

# Optimal Monetary Policy Rules: Theory and Estimation for OECD Countries

Inaugural Dissertation for a Doctoral Degree  
of the Faculty of Economics  
at the University of Bielefeld

submitted by

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Bielefeld, March 2004

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# Acknowledgement

I wish to thank the Faculty of Economics at the University of Bielefeld for the opportunity to write this dissertation and for the facilities which enabled me to do so.

I am very grateful to my supervisor, Professor Willi Semmler, for his kind and patient supervision of my dissertation. It is he who led me into the field of dynamic macroeconomics and economic policy and encouraged me to further my research, especially in the area of monetary policy. His broad knowledge of economics has shown me that economics is not only a branch of science, but also a world of wonder, which makes life more beautiful.

I am grateful to my second supervisor, Professor Peter Flaschel, not only for his advice on my dissertation, but also for his observations and instruction on dynamic systems.

I would like to thank the members of IMW in the University of Bielefeld, especially Professor Walter Trockel, for his encouragement and guidance. I am also grateful to Professor Gerd Ronning at the University of Tuebingen for his guidance and support. I thank Professor Lars Gruene for his instruction on the dynamic programming algorithm, and also Wolfgang Lemke and Chih-Ying Hsiao, for their friendship and valuable discussions on mathematical problems. I also would like to thank Professor Wenda Hong at Fudan University in China for his kind support. I am grateful to my friends in the US, Helen Ogden and Rick McGarrity, not only for their corrections in English, but, more importantly, for their more than ten years of encouraging friendship. I also wish to thank Miss Gaby Windhorst for her kind hospitality.

Last, but not least, I thank my family for their unflagging support and also my friends in China and Germany, Dr. Fulin Wu, Dr. Shaoze Hou, Dr. Shuncheng Mao, Dr Ningning Liu, and Mark Hahmeier, for example, for their friendship and encouragement.

# Contents

<b>Acknowledgement</b>	<b>ii</b>
<b>1 Introduction</b>	<b>1</b>
<b>2 Empirical Evidence of the IS and Phillips Curves</b>	<b>10</b>
2.1 Introduction . . . . .	10
2.2 The IS and Phillips Curves with Backward-Looking Behaviors	13
2.3 The IS and Phillips Curves with Forward-Looking Behaviors .	16
2.4 Time-Varying Phillips Curve . . . . .	25
2.5 Conclusion . . . . .	32
<b>3 Monetary Policy and Interest-Rate Rules</b>	<b>43</b>
3.1 Introduction . . . . .	43
3.2 The Money-Supply Rule . . . . .	44
3.3 The Interest-Rate Rules . . . . .	47
3.4 Conclusion . . . . .	62
<b>4 Time-Varying Monetary Policy Rules</b>	<b>64</b>
4.1 Introduction . . . . .	64
4.2 The OLS Regression and Chow Break-Point Tests of the Interest- Rate Rule . . . . .	65
4.3 Estimation of the Time-Varying Interest-Rate Rule with the Kalman Filter . . . . .	76
4.4 Euro-Area Monetary Policy Effects Using the Time-Varying US Monetary Policy Rule . . . . .	88

4.5	Conclusion . . . . .	94
<b>5</b>	<b>Monetary Policy Rules Under Uncertainty</b>	<b>95</b>
5.1	Introduction . . . . .	95
5.2	Empirical Evidence of Uncertainty: A State-Space Model with Markov-Switching . . . . .	98
5.3	Monetary Policy Rules with Adaptive Learning . . . . .	111
5.4	Monetary Policy Rules with Robust Control . . . . .	122
5.5	Conclusion . . . . .	130
<b>6</b>	<b>Monetary Policy Rules with Financial Markets</b>	<b>138</b>
6.1	Introduction . . . . .	138
6.2	The Basic Model . . . . .	141
6.3	Monetary Policy Rule in Practice: The Case of the Euro-Area	147
6.4	Endogenization of P and a Nonlinear Monetary Policy Rule .	151
6.5	The Zero Bound on the Nominal Interest Rate . . . . .	160
6.6	Conclusion . . . . .	168
<b>7</b>	<b>Concluding Remarks</b>	<b>169</b>
	<b>Bibliography</b>	<b>173</b>
	<b>List of Figures</b>	<b>192</b>
	<b>List of Tables</b>	<b>193</b>
	<b>CV</b>	<b>194</b>

# Chapter 1

## Introduction

The topic of monetary policy rules has a long history in macroeconomics. As stated by McCallum (1999), early contributions have been made by Wicksell (1898), Fisher (1920) and others. The last century, however, has seen many changes in monetary policy rules. I will not survey alternative monetary policy rules in history but will instead mention a few policy rules which may have played important roles. A short historical review of monetary policy rules can be found in Adema and Sterken (2003), Taylor (1999), McCallum (2000) and Svensson (2003a).

As surveyed by Adema and Sterken (2003, p.12), the early monetary theorists, Wicksell for instance, emphasized the “indirect monetary transmission mechanism”. Wicksell (1898), for example, proposed that the interest rate should be adjusted with the changes in the price level. Examples of monetary rules proposed or applied later include the price level targeting in Sweden in the 1930s and the constant money-growth-rate rule by Friedman (1960). As stated by Adema and Sterken (2003, p.15), the aim of the constant money-growth-rate rule is to eliminate inflation and the main problem is that it assumes a constant income velocity of money, which may, however, experience significant changes in practice.

In the 1980s the money supply began to be taken as the monetary policy instrument and it had been argued that the growth rate of the money supply should be the sum of the targeted inflation rate plus the desired growth

rate of output. The main disadvantage of the money-supply rule is that the velocity of the money supply has fluctuated too much and the demand for money is unstable. This problem has been analyzed by numerous researchers, see Mishkin (2003, Ch. 21), Blanchard (2003a, Ch. 25) and Semmler (2003, Ch. 1), for example. Therefore, at the beginning of the 1990s the short-term interest rate was proposed to be the monetary policy instrument. A typical interest-rate rule is the Taylor rule (Taylor, 1993), which proposes that the short-term interest rate should be a function of the inflation rate, output gap and long-run equilibrium short-term interest rate. Interest-rate rules have recently attracted much attention and have been employed by numerous central banks. Therefore, I will focus on interest-rate rules in this dissertation.

Although there may exist alternative definitions of a monetary policy rule, this dissertation adopts the definition of Taylor (1999):

... a monetary policy rule is defined as a description—expressed algebraically, numerically or graphically—of how the *instruments* of policy, such as the monetary base or the federal funds rate, change in response to economic variables (Taylor, 1999, p.319).

Moreover, some researchers, Svensson (1999a), for example, distinguish monetary policy rules as “instrument rules” and “targeting rules”. Svensson defines “instrument rules” and “targeting rules” as follows

An *instrument rule* expresses the instruments as a prescribed function of predetermined or forward-looking variables, or both. If the instruments are a prescribed function of predetermined variables only, that is, a prescribed reaction function, the rule is an *explicit* instrument rule. If the instruments are a prescribed function of forward-looking variables, that is, a prescribed implicit reaction function, the rule is an *implicit* instrument rule (Svensson, 1999a, p.614).

and



By a targeting rule, I mean, at the most general level, the assignment of a particular loss function to be minimized. More precisely, a *target(ing) rule* specifies a (vector of) target variable(s)  $Y_t$ , a (vector of) target level(s)  $Y^*$ , and a corresponding loss function...that is to be minimized (Svensson, 1999a, p.617).

In the research below I will not explore whether a monetary policy rule is an instrument rule or a targeting rule, since this requires much discussion which is out of the scope of this dissertation.

## Recent Literature on Monetary Policy Rules

Next, I make a brief sketch of the recent literature on monetary policy rules which can be roughly divided into four directions:

### **Theory and Empirical Evidence of Alternative Interest-Rate Rules**

Much research has been done on the Taylor rule since it was proposed in 1993. Alternative Taylor-type rules have, however, been proposed because of some drawbacks claimed of the simple Taylor rule. These papers include Kozicki (1999), Svensson (2003b), Taylor (1999), Sack and Wieland (2000) and others. Clarida, Gali and Gertler (1998), for instance, estimate a Taylor-type rule with expectations and interest-rate smoothing by way of the generalized method of moments (GMM). An important discussion on the Taylor-type rules is whether interest-rate smoothing is desirable. Sack and Wieland (2000, p.209-210), for example, argue that interest-rate smoothing can be desirable for at least three reasons: (a) forward-looking behaviors, (b) measurement error in macroeconomic variables, and (c) parameter uncertainty. Woodford (2003b) also shows that interest-rate smoothing may be desirable.

Judd and Rudebusch (1998) estimate a Taylor-type reaction function and claim that such a rule seems to “capture some important elements of monetary policy during Alan Greenspan’s tenure as Federal Reserve Chairman” (Judd and Rudebusch, 1998, p.12). Fair (2000b) examines the ability of the estimated, calibrated and optimal interest-rate rules to stabilize economic

fluctuations. Some researchers, Benhabib and Schmitt-Grohé (2001), for instance, argue that the Taylor rule cannot prevent an economy from falling into a “Liquidity Trap” when a zero bound on the nominal interest rate is taken into account. Benhabib and Schmitt-Grohé (2001, abstract) argue that active interest rate feedback rules can lead to “unexpected consequences” in the presence of a zero bound on the nominal rate. That is, there might exist infinite number of equilibrium trajectories converging to a Liquidity Trap even if there exists a unique equilibrium. Moreover, Benhabib, Schmitt-Grohé and Uribe (2001) find that active interest-rate feedback rules might lead to multiple equilibria.

**Monetary Policy Rules under Uncertainty** In the profession it has been increasingly recognized that formal modelling of monetary policy faces great challenges because of many kinds of uncertainties such as model uncertainty, data uncertainty and shock uncertainty. Recent literature dealing with these uncertainties can be found in Giannoni (2002), Söderström (1999), Meyer et al. (2001), Wieland (2000), Tetlow and von zur Muehlen (2001), Orphanides and Williams (2002), Hansen and Sargent (2002), Beck and Wieland (2002), Onatski and Williams (2002) and others. These papers explore, usually theoretically, how a certain kind of uncertainty affects the decisions of the central bank and regulatory agencies. Beck and Wieland (2002), for instance, explore how parameter uncertainty may affect the economy, assuming that the central bank designs the optimal monetary policy by learning through the Kalman filter mechanism. Orphanides and Williams (2002), however, analyze monetary policy with imperfect knowledge by employing the least squares learning algorithm.

On the other hand, Hansen and Sargent (2002) employ another approach to explore the economy under uncertainty, namely, robust control. Unlike the learning algorithm, which assumes that economic agents improve their knowledge of economic models by learning, robust control seeks a policy rule robust to uncertainty. That is, the economic agents seek the best rule from the “worst case”.

Another interesting topic concerning monetary policy-making under un-

certainty is whether the central bank should be bolder or more cautious than when no uncertainty exists. Employing a macroeconomic model with forward-looking behaviors, Giannoni (2002, abstract) claims that “...although it is commonly believed that monetary policy should be less responsive when there is parameter uncertainty, we show that *robust optimal Taylor rules* prescribe in general a stronger response of the interest rate to fluctuations in inflation and the output gap than is the case in the absence of uncertainty.”

**Asset Prices and Monetary Policy Rules** It is well known that the inflation rates in the industrial countries in the 1990s remained relatively stable and low, while the prices of equities, bonds, and foreign exchanges experienced strong volatility with the liberalization of financial markets. Some central banks, therefore, have become concerned with such volatility and doubt whether the volatility is justifiable on the basis of economic fundamentals. The question has arisen whether a monetary policy should be pursued that takes into account financial markets and asset price stabilization. In order to answer this question it is necessary to model the relationship between asset prices and the real economy. An early study of this type can be found in Blanchard (1981) who has analyzed the relationship between the stock value and output in “good news” and “bad news” cases. Recent papers on this topic include Bernanke and Gertler (2000), Smets (1997), Kent and Lowe (1997), Chiarella et al. (2001), Mehra (1998), Vickers (1999), Filardo (2000), Okina, Shirakawa and Shirats (2000), and Dupor (2001).

Among these papers, the work by Bernanke and Gertler (2000) has attracted much attention. Bernanke and Gertler (2000) employ a macroeconomic model and explore how the macroeconomy may be affected by alternative monetary policy rules which may, or may not, take into account the asset price bubble, and conclude that “asset prices become relevant only to the extent they may signal potential inflationary or deflationary forces” (Bernanke and Gertler, 2000, abstract). This argument is supported by Okina and Shiratsuka (2002) with Japan’s experience. In contrast, Smets (1997) argues that the optimal monetary-policy response to changes in asset prices depends on the role they play in the “monetary transmission mechanism” as well as

the sources of shocks.<sup>1</sup> He further explores the potential problems of the view that asset prices should not be considered in monetary policy-making.

Some empirical work has been done to explore whether the financial markets have been taken into account in monetary policy-making. Following Clarida, Gali and Gertler (1998) (CGG98), Smets (1997) estimates the monetary reaction function of Canada and Australia by adding three financial variables (the nominal trade-weighted exchange rate, ten-year nominal bond yield, and a broad stock market index) into the CGG98 model. He finds that the changes in the exchange rate and stock market index in Canada induce significant changes in the interest rate. The response coefficients are, however, insignificant in the case of Australia. By adding stock returns into the CGG98 model, Bernanke and Gertler (2000) test whether the short-term interest rate has responded to stock returns in the US and Japan and find that the federal funds rate did not show a significant response to stock returns from 1979-97. For Japan, however, they find different results: for the whole period from 1979-97 the interest rate seems not to have been affected by the stock market, but for the two subperiods from 1979-89 and from 1989-97 the response coefficients of stock returns are significant enough, but with different signs. Rigobon and Sack (2001), however, claim that the US monetary policy has reacted significantly to stock market movements for the sample 1985-1999.

**Monetary Policy Rules in Open Economies** How to design monetary policy rules in open economies is another important issue in macroeconomics. While in a closed economy the short-term interest rate is usually taken as the policy instrument and the inflation and output gap are taken as targeted variables, the exchange rate may play an important role in an open economy. Recent papers on monetary policy rules in open economies include Ball (1999), Devereux and Engel (2000), Leith and Wren-Lewis (2002), Svensson (1998), McCallum and Nelson (2001), Batini et al. (2001), Walsh (1999), Benigno and Benigno (2000), Clarida et al. (2002), and others. Ball (1999)

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<sup>1</sup>See Smets (1997, p.219).

extends the Svensson (1997)-Ball (1997) closed economy model to an open economy and finds that the optimal monetary policy rule in an open economy is different from that in a closed economy in two aspects: (a) the policy variable is a combination of the short-term interest rate and the exchange rate, rather than the interest rate alone, and (b) the inflation rate in the Taylor rule is replaced by a combination of inflation and the lagged exchange rate. Clarida et al. (2002) explore monetary policy between two countries with and without cooperation and find that under cooperation central banks should respond to the foreign inflation as well as the domestic inflation. Svensson (1998) presents a simple model and examines the properties of “strict” vs. “flexible inflation targeting”, and “domestic” vs. “CPI-inflation targeting”.

## The Goal and Organization of this Dissertation

**The Goal of this Dissertation** This dissertation focuses mainly on the following problems:

1. Time-varying behaviors in monetary policy rules. Although there is a large literature on monetary policy rules, few papers consider time-varying behaviors which may be caused by the changing economic environment. Therefore, time-varying monetary policy rules will be estimated and the results will be discussed.
2. Monetary policy rules under uncertainty. As surveyed in the literature, this is an important problem for central banks. I will employ economic models different from those in the literature and employ different approaches in numerical studies. I will study adaptive learning as well as robust control. A dynamic programming algorithm that is recently developed with adaptive grids will be applied.
3. Financial markets and monetary policy rules. Unlike other papers on this topic, I will use an optimal control framework and endogenize the probability for the asset price bubble to increase or decrease in the next period. Most researchers, Bernanke and Gertler (2000) and Smets (1997), for instance, either take such a probability as a constant or assume it to be

a linear function of the asset price bubble and interest rate. In my model such a probability is endogenized as a nonlinear function of the asset price bubble and interest rate because both positive and negative bubbles will be considered. The problem of a zero bound on the nominal interest rate will be considered in the context of financial markets, while most researchers have explored this problem only in a real economy.

Numerical studies and empirical evidence will be undertaken and explored using the data of OECD countries.

**The Organization of this Dissertation** The rest of this dissertation is organized as follows:

Chapter 2 presents some empirical evidence of the IS and Phillips curves which have been shown to be the baseline model of monetary policy. Both backward- and forward-looking behaviors will be considered, since numerous economists argue that inflation is influenced by forward-looking as well as backward-looking behaviors. A survey of monetary policy from the New Keynesian perspective can be found in Clarida, Gali and Gertler (1999). In this chapter a time-varying Phillips curve will also be estimated to explore regime changes in the economy.

Chapter 3 discusses monetary policy and interest-rate rules. While the money-supply rule was widely used in the 1980s, the short-term interest rate has been generally taken as the policy instrument since the 1990s. The derivation, advantages and disadvantages of these monetary policy rules will be explored in this chapter. Before deriving an interest-rate rule from a dynamic macroeconomic model, I will follow Woodford (2003a) and briefly discuss the loss function of the central bank in pursuing monetary policies. The traditional quadratic loss function will be used in the dissertation, since it has been claimed by some researchers, Svensson (2002) for instance, to dominate other alternatives such as the asymmetric LINEX function.

The empirical evidence of a time-varying Phillips curve in Chapter 2 and the derivation of the interest-rate rule in Chapter 3 indicate that the interest-rate rule may be state-dependent rather than invariant. Therefore, Chapter 4 illustrates a time-varying monetary policy reaction function by way of the

Kalman filter as well as the OLS regression and Chow break-point test. In order to explore whether the monetary policy in the Euro-area was too tight in the 1990s, some simulation of the Euro-area economy employing the time-varying US monetary policy rule will be undertaken.

The empirical evidence of the time-varying Phillips curve and monetary policy reaction function in the previous chapters indicates that there may exist uncertainties as well as structural changes in economic models. Monetary policy rules under uncertainty are, therefore, explored in Chapter 5. I will first present some empirical evidence of model uncertainty employing a State-Space model with Markov-Switching. With such a model I can explore shock uncertainty as well as parameter uncertainty. Based on this evidence, I will then explore monetary policy rules under uncertainty with two approaches: (a) the adaptive learning algorithm, and (b) robust control. By the former approach the central bank is assumed to improve its knowledge of an economic model by learning, while the latter assumes that the central bank seeks a monetary policy rule robust to uncertainty.

While the previous chapters focus on monetary policy rules in a real economy, Chapter 6 explores monetary policy rules with the financial markets. The difference between my model and others, that of Bernanke and Gertler (2000), for example, is that I will endogenize the probability for the asset price bubble to increase or decrease in the next period as a nonlinear function of the interest rate and the size of the bubble. I will also consider the effects of financial markets on the real economy in the presence of a zero bound on the nominal interest rate in the situation of a Liquidity Trap and deflation.

Chapter 7 presents some concluding remarks of this dissertation.

# Chapter 2

## Empirical Evidence of the IS and Phillips Curves

### 2.1 Introduction

The study of monetary policy is usually concerned with two important equations: the “IS” curve, which implies a negative relation between output gap and real interest rate, and the Phillips curve named after A.W.Phillips, which implies a positive relation between inflation and output gap. While the IS curve originally described the equilibrium in the goods market, the Phillips curve was originally developed by Phillips (1958) who explored the relation between the unemployment and the rate of change of money wage rates in the UK from 1861-1957.

While some researchers doubt whether the Phillips curve is dead, numerous researchers, Eller and Gordon (2003), Karanassou et al. (2003) and Mankiw (2000), for example, insist on the traditional view that there exists a tradeoff between inflation and output. Mankiw (2000), however, claims that what he means by “tradeoff between inflation and output” is somewhat different from the traditional view:

I do not mean that a scatterplot of these two variables produces a stable downward-sloping Phillips curve. Nor do I mean that any particular regression fits the data well or produces any particular



set of coefficients. The inflation-unemployment tradeoff is, at its heart, a statement about the effects of monetary policy. It is the claim that changes in monetary policy push these two variables in opposite directions (Mankiw, 2000, p.2).

Karanassou et al. (2003) argue that there exists a tradeoff between inflation and output even if there is no money illusion because of “frictional growth”. They further claim that there exists a *long-run* tradeoff between inflation and output.

Some researchers, Flaschel and Krolzig (2002), Chen and Flaschel (2004), Flaschel et al. (2004), and Fair (2000a), for example, argue that two Phillips curves, rather than a single one, should be considered. This has been stated by Flaschel and Krolzig (2002) as follows

Rarely, however, at least on the theoretical level, is note taken of the fact that there are in principle two relationships of the Phillips curve involved in the interaction of unemployment and inflation, namely one on the labor market, the Phillips (1958) curve, and one on the market for goods, normally not considered a separate Phillips curve, but merged with the other one by assuming that prices are a constant mark-up on wages or the like, an extreme case of the price Phillips curve that we shall consider in this paper (Flaschel and Krolzig, 2002, p.2).

Numerous researchers on macroeconomics and monetary policy, Rudebusch and Svensson (1999), Woodford (2001, 2003b), Clarida, Galí and Gertler (2000), Svensson (1997, 1999a, 1999b), and Ball (1997), for example, have, however, employed a single Phillips curve. One justification for this simplicity, as mentioned by Flaschel and Krolzig (2002, p.3), can be “rigid markup pricing”. Flaschel and Krolzig (2002, p.3), moreover, state that there may exist microfoundations to justify a single Phillips curve, especially in the case of the New Keynesian Phillips curve. In the research below I will employ a single Phillips curve just for simplicity, following the papers mentioned above.

While the traditional Phillips curve considers mainly backward-looking behaviors, the “New Keynesian” Phillips curve takes forward-looking behaviors into account. Because of the drawbacks claimed of the “New Keynesian” Phillips curve which will be discussed below, a so-called “hybrid New Keynesian Phillips curve” has been proposed. The hybrid Phillips curve considers backward- as well as forward-looking behaviors.

Another topic concerning the Phillips curve is its shape. While most papers in the literature have assumed a linear Phillips curve, some researchers have recently argued that the Phillips curve may be nonlinear. These papers include Dupasquier and Ricketts (1998a), Schaling (1999), Laxton, Rose and Tambakis (1998), Aguiar and Martins (2002) and others. Semmler and Zhang (2003), for example, explore monetary policy with different shapes of the Phillips curve. Flaschel et al. (2004) also claim to have detected nonlinearity in the Phillips curve.

This topic will be discussed in more detail in Chapter 5. In the present chapter I will focus on the linear Phillips curve, because there is no consensus on the form of nonlinearity in the Phillips curve yet. Some researchers, Schaling (1999), and Laxton, Rose and Tambakis (1998), for example, argue that it is convex, while other researchers, Stiglitz (1997), for instance, argue that it is concave. Filardo (1998), however, argues that the Phillips curve is convex in the case of positive output gaps and concave in the case of negative output gaps.

Next, I will present some empirical evidence of the IS and Phillips curves, since they are very often employed in the following chapters. While in Section 2 only backward-looking behaviors will be considered, in Section 3 I will estimate the two curves with both backward- and forward-looking behaviors. These two sections estimate the IS and Phillips curves under the assumption that the coefficients in the equations are invariant, in the fourth section, however, I will estimate the Phillips curve with time-varying coefficients, since there might exist regime changes in the economy.

## 2.2 The IS and Phillips Curves with Backward-Looking Behaviors

In this section I will estimate the traditional IS and Phillips curves, which consider only backward-looking behaviors, as shown in Rudebusch and Svensson (1999):

$$\pi_t = \alpha_0 + \sum_{i=1}^m \alpha_i \pi_{t-i} + \alpha_{m+1} y_{t-1} + \varepsilon_t, \quad (2.1)$$

$$y_t = \beta_0 + \sum_{i=1}^n \beta_i y_{t-i} + \beta_{n+1} (\bar{i}_{t-1} - \bar{\pi}_{t-1}) + \xi_t, \quad (2.2)$$

where  $\pi_t$  denotes the inflation rate,  $y_t$  is the output gap and  $i_t$  is the short-term interest rate.  $\varepsilon_t$  and  $\xi_t$  are shocks subject to normal distributions with zero mean and constant variances. The symbol “-” above  $i_t$  and  $\pi_t$  denotes the four-quarter average values of the corresponding variables. Quarterly data are used and the data source is the International Statistical Yearbook. The inflation rate is measured by changes in the Consumer Price Index (CPI, base year: 1995). The output gap is defined as the percentage deviation of the log of the Industrial Production Index (IPI, base year: 1995) from its polynomial trend, the same as in Clarida, Galí and Gertler (1998). The polynomial trend reads as

$$y^* = \sum_{i=0}^n c_i t^i,$$

with  $n=3$ .<sup>1</sup> Because the IPI of Italy is not available, I use the GDP at a constant price (base year: 1995) instead. The Akaike Information Criterion (AIC) is used to determine how many and which lags of the dependent variables should be used in the estimation. The estimation results are presented below with T-Statistics in parentheses. The equations are estimated separately with the ordinary least squares (OLS). I have also tried the estimation

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<sup>1</sup>As surveyed by Orphanides and van Norden (2002), there are different approaches to measure the potential output. In the following chapters I will try some other methods. While Clarida, Galí and Gertler (1998) use the quadratic trend to measure the potential output, I use the third-order trend because the data used here cover a much longer period and the third-order trend fits the data better.

with the seemingly unrelated regression (SUR) and find that the results are very similar to those of the separate OLS regressions, since the covariances of the errors are almost zero. The countries I will look at include Germany, France, the UK, Italy and the European Union (EU) as an aggregate economy.

**Germany** The short-term interest rate of Germany is measured by the 3-month treasury bill rate. The data from 1963.1-98.2 generate the following estimates:

$$\begin{aligned}\pi_t &= 0.004 + 1.082\pi_{t-1} - 0.179\pi_{t-2} + 0.184y_{t-1}, \quad R^2 = 0.907, \\ &\quad (3.314) \quad (13.049) \quad (2.215) \quad (3.796) \\ y_t &= 0.001 + 0.946y_{t-1} - 0.046(\bar{i}_{t-1} - \bar{\pi}_{t-1}), \quad R^2 = 0.868. \\ &\quad (1.727) \quad (29.896) \quad (2.330)\end{aligned}$$

**France** The short-term interest rate of France is measured by two different rates. From 1962-68 I take the call money rate and from 1969-99 I use the 3-month treasury bill rate, because the 3-month treasury bill rate before 1968 is unavailable. With the data from 1962.1-99.4 I obtain the following estimates:

$$\begin{aligned}\pi_t &= 0.003 + 1.402\pi_{t-1} - 0.440\pi_{t-2} + 0.165y_{t-1}, \quad R^2 = 0.979, \\ &\quad (3.158) \quad (19.120) \quad (6.108) \quad (3.167) \\ y_t &= -0.001 + 0.603y_{t-1} - 0.185y_{t-2} - 0.041(\bar{i}_{t-1} - \bar{\pi}_{t-1}), \quad R^2 = 0.683. \\ &\quad (0.980) \quad (7.521) \quad (2.351) \quad (2.227)\end{aligned}$$

**Italy** The short-term interest rate of Italy is measured by the official discount rate, because other interest rates are unavailable. The quarterly data from 1970.1-99.3 generate the following estimates:

$$\begin{aligned}\pi_t &= 0.002 + 1.412\pi_{t-1} - 0.446\pi_{t-2} + 0.236y_{t-1}, \quad R^2 = 0.964, \\ &\quad (1.094) \quad (16.761) \quad (5.243) \quad (2.250) \\ y_t &= 0.002 + 0.712y_{t-1} - 0.107y_{t-3} - 0.030(\bar{i}_{t-1} - \bar{\pi}_{t-1}), \quad R^2 = 0.572. \\ &\quad (2.689) \quad (9.964) \quad (1.596) \quad (1.912)\end{aligned}$$

**The UK** The short-term interest rate of the UK is measured by the 3-month treasury bill rate. The data from 1963.2-99.1 generate the following

estimates:

$$\pi_t = 0.004 + 1.397\pi_{t-1} - 0.413\pi_{t-2} - 0.216\pi_{t-3} + 0.192\pi_{t-4} + 0.494y_{t-1},$$

$(2.034) \quad (17.004) \quad (2.909) \quad (1.517) \quad (2.408) \quad (3.708)$   
 $R^2 = 0.954.$

$$y_t = 0.00003 + 0.849y_{t-1} - 0.015(\bar{i}_{t-4} - \bar{\pi}_{t-4}), \quad R^2 = 0.735.$$

$(0.076) \quad (19.706) \quad (1.810)$

From the estimation of the IS and Phillips curves of the four main European countries above one observes that the T-Statistics of the coefficients of  $y_t$  in the Phillips curve and the real interest rate in the IS curve are significant enough. This indicates that there exists a significant relation between the output and the inflation, and between the inflation and the real interest rate.

Next, I come to the aggregation of the EU economy. I undertake the estimation with the aggregate data of the four main countries of Germany, France, Italy and the UK (EU4) and then the three countries of Germany, France and Italy (EU3). The aggregate inflation rate and output gap are measured by the GDP-weighted sums of the inflation rates and output gaps of the individual countries. I use the German call money rate as the short-term interest rate of EU4 and EU3. Such aggregation of data can be found in Peersman and Smets (1998). There they have also justified using the German rate to measure the monetary policy in the aggregate economy of the Euro-area.

The aggregate data of EU4 and EU3 from 1978.4-98.3 generate the following estimates:

#### EU4

$$\pi_t = 0.003 + 1.175\pi_{t-1} - 0.469\pi_{t-3} + 0.265\pi_{t-4} + 0.396y_{t-1}, \quad R^2 = 0.974.$$

$(1.979) \quad (15.860) \quad (3.262) \quad (2.424) \quad (3.126)$   
 $y_t = 0.001 + 0.947y_{t-1} - 0.033(\bar{i}_{t-1} - \bar{\pi}_{t-1}), \quad R^2 = 0.900.$   
 $(1.280) \quad (26.242) \quad (2.055)$

#### EU3

$$\pi_t = 0.003 + 1.235\pi_{t-1} - 0.510\pi_{t-3} + 0.240\pi_{t-4} + 0.236y_{t-1}, \quad R^2 = 0.972,$$

$(1.652) \quad (17.182) \quad (3.438) \quad (2.121) \quad (2.025)$   
 $y_t = 0.001 + 0.969y_{t-1} - 0.039(\bar{i}_{t-1} - \bar{\pi}_{t-1}), \quad R^2 = 0.901.$   
 $(1.480) \quad (25.524) \quad (2.141)$

From these results one can come to the same conclusion as for the individual countries, that is, there exists a significant relation between  $\pi$  and  $y$ , and between  $y$  and the real interest rate.

## 2.3 The IS and Phillips Curves with Forward-Looking Behaviors

As mentioned by Clarida, Galí and Gertler (1999, p.1664), the New Keynesian IS and Phillips curves can be derived from a dynamic general equilibrium model with money and temporary nominal price rigidities. Clarida, Gali and Gertler (1999 p.1665) write the IS and Phillips curves with forward-looking behaviors as

$$y_t = E_t y_{t+1} - \varphi [i_t - E_t \pi_{t+1}] + g_t, \quad (2.3)$$

$$\pi_t = \lambda y_t + \beta E_t \pi_{t+1} + u_t, \quad (2.4)$$

where  $g_t$  and  $u_t$  are disturbances terms.  $i_t$  is the short-term interest rate and  $E$  denotes the expectation operator.

Numerous researchers, Galí and Gertler (1999), Galí, Gertler and López-Salido (2001a), Woodford (1996), and Chadha and Nolan (2002) for example, have derived the New Keynesian Phillips curve (2.4). While Galí and Gertler (1999) derive the New Keynesian Phillips curve under the assumption that firms face identical constant marginal costs, Galí, Gertler and López-Salido (2001a) derive the New Keynesian Phillips curve under the assumption of increasing real marginal costs. Although there exist some differences between their frameworks, their models do have something in common, that is, the Calvo (1983) pricing model and Dixit-Stiglitz consumption and production models are usually employed. In the appendix of this chapter I will make a brief sketch of Woodford's (1996) derivation of the New Keynesian IS and Phillips curves.

Clarida, Gali and Gertler (1999), moreover, describe the properties of the above two equations as follows:

Equation (2.3) is obtained by log-linearizing the consumption Euler equation that arises from the household's optimal saving decision, after imposing the equilibrium condition that consumption equals output minus government spending. The resulting expression differs from the traditional IS curve mainly because current output depends on expected future output as well as the interest rate. Higher expected future output raises current output: Because individuals prefer to smooth their consumption, expectation of higher consumption next period (associated with higher expected output) leads them to want to consume more today, which raises current output demand. ...

... Equation (2.4) is simply a log-linear approximation about the steady state of the aggregation of the individual firm pricing decisions. Since the equation relates the inflation rate to the output gap and expected inflation, it has the flavor of a traditional expectations-augmented Phillips curve. A key difference with the standard Phillips curve is that expected future inflation,  $E_t\pi_{t+1}$ , enters additively, as opposed to expected current inflation,  $E_{t-1}\pi_t$ . ... In contrast to the traditional Phillips curve, there is no lagged dependence in inflation. Roughly speaking, firms set nominal price based on the expectations of future marginal costs (Clarida, Gali and Gertler, 1999, p.1665-1667).

The virtues of the New Keynesian Phillips curve have been described by Mankiw (2000) as follows

First, it gives some microfoundations to the idea that the overall price level adjusts slowly to changing economic conditions. Second, it produces an expectations-augmented Phillips curve loosely resembling the model that Milton Friedman and Edmund Phelps pioneered in the 1960s and that remains the theoretical benchmark for inflation-unemployment dynamics. Third, it is simple enough to be useful for theoretical policy analysis (Mankiw, 2000, p.13).

Mankiw (2000, p.13-16), however, also mentions three failures of the New Keynesian Phillips curves: (a) disinflationary booms, (b) inflation persistence, and (c) impulse response functions to monetary policy shocks. Moreover, Eller and Gordon (2003) criticize the New Keynesian Phillips curve (NKPC) as follows

This paper shows that the NKPC approach is an empirical failure by every measure. Its residual unexplained error in inflation equation is between three and four times that of the mainstream model. In dynamic simulations its error over the 1993-2002 period is between three and ten times that of the mainstream model. Its only claim for attention, that it is tied to theoretical maximizing models, fades away when its central driving variable, expected future inflation, is shown to have no explanatory power beyond that contributed by lagged, backward-looking inflation. The NKPC variables that push future inflation up or down, the output gap and marginal costs, are shown by simple theoretical reasoning to have coefficients that are biased toward zero and are shown here in statistical tests to have the wrong sign and/or to contribute virtually nothing to the explanation of inflation (Eller and Gordon, 2003, abstract).

Because of the problems of the traditional and New Keynesian Phillips curves, a third type of Phillips curve, the so-called hybrid New Keynesian Phillips curve, has been derived and employed in macroeconomics. In the hybrid New Keynesian Phillips curve both backward- and forward-looking behaviors are considered. The IS curve (with backward- and forward-looking behaviors) and the hybrid Phillips curve have been written by Clarida, Gali and Gertler (1999, p.1691) as follows

$$y_t = \alpha_1 y_{t-1} + (1 - \alpha_1) E_t y_{t+1} - \alpha_2 (r_t - E_t \pi_{t+1}) + \varepsilon_t, \quad \alpha_i > 0, \quad (2.5)$$

$$\pi_t = \beta_1 \pi_{t-1} + (1 - \beta_1) \beta_2 E_t \pi_{t+1} + \beta_3 y_t + \xi_t, \quad \beta_i > 0, \quad (2.6)$$

where  $r_t$  is the short-term interest rate and  $\beta_2$  is the discount factor.  $\varepsilon_t$  and  $\xi_t$  are disturbances terms. The difference between the derivations of the



New Keynesian Phillips curve and the hybrid New Keynesian Phillips curve consists in a fundamental assumption of the models. The former assumes that each firm resets its price with probability  $(1-\theta)$  each period and keeps its price unchanged with probability  $\theta$ . The latter, however, further assumes that the firms can be divided into types, that is, a fraction  $1 - \omega$  of the firms are “forward-looking” and the remaining  $\omega$  of the firms are “backward-looking”. Some estimations of the hybrid New Keynesian Phillips curves have been undertaken. Using the real marginal costs rather than the output gap in the estimation, Galí and Gertler (1999), for example, come to the following conclusions

... (b) Forward looking behavior is very important: our model estimates suggest that roughly sixty to eighty percent of firms exhibit forward looking price setting behavior; (c) Backward looking behavior is statistically significant though, in our preferred specifications, is of limited quantitative importance. Thus, while the benchmark pure forward looking model is rejected on statistical grounds, it appears still to be a reasonable first approximation of reality ... (Galí and Gertler, 1999, p.197).

Moreover, Galí, Gertler and López-Salido (2003), employing different approaches (GMM, nonlinear instrumental variables and maximum likelihood estimation), estimate the hybrid New Keynesian Phillips curve with the US data and find that the estimation results are robust to the approaches employed. Galí, Gertler and López-Salido (2001b) estimate the hybrid New Keynesian Phillips curve with more lags of inflation and find that the additional lags of inflation do not greatly affect the results.

The hybrid New Keynesian Phillips curve given by Eq. (2.6) is, in fact, similar to the hybrid Phillips curve proposed by Fuhrer and Moore (1995), which reads

$$\pi_t = \phi\pi_{t-1} + (1 - \phi)E_t\pi_{t+1} + \delta y_t. \quad (2.7)$$

Although Eq. (2.7) looks similar to Eq. (2.6), the former is mainly an empirical issue. Fuhrer and Moore (1995) derive this hybrid Phillips curve from a model of relative wage hypothesis. Moreover, Fuhrer and Moore

(1995) set  $\phi = 0.5$ . In case  $\beta_2 = 1$ , the hybrid New Keynesian Phillips curve then looks the same as Eq. (2.7) except for a disturbance term in Eq. (2.6).

Next, I will estimate the system (2.5)-(2.6) with the generalized method of moments (GMM), following Clarida, Gali and Gertler (1998). In the estimation below, I find that  $\beta_2$  is always very close to one (0.985 in the case of Germany, 0.990 in France and 0.983 in the US, for example). Therefore, I will assume  $\beta_2 = 1$  for simplicity. Thus, the hybrid New Keynesian Phillips curve looks the same as the hybrid Phillips curve derived and employed by Fuhrer and Moore (1995) except that the former has a disturbance term.

Defining  $\Omega_t$  as the information available to economic agents when expectations of the output gap and inflation rate are formed, and assuming  $\varepsilon_t$  and  $\xi_t$  to be iid with zero mean and constant variances for simplicity, one has

$$y_t = \alpha_1 y_{t-1} + (1 - \alpha_1) E[y_{t+1} | \Omega_t] - \alpha_2 (r_t - E[\pi_{t+1} | \Omega_t]) + \varepsilon_t, \quad \alpha_i > 0, \quad (2.8)$$

$$\pi_t = \beta_1 \pi_{t-1} + (1 - \beta_1) E[\pi_{t+1} | \Omega_t] + \beta_3 y_t + \xi_t, \quad \beta_i > 0, \quad (2.9)$$

After eliminating the unobservable variables from the system one has the following new equations:

$$y_t = \alpha_1 y_{t-1} + (1 - \alpha_1) y_{t+1} - \alpha_2 (r_t - \pi_{t+1}) + \eta_t, \quad (2.10)$$

$$\pi_t = \beta_1 \pi_{t-1} + (1 - \beta_1) \pi_{t+1} + \beta_3 y_t + \epsilon_t, \quad (2.11)$$

with

$$\eta_t = (1 - \alpha_1)(E[y_{t+1} | \Omega_t] - y_{t+1}) + \alpha_2(E[\pi_{t+1} | \Omega_t] - \pi_{t+1}) + \varepsilon_t$$

$$\epsilon_t = (1 - \beta_1)(E[\pi_{t+1} | \Omega_t] - \pi_{t+1}) + \xi_t.$$

Let  $u_t (\in \Omega_t)$  be a vector of variables within the economic agents' information set at the time they form expectations of the inflation rate and output gap that are orthogonal to  $\eta_t$  and  $\epsilon_t$ , one has  $E[\eta_t | u_t] = 0$  and  $E[\epsilon_t | u_t] = 0$ .  $u_t$  includes any lagged variable that helps to forecast the output and inflation, as well as any contemporaneous variable that is uncorrelated with the current shocks  $\varepsilon_t$  and  $\xi_t$ . One now has the following equations:

$$E[y_t - \alpha_1 y_{t-1} - (1 - \alpha_1) y_{t+1} + \alpha_2 (r_t - \pi_{t+1}) | u_t] = 0, \quad (2.12)$$

$$E[\pi_t - \beta_1 \pi_{t-1} - (1 - \beta_1) \pi_{t+1} - \beta_3 y_t | u_t] = 0. \quad (2.13)$$

I will estimate this system by way of the GMM with quarterly data. The data source is the International Statistical Yearbook.<sup>2</sup> The measures of the inflation rate, output gap, and short-term interest rate are the same as in the previous section. The estimation results of several OECD countries are presented below with T-Statistics in parentheses. Because the number of instruments used for the estimation is larger than that of the parameters to be estimated, I present the J-statistics (J-St.) to illustrate the validity of the overidentifying restriction.<sup>3</sup>

**Germany** The estimation for Germany is undertaken with the data from 1970.1-98.4. The instruments include the 1-4 lags of the short-term interest rate, inflation rate, output gap, the percentage deviation of the real money supply (M3) from its HP-filtered trend, the log difference of the nominal DM/USD exchange rate, price changes in imports, energy and shares and a constant. Correction for MA(1) autocorrelation is undertaken. J-St.=0.388 and the residual covariance is  $1.11 \times 10^{-10}$ .

$$\begin{aligned} y_t &= \underset{(0.883)}{0.002} + \underset{(21.024)}{0.491} y_{t-1} + (1 - 0.491) E[y_{t+1}|u_t] - \underset{(1.956)}{0.011} (r_t - E[\pi_{t+1}|u_t]) + \epsilon_t \\ &= 0.002 + 0.491 y_{t-1} + 0.509 E[y_{t+1}|u_t] - 0.011 (r_t - E[\pi_{t+1}|u_t]) + \epsilon_t, \\ R^2 &= 0.662, \end{aligned} \tag{2.14}$$

$$\begin{aligned} \pi_t &= \underset{(2.236)}{0.001} + \underset{(4.162)}{0.147} y_t + \underset{(22.655)}{0.345} \pi_{t-1} + (1 - 0.345) E[\pi_{t+1}|u_t] + \xi_t \\ &= 0.001 + 0.147 y_t + 0.345 \pi_{t-1} + 0.655 E[\pi_{t+1}|u_t] + \xi_t, \quad R^2 = 0.954. \end{aligned} \tag{2.15}$$

**France** The estimation of France is undertaken with the data from 1970.1-99.4. The instruments include the 1-4 lags of the interest rate, output gap, inflation rate, log difference of index of unit value of import, log difference

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<sup>2</sup>I use the 2SLS to obtain the initial estimates of the parameters and then use these initial estimates to obtain the final estimates by way of the GMM with quarterly data.

