Replication in Consistent Binomial Models

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Abstract

The binomial model has been used to price a wide variety of equity and interest rate options for more than two decades. Originally developed by Cox, Ross, and Rubinstein to clarify the basic pricing principle of its continuous-time counterpart with reduced mathematical requirements, the approach became a numerical scheme to evaluate all kinds of contingent claims. Some of the algorithms have dissociated more and more from the basic principles. In this paper we turn to the foundations of the binomial model and elaborate the relation between real world processes, replicating strategies and martingales in a strict way.

Keywords: binomial model, martingale method, option pricing, trading strategy

JEL Classification: G13, C60

1 Introduction

Two perspectives on the nature of the binomial model are presented in this paper. First, the binomial model can be understood as a stand-alone pricing model, whose paths imitate the price behavior of a traded asset in a simplified way. If we start modelling the real world behavior and interpret the paths as the result of a particular state of the world, then we can determine a unique trading strategy that replicates an arbitrary contingent claim. If the law of one price holds, the price of this contingent claim must be equal to the market price of the replicating portfolio. According to the methodology introduced by HARRISON AND KREPS [3], it is possible to formulate the stock process in units of a numeraire (a money market fund) and to create a probability measure under which the process is a martingale. The price of any contingent claim — formulated in units of a numeraire — can be expressed as an expected value of its normed future payoffs under the same measure.

The other interpretation connects the binomial model with a related model, which is assumed to reflect the true processes for theoretical reasons or which simply behaves more like true price paths according to experience. In this context, the binomial model is applied as a numerical scheme to approximate the more adequate model. Since it is only an approximation, some important properties get lost. On the other hand, the reduction of complexity is sometimes the only way to obtain solutions. The binomial model is used to approximate Brownian motions or transformations of the process justified by the central limit theorem. The paths of the binomial model do not play the decisive role any longer; it is the distributional behavior and the convergence property that is of major interest.

In this paper the focus is not on the properties of the binomial model as an algorithmic tool for pricing derivatives; neither the valuation of a new contract nor the analysis of convergence are presented. The question we try to answer is what we can say about the relation between real world processes and martingales if we take the theoretical background and the assumptions of binomial model seriously. Most numerical schemes start modelling the basic processes under the martingale measure and do not bother about real world processes. But it is the real world process that determines the payoff of a derivative and martingales should reflect the replicating strategy.

2 The Valuation Framework

It is assumed that the participants of a financial market have clear and homogenous ideas on the price evolution of some securities (basis securities). In accordance with the modelling of COX, ROSS, AND RUBINSTEIN [2], the future prices are expressed as the outcome of a binomial process. Each path can be associated with an element in the sample space Ω. Together with a σ-Algebra $\mathcal{A}$ and a probability measure P it forms a probability space $(\Omega, \mathcal{F}, P)$. The probability space is equipped with a filtration $\{\mathcal{F}_{t_n}\}_{n=0}^{N}$ which has the characteristic property

$$\mathcal{F}_{t_0} \subseteq \mathcal{F}_{t_1} \subseteq \ldots \subseteq \mathcal{F}_{t_N} = \mathcal{F}$$

representing the evolution of information on the market, where no piece of information gets lost over time.

Trading only takes place at certain equidistant points in time contained in the set

$$\begin{aligned} \mathcal{T} &= \{0 = t_0, t_1, \ldots, t_N = T\} \\ &= \{t_0, t_0 + \Delta_t, \ldots, t_0 + N \cdot \Delta_t\} \end{aligned}$$

with the overall time interval from 0 to T being fixed. Suppose that the price of the money market fund is determined by the non-stochastic one-period interest rate $r \geq 0$, such that its price evolution can be described by

$$B_{t_n} = \begin{cases} B_{t_0}, & \text{if } t_n = t_0; \\ B_{t_{n-1}} \exp(r\Delta_t), & \text{if } t_0 < t_n \leq t_N. \end{cases} \tag{1}$$

The stochastic process that governs the evolution of the stock price is given by

$$S_{t_n} = \begin{cases} S_{t_0}, & \text{if } t_n = t_0; \\ S_{t_{n-1}} \exp(\mu\Delta_t + \sigma\sqrt{\Delta_t} X_{t_n}), & \text{if } t_0 < t_n \leq t_N; \end{cases} \tag{2}$$

where X_{t_n} is a sequence of independently identically distributed (i.i.d.) Bernoulli random variables

$$X_{t_n} : (\Omega_{t_n}, \mathcal{F}_{t_n}) \rightarrow (\mathcal{X}_{t_n}, \mathcal{B}_{t_n})$$

with outcomes in the state space $\mathcal{X}_{t_n} = \{-1, 1\}$. Given the information at t_n, the probability that $X_{t_{n+1}} = 1$ is $p, 0 < p < 1$, and that $X_{t_{n+1}} = -1$ is $(1-p)$. The parameter $\mu \in \mathbb{R}$ is referred to as the drift coefficient and the parameter $\sigma > 0, \sigma \in \mathbb{R}$, as the diffusion coefficient of the process.

The market is complete by construction. Furthermore, it is assumed that the market is arbitrage-free and remains arbitrage-free after introducing new assets or contracts. A necessary condition for the market being arbitrage-free is

$$\mu - \frac{\sigma}{\sqrt{\Delta_t}} < r < \mu + \frac{\sigma}{\sqrt{\Delta_t}}, \tag{3}$$

i.e. no asset price process dominates the other one.

Our aim is to formulate consistent binomial option pricing models. We give a definition how consistency is understood in this context.

Definition 2.1 (consistent binomial model) *Consider an arbitrage-free binomial model where the expected value of the payoff of an arbitrary derivative expressed in units of a numeraire equals the price of a replicating trading strategy based on real world processes. The model is consistent with respect to parameters ($\hat{\mu}$, $\hat{\sigma}$) if the expected real world logarithmic return over an interval of length T equals $\hat{\mu}T$ and the corresponding variance is $\hat{\sigma}^2 T$.*

The question we try to answer is how to specifiy the parameters of the processes under Q such that the real world processes have exactly the distributional properties of the desired kind.

