SONICFUNCTION: EXPERIMENTS WITH A FUNCTION BROWSER FOR THE VISUALLY IMPAIRED

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ABSTRACT

We present in this paper `SonicFunction`, a prototype for the interactive sonification of mathematical functions. Since many approaches to represent mathematical functions as auditory graphs exist already, we introduce in `SonicFunction` three new aspects related to sound design. Firstly, `SonicFunction` features a hybrid approach of discrete and continuous sonification of the function values $f(x)$. Secondly, the sonification includes information about the derivative of the function. Thirdly, `SonicFunction` includes information about the sign of the function value $f(x)$ within the timbre of the sonification and leaves the auditory graph context free for an acoustic representation of the bounding box. We discuss `SonicFunction` within the context of existing function sonifications, and report the results from an evaluation of the program with 14 partially sighted and blind students.

1. INTRODUCTION

Teaching material for the blind and partially sighted in mathematics is generally tactile using Braille notation or reliefs. When it comes to function analysis, this form of notation has some limitations. One of which is its involved method of production, which is the reason why appropriate teaching material is limited.

Due to the fact that the blind and partially sighted often posses heightened auditory capacities, there have been occasional efforts to develop auditory displays for teaching mathematics. In sonification, a big amount of research has been conducted on auditory graphs. Foundational work was laid in [1] and good overviews over the field can be found in [2] and [3]. In [4] there is an interesting study that contrasts the difference between discrete and continuous auditory graphs, giving evidence that both representation modes serve different purposes. Related work can further be found in [5]. A conceptual model of auditory graph comprehension can be found in [6], where particularly the consideration on the context information in a graph, i.e. axes and their +/- orientation, are relevant for us.

While the results from this field provides a good basis for the development of auditory displays for mathematical functions of one variable $f(x)$, we believe that there are still possibilities for further improvement of the sound design. This is important, first because new concepts, that illustrate how to include information in the sonification of $f(x)$ rather than putting it into the auditory context, extends the usefulness of auditory graphs. Second auditory rich and yet distinguishable information is usually more interesting to listen to, and hence user fatigue can be reduced. Third, in many of the studies above auditory graphs had limited interaction possibilities. However as stated in [7] interaction introduces new and exciting possibilities for a better understanding of sonification in general and also does so for auditory graphs in particular. Particularly questions about continuous and discrete sonifications for mathematical functions must be revisited with respect to new possibilities in interactive sonification.

As a new methodological contribution to the field of function sonification, we here introduce the idea of multi-parameter sonification of mathematical functions, which goes beyond the existing pitch mapping-based strategies in the aspect that they utilize the Taylor expansion of function f at location x as source for a stationary sonic representation. More specifically, we suggest to map the first m terms of the Taylor series $(f(x), f'(x), \ldots, f^{(n)}(x))$ at location x as *fingerprint* for the local characteristics of the function and derive a corresponding sonic counterpart.

Depending on the mapping, the main association of $f(x)$ to pitch can be maintained, but be extended to reflect slope for instance as pulse rate, curvature $f''(x)$ as attack time of events or whatever mapping seems appropriate. Since these attributes change systematically while traversing along the x-axis, a sort of recognizable sequenced auditory gestalt builds up when walking towards specific points of interest such as turning points, saddle points or local optima.

In the current empirical study, however, we adapted and restricted this more general sonification approach specifically to fit to the subject group. Second author Trixi Drossard who has a background as math teacher for the partially sighted and the blind conducted the study with pupils. We wanted to evaluate our sonification strategy with blind pupils from the very beginning and we were less interested to evaluate if pupils recognize already learned features of mathematical functions but more in whether sonic function representations work in a teaching situation. This imposed several constraints to the general multivariate representation concept so that in consequence we included only the first derivative, since the concept of higher derivatives are difficult to grasp and not part of the curriculum for pupils of the age of our test subjects.

One important application featuring interactive sonified graphs is the java program MathTrax, [8] which has been adapted to typical requirements for the blind and partially sighted, it works for instance together with screen readers and features shortcuts and hot-keys for efficient navigation. MathTrax presents visual, acoustic and descriptive information about mathematical functions. However, since we were interested in new sonification designs, we decided to implement our own minimal prototype called `SonicFunction`.

