t_i}^d \text{ for each } i \in \{0, \dots, k-1\}, c \in \{b, g\}. \quad (28)$$

Compared to (21) and (22) the orders (23) and (24) derive from the fact that (25) requires long positions in stocks and MMF certificates so that the taxation of MMF certificates, i.e. interest, does not reduce financing cost but interest revenue and, thus, means increased funding. If (28) holds, then strategies  $(\mathbf{x}_{t_i})_{i=0}^{k-1}$  and  $(\mathbf{y}_{t_i})_{i=0}^{k-1}$  allow for short positions in stocks so that tax scheme  $GI$  leads to an increased volume of short sales compared to tax scheme  $I$ . This explains the difference between the orders (26), (27) and (23), (24).

Under tax scheme  $I^+$  we have to look at strategies  $(-\mathbf{x}_{t_i})_{i=0}^{k-1}$  and  $(\mathbf{y}_{t_i})_{i=0}^{k-1}$ . As is revealed by (12), in the absence of taxes the implementation of trading strategy  $(-\mathbf{x}_{t_i})_{i=0}^{n-1}$  requires a long position in MMF certificates at time  $t_i$  if and only if  $\frac{b_{t_i}^d}{d_{t_i}} < \frac{b_{t_i}^u}{u_{t_i}}$  whereas the implementation of trading strategy  $(\mathbf{y}_{t_i})_{i=0}^{n-1}$  requires a long position in MMF certificates at time  $t_i$  if and only if  $\frac{g_{t_i}^u}{u_{t_i}} < \frac{g_{t_i}^d}{d_{t_i}}$ . Thus, given the definition

$$1_j^c := \begin{cases} 1 & \text{if } \frac{c_{t_j}^d}{d_{t_j}} < \frac{c_{t_j}^u}{u_{t_j}} \\ 0 & \text{otherwise} \end{cases} \quad (c \in \{b, g\})$$

we get

$$\begin{aligned} \underline{V}_{t_0}^{I^+}(C_{t_k}^{tax}) &= \prod_{j=0}^{k-1} (\alpha_j \beta_j)^{1_j^b} \underline{V}_{t_0}(C_{t_k}^{tax}) \\ &= \prod_{j=0}^{k-1} (\alpha_j \beta_j)^{1_j^b} \prod_{j=0}^{k-1} (\alpha_j \beta_j)^{-1} \underline{V}_{t_0}^I(C_{t_k}^{tax}) \\ &= \prod_{j=0}^{k-1} (\alpha_j \beta_j)^{1_j^b - 1} \underline{V}_{t_0}^I(C_{t_k}^{tax}) \\ \bar{V}_{t_0}^{I^+}(C_{t_k}^{tax}) &= \prod_{j=0}^{k-1} (\alpha_j \gamma_j)^{1 - 1_j^g} \bar{V}_{t_0}(C_{t_k}^{tax}) \\ &= \prod_{j=0}^{k-1} (\alpha_j \gamma_j)^{-1_j^g} \bar{V}_{t_0}^I(C_{t_k}^{tax}). \end{aligned}$$

Regarding

$$\alpha_j \beta_j \leq 1 \Leftrightarrow \frac{b_{t_j}^d}{d_{t_j}} \leq \frac{b_{t_j}^u}{u_{t_j}} \Leftrightarrow 1_j^b = \begin{cases} 1 \\ 0 \end{cases}$$

and

$$\alpha_j \gamma_j \leq 1 \Leftrightarrow \frac{g_{t_j}^d}{d_{t_j}} \leq \frac{g_{t_j}^u}{u_{t_j}} \Leftrightarrow 1_j^g = \begin{cases} 1 \\ 0 \end{cases}$$

we get to the following conclusions: If the payoff characteristic is convex for all  $t_i$ , then strategy  $(-\mathbf{x}_{t_i})_{i=0}^{k-1}$  is a *short stock - long MMF strategy* and strategy  $(\mathbf{y}_{t_i})_{i=0}^{k-1}$  is a *long stock - short MMF strategy*. Thus,