3 Process Parameters and Moments of Logarithmic Returns

Under the specified assumptions the local expected value of the logarithmic return equals

$$\mathrm{E}_P\left[\ln\left(\frac{S_{t_n}}{S_{t_{n-1}}}\right)\middle|\mathcal{F}_{t_{n-1}}\right] = \mu\Delta_t - (1-2p)\sigma\sqrt{\Delta_t}$$

and the local variance of the logarithmic return equals

$$\mathrm{Var}_P\left[\ln\left(\frac{S_{t_n}}{S_{t_{n-1}}}\right)\middle|\mathcal{F}_{t_{n-1}}\right] = 4p(1-p)\sigma^2\Delta_t\,.$$

Note that the diffusion parameter σ has no influence on the local expected value if and only if p equals $\frac{1}{2}$. In this case the local variance reduces to

$$\mathrm{Var}_P\left[\ln\left(\frac{S_{t_n}}{S_{t_{n-1}}}\right)\middle|\mathcal{F}_{t_{n-1}}\right]=\sigma^2\Delta_t\,.$$

Equal probabilities play a prominent role when interpreting μ and σ as distribution coefficients in a binomial model. Therefore, we frequently split a probability p into a reference probability of $\frac{1}{2}$ and a resulting deviation according to

$$p=\frac{1}{2}+\frac{1}{2}\eta_p\,,\;\eta_p\in(-1,1)\,,$$

which leads to

$$\mathrm{E}_P\left[\ln\left(\frac{S_{t_n}}{S_{t_{n-1}}}\right)\middle|\mathcal{F}_{t_{n-1}}\right]=\mu\Delta_t+\eta_p\sigma\sqrt{\Delta_t}$$

and

$$\mathrm{Var}_P\left[\ln\left(\frac{S_{t_n}}{S_{t_{n-1}}}\right)\middle|\mathcal{F}_{t_{n-1}}\right]=\left(1-\eta_p^2\right)\sigma^2\Delta_t\,.$$

For the fixed period from 0 to T one obtains an expected return of

$$\mathrm{E}_P\left[\ln\left(\frac{S_T}{S_0}\right)\middle|\mathcal{F}_0\right]=N\left(\mu\Delta_t+\eta_p\sigma\sqrt{\Delta_t}\right)=\mu T+\eta_p\sigma\sqrt{N\cdot T}\tag{4}$$

and a variance of

$$\mathrm{Var}_P\left[\ln\left(\frac{S_T}{S_0}\right)\middle|\mathcal{F}_0\right]=N\left(1-\eta_p^2\right)\sigma^2\Delta_t=\left(1-\eta_p^2\right)\sigma^2 T\,.\tag{5}$$

The expected value does not depend on the number of trading days if and only if $\eta_p=0$, which again underlines the importance of this specification. So far η_p has been interpreted as the spread between the probability of an up move and a down move. Note that the square of η_p has a meaning as well. It can be interpreted as the percentage deviation of the variance of the logarithmic returns from $\sigma^2 T$.

Let us assume that the expected value and the variance of the logarithmic return of the true stock process are known and denoted by $\hat{\mu}$ and $\hat{\sigma}^2$. If the binomial model is interpreted as a stand-alone model in the sense of the introductory section, then first of all the probability space is specified. The first two moments of the stock's logarithmic returns are then determined by all possible paths implied by the probability space. There is no degree of freedom to calibrate the moments. It is, of course, possible to specify the probability space such that the moments are met — this is the procedure used in practice. However, the probability measure is not really taken into account except for identifying the paths that occur with positive probability. This approach deals with paths and not with moments or distributions.

If we look at the second interpretation, then we are just interested in fitting some properties of the underlying distribution. If we concentrate on the first two moments, the parameters μ, σ and p offer one degree of freedom. Equations (4) and (5) show that the parameters μ and σ^2 can only be interpreted as the moments of the logarithmic returns, if $p = \frac{1}{2}$. Then we get

$$\mu T = \hat{\mu} T \text{ and } \sigma^2 T = \hat{\sigma}^2 T \, .$$

We can, of course, choose an arbitrary transition probability $p = \frac{1}{2} + \frac{1}{2}\eta_p$ and set

$$\mu = \hat{\mu} - \eta_p \frac{\sigma}{\sqrt{\Delta_t}} \tag{6}$$

and

$$\begin{aligned} \sigma &= \frac{1}{\sqrt{1-\eta_p^2}} \hat{\sigma} \\ &= \sqrt{\hat{\sigma}^2 + (\hat{\mu} - \mu)^2 \Delta_t} \end{aligned} \tag{7}$$

to obtain the given moments. Putting (7) into (6) yields

$$\mu = \hat{\mu} - \frac{\eta_p}{\sqrt{1-\eta_p^2}} \frac{\hat{\sigma}}{\sqrt{\Delta}} \, . \tag{8}$$

Solving for η_p^2 one obtains

$$\eta_p^2 = \frac{(\hat{\mu} - \mu)^2}{(\hat{\mu} - \mu)^2 + \frac{\hat{\sigma}^2}{\Delta_t}} \, . \tag{9}$$

This expression approves to be extraordinarily useful for determining consistent models. It allows calculating the transition probabilities for a given drift coefficient, such that the (real) distributional parameters are met.

Using (7) and (8) the real world stock price process can be specified as

$$S_{t_n} = \begin{cases} S_{t_0}, & \text{if } t_n = t_0; \\ S_{t_{n-1}} \exp\left(\left(\hat{\mu} - \frac{\eta_p}{\sqrt{1-\eta_p^2}\sqrt{\Delta_t}}\hat{\sigma}\right)\Delta_t + \frac{1}{\sqrt{1-\eta_p^2}}\hat{\sigma}\sqrt{\Delta_t}X_{t_n}\right), & \text{if } t_0 < t_n \leq t_N; \end{cases}$$

or equivalently as

$$S_{t_n} = \begin{cases} S_{t_0}, & \text{if } t_n = t_0; \\ S_{t_{n-1}} \exp\left(\hat{\mu}\Delta_t - \hat{\sigma}\sqrt{\Delta_t}X_{t_n}^{\eta_p}\right), & \text{if } t_0 < t_n \leq t_N; \end{cases}$$

with

$$X_{t_n}^{\eta_p} := \frac{X_{t_n} - \eta_p}{\sqrt{1-\eta_p^2}},$$

having the property that for any arbitrary transition probability $p = \frac{1}{2} + \frac{1}{2}\eta_p,\ 0 < p < 1$, its logarithmic return over the time interval $[0,T]$ has mean $\hat{\mu}T$ and variance $\hat{\sigma}^2 T$.