2. THE PROGRAM SONICFUNCTION

The program `SonicFunction` is implemented in python and Tcl/Tk for a minimal user interface and the calculations of the mathematical functions. Open Sound Control (OSC) [9] [10] provides the protocol to send the parameters to the `SuperCollider` soundserver [11] available at [12]. As an input device we decided to use the keyboard since it is a very familiar interface for the visually impaired. The user can interact with the program through the following keys:

- The arrow keys up and down control the volume of the sonification. This is important to adapt it to the volume of screen readers, such as Jaws for Windows.
- The arrow keys left and right allow to navigate on the x-axis. If the arrow keys were constantly pressed, the function can be browsed in a constant movement from left to right.
- The keys x,c and v set the step size for the navigation on the x-axis to $^1/_{30}$, $^1/_{10}$ and $^1/_6$ respectively. This allows for a quick overview of the function and for detailed inspection.
- The number keys 1 to 6 are the selectors for the test functions, which are described below in detail.
- By hitting the keys h, t, n and a markers for maxima, minima, f(x) = 0 and x = 0 respectively are registered in a protocol file.

While navigating the function on the x-axis the sonification was presented on the corresponding position within the stereo panorama. The interaction for placing markers was included since we wanted to evaluated the sonified function, by recording and analyzing user interaction.

3. DIFFERENCES TO MATHTRAX

As mentioned above, the program MathTrax is a popular reference for function sonification. In `SonicFunction` we try to to include the information that is connected to the function within the acoustic representation of the function itself. In this section, we want to highlight the differences in sound design between `SonicFunction` and MathTrax.

For the distinction of positive and negative function values, MathTrax employs for instance the auditory context by adding a constant level of noise. `SonicFunction` integrates this information within the sound that represents the function value of $f(x)$, by changing its timbre. Thereby leaving the context of the auditory scene free for an acoustic equivalent of a bounding box.

By choosing two noise sources with different center frequencies, this bounding box also helps to indicate, whether the $f(x)$ is currently beyond the upper or lower limit of the bounding box. This is helpful for approximate extrapolation before sounding function values within the box are encountered.

`SonicFunction` also makes use of the derivative as a parameter for for the sonification. This is important to support the exploration around minima and maxima.

`SonicFunction` also combined two sonification approaches using continuous and discrete acoustic representations. Thereby the discrete sonification event is used to give an appropriate feedback for the stepwise interaction when moving along the x-axis. The continuous standing sound that goes with a ramp from one function value to the next emphasizes the dense distribution of real numbers on the x-axis.

4. SOUND DESIGN

As mentioned above the interaction feedback was provided by a discrete sonification, whereas the continuity of the function was represented through a continuous sonification. Examples of the sonifications of all test functions can be found on our website [1].

4.1. The Discrete Sonification

The discrete sonification was played each time the user moved along the x-axis one step. In Figure 1 you find `SuperCollider` code for the synthesis definition of the discrete sonification.

```
SynthDef(\discrete,
  { arg out=0, midinote = 60, pan = 0.0, delay = 0.1,
        duration = 3.5, vol = 0.1, bwf = 1;
    var klank, harm, amp, ring, filter, lfo,
        noise_source, env, freq, sig;
  freq = midinote.midicps;
  harm = Control.names([\harm]).kr([1,2,3,4,5,6,7,8,9]);
  amp = Control.names([\amp]).kr(Array.series(9,1,1).reverse.normalizeSum);
  ring = Control.names([\ring]).kr(Array.fill(9,100.0));
  noise_source =
    EnvGen.ar(Env.new([0,0,1,0],[delay,0.0,duration],-3),1.0,doneAction:0) +
    (EnvGen.ar(Env.new([0,0,1,0],[delay,0.0,duration/20],-3),1.0,doneAction:0)
    * ClipNoise.ar(1));
  env = EnvGen.ar( Env.new([0.0,1.0,0.0],[0.0,delay+0.5],-3),1, doneAction:2);
  klank = Klank.ar(`[harm, amp, ring], noise_source, freq);
  sig = LPF.ar(klank, freq*bwf);
  OffsetOut.ar(0,Pan2.ar(sig*env*AmpComp.kr(freq,40.midicps), pan, vol));
  }  ).load(s);
```

Figure 1: The `SuperCollider` synthesis definition for the discrete sonification.