<sup>3</sup>The J-statistic reported here is the minimized value of the objective function in the GMM estimation. Hansen (1982) claims that  $n \cdot J \xrightarrow{L} \chi^2(m - s)$ , with  $n$  being the sample size,  $m$  the number of moment conditions and  $s$  the number of parameters to be estimated.

of the nominal Franc/USD exchange rate, the unemployment rate and a constant. Correction for MA(1) autocorrelation is undertaken. J-St.=0.303 and the residual covariance is  $9.27 \times 10^{-11}$ .

$$\begin{aligned} y_t &= 0.0004 + \underset{(2.198)}{0.361} y_{t-1} + (1 - 0.361) E[y_{t+1}|u_t] - \underset{(2.279)}{0.009} (r_t - E[\pi_{t+1}|u_t]) + \epsilon_t \\ &= 0.0004 + 0.361 y_{t-1} + 0.639 E[y_{t+1}|u_t] - 0.009 (r_t - E[\pi_{t+1}|u_t]) + \epsilon_t, \\ R^2 &= 0.615, \end{aligned} \quad (2.16)$$

$$\begin{aligned} \pi_t &= -0.0004 + \underset{(1.075)}{0.551} y_t + \underset{(6.682)}{0.709} \pi_{t-1} + (1 - 0.709) E[\pi_{t+1}|u_t] + \xi_t \\ &= -0.0004 + 0.551 y_t + 0.709 \pi_{t-1} + 0.291 E[\pi_{t+1}|u_t] + \xi_t, \quad R^2 = 0.991. \end{aligned} \quad (2.17)$$

**Italy** For Italy I undertake the estimation from 1971.1-99.3. The instruments include the 1-4 lags of the interest rate, inflation rate, output gap, the log difference of index of unit value of import, the log difference of nominal LIRA/USD exchange rate, the unemployment rate and a constant. J-St. is 0.193 and the residual covariance is  $1.12 \times 10^{-9}$ . Correction for MA(2) autocorrelation is undertaken.

$$\begin{aligned} y_t &= 0.001 + \underset{(7.387)}{0.357} y_{t-1} + (1 - 0.357) E[y_{t+1}|u_t] - \underset{(6.847)}{0.019} (r_t - E[\pi_{t+1}|u_t]) + \epsilon_t \\ &= 0.001 + 0.357 y_{t-1} + 0.643 E[y_{t+1}|u_t] - 0.019 (r_t - E[\pi_{t+1}|u_t]) + \epsilon_t, \\ R^2 &= 0.673, \end{aligned} \quad (2.18)$$

$$\begin{aligned} \pi_t &= -0.0004 + \underset{(1.232)}{0.106} y_t + \underset{(3.138)}{0.572} \pi_{t-1} + (1 - 0.572) E[\pi_{t+1}|u_t] + \xi_t \\ &= -0.0004 + 0.106 y_t + 0.572 \pi_{t-1} + 0.428 E[\pi_{t+1}|u_t] + \xi_t, \quad R^2 = 0.986. \end{aligned} \quad (2.19)$$

**The UK** The estimation of the UK is undertaken from 1962.4-99.1. The instruments include the 1-4 lags of the interest rate, inflation rate, output gap, price changes in imports, the log difference of the nominal Pound/USD exchange rate, the unemployment rate and a constant. Correction for MA(2) autocorrelation is undertaken. J-St. is 0.214 and the residual covariance is

$5.11 \times 10^{-10}$ .

$$\begin{aligned} y_t &= \underset{(1.150)}{0.0001} + \underset{(15.840)}{0.363} y_{t-1} + (1 - 0.363) E[y_{t+1}|u_t] - \underset{(3.443)}{0.007} (r_t - E[\pi_{t+1}|u_t]) + \epsilon_t \\ &= 0.0001 + 0.363 y_{t-1} + 0.637 E[y_{t+1}|u_t] - 0.007 (r_t - E[\pi_{t+1}|u_t]) + \epsilon_t, \\ R^2 &= 0.752, \end{aligned} \tag{2.20}$$

$$\begin{aligned} \pi_t &= \underset{(3.973)}{-0.002} + \underset{(3.893)}{0.333} y_t + \underset{(22.513)}{0.553} \pi_{t-1} + (1 - 0.553) E[\pi_{t+1}|u_t] + \xi_t \\ &= -0.002 + 0.333 y_t + 0.553 \pi_{t-1} + 0.447 E[\pi_{t+1}|u_t] + \xi_t, \quad R^2 = 0.980. \end{aligned} \tag{2.21}$$

**The EU4** As in the previous section I also undertake the estimation with the aggregate data of the Euro-area. The estimation for the EU4 is undertaken from 1979.1-98.3. The instruments include the 1-4 lags of the output gap, inflation rate, interest rate, GDP-weighted average price changes in imports, the GDP-weighted unemployment rate, the first difference of the GDP-weighted log of exchange rate and a constant. Correction for MA(1) autocorrelation is undertaken, the residual covariance is  $4.59 \times 10^{-11}$  and J-St.=0.389.

$$\begin{aligned} y_t &= \underset{(6.283)}{0.0004} + \underset{(46.290)}{0.811} y_{t-1} + (1 - 0.811) E[y_{t+1}|u_t] - \underset{(6.310)}{0.018} (r_t - E[\pi_{t+1}|u_t]) + \epsilon_t \\ &= 0.0004 + 0.811 y_{t-1} + 0.189 E[y_{t+1}|u_t] - 0.018 (r_t - E[\pi_{t+1}|u_t]) + \epsilon_t, \\ R^2 &= 0.739, \end{aligned} \tag{2.22}$$

$$\begin{aligned} \pi_t &= \underset{(1.715)}{0.0005} + \underset{(6.631)}{0.335} y_t + \underset{(47.103)}{0.610} \pi_{t-1} + (1 - 0.610) E[\pi_{t+1}|u_t] + \xi_t \\ &= 0.0005 + 0.335 y_t + 0.610 \pi_{t-1} + 0.390 E[\pi_{t+1}|u_t] + \xi_t, \quad R^2 = 0.987. \end{aligned} \tag{2.23}$$

**The US** Next, I undertake the estimation for the US from 1962.1-98.4. For the US I use two lags of the inflation rate in equation (2.8), since the estimates will have signs opposite to the definition in equation (2.8) and (2.9) if I just estimate Eq. (2.11) with one lag of the inflation rate. The inflation rate of the US is measured by changes in the CPI, the short-term interest rate is the federal funds rate, and the output gap is the percentage

deviation of the log of the IPI from its third-order polynomial trend. The instruments include the 1-4 lags of the interest rate, inflation rate, output gap, percentage deviation of the real money supply (M3) from its HP filtered trend, price changes in imports, the log difference of the nominal USD/SDR exchange rate, the unemployment rate and a constant. Correction for MA(1) autocorrelation is undertaken. J-St. is 0.298 and the residual covariance is  $2.16 \times 10^{-11}$ .

$$\begin{aligned}
y_t &= \underset{(2.650)}{0.0004} + \underset{(22.814)}{0.526}y_{t-1} + (1 - 0.526)E[y_{t+1}|u_t] \\
&\quad - \underset{(2.275)}{0.011}(r_t - E[\pi_{t+1}|u_t]) + \epsilon_t \\
&= 0.0004 + 0.526y_{t-1} + 0.474E[y_{t+1}|u_t] - 0.011(r_t - E[\pi_{t+1}|u_t]) + \epsilon_t, \\
R^2 &= 0.931, \tag{2.24}
\end{aligned}$$

$$\begin{aligned}
\pi_t &= \underset{(2.217)}{0.0004} + \underset{(2.548)}{0.042}y_t + \underset{(19.294)}{0.861}\pi_{t-1} - \underset{(7.427)}{0.235}\pi_{t-2} \\
&\quad + (1 - 0.861 + 0.235)E[\pi_{t+1}|u_t] + \xi_t \\
&= 0.0004 + 0.042y_t + 0.861\pi_{t-1} - 0.235\pi_{t-2} + 0.374E[\pi_{t+1}|u_t] + \xi_t, \\
R^2 &= 0.990. \tag{2.25}
\end{aligned}$$

**Japan** The estimation of Japan with the data from 1970.1-99.4 is shown below. The inflation rate is measured by changes in the CPI (base year: 1995), the short-term interest rate is the call money rate and the output gap is the percentage deviation of the IPI (base year: 1995) from its third-order polynomial trend. The instruments used for Japan include the 1-4 lags of the inflation rate, output gap, call money rate, changes in the import prices and a constant. MA(4) autocorrelation is undertaken and the residual covariance

is  $1.27 \times 10^{-8}$  with the J-St. being 0.149.

$$\begin{aligned}
 y_t &= \underset{(0.402)}{0.0001} + \underset{(38.226)}{0.463}y_{t-1} + (1 - 0.463)E[y_{t+1}|u_t] \\
 &\quad - \underset{(2.083)}{0.025}(r_t - E[\pi_{t+1}|u_t]) + \epsilon_t \\
 &= 0.0001 + 0.463y_{t-1} + 0.537E[y_{t+1}|u_t] - 0.025(r_t - E[\pi_{t+1}|u_t]) + \epsilon_t, \\
 R^2 &= 0.986, \tag{2.26}
 \end{aligned}$$

$$\begin{aligned}
 \pi_t &= \underset{(1.365)}{0.0008} + \underset{(11.857)}{0.143}y_t + \underset{(31.641)}{0.988}\pi_{t-1} + (1 - 0.988)E[\pi_{t+1}|u_t] + \xi_t \\
 &= 0.0008 + 0.143y_t + 0.988\pi_{t-1} + 0.012E[\pi_{t+1}|u_t] + \xi_t, \\
 R^2 &= 0.929. \tag{2.27}
 \end{aligned}$$

The estimation results above show that the expectations do play some roles in the equations, since the coefficients of the expected variables are usually large enough in comparison with the coefficients of the lagged variables.

## 2.4 Time-Varying Phillips Curve

Above I have estimated the IS and Phillips curves with both backward- and forward-looking behaviors. One crucial assumption is that the coefficients in the equations are invariant. Recently, there has been some discussion on whether there are regime changes in the economy. That is, the parameters in the model might not be constant but instead time-varying. Cogley and Sargent (2001, 2002), for example, study the inflation dynamics of the US after WWII by way of Bayesian Vector Autoregression with time-varying parameters and claim to have found regime changes. In this section I will consider this problem and estimate the Phillips curve with time-varying coefficients for several OECD countries. This concerns the time-varying reaction of the private sector to the unemployment gap as well as the time variation of what has been called the natural rate of unemployment (or the NAIRU). The time-varying NAIRU has been estimated by Semmler and Zhang (2003). Therefore I will estimate only the time-varying coefficients of the Phillips curve with the NAIRU taken as a constant.

There are different approaches to estimate time-varying parameters, among which are the Recursive Least Squares (RLS), Flexible Least Squares (FLS) and the Kalman filter. In this section I will apply the Kalman filter because of the drawbacks of the FLS and RLS. By the RLS algorithm, the coefficient usually experiences significant changes at the beginning and becomes relatively stable at the end of the sample because old observations are assigned larger weights than new ones. Therefore, the RLS estimates tend to be relatively smooth at the end of the sample, and the real changes in coefficients are not properly shown.

The FLS is developed under the assumption that the coefficients evolve only “slowly”. In this approach two kinds of model specification errors can be associated with each choice of an estimate  $b = (b_1, \dots, b_N)$  for the sequence of coefficient vectors  $b_n$ : the residual “measurement error” which is the difference between dependent variable  $y_n$  and the estimated model  $x_n^T b_n$ , and the residual “dynamic error” which is computed as  $[b_{n+1} - b_n]$ .<sup>4</sup> One of the most important variables in the FLS estimation is the weight  $\mu$  (can be vector or scalar) given to the dynamic errors. The smaller the  $\mu$  is, the larger the changes in the coefficients, and vice versa. In the extreme, when  $\mu$  tends to infinity, the coefficients do not change at all. It is quite difficult to assign an appropriate value to  $\mu$  and, therefore, it is hard to figure out the real changes of the coefficients. Moreover, there are not only “slow” but also drastic changes in the coefficients in economic models and, therefore, on the basis of the FLS, Luetkepohl and Herwartz (1996) develop the Generalized Flexible Least Squares (GFLS) method to estimate the seasonal changes in coefficients.

In fact, Tucci (1990) finds that the FLS and the Kalman filter are equivalent under some assumptions, that is, under certain conditions there is no difference between these two methods. The Kalman filter undoubtedly has disadvantages too. One example is that it requires the specification of probabilistic properties for residual error terms. It is usually assumed that the

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<sup>4</sup> $N$  denotes the number of observations and  $x$  is the vector of independent variables.  $b$  is the vector of time-varying parameters. The reader can refer to Kalaba and Tesfatsion (1988) for the FLS.



error terms have Gaussian distributions, which is not necessarily satisfied in practice. A brief sketch of the Kalman filter can be found in the appendix of this chapter.

In order to simplify the estimation I do not consider forward-looking behaviors in the Phillips curve below. Replacing the output with the unemployment rate, one has the following Phillips curve with time-varying reaction

$$\pi_t = \alpha_0 + \sum_{i=1}^n \alpha_i \pi_{t-i} + \alpha_{ut}(U_t - U_t^N) + \xi_t, \quad (2.28)$$

$$\alpha_{ut} = \alpha_{ut-1} + \eta_t, \quad (2.29)$$

where  $\pi_t$  is the inflation rate,  $U_t$  is the unemployment rate and  $U_t^N$  denotes the so-called NAIRU.  $\xi_t$  and  $\eta_t$  are shocks subject to normal distributions with zero mean and variance  $\sigma_\xi^2$  and  $\sigma_\eta^2$  respectively. The  $\alpha_{ut}$  is expected to be smaller than zero. The number of lags depends on the T-Statistics of the corresponding coefficients, namely, the lags with insignificant T-Statistics will be excluded. Equation (2.29) assumes that  $\alpha_{ut}$  is time-varying and follows a random-walk path. In order to estimate the time-varying path of  $\alpha_{ut}$ , I employ the maximum likelihood estimation by way of the Kalman filter.<sup>5</sup> The countries to be examined include Germany, France, the UK, Italy, the US and Japan. Quarterly data are used. The data source is the International Statistical Yearbook. T-Statistics of the estimation are shown in parentheses.

The inflation rate of Germany is measured by changes in the CPI. The NAIRU is assumed to be fixed at 6 percent. This is undoubtedly a simplification, since the NAIRU may change over time too.<sup>6</sup> The data from

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<sup>5</sup>The reader can also refer to Hamilton (1994, Ch. 13) for the details of the Kalman filter. In this section I apply the random-walk model (shown in the appendix) to estimate the time-varying coefficients.

<sup>6</sup>Here I assume that the NAIRU is fixed for all countries, close to the average values of the unemployment rates in these countries. It is obvious that the value of the constant NAIRU does not essentially affect the estimation. Semmler and Zhang (2003) estimate the time-varying NAIRU with the Kalman filter, following Gordon (1997).

1963.4-98.4 generate the following estimation results:

$$\pi_t = 0.005 + 1.047\pi_{t-1} - 0.181\pi_{t-2} + \alpha_{ut}(U_t - U_t^N).$$

(1.495)
(9.922)
(2.268)

The path of  $\alpha_{ut}$  is presented in Figure 2.1A.

The inflation rate of France is measured by the log difference of the GDP deflator. The NAIRU is also assumed to be 6 percent. The data from 1969.1-99.4 generate the following estimation results

$$\pi_t = 0.008 + 0.901\pi_{t-1} - 0.003\pi_{t-2} + \alpha_{ut}(U_t - U_t^N).$$

(0.566)
(6.070)
(0.045)

The path of  $\alpha_{ut}$  is presented in Figure 2.1B.

The inflation rate of the UK is measured by changes in the CPI. The NAIRU is assumed to be 6 percent. The data from 1964.1-99.4 generate the following estimation results

$$\pi_t = 0.007 + 1.384\pi_{t-1} - 0.491\pi_{t-2} + \alpha_{ut}(U_t - U_t^N).$$

(2.403)
(15.845)
(6.695)

The path of  $\alpha_{ut}$  is presented in Figure 2.1C.

The inflation rate of Italy is also measured by changes in the CPI and the NAIRU is assumed to be 5 percent. With the data from 1962-99 the changes of  $\alpha_{ut}$  are insignificant, but for the period from 1962-94 the changes are significant enough, therefore the estimation is undertaken from 1962.3-94.3 and the result reads

$$\pi_t = 0.004 + 1.409\pi_{t-1} - 0.448\pi_{t-2} + \alpha_{ut}(U_t - U_t^N).$$

(0.887)
(14.111)
(2.870)

The path of  $\alpha_{ut}$  is presented in Figure 2.1D.

Next, I undertake the estimation for the US and Japan. The inflation rate of the US is measured by changes in the CPI and the NAIRU is taken to be 5 percent. The data from 1961.1-99.4 generate the following estimation results

$$\pi_t = 0.004 + 1.198\pi_{t-1} - 0.298\pi_{t-2} + 0.203\pi_{t-3} - 0.202\pi_{t-4} + \alpha_{ut}(U_t - U_t^N).$$

(2.665)
(12.242)
(2.119)
(1.589)
(2.275)

The path of  $\alpha_{ut}$  is shown in Figure 2.1E. In Figure 2.1E one finds that for many years  $\alpha_{ut}$  is positive, which is inconsistent with the traditional view that

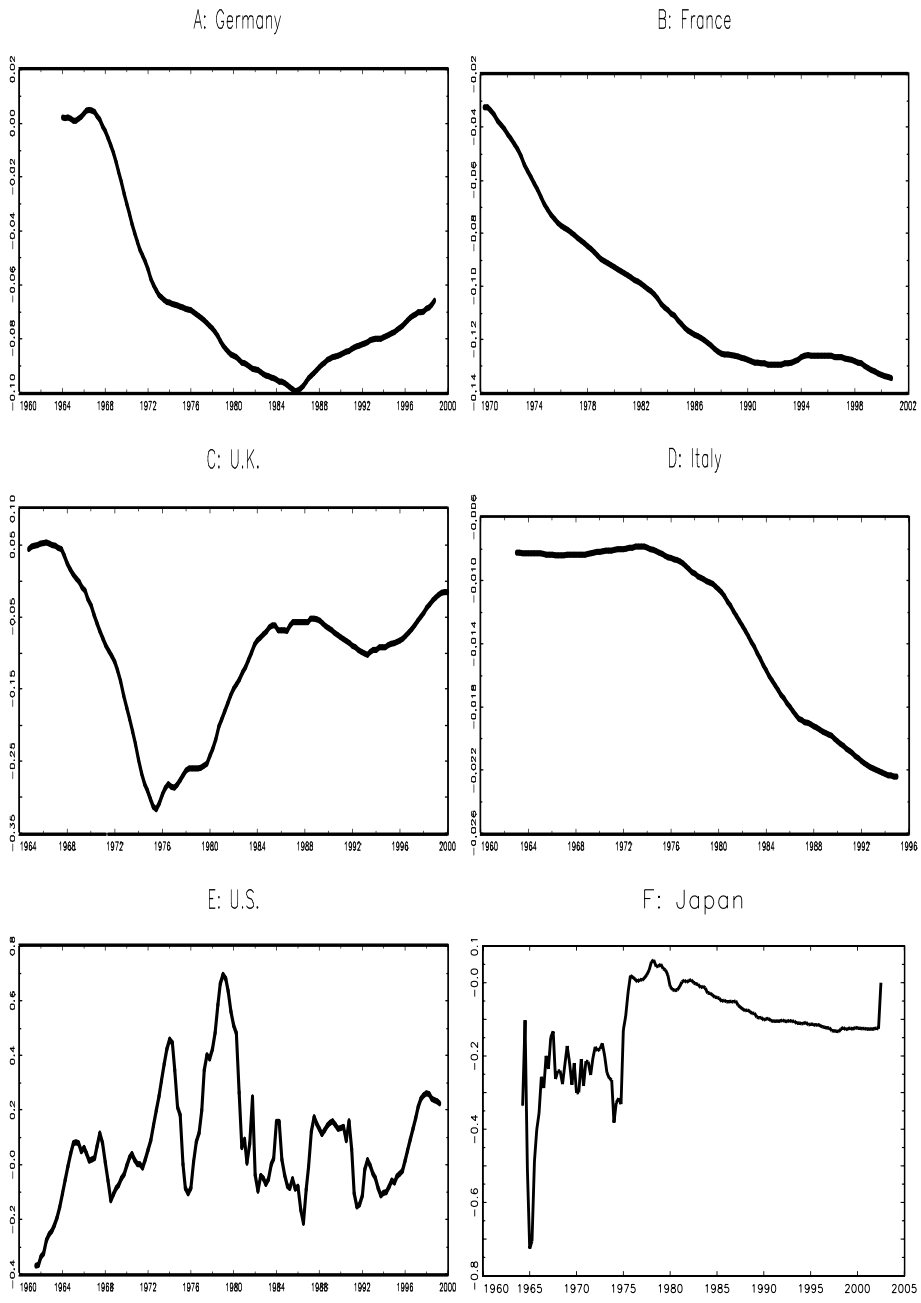


Figure 2.1: Time-Varying  $\alpha_{ut}$

there is a negative relation between the inflation rate and the unemployment rate. One reason may be the value of the NAIRU, which is assumed to be fixed at 5 percent here. The unemployment rate in the US was quite high in the 1970s and 1980s, attaining 11% around 1983. It experienced significant changes from the 1960s to the 1990s. Therefore, assuming a fixed NAIRU of 5% does not seem to be a good choice.

The inflation rate of Japan is measured by changes in the CPI and the NAIRU is assumed to be 3 percent which is close to its average value from the middle of the 1960s to the end of the 1990s. The estimation result with the Japanese data from 1964.1-2002.4 reads

$$\pi_t = 0.006 + 1.216\pi_{t-1} - 0.290\pi_{t-2} + \alpha_{ut}(U_t - U_t^N).$$

(2.208)      (22.081)      (5.225)

The path of Japanese  $\alpha_{ut}$  is presented in Figure 2.1F. It is negative most of the time and experienced some structural changes before the 1980s and remained relatively stable thereafter. This is consistent with the fact that the inflation rate also experienced some significant changes before the 1980s and remained relatively stable thereafter. The inflation rate and unemployment rate of Japan are presented in Figure 2.2.

From the empirical evidence above one finds that the  $\alpha_{ut}$  in Eq.(2.28) did experience some changes. For the three EU countries of Germany, France and Italy, one finds that the changes of  $\alpha_{ut}$  are to some extent similar.  $\alpha_{ut}$  of France and Italy have been decreasing persistently since the 1960s. In the case of Germany, however, it has been increasing slowly since the middle of the 1980s. As regards the UK, the change of  $\alpha_{ut}$  is relatively different from those of the other three countries. It decreased very fast in the 1960s and started to increase in 1975. In order to analyze the causes of the differences of the evolution of  $\alpha_{ut}$ , I present the inflation and unemployment rates of the four EU countries from 1970 to 1999 in Figure 2.3 and 2.4 respectively. It is obvious that the changes in inflation rates of the four countries are similar.  $\pi_t$  attained its highest point around 1975, decreased to a low value in about 4 years, increased to another peak at the end of the 1970s and then continued to go down before 1987, after which it evolved smoothly

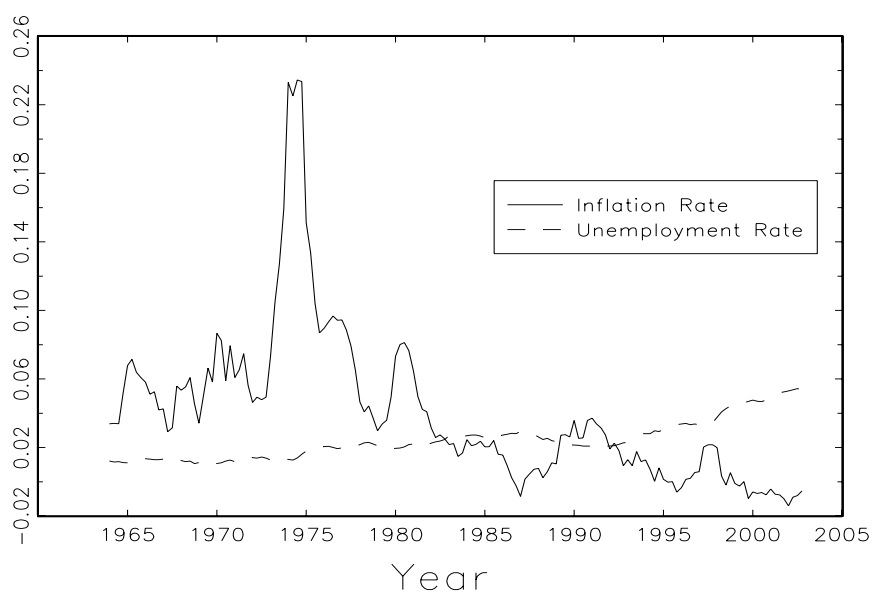


Figure 2.2: Inflation Rate and Unemployment Rate of Japan

and stayed below 10 percent. The evolution of the inflation rate does not seem to be responsible for the differences in the paths of  $\alpha_{ut}$  of the four countries. The evolution of the unemployment rates in Figure 2.4, however, may partly explain why the change of  $\alpha_{ut}$  in the UK is somewhat different from those of the other three countries. Before 1986 the unemployment rates of the four countries increased almost simultaneously, while after 1986 there existed some differences. The evolution of  $U_t$  in the UK was not completely consistent with those of the other three countries. After 1992 the  $U_t$  of the UK decreased rapidly from about 10 percent to 4 percent, while those of the other three countries remained relatively high during the whole of the 1990s and did not begin to go down until 1998.

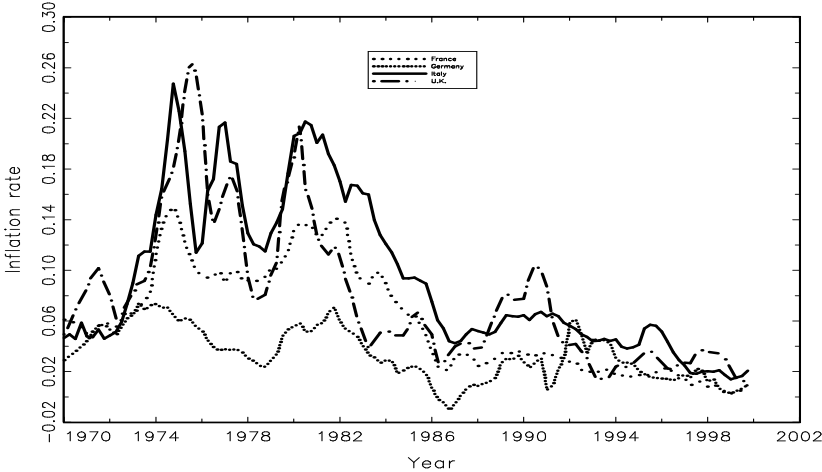


Figure 2.3: Inflation Rates of Germany, France, Italy and the UK

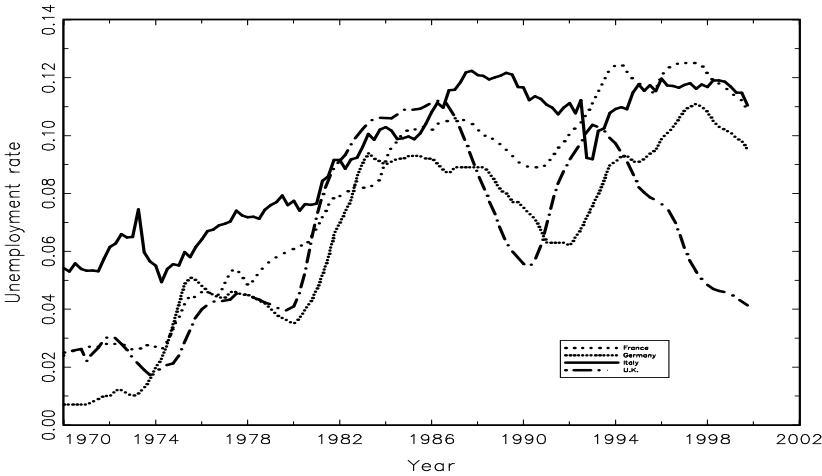


Figure 2.4: Unemployment Rates of Germany, France, Italy and the UK

## 2.5 Conclusion

This chapter presents some empirical evidence of the baseline model of monetary policy, the IS and Phillips curves. Both backward- and forward-looking

behaviors have been considered. The evidence of the countries studied shows that there do exist some significant relations between the output gap and real interest rate, and between the inflation and the output gap. In order to explore regime changes in the economy I have also estimated a time-varying Phillips curve. The estimation results show that the reaction to the unemployment gap has been changing, indicating regime changes in the economy.

## Appendix A: The State-Space Model and Kalman Filter

Here I make a brief sketch of the State-Space model (SSM) and Kalman filter, following Harvey (1989, 1990) and Hamilton (1994).<sup>7</sup> After arranging a model in a State-Space form, one can use the Kalman filter to obtain the paths of time-varying parameters.

### The State-Space Model

The State-Space model applies to a multivariate time series,  $y_t$ , containing  $N$  elements. These observable variables are, via a so-called “measurement equation”, related to an  $m \times 1$  vector,  $\alpha_t$  which is known as the “state vector”,

$$y_t = Z_t \alpha_t + d_t + \epsilon_t, \quad (2.30)$$

with  $t = 1, \dots, T$ ,  $Z_t$  is an  $N \times m$  matrix,  $d_t$  is an  $N \times 1$  vector and  $\epsilon_t$  is an  $N \times 1$  vector of serially uncorrelated disturbances with zero mean and covariance matrix  $H_t$ . Usually the elements of  $\alpha_t$  are not observable but are known or assumed to be generated by a first-order Markov process, which is known as the “transition equation”

$$\alpha_t = T_t \alpha_{t-1} + c_t + R_t \eta_t, \quad (2.31)$$

with  $t = 1, \dots, T$ .  $T_t$  is an  $m \times m$  matrix,  $c_t$  is an  $m \times 1$  vector,  $R_t$  is an  $m \times g$  matrix and  $\eta_t$  is a  $g \times 1$  vector of serially uncorrelated disturbances with zero mean and covariance  $Q_t$ . If the system matrices  $Z_t, d_t, H_t, T_t, c_t, R_t$  and  $Q_t$  do not change over time, the model is said to be time-invariant, otherwise, it is time-variant.

### The Kalman Filter

The Kalman filter estimates time-varying parameters in three steps. Given all the information currently available, the first step forms the optimal predictor

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<sup>7</sup>Although there are numerous books dealing with the Kalman filter, the framework in this appendix is mainly based on Harvey (1989, 1990).



of the next observation via the so-called “prediction equations”. The second step is to update the estimator by incorporating the new observation via the “updating equations”. These two steps use only the past and current information, disregarding the future information which may also affect the estimation. Therefore, the third step is to “smooth” the estimators based on all of the observations to get a more reasonable result.

**Prediction** Let  $a_{t-1}$  denote the optimal estimate of  $\alpha_{t-1}$  based on the observations up to and including  $y_{t-1}$ . Let  $P_{t-1}$  denote the  $m \times m$  covariance matrix of the estimate error, i.e.

$$P_{t-1} = E[(\alpha_{t-1} - a_{t-1})(\alpha_{t-1} - a_{t-1})'].$$

Given  $a_{t-1}$  and  $P_{t-1}$ , the optimal estimate of  $\alpha_t$  is given by

$$a_{t|t-1} = T_t a_{t-1} + c_t, \quad (2.32)$$

while the covariance matrix of the measurement error is

$$P_{t|t-1} = T_t P_{t-1} T_t' + R_t Q_t R_t', \quad t = 1, \dots, T. \quad (2.33)$$

These two equations are called the prediction equations.

**Updating** Once the new observations of  $y_t$  become available, the estimate of  $\alpha_t$ ,  $a_{t|t-1}$ , can be updated with the following equations

$$a_t = a_{t|t-1} + P_{t|t-1} Z_t' F_t^{-1} v_t, \quad (2.34)$$

and

$$P_t = P_{t|t-1} - P_{t|t-1} Z_t' F_t^{-1} Z_t P_{t|t-1}, \quad (2.35)$$

where  $v_t = y_t - Z_t a_{t|t-1} - d_t$ , which is called the prediction error, and  $F_t = Z_t P_{t|t-1} Z_t' + H_t$ , for  $t = 1, \dots, T$ .

**Smoothing** The prediction and updating equations estimate the state vector,  $\alpha_t$ , conditional on the information available at time  $t$ . The aim of

smoothing is to take account of the information available after time  $t$ .<sup>8</sup> The smoothing algorithms consist of a set of recursions that start with the final quantities ( $a_T$  and  $P_T$ ) and work backwards. The equations are

$$a_{t|T} = a_t + P_t^*(a_{t+1|T} - T_{t+1}a_t - c_{t+1}), \quad (2.36)$$

and

$$P_{t|T} = p_t + P_t^*(P_{t+1|T} - P_{t+1|t})P_t^{*'}, \quad (2.37)$$

where

$$P_t^* = P_t T_{t+1}' P_{t+1|t}^{-1}, \quad t = T - 1, \dots, 1,$$

with  $a_{T|T} = a_T$  and  $P_{T|T} = P_T$ .

**The Maximum Likelihood Function** In order to estimate the state vector, one must first estimate a set of unknown parameters ( $n \times 1$  vector  $\psi$ , referred to as “hyperparameters”) with the maximum likelihood function. For a multivariate model the maximum likelihood function reads

$$L(y; \psi) = \prod_{t=1}^T p(y_t | Y_{t-1}),$$

where  $p(y_t | Y_{t-1})$  denotes the distribution of  $y_t$  conditional on the information set at time  $t - 1$ , that is,  $Y_{t-1} = (y_{t-1}, y_{t-2}, \dots, y_1)$ . The likelihood function for a Gaussian model can be written as

$$\log L(\psi) = -(1/2)(NT \log 2\pi + \sum_{t=1}^T \log |F_t| + \sum_{t=1}^T v_t' F_t^{-1} v_t), \quad (2.38)$$

where  $F_t$  and  $v_t$  are the same as those defined in the Kalman filter.

In sum, one has to do the following to estimate the state vector with the Kalman filter. (a) Write the model in a State-Space form of Eq. (2.30)-(2.31), run the Kalman filter of Eq. (2.32)-(2.35) and store all  $v_t$  and  $F_t$  for future use. (b) Estimate the hyperparameters with the maximum likelihood

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<sup>8</sup>Harvey (1989) points out three smoothing algorithms: “Fixed-point” smoothing, “Fixed-lag” smoothing and “Fixed-interval” smoothing. In this dissertation I use the third one, which is widely used in economic problems.

function presented in Eq. (2.38). (c) Run the Kalman filter again with the estimates of the hyperparameters to get the non-smoothed estimates of the state vector. (d) Smooth the state vector with the smoothing equations Eq. (2.36)-(2.37).

In order to run the Kalman filter one needs starting values of  $a_t$  and  $P_t$ , that is, one needs to know  $a_0$  and  $P_0$ . For a stationary and time invariant transition equation, the starting values are given as follows:

$$a_0 = (I - T)^{-1}c, \quad (2.39)$$

and

$$vec(P_0) = [I - T \otimes T]^{-1}vec(RQR'). \quad (2.40)$$

If the transition equation is non-stationary, the initial conditions must be estimated from the model. There are usually two approaches to deal with this problem. The first approach assumes that the initial state is fixed with  $P_0 = 0$  (or a zero matrix) and the initial state is treated as unknown parameters that will be estimated from the model. The second approach assumes that the initial state is random and has a diffuse distribution, that is, its covariance matrix is  $P_0 = \kappa I$ , with  $\kappa$  being a large number.

**Time-Varying Coefficient Estimation** Consider a linear model

$$y_t = x_t' \beta_t + \epsilon_t, \quad t = 1, \dots, T,$$

where  $x_t$  is a  $k \times 1$  vector of exogenous variables and  $\beta_t$  is the corresponding  $k \times 1$  vector of unknown parameters which evolve over time according to certain stochastic processes. Defining  $\beta_t$  as the state vector, one can use the State-Space model and Kalman filter to estimate the time-varying parameters. There are basically three classes of models that can be used for the time-varying coefficient estimation:

**The Random-Coefficient Model** In this model the coefficients vary randomly about a fixed, but unknown mean,  $\bar{\beta}$ . The State-Space form is

$$y_t = x_t' \beta_t$$

$$\beta_t = \bar{\beta} + \epsilon_t, \quad \epsilon_t \sim NID(0, Q),$$

for all  $t$ . The time-varying coefficients in this model are stationary and do not show structural changes.

**The Random-Walk Model** In the random-walk model the coefficients are non-stationary and follow a random-walk path. The State-Space form reads:

$$y_t = x_t' \beta_t + \epsilon_t, \quad t = 1, \dots, T$$

where  $\epsilon_t \sim NID(0, H)$  and the vector  $\beta_t$  is generated by the process

$$\beta_t = \beta_{t-1} + \eta_t,$$

where  $\eta_t \sim NID(0, Q)$ .

**The Return-to-Normality Model** In this model the coefficients are generated by a stationary multivariate AR(1) process. The State-Space form reads

$$y_t = x_t' \beta_t + \epsilon_t, \quad t = 1, \dots, T, \quad (2.41)$$

$$\beta_t - \bar{\beta} = \phi(\beta_{t-1} - \bar{\beta}) + \eta_t, \quad (2.42)$$

where  $\epsilon_t \sim NID(0, H)$ , and  $\eta_t \sim NID(0, Q)$ . The coefficients are stationary and evolve around a mean,  $\bar{\beta}$ . It is clear that the random-coefficient and random-walk models are just two special cases of the return-to-normality model.

In order to apply the Kalman filter one has to rearrange the return-to-normality model in a standard State-Space form. Let  $\beta_t^* = \beta_t - \bar{\beta}$ , one has

$$y_t = (x_t' \quad x_t') \alpha_t + \epsilon_t, \quad t = 1, \dots, T \quad (2.43)$$

and

$$\alpha_t = \begin{bmatrix} \bar{\beta}_t \\ \beta_t^* \end{bmatrix} = \begin{bmatrix} I & 0 \\ 0 & \phi \end{bmatrix} \begin{bmatrix} \bar{\beta}_{t-1} \\ \beta_{t-1}^* \end{bmatrix} + \begin{bmatrix} 0 \\ \eta_t \end{bmatrix}. \quad (2.44)$$

A diffuse prior is used for  $\bar{\beta}_t$ , implying that the starting values are constructed from the first  $k$  observations. The starting value of  $\beta_t^*$  is given by a zero vector with the starting covariance matrix given by Eq. (2.40).

## Appendix B: Derivation of the New Keynesian Phillips Curve by Woodford (1996)

Here I make a brief sketch of Woodford's (1996) derivation of the New Keynesian Phillips (and IS) curve. The details of the derivation can be found in Woodford (1996, p.3-14).

The economy consists of a continuum of identical infinite-lived households indexed by  $j \in [0, 1]$ , and  $z \in [0, 1]$  denotes a continuum of differentiated goods produced by the households. The objective of each household is assumed to maximize the following function

$$E \left\{ \sum_{t=0}^{\infty} \beta^t \langle u(C_t^j + G_t) + v(M_t^j / P_t) - \omega[y_t(j)] \rangle \right\}, \quad (2.45)$$

where  $u$  and  $v$  are increasing concave functions and  $\omega$  is an increasing convex function.  $\beta$  denotes the discount factor between 0 and 1.  $y_t(j)$  denotes the product supplied by household  $j$ . The term  $v$  "indicates the existence of liquidity services from wealth held in the form of money" (Woodford, 1996, p.5).  $C_t^j$  is the consumption of household  $j$

$$C_t^j \equiv \left( \int_0^1 c_t^j(z)^{\frac{\theta-1}{\theta}} dz \right)^{\frac{\theta}{\theta-1}}, \quad (2.46)$$

where  $c_t^j(z)$  denotes household  $j$ 's consumption of good  $z$  at time  $t$ , and  $\theta > 1$  is the constant elasticity of substitution among alternative goods.  $G_t$  denotes the public goods.  $M_t^j$  denotes the household's money balances at the end of period  $t$ , and  $P_t$  is the price index of goods

$$P_t \equiv \left( \int_0^1 p_t(z)^{1-\theta} dz \right)^{\frac{1}{1-\theta}}, \quad (2.47)$$

with  $p_t(z)$  being the price of good  $z$  at time  $t$ . The budget constraint of each household reads

$$\int_0^1 p_t(z) c_t^j(z) dz + M_t^j + E_t(R_{t,t+1} B_{t+1}^j) \leq W_t^j + p_t(j) y_t(j) - T_t, \quad (2.48)$$

where  $B_{t+1}^j$  denotes the bond portfolio at date  $t$  and  $R_{t,T}$  is the stochastic discount factor.  $W_t^j$  denotes the nominal value of the household's financial wealth at the beginning of period  $t$ , that is,

$$W_t^j = M_{t-1}^j + B_t^j, \quad (2.49)$$

and  $T_t$  is the net nominal lump-sum tax. Woodford (1996, p.6) further claims that the budget constraint (2.48) is equivalent to the following expression

$$\begin{aligned} \sum_{T=t}^{\infty} E_t \left\{ R_{t,T} \left[ \int_0^1 p_T(z) c_T^j(z) dz + \frac{i_T}{1+i_T} M_T^j \right] \right\} \\ \leq \sum_{T=t}^{\infty} E_t \{ R_{t,T} [p_T(j) y_T(j) dz - T_T] \} + W_t^j, \end{aligned} \quad (2.50)$$

with  $i_t$  denoting the nominal interest rate on a riskless bond, therefore

$$1 + i_t \equiv \frac{1}{E_t(R_{t,t+1})}. \quad (2.51)$$

The consumption of good  $z$  in line with expenditure minimization and the demand of good  $j$  in line with cost minimization turn out to be

$$c_t^j(z) = C_t^j \left( \frac{p_t(z)}{P_t} \right)^{-\theta} \quad (2.52)$$

and

$$y_t(j) = Y_t \left( \frac{p_t(j)}{P_t} \right)^{-\theta}, \quad (2.53)$$

with  $Y_t = C_t + G_t$  and  $C_t = \int_0^1 C_t^h dh$ . Woodford (1996, p.7) further gives three necessary and sufficient conditions for an optimal consumption and portfolio plan of a household, that is,

$$\beta^{T-t} \frac{u'(Y_T)}{u'(Y_t)} \frac{P_t}{P_T} = R_{t,T} \quad (2.54)$$

$$\frac{v'(M_t/P_t)}{u'(Y_t)} = \frac{i_t}{1+i_t} \quad (2.55)$$

and that (2.50) holds with equality at date 0. From (2.54) one knows

$$\beta E_t \frac{u'(Y_{t+1})}{u'(Y_t)} \frac{P_t}{P_{t+1}} = \frac{1}{1+i_t}. \quad (2.56)$$

Next, Woodford (1996, p.8-9) shows how to set the price. Following the Calvo (1983) price-setting model, namely, each period a fraction  $1 - \alpha$  of goods suppliers set a new price and the remaining  $\alpha$  keep the old price, Woodford (1996, p.8) shows that the price  $p$  must be set to maximize

$$\sum_{k=0}^{\infty} \alpha^k \{ \Lambda_t E_t [R_{t,t+k} p y_{t+k}(p)] - \beta^k E_t [\omega(y_{t+k}(p))] \},$$

with  $y_T(p)$  being the demand at date  $T$  given by (2.53).  $\Lambda_t$  denotes the marginal utility of holding money. The optimal price  $\mathcal{P}_t$  satisfies the first-order condition

$$\sum_{k=0}^{\infty} \alpha^k E_t \{ R_{t,t+k} Y_{t+k} (\mathcal{P}_t / P_{t+k})^{-\theta} [\mathcal{P}_t - \mu S_{t+k,t}] \} = 0, \quad (2.57)$$

where  $\mu \equiv \frac{\theta}{\theta-1}$  and  $S_{T,t}$  denotes the marginal cost of production at date  $T$ :

$$S_{T,t} = \frac{\omega' [Y_T (\mathcal{P}_t / P_T)^{-\theta}]}{u'(Y_T)} P_T. \quad (2.58)$$

Employing Eq. (2.47), one finds that

$$P_t = [\alpha P_{t-1}^{1-\theta} + (1-\alpha) \mathcal{P}_t^{1-\theta}]^{\frac{1}{1-\theta}}. \quad (2.59)$$

On the basis of the analysis above, Woodford (1996) then explores how fiscal policy may affect macroeconomic instability. I will not sketch his analysis of this problem here, since this is not of much interest in my dissertation. Defining  $x_t$  as the percentage deviation of  $Y_t$  from its stationary value  $Y^*$  (namely,  $x_t = \frac{Y_t - Y^*}{Y^*}$ ) and  $\hat{\pi}_t$  as the percentage deviation of  $\pi_t$  from its stationary value,<sup>9</sup> and linearizing (2.56) at the stationary values of  $Y_t$ ,  $\pi_t$  and  $i_t$ , one then obtains the following IS curve<sup>10</sup>

$$x_t = E_t x_{t+1} - \sigma (\hat{i}_t - E_t \hat{\pi}_{t+1}), \quad (2.60)$$

with  $\hat{i}_t$  being the percentage deviation of the nominal interest rate from its stationary value, and

$$\sigma \equiv - \frac{u'(Y^*)}{u''(Y^*) Y^*}.$$

<sup>9</sup> $\pi_t$  is defined as  $\frac{P_t}{P_{t-1}}$ , since the stationary value of  $\pi_t$  is 1,  $\hat{\pi}_t$  is then equal to  $\frac{P_t - P_{t-1}}{P_{t-1}}$ .

<sup>10</sup>The stationary value of  $i_t$  is found to be  $\beta^{-1} - 1$ .