$$\underline{V}_{t_0}^I (C_{t_k}^{tax}) = \underline{V}_{t_0}^{I^+} (C_{t_k}^{tax}) < \underline{V}_{t_0} (C_{t_k}^{tax}) < \underline{V}_{t_0}^{GI} (C_{t_k}^{tax})$$

and

$$\bar{V}_{t_0}^I (C_{t_k}^{tax}) < \bar{V}_{t_0}^{I^+} (C_{t_k}^{tax}) = \bar{V}_{t_0} (C_{t_k}^{tax}) < \bar{V}_{t_0}^{GI} (C_{t_k}^{tax}).$$

If the payoff characteristic is concave constantly over time we get

$$\begin{aligned} \underline{V}_{t_0} (C_{t_k}^{tax}) &= \underline{V}_{t_0}^{I^+} (C_{t_k}^{tax}) < \underline{V}_{t_0}^I (C_{t_k}^{tax}) < \underline{V}_{t_0}^{GI} (C_{t_k}^{tax}) \\ \bar{V}_{t_0} (C_{t_k}^{tax}) &< \bar{V}_{t_0}^{GI} (C_{t_k}^{tax}) < \bar{V}_{t_0}^{I^+} (C_{t_k}^{tax}) = \bar{V}_{t_0}^I (C_{t_k}^{tax}) \end{aligned}$$

if

$$c_{t_i}^u \frac{d_{t_i}}{u_{t_i}} < c_{t_i}^d < c_{t_i}^u \text{ for each } i \in \{0, \dots, k-1\}, c \in \{b, g\}$$

because strategy  $(-\mathbf{x}_{t_i})_{i=0}^{k-1}$  is a *short stock - short MMF strategy* and strategy  $(\mathbf{y}_{t_i})_{i=0}^{k-1}$  is a *long stock - long MMF strategy* and

$$\begin{aligned} \underline{V}_{t_0} (C_{t_k}^{tax}) &= \underline{V}_{t_0}^{I^+} (C_{t_k}^{tax}) < \underline{V}_{t_0}^{GI} (C_{t_k}^{tax}) < \underline{V}_{t_0}^I (C_{t_k}^{tax}) \\ \bar{V}_{t_0} (C_{t_k}^{tax}) &< \bar{V}_{t_0}^{GI} (C_{t_k}^{tax}) = \bar{V}_{t_0}^{I^+} (C_{t_k}^{tax}) = \bar{V}_{t_0}^I (C_{t_k}^{tax}) \end{aligned}$$

if

$$c_{t_i}^u \leq c_{t_i}^d \text{ for each } i \in \{0, \dots, k-1\}, c \in \{b, g\}$$

because strategy  $(-\mathbf{x}_{t_i})_{i=0}^{k-1}$  is a *long stock - short MMF strategy* and strategy  $(\mathbf{y}_{t_i})_{i=0}^{k-1}$  is a *short stock - long MMF strategy*.

## 5 Summary

We have developed a simple yet robust approach to determine a range of no-arbitrage prices for an enterprise. The approach is simple and easy to implement because, firstly, its formal structure is an obvious generalization of the so called GORDON growth formula and, secondly, the required inputs are readily observable interest rate curves and volatilities as much as market data is concerned. The approach is robust as it is based on pure no-arbitrage conditions.

It is shown that under such conditions convexity is a crucial determinant for the impact of taxation. What counts is how the enterprise cash flows behave relative to the price of some exchange-traded reference asset in the best and in the worst of all possible worlds, respectively. If in principal there is no difference between these scenarios as to this behavior the upper and lower bound always move to the same direction as long as you switch between symmetric<sup>10</sup> tax schemes. This does not hold however for asymmetric tax schemes.

Although based on stylized facts concerning the tax scheme these results seem to have a potential for guiding further research in this field.

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<sup>10</sup>This means that short positions and long positions induce exactly the same tax payments in absolute terms.

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