The sonification was essentially a sound made of subtractive synthesis (the unit generator `Klank.ar`) with a base frequency and a series of overtones of decaying gain. The frequency of the base frequency covered the range from 46,25 to 698,46 Hz, (approx. 4 octaves). The considerably low range was chosen to have enough overhead in the spectrum for the 9 overtones, which helped to identify the pitch of the sound even for low base frequencies.

The excitation of the `Klank` filter was an attack decay envelope with a noise component in the attack phase. The filtered sound was multiplied with an envelope which also had an attack decay characteristic.

The sound was played after a delay, that allowed the continuous sonification to ramp to the target frequency. The discrete sounds were played back within the stereo panorama corresponding to the actual position on the x axis within the bounding box. Basic psychoacoustic amplitude compensation `AmpComp.kr` was additionally implemented.

4.2. The Continuous Sonification and the Derivative

With respect to spectral characteristics, the continuous sonification resembled very much the discrete sonification, except it was implemented as additive synthesis using the unit generator `Klang.ar`.

[1] `http://www.techfak.uni-bielefeld.de/ags/ami/publications/GDH2010-SEW/`

In Figure 2 you find the corresponding `SuperCollider` synthesis definition.

```
SynthDef(\continuous,
  { arg out=0, midinote = 60, pan = 0.0, lg = 0.1,
        vol = 0.1, bwf = 1, modf = 5, moda = 0.1;
  var klank, harm, amp, phase, freq, sig;
  freq = midinote.midicps;
  harm = Control.names([\harm]).kr([1,2,3,4,5,6,7,8,9]);
  amp = Control.names([\amp]).kr(Array.fill(9,{1}));
  phase = Control.names([\pi]).kr( Array.geom(9,1,9).reverse.normalize);
  klank = DynKlang.ar(
        `[harm.lag(0.1)*freq.lag(0.1),amp.lag(0.1),phase.lag(0.1)]);
  sig = LPF.ar(klank, freq*bwf);
  OffsetOut.ar(0,
              Pan2.ar(sig * AmpComp.kr(freq.lag(0.1), 40.midicps),
              pan,
              vol * SinOsc.kr(modf,0,moda,1)  ));
  }).load(s);
```

Figure 2: The `SuperCollider` synthesis definition for the continuous sonification.

The continuous sonification was also the carrier of the information about the derivative $f(x)/dx$ which was mapped to an Amplitude oscillation, where the oscillation of the amplitude approached 0 if the derivative approached 0. You find the corresponding implementation detail in Figure 2 as `SinOsc.ar(modf,0,moda,1)`.

4.3. The Difference between Positive and Negative $f(x)$

For the distinction between positive and negative function values $f(x)$, the sound was send through a 2nd order Butterworth lowpass filter, `LPF.ar`, that allowed to control the brightness. By controlling the cutoff frequency (5 or 2.5 times the base frequency) two different brightness modes were selected, with the brighter one indicating positive function values.

4.4. The Acoustic Boundig Box

For the upper and the lower limit of the bounding box noise was send through a band pass filter (BPF). The metaphor of upper and lower was mapped to high and low for the center frequency of the BPF. The center frequency for the upper limit was set to 5000, and for the lower 200 Hz. The noise source was played back on the actual x position within the stereo panorama. The left and right bounding box limit was indicated through noise played back on the corresponding stereo channels. For all the functions the bounding box was set from -10 - 10 in x and -5 to 5 for $f(x)$.

We think that the acoustic bounding box is particularly instructive at singularities, where the function graph would first have a ascending frequency, then it would audibly cross the upper bounding limit, then at the singularity the center-frequency would change to low and finally the function is audible again at low frequencies.

4.5. Clicks as Tick-Marks on the x-Axis

In order to indicate tick-marks at each integer value on the x-axis, simple clicks were used. They were synthesized through short envelopes over an additive synthesis of 4 overtones with a base frequency of 1.000 Hz . The tick-marks were played back on the stereo panorama according to their position. The tick-mark at the position $x = 0$ was highlighted by n elevated base frequency of 1.600 Hz.

5. THE EXPERIMENT

Fourteen (7 female, 7 male) blind and partially sighted German students from the age range 17-19 participated in the study. Seven participants were blind, four were partially sighted, and three high-grade partially sighted, as stated by the participants themselves. For eleven of the participants their vision was constantly restricted or absent since their birth. Two of the participants with strongly restricted vision and one blind participants reported a degradation of their vision over the years.