After linearizing Eq. (2.57)-(2.59) around the stationary values of the variables and rearranging the terms, one obtains

$$\hat{\mathcal{P}}_t = \frac{\kappa\alpha}{1-\alpha} \sum_{k=0}^{\infty} (\alpha\beta)^k E_t x_{t+k} + \sum_{k=1}^{\infty} (\alpha\beta)^k E_t \hat{\pi}_{t+k}, \quad (2.61)$$

$$\hat{\pi}_t = \frac{1-\alpha}{\alpha} \hat{\mathcal{P}}_t, \quad (2.62)$$

with

$$\kappa \equiv \frac{(1-\alpha)(1-\alpha\beta)}{\alpha} \frac{\varpi + \sigma}{\sigma(\varpi + \theta)} \quad \text{and} \quad \varpi \equiv \frac{\omega'(Y^*)}{\omega''(Y^*)Y^*},$$

where  $\hat{\mathcal{P}}_t$  is the percentage deviation of  $\mathcal{P}_t/P_t$  from its stationary value, which is 1. After rearranging Eq. (2.61) as

$$\hat{\mathcal{P}}_t = \alpha\beta E_t \hat{\mathcal{P}}_{t+1} + \frac{\kappa\alpha}{1-\alpha} x_t + \alpha\beta E_t \hat{\pi}_{t+1} \quad (2.63)$$

and substituting (2.62) into (2.63), one finally obtains the following Phillips curve:

$$\hat{\pi}_t = \beta E_t \hat{\pi}_{t+1} + \kappa x_t. \quad (2.64)$$



# Chapter 3

## Monetary Policy and Interest-Rate Rules

### 3.1 Introduction

The topic of monetary policy rules has a long history in macroeconomics. The last century has, however, seen much discussion of this topic and many changes in monetary policy rules. There are, in fact, two important monetary policy rules that have been recently discussed. The first rule takes money supply as the policy instrument and proposes that the growth rate of the money supply should be the sum of the target inflation and the desired growth rate of output. The second rule, however, proposes that the short-term interest rate should be taken as the policy instrument and the interest rate can be determined as a function of the output gap and the deviation of the inflation rate from its target. While the first rule was mainly applied in the 1980s, the second rule began to be adopted at the beginning of the 1990s. In this chapter I will briefly discuss these two monetary policy rules with more emphasis on the second one, since it has been proposed to have some advantages over the first one and has been adopted by numerous central banks recently.

Moreover, some researchers, Svensson (2003b), for example, distinguish monetary policy rules as “instrument rules” and “targeting rules”. As men-

tioned by Svensson (1999a), most of the literature focuses on instrument rules, by which the policy instrument is prescribed as a function of a small subset of the information available to the central bank. The Taylor rule (Taylor 1993) is a typical instrument rule with the subset of information being the output gap, actual inflation and its target. In the research below I will not explore whether a monetary policy rule is an instrument rule or targeting rule, since this requires much discussion which is out of the scope of this dissertation.

### 3.2 The Money-Supply Rule

The money-supply rule originated in the monetarist view of the working of a monetary economy (Semmler, 2003, p.11). According to this rule money supply should be taken as the policy instrument and the rate of the nominal money growth should be equal to the target inflation rate plus the desired growth rate of output. To be precise,

$$\hat{m} = \hat{p} + \hat{y},$$

where  $\hat{m}$  denotes the nominal money growth rate,  $\hat{p}$  is the target inflation rate and  $\hat{y}$  is the desired growth rate of output. As mentioned by Semmler (2003, p.11), this view prevailed during a short period in the 1980s in the US and until recently at the German Bundesbank. The derivation of this rule is shown below.

According to Fisher's quantity theory of money, the equation of exchange can be written as

$$MV = PY, \tag{3.1}$$

where  $M$  denotes the total quantity of money (money supply),  $V$  is the velocity of money,  $P$  is the price level and  $Y$  denotes the aggregate output. As mentioned by Mishkin (2003, p.539), it has been claimed that the velocity of money is relatively constant in the short run and changes in the price level are mainly caused by changes in the quantity of money.

Let  $\Delta M_{t+1} = M_{t+1} - M_t$ , the growth rate of  $M$  is then  $\frac{\Delta M_{t+1}}{M_t}$ . In order to derive the money-supply rule I first show that

$$\frac{\Delta M_{t+1}}{M_t} \simeq \ln M_{t+1} - \ln M_t.$$

It is obvious

$$\ln M_{t+1} - \ln M_t = \ln \frac{M_{t+1}}{M_t} = \ln \left[ \frac{\Delta M_{t+1}}{M_t} + 1 \right]. \quad (3.2)$$

Define  $x = \frac{\Delta M_{t+1}}{M_t} + 1$ , the Taylor expansion tells us that

$$f(x) \simeq f(a) + f'(a)(x - a),$$

where  $a$  is a constant and  $f'(a)$  denotes  $\frac{df(x)}{dx}$  evaluated at  $a$ . Taking  $f(x) = \ln x$  and letting  $a = 1$ , one obtains

$$\ln \left[ \frac{\Delta M_{t+1}}{M_t} + 1 \right] = \ln x = \ln 1 + \frac{\Delta M_{t+1}}{M_t} = \frac{\Delta M_{t+1}}{M_t}. \quad (3.3)$$

Equation (3.2) and (3.3) together tell us that  $\frac{\Delta M_{t+1}}{M_t} \simeq \ln M_{t+1} - \ln M_t$ . Taking log of both sides of (3.1), one obtains

$$\ln M_t + \ln V_t = \ln P_t + \ln Y_t$$

and

$$\ln M_{t+1} + \ln V_{t+1} = \ln P_{t+1} + \ln Y_{t+1},$$

If  $V_t$  is assumed to be constant, one has

$$\frac{\Delta M_{t+1}}{M_t} = \frac{\Delta P_{t+1}}{P_t} + \frac{\Delta Y_{t+1}}{Y_t},$$

namely,

$$\hat{m} = \hat{p} + \hat{y}. \quad (3.4)$$

This monetary policy rule has been widely applied since the 1980s, but has been given up by numerous central banks in the past decade. The derivation of the rule above assumes that the velocity of money is constant. This has, however, been a strong assumption. Mishkin (2003, Ch. 21) shows that the velocity of both  $M_1$  and  $M_2$  has fluctuated too much to be seen as a

constant in the US from 1915 to 2002. Moreover, Mendizábal (2004) explores the behavior of money velocity in low and high inflation countries by endogenizing the money velocity which can be influenced by fluctuations in the interest rate. There it is found that there exists a significant correlation between the velocity and inflation rate if transaction costs are considered.

Another assumption of this rule is that there exists a close relation between inflation and nominal money growth. But this relation has not been found to be close in practice because money demand may experience large volatility. Recently, numerous papers have been contributed to this problem and the conclusions differ across countries. Wolfers et al. (1998), for example, test the stability of the money demand in Germany from 1976 to 1994 and find that money demand has been stable except for a structural break around 1990 when the German monetary union was formed. Lütkepohl and Wolters (1998) further explore the stability of the German M3 by way of a system estimation rather than a single-equation estimation and find that there does not exist a strong relation between money and inflation and therefore the money growth appears not to be a good instrument to control inflation. By using different estimation techniques and testing procedures for long-run stability, Scharnagl (1998) also claims to have found stability in the German money demand. Tullio et al. (1996), however, claim that there is empirical evidence that the money demand in Germany has been unstable after the German monetary union was formed. Moreover, Choi and Jung (2003) test for the stability of money demand in the US from 1959 to 2000 and claim that a stable long-run money demand does not exist for the whole period, but a stable long-run money demand is claimed to exist for the subperiods 1959-1974, 1974-1986 and 1986-2000. Vega (1998) explores the stability of money demand in Spain and claims that the long-run properties of the money demand have been altered.

### 3.3 The Interest-Rate Rules

#### The Taylor Rule

Because of the drawbacks of the money-supply rule mentioned above, another type of monetary policy rule, which takes the short-term interest rate as the policy instrument, has been proposed. The most popular interest-rate rule is the so-called Taylor rule (Taylor, 1993), named after John B. Taylor. The Taylor rule can be written as

$$r_t = \bar{r} + \pi_t + \beta_1(\pi_t - \pi^*) + \beta_2 y_t, \quad \beta_1, \beta_2 > 0, \quad (3.5)$$

where  $r_t$  denotes the nominal interest rate,  $\bar{r}$  is the equilibrium real rate of interest,  $\pi_t$  is the inflation rate,  $\pi^*$  is the target inflation and  $y_t$  denotes the deviation of the actual output from its potential level.  $\beta_1$  and  $\beta_2$  are reaction coefficients that determine how strongly the monetary authority stresses inflation stabilization and output stabilization.<sup>1</sup>

Taking  $\pi^*$  as 2 percent and using a linear trend of the real GDP to measure the potential output, Taylor (1993) finds that with  $\beta_1 = 0.5$ ,  $\bar{r} = 2$  and  $\beta_2 = 0.5$  this rule can accurately simulate the short-term nominal interest rate of the US from 1984-1992. Taylor (1999), however, keeps  $\beta_1$  at 0.5 but raises  $\beta_2$  to 1.0.

Taylor (1999) describes briefly how the Taylor rule can be derived from the quantity equation of money (3.1). In deriving the money-supply rule the velocity of money ( $V$ ) is assumed to be constant and the money supply ( $M$ ) is assumed to be a variable. In deriving the Taylor rule, however, Taylor

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<sup>1</sup>Note that  $r_t$  in Eq. (3.5) denotes the *nominal* rate and  $\bar{r}$  the equilibrium *real* rate. One can also express Eq. (3.5) as

$$r_t = r^* + (1 + \beta_1)(\pi_t - \pi^*) + \beta_2 y_t,$$

where  $r_t$  still denotes the nominal rate, but  $r^*$  denotes the equilibrium *nominal* rate rather than the equilibrium *real* rate. Note that the Taylor rule is an “active” monetary policy rule, because its response to the inflation deviation is  $1 + \beta_1 (> 1)$ . Leeper (1991) describes a monetary policy as “active” if its response coefficient to the inflation is larger than one, otherwise it is “passive”.

assumes the money supply to be fixed or growing at a constant rate. The velocity of money, on the contrary, is assumed to depend on the interest rate  $r$  and real output or income ( $Y$ ). From the following paragraph one can get a general idea of how Taylor derives the policy rule:

... First imagine that the money supply is either fixed or growing at a constant rate. We know that velocity depends on the interest rate ( $r$ ) and on real output or income ( $Y$ ). Substituting for  $V$  in the quantity equation one thus gets a relationship between the interest rate, the price level ( $P$ ) and real output. If we isolate the interest rate on the left-hand side of this relationship, we see a function of two variables: the interest rate as a function of the price level and real output. Shifts in this function would occur when either velocity growth or money growth shifts. Note also that such a function relating the interest rate to the price level and real output will still emerge if the money stock is not growing at a fixed rate, but rather responds in a systematic way to the interest rate or to real output; the response of money will simply change the parameters of the relationship.

The functional form of the relationship depends on many factors including the functional form of the relationship between velocity and the interest rate and the adjustment time between changes in the interest rate and changes in velocity. The functional form I use is linear in the interest rate and in the logarithms of the price level and real output. I make the latter two variables stationary by considering the derivation of real output from a possibly stochastic trend and considering the first difference of the log of the price level—or the inflation rate. I also abstract from lags in the response of velocity to interest rate or income. These assumptions result in the following linear equation:

$$r = \pi + gy + h(\pi - \pi^*) + r^f, \quad (3.6)$$

where the variables are  $r$  =the short-term interest rate,  $\pi$  =the inflation rate (percentage change in  $P$ ), and  $y$  =the percentage

deviation of real output ( $Y$ ) from trend and the constants are  $g$ ,  $h$ ,  $\pi^*$ , and  $r^f$ ... (Taylor, 1999, p.322-323).

The  $\pi^*$  is interpreted as the inflation target and  $r^f$  is the central bank's estimate of the equilibrium real rate of interest.

Svensson (2003b, p.19-20) specifies the idea of a commitment to a simple instrument rule such as the Taylor rule as three steps. The first step is to consider a class of reaction functions in which the policy instrument is set as a function of a subset of variables,  $\bar{I}_t$ , of the central bank's information,  $I_t$ ,

$$i_t = f(\bar{I}_t),$$

where  $i_t$  is the instrument ( $r_t$  in the Taylor rule). Usually the instrument is set as a linear function of target variables (inflation and output gap in the Taylor rule) and the lagged instrument.<sup>2</sup> The second step is to determine the numerical values of its parameters ( $g$ ,  $h$ ,  $\pi^*$ , and  $r^f$  in the Taylor rule, for example). The third step is to commit to the particular simple instrument rule chosen until a new rule is determined.

## Comments on the Taylor Rule

Svensson (2003b, p.21) points out that the advantages of a commitment to an interest-rate rule such as the Taylor rule are (1) the simplicity of the instrument rule makes commitment technically feasible, and (2) simple instrument rules may be relatively robust. As regards robustness, he quotes Levin, Wieland and Williams (1999) as an example, who find that a Taylor-type rule with interest-smoothing is robust for different models of the US economy.

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<sup>2</sup>Some Taylor-type rules with interest-smoothing have been proposed in the literature, with the example from Sack and Wieland (2000) being:

$$r_t = \rho r_{t-1} + (1 - \rho)[\bar{r} + \pi_t + \beta_1(\pi_t - \pi^*) + \beta_2 y_t],$$

where  $0 < \rho < 1$  is the smoothing parameter. Sack and Wieland (2000, p.209-210) argue that interest-rate smoothing is desirable for at least three reasons: (a) forward-looking behavior, (b) measurement error in macroeconomic variables, and (c) parameter uncertainty.

Svensson (2003b, p.22-25) also points out that such a simple instrument rule may have some problems, three of which are: (a) other state variables than inflation and output gap might also be important. Asset prices, for instance, might play an important role in an economy. (b) New information about the economy is not allowed for. (c) Such a rule does not seem to describe the current monetary policy accurately.

The recent literature on monetary policy rules, moreover, has proposed two further disadvantages of the Taylor rule.

The first disadvantage is that it has been mostly concerned with a closed economy. Ball (1999), therefore, extends the Svensson (1997)-Ball (1997) closed economy model to an open economy and explores how the optimal policies may change. Ball (1999) finds that the optimal monetary policy rule in an open economy is changed in two ways. First, the policy variable is a combination of the short-term interest rate and the exchange rate, rather than the interest rate alone. This finding supports using the “monetary conditions index” (MCI) as the policy instrument as in the cases of Canada, New Zealand and Sweden.<sup>3</sup> Second, inflation in the Taylor rule is replaced by a combination of inflation and the lagged exchange rate. Therefore, different rules are required for closed and open economies because in open economies monetary policy can influence the economy through the exchange rate channel.

The second disadvantage of the Taylor rule, as explored by Benhabib et al. (2001), is that it may not prevent the economy from falling into a “deflationary spiral”. Benhabib et al. (2001, abstract) argue that active interest-rate rules can lead to “unexpected consequences” in the presence of the zero bound on the nominal rate. That is, there might exist infinite trajectories converging to a Liquidity Trap even if there is an unique equilibrium.

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<sup>3</sup>Deutsche Bundesbank Monthly Report (April 1999, p.54) describes the MCI as “... the MCI is, at a given time  $t$ , the weighted sum of the (relative) change in the effective real exchange rate and the (absolute) change in the short-term real rate of interest compared with a base period...” Some research on the MCI can also be found in Gerlach and Smets (2000).



## Deriving the Interest-Rate Rule from a Dynamic Macroeconomic Model

As mentioned before, Taylor derives the simple Taylor rule from the Fisher equation with the velocity of money defined as a function of the interest rate. In fact, an interest-rate rule that is akin to the Taylor rule can be derived from a simple dynamic macroeconomic model. Before deriving such a monetary policy rule, I will discuss briefly the goal of monetary policy. There are usually two types of objective functions in monetary policy models. Some researchers claim that monetary policy should be pursued to maximize utility functions of the households and firms. This type of objective function is usually employed by the New Classical economists. The other researchers, however, claim that the goal of monetary policy is to minimize a loss function of the monetary authority. This type of objective function is usually employed by the Keynesian economists. But even if it is agreed that monetary policy should be pursued to minimize a loss function of the central bank, there is still disagreement on what kind of loss functions should be minimized. This problem has been explored by Woodford (2003a) in detail. There he finds that the maximization of a utility function of the households can be shown to be consistent with the minimization of loss functions of the central bank. Next, I will make a brief sketch of his analysis, the details can be found in Woodford (2003a, Ch. 6).

**The Goal of Monetary Policy** In the basic analysis Woodford (2003a) assumes that there are no monetary frictions. The level of the representative household's expected utility can be written as

$$E \left\{ \sum_{t=0}^{\infty} \beta^t U_t \right\}, \quad (3.7)$$

where  $\beta$  denotes the discount factor between 0 and 1, and  $U_t$  is the utility function in period  $t$ , which is assumed to have the specific form

$$U_t = u(C_t; \xi_t) - \int_0^1 v(h_t(i); \xi_t) di, \quad (3.8)$$

where  $C_t$  denotes the Dixit-Stiglitz consumption,

$$C_t \equiv \left[ \int_0^1 c_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}},$$

where  $c_t(i)$  denotes the consumption of differentiated goods  $i$  in period  $t$ .  $\theta (> 1)$  is the constant elasticity of substitution between goods.  $\xi_t$  is a vector of preferences shocks and  $h_t(i)$  is the supply of labor used in sector  $i$ . Letting  $G_t$  denote the government purchase and  $y_t(i)$  the production in period  $t$  of differentiated goods  $i$ , and using  $C_t + G_t = Y_t$  and  $y_t(i) = A_t f(h_t(i))$ , one can rewrite the utility function above as

$$U_t = \tilde{u}(Y_t; \tilde{\xi}_t) - \int_0^1 \tilde{v}(y_t(i); \tilde{\xi}_t) di, \quad (3.9)$$

where  $A_t (> 0)$  is a time-varying exogenous technology factor and

$$\tilde{u}(Y; \tilde{\xi}) \equiv u(Y - G; \xi) \quad (3.10)$$

$$\tilde{v}(y; \tilde{\xi}) \equiv v(f^{-1}(y/A); \xi), \quad (3.11)$$

with  $\tilde{\xi}_t$  denoting the complete vector of exogenous disturbances ( $\xi_t$ ,  $G_t$  and  $A_t$ ) and

$$Y_t \equiv \left[ \int_0^1 y_t(i)^{\frac{\theta-1}{\theta}} di \right]^{\frac{\theta}{\theta-1}}. \quad (3.12)$$

Assuming small enough fluctuations in the production, small disturbances and small value of distortion in the steady-state output level and applying the Taylor-series expansion, Woodford (2003a) finds that  $U_t$  can be approximately written as

$$U_t = -\frac{\bar{Y} u_c}{2} \{ (\sigma^{-1} + \omega)(x_t - x^*)^2 + \theta(1 + \omega\theta) \text{var}_i \log p_t(i) \} + t.i.p. + o(\|\bullet\|^3), \quad (3.13)$$

where  $x_t$  denotes the output gap,<sup>4</sup>  $p_t(i)$  is the price level of goods  $i$  and  $x^*$  denotes the efficient level of output gap. *t.i.p.* denotes the terms independent

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<sup>4</sup>Note that the output gap defined by Woodford (2001, 2003a) is the gap between actual output and the natural rate of output, not the same as in Taylor (1993). In Taylor (1993) the output gap is measured by the real GDP relative to a deterministic trend. Woodford (2001, p.234) defines “the natural rate of output as the equilibrium level of

of policy.  $o(\cdot)$  denotes higher-order terms.<sup>5</sup> Woodford (2003a, p.396) further claims that the approximation above “applies to any model with no frictions other than those due to monopolistic competition and sticky prices.”

Considering alternative types of price-setting, Woodford (2003a) finds that the approximation of the utility function above can be written as a quadratic function of the inflation rate and output gap. Examples considered are:<sup>6</sup>

(1) *Case 1*: A fraction of goods prices are fully flexible, while the remaining fraction must be fixed a period in advance. In this case  $U_t$  can be approximated as

$$U_t = -\Omega L_t + t.i.p. + o(\|\bullet\|^3),$$

where  $\Omega$  is a positive constant and  $L_t$  is a quadratic loss function of the form

$$L_t = (\pi_t - E_{t-1}\pi_t)^2 + \lambda(x_t - x^*)^2, \quad (3.14)$$

with  $\pi_t$  denoting the inflation and  $E$  being the expectations operator.  $\lambda$  is the weight of output-gap stabilization.

(2) *Case 2*: Discrete-time version of the Calvo (1983) pricing model. It turns

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output that would obtain in the event of perfectly flexible prices”. Moreover, he claims that “in general, this will not grow with a smooth trend, as a result of real disturbances of many kinds.” Three other concepts concerning output are the steady-state level of output, the efficient level of output and the equilibrium level of output. Let  $s(y, Y; \tilde{\xi})$  denote the real marginal cost function, Woodford (2003a, p.393-394) defines the first two concepts as follows. The steady-state level of output associated with zero inflation in the absence of real disturbances (i.e. when  $\tilde{\xi} = 0$  at all times) is the quantity  $\bar{Y}$  that satisfies  $s(\bar{Y}, \bar{Y}; 0) = (1 - \tau)/\mu$  with  $\tau$  being the constant proportional tax rate on sales proceeds and  $\mu$  the desired markup as a result of suppliers’ market power. The efficient level of output is the quantity  $Y^*$  that satisfies  $s(Y^*, Y^*; 0) = 1$ . Woodford (2003a, p.151) defines the equilibrium level of output  $Y_t^n$  as the quantity that satisfies  $s(Y_t^n, Y_t^n; \tilde{\xi}_t) = \mu^{-1}$ . The efficient level of output gap  $x^*$  is the difference between the efficient level of output and the natural rate of output (see also Woodford (2001)). As for the details of the economic models, the reader is referred to Chapter 3 and Chapter 6 of Woodford (2003a).

<sup>5</sup>This is equation (2.13) in Woodford (2003a, p.396). The reader is referred to Woodford (2003a, Ch.6) for the details of the other parameters and variables in Eq. (3.13).

<sup>6</sup>The reader is referred to Woodford (2003a, Ch. 6) for the details of the derivation of these results.

out that

$$\sum_{t=0}^{\infty} \beta^t U_t = -\Omega \sum_{t=0}^{\infty} \beta^t L_t + t.i.p. + o(\|\bullet\|^3), \quad (3.15)$$

where  $L_t$  is given by

$$L_t = \pi_t^2 + \lambda(x_t - x^*)^2. \quad (3.16)$$

(3) *Case 3: Inflation Inertia.* Eq. (3.15) now holds with  $L_t$  given by

$$L_t = (\pi_t - \gamma\pi_{t-1})^2 + \lambda(x_t - x^*)^2.$$

In the basic analysis Woodford (2003a) also considers the case of habit persistence in the preferences of the representative household and finds that Eq. (3.13) can be modified to include a term of  $x_{t-1}$ .<sup>7</sup> He further shows that the modified equation can also be written in the form of quadratic functions of the inflation rate, output gap and its lag.

While the models above are discussed in a cashless economy, in the extensions of the basic analysis Woodford (2003a) considers the effect of transaction frictions. Therefore, in the extended models interest rates will be taken into account. The approximation of the representative household's utility function is, as a result, correspondingly modified. Under certain assumptions, for example, the approximation in Eq. (3.15) is changed with  $L_t$  now given as follows

$$L_t = \pi_t^2 + \lambda_x(x_t - x^*)^2 + \lambda_i(\hat{i}_t - i^*)^2,$$

where  $\hat{i}_t$  denotes the nominal rate and  $i^*$  is an optimal nominal interest rate. Woodford (2003a) extends the basic analysis by considering not only transaction frictions, but also the zero-interest-rate bound, asymmetric disturbances, sticky wages and prices and time-varying tax wedges or markups. In all cases he finds that the utility function of the representative household can be approximated with the Taylor-series expansion and, as a result, be written in alternative forms of a quadratic loss function of the inflation rate, output gap and interest rate. I will not present all of his analysis here, since

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<sup>7</sup>The reader is referred to Woodford (2003a, Chapter 5, p.332-335) for the discussion of habit persistence.

this requires much discussion and the reader can refer to Chapter 6 of his book for details.

Recently, some researchers, Nobay and Peel (2003), for example, argue that the loss function of the central bank may be asymmetric rather than symmetric. Therefore, the quadratic loss functions proposed above may not appropriately express the central bank's preferences. Therefore, some research has been done in the framework of an asymmetric loss function. A typical asymmetric loss function is the so-called LINEX function.<sup>8</sup> To be precise, it is argued that the central bank may suffer less loss when the inflation is under its target than when it is above its target and the opposite is true of the output gap. Dolado et al. (2001) show that most central banks show a stronger reaction to the positive inflation deviation than to the negative one, but no asymmetric behavior with respect to the output gap is found except for the Federal Reserve.

Tambakis (1998), however, explores monetary policy with a convex Phillips curve and an asymmetric loss function and finds that "for parameters estimates relevant to the United States, the symmetric loss function dominates the asymmetric alternative" (Tambakis, 1998, abstract). Schellekens and Chadha (1998) also explore monetary policy with an asymmetric loss function and argue that asymmetries affect the optimal rule under both additive and multiplicative uncertainty, but the policy rule is shown to be similar or equivalent to that obtained in the case of a quadratic loss function. Moreover, they further claim that the assumption of quadratic loss functions may not be so drastic in monetary policy-making. Svensson (2002, p.5 and footnote 6) also claims that a symmetric loss function for monetary policy is very intuitive, because too low inflation can be as great a problem as too high inflation, since the former may lead to the problem of the Liquidity Trap and deflationary spirals, as has happened in Japan. He further argues that "asymmetric loss functions are frequently motivated from a descriptive rather than perspective point of view," and that a competent monetary policy committee should make decisions from a perspective point of view (Svensson,

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<sup>8</sup>The graph of this function is shown in Figure 6.3.

2002, p.5 and footnote 6).

Because of the literature mentioned above, in the research below I will assume that the central bank pursues monetary policy to minimize a quadratic loss function.

**Derivation of an Interest-Rate Rule** Next, I show how to derive an interest-rate rule from a dynamic macroeconomic model. The simple model reads

$$\underset{\{r_t\}_0^\infty}{Min} \sum_{t=0}^{\infty} \rho^t L_t \quad (3.17)$$

with<sup>9</sup>

$$L_t = (\pi_t - \pi^*)^2 + \lambda y_t^2, \quad \lambda > 0,$$

subject to

$$\pi_{t+1} = \alpha_1 \pi_t + \alpha_2 y_t, \quad \alpha_i > 0 \quad (3.18)$$

$$y_{t+1} = \beta_1 y_t - \beta_2 (r_t - \pi_t), \quad \beta_i > 0, \quad (3.19)$$

where  $\pi_t$  denotes the deviation of the inflation rate from its target  $\pi^*$  (assumed to be zero in the model),  $y_t$  is the output gap,  $r_t$  denotes the gap between the short-term nominal rate  $R_t$  and the long-run level of the short-term rate  $\bar{R}$ , namely  $r_t = R_t - \bar{R}$ .  $\rho$  is the discount factor bounded between 0 and 1. (3.18) is the Phillips curve and (3.19) is the IS curve.<sup>10</sup>

Following Svensson (1997, 1999b), I will derive the optimal monetary policy rule from the above model.<sup>11</sup> Let's ignore the state equation of  $y_t$  at the moment. The problem now turns out to be

$$V(\pi_t) = \underset{y_t}{Min} [(\pi_t^2 + \lambda y_t^2) + \rho V(\pi_{t+1})] \quad (3.20)$$

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<sup>9</sup>If  $\lambda = 0$ , the model is referred to as "strict inflation targeting", here I assume  $\lambda > 0$ , therefore, it is "flexible inflation targeting".

<sup>10</sup>In order for consistent expectations to exist,  $\alpha_1$  is usually assumed to be 1. The loss function here is similar to that in the second case of Woodford (2003a) shown above with  $x^*$  equaling zero. The discussion about  $x^* = 0$  can be found in Woodford (2003a, p.407).

<sup>11</sup>The reader can also refer to Svensson (1997) and the appendix of Svensson (1999b) for the derivation below.

subject to

$$\pi_{t+1} = \alpha_1 \pi_t + \alpha_2 y_t \quad (3.21)$$

Equation (3.20) is the so-called Hamilton-Jacobi-Bellman (HJB) equation and  $V(\pi_t)$  is the value function, with  $y_t$  being the control variable now. For a linear-quadratic (LQ) control problem above, it is clear that the value function must be quadratic. Therefore, I assume that the value function takes the form

$$V(\pi_t) = \Omega_0 + \Omega_1 \pi_t^2, \quad (3.22)$$

where  $\Omega_0$  and  $\Omega_1$  remain to be determined. The first-order condition turns out to be

$$\lambda y_t + \rho \alpha_2 \Omega_1 \pi_{t+1} = 0,$$

from which one has

$$\pi_{t+1} = -\frac{\lambda}{\rho \alpha_2 \Omega_1} y_t. \quad (3.23)$$

Substituting (3.23) into (3.19) gives

$$y_t = -\frac{\rho \alpha_1 \alpha_2 \Omega_1}{\lambda + \rho \alpha_2^2 \Omega_1} \pi_t, \quad (3.24)$$

and after substituting this equation back into (3.23), one has

$$\pi_{t+1} = \frac{\alpha_1 \lambda}{\lambda + \rho \alpha_2^2 \Omega_1} \pi_t. \quad (3.25)$$

By applying (3.20), (3.22) and (3.24), the envelop theorem gives us the following equation

$$V_\pi(\pi_t) = 2 \left( 1 + \frac{\alpha_1^2 \rho \lambda \Omega_1}{\lambda + \rho \alpha_2^2 \Omega_1} \right) \pi_t,$$

and from (3.22), one has

$$V_\pi(\pi_t) = 2\Omega_1 \pi_t,$$

these two equations tell us that

$$\Omega_1 = 1 + \frac{\alpha_1^2 \rho \lambda \Omega_1}{\lambda + \rho \alpha_2^2 \Omega_1}.$$

The right-hand side of this equation has the limit  $1 + \frac{\alpha_1^2 \lambda}{\alpha_2^2}$  as  $\Omega_1 \rightarrow \infty$ . The root of  $\Omega_1$  larger than one can therefore be solved from the equation

$$\Omega_1^2 - \left[ 1 - \frac{(1 - \rho\alpha_1^2)\lambda}{\rho\alpha_2^2} \right] \Omega_1 - \frac{\lambda}{\rho\alpha_2^2} = 0,$$

which gives the solution of  $\Omega_1$ :

$$\Omega_1 = \frac{1}{2} \left( 1 - \frac{\lambda(1 - \rho\alpha_1^2)}{\rho\alpha_2^2} + \sqrt{\left( 1 - \frac{\lambda(1 - \rho\alpha_1^2)}{\rho\alpha_2^2} \right)^2 + \frac{4\lambda}{\rho\alpha_2^2}} \right). \quad (3.26)$$

By substituting  $t + 1$  for  $t$  into (3.24), one has

$$y_{t+1} = -\frac{\rho\alpha_1\alpha_2\Omega_1}{\lambda + \rho\alpha_2^2\Omega_1}\pi_{t+1}. \quad (3.27)$$

Substituting (3.18) and (3.19) into (3.27) with some computation, one obtains the optimal decision rule for the short-term interest rate:

$$R_t = \bar{R} + f_1\pi_t + f_2y_t, \quad (3.28)$$

with

$$f_1 = 1 + \frac{\rho\alpha_1^2\alpha_2\Omega_1}{(\lambda + \rho\alpha_2^2\Omega_1)\beta_2}, \quad (3.29)$$

$$f_2 = \frac{\beta_1}{\beta_2} + \frac{\rho\alpha_2^2\alpha_1\Omega_1}{(\lambda + \rho\alpha_2^2\Omega_1)\beta_2}; \quad (3.30)$$

Equation (3.28) shows that the optimal short-term interest rate should be a linear function of the inflation rate and output gap. This is similar to the Taylor rule presented before in the sense that the short-term interest rate is a linear function of the output gap and inflation deviation. Note that  $f_1 > 1$ , indicating the optimal monetary policy should be “active”. That is, there is a more than one-for-one increase in the nominal interest rate with the increase in inflation.

**Simulation of the Model** Next, I undertake some simulations with the US quarterly data from 1961.1-99.4. The seemingly uncorrelated regression



(SUR) estimation of the IS and Phillips curves reads<sup>12</sup>

$$\pi_{t+1} = \underset{(0.800)}{0.0007} + \underset{(59.406)}{0.984}\pi_t + \underset{(3.948)}{0.066}y_t, \quad R^2 = 0.958, \quad (3.31)$$

$$y_{t+1} = \underset{(0.529)}{-0.0006} + \underset{(20.203)}{0.960}y_t - \underset{(2.662)}{0.157}\{(R_t - \pi_t) - \bar{R}\}, \quad R^2 = 0.788. \quad (3.32)$$

With the parameters estimated above and  $\lambda=0.1$ ,  $\rho=0.985$ , one obtains  $\Omega_1=4.93$  and the following optimal policy reaction function

$$R_t = \bar{R} + 17.50\pi_t + 7.22y_t. \quad (3.33)$$

Let both  $\pi_0$  and  $y_0$  be 0.03, the simulations with  $\lambda = 0.1$  are presented in Figure 3.1. Next, I undertake the simulation with a larger  $\lambda$ . Let  $\lambda=10$ , one obtains  $\Omega_1=22.76$  and the following optimal interest rate reaction function

$$R_t = \bar{R} + 1.92\pi_t + 6.18y_t, \quad (3.34)$$

with the simulations presented in Figure 3.2. The response coefficients of the inflation deviation and output gap are relatively large, because the estimate of  $\beta_1$  is relatively larger than that of  $\beta_2$ .

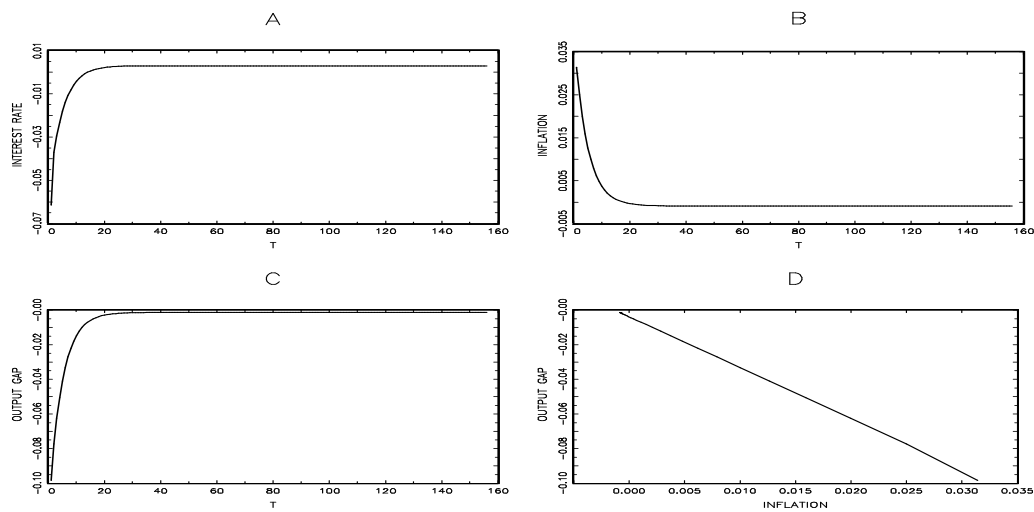
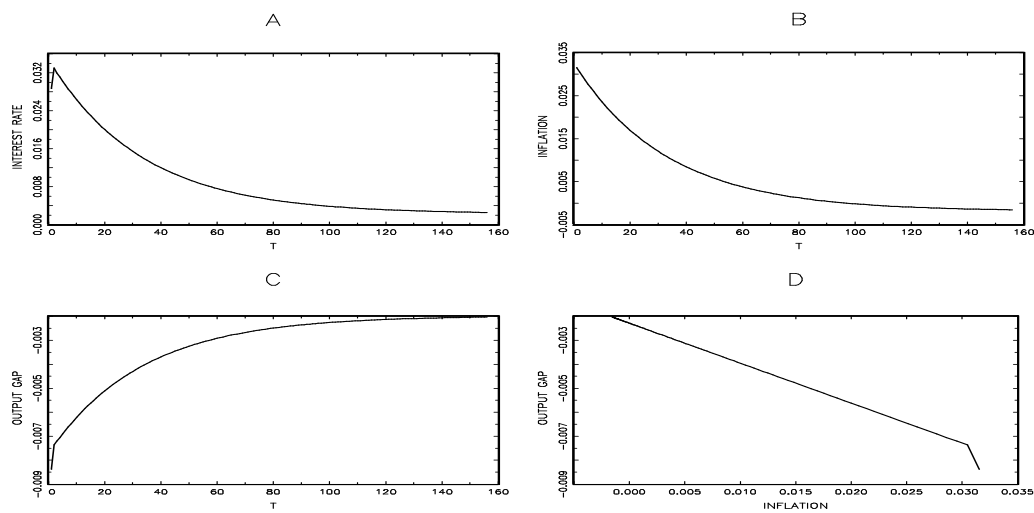
Figure 3.1A and 3.2A represent the path of the optimal interest rate, Figure 3.1B-C and 3.2B-C are the optimal trajectories of  $\pi_t$  and  $y_t$ , and Figure 3.1D and 3.2D are the phase diagrams of the inflation deviation and output gap with starting values (0.03, 0.03). Both Figure 3.1 and 3.2 show that the optimal trajectories of the inflation deviation and output gap converge to zero over time. As the inflation deviation and output gap converge to zero, the optimal feedback rule converges to the long run equilibrium interest rate  $\bar{R}$ . From (3.19) one knows that as  $\pi_{t+1}$ ,  $\pi_t$ ,  $y_{t+1}$  and  $y_t$  converge to zero,  $R_t \rightarrow \bar{R}$ .

Next, I explore how the relative weight of output stabilization,  $\lambda$ , influences the optimal monetary policy rule. Denoting  $f = \frac{f_1}{f_2}$ , one has

$$f = \frac{1}{\Theta}[(\lambda + \rho\alpha_2^2\Omega_1)\beta_2 + \rho\alpha_1^2\alpha_2\Omega_1], \quad (3.35)$$

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<sup>12</sup>I assume  $\bar{R}$  to be zero for simplicity. The inflation rate is measured by changes in the CPI, the output gap is measured by the percentage deviation of the log of the Introduction Production Index (base year: 1995) from its HP filtered trend.  $R_t$  is the federal funds rate. Data source: International Statistical Yearbook.

Figure 3.1: Simulation with  $\lambda=0.1$ Figure 3.2: Simulation with  $\lambda=10$

with  $\Theta = (\lambda + \rho\alpha_2^2\Omega_1)\beta_1 + \rho\alpha_2^2\alpha_1\Omega_1$ , and

$$\frac{df}{d\lambda} = \frac{1}{\Theta^2}[\rho\alpha_1\alpha_2\Omega_1(\alpha_2\beta_2 - \alpha_1\beta_1)]. \quad (3.36)$$

It is clear that  $\frac{df}{d\lambda} < 0$  ( $> 0$ ) if  $\alpha_2\beta_2 - \alpha_1\beta_1 < 0$  ( $> 0$ ). As long as the inflation and output are greatly influenced by their lags, as is usually true in estimations, one has  $\alpha_2\beta_2 - \alpha_1\beta_1 < 0$ . This implies that if  $\lambda$  increases, namely, if more emphasis is put on the output stabilization than on the inflation, the ratio of the reaction coefficient on the output gap and that on the inflation in the optimal monetary policy rule is correspondingly relatively larger. In the simulation above  $f = 0.41$  if  $\lambda = 0.1$ , and  $f = 3.22$  if  $\lambda = 10$ .

Svensson (2003b, p.39), however, points out that such an interest-rate rule may have the following problems: (a) the objectives may not be sufficiently well specified. It is not clear, for example, what the relative weight on the output-gap stabilization should be. (b) Such discretionary optimization is argued not to be fully optimal in a situation with forward-looking variables.

Another interesting topic concerning money-supply and interest-rate rules is price-level (in)determinacy. This problem originated from Wicksell (1898) as follows

At any moment and in every economic situation there is a certain level of the average rate of interest which is such that the general level of prices has no tendency to move either upwards or downwards. This we call the *normal* rate of interest. Its magnitude is determined by the current level of the natural capital rate, and rises and falls with it.

If, for any reason whatever, the average rate of interest is set and maintained *below* this normal level, no matter how small the gap, prices will rise and will go on rising; or if they were already in process of falling, they will fall more slowly and eventually begin to rise.

If, on the other hand, the rate of interest is maintained no matter

how little *above* the current level of the natural rate, prices will fall continuously and without limit (Wicksell, 1898, p.120).<sup>13</sup>

This problem has been discussed by numerous researchers, see Sargent and Wallace (1975), Carlstrom and Fuerst (2000), Benhabib et al. (2001) and Woodford (2001, 2003a), for example. Sargent and Wallace (1975) argue that while money-supply rules lead to a determinate rational-expectations equilibrium, none of the interest-rate rules do. Carlstrom and Fuerst (2000) also show that money-growth rules can produce real determinacy and interest-rate rules may not necessarily do so. As mentioned before, Benhabib et al. (2001) argue that even active interest-rate rules can lead to indeterminacy. Woodford (1994) specify sufficient conditions for price-level determinacy for both money-supply and interest-rate rules in a cash-in-advance model.

Woodford (2003a) discusses the problem of price-level determinacy in detail and claims that interest-rate rules can lead to price-level determinacy when some conditions are satisfied. Woodford (2003a, Ch. 2) analyzes both *local* and *global* price-level determinacy in a model, assuming that prices are completely flexible and the supply of goods is given by an exogenous endowment. There he finds that interest-rate rules can lead to price-level determinacy *locally* if certain conditions are satisfied. Moreover, he finds that interest-rate rules can lead to *global* price-level determinacy under certain fiscal-policy regimes. Woodford (2003a, Ch. 4) discusses this problem further in the so-called “neo-Wicksellian” model and specify conditions under which price-level determinacy can be obtained.

### 3.4 Conclusion

In this chapter I have discussed the money-supply rule and the interest-rate rule, with more attention given to the latter. The European Central

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<sup>13</sup>Wicksell (1898, p.102) describes the natural rate of interest as “There is a certain rate of interest on loans which is neutral in respect to commodity prices, and tends neither to raise nor to lower them.” Woodford (2003a, p.248) defines it explicitly as the equilibrium real rate of return in the case of fully flexible prices.

Bank (ECB) originally followed the money-supply rule. It had been argued that the German Bundesbank had achieved a solid reputation in keeping the inflation rate down with monetary targeting (Semmler, 2003, p.12). Interest-rate rules have, however, attracted more attention since the 1990s. The stabilizing properties of these two monetary policy rules are studied in a macroeconomic framework in Flaschel, Semmler and Gong (2001), where it is found that, by and large, the interest-rate rule has better stabilizing properties in both stable and unstable cases. In the medium run, with the Taylor rule, employment, inflation, expected inflation and output experience smaller fluctuations than with the money-supply rule. In line with most recent research on monetary policy rules, this dissertation focuses on the interest-rate rules in the following chapters.

# Chapter 4

## Time-Varying Monetary Policy Rules

### 4.1 Introduction

As shown in the previous chapter, interest-rate rules propose that the short-term interest rate can be determined as a function of the output gap and the deviation of the inflation rate from its target. A monetary policy rule can be referred to as “active” or “passive”, depending on whether the coefficient of the inflation rate is larger or smaller than one. Up to now I have assumed the coefficients in the interest-rate rule to be invariant. In practice, however, the coefficients can be state-dependent and time-varying. It is obvious that the reaction coefficients of the inflation rate and output gap in the instrument rule derived from the dynamic macroeconomic model (3.17)-(3.19) depend on the parameters in the IS and Phillips curves and the loss function of the central bank. Therefore, the reaction coefficients in the optimal interest-rate rule change with the changes of the parameters in the IS and Phillips curves and the loss function of the central bank.

The empirical evidence of the time-varying Phillips curve in Chapter 2, as a result, indicates that the coefficients in the policy reaction function may be time-varying rather than invariant. Greiner and Semmler (2002), moreover, claim that the weight of output stabilization ( $\lambda$ ) in the central bank’s loss

function can be state-dependent. The change of  $\lambda$ , as a result, can also induce the change of the response coefficients in the optimal monetary policy rule. As quoted in Chapter 3, Taylor (1999) states that shifts in the monetary policy reaction function relating the interest rate and the price level and real output would occur when either velocity growth or money growth shifts. Therefore, he suggests different values of parameters in the Taylor rule for different periods.

This chapter presents some empirical evidence on structural changes in the coefficients in the monetary policy reaction function in the past decades in several OECD countries. In Section 2 I will first present the OLS estimation of an interest-rate rule and undertake the Chow break-point tests to study structural changes in the coefficients. While the Chow break-point tests can only explore structural changes at certain predetermined points, the Kalman filter can explore all possible changes in the coefficients. Therefore, in the third section I will estimate the interest-rate rule by employing the Kalman filter. In Section 4 I will explore whether the monetary policy was too tight in the Euro-area in the 1990s by undertaking some simulations, assuming that the time-varying US monetary policy rule had been followed by the Euro-area.

## 4.2 The OLS Regression and Chow Break-Point Tests of the Interest-Rate Rule

Let us write the interest-rate rule as:

$$r_t = \beta_c + \beta_\pi \pi_t + \beta_y y_t, \quad (4.1)$$

where  $r_t$  is the short-term interest rate,  $\pi_t$  is the deviation of the inflation rate from its target and  $y_t$  denotes the output gap. Because the inflation targets are unavailable, I will take it as a constant and refer to the research of Clarida, Gali and Gertler (1998) (CGG98) who estimate the inflation target for several countries.  $y_t$  is the output gap which is measured by the percentage deviation of the Industrial Production Index (IPI, base year: 1995) from

its HP-filtered trend. There are alternative methods to measure the output gap, a discussion of this problem can be found in Orphanides and van Norden (2002). I find that the potential output measured with the Band-Pass filter is not essentially changed from that computed with the HP filter. The countries to be examined include Germany, France, Italy, the UK, Japan and the US.