4 Replicating Trading Strategies

A contingent claim is a contract whose payoff structure depends on the state dependent prices of one or more other assets. It is attainable if its future payoff can be generated by a dynamic portfolio strategy. A dynamic portfolio strategy is a previsible process $\phi_{t_n}, n = 0,\ldots,N$, representing the number of basis securities that are held in each of the periods $[t_{n-1},t_n)$. Since the market is complete, the payoff structure of any security contract can be generated by a dynamic portfolio strategy.

In the binomial model the dynamic portfolio strategy is determined by

$$\begin{pmatrix} S_{t_n}(+1) & B_{t_n} \\ S_{t_n}(-1) & B_{t_n} \end{pmatrix} \begin{pmatrix} \phi_{t_n}^S \\ \phi_{t_n}^B \end{pmatrix} = \begin{pmatrix} \Pi_{t_n}(+1) \\ \Pi_{t_n}(-1) \end{pmatrix}, \tag{10}$$

where Π_{t_n} is the price of a portfolio that replicates the (dynamic) payoff structure of the contract to be valued. The price of this replicating portfolio in t_{n-1} is then given by

$$\Pi_{t_{n-1}} = \phi^S_{t_n} S_{t_{n-1}} + \phi^B_{t_n} B_{t_{n-1}} \,.$$

We follow the martingale method by HARRISON AND KREPS [3] and express the prices in units of a numeraire by premultiplying both sides of equation (10) by the matrix

$$\mathbf{D}_{t_n} = \begin{pmatrix} B^{-1}_{t_n} & 0 \\ 0 & B^{-1}_{t_n} \end{pmatrix} .$$

Let $\hat{X}_{t_n}$ denote the price of an arbitrary asset or contract X in units of the numeraire, i.e.

$$\hat{X}_{t_n} = B^{-1}_{t_n} X_{t_n} \,.$$

The number of assets is given by

$$\begin{pmatrix} \phi^S_{t_n} \\ \phi^B_{t_n} \end{pmatrix} = \begin{pmatrix} \frac{\hat{\Pi}_{t_n}(+1) - \hat{\Pi}_{t_n}(-1)}{\hat{S}_{t_n}(+1) - \hat{S}_{t_n}(-1)} \\ \frac{\hat{S}_{t_n}(+1)\hat{\Pi}_{t_n}(-1) - \hat{S}_{t_n}(-1)\hat{\Pi}_{t_n}(+1)}{\hat{S}_{t_n}(+1) - \hat{S}_{t_n}(-1)} \end{pmatrix}$$

which results in a portfolio price in t_{n-1} of

$$\begin{aligned} \hat{\Pi}_{t_{n-1}} &= \frac{\hat{\Pi}_{t_n}(+1) - \hat{\Pi}_{t_n}(-1)}{\hat{S}_{t_n}(+1) - \hat{S}_{t_n}(-1)} \hat{S}_{t_{n-1}} + \frac{\hat{S}_{t_n}(+1)\hat{\Pi}_{t_n}(-1) - \hat{S}_{t_n}(-1)\hat{\Pi}_{t_n}(+1)}{\hat{S}_{t_n}(+1) - \hat{S}_{t_n}(-1)} \\ &= \frac{\hat{S}_{t_n}(+1) - \hat{S}_{t_{n-1}}}{\hat{S}_{t_n}(+1) - \hat{S}_{t_n}(-1)} \hat{\Pi}_{t_n}(-1) + \frac{\hat{S}_{t_{n-1}} - \hat{S}_{t_n}(-1)}{\hat{S}_{t_n}(+1) - \hat{S}_{t_n}(-1)} \hat{\Pi}_{t_n}(+1) \,. \end{aligned}$$

It is well known that the normed price of the replicating portfolio in t_{n-1} can be formulated as the weighted normed prices of the replicating portfolio in t_n and that these weights have the properties of probabilities. Moreover, there is a one-to-one relation between each of the portfolio weights and these pseudo-probabilities, since

$$\phi^S_{t_n} = q \frac{\hat{\Pi}_{t_n}(+1) - \hat{\Pi}_{t_n}(-1)}{\hat{S}_{t_{n-1}} - \hat{S}_{t_n}(-1)}$$

and

$$\phi^B_{t_n} = q \frac{\hat{S}_{t_n}(+1)\hat{\Pi}_{t_n}(-1) - \hat{S}_{t_n}(-1)\hat{\Pi}_{t_n}(+1)}{\hat{S}_{t_{n-1}} - \hat{S}_{t_n}(-1)} \,.$$

The structure of

$$q := \frac{\hat{S}_{t_{n-1}} - \hat{S}_{t_n}(-1)}{\hat{S}_{t_n}(+1) - \hat{S}_{t_n}(-1)}$$

if asset price processes are modelled according to (1) and (2) is another point we want to record here. For the specified processes the weight q must be

$$q = \frac{\exp(-(\mu - r)\Delta_t) - \exp\left(-\sigma\sqrt{\Delta_t}\right)}{\exp\left(\sigma\sqrt{\Delta_t}\right) - \exp\left(\sigma\sqrt{\Delta_t}\right)} \tag{11}$$

$$= \frac{1}{2} + \frac{\exp(-(\mu - r)\Delta_t) - \cosh\left(\sigma\sqrt{\Delta_t}\right)}{2\sinh\left(\sigma\sqrt{\Delta_t}\right)} \tag{12}$$

to reflect a replicating trading strategy.