Figure 3: Photo from the experiment: the test subject sits in the foreground on the right following the instructions by coauthor — ________

The experiment was conducted with each student individually in a quiet room in order to avoid acoustic disturbance. The assisting conductor of the study, coauthor ________ instructed the students how to use the program `SonicFunction`. For the acoustic display, regular headphones were used.

During the instruction period the students were encouraged to ask the instructor about the meaning of the sounds and the possibilities of interacting with `SonicFunction`. The instructor made sure that all acoustic features relevant for the tasks were understood.

The participants were browsing a selected function and reported verbally what kind of features they encountered. Each time they reported minima, maxima or values for $x = 0$ or $f(x) = 0$, the conductor marked the finding on the keyboard and the data were recorded in a file.

The students were also asked to guess and describe with words, what kind of function they thought they heard. At the end of the experiment they were asked to give feedback about the program `SonicFunction`. The participants were further asked what kind of learning type they are (visual, auditory or haptic), according to their preferences for learning most efectively. From the statements we could concloude, that ten students are visual learners while the other four are auditory learners.

6. TYPICAL FUNCTIONS AS TEST CASES

The following functions eq. 1 - eq. 6 were selected as test cases for the participants. The choice was primarily motivated by pedagog-

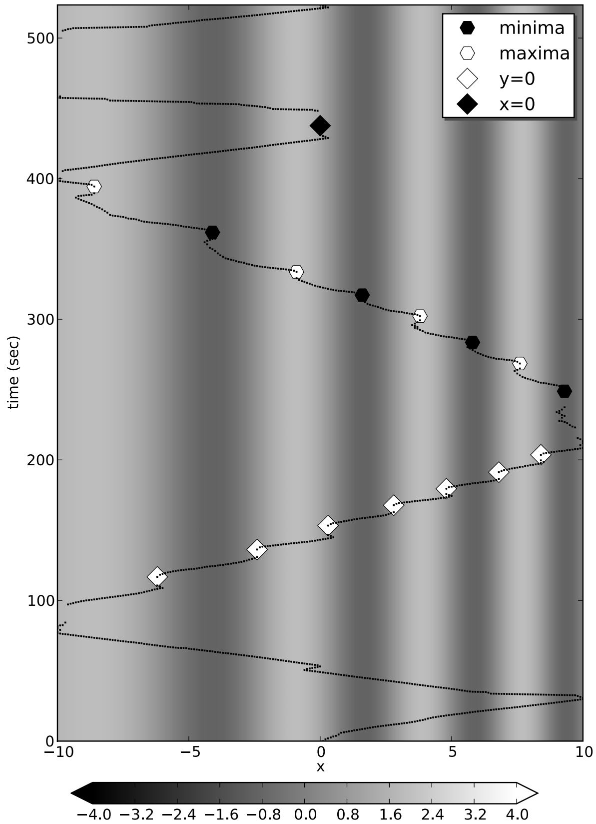

Figure 4: Example of a typical exploration of $f_5(x)$. The function values $f_5(x)$ are encoded in grey. The participant started in the middle and explored the function to both limits of the bounding box. Then $f(x) = 0$, further extrema and finally, $x = 0$ were marked.

ical aspects with regards to function analysis.

$$
\begin{aligned}
f_1(x) &= {}^3/_4\,(t+1)^2 - 2 && (1)\\
f_2(x) &= 2t + 3 && (2)\\
f_3(x) &= t^2 + 1 && (3)\\
f_4(x) &= 0.5/t && (4)\\
f_5(x) &= \sin((0.2t+3)^2)1.5 && (5)\\
f_6(x) &= {}^1/_{\sin(t)} && (6)
\end{aligned}
$$

The function from eq. 1 was selected to introduce the test-subjects to all the audible features of the auditory function graph. The values for x and $f(x)$ cover positive and negative values. Hence the test-subject hears the click for $x = 0$ and the change in timbre at the transition from negative to positive function values $f(x) = 0$. The minimum at $x = -1$ makes the LFO oscillation of the base frequency audible, which is controlled by the derivative $df_1(x)/dx$.

The second function eq. 2 was used to verify if the test subjects had understood the concept $x = 0$ as well as the concept of $f(x) = 0$ at $x = -3$.