**Germany** CGG98 explore monetary policy rules under the assumption that, while making monetary policy the monetary authorities take into account the expected inflation rate rather than the lagged inflation rate or the current inflation rate. A by-product of their model is the inflation target. Their estimate of the German target inflation rate from 1979-1993 is 1.97 percent. This seems consistent with the official German target inflation rate, which is usually declared to be 2 percent. Therefore, in the estimation below I assume the inflation target of Germany to be 2 percent.<sup>1</sup> The short-term interest rate (3 month FIBOR, denoted by  $r$ ), inflation rate (denoted by  $\text{inf}$ , measured by changes in the CPI) and output gap (denoted by  $\text{gap}$ ) of Germany are shown in Figure 4.1 (Data Source: International Statistical Yearbook).

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<sup>1</sup>The inflation target does not affect the regression much as long as it is assumed to be a constant.



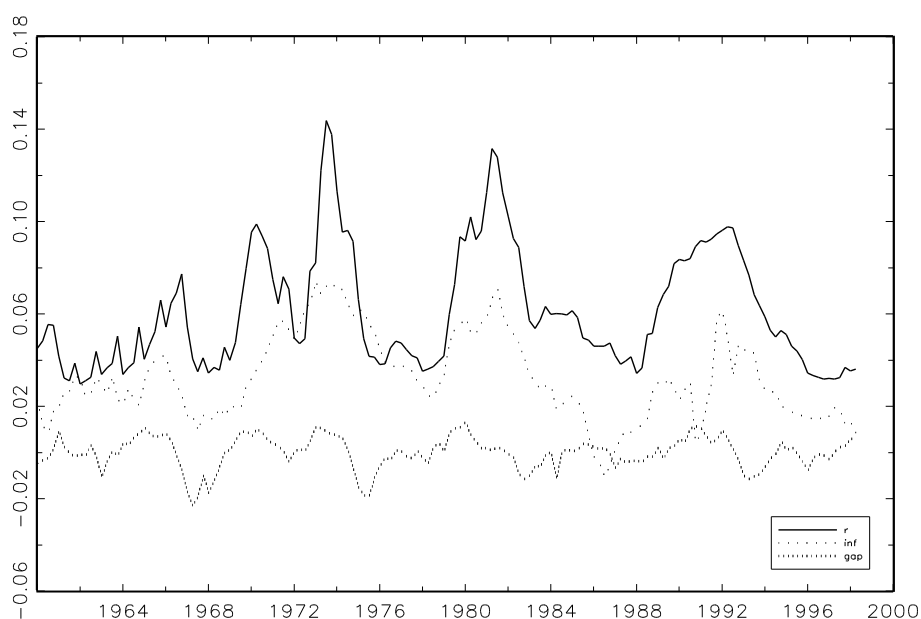


Figure 4.1: Inflation Rate, Output Gap and Short-Term Interest Rate of Germany

The estimation results of the policy reaction function for Germany from 1960-1998 are shown in Table 4.1. I will, for simplicity, not present the estimate of  $\beta_c$ .

Table 4.1: OLS Estimation of the Interest-Rate Rule of Germany

Sample	$\beta_\pi$		$\beta_y$		$R^2$
	Estimate	T-St.	Estimate	T-St.	
1960.1-69.4	0.052	0.181	0.372	1.300	0.070
1970.1-79.4	1.170	6.028	1.937	5.140	0.660
1980.1-89.4	1.086	14.414	0.713	2.179	0.148
1990.1-98.2	1.201	5.905	1.766	3.723	0.579
1960.1-98.2	0.841	10.337	0.972	4.480	0.494

The estimates above indicate some changes in the coefficients for different subperiods. The inflation rate seems to have played a more important role in monetary policy-making in the 1970s and 1980s than the output, while in

the 1960s and 1990s the output may have had larger effects on the monetary policy. This is consistent with the fact that the inflation rate was relatively low in the 1960s and has been decreasing since the beginning of the 1990s. In order to explore whether there are structural changes in the policy reaction function, I will undertake the Chow break-point test for the regression. I choose 1979 and 1990 as two break-points, when the EMS started and the reunification of Germany took place. The F-Statistics of the break-point tests for 1979.4 and 1989.2 are 15.913 and 4.044 respectively, significant enough to indicate structural changes around these two points (the critical value at 5 percent level of significance lies between 2.60 and 2.68).

**Japan** The estimate of CGG98 of the inflation target of Japan for the period from 1979.4-94.12 is 2.03 percent. I will, therefore, assume it to be 2 percent in the estimation below, since the average inflation rate of the period from 1960-1997 is not higher than that of the period from 1979-1994. The short-term interest rate (call money rate), inflation rate (changes in the CPI) and output gap of Japan are presented in Figure 4.2 (Data Source: International Statistical Yearbook).

The estimation for Japan from 1960.1-1997.4 is shown in Table 4.2.

Table 4.2: OLS Estimation of the Interest-Rate Rule of Japan

Sample	$\beta_\pi$		$\beta_y$		$R^2$
	Estimate	T-St.	Estimate	T-St.	
1960.1-64.4	0.075	0.377	0.740	2.493	0.269
1965.1-69.4	0.334	1.206	0.738	4.576	0.560
1970.1-79.4	0.430	4.825	0.344	1.234	0.376
1980.1-89.4	0.598	4.676	2.171	4.976	0.472
1990.1-97.4	0.405	2.128	1.398	2.541	0.494
1960.1-1997.4	0.216	4.055	0.657	3.008	0.131

The changes in  $\beta_\pi$  are not very significant, but the changes in  $\beta_y$ , however, are relatively large. It was smaller than one before 1980, but higher than one

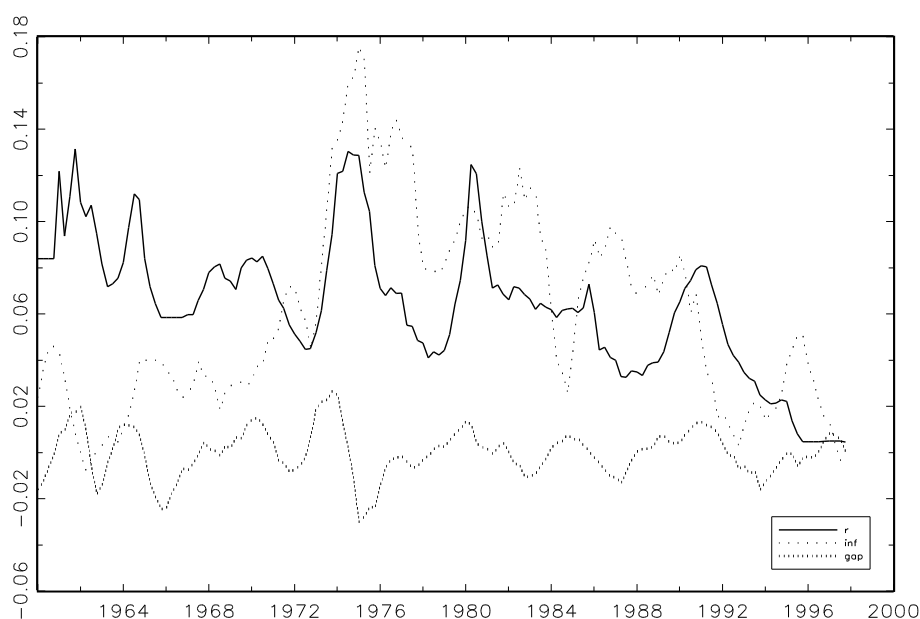


Figure 4.2: Inflation Rate, Output Gap and Short-Term Interest Rate of Japan

after 1980, especially in the 1980s. Next, I will undertake the Chow break-point test for 1974.4 and 1980.4 around which there were great changes in both the inflation rate and interest rate. The F-Statistics are 43.492 and 33.944 respectively, significant enough to indicate structural changes around these two points (the critical value at 5 percent level of significance lies between 2.60 and 2.68). I have also undertaken the Chow break-point test for 1965.1 and the F-Statistic is 28.400, significant enough to indicate a structural change at this point.

**The US** The estimate by CGG98 of the US inflation target is 4.04 percent for the period from 1979-1994. As stated by the authors, a target of 4 percent seems to be too high for the US, given a sample average real rate of 3.48 percent. In the estimation below I therefore simply assume the target inflation to be 2.5 percent, a little higher than that of Germany. The short-term interest rate (the federal funds rate), inflation rate (changes in the CPI) and output gap of the US are presented in Figure 4.3 (Data Source: International Statistical Yearbook) and the estimation results of the policy

reaction function for different periods are shown in Table 4.3.

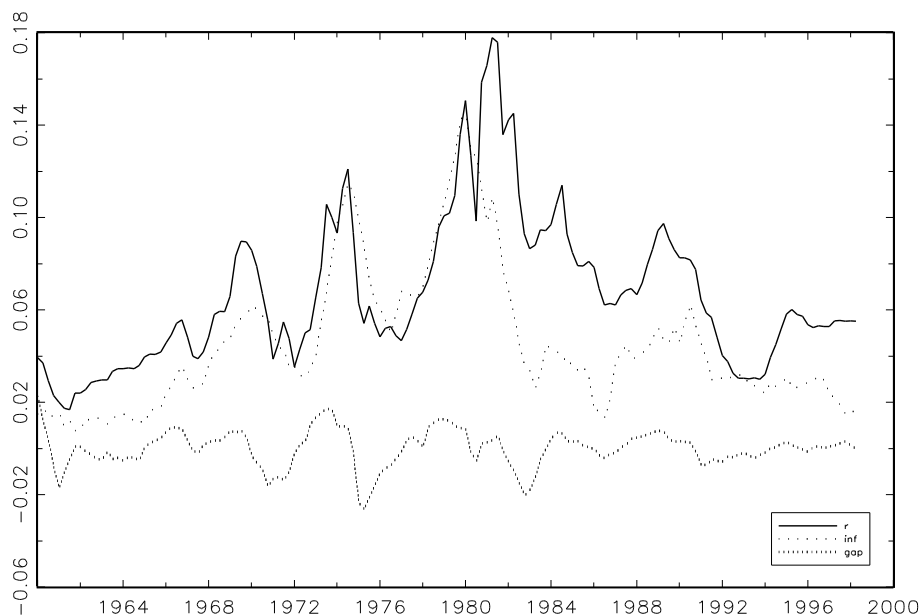


Figure 4.3: Inflation Rate, Output Gap and Short-Term Interest Rate of the US

Table 4.3: OLS Estimation of the Interest-Rate Rule of the US

Sample	$\beta_{\pi}$		$\beta_y$		$R^2$
	Estimate	T-St.	Estimate	T-St.	
1960.1-69.4	1.047	14.913	0.443	2.965	0.903
1970.1-79.4	0.643	9.975	0.808	5.245	0.825
1980.1-84.4	0.489	3.152	0.723	1.053	0.493
1985.1-89.4	-0.027	0.097	2.230	2.439	0.481
1990.1-98.2	0.854	5.034	2.389	3.965	0.556
1960.1-98.2	0.772	13.117	0.527	2.333	0.562

One can observe some significant changes in the coefficients for the US. In the middle of the 1980s the coefficient of the inflation rate changed even from positive to negative. In the first half of the 1980s  $\beta_{\pi}$  was much larger

than  $\beta_y$  with a significant T-Statistic, but in the second half of the 1980s  $\beta_\pi$  became negative with an insignificant T-Statistic of 0.097. This indicates that the inflation rate may have played a more important role in monetary policy-making in the first half than in the second half of the 1980s. This should not be surprising, since the US experienced very high inflation rate in the first half of the 1980s and the interest rate was raised to deal with this problem after Volcker was appointed the chair of the Fed.

Next, I undertake the Chow break-point test for 1982.1 because there were significant changes in the inflation rate and interest rate around this point. The F-Statistic is 18.920, significant enough to indicate a structural change at this point (the critical value at 5 percent level of significance lies between 2.60 and 2.68).

**France** CGG98 fail to obtain a reasonable estimate of the inflation target for France and it is then assumed it to be 2 percent for the period 1983-1989. Since the data used here cover a much longer period (1970-96) than that of CGG98, I assume the inflation target to be 2.5 percent for France, since France experienced a high inflation rate from the beginning of the 1970s to the middle of the 1980s, with the average rate higher than 8 percent. The inflation rate (changes in the CPI), short-term interest rate (3-month treasury bill rate) and output gap of France are presented in Figure 4.4 (Data Source: International Statistical Yearbook). The output gap was quite smooth during the whole period except a relatively significant change in the middle of the 1970s. The inflation rate was quite high before the middle of the 1980s and decreased to a relatively lower level around 1985. The regression results are shown in Table 4.4.

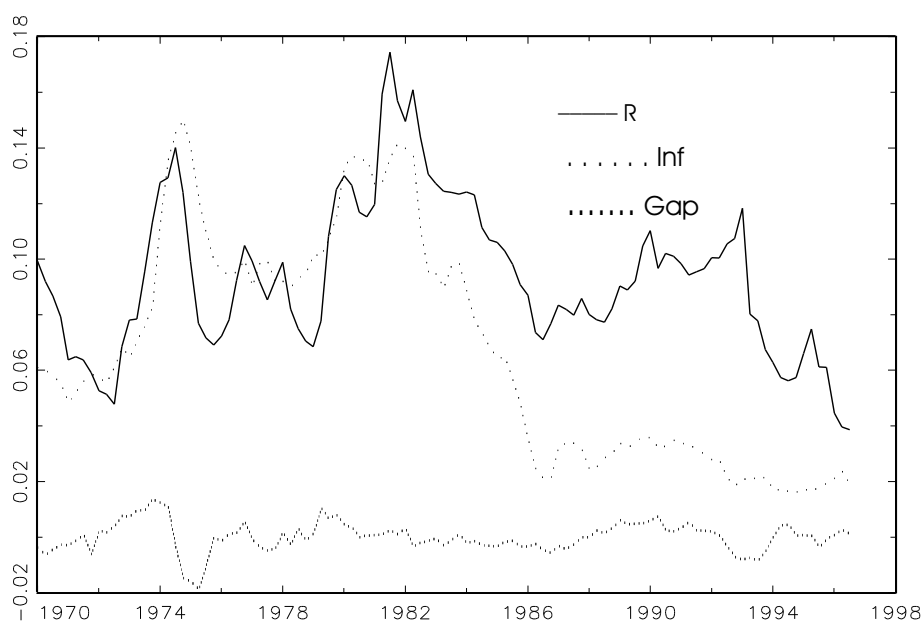


Figure 4.4: Inflation Rate, Output Gap and Short-Term Interest Rate of France

Table 4.4: OLS Estimation of the Interest-Rate Rule of France

Sample	$\beta_{\pi}$		$\beta_y$		$R^2$
	Estimate	T-St.	Estimate	T-St.	
1970.1-79.4	0.603	6.257	0.835	2.391	0.523
1980.1-89.4	0.570	12.842	0.180	0.280	0.822
1990.1-96.3	2.345	4.142	0.778	0.832	0.507
1970.1-96.3	0.425	8.207	0.365	0.930	0.395

One can observe a significant change in the  $\beta_{\pi}$ . It was about 0.60 before 1990, but rose to 2.345 in the 1990s. Unfortunately, the estimate of  $\beta_y$  has insignificant T-Statistics most of the time. This may suggest either model misspecification or problems in the output gap measurement. The Chow break-point test for 1979.4 has an F-Statistic of 29.143, significant enough to indicate a structural change at this point (the critical value at 5 percent

level of significance is about 2.70). One can observe some large changes in the interest rate and inflation rate around this point in Figure 4.4.

**The UK** CGG98 are also unable to obtain a reasonable estimate of the inflation target for the UK. I assume it to be 2.5 percent in the estimation for the period from 1960.1-1997.4. The short-term interest rate (3-month treasury bill rate), inflation rate (changes in the CPI) and output gap of the UK are presented in Figure 4.5 (Data Source: International Statistical Yearbook). The regression results of the interest-rate rule are shown in Table 4.5.

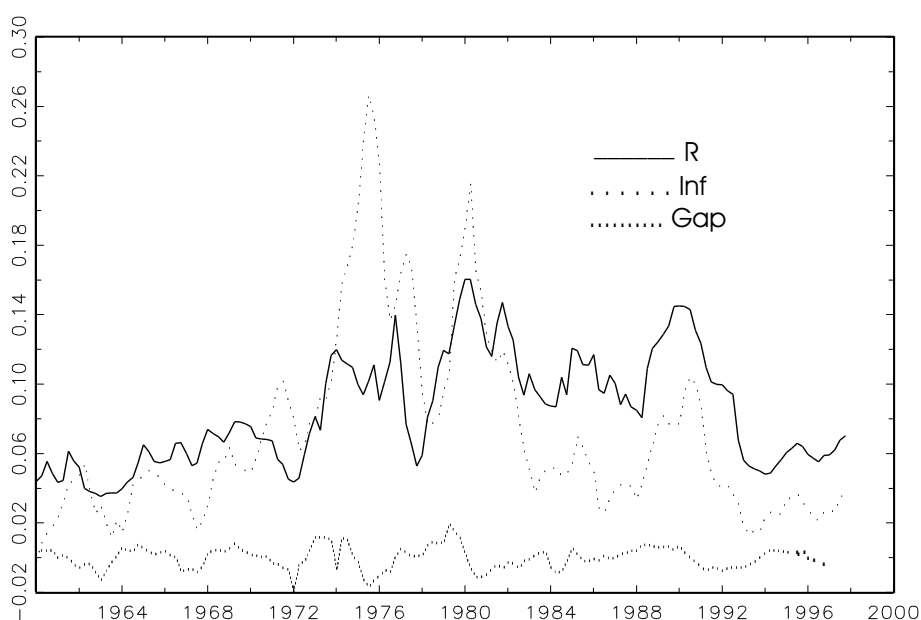


Figure 4.5: Inflation Rate, Output Gap and Short-Term Interest Rate of the UK

Table 4.5: OLS Estimation of the Interest-Rate Rule of the UK

Sample	$\beta_\pi$		$\beta_y$		$R^2$
	Estimate	T-St.	Estimate	T-St.	
1960.1-69.4	0.440	4.072	0.644	2.045	0.409
1970.1-79.4	0.322	5.496	1.790	4.812	0.520
1980.1-89.4	0.453	10.075	0.886	2.319	0.745
1990.1-97.4	1.144	16.596	-1.252	2.858	0.910
1960.1-97.4	0.358	9.151	0.802	2.391	0.363

It is surprising that  $\beta_y$  was negative with a significant T-Statistic in the 1990s. This may be due to model misspecification or the computation of the output gap. The  $\beta_y$  seems to have experienced more significant changes than the  $\beta_\pi$ . I undertake the Chow break-point test for 1979.1 and obtain an F-Statistic of 72.900, significant enough to indicate a structural change at this point (the critical value at 5 percent level of significance lies between 2.60 and 2.68).

**Italy** CGG98 explore the monetary policy of Italy for the period from 1981-89 and fail to obtain a reasonable inflation target. My estimation covers the period from 1970-98. The inflation rate was quite high during this period, evolving between 1.18 percent and 24.75 percent with the average value being 9.72 percent. Therefore, I assume the target inflation to be 3.0 percent, a little higher than those of the other European countries. I present the short-term interest rate (official discount rate), inflation rate (changes in the CPI) and output gap of Italy in Figure 4.6 (Data Source: International Statistical Yearbook) and the regression results in Table 4.6.



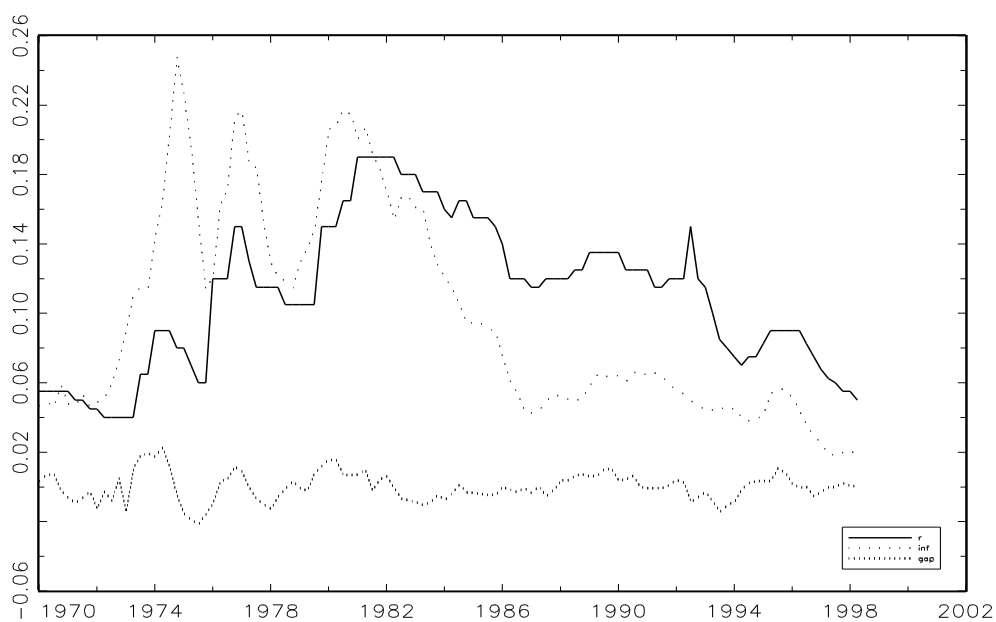


Figure 4.6: Inflation Rate, Output Gap and Short-Term Interest Rate of Italy

Table 4.6: OLS Estimation of the Interest-Rate Rule of Italy

Sample	$\beta_{\pi}$		$\beta_y$		$R^2$
	Estimate	T-St.	Estimate	T-St.	
1970.1-79.4	0.401	5.937	0.468	1.294	0.513
1980.1-89.4	0.354	8.120	0.073	0.184	0.707
1990.1-98.2	1.361	7.730	0.696	1.593	0.729
1970.1-98.2	0.340	5.889	0.301	0.700	0.248

The F-Statistic of the Chow break-point test for 1979.4 is 67.473, significant enough to indicate a structural change at this point (the critical value at 5 percent level of significance is about 2.50).

### 4.3 Estimation of the Time-Varying Interest-Rate Rule with the Kalman Filter

From the OLS regression and Chow break-point tests one finds that there are some structural changes in the monetary reaction function. The drawback of the Chow break-point test is that one can only explore whether there are structural changes at some predetermined points. This approach is not of much help if one wants to explore all structural changes or wants to obtain the path of a time-varying parameter. In order to explore how the coefficients in the monetary policy reaction function may have changed over time, I will estimate the time-varying interest-rate rule with the Kalman filter in this section. In Chapter 2 I have estimated the time-varying Phillips curve with the Kalman filter, assuming that the coefficient in the Phillips curve follows a random-walk path.

Somewhat different from the estimation in Chapter 2, however, I will employ the so-called “Return-to-Normality” (mean-reversion) model in this section, that is, I assume that the time-varying parameters are stationary and evolve around a mean. If the parameter is found to be non-stationary, I will give up the mean-reversion model and resort to the random-walk model as in Chapter 2. A brief introduction to the “Return-to-Normality” model is shown in the appendix of Chapter 2.

#### Empirical Evidence

Let’s define the variables as follows:

$$x_t = \begin{pmatrix} 1 \\ \pi_t \\ y_t \end{pmatrix} \text{ and } \beta_t = \begin{pmatrix} \beta_{ct} \\ \beta_{\pi t} \\ \beta_{yt} \end{pmatrix}.$$

In the “Return-to-Normality” model the time-varying coefficients are assumed to be generated by a stationary multivariate AR(1) process. The interest-rate rule can then be written in the following State-Space form

$$r_t = x_t' \beta_t + \epsilon_t, \quad t = 1, \dots, T,$$

$$\beta_t - \bar{\beta} = \phi(\beta_{t-1} - \bar{\beta}) + \eta_t,$$

where  $\epsilon_t \sim NID(0, H)$ , and  $\eta_t \sim NID(0, Q)$ . The coefficients are stationary and evolve around the mean,  $\bar{\beta}$ . After arranging the interest-rate rule in an SSM one can use the Kalman filter to estimate  $\phi$ ,  $\bar{\beta}$ ,  $\beta_t$  and, as a result, obtains a path of  $\alpha_t$ . The estimation results of Germany, France, Italy, Japan, the UK and the US are presented below. If the elements of the matrix  $\phi$  are larger than one in absolute value, the “Return-to-Normality” model has to be abandoned and the random-walk model should be employed.

**Germany** The German data from 1960-98 generate the  $\phi$  as

$$\begin{pmatrix} 0.935 & 0 & 0 \\ 0 & 0.892 & 0 \\ 0 & 0 & 0.925 \end{pmatrix}.$$

All elements of  $\phi$  are smaller than one, indicating that the coefficients are stationary. The  $\bar{\beta}$  is  $\begin{pmatrix} 0.052 \\ 0.260 \\ 0.294 \end{pmatrix}$ , indicating that  $\beta_c$  evolves around 0.052,  $\beta_\pi$  around 0.260 and  $\beta_y$  around 0.294. The paths of  $\beta_\pi$  and  $\beta_y$  are shown in Figure 4.7A-B. The path of  $\beta_c$  is not shown here just for simplicity.

As shown in Figure 4.7A,  $\beta_\pi$  experiences significant changes. Comparing Figure 4.7A with Figure 4.1, one finds that the switching of  $\beta_\pi$  was similar to that of  $\pi_t$ , except in the 1960s. That is, when the inflation rate was high,  $\beta_\pi$  was also high and vice versa. In 1970, 1974 and 1981,  $\beta_\pi$  reached some peaks, when the interest rate and inflation rate were also at their peaks. In the 1960s  $\beta_\pi$  and  $\pi_t$  evolved in opposite directions most of the time, especially from 1965-1970. The fact that the changes of  $\beta_\pi$  and  $\pi_t$  are inconsistent with each other in the 1960s may be caused by the initial startup idiosyncracies of the Kalman filter algorithm. From 1960-1965  $\beta_\pi$  was below zero most of the time, consistent with the OLS regression ( $\beta_\pi = -0.804$  from 1960.1-1964.4). Figure 4.7A shows that  $\beta_\pi$  experienced a significant structural change around 1979 and a small change around 1989, consistent with the Chow break-point

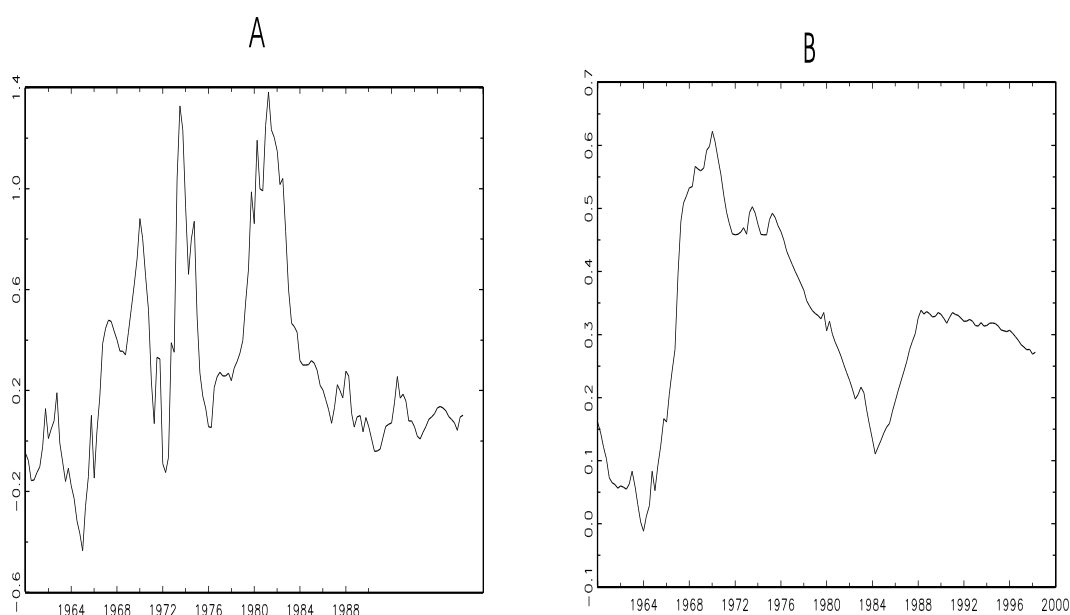


Figure 4.7: Time-Varying  $\beta_\pi$  and  $\beta_y$  of Germany

tests in the previous section.  $\beta_y$  experienced significant changes around 1970 and 1984.

**France** The French data from 1970-96 generate the  $\phi$  as

$$\begin{pmatrix} 0.967 & 0 & 0 \\ 0 & 0.826 & 0 \\ 0 & 0 & 0.575 \end{pmatrix}$$

with all elements smaller than one, indicating that the return-to-normality model is the right choice.  $\bar{\beta}$  equals  $\begin{pmatrix} 0.064 \\ 0.631 \\ 0.091 \end{pmatrix}$ , indicating that  $\beta_c$ ,  $\beta_\pi$  and  $\beta_y$  evolve around 0.064, 0.631 and 0.091 respectively. The paths of the  $\beta_\pi$  and  $\beta_y$  are presented in Figure 4.8A-B.

Figure 4.8A shows that  $\beta_\pi$  experienced significant changes in the 1970s and has been staying at a relatively stable level since the middle of the 1980s. It decreased to the lowest point in 1979 and reached the highest point in 1981, when the interest rate also reached the highest point.  $\beta_\pi$  remained at a

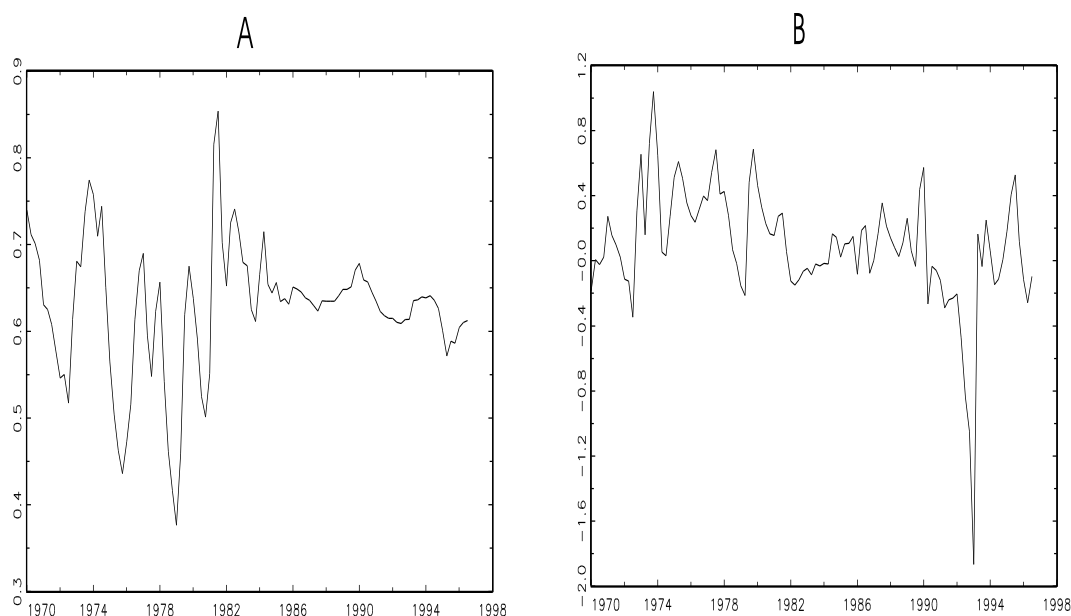


Figure 4.8: Time-Varying  $\beta_\pi$  and  $\beta_y$  of France

relatively high level after the 1980s, even if the inflation rate has been quite low since the middle of the 1980s, which may indicate the effect of the EMS on the monetary policies of member countries.  $\beta_y$  also experienced a change in 1979. This is consistent with the conclusion of the Chow break-point test in the previous section. Note that  $\beta_y$  had a negative mean ( $-0.153$ ) in the 1990s and decreased to the lowest point of  $-1.867$  in 1993, consistent with the fact that  $\beta_y$  in the OLS regression was negative in the 1990s.

**The UK** The UK data from 1960-97 generate the  $\phi$  as

$$\begin{pmatrix} 0.956 & 0 & 0 \\ 0 & 0.931 & 0 \\ 0 & 0 & 0.049 \end{pmatrix}$$

with all elements smaller than one. Note that the last element is very small (0.049), indicating that  $\beta_y$  may not have experienced significant structural changes.  $\bar{\beta}$  is  $\begin{pmatrix} 0.069 \\ 0.353 \\ 0.330 \end{pmatrix}$ , indicating that  $\beta_c$ ,  $\beta_\pi$  and  $\beta_y$  evolve around 0.069,

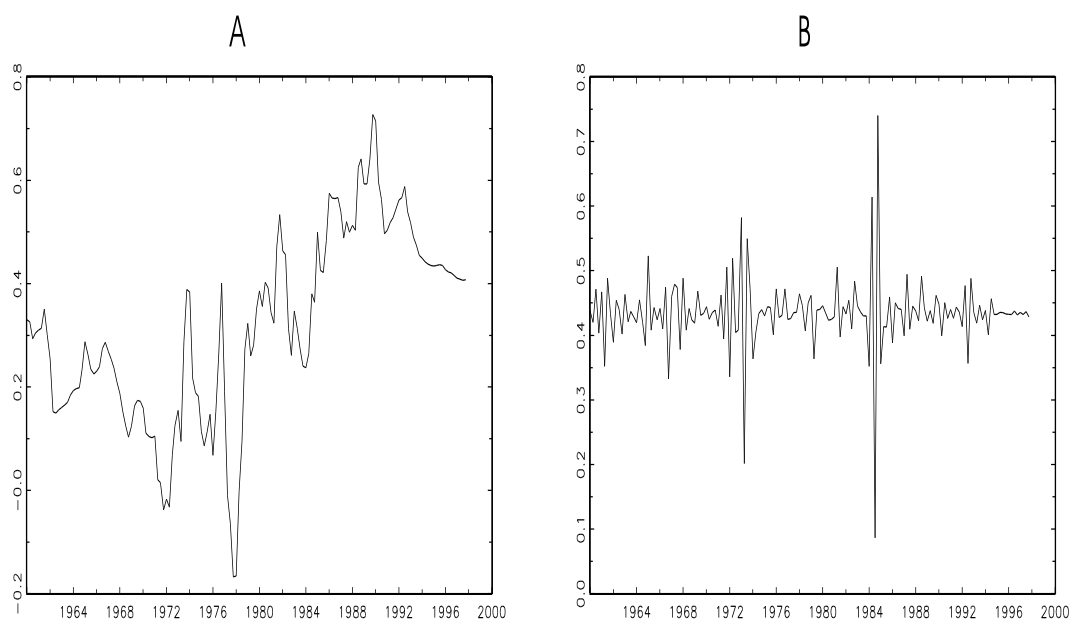


Figure 4.9: Time-Varying  $\beta_\pi$  and  $\beta_y$  of the UK

0.353 and 0.330 respectively. The paths of  $\beta_\pi$  and  $\beta_y$  are presented in Figure 4.9A-B respectively.

Figure 4.9A shows that  $\beta_\pi$  experienced significant changes in the 1970s and remained at a relatively high and stable level afterwards. Note that the switching of  $\beta_\pi$  is similar in France and the UK: it experienced similar changes in the 1970s and then stayed at a relatively high level without significant changes after the 1980s.

Figure 4.9B shows that  $\beta_y$  did not experience such significant changes as those of the other European countries. This is consistent with the fact that the last element in  $\phi$  is not large (0.049).

**Italy** The Italian data from 1970-98 generate the  $\phi$  as

$$\begin{pmatrix} 0.992 & 0 & 0 \\ 0 & 1.021 & 0 \\ 0 & 0 & 0.400 \end{pmatrix}$$

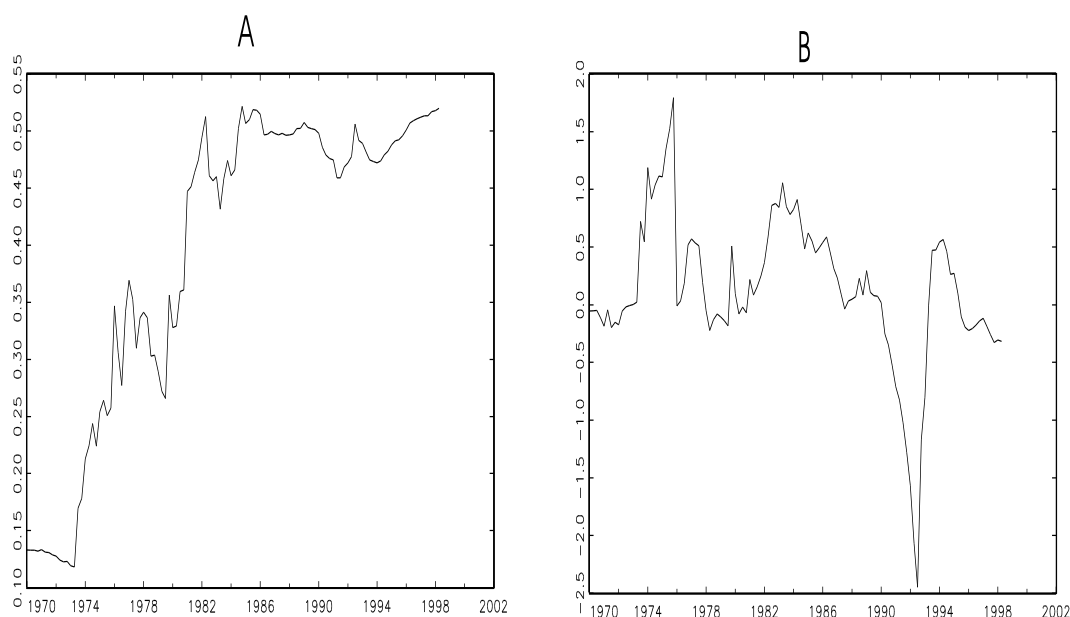


Figure 4.10: Time-Varying  $\beta_\pi$  and  $\beta_y$  of Italy

and  $\bar{\beta}$  as  $\begin{pmatrix} 0.066 \\ 0.059 \\ 0.238 \end{pmatrix}$ . Because the second diagonal element of  $\phi$  is larger than one,  $\beta_\pi$  is therefore non-stationary and I have to employ the random-walk model instead of the “Return-to-Normality” model. The paths of  $\beta_\pi$  and  $\beta_y$  estimated with the random-walk model are presented in Figure 4.10A-B.

Figure 4.10A shows that  $\beta_\pi$  has been increasing since the middle of the 1970s. It experienced a structural change in 1979 and then increased to a relatively stable and high level, similar to the cases of France and the UK.  $\beta_y$  of Italy also experienced a large decrease around 1993, similar to the case of France.

**Japan** The data of Japan from 1960-97 generate the  $\phi$  as

$$\begin{pmatrix} 1.013 & 0 & 0 \\ 0 & 0.935 & 0 \\ 0 & 0 & 0.715 \end{pmatrix}.$$

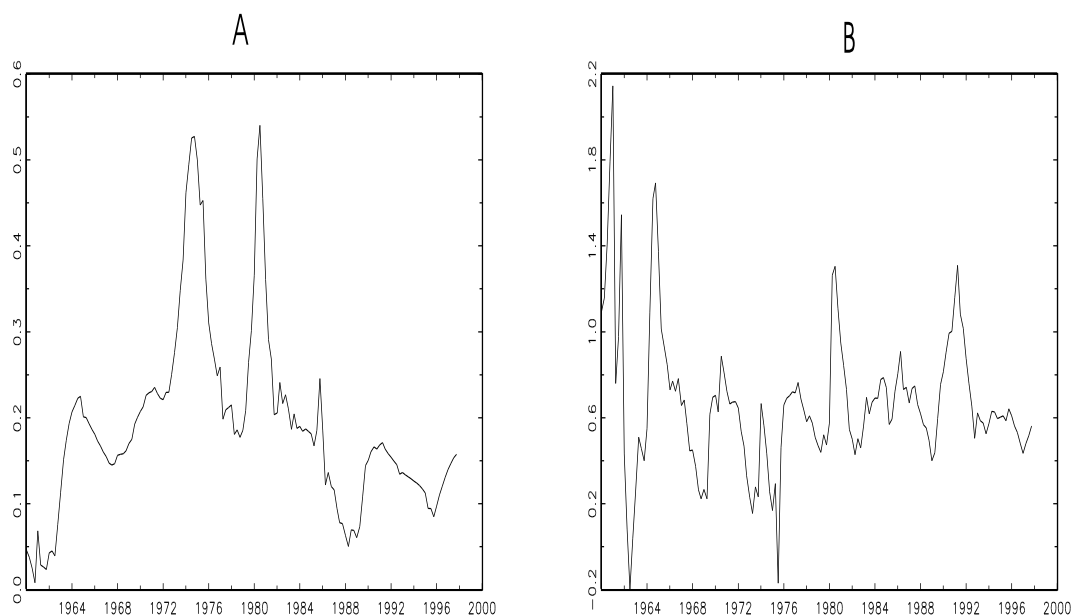


Figure 4.11: Time-Varying  $\beta_\pi$  and  $\beta_y$  of Japan

One element of  $\phi$  is larger than one and the other two are smaller than one. This implies that  $\beta_c$  is non-stationary, but  $\beta_\pi$  and  $\beta_y$  are stationary. Because the intercept is not of much interest, I stick to the “Return-to-Normality”

model.  $\bar{\beta}$  is  $\begin{pmatrix} -0.258 \\ 0.177 \\ 0.674 \end{pmatrix}$ , implying that  $\beta_c$  evolves around  $-0.258$ ,  $\beta_\pi$  around

$0.177$  and  $\beta_y$  around  $0.674$ . The paths of  $\beta_\pi$  and  $\beta_y$  are presented in Figure 4.11A-B.  $\beta_\pi$  experienced large changes around 1974 and 1980, attaining the highest point of about 0.55. This is consistent with the switching of the interest rate and inflation rate, which also attained their highest values around these two points.

In the previous section I have undertaken the Chow break-point test for 1974.4 and 1980.4 when there were great changes in the interest rate and conclude that there are indeed structural changes in the model. Figure 4.11A-B confirm this conclusion:  $\beta_\pi$  attained its second highest value around 1974 and  $\beta_y$  also increased to a high value. Figure 4.11A-B also show that there were structural changes in both coefficients between 1980 and 1981, when



the interest rate and inflation increased to some large values. In 1964 there were also break-points in both  $\beta_\pi$  and  $\beta_y$ , consistent with the conclusion of the Chow break-point test.

**The US** The US data from 1960-98 generate the  $\phi$  as

$$\begin{pmatrix} 0.991 & 0 & 0 \\ 0 & 0.893 & 0 \\ 0 & 0 & 0.674 \end{pmatrix},$$

with all elements smaller than one, indicating that the coefficients are all stationary.  $\bar{\beta}$  is  $\begin{pmatrix} 0.050 \\ 0.448 \\ 0.705 \end{pmatrix}$ , indicating that  $\beta_c$  evolves around 0.050,  $\beta_\pi$  around 0.448 and  $\beta_y$  around 0.705. The paths of  $\beta_\pi$  and  $\beta_y$  are presented in Figure 4.12A-B respectively.

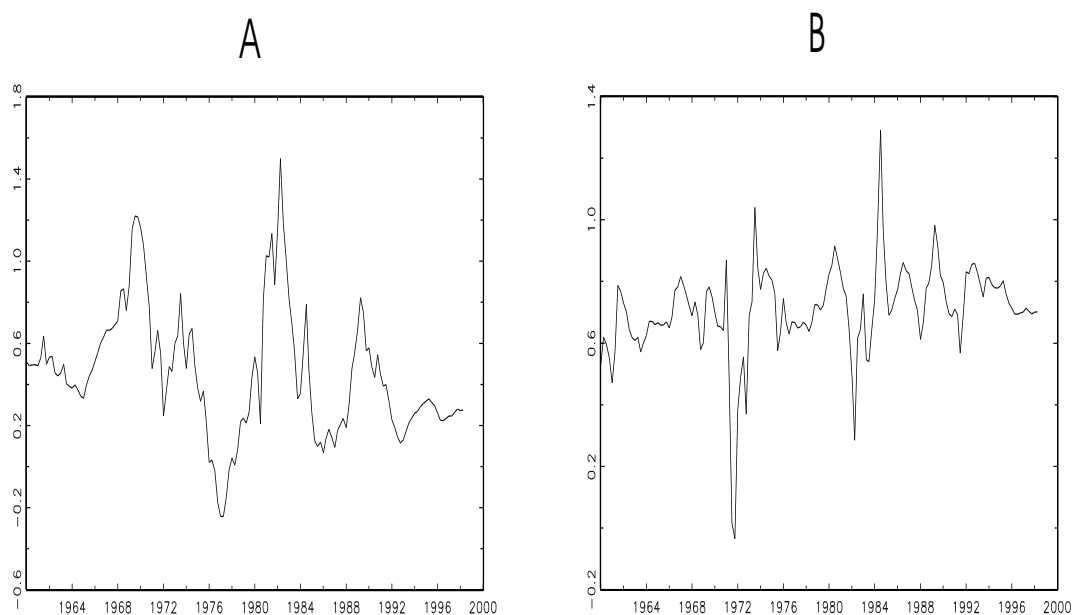


Figure 4.12: Time-Varying  $\beta_\pi$  and  $\beta_y$  of the US

Figure 4.12A shows that the switching of  $\beta_\pi$  is very similar to that of the inflation rate and interest rate. That is, when the inflation rate was high  $\beta_\pi$  was also high.