5 Martingale processes

5.1 Parameter restrictions

The aim is to find a martingale measure under which the normed prices of all traded assets are martingales. We follow the standard procedure and formulate the prices of the stock in units of the money market fund. We obtain

$$\begin{aligned}\hat{S}_{t_n} &:= B_{t_n}^{-1} S_{t_n} \\ &= \hat{S}_{t_0} \exp\left((\alpha - r)(t_n - t_0) + \beta\sqrt{\Delta_t}\sum_{i=1}^{n} X_{t_i}\right).\end{aligned}$$

Obviously, the expected value exists and hence, the process is a martingale if the following condition is satisfied:

$$\begin{aligned}\mathrm{E}_Q\left[\hat{S}_{t_{n+1}}|\mathcal{F}_{t_n}\right] &= B_{t_{n+1}}^{-1}\left(qS_{t_n}\exp\left(\alpha\Delta_t + \beta\sqrt{\Delta_t}\right) + (1-q)\,S_{t_n}\exp\left(\alpha\Delta_t - \beta\sqrt{\Delta_t}\right)\right) \\ &\stackrel{!}{=} B_{t_n}^{-1} S_{t_n} = \hat{S}_{t_n}.\end{aligned} \tag{13}$$

This is equivalent to

$$\exp(-(\alpha - r)\Delta_t) = \left[q\exp\left(\beta\sqrt{\Delta_t}\right) + (1-q)\exp\left(-\beta\sqrt{\Delta_t}\right)\right]$$

which can be analyzed more conveniently if we split up the transition probability q induced by the martingale measure according to

$$q = \frac{1}{2} + \frac{1}{2}\eta_q .$$

The expression simplifies to

$$\exp(-(\alpha - r)\Delta_t) = \left(\cosh\left(\beta\sqrt{\Delta_t}\right) + \eta_q \sinh\left(\beta\sqrt{\Delta_t}\right)\right)$$

and results in a condition for

$$\eta_q = \frac{\exp(-(\alpha - r)\Delta_t) - \cosh\left(\beta\sqrt{\Delta_t}\right)}{\sinh\left(\beta\sqrt{\Delta_t}\right)} .$$

The quantity q is a probability if

$$\alpha - \frac{\beta}{\sqrt{\Delta_t}} < r < \alpha + \frac{\beta}{\sqrt{\Delta_t}}$$

is satisfied. A detailed discussion of the relationship of drift parameters and transition probabilities in arbitrage-free binomial models can be found in WÖSTER [7].

5.2 Replicating a given real world process

This interpretation starts with a given probability space $(\Omega, \mathcal{F}, P)$ describing the states of the real world. Since the probability measure P is given, the transition probabilities and henceforth the probability spread η_p is determined. If the stock price evolves according to (2), then for a given mean $\hat{\mu}$ and a given variance $\hat{\sigma}^2$ the drift coefficient μ and the diffusion coefficient σ are determined by (6) and (7).

Given the real world probability space and the given structure of the basic asset price processes, there is a unique trading strategy replicating the payoff of a specified contract. Since the payoff of a derivative is determined by the real world stock process, we have to choose

$$\beta = \sigma = \frac{1}{\sqrt{1 - \eta_p^2}}\hat{\sigma}$$

and

$$\alpha = \mu = \hat{\mu} - \frac{\eta_p}{\sqrt{\left(1-\eta_p^2\right)}} \frac{\hat{\sigma}}{\sqrt{\Delta_t}}.$$

The martingale measure is then given by

$$q = \frac{1}{2} + \frac{\exp(-(\mu - r)\Delta_t) - \cosh\left(\sigma\sqrt{\Delta_t}\right)}{2\sinh\left(\sigma\sqrt{\Delta_t}\right)} \tag{14}$$

provided that condition (3) holds. A comparison with (11) shows that the transition probability q given by (14) reflects a replicating trading strategy.

5.3 The real world background process of a given martingale process

The standard procedure is to model martingales without thinking about the real world processes that back them, i.e. those real world processes on which replicating strategies are based. In this section we analyze approaches starting with martingales.

There are two main procedures to obtain the martingale property of a binomial process. The first one is to fix a transition probability q and the diffusion parameter β under the martingale measure. The drift parameter is then uniquely determined by

$$\alpha(q,\beta) = r - \frac{1}{\Delta_t} \ln\left(\cosh\left(\beta\sqrt{\Delta_t} + \eta_q \sinh\left(\beta\sqrt{\Delta_t}\right)\right)\right).$$

A very convenient choice is $q = \frac{1}{2}$, which results in

$$\alpha\left(\tfrac{1}{2},\beta\right) = r - \frac{1}{\Delta_t}\left(\cosh\left(\beta\sqrt{\Delta_t}\right)\right).$$

Equal transition probabilities have been applied to stock processes in option pricing models by AMIN [1] and to interest rate processes in term structure by JARROW [6] and HEATH, JARROW, AND MORTON [4, 5].

The second approach fixes the drift parameter α and the diffusion parameter β under the martingale measure. The corresponding transition probability must be

$$q(\alpha,\beta) = \frac{1}{2} + \frac{\exp(-(\alpha - r)\Delta_t) - \cosh\left(\beta\sqrt{\Delta_t}\right)}{2\sinh\left(\beta\sqrt{\Delta_t}\right)}$$

under the given parameter constellation. This method was proposed by COX, ROSS, AND RUBINSTEIN [2] setting $\alpha = 0$.

The question arises how to determine parameter β. It is often argued that β should correspond to the standard deviations of real world logarithmic returns. However, $\beta = \hat{\sigma}$ implies

$$\sigma = \frac{\beta}{\sqrt{1-\eta_p^2}}\,.$$

This in turn means the process under the risk neutral measure does not reflect trading strategies under the real world measure unless the transition probability equals $\frac{1}{2}$. The problem is that the real world distributional behavior (and hence the standard deviation $\hat{\sigma}$) depends on the real world measure which is typically never specified in these binomial model implementations.

There is a consistent way to specify the parameters of a martingale process for fixed transition probabilities that retains the fundamental principle of replicating payoffs in the real world. The solution is to set $\alpha = \mu$ and $\beta = \sigma$. Under these specifications it is consistent to calculate the derivative on the dynamics of the martingale asset price. Given a drift coefficient α, the diffusion coefficient is determined by (8) to be compatible with predefined distribution parameters. Thus, the coefficient reads

$$\beta = \sqrt{(\hat{\mu}-\alpha)^2\Delta + \hat{\sigma}^2}\,.$$

On the one hand the difference between β and $\hat{\sigma}$ is very small in practice and there seems to be no need for an adjustment, on the other hand this correction is easily done.