In the third function, the symmetric parabola from eq. 3, test subjects were asked to identify the minimum and the position with $x = 0$.

With including function 4 we wanted to find out if test-subjects were able to make sense of an acoustically represented singularity.

Function 5 was included because we were interested if and how the precision of the extrema identification depends on the curvature i.e. the acoustic contrast around $df(x)/dx = 0$.

By including function 6 we wanted to find out how the concept of minima and maxima is perceived between singularities. These extrema are located at $\pi/2 \cdot m$ with $m \in | -5, -3 - 1, 1, 3, 5|$.

The test case functions together with the recorded markers for $f(x) = 0$ and $x = 0$ can be found in Figure 5, the markers for minima and maxima in Figure 6.

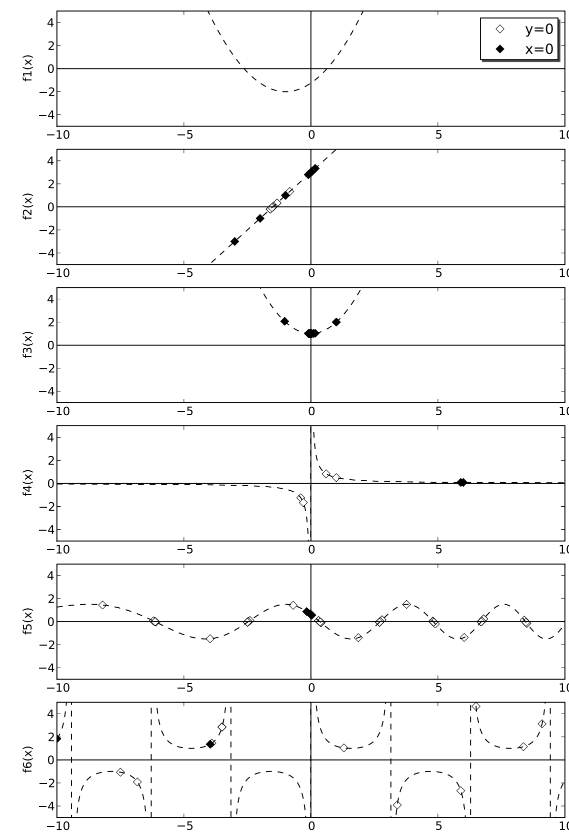

Figure 5: The test case functions with the $f(x) = 0$ and $x = 0$ markers

6.1. Discussion of Figure 5 and 6

6.1.1. Markers for $x = 0$ and $f(x) = 0$ in Figure 5:

1. $f_1(x)$ shows no markers since its sole purpose was to instruct the participants.
2. $f_2(x)$ shows that most of the markers were placed around $x = 0$ and $f(x) = 0$. There were outliers for $x = 0$. It seems that ordinary tick-marks on the x-axis were believed to be the distict tick-mark at $x = 0$.

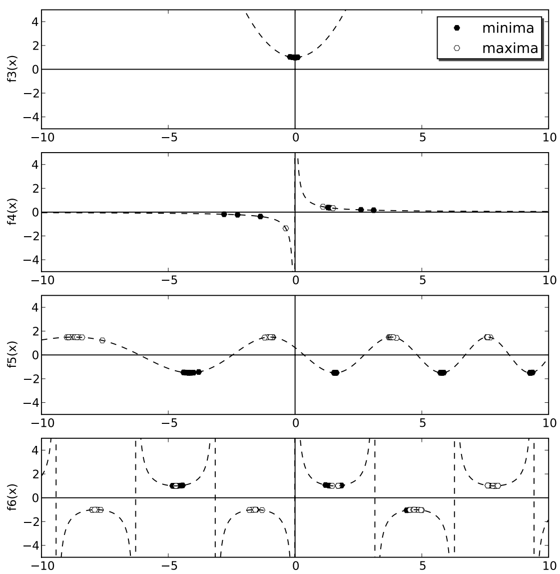

Figure 6: The test case functions with the minima and maxima markers

3. $f_3(x)$, here most of the markers have been placed at $x = 0$, again two outliers are found, which suggest similar problems as in $f_2(x)$.
4. $f_4(x)$ was a real challenge for the participants since none of the concepts $x = 0$ or $f(x) = 0$ were explicitly present. Interestingly some markers were placed approximately where the function has the strongest curvature.
5. $f_5(x)$ shows that most of the participants became familiar with the sonification and identified well the tested position except one person that marked the extrema as $f(x) = 0$.
6. $f_6(x)$ was a similar challenge as $f_4(x)$, and no particular pattern in the positioning of the markers can be found.