Above I have estimated the time-varying coefficients in the interest-rate rule and find that there do exist some structural changes. One may propose that the policy reaction coefficients of the inflation rate and output gap are state-dependent. That is, the changes of the economic environment may have caused the changes in the coefficients. One observes that the changes in the coefficients seem to have been more or less consistent with the changes in the corresponding economic variables, the inflation rate and output gap. In order to explore whether there is some empirical evidence for this argument, I will estimate the following two equations, taking the US as an example:

$$\beta_\pi = c_1 + c_2\pi_t, \quad (4.2)$$

$$\beta_y = \tau_1 + \tau_2 y_t. \quad (4.3)$$

The estimation results for different subperiods are shown in Table 4.7 and 4.8.

Table 4.7: State-Dependent Evidence of the US  $\beta_\pi$

Sample	$c_1$		$c_2$		$R^2$
	Estimate	T-St.	Estimate	T-St.	
1960.1-69.4	0.615	41.781	13.989	13.790	0.833
1970.1-74.4	0.584	6.799	1.468	0.765	0.032
1975.1-79.4	-0.217	2.474	5.528	3.932	0.462
1980.1-89.4	0.423	5.651	4.180	2.385	0.130
1990.1-98.2	0.255	15.458	6.575	4.750	0.414
1960.1-98.2	0.428	14.278	1.303	1.584	0.016

The state-dependent evidence of  $\beta_\pi$  seems more obvious than that of  $\beta_y$ , since the estimates of Eq. (4.2) usually have more significant T-Statistics and higher  $R^2$  than those of Eq. (4.3). In fact, comparing Figure 4.12 and Figure 4.3 one can find some similar evidence. The change of the  $\beta_\pi$  seems to be more consistent with the change of the inflation rate than the  $\beta_y$  with the output gap.

Table 4.8: State-Dependent Evidence of the US  $\beta_y$ 

Sample	$\tau_1$		$\tau_2$		$R^2$
	Estimate	T-St.	Estimate	T-St.	
1960.1-69.4	0.667	52.160	3.217	1.728	0.073
1970.1-79.4	0.654	21.442	4.821	1.854	0.083
1980.1-89.4	0.768	33.552	11.201	3.353	0.228
1990.1-94.4	0.772	38.250	3.586	0.679	0.025
1995.1-98.2	0.716	56.978	5.363	0.648	0.034
1960.1-98.2	0.650	36.000	4.500	1.600	0.070

## Comparison of E3-Countries

CGG98 refer to France, Italy and the UK as the E3 countries, in contrast to the so-called G3 countries of Germany, Japan and the US whose central banks have virtually autonomous control over the domestic monetary policies. Above I have presented the estimation results of the time-varying coefficients in the interest-rate rule of the E3 countries. As mentioned before, the changes in the coefficients in the monetary reaction function in the case of these three countries are, to some extent, similar. I will analyze this problem briefly below.  $\beta_\pi$  of the three countries are shown in Figure 4.13. The  $\beta_\pi$  of the UK is presented from 1970-98, so that it is consistent with the time period of the estimation of the other two countries.

Figure 4.13 shows that the  $\beta_\pi$  of the three countries experienced some significant changes in the 1970s and then remained at a relatively stable and high level after the middle of the 1980s. This indicates that the inflation deviation may have played an important role in the three countries' policy making after 1980. Moreover, the switching of  $\beta_\pi$  in the cases of the UK and France is very similar before 1985, though the  $\beta_\pi$  of France stayed at a higher level than that of the UK in this period. I also present the inflation rates of the three countries in Figure 4.14.

Figure 4.14 shows that the inflation rates of the three countries also ex-

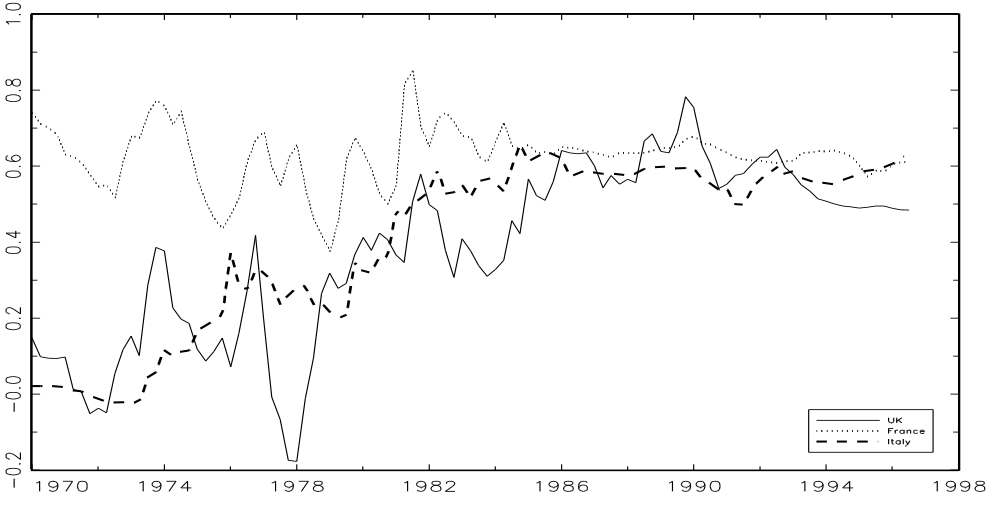


Figure 4.13:  $\beta_\pi$  of E3 Countries

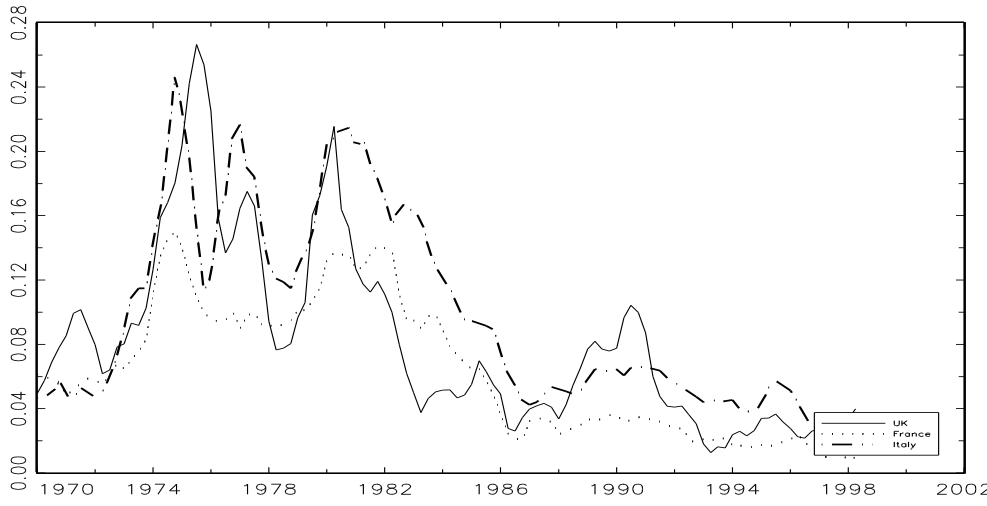


Figure 4.14: Inflation Rates of E3 Countries

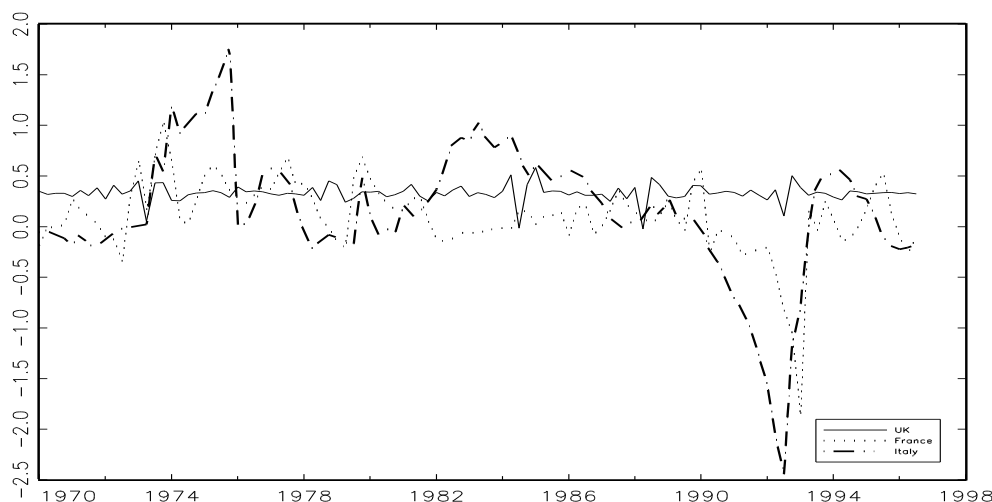


Figure 4.15:  $\beta_y$  of E3 Countries

perienced some similar changes: they increased to the highest value around 1975, went down at the end of the 1970s, increased to another high point at the beginning of the 1980s and then decreased persistently with some small increases around 1990 and 1995. The similarity of the inflation rates among the three countries may explain the consistency of  $\beta_\pi$  to some extent. But the EMS may also have some common effects on the monetary policy of the three countries. I present  $\beta_y$  of the three countries in Figure 4.15.

The switching of the  $\beta_y$  in Italy and France is also similar most of the time. That is, both decreased to the lowest point between 1992 and 1993 when the crisis of the EMS occurred. The  $\beta_y$  of the UK is very smooth, as mentioned before. The output gaps for the three countries are presented in 4.16.

Figure 4.16 shows that the output gaps of the three countries also experienced some similar changes, especially in the cases of the UK and France. This evidence seems to indicate some consistency between the monetary policies of the E3 countries. One can observe that for all three countries the response coefficient of the inflation deviation moved up and stayed high in the 1990s and that the response coefficient of the output gap is almost constant except when Germany raised the interest rate after the German reunification and the other countries had to raise the interest rate too, in spite of a

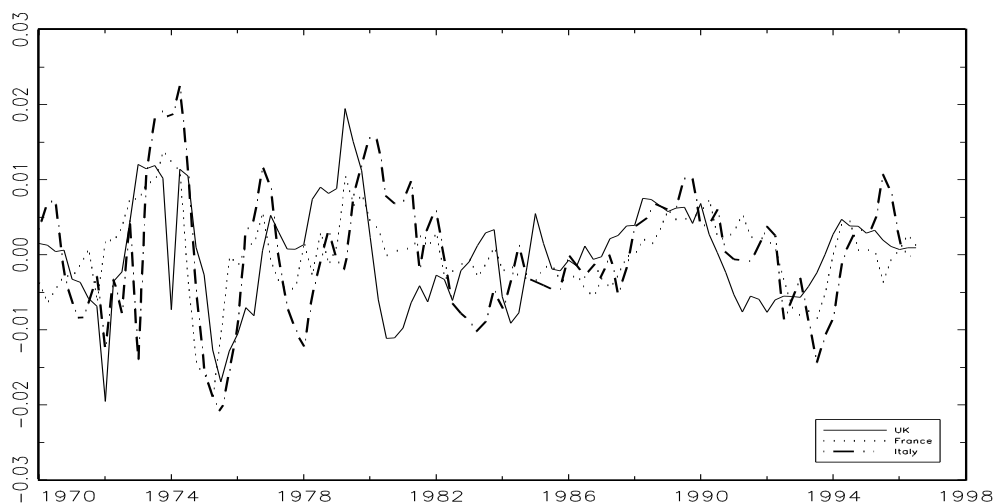


Figure 4.16: Output Gaps of E3 Countries

negative output gap.

#### 4.4 Euro-Area Monetary Policy Effects Using the Time-Varying US Monetary Policy Rule

It is well known that in the 1990s the economy of the Euro-area has performed worse than the US economy. The difference in the growth and unemployment performance of the Euro-area and the US may be seen to have been caused by differences in monetary policies. Difference in the interest rates can be seen in Figure 4.17. Similar to Peersman and Smets (1998), I use the German call money rate to study the monetary policy in the Euro-area.<sup>2</sup> The aggregate inflation rate and output gap of the Euro-area are measured respectively by the GDP-weighted sums of the inflation rates and output gaps of Germany, France and Italy (referred to as the EU3). In particular, before 1994 the interest rate of the US was much lower (4.9 percent on average) than that of the Euro-area (8.5 percent on average). For the whole decade of the 1990s the

<sup>2</sup>Peersman and Smets (1998) justify using the German day-to-day rate to measure the monetary policy in the Euro-area.

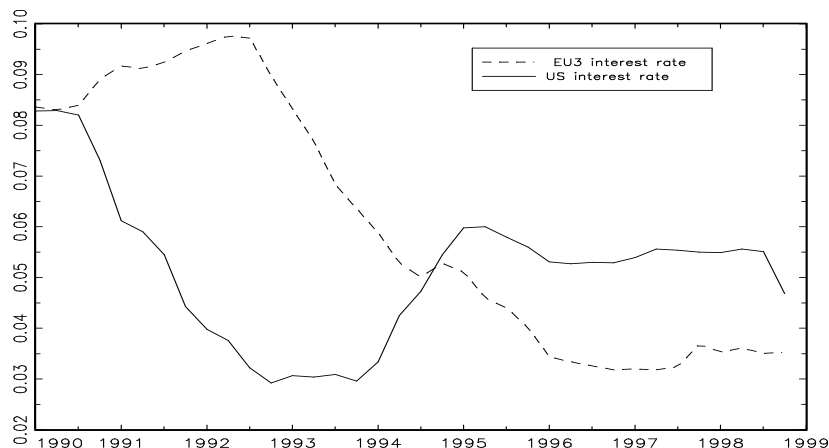


Figure 4.17: Interest Rates of the US and Euro-Area

average rate of the US was 5.1 percent, while that of the Euro-area was 6.1 percent. One finds similar results for the real interest rate. The real interest rate of the US in the 1990s was 1.8 percent, while that of the Euro-area was 3.2 percent. So an interesting question is: what would have happened if the Euro-area had followed the monetary policy rule of the US in the 1990s? In this section I will undertake some simulation of the Euro-area economy with the time-varying US monetary policy rule estimated in the previous section.

I will simulate the inflation rate and output gap for the Euro-area from 1990-98, assuming that the Euro-area had followed the monetary policy of the US. A similar counterfactual study has been undertaken by Taylor (1999) using the pre-Volcker policy interest rate reaction function to study the macroeconomic performance of the Volcker and post-Volcker periods.

Let us write the interest-rate rule with time-varying response coefficients as:

$$r_t = \bar{r} + \beta_{\pi t} \pi_t + \beta_{y_t} y_t, \quad (4.4)$$

where  $\bar{r}$  is the long-run equilibrium interest rate, and other variables are interpreted the same as in the previous sections. The time-varying paths of

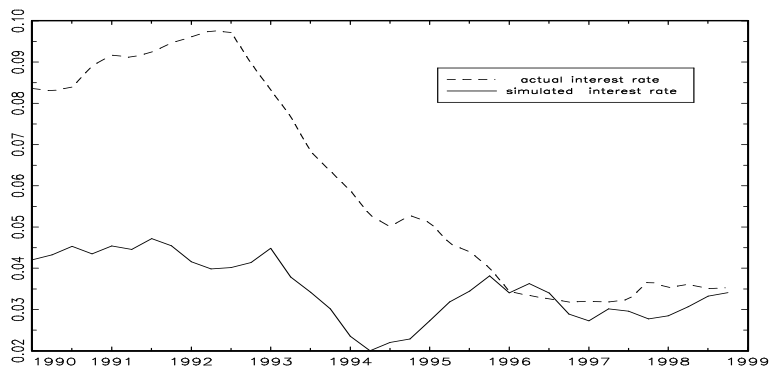


Figure 4.18: Actual and Simulated Interest Rates of the Euro-Area

$\beta_\pi$  and  $\beta_y$  of the US are presented in Figure 4.12A-B.

Next, I assume that the Euro-area follows the monetary policy of the US by determining the interest rate according to the HP-filtered trends of  $\beta_{\pi t}$  and  $\beta_{y t}$  of the US instead of the exact paths of  $\beta_{\pi t}$  and  $\beta_{y t}$ , since I assume that the Euro-area had followed only approximately the US monetary policy rule. I simulate the Euro-area interest rate with equation (4.4) by measuring  $\bar{r}$  with the average real interest rate of the Euro-area of the 1990s and substituting the US  $\beta_{\pi t}$  and  $\beta_{y t}$  trends for  $\beta_{\pi t}$  and  $\beta_{y t}$ . The inflation target is assumed to be 2.5 percent. The simulated Euro-area interest rate is presented in Figure 4.18, together with the actual Euro-area interest rate. The simulated rate is much lower (3.56 percent on average before 1995) than the actual rate in the first half of the 1990s and close to the actual rate after 1994. The average value of the simulated rate is 3.39 percent from 1990-98. The simulations of the Euro-area inflation rate and output gap will be undertaken with the IS-Phillips curves:<sup>3</sup>

$$\pi_t = a_1 + a_2\pi_{t-1} + a_3\pi_{t-2} + a_4\pi_{t-3} + a_5y_{t-1}, \quad (4.5)$$

$$y_t = b_1 + b_2y_{t-1} + b_3y_{t-2} + b_4(r_{t-1} - \pi_{t-1}), \quad (4.6)$$

<sup>3</sup>The lags of the inflation rate and output gap with insignificant T-Statistics are excluded. This model is similar to that of Rudebusch and Svensson (1999).



where all variables have the same interpretations as in equation (4.4). In order to simulate the  $\pi_t$  and  $y_t$  from equation (4.5)-(4.6), one needs to know the values of the coefficients,  $a_i$  and  $b_i$ , which will be generated by estimating equation (4.5)-(4.6) by way of the SUR with the quarterly data from 1990-98. The estimation results for this system are presented as follows, with T-Statistics in parentheses:<sup>4</sup>

$$\begin{aligned}\pi_t &= \underset{(0.254)}{-0.0006} + \underset{(6.165)}{0.984}\pi_{t-1} - \underset{(1.307)}{0.291}\pi_{t-2} + \underset{(1.743)}{0.286}\pi_{t-3} + \underset{(1.014)}{0.149}y_{t-1} \\ R^2 &= 0.852 \\ y_t &= \underset{(0.277)}{0.0002} + \underset{(7.841)}{1.229}y_{t-1} - \underset{(2.467)}{0.403}y_{t-2} - \underset{(0.334)}{0.008}(r_{t-1} - \pi_{t-1}) \\ R^2 &= 0.799.\end{aligned}$$

The determinant residual covariance is  $6.82 \times 10^{-11}$ . After substituting the simulated interest rate into these equations, one obtains the simulations of  $\pi_t$  and  $y_t$ .<sup>5</sup> The simulated output gap is presented in Figure 4.19. It is clear that the simulated output gap declined very rapidly at the beginning of the 1990s and increased a little in 1994. The simulated and actual output gaps are presented in Figure 4.20. Unlike the actual output gap, which experienced significant decreases during 1992-94 and 1995-97, the simulated output gap is

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<sup>4</sup>The T-Statistics of the last terms of these two equations are unfortunately insignificant. The results seem sensitive to the period studied and how potential output is computed. If one uses the linear quadratic trend of the log value of the Industrial Production Index as the potential output, for example, one can obtain the following results with the data from 1986-98:

$$\begin{aligned}\pi_t &= \underset{(1.806)}{0.004} + \underset{(8.150)}{1.117}\pi_{t-1} - \underset{(1.719)}{0.341}\pi_{t-2} + \underset{(0.556)}{0.072}\pi_{t-3} + \underset{(1.934)}{0.184}y_{t-1} \\ R^2 &= 0.830 \\ y_t &= \underset{(1.772)}{0.001} + \underset{(9.376)}{1.254}y_{t-1} - \underset{(1.835)}{0.275}y_{t-2} - \underset{(1.714)}{0.045}(r_{t-1} - \pi_{t-1}) \\ R^2 &= 0.909\end{aligned}$$

with the determinant residual covariance being  $6.44 \times 10^{-11}$ . The T-Statistics of the last terms are now more significant. Since the T-Statistics significance of these terms has little effect on the simulations below, I do not discuss how output gap should be defined here.

<sup>5</sup>As for the 3 initial lags of inflation rate and 2 initial lags of output gap, I just take the actual inflation rate from 1989.2-89.4 and output gap from 1989.3-89.4.

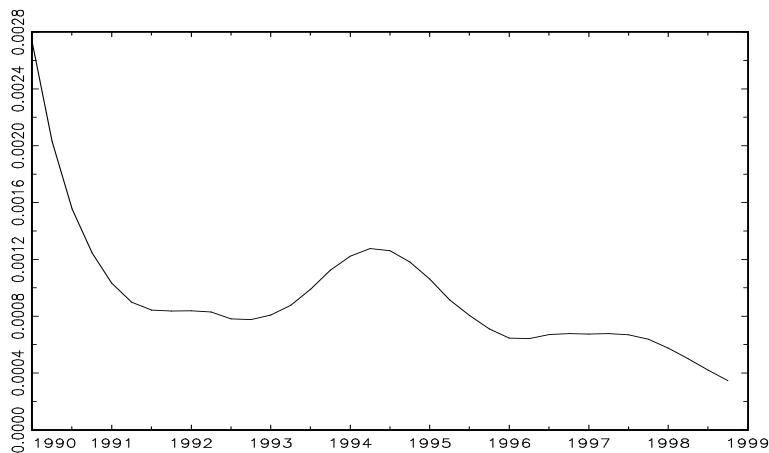


Figure 4.19: Simulated Output Gap of the Euro-Area

always positive and smoother than the actual one. The simulated and actual inflation rates are presented in Figure 4.21. One finds that the simulated inflation rate is almost a straight line and lower than the actual inflation rate most of the time.

The simulation above suggests that if the Euro-area had followed the US monetary policy rule in the 1990s, the output would not have experienced such significant decreases and moreover, the simulated inflation is very similar to the actual inflation. The monetary policy in the Euro-area seems to have been too tight in the 1990s. Many observers, of course, would argue that lowering the interest rate was not a feasible policy since this would have led to an accelerated depreciation of the European currencies and later of the Euro. However, as shown by Semmler (2002), the Euro-area has large net foreign assets and thus large foreign currencies reserves, so that an accelerated depreciation would not have occurred. Moreover, as recently shown by Corsetti and Pesenti (1999) the high value of the dollar is strongly positively correlated with the growth differentials of the US and Euro economies. One might conjecture that a lower interest rate and thus a higher expected

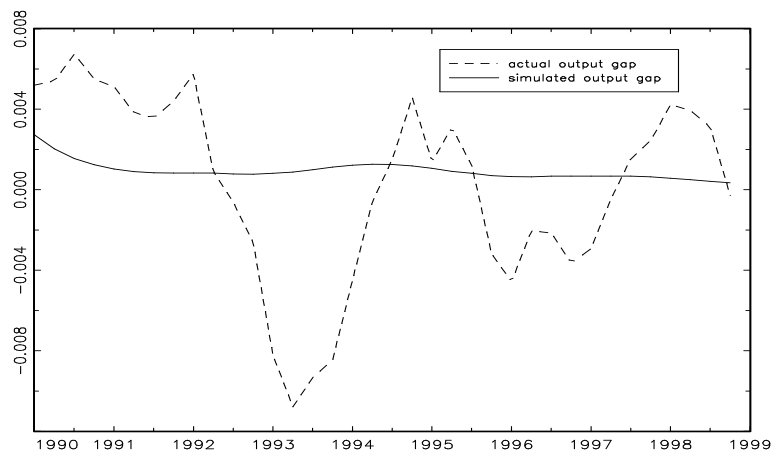


Figure 4.20: Actual and Simulated Output Gaps of the Euro-Area

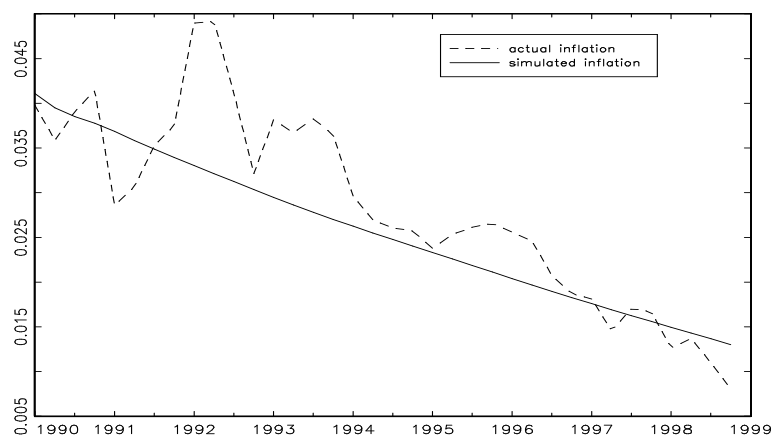


Figure 4.21: Actual and Simulated Inflation Rates of the Euro-Area

growth rate of the Euro-area would have attracted capital inflows into the Euro-area, and would have also prevented the Euro from being depreciated.

## 4.5 Conclusion

This chapter presents some empirical evidence of a time-varying monetary policy reaction function. Both the Chow break-point tests and the Kalman filter estimation indicate that there are really some structural changes in the interest-rate rule in the countries studied.<sup>6</sup> This is consistent with the estimation of the time-varying Phillips curve in Chapter 2, since time-varying parameters in the Phillips curve may indicate time-varying behaviors in monetary policy rules. In addition, this chapter undertakes some simulation of the Euro-area economy, assuming that the Euro-area had followed the time-varying US monetary policy rule in the 1990s. The simulation suggests that the Euro-area would not have experienced so significant a decrease in the output if the US monetary policy had been followed.

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<sup>6</sup>The OLS and time-varying-parameter estimations show that  $\beta_\pi$  may be smaller than one in practice. This seems to be inconsistent with the optimal interest-rate rule derived from the dynamic model shown in Chapter 3 and the original Taylor rule presented in footnote 1 of Chapter 3. This problem has been briefly discussed by Woodford (2003a, p.93).

# Chapter 5

## Monetary Policy Rules Under Uncertainty

### 5.1 Introduction

In the profession it has increasingly been recognized that formal modelling of monetary policy faces great challenges because of many kinds of uncertainties such as model uncertainty, shock uncertainty and data uncertainty. Recent studies dealing with these uncertainties can be found in Isard et al. (1999), Söderström (1999), Giannoni (2002), Meyer et al. (2001), Wieland (2000), Tetlow and von zur Muehlen (2001), Orphanides and Williams (2002), Svensson (1999b) and Martin and Salmon (1999).<sup>1</sup> Those papers explore, usually theoretically, how certain kinds of uncertainties may affect the decisions of the central banks or regulatory agencies.<sup>2</sup>

Empirical work on parameter uncertainty and how to capture them by modelling and estimating parameter shifts can be found in Cogley and Sargent (2001) who study the inflation dynamics of the US after WWII by way of Bayesian VAR with time-varying parameters without stochastic volatil-

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<sup>1</sup>Several contributions to this problem can also be found in *Macroeconomic Dynamics*, No. 6, 2002.

<sup>2</sup>For a study of the effect of model uncertainty in the context of ecological management problem, see Brock and Xepapadeas (2003).

ity. Yet, Sims (2001b) points out that the monetary policy behavior may not have experienced such a sharp change as shown by Cogley and Sargent (2001). Sims and Zha (2002) also study parameter shifts in estimates of the US economy and find more evidence in favor of stable dynamics with unstable variance of the disturbance than of clear changes in model structure. In contrast to Sims (2001b), Cogley and Sargent (2002) study the drifts and volatilities of the US monetary policies after WWII through a Bayesian VAR with time-varying parameters and stochastic volatility and claim to have found regime changes.

Thus, given such evidence on model and shock uncertainties, economic agents (central banks for example) may resort to different strategies: they may either reduce uncertainty by learning or just seek a policy robust to model uncertainty without learning. As to the former, among the important research, the work by Sargent (1993, 1999) has attracted much attention. Although Sargent (1999) explores monetary policy rules with the adaptive learning in an optimal control framework, he assumes that once the uncertain parameter is updated, the government pretends that the updated parameters will govern the dynamics forever. His analysis is undertaken in the traditional LQ framework. With such an approach there may exist convergence to an ergodic distribution in a stochastic model and convergence w.p.1 in a non-stochastic model. But, as mentioned by Tetlow and von zur Muehlen (2003), the above problem in Sargent's assumption represents an inconsistency in his adaptive learning mechanism.

Because of this problem in Sargent's approach it is a challenge to explore the learning algorithm in an appropriate manner. Therefore, in this chapter I will study monetary policy rules under uncertainty with adaptive learning by using a dynamic programming algorithm recently developed by Grüne (1997) and Grüne and Semmler (2002). Different from Sargent's approach, I am able to endogenize the learning of uncertain parameters and explore the problem with a dynamic-programming algorithm with adaptive grids which can deal with nonlinear constraints.

As stated above, the alternative for monetary authorities is to resort to a monetary policy rule robust to uncertainty. This is a strategy different from

adaptive learning. In this approach the central bank considers the economic model only as an approximation to another model that it cannot specify. With a so-called robustness parameter it pursues a monetary policy rule allowing for a “worst case” scenario. While adaptive learning considers mainly parameter uncertainty, robust control might consider more general uncertainties. The robustness parameter, as stated by Gonzalez and Rodriguez (2003), can be considered as a measure of uncertainty and may affect the robust monetary policy.

The important question concerning monetary policy under uncertainty is whether uncertainty requires caution. Brainard (1967), for example, proposes that parameter uncertainty should imply a more “cautious” policy. This argument has been supported by the recent research of Martin and Salmon (1999) who explore the monetary policy of the UK employing a VAR model with and without parameter uncertainty. They find that the optimal rule in the presence of parameter uncertainty implies a less aggressive path for official interest rates than when no parameter uncertainty is considered. Other researchers, Gonzalez and Rodriguez (2003) and Giannoni (2002), for example, however, argue that uncertainty does not necessarily require caution. I will also study this problem using robust control theory, obtaining similar results to Giannoni (2002).

The remainder of this chapter is organized as follows. In the second section I present some empirical evidence of model and shock uncertainties in the IS and Phillips curves by way of a State-Space model with Markov-Switching. In Section 3 I explore monetary policy rules under model uncertainty with adaptive learning. Section 4 explores monetary policy rules with robust control and Section 5 concludes this chapter.

## 5.2 Empirical Evidence of Uncertainty: A State-Space Model with Markov-Switching

Consider an economic model

$$\text{Min}_{\{u_t\}_0^\infty} E_0 \sum_{t=0}^{\infty} \rho^t L(x_t, u_t), \quad (5.1)$$

subject to

$$x_{t+1} = f(x_t, u_t, \varepsilon_t), \quad (5.2)$$

where  $\rho$  is the discount factor bounded between 0 and 1,  $L(x_t, u_t)$  denotes a loss function of an economic agent (central bank for instance),  $x_t$  is a vector of state variables,  $u_t$  is a vector of control variables,  $\varepsilon_t$  is a vector of shocks and  $E_0$  denotes the mathematical expectation operator upon the initial values of the state variables. This kind of model represents the basic monetary control model employed by Svensson (1997, 1999b), Beck and Wieland (2002) and Clarida, Gali and Gertler (1999) and others, where the constraint equations are usually the IS and Phillips curves. Given the loss function  $L(x, u)$  and the state equation (5.2), the problem is to derive a path of the control variable,  $u_t$ , to satisfy (5.1). The question arising is, however, whether the state equation (5.2) can be correctly specified with time series estimates. The uncertainty of the state equation can be caused by the uncertainty in the shock  $\varepsilon_t$  and uncertainty in parameters and data. Following Svensson (1997, 1999b), in Chapter 3 I have shown how to derive an optimal monetary policy rule from an optimal control problem similar to the model above and find that the optimal monetary policy rule is greatly affected by the estimated parameters of the model. Therefore, if the parameters in the model are uncertain, the derived optimal monetary policy rule may not be very reliable. The empirical evidence of a time-varying Phillips curve and a monetary policy rule has been shown in Chapter 2 and Chapter 4.

As has been done in Chapter 2, one can estimate time-varying parameters



of the traditional Phillips curve with the following State-Space model:

$$\pi_t = v_t' \beta_t + \varepsilon_t, \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \quad (5.3)$$

$$\beta_t = \beta_{t-1} + \eta_t, \quad \eta_t \sim N(0, \sigma_\eta^2) \quad (5.4)$$

where  $\pi_t$  denotes the inflation rate and  $v_t$  is a vector of the lags of the inflation rate and output gap.  $\beta_t$  is a vector of time-varying parameters. Note that in this model it is assumed that the shocks have constant variances and only  $\beta_t$  is uncertain. Cogley and Sargent (2001) study the inflation dynamics of the US after WWII by way of Bayesian VAR with time-varying parameters without stochastic volatility. Sims (2001b), however, claims that the monetary policy behavior may not have experienced such a sharp change as demonstrated by Cogley and Sargent (2001). Sims and Zha (2002) also study macroeconomic switching of the US policy and find more evidence in favor of stable dynamics with unstable disturbance variance than of clear changes in model dynamics. Therefore, Cogley and Sargent (2002) modify the model by considering both time-varying parameters and stochastic volatility and claim to have found regime switching.

A drawback of the traditional State-Space model such as (5.3) and (5.4) is that the changes of the time-varying parameters may be exaggerated, because the shocks are assumed to have constant variances. This is the reason why Cogley and Sargent (2002) assume stochastic volatility. Therefore, in the research below I assume that  $\varepsilon_t$  has a state-dependent variance. This is similar to the assumption of Cogley and Sargent (2002). But unlike Cogley and Sargent (2002), who assume the variances of the shocks to change from period to period, I assume that there are only two states of disturbance variance with Markov property. This is, to some extent, similar to the assumption of Sims and Zha (2002) who assume that there are three states of economy. With such an assumption one can explore the probability of regime switching. Another advantage of the State-Space model with Markov-Switching is that, as will be seen below, it can explore not only parameter uncertainty but also shock uncertainty.

Following Kim and Nelson (1999), I simply assume that  $\varepsilon_t$  in (5.3) has

two states of variance with Markov property,<sup>3</sup> namely,

$$\varepsilon_t \sim N(0, \sigma_{\varepsilon, S_t}^2), \quad (5.5)$$

with

$$\sigma_{\varepsilon, S_t}^2 = \sigma_{\varepsilon, 0}^2 + (\sigma_{\varepsilon, 1}^2 - \sigma_{\varepsilon, 0}^2)S_t, \quad \sigma_{\varepsilon, 1}^2 > \sigma_{\varepsilon, 0}^2,$$

and

$$\begin{aligned} Pr[S_t = 1 | S_{t-1} = 1] &= p, \\ Pr[S_t = 0 | S_{t-1} = 0] &= q, \end{aligned}$$

where  $S_t = 0$  or  $1$  indicates the states of the variance of  $\varepsilon_t$  and  $Pr$  stands for probability. In the research below I explore uncertainty in the IS and Phillips curves, since these two curves form the core of a monetary policy model.

## Evidence of Uncertainty in the Traditional IS and Phillips Curves

I will first explore uncertainty in the traditional IS and Phillips curves which have often been taken as constraints in an optimal control model such as (5.1) and (5.2). In order to reduce the dimension of the model, I estimate the Phillips and IS curves with only one lag of the inflation rate and output gap:

$$\pi_t = \alpha_{1t} + \alpha_{2t}\pi_{t-1} + \alpha_{3t}y_{t-1} + \varepsilon_{\pi, t}, \quad (5.6)$$

$$y_t = \beta_{1t} + \beta_{2t}y_{t-1} + \beta_{3t}(R_{t-1} - \pi_{t-1}) + \varepsilon_{y, t}, \quad (5.7)$$

where  $\pi_t$  is the inflation rate,  $y_t$  is the output gap,  $R_t$  denotes the short-term nominal interest rate, and  $\varepsilon_{\pi, t}$  and  $\varepsilon_{y, t}$  are shocks subject to Gaussian

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<sup>3</sup>Vázquez (2003) estimates an augmented Taylor rule with a Markov-Switching VAR model with the US data from 1967-2002 and finds that there is no essential difference between the model with two regimes and three regimes and therefore the model with two regimes can accurately describe the economy.

distributions with zero mean and Markov-Switching variances.<sup>4</sup> The  $\beta_{3t}$  is expected to be negative. Let  $r_t$  denote the real interest rate, namely,  $r_t = R_t - \pi_t$ , the model can be rewritten in a State-Space form as follows:

$$Y_t = X_t\phi_t + \varepsilon_t, \quad (5.8)$$

$$\phi_t = \bar{\Phi}_{S_t} + F\phi_{t-1} + \eta_t, \quad (5.9)$$

where  $Y_t$  denotes the dependent variables ( $\pi_t, y_t$ ) and  $X_t$  denotes the independent variables ( $\pi_{t-1}, y_{t-1}$  and  $r_{t-1}$ ).  $\phi_t$  denotes the time-varying parameters  $\alpha_{n,t}$  and  $\beta_{n,t}$  ( $n = 1, 2, 3$ ).  $\bar{\Phi}_{S_t}$  ( $S_t=0$  or  $1$ ) is the drift of  $\phi_t$  and  $F$  is a diagonal matrix with constant elements to be estimated from the model.  $\eta_t$  has the distribution shown in Eq. (5.4).  $\varepsilon_t$  is now assumed to have the distribution presented in Eq. (5.5).<sup>5</sup> A brief sketch of the State-Space model with Markov-Switching is presented in Appendix A of this chapter.<sup>6</sup>

The estimation will be undertaken with the US quarterly data from 1964.1-2003.1. The inflation rate is measured by changes in the GDP deflator, the output gap is measured by the percentage deviation of the Industrial Production Index (IPI, base year: 1995) from its fourth-order polynomial trend.<sup>7</sup>  $R_t$  is the federal funds rate. The data source is the International Statistical Yearbook 2003. The estimates of the hyperparameters are shown

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<sup>4</sup>Forward-looking behaviors have been frequently taken into account in the Phillips curve, as explored in Chapter 2. Because it is quite difficult to estimate a State-Space model with forward-looking behaviors, I just consider backward-looking behaviors in this section. A justification of the above type of backward-looking model can be found in Rudebusch and Svensson (1999).

<sup>5</sup>Theoretically, the elements of  $F$  and the variance of  $\eta_t$  may also have Markov property, but since there are already many parameters to estimate, I just ignore this possibility to improve the efficiency of estimation. Note that if the elements of  $F$  are larger than 1 in absolute value, that is, if the time-varying parameters are non-stationary, the transition equation should be altered into the form of Eq. (5.4). Because the Phillips and IS curves contain only lags of variables and have uncorrelated noise, I can estimate the Phillips and IS curves separately. In this case,  $\phi_t$ ,  $\bar{\Phi}_{S_t}$  ( $S_t = 0$  or  $1$ ) and  $\eta_t$  are  $3 \times 1$  vectors and  $\varepsilon_t$  is a scalar.

<sup>6</sup>As for the details of the State-Space model with Markov-Switching, the reader is referred to Kim and Nelson (1999, Ch. 5). The program applied below is based on the Gauss Programs developed by Kim and Nelson (1999).

<sup>7</sup>The IPI has also been used by Clarida, Gali and Gertler (1998) to measure the output

in Table 5.1.  $\sigma_{\varepsilon\pi,1}$  (0.0021) is almost twice as large as  $\sigma_{\varepsilon\pi,0}$  (0.0011). The difference between  $\sigma_{\varepsilon y,0}$  (0.0000) and  $\sigma_{\varepsilon y,1}$  (0.0199) is still more significant. The difference between  $\bar{\Phi}_{\alpha_2,1}$  (0.4704) and  $\bar{\Phi}_{\alpha_2,0}$  (0.6406) is less significant than that between  $\bar{\Phi}_{\alpha_3,1}$  (0.0502) and  $\bar{\Phi}_{\alpha_3,0}$  (0.0021). The difference between  $\bar{\Phi}_{\beta_3,0}$  (0.0873) and  $\bar{\Phi}_{\beta_3,1}$  (-0.3364) is obvious not only in magnitude and T-Statistics but also in signs. Therefore the estimation results confirm a state of economy with high volatility (state 1) and a state with low volatility (state 0). The fact that all the elements of F are smaller than 1 indicates that the time-varying parameters are stationary and therefore justifies the adoption of Eq. (5.9).

The paths of  $\alpha_{2t}$  are shown in Figure 5.1A. I leave aside the paths of the intercepts in the IS and Philips curves just for simplicity. In Figure 5.1A, “Alpha\_2t,0” and “Alpha\_2t,1” denote the paths of  $\alpha_{2t}$  when  $[S_t = 0|\psi_t]$  (namely  $\alpha_{2t,0}$ ) and  $[S_t = 1|\psi_t]$  (namely  $\alpha_{2t,1}$ ) respectively. “Alpha\_2t” denotes the weighted sum of  $\alpha_{2t,0}$  and  $\alpha_{2t,1}$ . That is,

$$\alpha_{2t} = Pr[S_t = 0|\psi_t]\alpha_{2t,0} + Pr[S_t = 1|\psi_t]\alpha_{2t,1}.$$

The paths of  $\alpha_{3t}$  are shown in Figure 5.1B, where “Alpha\_3t,0” and “Alpha\_3t,1” denote the paths of  $\alpha_{3t}$  when  $[S_t = 0|\psi_t]$  (namely  $\alpha_{3t,0}$ ) and  $[S_t = 1|\psi_t]$  (namely  $\alpha_{3t,1}$ ) respectively. Similarly, “Alpha\_3t” denotes the weighted sum of  $\alpha_{3t,0}$  and  $\alpha_{3t,1}$ . Figure 5.1C represents the path of  $Pr[S_t = 1|\psi_t]$ . It is clear that the economy was probably in state 0 most of the time, since  $Pr[S_t = 1|\psi_t]$  was very low except at the beginning of the 1970s. Therefore  $\alpha_{2t}$  is relatively close to  $\alpha_{2t,0}$  and  $\alpha_{3t}$  relatively close to  $\alpha_{3t,0}$  most of the time.  $\alpha_{2t}$  experienced some significant changes between 1970 and 1975 and at the beginning of the 1980s.  $\alpha_{3t}$  also experienced significant changes in the first

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for Germany, France, the US, the UK, Japan and Italy. Clarida, Gali and Gertler (1998) use the quadratic trend of the IPI to measure the potential output for the US of the period 1979-1993. I use the fourth-order trend of the IPI as the potential output because my research covers a much longer period and the fourth-order trend fits the data better than the quadratic one. As surveyed by Orphanides and van Norden (2002), there are many approaches to measure the output gap.