Of course, the measure of the real world probabilities have to be adjusted. The probability spread can be computed according to (9) and results in

$$\eta_p = \frac{\hat{\mu}-\alpha}{\sqrt{(\hat{\mu}-\alpha)^2 + \frac{\hat{\sigma}^2}{\Delta_t}}}\,. \qquad (15)$$

6 An Example: An Up-and-Out Binary Option

It is assumed that a money market fund can be purchased promising a fixed interest rate quoted as an instantaneously compounded rate

$$r = 0.06\,.$$

Moreover, the expected logarithmic returns of the stock and their corresponding variances are independent of time and known with certainty. Suppose that the values are given by

$$\hat{\mu} = 0.12$$

and

$$\hat{\sigma}^2 = 0.09\,.$$

As we know from section 3, the stock price process must be of the structure

$$S_{t_n} = \begin{cases} S_{t_0}, & \text{if } t_n = t_0; \\ S_{t_{n-1}} \exp\left(\left(0.12 - \frac{\eta_p}{\sqrt{1-\eta_p^2}\sqrt{\Delta_t}} 0.3\right)\Delta_t + \frac{1}{\sqrt{1-\eta_p^2}} 0.3\sqrt{\Delta_t} X_{t_n}\right), & \text{if } t_0 < t_n \leq t_N. \end{cases}$$

In t_0 the price of one share of the money market fund is normalized to a price of 1, the stock quotes at a price of 20.

Suppose the arbitrage-free price of an up-and-out stock-or-nothing binary option with a hurdle $H = 24$ is to be determined. The contract matures in 0.25 time units. There are three equidistant trading days (excluding the current day, including the maturity day), so $\Delta_t = \frac{1}{12}$. The option's payoff at maturity $C_{t_N}(H)$ is given by

$$C_{t_3}(24) = \begin{cases} S_{t_3}, & S_{t_n} < 24 \; \forall t_n \in \mathfrak{T}; \\ 0, & \text{otherwise.} \end{cases}$$

in this example. The contract does not pay any money before maturity.

6.1 Security Processes Under the Real World Measure P

Let us start with the processes representing the security price behavior in the real world. Suppose that the probability for an up move equals the probability for a down move. These transition probabilities have been assigned to the edges of the tree in figure 1. We have to stress that they are neither necessary to build a replicating trading strategy nor to determine the value of an option using the martingale method.

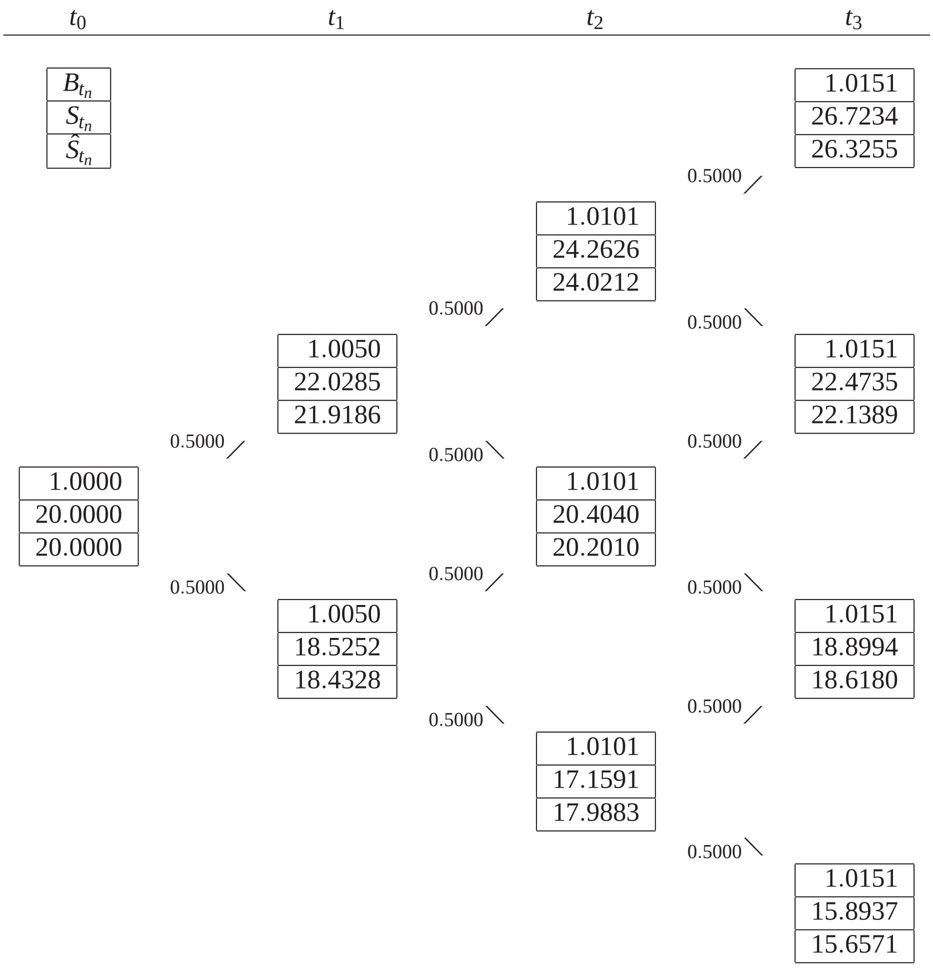

Figure 1: Binomial tree under the real world probability measure

However, the transition probabilities are implicitly determined if the distributional parameters $(\hat{\mu}, \hat{\sigma})$ and the process parameters (μ, σ) are given. Since we wish to have the equality of $\hat{\mu}$ and μ on the one hand and $\hat{\sigma}$ and σ on the other hand we have to choose a transition probability $p = \frac{1}{2}$.

The last entries refer to the stock prices in units of the numeraire. In this case the money market fund has been used as the reference quantity.

6.2 The Replicating Portfolio Strategy

If we assume that the processes formed in the previous subsections reflect the possible outcome of the processes given the real world probability space, then the trading strategy formulated in figure 2 is the unique portfolio process generating the payoff of the binary option.