6.1.2. Markers for minima and maxima in Figure 5 :

1. $f_3(x)$ the minimum was well identified by all participants.
2. $f_4(x)$ some minima were wrongly identified were the function approached the x-axis.
3. $f_5(x)$ minima and maxima were well identified. note the broader distribution at extrema with lower curvature.
4. $f_6(x)$ minima and maxima were identified however the concept of both was confused.

6.2. A closer look on function $f_5(x)$

For the evaluation of the questions regarding $f_5(x)$ the first and the second derivative was calculated as in eq. 7 and eq. 8 respectively.

$$\frac{df_5(x)}{dx} = \frac{3}{25}\,(15+x)\,\cos(\left(3+\frac{x}{5}\right)^2) \tag{7}$$

$$\frac{df_5(x)}{dx^2} = \frac{-3}{625}\,\left(-25\,\cos(\left(3+\tfrac{x}{5}\right)^2) + 2\,(15+x)^2\,\sin(\left(3+\tfrac{x}{5}\right)^2)\right) \tag{8}$$

By using numerical methods[2] to solve the equation $\frac{f_5(x)}{dx} = 0$, values for x were obtained within the interval from -10 to 10. Those values together with corresponding curvature are compiled in Table 1.

		$f_5(x)/dx = 0$	$f_5(x_i)/dx^2$
x_1	max	-8.733	-0.377
x_2	min	-4.146	1.131
x_3	max	-0.988	-1.885
x_4	min	1.580	2.639
x_5	max	3.799	-3.393
x_6	min	5.784	4.147
x_7	max	7.594	-4.901
x_8	min	9.270	5.655

Table 1: extrema and curvature values for $f_5(x)$

7. STATISTICS OF THE MARKER DISTRIBUTION

We calculated for some of the interesting cases the mean value and the standard deviation for the marker distribution. The results are compiled in Table 2.

fuction	marker	position	numeric value	mean	standard deviation
$f_2(x)$					
	x_0	0.0	0.0	-0.378	0.834
	y_0	$-3/2$	-1.5	-1.448	0.188
$f_3(x)$					
	min	0.0	0.0	5.3^{-15}	0.076
	x_0	0.0	0.0	4.4^{-3}	0.376
$f_5(x)$					
	max		-8.733	-8.640	0.377
	min		-4.146	-4.166	0.165
	max		-0.988	-0.993	0.106
	min		1.580	1.585	0.028
	max		3.799	3.806	0.077
	min		5.784	5.792	0.038
	max		7.594	7.604	0.042
	min		9.270	9.295	0.033
$f_6(x)$					
	min	$-5\pi/2$	-7.854	-7.862	0.112
	max	$-3\pi/2$	-4.712	-4.666	0.132
	min	$-\pi/2$	-1.571	-1.604	0.131
	max	$\pi/2$	1.571	1.566	0.208
	min	$3\pi/2$	4.712	4.710	0.190
	max	$5\pi/2$	7.853	7.830	0.151

Table 2: Results for the mean value and standard deviation for some of the markers in $f_2(x)$, $f_3(x)$, $f_5(x)$ and $f_6(x)$

The high values for the standard-deviation of x_0 for $f_2(x)$ and for $f_3(x)$ are due to the outliers as discussed in 6.1.1. The distribution of the markers around the maxima and minima of $f_6(x)$, which were all treated as extrema, is quite uniform. The function

[2] such as damped Newton's Method, as implemented in the software package Mathematica

$f_5(x)$ is an interesting case. Here we can see how the standard-deviation of the extrema decreases as we go along the x-axis from left to right. This seems to correspond to the increasing absolute curvature of the extrema in Table 1. In Figure 7 a correlation plot of the absolute value of the curvature against the standard-deviation $\sigma$and also against the standard deviation of $(x_i^k - x_0^k)$ denoted as and $\hat{\sigma}$ with x_0^k being the exact position of the extremum can be found.