Table 5.1: Estimates of Hyperparameters in the Time-Varying Phillips and IS Curves

Phillips Curve			IS Curve		
Parameter	Estimate	S.D.	Parameter	Estimate	S.D.
$\sigma_{\varepsilon\pi,0}$	0.0011	0.0021	$\sigma_{\varepsilon y,0}$	0.0000	0.0024
$\sigma_{\varepsilon\pi,1}$	0.0021	0.0018	$\sigma_{\varepsilon y,1}$	0.0199	0.0064
$\sigma_{\eta\alpha_1}$	0.0020	0.0013	$\sigma_{\eta\beta_1}$	0.0073	0.0007
$\sigma_{\eta\alpha_2}$	0.0434	0.0106	$\sigma_{\eta\beta_2}$	0.0000	0.0334
$\sigma_{\eta\alpha_3}$	0.0000	0.0069	$\sigma_{\eta\beta_3}$	0.0000	0.0196
$\bar{\Phi}_{\alpha_1,0}$	0.0005	0.0005	$\bar{\Phi}_{\beta_1,0}$	-0.0003	0.0010
$\bar{\Phi}_{\alpha_1,1}$	0.0080	0.0052	$\bar{\Phi}_{\beta_1,1}$	0.0023	0.0026
$\bar{\Phi}_{\alpha_2,0}$	0.6406	0.2208	$\bar{\Phi}_{\beta_2,0}$	0.5893	0.3372
$\bar{\Phi}_{\alpha_2,1}$	0.4704	0.2183	$\bar{\Phi}_{\beta_2,1}$	0.7272	0.3836
$\bar{\Phi}_{\alpha_3,0}$	0.0021	0.0020	$\bar{\Phi}_{\beta_3,0}$	0.0873	0.1881
$\bar{\Phi}_{\alpha_3,1}$	0.0502	0.0337	$\bar{\Phi}_{\beta_3,1}$	-0.3364	0.1809
$f_{\alpha_1}$	0.3677	0.3710	$f_{\beta_1}$	0.7688	0.0810
$f_{\alpha_2}$	0.3431	0.2205	$f_{\beta_2}$	0.2455	0.4093
$f_{\alpha_3}$	0.8757	0.0644	$f_{\beta_1}$	-0.5817	0.2662
$p$	0.9875	0.0147	$p$	0.8451	0.1589
$q$	0.7586	0.2323	$q$	0.9591	0.0248
Likelihood	-660.6778		Likelihood	-472.5914	

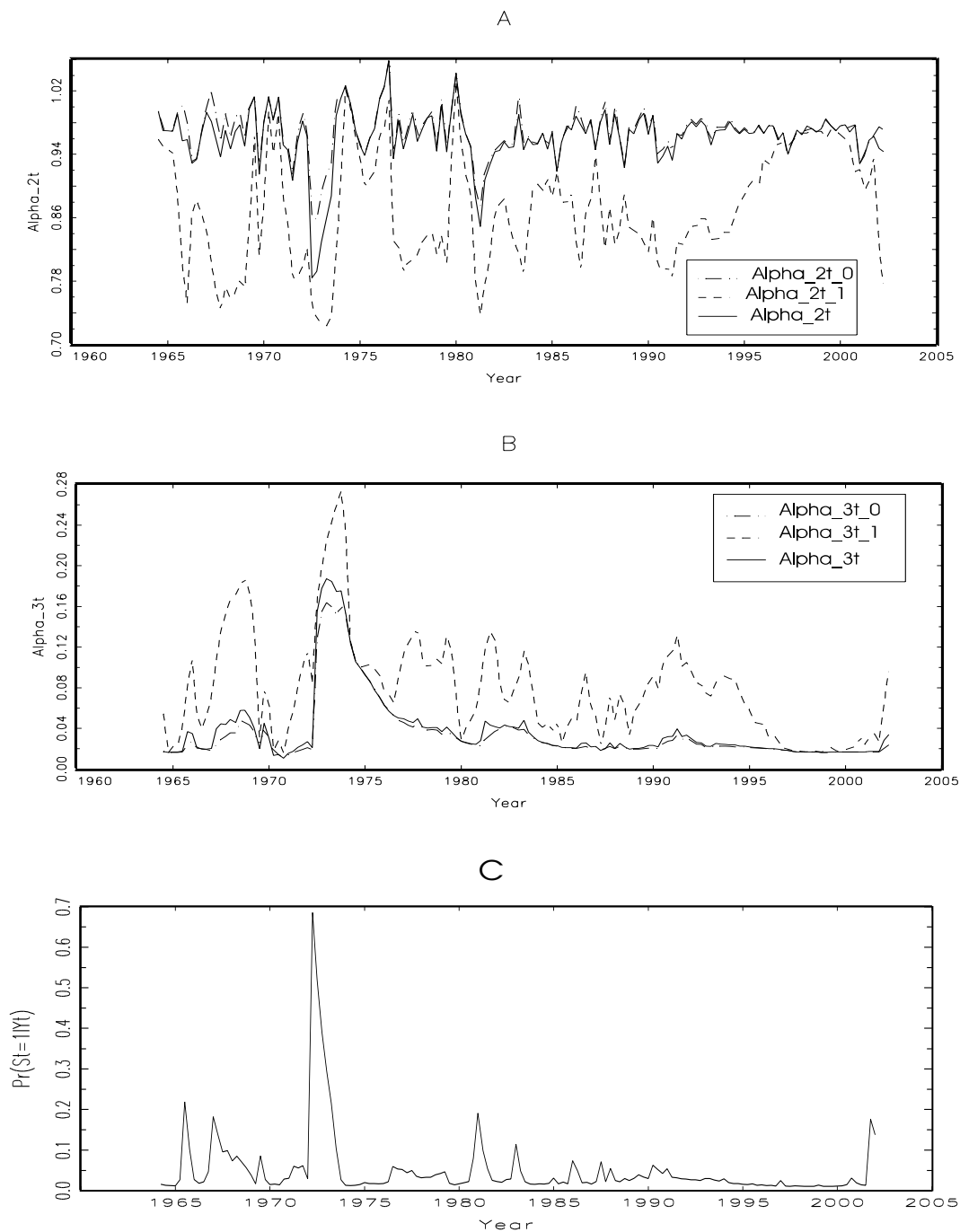


Figure 5.1: Empirical Evidence of Uncertainty in the Phillips Curve

half of the 1970s, increasing from about 0.02 to 0.20. One can also illustrate these structural changes by way of the Chow break-point test of the following regression (T-Statistics in parentheses):

$$\pi_t = 0.001 + 1.396\pi_{t-1} - 0.316\pi_{t-2} - 0.171\pi_{t-3} + 0.066\pi_{t-4} + 0.023y_{t-1}$$

(1.591)
(16.566)
(2.197)
(1.181)
(0.805)
(3.036)

with  $R^2 = 0.979$  and  $DW = 2.026$ . The F-Statistics of the Chow break-point test for 1971.4 and 1981.4 are 2.325 and 4.712 respectively, significant enough to indicate structural changes at these two points (the critical value at 5 percent level of significance lies between 2.10 and 2.19).

The estimation result of the IS curve is demonstrated in Figure 5.2. The paths of  $\beta_{2t}$  and  $\beta_{3t}$  are presented in Figure 5.2A and 5.2B. It is clear that  $\beta_{3t}$  is lower than zero most of the time. Figure 5.2C is the path of  $Pr[S_t = 1|\psi_t]$ . From Figure 5.2C one finds that the IS curve has probably been in state 0 since the middle of the 1980s. Therefore,  $\beta_{2t}$  was close to  $\beta_{2t,0}$  after 1984. The same is true of  $\beta_{3t}$ . Both  $\beta_{2t}$  and  $\beta_{3t}$  experienced some significant structural changes in the 1970s and small changes around 1990. It is also obvious that the time-varying parameters show some structural changes at the beginning of the 1980s which coincides with the beginning of the post-Volcker period. Then, after that, for the period from 1984-1990 the time-varying parameters seem to be relatively stable.<sup>8</sup>

## Evidence of Uncertainty in a Convex Phillips Curve

In the previous subsection I have explored uncertainty in the traditional IS and Phillips curves. The 1990s, however, has seen the development of the literature on the so-called nonlinear Phillips curve. Dupasquier and Ricketts (1998a) survey several models of the nonlinearity in the Phillips curve. The five models surveyed are the *capacity constraint model*, the *mis-perception* or *signal extraction model*, the *costly adjustment model*, the *downward nominal wage rigidity model* and the *monopolistically competitive model*. As mentioned by Akerlof (2002), the nonlinearity of the Phillips curve has been an

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<sup>8</sup>The differences of the monetary policy rules of the US across periods have also been explored by Clarida, Galí and Gertler (2000).

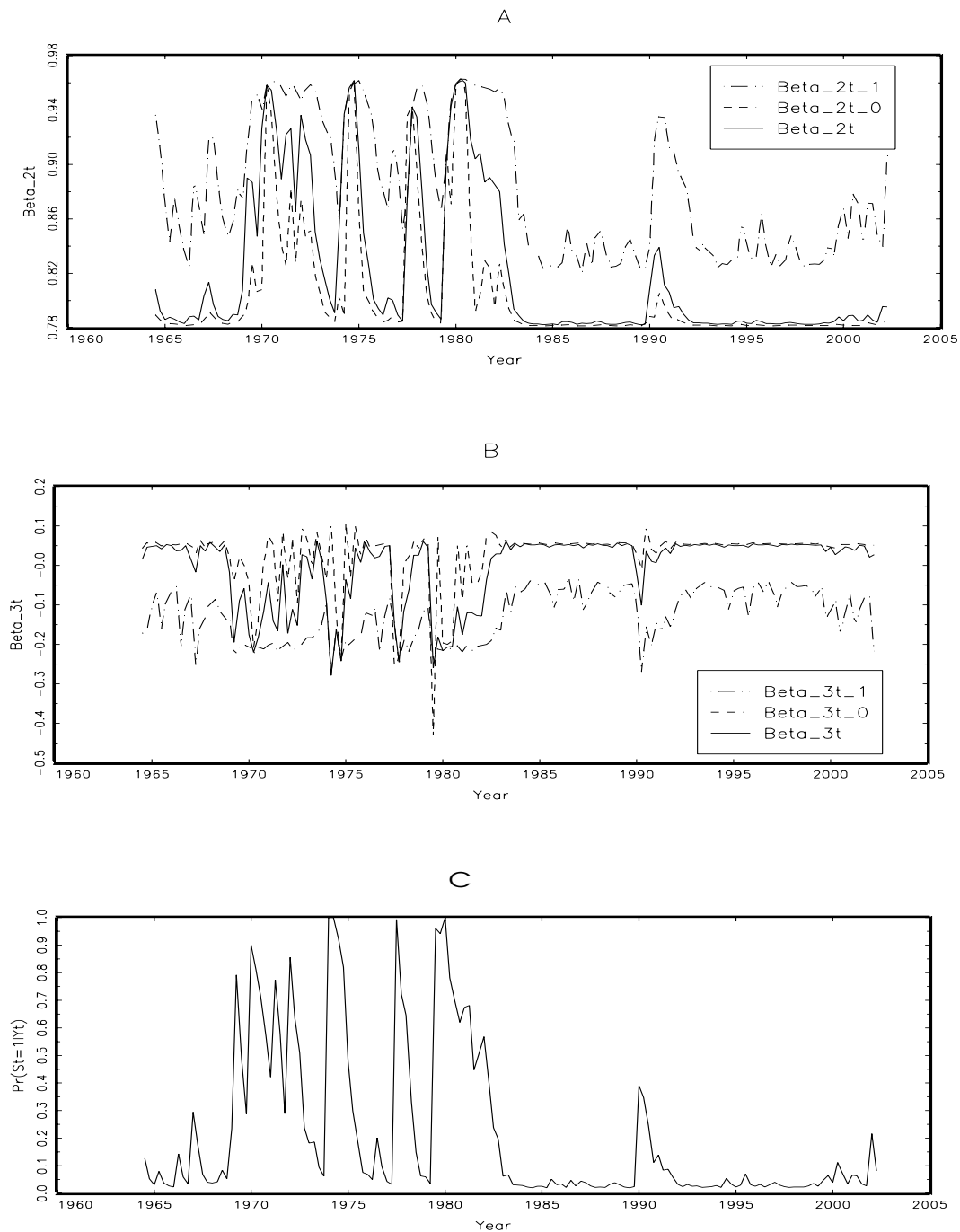


Figure 5.2: Empirical Evidence of Uncertainty in the IS Curve



important issue of macroeconomics. Aguiar and Martins (2002), for example, test three kinds of nonlinearities (quadratic, hyperbole and exponential) in the Phillips curve and Okun's law with the aggregate Euro-area macroeconomic data and find that the Phillips curve turns out to be linear, but the Okun's law is nonlinear. Many empirical studies have been undertaken to explore the Phillips-curve nonlinearity. Dupasquier and Ricketts (1998a) explore nonlinearity in the Phillips curve for Canada and the US and conclude that there is stronger evidence in favor of nonlinearity for the US than for Canada. Other studies on the nonlinearity of the Phillips curve include Dupasquier and Ricketts (1998b) and Bean (2000). Monetary policy with a nonlinear Phillips curve has also been explored by numerous researchers, see Schaling (1999), Tambakis (1998), and Semmler and Zhang (2003), for example. Since monetary policy with a linear Phillips curve can be different from that with a nonlinear Phillips curve, I will explore uncertainty in such a Phillips curve below.

As discussed by Aguiar and Martins (2002), there may be different forms of nonlinearity in the Phillips curve. Laxton, Rose and Tambakis (1998) explore alternative shapes (concave, linear and convex) of the US Phillips curve and argue that the Fed should assume the convex form. To be precise, it is argued that the negative output gap may be less deflationary than the positive output gap is inflationary. Therefore, I just follow Schaling (1999) and assume that the nonlinear form of the output gap in the Phillips curve reads as<sup>9</sup>

$$f(y_t) = \frac{\alpha y_t}{1 - \alpha \beta y_t}, \quad \alpha > 0, 1 > \beta \geq 0, \quad (5.10)$$

where  $y_t$  denotes the output gap and the parameter  $\beta$  indexes the curvature of the curve. When  $\beta$  is very small, the curve approaches a linear relationship. Assuming  $\alpha=10$  and  $\beta=0.99$ , I present  $f(y_t)$  with the US quarterly data in Figure 5.3. It is obvious that when the actual output is lower than the potential output, the curve of  $f(y_t)$  is flatter. From this figure one finds that

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<sup>9</sup>Note that this function is not continuous with a breaking point at  $y_t = \frac{1}{\alpha\beta}$ . When  $y_t < \frac{1}{\alpha\beta}$ ,  $f''(y_t) > 0$  and if  $y_t > \frac{1}{\alpha\beta}$ ,  $f''(y_t) < 0$ . In the research below I choose appropriate values of  $\alpha$  and  $\beta$  so that with the US output gap data, one has  $f''(y_t) > 0$ .

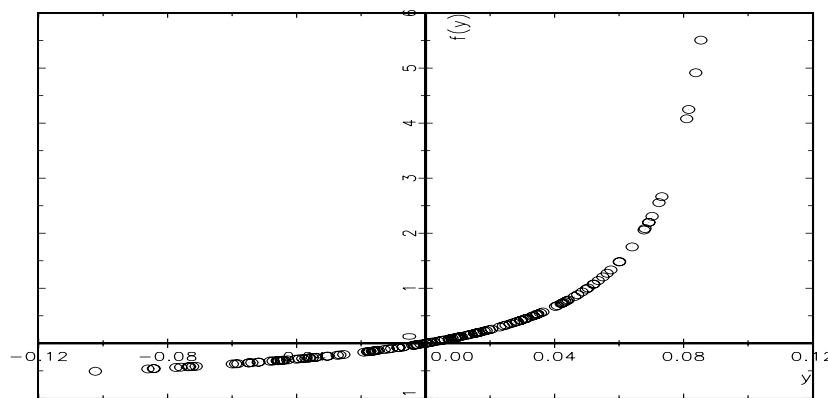


Figure 5.3: An Example of  $f(y_t)$  with the US Data from 1964-2003

this function describes very well the idea that the negative output gap is less deflationary than the positive output gap is inflationary.

Substituting  $f(y_t)$  for  $y_t$  in the Phillips curve, one has now

$$\pi_t = \alpha_{1t} + \alpha_{2t}\pi_{t-1} + \alpha_{3t}f(y_{t-1}) + \varepsilon_{\pi,t}. \quad (5.11)$$

Following the same procedure in the previous subsection, I present the estimation results of the State-Space model of Eq. (5.11) in Table 5.2 and Figure 5.4.

In Figure 5.4 one also observes some structural changes in the coefficients. The difference between the traditional Phillips curve and the convex one can be obviously seen in the paths of  $Pr(S_t = 1|\psi_t)$  and  $\alpha_{3t}$ . With the traditional Phillips curve one finds that the economy was in state 0 most of the time except around 1973, while the convex one shows that the economy was probably in state 1 most of the time except in the middle of the 1970s and the 1990s. The time-varying paths of  $\alpha_{2t}$  and  $\alpha_{3t}$  in the traditional and convex Phillips curves are shown in Figure 5.5. The main difference lies in  $\alpha_{3t}$ : although it experienced some changes in both cases in almost the same periods, the changes in the traditional Phillips curve are much more significant than those in the convex one:  $\alpha_{3t}$  evolves between 0.01 and 0.19 in the former case and between 0 and 0.02 in the latter case. The differences

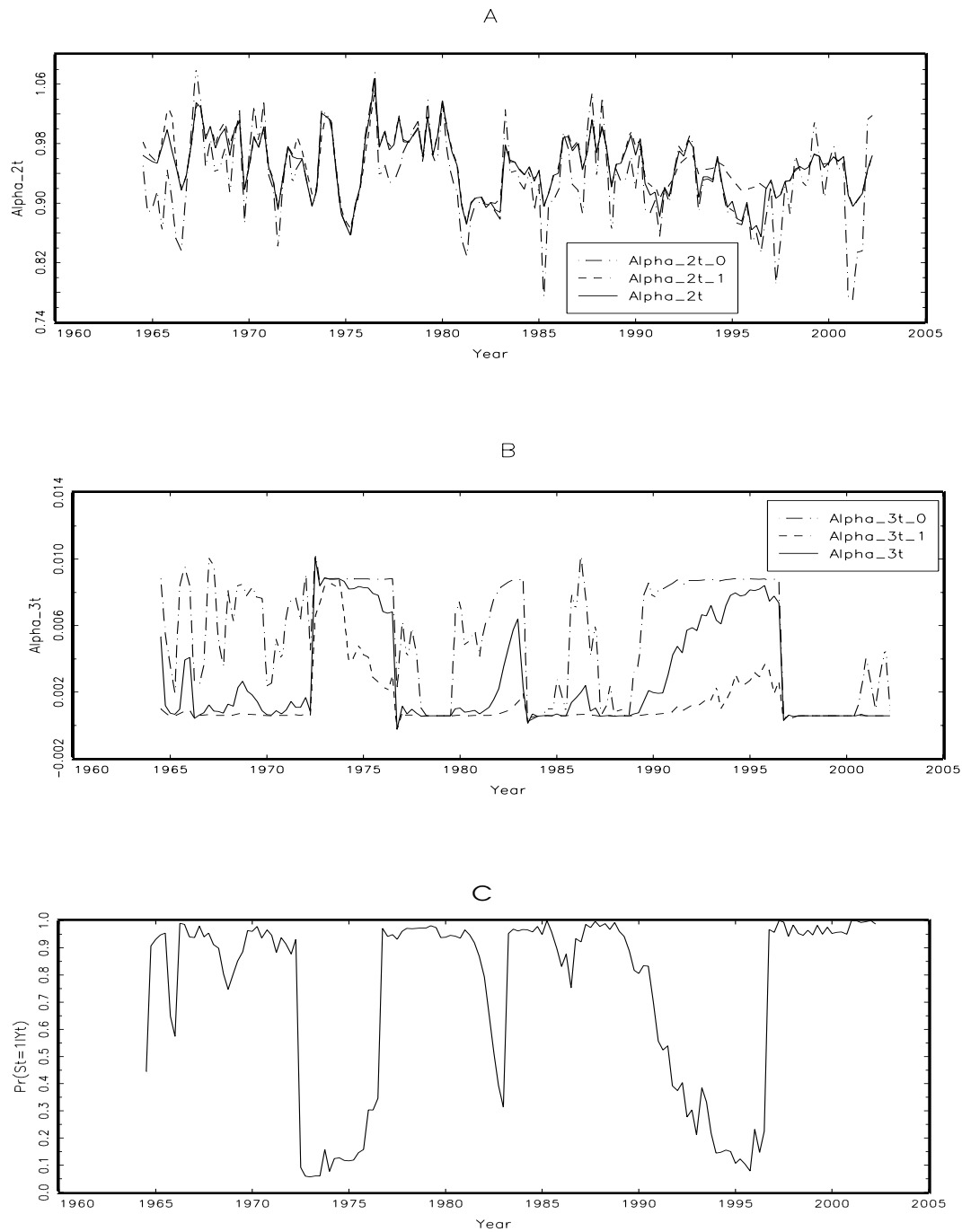


Figure 5.4: Empirical Evidence of Uncertainty in the Convex Phillips Curve

Table 5.2: Estimates of the Hyperparameters in the Convex Time-Varying Phillips Curve

Parameter	Estimate	S.D.	Parameter	Estimate	S.D.
$\sigma_{\varepsilon\pi,1}$	0.0026	0.0003	$\sigma_{\varepsilon\pi,0}$	0.0009	0.0003
$\sigma_{\eta\alpha_1}$	0.0000	0.0005	$\sigma_{\eta\alpha_2}$	0.0419	0.0071
$\sigma_{\eta\alpha_3}$	0.0000	0.0003	$\bar{\Phi}_{\alpha_1,1}$	0.0017	0.0015
$\bar{\Phi}_{\alpha_1,0}$	0.0050	0.0019	$\bar{\Phi}_{\alpha_2,1}$	0.3382	0.1388
$\bar{\Phi}_{\alpha_2,0}$	0.3245	0.1350	$\bar{\Phi}_{\alpha_3,1}$	0.0007	0.0006
$\bar{\Phi}_{\alpha_3,0}$	0.0110	0.0032	$f_{\alpha_1}$	-0.5193	0.2683
$f_{\alpha_2}$	0.6481	0.1393	$f_{\alpha_3}$	-0.2419	0.3699
$p$	0.9756	0.0187	$q$	0.9313	0.0425
Likelihood	-662.5140				

between the  $\alpha_{2t}$  in the two Phillips curves are not so large as those of the  $\alpha_{3t}$

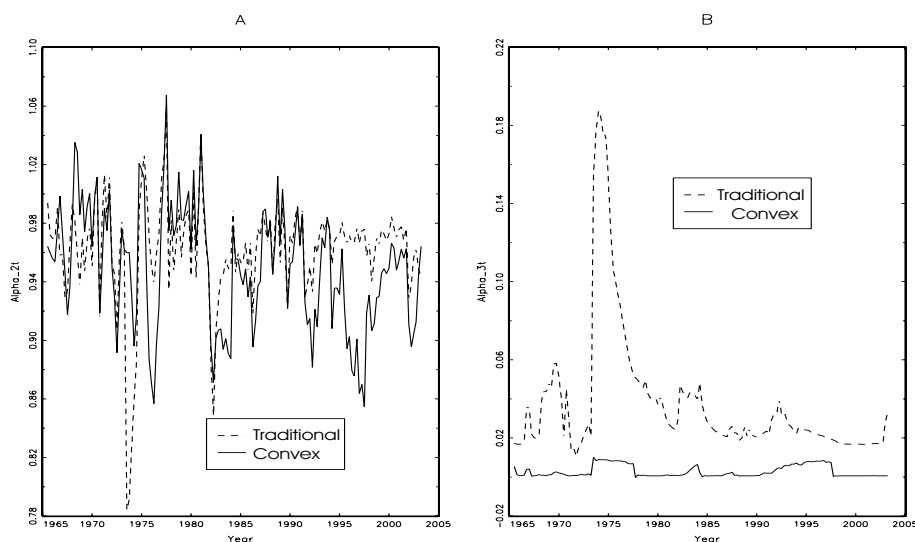


Figure 5.5:  $\alpha_{2t}$  and  $\alpha_{3t}$  in the Traditional and Convex Phillips Curves

Above I have explored model and shock uncertainties in the IS-Phillips curves with the US data. The results are consistent with the line of research

that maintains that there were regime changes in the US economy.<sup>10</sup> Overall, the uncertainty of parameters and shocks, and their impact on monetary policy rules suggest exploring monetary policy rules with learning and robust control.

### 5.3 Monetary Policy Rules with Adaptive Learning

The question arising is, what is the optimal monetary policy rule in case some parameters or shocks in an economic model as presented by Eq. (5.1)-(5.2) are uncertain? Recently numerous papers have been contributed to this topic. Svensson (1999b), Orphanides and Williams (2002), Tetlow and von zur Muehlen (2001), Söderström (1999), Beck and Wieland (2002) and McGough (2003), for example, explore optimal monetary policy rules under the assumption that the economic agents learn the parameters in the model through a certain mechanism. One approach is that the economic agents may learn the parameters using the Kalman filter. This approach has been pursued by Tucci (1997) and Beck and Wieland (2002). Another learning mechanism which is also applied frequently is the recursive least squares (RLS) algorithm. This kind of learning mechanism has been applied by Sargent (1999) and Orphanides and Williams (2002). By intuition one would expect that economic agents reduce uncertainty and therefore improve economic models by learning over time using all information available.

Beck and Wieland (2002) and Orphanides and Williams (2002) have explored monetary policy with adaptive learning. Besides the difference in the learning algorithm they use, another difference between Beck and Wieland (2002) and Orphanides and Williams (2002) is that the former do not consider the role of expectations in the model, while the latter take into account expectations in the Phillips curve. Unlike Beck and Wieland (2002), Orphanides and Williams (2002) do not employ an intertemporal framework. They provide a learning algorithm with a constant gain but do not use a

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<sup>10</sup>See Cogley and Sargent (2001, 2002), for example.

discounted loss function. Moreover, Orphanides and Williams (2002) assume that the government knows the true model, but the private agents do not know the true model and have to learn the parameters with the RLS algorithm. In their case the government and the private agents are treated differently.

Among the research of monetary policy with adaptive learning, the work of Sargent (1999) has attracted much attention. Sargent (1999) employs both a learning algorithm as well as a discounted loss function, but in an LQ model. Yet, Sargent (1999) constructs his results in two steps. First, assuming the RLS learning algorithm with a decreasing or constant gain, the agents estimate a model of the economy using the latest available data and update parameter estimates from period to period. Second, once the unknown parameter is updated, an optimal policy is derived from an LQ control model under the assumption that the updated parameter will govern the dynamics forever. As remarked by Tetlow and von zur Muehlen (2003), however, the two steps are inconsistent with each other.

Because of this problem in Sargent's approach, it is required to explore such models by employing appropriate solution techniques. Therefore in this section I will explore monetary policy with adaptive learning by using a recently developed dynamic programming algorithm.<sup>11</sup> In order to overcome the problem of Sargent (1999) I will endogenize the changing parameters in a nonlinear optimal control problem and explore how my conclusion may be different from that of Sargent (1999).

Thus, the difference of my model from that of Beck and Wieland (2002) can be summarized in three points: (a) I consider both linear and nonlinear Phillips curves. (b) I take into account expectations. This is consistent with the model of Orphanides and Williams (2002). (c) I employ the RLS learning algorithm instead of the Kalman filter algorithm. In fact, Harvey (1989) and Sargent (1999) prove that RLS is a specific form of the Kalman filter.

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<sup>11</sup>Evans and Honkapohja (2001) analyze expectations and learning mechanisms in macroeconomics in detail.

## RLS Learning in a Linear Phillips Curve

Orphanides and Williams (2002) assume that the current inflation rate is not only affected by the lagged inflation rate but also by inflation expectations. Following Orphanides and Williams (2002), I assume that the linear Phillips curve takes the following form:

$$\pi_t = \gamma_1 \pi_{t-1} + \gamma_2 \pi_t^e + \gamma_3 y_t + \varepsilon_t, \quad \varepsilon \sim iid(0, \sigma_\varepsilon^2), \quad (5.12)$$

where  $\pi_t^e$  denotes the agents' (including the central bank) expected inflation rate based on the time  $t$  information,  $\gamma_1, \gamma_2 \in (0,1)$ ,  $\gamma_3 > 0$  and  $\varepsilon$  is a serially uncorrelated innovation. In order to simplify the analysis, I further assume the IS equation to be deterministic in the following form:<sup>12</sup>

$$y_t = -\theta r_{t-1}, \quad \theta > 0, \quad (5.13)$$

where  $r_t$  denotes the real interest rate. Substituting Eq. (5.13) into (5.12), one has

$$\pi_t = \gamma_1 \pi_{t-1} + \gamma_2 \pi_t^e - \gamma_3 \theta r_{t-1} + \varepsilon_t, \quad \varepsilon \sim iid(0, \sigma_\varepsilon^2). \quad (5.14)$$

In the case of rational expectations, namely,  $\pi_t^e = E_{t-1} \pi_t$ , one obtains

$$E_{t-1} \pi_t = \gamma_1 \pi_{t-1} + \gamma_2 E_{t-1} \pi_t - \gamma_3 \theta r_{t-1},$$

that is,

$$E_{t-1} \pi_t = \bar{a} \pi_{t-1} + \bar{b} r_{t-1},$$

with

$$\bar{a} = \frac{\gamma_1}{1 - \gamma_2} \quad (5.15)$$

$$\bar{b} = -\frac{\gamma_3 \theta}{1 - \gamma_2}. \quad (5.16)$$

With these results one obtains the rational expectations equilibrium (REE)

$$\pi_t = \bar{a} \pi_{t-1} + \bar{b} r_{t-1} + \varepsilon_t. \quad (5.17)$$

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<sup>12</sup>This is the same as Orphanides and Williams (2002), except that they include a noise in the equation.

Now suppose that the agents believe the inflation rate follows the process

$$\pi_t = a\pi_{t-1} + br_{t-1} + \varepsilon_t,$$

corresponding to the REE, but that  $a$  and  $b$  are unknown and have to be learned. Suppose that the agents have data on the economy from periods  $i = 0, \dots, t-1$ . Thus the time-( $t-1$ ) information set is  $\{\pi_i, r_i\}_{i=0}^{t-1}$ . Further suppose that agents estimate  $a$  and  $b$  by a least squares regression of  $\pi_i$  on  $\pi_{i-1}$  and  $r_{i-1}$ . The estimates will be updated over time as more information is collected. Let  $(a_{t-1}, b_{t-1})$  denote the estimates through time  $t-1$ , the forecast of the inflation rate is then given by

$$\pi_t^e = a_{t-1}\pi_{t-1} + b_{t-1}r_{t-1}. \quad (5.18)$$

The standard least squares formula gives the equations

$$\begin{pmatrix} a_t \\ b_t \end{pmatrix} = \left( \sum_{i=1}^t z_i' z_i \right)^{-1} \left( \sum_{i=1}^t z_i' \pi_i \right), \quad (5.19)$$

where  $z_i = \begin{pmatrix} \pi_{i-1} & r_{i-1} \end{pmatrix}'$ .

Defining  $c_t = \begin{pmatrix} a_t \\ b_t \end{pmatrix}$ , one can also compute Eq. (5.19) using the stochastic approximation of the recursive least squares equations

$$c_t = c_{t-1} + \kappa_t V_t^{-1} z_t (\pi_t - z_t' c_{t-1}), \quad (5.20)$$

$$V_t = V_{t-1} + \kappa_t (z_t z_t' - V_{t-1}), \quad (5.21)$$

where  $c_t$  and  $V_t$  denote the coefficient vector and the moment matrix for  $z_t$  using data  $i = 1, \dots, t$ .  $\kappa_t$  is the gain. To generate the least squares values, one must set the initial values of  $c_t$  and  $V_t$  approximately.<sup>13</sup> The gain  $\kappa_t$  is an important variable. According to Evans and Honkapohja (2001), the assumption that  $\kappa_t = t^{-1}$  (decreasing gain) together with the condition

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<sup>13</sup>Evans and Honkapohja (2001, Ch. 2, footnote 4) explain how to set the starting values of  $c_t$  and  $V_t$  as follows. Assuming  $Z_k = (z_1, \dots, z_k)'$  is of full rank and letting  $\pi^k$  denote  $\pi^k = (\pi_1, \dots, \pi_k)'$ , the initial value  $c_k$  is given by  $c_k = Z_k^{-1} \pi^k$  and the initial value  $V_k$  is given by  $V_k = k^{-1} \sum_{i=1}^k z_i z_i'$ .



$\gamma_2 < 1$  ensures the convergence of  $c_t$  as  $t \rightarrow \infty$ . That is, as  $t \rightarrow \infty$ ,  $c_t \rightarrow \bar{c}$  with probability 1, with  $\bar{c} = \begin{pmatrix} \bar{a} \\ \bar{b} \end{pmatrix}$  and therefore  $\pi_t^e \rightarrow \text{REE}$ .

As indicated by Sargent (1999) and Evans and Honkapohja (2001), if  $\kappa_t$  is a constant, however, there might be difficulties of convergence to the REE. If the model is non-stochastic and  $\kappa_t$  sufficiently small,  $\pi_t^e$  converges to REE under the condition  $\gamma_2 < 1$ . However, if the model is stochastic and  $\gamma_2 < 1$ , the belief does not converge to REE, but to an ergodic distribution around it. Here I follow Orphanides and Williams (2002) and assume that agents are constantly learning in a changing environment. The assumption of a constant gain implies that the agents believe the Phillips curve might exhibit structural changes and allocate larger weights to the recent observations of the inflation rate than to the earlier ones. Orphanides and Williams (2002) denote the case of  $\kappa_t = \frac{1}{t}$  as “infinite memory” and the case of a constant  $\kappa_t$  as “finite memory”. Following Svensson (1997, 1999b) I assume that the central bank pursues a monetary policy by minimizing a quadratic loss function. The problem reads as

$$\text{Min}_{\{r_t\}_0^\infty} E_0 \sum_{t=0}^{\infty} \rho^t L(\pi_t, r_t), \quad L(\pi_t, r_t) = (\pi_t - \pi^*)^2, \quad (5.22)$$

subject to eqs. (5.14), (5.18), (5.20) and (5.21).  $\pi^*$  is the target inflation rate which is assumed to be zero.<sup>14</sup>

As mentioned above, if the unknown parameters are adaptively estimated by way of the RLS learning algorithm with a small and constant gain, they will converge in distributions in a stochastic model and converge w.p.1 in a non-stochastic model. But an optimal control problem such as (5.22) with nonlinear state equations, embedded in eqs. (5.14), (5.18), (5.20) and (5.21), is difficult to solve, and using appropriate solution techniques indicates that the model will not necessarily converge even if the state equations are non-stochastic.

Next, I undertake an appropriate numerical study of the model. Though the return function is quadratic and the Phillips curve is linear, the problem

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<sup>14</sup>In order to simplify the problem I assume strict inflation targeting, that is, the central bank is concerned only with the inflation stabilization.

falls outside the scope of LQ optimal control problems, since some parameters in the Phillips curve are time-varying and follow nonlinear paths. The problem cannot be solved analytically and numerical solutions have to be explored. In the numerical study below I resort to the algorithm developed by Grüne (1997), who applies adaptive rather than uniform grids.<sup>15</sup> The numerical study is undertaken for the deterministic case. In order to simplify the numerical study, I assume that  $a_t = \bar{a}$  and only  $b_t$  has to be learned in the model. In this case one has  $c_t = b_t$  and  $z_i = r_{i-1}$ . As mentioned by Beck and Wieland (2002), the reason for focusing on the unknown parameter  $b$  is that this parameter is multiplicative to the decision variable  $r_t$  and therefore central to the trade-off between current control and estimation.

### *Numerical Study*

In this numerical study I assume  $\gamma_1 = 0.6$ ,  $\gamma_2 = 0.4$ ,  $\gamma_3 = 0.5$ ,  $\theta = 0.4$ ,  $\rho = 0.985$  and  $\kappa_t = 0.05$ . The initial values of  $\pi_t$ ,  $b_t$  and  $V_t$  are 0.2,  $-0.6$  and 0.04. The paths of  $\pi_t$ ,  $b_t$ ,  $V_t$  and  $r_t$  are shown in Figure 5.6A-D respectively. Figure 5.6E is the phase diagram of  $\pi_t$  and  $r_t$ . Neither the state variables nor the control variable converges. In fact, they fluctuate cyclically. I explore solution paths with many different initial values of the state variables and smaller  $\kappa_t$  (0.01 for example) and find that in no case do the variables converge. Similar results are obtained with different values for  $\gamma_1$  (e.g. 0.9 and 0.3) and  $\gamma_2$  (e.g. 0.1 and 0.7).

Given the above parameters, one has  $\bar{a} = 1$ ,  $\bar{b} = -0.33$ , therefore the REE is

$$\pi_t = \pi_{t-1} - 0.33r_{t-1} + \varepsilon_t. \quad (5.23)$$

In the case of RLS learning, however, one has

$$\pi_t = \pi_{t-1} + \tilde{b}_t r_{t-1} + \varepsilon_t,$$

with

$$\tilde{b}_t = \gamma_2 b_{t-1} - \gamma_3 \theta.$$

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<sup>15</sup>A less technical description and economic applications of this algorithm can be found in Grüne and Semmler (2002).

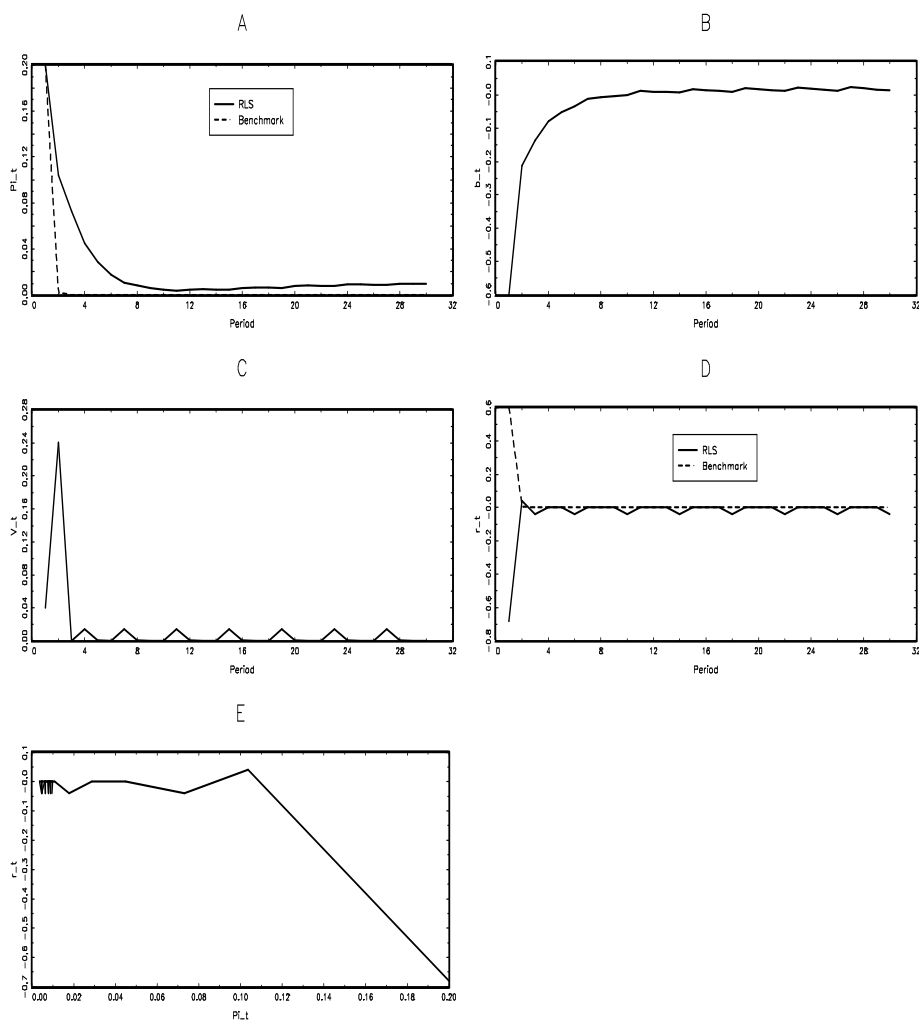


Figure 5.6: Simulations of RLS Learning (solid line) and Benchmark Model (dashed line) with Linear Phillips Curve

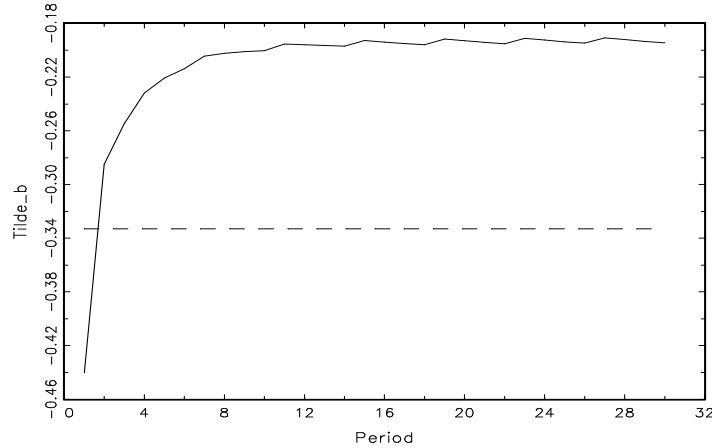


Figure 5.7: Path of  $\tilde{b}_t$  (solid line) in the Linear Phillips Curve

The path of  $\tilde{b}_t$  is presented in Figure 5.7.  $\tilde{b}_t$  evolves at a higher level than  $\bar{b}$ . Simulations are undertaken with different initial values of the state variables and similar results for  $\tilde{b}_t$  are obtained.

If there is perfect knowledge, namely, the agents have rational expectation,  $\pi_t$  can converge to its target value  $\pi^*$  (zero here), since the model then becomes a typical LQ control problem which has converging state and control variables in a non-stochastic model. I define this case as the benchmark model. The results of the benchmark model are shown in Figure 5.6A and 5.6D with dashed lines. Note that the benchmark model contains only one state variable, namely  $\pi_t$ , with dynamics denoted by (5.23). In the non-stochastic benchmark model the optimal monetary policy rule turns out to be  $r_t = 3.00\pi_t$  and the optimal trajectory of  $\pi_t$  is  $\pi_t = 0.01\pi_{t-1}$ . From Figure 5.6A and 5.6D one observes that  $\pi_t$  and  $r_t$  converge to zero over time in the benchmark model.

## RLS Learning in Nonlinear Phillips Curve

As surveyed in Section 2, the Phillips curve can be convex. Given such a convex Phillips curve, Eq. (5.12) reads as,

$$\pi_t = \gamma_1 \pi_{t-1} + \gamma_2 \pi_t^e + \gamma_3 f(y_t) + \varepsilon_t, \quad \varepsilon \sim iid(0, \sigma_\varepsilon^2), \quad (5.24)$$

with  $f(y_t)$  given by Eq. (5.10). Substituting Eq. (5.13) into Eq. (5.10), and then (5.10) into (5.24), one obtains the following nonlinear Phillips curve

$$\pi_t = \gamma_1 \pi_{t-1} + \gamma_2 \pi_t^e - \gamma_3 g(r_{t-1}) + \varepsilon_t, \quad \varepsilon \sim iid(0, \sigma_\varepsilon^2), \quad (5.25)$$

where

$$g(r_t) = \frac{\alpha \theta r_t}{1 + \alpha \beta \theta r_t}.$$

The REE turns out to be

$$\pi_t = \bar{a} \pi_{t-1} + \bar{b} g(r_{t-1}) + \varepsilon_t, \quad (5.26)$$

where  $\bar{a}$  is defined in (5.15) but  $\bar{b}$  is changed to be  $-\frac{\gamma_3}{1-\gamma_2}$ . The forecast of the inflation rate is now given by

$$\pi_t^e = a_{t-1} \pi_{t-1} + b_{t-1} g(r_{t-1}). \quad (5.27)$$

The RLS learning mechanism is the same as the case of the linear Phillips curve, except that  $z_i$  is now modified as

$$z_i = \left( \pi_{i-1} \quad g(r_{i-1}) \right)'$$

The optimal control problem (5.22) now turns out to have constraints (5.25), (5.27), (5.20) and (5.21).

### *Numerical Study*

In this version I assume  $\alpha = 10$  and  $\beta = 0.99$ . The solution paths with the same starting values of the state variables as in the previous subsection are presented in Figure 5.8A-D. Figure 5.8A represents the path of  $\pi_t$ , 5.8B is the path of  $b_t$ , 5.8C is the path of  $V_t$  and 5.8D is the path of  $r_t$ . The

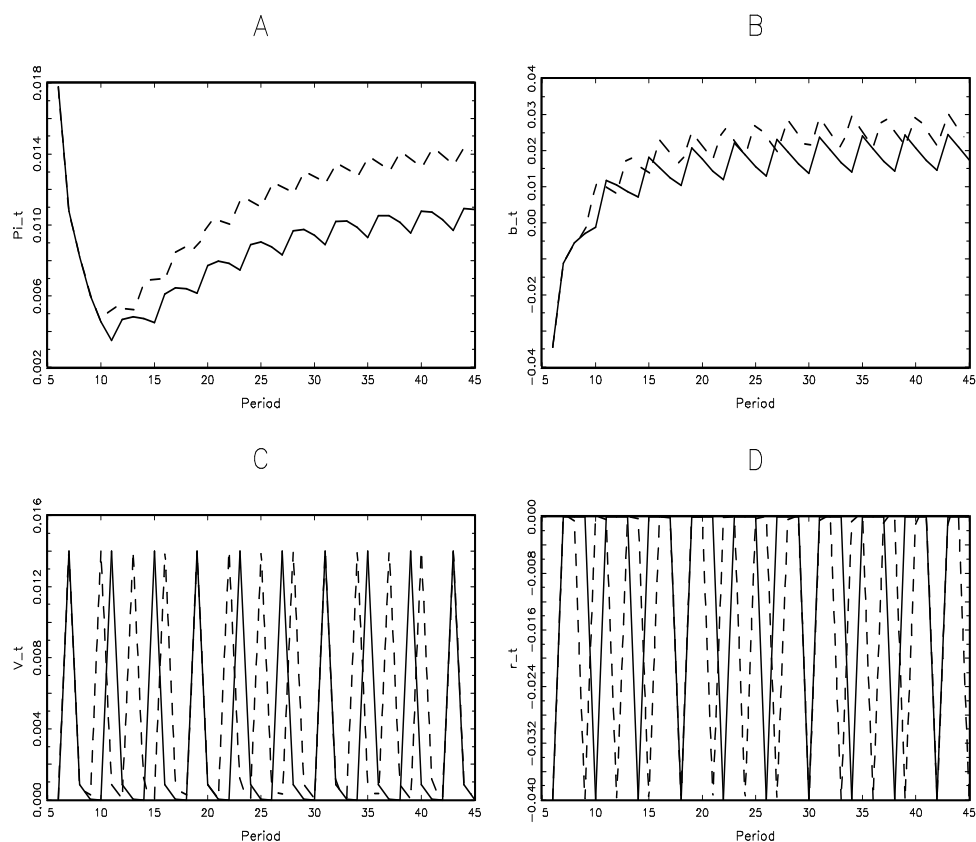


Figure 5.8: RLS Learning with Linear (solid line) and Nonlinear (dashed line) Phillips Curves

results of this subsection (nonlinear Phillips curve) are presented by dashed lines, while the results from the previous subsection (linear Phillips curve) are indicated by solid lines.<sup>16</sup>

One finds that the state variables also do not converge in the optimal control problem with the nonlinear Phillips curve. Similar to the case of the linear Phillips curve, the state and control variables fluctuate cyclically. Experiments with many different initial values of state variables were un-

<sup>16</sup>In order to see the differences of the simulations clearly, I just present the results from  $t=6$  on.