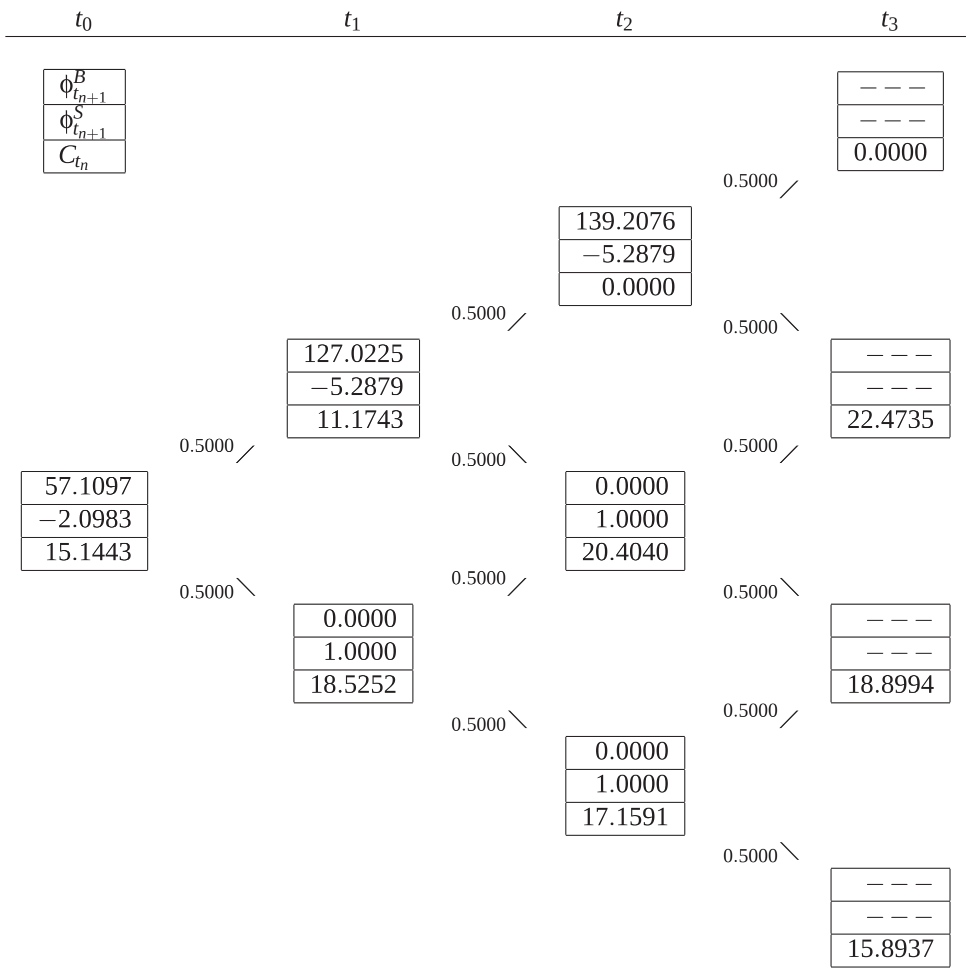

Figure 2: Replicating strategy for the binary option

The trading strategy is determined recursively by the formulae introduced in section 4. If the payoff at maturity is not equal to 0, then it must be equal to the corresponding stock price. The standard procedure is to calculate the value of the option at a certain time independently of the stock price up to this time. The continuation value is replaced by 0 if the stock price at the corresponding node hits or exceeds the hurdle H. This is the case at time t_2 in the uppermost node.

6.3 Security Processes Under the Martingale Measure Q

Now our view will change to the so-called risk neutral world where we consider martingale processes rather than real world processes. The first example keeps the real world processes in mind and models martingales that are consistent with the real world described above. The second example is an application of the CRR model specification.