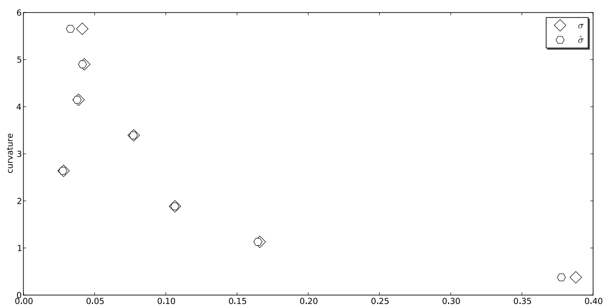

Figure 7: curvature versus the σ and $\hat{\sigma}$. It can be seen that low curvature tends to go with a broader distribution of clicks around the position of the function extrema

	correlation coefficient	p-value
$\|curvature\|$ versus σ	-0.7381	0.0366
$\|curvature\|$ versus $\hat{\sigma}$	-0.6905	0.0580

Table 3: Results from the Spearman rank correlation test

In Table 3 you find the results of the Spearman rank correlation coefficient and the two-sided p-value for a hypothesis test whose null hypothesis is that the two sets of data are uncorrelated. If we accept as a threshold for significance of 5% only the correlation with σ is below. None the less, we think that given the low amount of data (8 extrema), a general correlation between the curvature i.e. the acoustic contrast and the precision with which extrema are identified, can be established.

8. DISCUSSION

Looking at the results from the analysis of the markers set by the participants, we need to take into account that some of the mathematical concepts that were tested had maybe not been properly understood. One example is the misunderstanding of the definition of minima and maxima in function f_6. Their confusion in f_6 might be explained by the fact that the function values for maxima and hence their corresponding pitch was lower than the one for minima. Maximum and minimum seems to have been related to the absolute function value at $df_6/dx = 0$ and not to the sign of the curvature at that point.

However the evaluation of function f_5 lead to interesting insights. The precision with which extrema can be localized depends on the acoustic contrast i.e. the curvature around the extremum.

If we quickly summarize what the participants reported verbally about function f_5, none of them reported explicitly the increase of frequency while exploring $f_5(x)$ along the x-axis. In brief the participants said that the function appears as ”something sinus like”. This is explainable since the functions were all explored interactively and the progression along the x-axis was not necessarily constant. Therefore the change in the frequency of oscillations between the extrema have not been perceived or interpreted by the participants.

9. CONCLUSION

The evaluation of SonicFunction should be considered as a preliminary study of sonified graphs in a real-world teaching situation with partially sighted. From the experience of using SoniFunction in school, we can conclude that particularly for students who are either strongly partially sighted and use media specific for the blind or who are primarily an auditory learning type, the sonified functions are very supportive to grasp important characteristics of a mathematical function. The auditory graphs are especially well suited to be an alternative offer for strongly partially sighted or for students who are in the in the process of loosing sight. In these cases the sense of touch is not yet differentiated enough and those people often cannot handle braille yet. However auditory graphs are not meant to be a replacement for tactile graphs, but rather an addition to them to facilitated understanding for different learning types.

As far as the sonification design is concerned, we can not yet prove nor measure its utility, and the experimental results do not permit to compare our design to other sonification designs. This is mostly due to the heterogeneous population of our test subjects in terms of the restriction of their eyesight. Furthermore, the pupils have initially not been familiar with the idea of acoustic representation so that this was already a challenge and novelty, although it was generally much appreciated.

For future studies we therefore consider two directions: (a) using different sonification designs according to our new Taylor-based multi-parameter mapping concept with subjects that are already familiar with the mathematical background, e.g. math students, and (b) testing the winning design in a longitudinal study together with pupils who learn mathematical functions with the aid of sonification.

From our experience so far we found that the strategy to move information from the context to the sonification itself is promising: the integration of the transition from negative to positive function values as timbre filter leaves the noise stream available for the bounding box information. The successful integration of derivatives into the sonification is particularly important in case of interactive exploration where it is not assured that the user receives a proper overview over the progression of the function along the x-axis with a constant rate and therefore has more challenges to deduce information such as the curvature.

In summary, SonicFunction introduced a new mapping rationale and demonstrated hand-tuned contextual elements for the auditory display of mathematical functions for the visually impaired. We plan to address the open questions in our future research.

10. ACKNOWLEDGMENT

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in the study.

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