Table 5.3: Mean and S.D. of State and Control Variables (L and NL stand for linear and nonlinear Phillips curves respectively).

	$\pi_t$		$b_t$		$V_t$		$r_t$	
	L	NL	L	NL	L	NL	L	NL
Mean	0.0102	0.0135	0.0181	0.0243	0.0037	0.0049	-0.0101	-0.0135
S.D.	0.0016	0.0022	0.0069	0.0077	0.0060	0.0064	0.0174	0.0190

undertaken and in no case are the state variables found to converge. But the difference between the simulations with linear and nonlinear Phillips curves cannot be ignored. Figure 5.8 indicates that both  $\pi_t$  (Figure 5.8A) and  $b_t$  (Figure 5.8B) evolve at a higher level in the case of a nonlinear Phillips curve than in the case of a linear one. The mean and standard deviation of  $\pi_t$ ,  $b_t$ ,  $V_t$  and  $r_t$  from the two experiments are shown in Table 5.3. The S.D. and absolute values of the mean of these variables are larger in the case of the nonlinear Phillips curve than in the case of the linear one. As in the previous subsection the experiments are undertaken with different  $\gamma_1$  and  $\gamma_2$  and the results are found to be similar. The fact that the inflation rate has a higher mean and experiences larger changes in the nonlinear Phillips curve than in the linear one seems to be consistent with the research of Tambakis (1998) who analyzes the single-period Barro-Gordon optimal monetary problem with a convex Phillips curve and an asymmetric loss function. Tambakis (1998) finds that, both symmetric and asymmetric loss functions with a convex Phillips curve yield a positive expected inflation bias.

Next, I show the  $\tilde{b}$  in the nonlinear Phillips curve in Figure 5.9.  $\tilde{b}$  in the nonlinear Phillips curve equals  $\gamma_2 b_{t-1} - \gamma_3$ . The  $\tilde{b}$  and  $\bar{b}$  from the simulations with the linear Phillips curve are also shown in Figure 5.9, from which one finds that the  $\tilde{b}$  evolves at a higher level than  $\bar{b}$  in both linear and nonlinear Phillips curves.

Above I have explored optimal monetary policy rules with adaptive learning. The simulations indicate that the state variables do not converge no

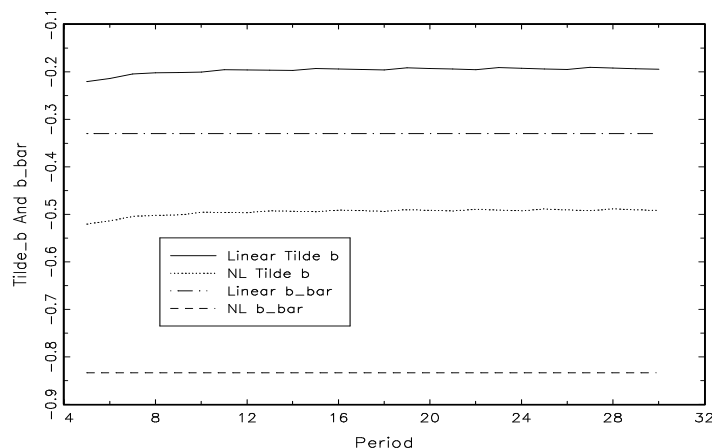


Figure 5.9: Paths of  $\tilde{b}_t$  and  $\bar{b}$  in Linear and Nonlinear Phillips Curves (NL stands for nonlinear)

matter whether the linear or nonlinear Phillips curve is employed. This is different from the conclusion of Sargent (1999), who claims that the state variables can converge in such a non-stochastic model. The problem of Sargent (1999), as mentioned before, is that he employs two assumptions which turn out to be inconsistent with each other. This is because he explores the problem in a traditional LQ framework which fails to endogenize the uncertain parameter.

## 5.4 Monetary Policy Rules with Robust Control

A disadvantage of adaptive learning analyzed in the previous section is that I have considered only parameter uncertainty. Monetary authorities have to guard against uncertainties such as model misspecification in a more general way. For this purpose robust control theory has been applied. Robust control induces the economic agents to seek a strategy for the “worst case” and can deal with more general uncertainties than the adaptive learning does.



On the basis of some earlier papers (see Hansen and Sargent 1999, 2001a, 2001b), Hansen and Sargent (2002) explore robust control in macroeconomics in detail. Cagetti, Hansen, Sargent and Williams (2002) also employ robust control in macroeconomics. Svensson (2000) applies the idea of robust control in a simpler framework. Giordani and Söderlind (2002) and Giannoni (2002), however, extend the robust control theory by considering forward-looking behaviors.

The interesting question concerning monetary policy under uncertainty is whether the central bank should show a stronger or weaker response to the fluctuations of economic variables than when no uncertainty exists. Brainard (1967), for example, proposes that parameter uncertainty should incur a more “cautious” policy. This argument has been supported by recent research of Martin and Salmon (1999) who explore the monetary policy of the UK, employing a VAR model with and without parameter uncertainty. They find that the optimal rule in the presence of parameter uncertainty incurs a less aggressive path for official interest rates than when no parameter uncertainty is considered. Other researchers, Gonzalez and Rodriguez (2003) and Giannoni (2002), for example, however, argue that uncertainty does not necessarily require caution. Following Hansen and Sargent (2002), in this section I will explore this problem with robust control and take up the problem of central bank’s response under uncertainty with respect to model misspecification.<sup>17</sup>

The research undertaken below is based on the framework of Hansen and Sargent (2002). A brief sketch of the robust control theory developed by Hansen and Sargent (2002) is presented in Appendix B of this chapter. Note that the so-called robustness parameter  $\theta$  plays an important role. It reflects the agents’ preferences of robustness and plays an important role in the problem’s solution. If  $\theta$  is  $+\infty$ , the problem collapses to the traditional optimal control without model misspecification. Gonzalez and Rodriguez (2003) explore how the robustness parameter  $\theta$  affects the control variable and prove

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<sup>17</sup>Brock et al. (2003) and Brock and Xepapadeas (2003) also explore policy in an uncertain economic environment. Whereas they have discussed how changes of parameters in a model may affect the policy response, here I discuss how the so-called robustness parameter may influence the policy rules.

that in a one-state and one-control model, the response is characterized by a hyperbolic function with a discontinuity at  $\underline{\theta}$ . Namely, the response presents a concave shape on the right side of the discontinuity and a convex one on the left.

Related to the above is the concept of a detection error probability. It is a statistic concept designed by Hansen and Sargent (2002, Ch. 13) to spell out how difficult it is to tell the approximating model apart from the distorted one. The larger the detection error probability, the more difficult to tell the two models apart. The design and interpretation of the detection error probability are shown in the appendix.

Semmler, Greiner and Zhang (2003) estimate the IS and Phillips curves with the US quarterly data for the period 1961-1999.<sup>18</sup> Next I will undertake some simulations of the robust control with the parameters estimated by Semmler, Greiner and Zhang (2003). Let  $A_{11}$  be the sum of the coefficients of the lagged inflation rates in the Phillips curves (0.965) and  $A_{22}$  be the sum of the coefficients of the lagged output gaps in the IS curve (0.864), one has

$$\mathbf{A} = \begin{pmatrix} 0.965 & 0.045 \\ 0.074 & 0.864 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 \\ -0.074 \end{pmatrix}, \quad x_t = \begin{pmatrix} \pi_t \\ y_t \end{pmatrix},$$

and  $u_t = r_t$  with  $\pi_t$ ,  $y_t$  and  $r_t$  being the inflation rate, output gap and deviation of the interest rate from its long-run equilibrium level (assumed to be zero in the simulation below) respectively. The problem then turns out to be

$$\underset{\{R_t\}_{t=0}^{\infty}}{Max} \underset{\{\omega_{t+1}\}_{t=0}^{\infty}}{Min} E_0 \sum_{t=0}^{\infty} \rho^t [ -(\pi_t^2 + \lambda y_t^2) + \rho \theta \omega'_{t+1} \omega_{t+1} ]$$

subject to

$$x_{t+1} = Ax_t + Br_t + C(\epsilon_{t+1} + \omega_{t+1}).$$

With the parameters above and the starting values of  $\pi_0$  and  $y_0$  both being 0.02,  $\lambda = 1$ ,  $\rho = 0.985$  and  $C = \begin{pmatrix} 0.01 & 0 \\ 0 & 0.01 \end{pmatrix}$ , the detection error probability

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<sup>18</sup>The data source is the International Statistical Yearbook. The output gap is measured by the percentage deviation of the IPI from its HP-filtered trend. The inflation rate is measured by changes in the GDP deflator and the short-term interest rate is measured by the federal funds rate.

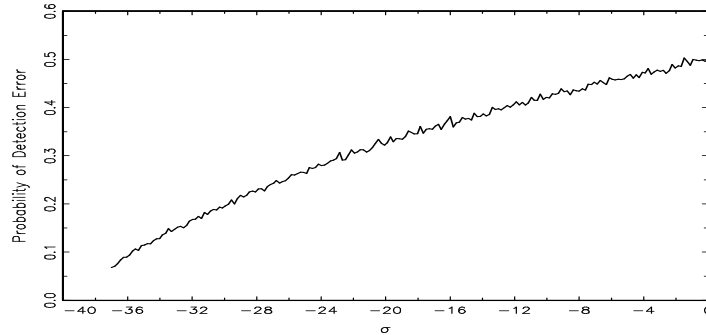


Figure 5.10: Detection Error Probability

is shown in Figure 5.10.<sup>19</sup> If one wants a detection error probability of about 0.15, then  $\sigma = -33$ , that is  $\theta = 0.03$ . With  $\theta = 0.03$ , one has

$$F = \begin{pmatrix} 10.462 & 12.117 \end{pmatrix}, \quad K = \begin{pmatrix} 5.291 & 0.247 \\ 4.737 \times 10^{-7} & 5.486 \times 10^{-7} \end{pmatrix},$$

and the value function turns out to be

$$V(\pi, y) = 16.240\pi^2 + 1.033y^2 + 1.421\pi y + 0.113.$$

If one wants a higher detection error probability, 0.40 for example, one has  $\sigma = -11$  ( $\theta = 0.091$ ), and

$$F = \begin{pmatrix} 7.103 & 11.960 \end{pmatrix}, \quad K = \begin{pmatrix} 1.173 & 0.055 \\ 1.072 \times 10^{-7} & 1.805 \times 10^{-7} \end{pmatrix},$$

and  $V(\pi, y) = 11.134\pi^2 + 1.022y^2 + 0.945\pi y + 0.080$ . In case  $\theta = +\infty$ , one has  $F = \begin{pmatrix} 6.438 & 11.929 \end{pmatrix}$  and  $V(\pi, y) = 10.120\pi^2 + 1.020y^2 + 0.850\pi y + 0.073$ . Comparing the elements in  $F$  obtained with different values of  $\theta$ , one finds that the lower the  $\theta$ , the higher the coefficients of the inflation rate and output gap in the interest-rate rule. That is, the farther the distorted model

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<sup>19</sup>The numerical studies in this section are done with the algorithms developed by Hansen and Sargent (2002). In the computation of the detection error probability,  $T$  (number of periods) is set to be 150 and 5000 simulations are undertaken here.

stays away from the approximating one, the stronger the response of the interest rate to the inflation and output gap. This is consistent with the conclusion of Gonzalez and Rodriguez (2003) who deal with a one-state and one-control model and prove that more uncertainty with respect to model misspecification requires a stronger response of the control variable. This is also consistent with the conclusion of Giannoni (2002) who shows that uncertainty does not necessarily require caution in a forward-looking model with robust control.

I present the paths of the inflation rate, output gap and interest rate with different values of  $\theta$  in Figure 5.11A-C. One finds that the lower the  $\theta$ , the larger the volatility of the state and control variables. The standard deviations of the state and control variables are shown in Table 5.4, which indicates that the standard deviations of the state and control variables increase if  $\theta$  decreases.

Table 5.4: Standard Deviations of the State and Control Variables with Different Values of  $\theta$

$\theta$	S.D. of $\pi_t$	S.D. of $y_t$	S.D. of $r_t$
0.03	0.038	0.028	0.223
0.09	0.032	0.017	0.186
$+\infty$	0.030	0.015	0.179

Next, I come to a special case, namely the case of zero shocks. What do the state and control variables look like and how can the robustness parameter  $\theta$  affect the state variables and the value function? According to the certainty equivalence principle, the optimal rules of the robust control with zero shocks are the same as when there are non-zero shocks. That is,  $F$  and  $K$  in Eq. (5.48) and (5.49) do not change no matter whether there are shocks or not. The difference lies in the value function. The simulations for zero shocks and with the same parameters as in the case of non-zero shocks are shown in Figure 5.12. Figure 5.12A-C present the paths of the state and control variables with different  $\theta$  respectively. In Figure 5.12 one finds that

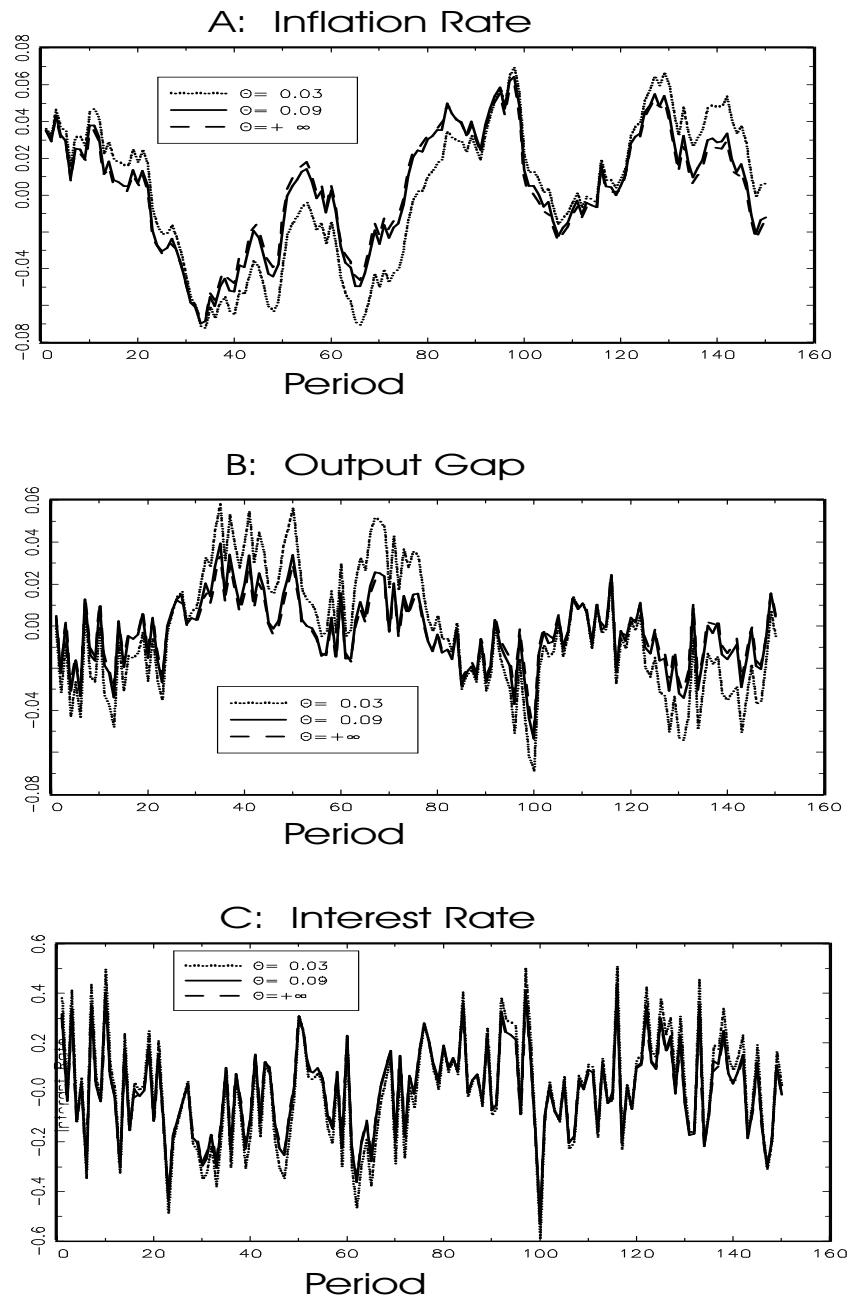


Figure 5.11: Simulation of the Robust Control with  $\pi_0 = 0.02$  and  $y_0 = 0.02$

the state variables converge to their equilibria zero no matter whether the robustness parameter is small or large. But in the case of a small robustness parameter, the state variables evolve at a higher level and converge more slowly to zero than when the robustness parameter is large. The simulations tell us that the larger the robustness parameter  $\theta$ , the lower the  $\pi_t$ ,  $y_t$  and  $r_t$ , and moreover, the faster the state variables converge to their equilibria. And in case  $\theta = +\infty$ , the state variables reach their lowest values and attain the equilibria at the highest speed.

In sum, I have shown that uncertainty with respect to model misspecification might not necessarily require caution. Though robust control can deal with problems that cannot be solved with the classical optimal control theory, some researchers have cast doubt on robust control. Chen and Epstein (2000) and Epstein and Schneider (2001), for example, criticize the application of the robust control theory for problems of time-inconsistency in preferences. Therefore, Hansen and Sargent (2001b) discuss the time-consistency of the alternative representations of preferences that underlie the robust control theory. An important criticism of robust control comes from Sims (2001a). He criticizes the robust control approach on conceptual grounds. As pointed out by Sims (2001a), there are major sources of more fundamental types of uncertainties that the robust control theory does not address.<sup>20</sup> One major uncertainty is the extent to which there is a medium run trade-off between inflation and output. Sims (2001a) shows that long run effects of inflation on output may not need to be completely permanent in order to be important. On the other hand, deflation may have strong destabilizing effects while interest rates are already very low. Thus, there may, in fact, be a long-run non-vertical Phillips curve.<sup>21</sup> Yet, the robust control approach developed so far seems to follow the neutrality postulate, implying a vertical long-run Phillips curve.

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<sup>20</sup>Moreover, steady states might not be optimal, if multiple steady states exist, see Greiner and Semmler (2002).

<sup>21</sup>See Graham and Snower (2002) and Blanchard (2003b), for example.

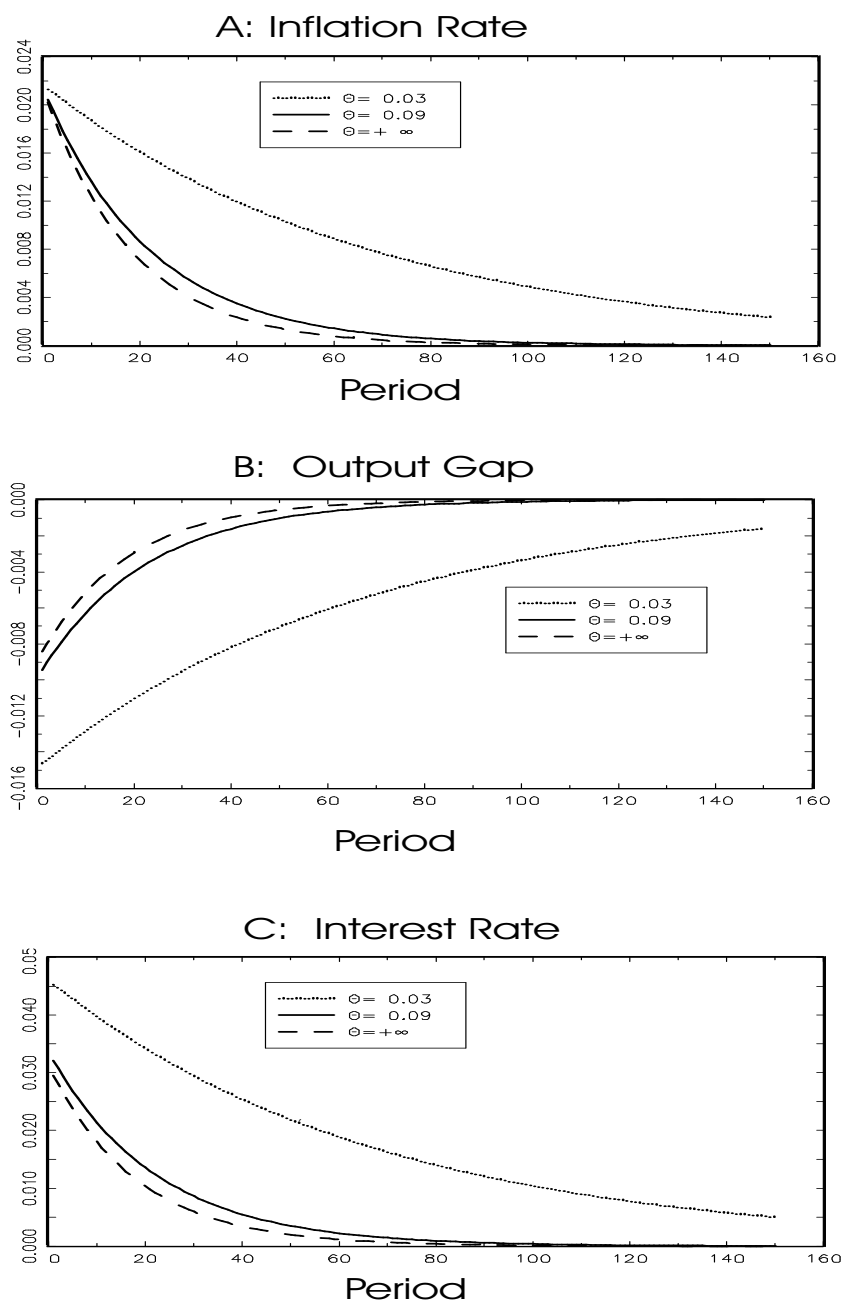


Figure 5.12: Results of the Robust Control with Zero Shocks

## 5.5 Conclusion

This chapter is concerned with monetary policy rules under uncertainty. I have first presented some empirical evidence of uncertainty using a State-Space model with Markov-Switching. The empirical model using the US data indicates that there have been regime changes in both parameters and shocks. Based on this empirical evidence I have then explored two approaches to deal with monetary policy under uncertainty: (a) adaptive learning, and (b) robust control. In the former case the central bank is assumed to improve its knowledge of economic models by learning from the information available. While the adaptive learning considers mainly parameter uncertainty, robust control admits more general uncertainties.

As regards adaptive learning, in contrast to Sargent (1999), who explores monetary policy with adaptive learning in a two-step decision process, I have endogenized the uncertain parameter and employed a dynamic programming algorithm with adaptive grids, which can deal with nonlinear state equations, so that I have solved the model appropriately. With such an approach I was able to overcome the problem of inconsistency in the two-step decision process of Sargent (1999). Yet, different from the results of Sargent (1999), I show that, when the learning of coefficients is fully endogenized, the state variables do not necessarily converge even in a non-stochastic model with adaptive learning.

As regards robust control, which is a more general approach to guard policy against uncertainty, I have focused on model misspecification and shock uncertainty and explored whether uncertainty requires caution. Different from the common view that monetary policy under uncertainty should be more cautious as compared to no uncertainty, my research indicates that the robust policy rule may respond more strongly to the economic variables in the presence of uncertainty and therefore implies that uncertainty does not necessarily require caution.

Finally I want to note that there might exist other kinds of uncertainties, among which the data uncertainty has attracted much attention. The usually discussed data uncertainty is concerned with the output gap or NAIRU



uncertainty. A discussion of the unreliability of output gap estimated with real-time data can be found in Orphanides and van Norden (2002). The effect of data uncertainty on monetary policy may be different from those of model and shock uncertainties.<sup>22</sup> In contrast to my above results on robust control, in the research on data uncertainty it is frequently found that the central bank should respond with greater caution to a variable estimated with error than it would in the absence of data uncertainty.<sup>23</sup> More research on data uncertainty and more succinct results are surely expected to be forthcoming in the future.

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<sup>22</sup>Jenkins (2002) claims that, the policy to be pursued may not be affected by pure data uncertainty in the case of additive-shock uncertainty. But in case the central bank follows an interest-rate rule that is a function of a small number of variables, the monetary policy rule may be largely affected by data uncertainty.

<sup>23</sup>Rudebusch (2001), for example, shows that an increase in output-gap uncertainty may reduce the coefficient on the output gap in the best simple rule.

## Appendix A: State-Space Model with Markov-Switching

Below I make a brief sketch of the State-Space model with Markov-Switching explored by Kim and Nelson (1999, Ch. 5).

Let  $\psi_{t-1}$  denote the vector of observations available as of time  $t-1$ . In the usual derivation of the Kalman filter in a State-Space model without Markov-Switching, the forecast of  $\phi_t$  based on  $\psi_{t-1}$  can be denoted by  $\phi_{t|t-1}$ . Similarly, the matrix denoting the mean squared error of the forecast can be written as

$$P_{t|t-1} = E[(\phi_t - \phi_{t|t-1})(\phi_t - \phi_{t|t-1})' | \psi_{t-1}],$$

where  $E$  is the expectation operator.

In the State-Space model with Markov-Switching, the goal is to form a forecast of  $\phi_t$  based not only on  $\psi_{t-1}$  but also conditional on the random variable  $S_t$  taking on the value  $j$  and on  $S_{t-1}$  taking on the value  $i$  ( $i$  and  $j$  equal 0 or 1):

$$\phi_{t|t-1}^{(i,j)} = E[\phi_t | \psi_{t-1}, S_t = j, S_{t-1} = i],$$

and correspondingly the mean squared error of the forecast is

$$P_{t|t-1}^{(i,j)} = E[(\phi_t - \phi_{t|t-1})(\phi_t - \phi_{t|t-1})' | \psi_{t-1}, S_t = j, S_{t-1} = i].$$

Conditional on  $S_{t-1} = i$  and  $S_t = j$  ( $i, j = 0, 1$ ), the Kalman filter algorithm for our model is as follows:

$$\phi_{t|t-1}^{(i,j)} = \bar{\Phi}_j + F \phi_{t-1|t-1}^i, \quad (5.28)$$

$$P_{t|t-1}^{(i,j)} = F P_{t-1|t-1}^i F' + \sigma_\eta^2, \quad (5.29)$$

$$\xi_{t|t-1}^{(i,j)} = Y_t - X_t \phi_{t|t-1}^{(i,j)}, \quad (5.30)$$

$$\nu_{t|t-1}^{(i,j)} = X_t P_{t|t-1}^{(i,j)} X_t' + \sigma_{\varepsilon,j}^2, \quad (5.31)$$

$$\phi_{t|t}^{(i,j)} = \phi_{t|t-1}^{(i,j)} + P_{t|t-1}^{(i,j)} X_t' [\nu_{t|t-1}^{(i,j)}]^{-1} \xi_{t|t-1}^{(i,j)}, \quad (5.32)$$

$$P_{t|t}^{(i,j)} = (I - P_{t|t-1}^{(i,j)} X_t' [\nu_{t|t-1}^{(i,j)}]^{-1} X_t) P_{t|t-1}^{(i,j)}, \quad (5.33)$$

where  $\xi_{t|t-1}^{(i,j)}$  is the conditional forecast error of  $Y_t$  based on information up to time  $t-1$  and  $\nu_{t|t-1}^{(i,j)}$  is the conditional variance of the forecast error  $\xi_{t|t-1}^{(i,j)}$ . In order to make the above Kalman filter algorithm operable, Kim and Nelson (1999) develop some approximations and manage to collapse  $\phi_{t|t}^{(i,j)}$  and  $P_{t|t}^{(i,j)}$  into  $\phi_{t|t}^j$  and  $P_{t|t}^j$  respectively.

Because the Phillips and IS curves contain only lags of variables and have uncorrelated noise, one can estimate the two equations separately. For the Phillips curve one has the following State-Space model

$$Y_t = \pi_t, \quad X_t = (1 \quad \pi_{t-1} \quad y_{t-1}), \quad \phi_t = (\alpha_{1t} \quad \alpha_{2t} \quad \alpha_{3t})', \quad \varepsilon_t = \varepsilon_{\pi t},$$

with

$$\begin{aligned} \varepsilon_{\pi t} &\sim N(0, \sigma_{\varepsilon\pi, S_t}^2), \\ \sigma_{\varepsilon\pi, S_t}^2 &= \sigma_{\varepsilon\pi, 0}^2 + (\sigma_{\varepsilon\pi, 1}^2 - \sigma_{\varepsilon\pi, 0}^2)S_t, \quad \sigma_{\varepsilon\pi, 1}^2 > \sigma_{\varepsilon\pi, 0}^2, \end{aligned}$$

and

$$\begin{aligned} \eta_t &= (\eta_{\alpha 1t} \quad \eta_{\alpha 2t} \quad \eta_{\alpha 3t})', \\ \sigma_\eta^2 &= (\sigma_{\eta\alpha 1}^2 \quad \sigma_{\eta\alpha 2}^2 \quad \sigma_{\eta\alpha 3}^2)', \\ \bar{\Phi}_{S_t} &= (\bar{\Phi}_{\alpha 1, S_t} \quad \bar{\Phi}_{\alpha 2, S_t} \quad \bar{\Phi}_{\alpha 3, S_t})', \\ F &= \begin{pmatrix} f_{\alpha 1} & 0 & 0 \\ 0 & f_{\alpha 2} & 0 \\ 0 & 0 & f_{\alpha 3} \end{pmatrix}, \end{aligned}$$

and similarly for the IS curve, one has

$$Y_t = y_t, \quad X_t = (1 \quad y_{t-1} \quad r_{t-1}), \quad \phi_t = (\beta_{1t} \quad \beta_{2t} \quad \beta_{3t})', \quad \varepsilon_t = \varepsilon_{yt},$$

with

$$\begin{aligned} \varepsilon_{yt} &\sim N(0, \sigma_{\varepsilon y, S_t}^2), \\ \sigma_{\varepsilon y, S_t}^2 &= \sigma_{\varepsilon y, 0}^2 + (\sigma_{\varepsilon y, 1}^2 - \sigma_{\varepsilon y, 0}^2)S_t, \quad \sigma_{\varepsilon y, 1}^2 > \sigma_{\varepsilon y, 0}^2, \end{aligned}$$

and

$$\begin{aligned}\eta_t &= (\eta_{\beta_1 t} \ \eta_{\beta_2 t} \ \eta_{\beta_3 t})', \\ \sigma_\eta^2 &= (\sigma_{\eta_{\beta_1}}^2 \ \sigma_{\eta_{\beta_2}}^2 \ \sigma_{\eta_{\beta_3}}^2)', \\ \bar{\Phi}_{S_t} &= (\bar{\Phi}_{\beta_1, S_t} \ \bar{\Phi}_{\beta_2, S_t} \ \bar{\Phi}_{\beta_3, S_t})' \\ F &= \begin{pmatrix} f_{\beta_1} & 0 & 0 \\ 0 & f_{\beta_2} & 0 \\ 0 & 0 & f_{\beta_3} \end{pmatrix}.\end{aligned}$$

## Appendix B: A Brief Sketch of Robust Control

Here I present a brief sketch of the robust control theory developed by Hansen and Sargent (2002). Let the one-period loss function be  $L(y, u) = -(x'Qx + u'Ru)$ , with  $Q$  being positive semi-definite and  $R$  positive definite matrices. The optimal linear regulator problem without model misspecification is

$$\text{Max}_{\{u_t\}_{t=0}^{\infty}} E_0 \sum_{t=0}^{\infty} \rho^t L(x_t, u_t), \quad 0 < \rho < 1, \quad (5.34)$$

subject to the so-called approximating model<sup>24</sup>

$$x_{t+1} = Ax_t + Bu_t + C\tilde{\epsilon}_{t+1}, \quad x_0 \text{ given}, \quad (5.35)$$

where  $\{\tilde{\epsilon}\}$  is an iid Gaussian vector process with mean zero and identity contemporaneous covariance matrix. If there is some model misspecification, the policy maker will not regard the model above as true but only as a good approximation to another model that cannot be specified. In order to express the misspecification which cannot be depicted by  $\tilde{\epsilon}$  because of its iid nature,

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<sup>24</sup>The matrices  $A$ ,  $B$ ,  $Q$  and  $R$  are assumed to satisfy the assumptions stated in Hansen and Sargent (2002, Ch. 3).

Hansen and Sargent (2002) take a set of models surrounding Eq. (5.35) of the form (the so-called distorted model)

$$x_{t+1} = Ax_t + Bu_t + C(\epsilon_{t+1} + \omega_{t+1}), \quad (5.36)$$

where  $\{\epsilon_t\}$  is another iid Gaussian process with mean zero and identity covariance matrix and  $\omega_{t+1}$  a vector process that can feed back in a general way on the history of  $x$ :

$$\omega_{t+1} = g_t(x_t, x_{t-1}, \dots), \quad (5.37)$$

where  $\{g_t\}$  is a sequence of measurable functions. When Eq. (5.36) generates the data, the errors  $\check{\epsilon}$  in (5.35) are distributed as  $\mathcal{N}(\omega_{t+1}, I)$  rather than as  $\mathcal{N}(0, I)$ . Hansen and Sargent (2002) further restrain the approximation errors by

$$E_0 \sum_{t=0}^{\infty} \rho^{t+1} \omega'_{t+1} \omega_{t+1} \leq \eta_0. \quad (5.38)$$

to express the idea that Eq. (5.35) is a good approximation when Eq. (5.36) generates the data. In order to solve the robust control problem (5.34) subject to Eq. (5.36) and (5.38), Hansen and Sargent (2002) consider two kinds of robust control problems, the constraint problem and the multiplier problem, which differ in how they implement the constraint (5.38). The constraint problem is

$$\underset{\{u_t\}_{t=0}^{\infty}}{\text{Max}} \underset{\{\omega_{t+1}\}_{t=0}^{\infty}}{\text{Min}} E_0 \sum_{t=0}^{\infty} \rho^t U(x_t, u_t), \quad (5.39)$$

subject to Eq. (5.36) and (5.38). Given  $\theta \in (\underline{\theta}, +\infty)$  with  $\underline{\theta} > 0$ , the multiplier problem can be presented as

$$\underset{\{u_t\}_{t=0}^{\infty}}{\text{Max}} \underset{\{\omega_{t+1}\}_{t=0}^{\infty}}{\text{Min}} E_0 \sum_{t=0}^{\infty} \rho^t \{U(x_t, u_t) + \rho\theta \omega'_{t+1} \omega_{t+1}\}, \quad (5.40)$$

subject to Eq. (5.36). Hansen and Sargent (2002, Ch. 6) prove that under certain conditions the two problems have the same outcomes. Therefore, solving one of the two problems is sufficient.

The robustness parameter  $\theta$  reflects the agents' preferences of robustness and plays an important role in the problem's solution. If  $\theta$  is  $+\infty$ , the

problem collapses to the traditional optimal control without model misspecification. In order to find a reasonable value for  $\theta$ , Hansen and Sargent (2002, Ch. 13) design a detection error probability function by a likelihood ratio. Consider a fixed sample of observations on the state  $x_t$ ,  $t = 0, \dots, T - 1$ , and let  $L_{ij}$  be the likelihood of that sample for model  $j$  assuming that model  $i$  generates the data, the likelihood ratio is

$$r_i \equiv \log \frac{L_{ii}}{L_{ij}}, \quad (5.41)$$

where  $i \neq j$ . When model  $i$  generates the data,  $r_i$  should be positive. Define

$$\begin{aligned} p_A &= \text{Prob}(\text{mistake}|A) = \text{freq}(r_A \leq 0), \\ p_B &= \text{Prob}(\text{mistake}|B) = \text{freq}(r_B \leq 0). \end{aligned}$$

Thus  $p_A$  is the frequency of negative log likelihood ratios  $r_A$  when model A is true and  $p_B$  is the frequency of negative log likelihood ratios  $r_B$  when model B is true. Attach equal prior weights to model A and B, the detection error probability can be defined as

$$p(\theta) = \frac{1}{2}(p_A + p_B). \quad (5.42)$$

When a reasonable value of  $p(\theta)$  is chosen, a corresponding value of  $\theta$  can be determined by inverting the probability function defined in (5.42). Hansen and Sargent (2002, Ch. 7) find that  $\theta$  can be defined as the negative inverse value of the so-called risk-sensitivity parameter  $\sigma$ , that is  $\theta = -\frac{1}{\sigma}$ .

Note the interpretation of the detection error probability. As seen above, it is a statistic concept designed to spell out how difficult it is to tell the approximating model apart from the distorted one. The larger the detection error probability, the more difficult to tell the two models apart. In the extreme case, when it is 0.5 ( $\theta = +\infty$ ), the two models are the same. So a central bank can choose a  $\theta$  according to how large a detection error probability it wants. If the detection error probability is very small, that means, if it is quite easy to tell the two models apart, it does not make much sense to design a robust rule. As stated by Anderson, Hansen and Sargent (2000), the aim of the detection error probability is to eliminate models that

are easy to tell apart statistically. Note that the higher the  $\theta$ , the lower the robustness, not the opposite.

Next, I present the solution of the multiplier problem.<sup>25</sup> Define

$$\mathcal{D}(P) = P + PC(\theta I - C'PC)^{-1}C'P, \quad (5.43)$$

$$\mathcal{F}(\Omega) = \rho[R + \rho B'\Omega B]^{-1}B'\Omega A, \quad (5.44)$$

$$T(P) = Q + \rho A (P - \rho PB(R + \rho B'PB)^{-1}B'P) A. \quad (5.45)$$

Let  $P$  be the fixed point of iterations on  $T \circ \mathcal{D}$ :

$$P = T \circ \mathcal{D}(P),$$

then the solution of the multiplier problem (5.40) is

$$u = -Fx, \quad (5.46)$$

$$\omega = Kx, \quad (5.47)$$

with

$$F = \mathcal{F} \circ \mathcal{D}(P), \quad (5.48)$$

$$K = \theta^{-1}(I - \theta^{-1}C'PC)^{-1}C'P[A - BF]. \quad (5.49)$$

It is obvious that in case  $\theta = +\infty$ ,  $\mathcal{D}(P) = P$  and the problem collapses into the traditional LQ problem.

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<sup>25</sup>See Hansen and Sargent (2002, Ch. 6) for details.

# Chapter 6

## Monetary Policy Rules with Financial Markets

### 6.1 Introduction

It is clear that the inflation rates in the industrial countries in the 1990s remained relatively stable and low, while the prices of equities, bonds, and foreign exchanges experienced a strong volatility with the liberalization of financial markets. Some central banks, therefore, have become concerned with such volatility and doubt whether it is justifiable on the basis of economic fundamentals. The question has arisen whether a monetary policy should be pursued that takes into account financial markets and asset price stabilization. In order to answer this question, it is necessary to model the relationship between asset prices and the real economy. An early study of this type can be found in Blanchard (1981) who has analyzed the relation between the stock value and output in “good news” and “bad news” cases. Recent examples include Bernanke and Gertler (2000), Smets (1997), Kent and Lowe (1997), Chiarella et al. (2001), Mehra (1998), Vickers (1999), Filardo (2000), Okina, Shirakawa and Shirats (2000) and Dupor (2001).

Among these papers, the research by Bernanke and Gertler (2000) has attracted much attention. Bernanke and Gertler (2000) employ a macroeconomic model and explore how the macroeconomy may be affected by alter-



native monetary policy rules which may, or may not, take into account the asset-price bubble. There they conclude that it is desirable for central banks to focus on underlying inflationary pressures and “asset prices become relevant only to the extent they may signal potential inflationary or deflationary forces” (Bernanke and Gertler, 2000, abstract).

The shortcomings of the position by Bernanke and Gertler (2000) may, however, be expressed as follows. First, they do not derive monetary policy rules from certain estimated models, but instead design artificially alternative monetary policy rules which may or may not consider asset-price bubbles and then explore the effects of these rules on the economy. Second, Bernanke and Gertler (2000) assume that the asset-price bubble always grows at a certain rate before breaking. However, the asset-price bubble in reality might not break suddenly, but may instead increase or decrease at a certain rate before becoming zero. Third, they assume that the bubble can exist for a few periods and will not occur again after breaking. Therefore, they explore the effects of the asset-price bubble on the real economy in the short-run. Fourth, they do not endogenize the probability that the asset-price bubble will break in the next period because little is known about the market psychology. Monetary policy with endogenized probability for bubbles to break may be different from that with an exogenous probability.

The difference between my model below and that of Bernanke and Gertler (2000) consists in the following points. First, I employ an intertemporal framework to explore what the optimal monetary policy should be with and without the financial markets taken into account. Second, I assume that the bubble does not break suddenly and does not have to always grow at a certain rate; on the contrary, it may increase or decrease at a certain rate with some probability. The bubble does not have to break in certain periods and moreover, it can occur again even after breaking. Third, I assume that the probability that the asset-price bubble will increase or decrease in the next period can be endogenized. This assumption has also been made by Kent and Lowe (1997). They assume that the probability for an asset-price bubble to break is a function of the current asset-price bubble and the monetary policy. The drawback of Kent and Lowe (1997), however,

is that they explore only positive bubbles and assume a linear probability function, which is not bounded between 0 and 1. Following Bernanke and Gertler (2000), I consider both positive and negative bubbles and employ a nonlinear probability function which lies between 0 and 1.

What, however, complicates the response of monetary policy to asset price volatility is the relationship of asset prices and product prices, the latter being mainly the concern of the central banks. Low asset prices may be accompanied by low or negative inflation rates. Yet, there is a zero bound on the nominal interest rate. The danger of deflation and the so-called “Liquidity Trap” has recently attracted much attention because there exists, for example, a severe deflation and recession in Japan and monetary policy seems to be of little help since the nominal rate is almost zero and can hardly be lowered further. On the other hand, the financial market of Japan has also been in a depression for a long time. Although some researchers have discussed the zero interest-rate bound and Liquidity Trap in Japan, little attention has been paid to the asset price depression in the presence of a zero bound on the nominal rate. I will explore this problem with some simulations of a simple model.

The remainder of this chapter is organized as follows. In Section 2 I set up the basic model under the assumption that central banks pursue monetary policy to minimize a quadratic loss function. I will derive a monetary policy rule from the basic model by assuming that the output can be affected by the asset-price bubbles. The probability for the asset-price bubble to increase or decrease in the next period is assumed to be a constant. Section 3 explores evidence of the monetary policy with asset price in the Euro-area with a model set up by Clarida, Gali and Gertler (1998). Section 4 extends the model by assuming that the probability that the asset-price bubble will increase or decrease in the next period is influenced by the size of the bubble and the current interest rate. Section 5 explores how the asset price may affect the real economy in the presence of the danger of deflation and a zero bound on the nominal rate. The last section concludes this chapter.

## 6.2 The Basic Model

### Monetary Policy Rule from a Traditional Model

Let's rewrite the simple model explored in Chapter 3:

$$\text{Min}_{\{r_t\}_0^\infty} \sum_{t=0}^{\infty} \rho^t L_t$$

with

$$L_t = (\pi_t - \pi^*)^2 + \lambda y_t^2, \quad \lambda > 0,$$

subject to

$$\pi_{t+1} = \alpha_1 \pi_t + \alpha_2 y_t, \quad \alpha_i > 0 \quad (6.1)$$

$$y_{t+1} = \beta_1 y_t - \beta_2 (r_t - \pi_t), \quad \beta_i > 0, \quad (6.2)$$

where  $\pi_t$  denotes the deviation of the inflation rate from its target  $\pi^*$  (assumed to be zero here),  $y_t$  is output gap, and  $r_t$  denotes the gap between the short-term nominal rate  $R_t$  and the long-run level of the short-term rate  $\bar{R}$  (i.e.  $r_t = R_t - \bar{R}$ ).  $\rho$  is the discount factor bounded between 0 and 1. In order for consistent expectations to exist,  $\alpha_1$  is usually assumed to be 1.