6.3.1 A Model Based on Real World Processes

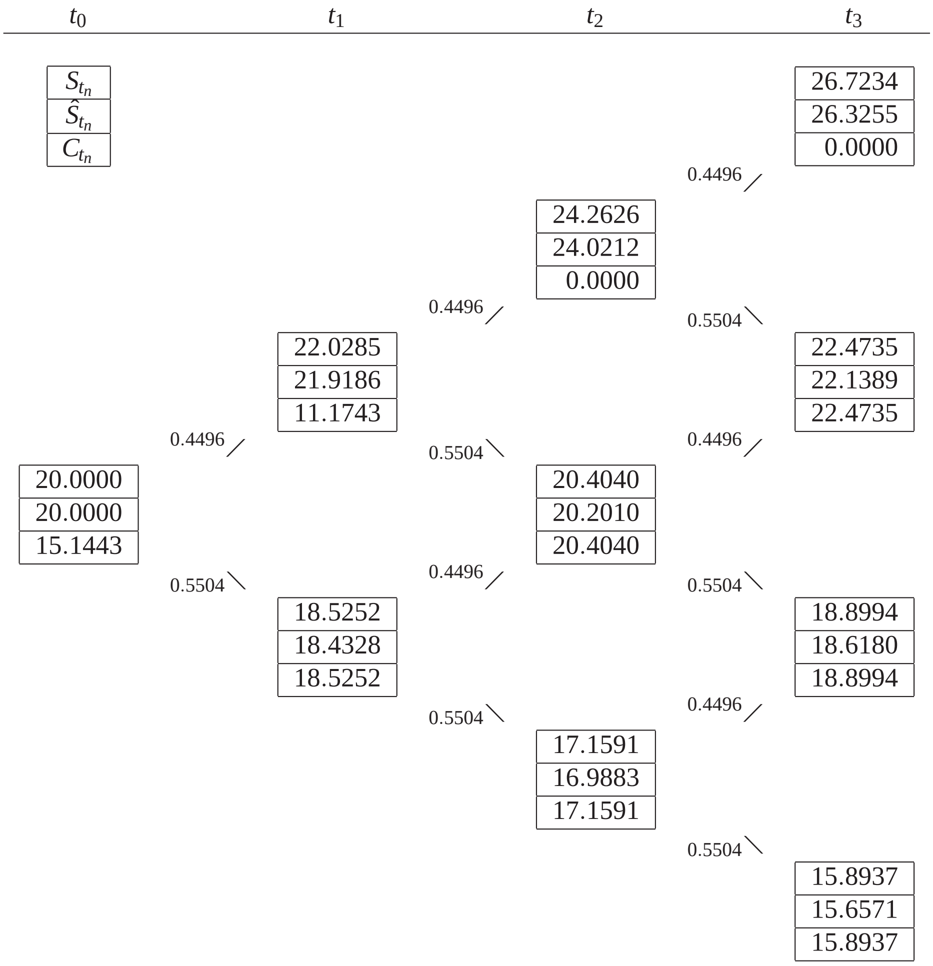

Figure 3: Binomial tree with a drift $\alpha = \mu$

Since the payoff of the derivative for which the price is to be determined depends on the stock price in the real world and not in the risk neutral world, it seems to be a natural approach to retain the dynamics of the real world process, to calculate the derivative payoffs and to change the measure such that the prices in the numeraire are martingales.

Figure 3 shows that the arbitrage-free price process of the derivative is compatible with the price process of the replicating trading strategy in the previous section, especially the prices in t_0 coincide.

6.3.2 The Cox-Ross-Rubinstein Model

Several proposals have been made in the financial literature how to implement binomial option pricing models. The most famous model has been developed by COX, ROSS, AND RUBINSTEIN [2]. The standard way to calibrate this model to the real world data of our example is to set $\alpha = 0$ and

$$\beta = \sigma = 0.3 .$$

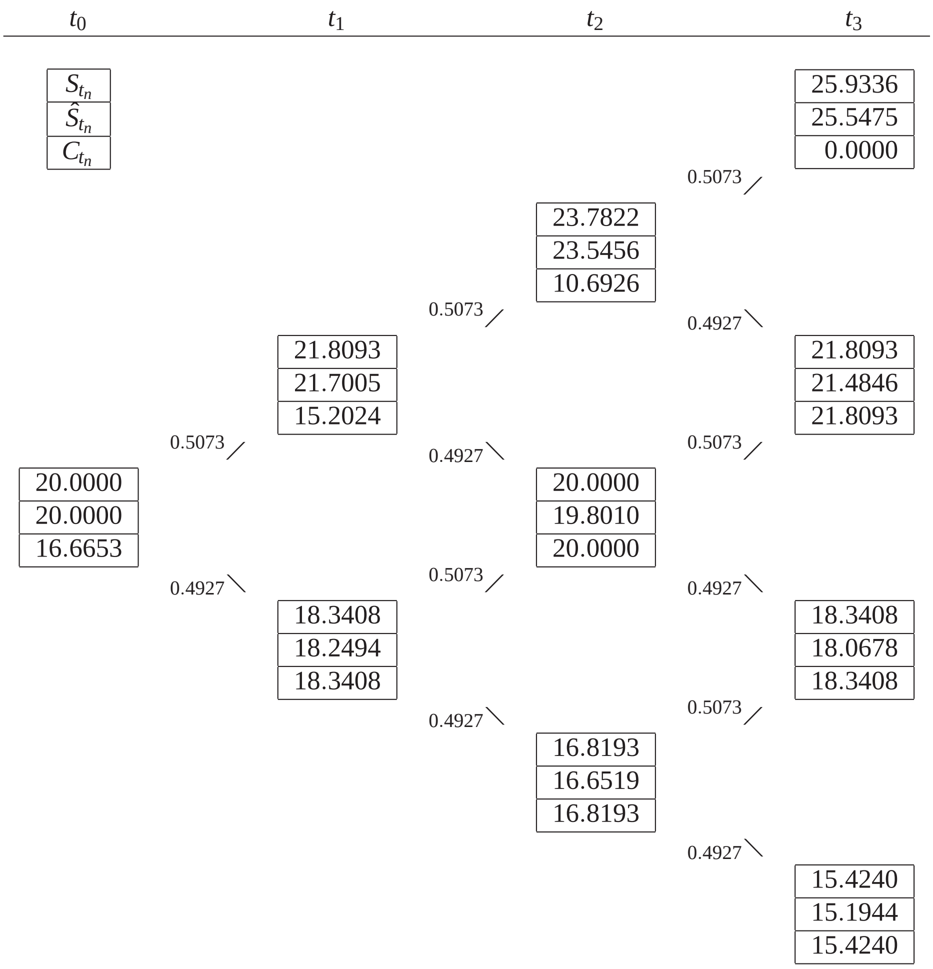

Figure 4: Binomial tree with a drift $\alpha = 0$

This yields a probability q under the martingale measure of

$$q(0,0.3) = \frac{1}{2} + \frac{\exp\left(\frac{0.06}{12}\right) - \cosh\left(0.3\sqrt{\frac{1}{12}}\right)}{2\sinh\left(0.3\sqrt{\frac{1}{12}}\right)} \approx 0.5073\,.$$

Note that this process does not reflect the trading strategy of section 4. The arbitrage-free price of the barrier option differs significantly from the price of the replicating strategy. There is a remarkable difference at the uppermost node at time t_2. Whereas the option value is strictly positive in the CRR model, it is 0 in this state at the same time in the preceding model. Given our real world process the option value must not have a positive value since the price of the underlying is 24.2626 and hence quoting above the hurdle which makes the option worthless. The CRR model has taken the martingale process multiplied by B_{t_2} to check if the stock price has hit or exceeded the hurdle. But this process has nothing in common with our real world processes which are responsible for the derivative payoff. Thus, if the CRR model does not represent our real world process, then what does it represent?

7 Interpreting the Approaches

We have to be careful when interpreting the results of the last sections. At a first glance the model seems to be inadequate to reflect the trading strategies based on real world processes. However, such a conclusion would be overhasty. The right formulation is that there exists a real world process associated with a real world probability measure such that trading strategies based on these processes are reflected by the CRR model.

If we use the stock process having a drift parameter α and a diffusion parameter β to determine the payoff of a derivative contract, then these parameters are not only the coefficients under the martingale measure Q but also under the real world measure P. Now, we can determine the undistorted diffusion coefficient beta, which is given by

$$\begin{aligned}\beta &= \sqrt{(\hat{\mu} - \alpha)\,\Delta_t + \hat{\sigma}^2} \\ &= \sqrt{(0.12 - 0)\cdot\tfrac{1}{12} + 0.09} = 0.3020\,.\end{aligned}$$

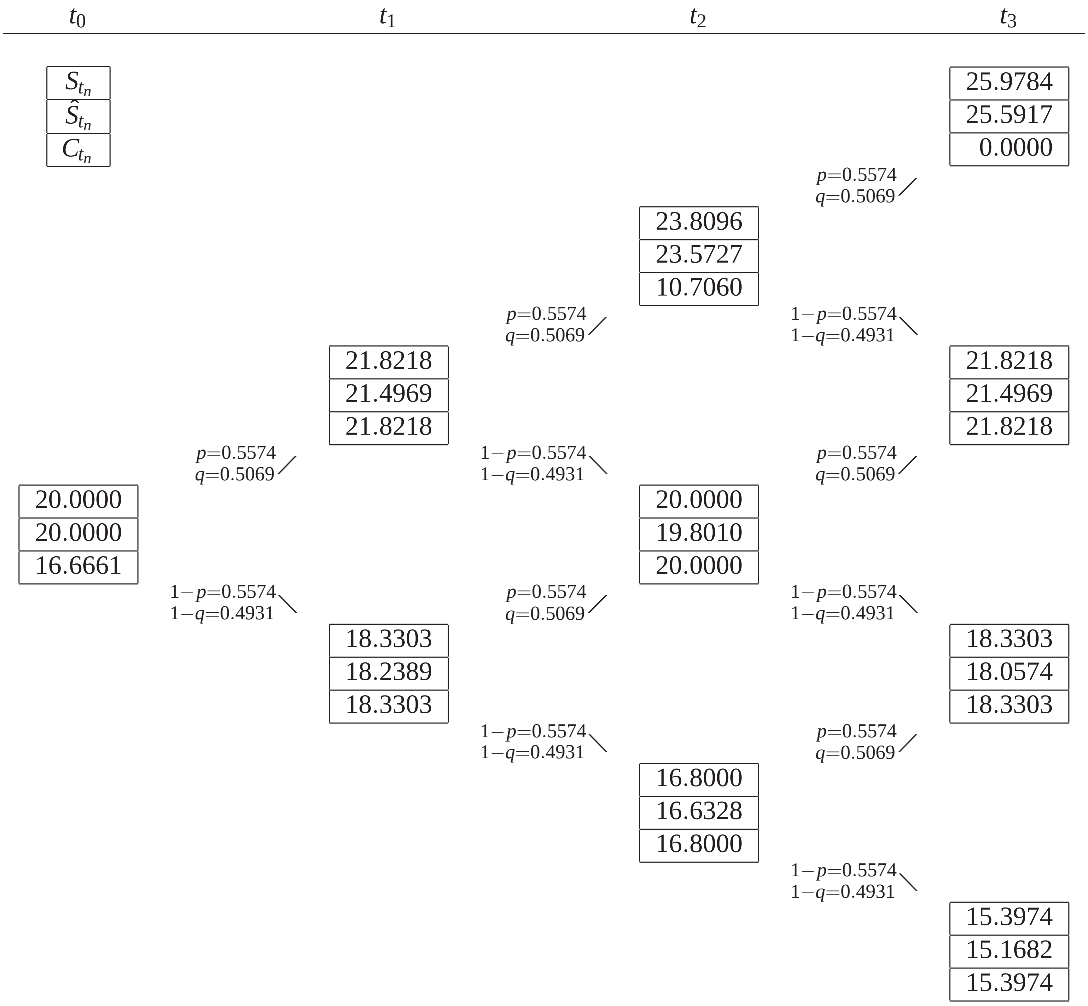

Figure 5: A consistent CRR binomial tree

The deviation from the real world diffusion parameter $\hat{\sigma}$ is very small, even though the time discretization is rather coarse. The more interesting result is that we can identify in straightforward way the real world process for which the replicating trading strategy is reflected by martingales in the CRR model. The probability spread is given by

$$\eta_p = \frac{\hat{\mu} - \alpha}{\sqrt{(\hat{\mu} - \alpha)^2 + \frac{\hat{\sigma}^2}{\Delta_t}}} = \frac{0.12}{\sqrt{0.12 + 0.09 \cdot 12}} = 0.1147\,,$$

and henceforth the transition probability by

$$p = 0.5 + 0.5 \cdot \eta_p = 0.5574\,.$$

To summarize, the CRR binomial process in figure 5 satisfies three properties: First, all prices in units of the numeraire are martingales. Secondly, the logarithmic returns of the price processes have the predetermined distributional parameters. Thirdly, the processes under the martingale measure reflect replicating trading strategies since the process parameters in both worlds coincide. Hence, the model is consistent in the above mentioned sense.

8 Concluding Remarks

This paper tries to clarify the relation between distributional parameters in the real world, namely the expected value and variance of logarithmic returns, the process parameters in the real world and the process parameters in the risk neutral world.

There is a continuum of process parameters (μ, σ) and transition probabilities p that result in a process with given distributional parameters $(\hat{\mu}, \hat{\sigma})$. The exact relations have been shown in section 3. If we fix the process parameters and the transition probabilities (probably with certain distributional parameters in mind), then the trading strategy of a certain derivative payoff structure is uniquely determined. Furthermore, there is a one-to-one relation between the number of assets held in this strategy and transition probabilities under the martingale measure. Thus, the risk-neutral world is determined as well.

The usual procedure is to model the other way round. The starting point is the stock process in units of a numeraire which is required to be a martingale. However, if the calculation of the derivative payoff is based on this process, then the real world process must have the same structure. In other words, the process parameters of the real world and the risk neutral world have been determined simultaneously, the real world model can then be completed by identifying the probability measure that yields the actual distributional parameters. In general, this means that the martingale diffusion parameter can only be determined after specifying the consistent real world model. However, the distortion that arises if we set the risk neutral diffusion parameter to the standard deviation of the logarithmic returns will be negligibly small in nearly all applications.

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