From Chapter 3 one knows the optimal policy rule reads

$$r_t = f_1 \pi_t + f_2 y_t, \quad (6.3)$$

with

$$f_1 = 1 + \frac{\rho \alpha_1^2 \alpha_2 \Omega_1}{(\lambda + \rho \alpha_2^2 \Omega_1) \beta_2}, \quad (6.4)$$

$$f_2 = \frac{\beta_1}{\beta_2} + \frac{\rho \alpha_2^2 \alpha_1 \Omega_1}{(\lambda + \rho \alpha_2^2 \Omega_1) \beta_2}; \quad (6.5)$$

and

$$\Omega_1 = \frac{1}{2} \left( 1 - \frac{\lambda(1 - \rho \alpha_1^2)}{\rho \alpha_2^2} + \sqrt{\left( 1 - \frac{\lambda(1 - \rho \alpha_1^2)}{\rho \alpha_2^2} \right)^2 + \frac{4\lambda}{\rho \alpha_2^2}} \right). \quad (6.6)$$

Equation (6.3) shows that the optimal short-term interest rate is a linear function of the inflation rate and output gap. It is similar to the Taylor rule (Taylor, 1993). The simulations undertaken in Chapter 3 show that the state and control variables converge to zero over time.

## Monetary Policy Rule with Asset-Price Bubbles

The model explored above does not take asset prices into account. Recently, however, some researchers argue that the financial markets can probably influence the inflation and output. Filardo (2000), for example, surveys some research which argues that the stock price may influence the inflation. Bernanke and Gertler (2000) explore how asset-price bubbles can affect the real economy with alternative monetary policy rules. Smets (1997) derives an optimal monetary policy rule from an intertemporal model under the assumption that the stock price can affect output. In the research below I also take into account the effects of the financial markets on the output and explore what the monetary policy rule should be. Before setting up the model I will explain some basic concepts.

In the research below I assume that the stock price  $s_t$  consists of the fundamental value  $\tilde{s}_t$  and the asset-price bubble  $b_t$ . I will not discuss how to compute the asset-price bubble or the fundamental value here, because this requires much work which is out of the scope of this chapter.<sup>1</sup> The stock price reads

$$s_t = \tilde{s}_t + b_t.$$

I further assume that if the stock price equals its fundamental value, the financial market exerts no effects on the output gap, that is, the financial market affects the output gap only through the asset-price bubbles. The asset-price bubble can be either positive or negative. The difference between the bubble in my research and those of Blanchard and Watson (1982), Bernanke and Gertler (2000), and Kent and Lowe (1997) is briefly stated below.

The so-called “rational bubble” defined by Blanchard and Watson (1982) cannot be negative because a negative bubble can lead to negative expected stock prices. Another difference between the bubble in this chapter and a rational bubble is that the latter always increases before breaking. Therefore, a rational bubble is non-stationary. Bernanke and Gertler (2000) also define

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<sup>1</sup>Alternative approaches have been proposed to compute the fundamental value and bubbles of the asset price. One example can be found in Shiller (1984).

the bubble as the gap between the stock price and its fundamental value. It can be positive or negative. The reason that they do not assume a rational bubble is that the non-stationarity of a rational bubble leads to technical problems in their framework. Kent and Lowe (1997) explore only positive bubbles.

Bernanke and Gertler (2000) and Kent and Lowe (1997), however, have something in common: they all assume that the bubble will break in a few periods (4 or 5 periods) from a certain value to zero *suddenly* rather than *gradually*. Moreover, if the bubble is broken, it will not occur again. This is, in fact, not true in practice, because in reality the bubble does not necessarily break suddenly from a large or low value, but may decrease or increase step by step before becoming zero rapidly or slowly. Especially, if the bubble is negative, it is implausible that the stock price will return to its fundamental value suddenly. A common assumption of the rational bubble and those definitions of Bernanke and Gertler (2000) and Kent and Lowe (1997) is that they all assume that the bubble will grow at a certain rate before it bursts.

Although I also define the asset-price bubble as the deviation of the asset price from its fundamental value, the differences between the bubble in this chapter and those mentioned above are obvious. To be precise, the bubble in my research below has the following properties: (a) it can be positive or negative, (b) it can increase or decrease before becoming zero or may even change from a positive (negative) one to a negative (positive) one and does not have to burst suddenly, (c) nobody knows when it will burst and, (d) it can occur again in the next period even if it becomes zero in the current period. Therefore, I assume the asset-price bubble evolves in the following way

$$b_{t+1} = \begin{cases} b_t(1 + g_1) + \varepsilon_{t+1}, & \text{with probability } p \\ b_t(1 - g_2) + \varepsilon_{t+1}, & \text{with probability } 1 - p \end{cases} \quad (6.7)$$

where  $g_1, g_2 (\geq 0)$  are the growth rate or decrease rate of the bubble.  $g_1$  can, of course, equal  $g_2$ .  $\varepsilon_t$  is an iid noise with zero mean and a constant variance. *Eq. (6.7) indicates that if the asset-price bubble  $b_t$  is positive, it may increase at rate  $g_1$  with probability  $p$  and decrease at rate  $g_2$  with probability  $1 - p$  in*

the next period. If the bubble is negative, however, it may decrease at rate  $g_1$  with probability  $p$  and increase at rate  $g_2$  with probability  $1 - p$  in the next period. The probability  $p$  is assumed to be a constant in this section, but state-dependent in the fourth section. From this equation one finds that even if the bubble is zero in the current period, it might not be zero in the next period.

Before exploring the monetary policy with asset-price bubbles theoretically, I explore some empirical evidence of the effects of the share bubbles on output gap. To be precise, I estimate the following equation by way of the OLS with the quarterly data of several OECD countries:

$$y_t = c_0 + c_1 y_{t-1} + c_2 b_{t-1} + \epsilon_t, \quad \epsilon_t \sim N(0, \sigma_\epsilon^2) \quad (6.8)$$

with  $y_t$  denoting the output gap. Following Clarida, Gali and Gertler (1998) I use the Industrial Production Index (IPI) to measure the output. The output gap is measured by the percentage deviation of the IPI (base year: 1995) from its Band-Pass filtered trend.<sup>2</sup> Similarly the asset-price bubble is measured by the percentage deviation of the share price index (base year: 1995) from its Band-Pass filtered trend just for simplicity. The estimation of Eq. (6.8) is shown in Table 6.1 with T-Statistics in parentheses. The estimate of  $c_0$  is not shown just for simplicity. The estimation is undertaken for two samples: (a) 1980-1999, and (b) 1990-1999.

From Table 6.1 one finds that  $c_2$  is significant enough in most cases. For the sample from 1990-99 it is significant enough in the cases of all countries except the US, but for the sample from 1980-99 it is significant enough in the case of the US. For the sample from 1980-99 it is insignificant in the cases of France and Italy, but significant enough in the cases of both countries in the period from 1990-99. It is significant enough in both samples of Japan. In short, the evidence in Table 6.1 does show some positive relation between

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<sup>2</sup>The reader is referred to Baxter and King (1995) for the Band-Pass filter. As surveyed by Orphanides and van Norden (2002), there are many methods to measure the output gap. I find that filtering the IPI using the Band-Pass filter leaves the measure of the output gap essentially unchanged from the measure with the HP-filter. The Band-Pass filter has also been used by Sargent (1999).

Table 6.1: Estimation of Eq. (6.8)

Para.	Sample	US	UK*	France	Germany	Italy	Japan
$c_1$	80.1-99.1	0.902 (22.218)	0.827 (16.821)	0.879 (19.170)	0.855 (19.313)	0.912 (22.024)	0.865 (18.038)
	90.1-99.1	0.925 (15.790)	0.918 (22.362)	0.836 (12.153)	0.808 (16.267)	0.843 (11.666)	0.864 (12.889)
$c_2$	80.1-99.1	0.064 (5.158)	0.050 (2.898)	0.005 (0.713)	0.021 (2.506)	0.002 (0.385)	0.045 (3.505)
	90.1-99.1	0.0005 (0.035)	0.099 (5.517)	0.032 (2.328)	0.075 (6.085)	0.020 (1.921)	0.063 (3.220)
$R^2$	80.1-99.1	0.875	0.824	0.845	0.864	0.869	0.835
	90.1-99.1	0.886	0.953	0.849	0.928	0.819	0.858

\*The estimation of the UK is undertaken for 80.1-97.1 and 90.1-97.1 because the share price index after 1997 is unavailable. Data source: International Statistical Yearbook.

the share bubbles and the output gap.

In the estimation above I have considered only the effect of the lagged asset-price bubble on output for simplicity, but in reality the expectation of financial markets may also influence the output. As regards how financial variables may influence the output, the basic argument is that the changes of the asset price may influence consumption (see Ludvigson and Steindel (1999), for example) and investment, which may in turn affect the inflation and output. The investment, however, can be affected by both current and forward-looking behaviors.

Therefore, in the model below I assume that the output gap can be influenced not only by the lagged asset-price bubble but also by expectations of asset-price bubbles formed in the previous period, that is,

$$y_{t+1} = \beta_1 y_t - \beta_2 (r_t - \pi_t) + \beta_3 b_t + (1 - \beta_3) E b_{t+1|t}, \quad 1 > \beta_3 > 0, \quad (6.9)$$

where  $E b_{t+1|t}$  denotes the expectation of  $b_{t+1}$  formed at time  $t$ . From Eq. (6.7) and  $E \varepsilon_{t+1|t} = 0$  one knows

$$E b_{t+1|t} = [1 - g_2 + p(g_1 + g_2)] b_t. \quad (6.10)$$

As a result, Eq. (6.9) turns out to be

$$y_{t+1} = \beta_1 y_t - \beta_2 \{r_t - \pi_t\} + \{1 + (1 - \beta_3)[p(g_1 + g_2) - g_2]\} b_t. \quad (6.11)$$

One can follow the same procedure as in Chapter 3 to solve the optimal control problem, since the bubble is taken as an exogenous variable. After replacing Eq. (6.2) with Eq. (6.11) one obtains the following monetary policy rule for the central bank

$$r_t = f_1 \pi_t + f_2 y_t + f_3 b_t, \quad (6.12)$$

with  $f_1$  and  $f_2$  given by (6.4)–(6.5) and

$$f_3 = \frac{1}{\beta_2} \{1 + (1 - \beta_3)[p(g_1 + g_2) - g_2]\}. \quad (6.13)$$

This rule is similar to the one obtained before except that there is an additional term of the bubble. The effect of  $p$  on the monetary policy rule can be explored from the following derivative

$$\frac{df_3}{dp} = \frac{1}{\beta_2} [(1 - \beta_3)(g_1 + g_2)] \geq 0. \quad (6.14)$$

The interpretation of (6.14) depends on whether the bubble is positive or negative. If the bubble is positive, a larger  $p$  leads to a higher  $f_3$  and as a result, a higher  $r_t$ . This is consistent with intuition, because in order to eliminate a positive bubble which is likely to continue to increase, it is necessary to raise the interest rate, since it is usually argued that there exists a negative relation between the interest rate and stock price.

Some empirical evidence on the effects of interest rate on the stock price can be obtained from the estimation of the following equation

$$b_t = \gamma_0 + \gamma_1 b_{t-1} + \gamma_2 r_t + \xi_t, \quad \gamma_2 < 0, \quad \xi_t \sim N(0, \sigma_\xi^2). \quad (6.15)$$

The estimation results with the quarterly data of several OECD countries are shown in Table 6.2 with T-Statistics in parentheses. From Table 6.2 one finds that the estimates of  $\gamma_2$  always have the correct sign with relatively significant T-Statistics except Italy. If one tries the sample from 1990-99 for



Table 6.2: Estimation of Eq. (6.15)

Parameter	US	UK	France	Germany	Italy	Japan
$\gamma_1$	0.902 (21.877)	0.874 (20.125)	0.909 (24.227)	0.898 (23.050)	0.921 (25.636)	0.858 (17.429)
$\gamma_2$	-0.288 (2.738)	-0.450 (2.591)	-0.363 (2.583)	-0.339 (2.067)	-0.070 (0.410)	-0.353 (1.712)
$R^2$	0.809	0.815	0.838	0.831	0.854	0.728

Data source: International Statistical Yearbook. Sample 1970.1-99.1 (UK 70.1-97.1). The short-term interest rates of the US, the UK, France, Germany, Japan and Italy are the federal funds rate, the treasury bill rate (UK and France), call money rate (Germany and Japan) and official discount rate respectively.

Italy, however, one obtains a significant T-Statistic (2.923) of  $\gamma_2$  with the correct sign.<sup>3</sup>

If the bubble is negative, however, a larger  $p$  also leads to a higher  $f_3$  but a lower  $r_t$ , since  $b_t$  is negative. That is, in order to eliminate a negative bubble which is likely to continue to decrease further, the interest rate should be decreased because of the negative relation between the interest rate and asset price. As stated before, although  $p$  may be state-dependent, I do not consider this possibility in this section.

### 6.3 Monetary Policy Rule in Practice: The Case of the Euro-Area

So far I have explored theoretically the monetary policy rule with the asset price volatility considered. The question is then whether asset-price bubbles have been taken into account in practice. This section presents some

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<sup>3</sup>I have also estimated Eq. (6.15) with  $r_{t-1}$  instead of  $r_t$  and find that the estimates of  $\gamma_2$  have correct signs but with smaller T-Statistics than those shown in Table 6.2, lying between 1.51 and 2.59 for all countries except Italy. The empirical evidence above suggests that the asset-price bubble can be an endogenous variable rather than an exogenous one. I will not discuss this possibility below, since this may make the model much more complicated to analyze.

empirical evidence on this problem.

Following Clarida, Gali and Gertler (1998) (CGG98 for short), Smets (1997) estimates the monetary reaction function of Canada and Australia by adding three financial variables into the CGG98 model, namely, the nominal trade-weighted exchange rate, ten-year nominal bond yield and a broad stock market index. His conclusion is that an appreciation of the exchange rate induces a significant change in the interest rates of the Bank of Canada. Moreover, he finds that changes in the stock market index also induces significant changes in the policy reaction function. The response coefficients in the case of Australia are, however, insignificant.

Bernanke and Gertler (2000) also follow CGG98 by adding stock returns into the model to test whether interest rates respond to stock returns in the US and Japan. Their conclusion is that the federal funds rate did not show a significant response to stock returns from 1979-97. For Japan, however, they find different results. To be precise, for the whole period 1979-97, there is little evidence that the stock market played a role in the interest-rate setting, but for the two subperiods, 1979-89 and 1989-97, the coefficients of stock returns have enough significant T-Statistics, but with different signs. Rigobon and Sack (2001), however, claim that the US monetary policy has reacted significantly to stock market movements.

In this section I also follow CGG98 to test whether the Euro-area monetary policy shows a significant response to the stock market.<sup>4</sup>

CGG98 assume that the short-term interest rate has the following path:

$$R_t = (1 - \kappa)R_t^* + \kappa R_{t-1} + v_t, \quad (6.16)$$

where  $R_t$  denotes the short-term interest rate,  $R_t^*$  is the target interest rate,  $v_t$  denotes an iid noise with zero mean and a constant variance, and  $\kappa$  captures the degree of interest-rate smoothing. The target interest rate is assumed to be determined in the following way:

$$R_t^* = \bar{R} + \beta(E[\pi_{t+n}|\Omega_t] - \pi^*) + \gamma(E[y_t|\Omega_t] - y_t^*),$$

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<sup>4</sup>The aggregation of data is the same as in Chapter 2.

where  $\bar{R}$  is the long-run equilibrium nominal rate,  $\pi_{t+n}$  is the rate of inflation between periods of  $t$  and  $t+n$ ,  $y_t$  is the real output, and  $\pi^*$  and  $y^*$  are targets of the inflation and output respectively.  $E$  is the expectation operator and  $\Omega_t$  is the information available to the central bank at the time it sets the interest rate. After adding the stock market into the equation above one obtains

$$R_t^* = \bar{R} + \beta(E[\pi_{t+n}|\Omega_t] - \pi^*) + \gamma(E[y_t|\Omega_t] - y_t^*) + \theta(E[s_{t+n}|\Omega_t] - \tilde{s}_{t+n}), \quad (6.17)$$

where  $s_{t+n}$  is the asset price in period  $t+n$  and  $\tilde{s}_t$  denotes the fundamental value of the asset price.  $\theta$  is expected to be positive, since I assume that central banks try to stabilize the stock market with the interest rate as the instrument. Define  $\alpha = \bar{R} - \beta\pi^*$ ,  $x_t = y_t - y_t^*$  and  $b_{t+n} = s_{t+n} - \tilde{s}_{t+n}$  (namely the asset-price bubble), Eq. (6.17) can be rewritten as

$$R_t^* = \alpha + \beta E[\pi_{t+n}|\Omega_t] + \gamma E[x_t|\Omega_t] + \theta E[b_{t+n}|\Omega_t], \quad (6.18)$$

after substituting Eq. (6.18) into (6.16), one has the following path for  $R_t$ :

$$R_t = (1 - \kappa)\alpha + (1 - \kappa)\beta E[\pi_{t+n}|\Omega_t] + (1 - \kappa)\gamma E[x_t|\Omega_t] + (1 - \kappa)\theta E[b_{t+n}|\Omega_t] + \kappa R_{t-1} + v_t. \quad (6.19)$$

After eliminating the unobserved forecast variables from the expression, one obtains the following presentation:

$$R_t = (1 - \kappa)\alpha + (1 - \kappa)\beta\pi_{t+n} + (1 - \kappa)\gamma x_t + (1 - \kappa)\theta b_{t+n} + \kappa R_{t-1} + \eta_t, \quad (6.20)$$

where  $\eta_t = -(1 - \kappa)\{\beta(\pi_{t+n} - E[\pi_{t+n}|\Omega_t]) + \gamma(x_t - E[x_t|\Omega_t]) + \theta(b_{t+n} - E[b_{t+n}|\Omega_t])\} + v_t$  is a linear combination of the forecast errors of the inflation, output gap, asset-price bubbles and the iid  $v_t$ . Let  $\mu_t$  be a vector of variables within the central bank's information set at the time it chooses the interest rate that are orthogonal to  $\eta_t$ , one has

$$E[R_t - (1 - \kappa)\alpha - (1 - \kappa)\beta\pi_{t+n} - (1 - \kappa)\gamma x_t - (1 - \kappa)\theta b_{t+n} - \kappa R_{t-1} | \mu_t] = 0. \quad (6.21)$$

Table 6.3: GMM Estimation of Eq. (6.21) with Different n for  $b_{t+n}$ 

Parameter	Estimates				
	n=0	n=1	n=2	n=3	n=4
$\kappa$	0.813 (19.792)	0.811 (18.561)	0.894 (30.224)	0.833 (15.870)	0.832 (17.089)
$\alpha$	0.030 (4.581)	0.028 (3.920)	0.007 (0.466)	0.020 (1.918)	0.021 (2.074)
$\beta$	0.748 (5.446)	0.777 (5.343)	1.522 (3.921)	0.940 (4.410)	0.890 (4.567)
$\gamma$	2.046 (5.679)	2.011 (5.300)	1.626 (3.234)	2.345 (3.990)	2.363 (4.203)
$\theta$	0.014 (0.509)	0.030 (0.927)	0.240 (2.328)	0.081 (1.264)	0.082 (1.100)
$R^2$	0.914	0.913	0.930	0.904	0.904
$J - Stat.$	0.088	0.087	0.111	0.069	0.074

Following CGG98 and the estimation in Chapter 2 I use the GMM to estimate this equation with the EU3 quarterly data.<sup>5</sup> Let  $\pi_{t+n} = \pi_{t+4}$ , as for  $b_{t+n}$  I will try the estimation with different n (0,1,..4).<sup>6</sup> The estimates with different n of  $b_{t+n}$  are presented in Table 6.3, with T-Statistics in parentheses.

As shown in Table 6.3,  $\beta$  and  $\gamma$  always have the correct signs and significant T-Statistics, indicating that the inflation and output always play important roles in the interest-rate setting. As for  $\theta$ , one finds that it always has the correct sign, but the T-Statistics are not always significant enough. When n=0 and 1, it is insignificant, when n=3 and 4, it is not enough significant, but when n=2 it is significant enough. Therefore, one may say that the asset price may have played a role (although not necessarily an important

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<sup>5</sup>In order to get the initial estimates of the parameters, I estimate the equation with traditional non-linear 2SLS methods first. The instruments include the 1-4 lags of the output gap, inflation rate, German call money rate, asset-price bubbles, nominal USD/ECU exchange rate and a constant. The instruments are pre-whitened before the estimation. Data source: International Statistical Yearbook.

<sup>6</sup>Correction for MA(4) autocorrelation is undertaken, and J-statistics are presented to illustrate the validity of the overidentifying restrictions. A brief explanation of the J-statistic is given in footnote 3 in Chapter 2.

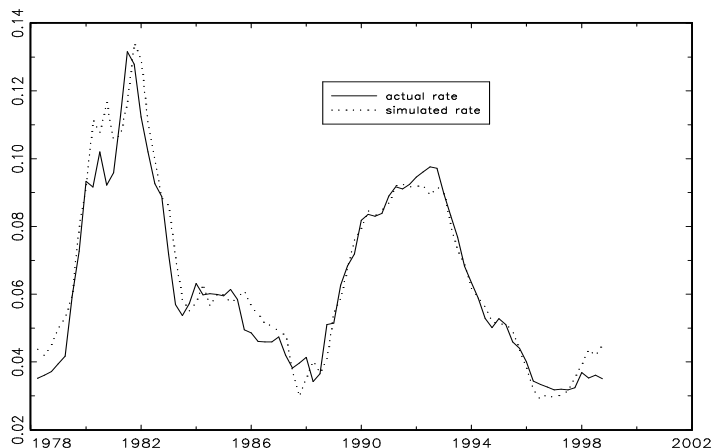


Figure 6.1: Actual and Simulated Interest Rates of EU3 (1978.1-98.4)

one) in the interest-rate setting in the Euro-area. The simulated interest rate with  $b_{t+n} = b_{t+2}$  is presented together with the actual interest rate in Figure 6.1. It is clear that the two rates are close to each other, especially after the second half of the 1980s.

## 6.4 Endogenization of $P$ and a Nonlinear Monetary Policy Rule

Up to now I have explored monetary policy with a constant probability for the asset-price bubble to increase or decrease in the next period. This is, in fact, a simplified assumption. Monetary policy and other economic variables can probably influence the path of  $p$ . Bernanke and Gertler (2000) take it as an exogenous variable because so little is known about the effects of policy actions on  $p$  that it is hard to endogenize  $p$ . Kent and Lowe (1997), however, endogenize the probability for the bubble to break as follows:

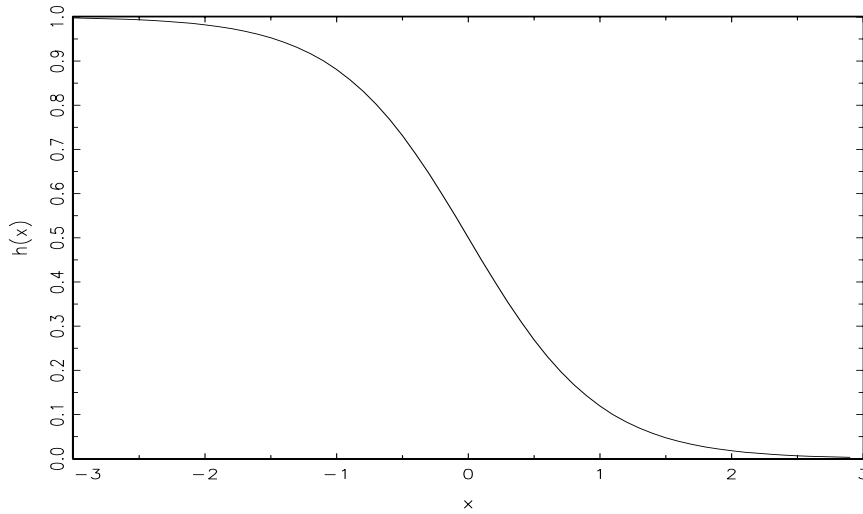
$$p_{t+1} = \phi_0 + \phi_1 b_t + \phi_2 r_t, \quad \phi_i > 0. \quad (6.22)$$

This function implies that the probability for the asset-price bubble to break in the next period depends on three factors: (a) an exogenous probability  $\phi_0$ , (b) the size of the current bubble, and (c) the level of the current interest rate. The larger the size of the current bubble and the higher the current interest rate, the larger the probability for the bubble to break in the next period. Note that, as mentioned before, Kent and Lowe (1997) analyze only positive asset-price bubbles. Kent and Lowe (1997) describe the effect of the size of the current bubble on  $p$  as follows:

... as the bubble becomes larger and larger, more and more people identify the increase in asset prices as a bubble and become increasingly reluctant to purchase the asset; this makes it more likely that a correction will occur (Kent and Lowe, 1997, p.16).

The effect of the current interest rate level on  $p$  is clear. That is, as the interest rate increases, the economic agents may expect the asset price to decrease, which raises the probability that the bubble will break in the next period.

In this section I will endogenize the  $p$ . Although the function given by Eq. (6.22) seems to be a reasonable choice, I will not employ it below for the following reasons: (a) as stated above, Kent and Lowe (1997) explore only positive bubbles, while I consider both positive and negative ones. When the asset-price bubble is positive, Eq. (6.22) is a reasonable choice. If the bubble is negative, however, this function has problems. (b) A probability function should be bounded between 0 and 1, but Eq. (6.22) is an increasing function without bounds. (c) Eq. (6.22) is a linear function, indicating that  $p$  changes proportionally to the changes of the bubble size and the interest rate. This may not be true in reality. (d) The  $p$  in our model describes the probability that the bubble will increase (if the bubble is positive) or decrease (if the bubble is negative) in the next period, while that in the model of Kent and Lowe (1997) describes the probability that the positive bubble will break in the next period.

Figure 6.2:  $h(x)$ 

Before designing the probability function, I introduce a function  $h(x)$  that will be used below. To be precise, define

$$h(x) = \frac{1}{2}[1 - \tanh(x)]. \quad (6.23)$$

It is clear that  $\frac{dh(x)}{dx} = -\frac{1}{2 \cosh^2(x)} < 0$ , with  $\lim_{x \rightarrow \infty} h(x) = 0$  and  $\lim_{x \rightarrow -\infty} h(x) = 1$ . The function  $h(x)$  is shown in Figure 6.2.

Next, I define the probability function  $p_{t+1}$  as

$$p_{t+1} = \frac{1}{2}\{1 - \tanh[\vartheta(b_t, r_t)]\}, \quad (6.24)$$

with

$$\vartheta(b_t, r_t) = \phi_1 f(b_t) + \phi_2 \text{sign}(b_t) r_t, \quad \phi_i > 0,$$

where  $\text{sign}(b_t)$  is the sign function which reads

$$\text{sign}(b_t) = \begin{cases} 1, & \text{if } b_t > 0; \\ 0, & \text{if } b_t = 0; \\ -1, & \text{if } b_t < 0, \end{cases} \quad (6.25)$$

and  $f(b_t)$  is the so-called LINEX function which is nonnegative and asymmetric around 0. The LINEX function, which can be found in Varian (1975)

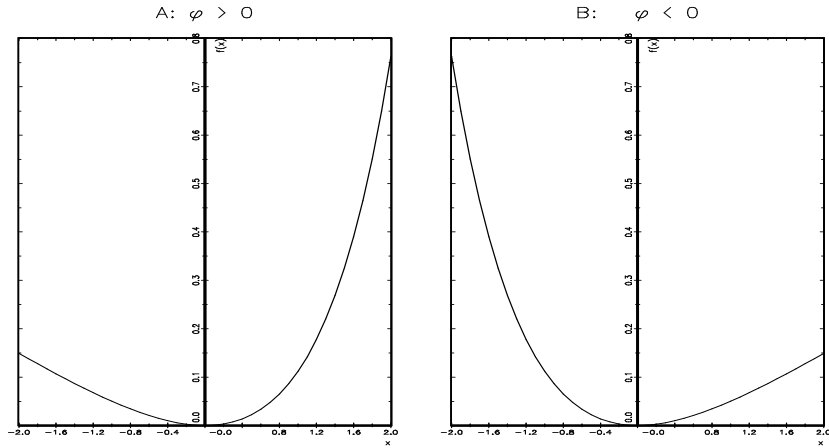


Figure 6.3: The LINEX Function

and Nobay and Peel (2003), reads

$$f(x) = \kappa[e^{\varphi x} - \varphi x - 1], \quad \kappa > 0, \varphi \neq 0. \quad (6.26)$$

$\kappa$  scales the function and  $\varphi$  determines the asymmetry of the function. An example of  $f(x)$  with  $\kappa = 0.1$  and  $\varphi = \pm 1.2$  is shown in Figure 6.3. In the work below I take  $\kappa = 1$  and  $\varphi > 0$ . The function  $f(x)$  with a positive  $\varphi$  is flatter when  $x$  is negative than when  $x$  is positive.

It is clear that

$$\frac{\partial p_{t+1}}{\partial b_t} = -\frac{\phi_1 \varphi (e^{\varphi b_t} - 1)}{2 \cosh^2[\vartheta(b_t, r_t)]} \begin{cases} < 0, & \text{if } b_t > 0, \\ > 0, & \text{if } b_t < 0. \end{cases} \quad (6.27)$$

Therefore, the probability function given by Eq. (6.24) indicates that the effects of the current asset-price bubble  $b_t$  on  $p_{t+1}$  depends on whether the bubble is positive or negative. In fact, the probability function defined above is asymmetric around  $b_t = 0$ . If it is positive, a larger bubble in the current period implies a lower probability that it will increase in the next period. This is consistent with the implication of the model of Kent and Lowe (1997): as more and more economic agents realize the bubble, they will become



reluctant to buy the asset as the stock price becomes higher and higher. This in turn prevents the stock price from increasing further. Note that if the bubble is negative,  $p$  represents the probability that  $b_t$  will decrease in the next period. In the case of a negative bubble, Eq. (6.27) indicates that the lower the stock price (but the larger the absolute value of the bubble in this case), the lower the probability that the (negative) bubble will continue to decrease in the next period. The justification is the same as for the positive bubble. As the stock price becomes lower and lower, it is also closer and closer to its lowest point (stock price does not decrease without end!) and may, therefore, be more and more likely to increase in the future. But I assume that the negative bubble does not influence  $p_{t+1}$  as strongly as a positive one, because in reality economic agents are usually more pessimistic in a bear market than optimistic in a bull market.

Moreover, it seems more difficult to activate a financial market when it is in recession than to hold it down when it is booming. This is what the function  $f(b_t)$  implies. It is flatter when  $b_t < 0$  than when  $b_t$  is positive. An example of  $p_{t+1}$  with  $\phi_1 = 0.4$ ,  $\varphi = 10$  and  $r_t = 0$  is shown in Figure 6.4, it is flatter when  $b_t$  is negative than when  $b_t$  is positive. Note that in Figure 6.4 one finds if  $b_t = 0$ , then  $p_{t+1} = 0.5$ . From the process of the bubble one knows if  $b_t = 0$  and  $r_t = 0$ ,  $b_{t+1}$  is  $\varepsilon_{t+1}$  which can be either positive or negative. Because little is known about the sign of the noise  $\varepsilon_{t+1}$ , the economic agents then expect it to be positive or negative with an equal probability of 0.5.

The effect of  $r_t$  on  $p_{t+1}$  can be seen from below:

$$\frac{\partial p_{t+1}}{\partial r_t} = -\frac{\phi_2 \text{sign}(b_t)}{2 \cosh^2[\vartheta(b_t, r_t)]} \begin{cases} < 0, & \text{if } b_t > 0, \\ > 0, & \text{if } b_t < 0. \end{cases} \quad (6.28)$$

This indicates that if the asset-price bubble is positive, an increase in the interest rate will lower the probability that the bubble will increase in the next period. If the bubble is negative, however, an increase in  $r_t$  will increase the probability that the bubble will decrease in the next period. This is consistent with the analysis in the previous section that an increase in the interest rate will lower the stock price. The probability function with  $\phi_1 = 0.4$ ,  $\phi_2 = 0.8$  and  $\varphi = 10$  is shown in Figure 6.5.

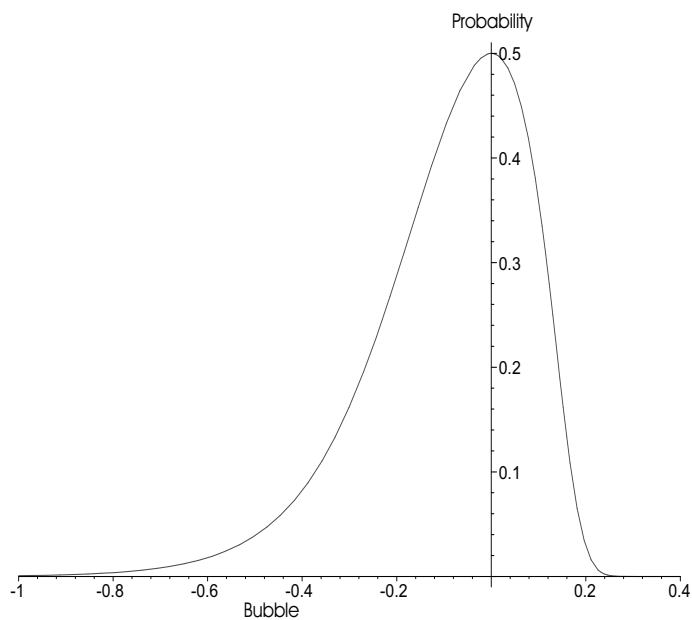


Figure 6.4: An Example of  $p_{t+1}$  with  $r_t = 0$

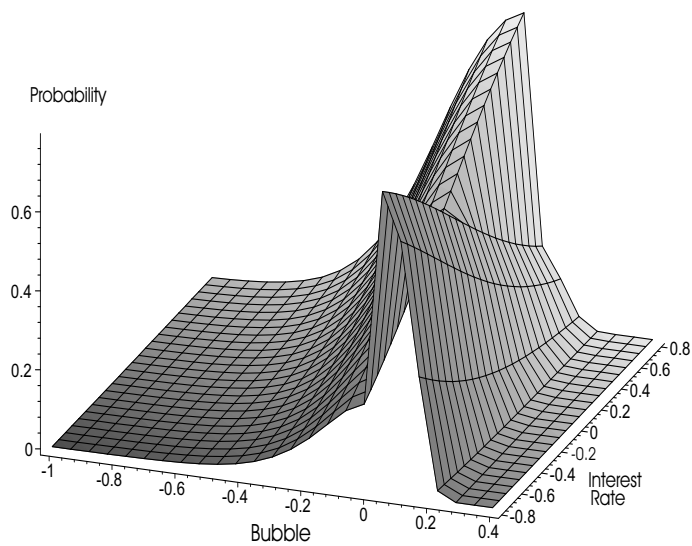


Figure 6.5:  $p_{t+1}$  with  $\phi_1 = 0.4$ ,  $\phi_2 = 0.8$  and  $\varphi = 10$

With the probability function defined by Eq. (6.24) one knows that

$$Eb_{t+1|t} = [1 - g_2 + \frac{1}{2}\{1 - \tanh[\vartheta(b_t, r_t)]\}(g_1 + g_2)]b_t. \quad (6.29)$$

Following the same procedure as in Section 2, one finds that the optimal monetary policy rule must satisfy the following equation

$$r_t = f_1\pi_t + f_2y_t + \frac{1}{2\beta_2}\{2 + (1 - \beta_3)\langle g_1 - g_2 - (g_1 + g_2)\tanh[\vartheta(b_t, r_t)]\rangle\}b_t, \quad (6.30)$$

with  $f_1$  and  $f_2$  given by (6.4) and (6.5). Different from the monetary policy rule given by (6.12), in which the optimal interest-rate rule is a linear function of the inflation rate, output gap and asset-price bubble,  $r_t$  is now a nonlinear function of  $\pi_t$ ,  $y_t$  and  $b_t$ . Moreover, the effects of  $\pi_t$ ,  $y_t$  and  $b_t$  on  $r_t$  are much more complicated than in the previous section.  $r_t$  can be affected not only by parameters such as  $g_1$  and  $g_2$ , but also by the parameters,  $\phi_1$ ,  $\phi_2$  and  $\varphi$  which measure the effects of the size of the bubble and the interest rate on the probability function. Because  $r_t$  is nonlinear in  $\pi_t$ ,  $y_t$  and  $b_t$ , there might exist multiple equilibria in such a model. It is difficult to obtain an analytical solution of the optimal interest-rate rule from (6.30), I will, therefore, undertake some numerical computation.

Assuming  $\pi_t = y_t = 0$  just for simplicity, Figure 6.6 presents Eq. (6.30) with alternative values of the parameters with the horizontal axis denoting the asset-price bubble and the vertical axis denoting the interest rate. It is clear that the response of  $r_t$  to  $b_t$  changes with the parameters.  $r_t$  is a monotonic function of  $b_t$  when the parameters are assigned some values (see Figure 6.6-(5) and (6)). When the parameters are assigned some other values, however,  $r_t$  can be a non-monotonic function of  $b_t$ . In Figure 6.6-(1) and 6.6-(4) the curve cuts the horizontal axis three times, indicating that there may exist multiple equilibria in the model. The parameters for Figure 6.6 are set as follows:  $\beta_2 = 0.30$ ,  $\phi_1 = 1.0$ ,  $\phi_2 = 0.80$  and  $\varphi = 10$ . The other parameters of  $\beta_3$ ,  $g_1$  and  $g_2$  are assigned different values in different figures as follows: (1)  $\beta_3 = 0.005$ ,  $g_1 = 0.001$  and  $g_2 = 1.05$ ; (2)  $\beta_3 = 0.10$ ,  $g_1 = 0.01$  and  $g_2 = 0.90$ ; (3)  $\beta_3 = 0.005$ ,  $g_1 = 0.001$  and  $g_2 = 0.95$ ; (4)  $\beta_3 = 0.005$ ,  $g_1 = 0.001$  and  $g_2 = 1.50$ ; (5)  $\beta_3 = 0.25$ ,  $g_1 = 0.10$  and  $g_2 = 6.50$ ;

(6)  $\beta_3 = 0.25$ ,  $g_1 = 0.01$  and  $g_2 = 0.70$ . The effects of  $g_1$  and  $g_2$  on  $r_t$  can be seen from 6.6-(3) and 6.6-(4). With other parameters unchanged, the values of  $g_1$  and  $g_2$  may determine the direction of how  $r_t$  moves.

This section endogenizes the probability that the asset-price bubble will increase or decrease in the next period. Defining  $p$  as a function of the asset-price bubble and the current interest rate, one finds that the monetary policy turns out to be a nonlinear function of the inflation rate, output gap and asset-price bubble, and there might exist multiple equilibria in the economy.

Recently, some researchers argue that the linear interest-rate rules may not have captured the truth of monetary policy. Meyer (2000), for example, claims that nonlinear monetary policy rules are likely to arise under uncertainty. He argues that "... a nonlinear rule could be justified by nonlinearities in the economy or by a non-normal distribution of policymakers' prior beliefs about the NAIRU." Meyer et al. (2001) provide a theoretical justification for this argument and show some empirical evidence on the relative performance of linear and nonlinear rules. Nonlinear monetary policy rules can also be induced by a nonlinear Phillips curve and a non-quadratic loss function of central banks. Monetary policy with nonlinear Phillips curves have been studied by Semmler and Zhang (2003) and Dolado et al. (2002), for example. Dolado et al. (2002) find that the US monetary policy can be characterized by a nonlinear policy rule after 1983, but not before 1979. Kim et al. (2002), however, find that the US monetary policy rule has been nonlinear before 1979 and little evidence of nonlinearity has been found for the period after 1979. My research above shows that a nonlinear monetary policy rule can also arise in a model with financial markets, assuming an endogenous probability for the asset-price bubble to increase or decrease in the next period.

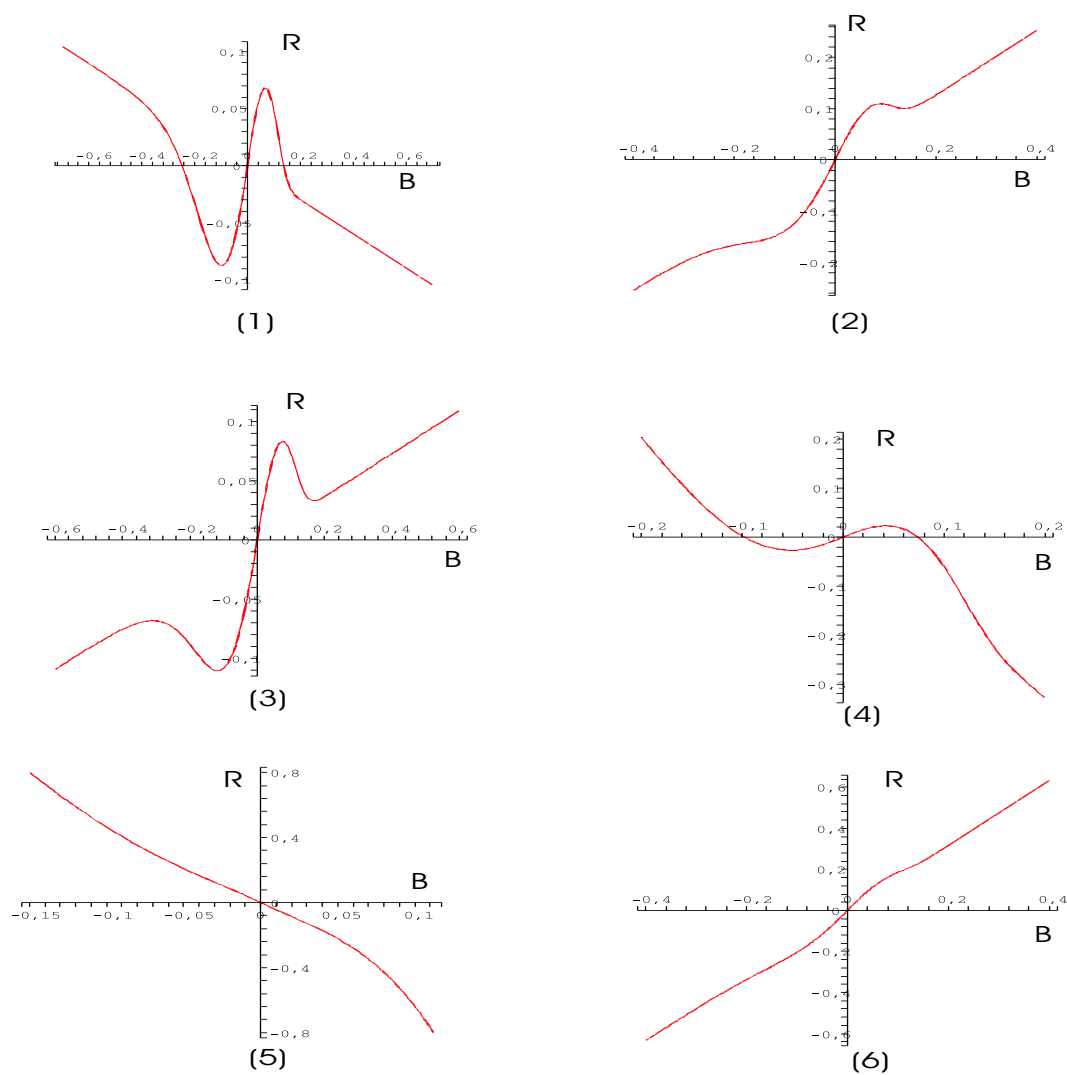


Figure 6.6: The Response of  $r_t$  to  $b_t$  with Alternative Values of Parameters

## 6.5 The Zero Bound on the Nominal Interest Rate

Above I have discussed the monetary policy rule with asset prices considered. In the case of a constant probability ( $p$ ) for the asset-price bubble to increase or decrease in the next period, the optimal monetary policy turns out to be a linear function of the inflation, output gap and asset-price bubbles, similar to the simple Taylor rule except that the asset-price bubble is added as an additional term. However, if  $p$  is assumed to be an endogenous variable depending on the monetary policy and the asset-price bubble size, the monetary policy rule turns out to be a nonlinear function of the inflation rate, output gap and asset-price bubble.

A drawback of the Taylor rule, and also of the monetary policy discussed above, is that the monetary policy instrument—the short-term interest rate—is assumed to be able to move without bounds. This is, however, not true in practice and one example is the so-called Liquidity Trap in which a monetary policy cannot be of much help because the short-term nominal interest rate is almost zero and cannot be lowered further. This problem has recently become important because of the Liquidity Trap in Japan and the low interest rate in the US. If, furthermore, there is deflation, the real interest rate will rise. Considering the zero bound on the short-term interest rate and the possibility of deflation at very low interest rates, the monetary policy can be very different from that without bounds on the interest rate.

Benhabbib and Schmitt-Grohé (2001), for example, argue that once the zero bound on nominal interest rates is taken into account, the active Taylor rule can easily lead to “unexpected consequences”. To be precise, they find that there may exist an infinite number of trajectories converging to a Liquidity Trap even if there exist a unique equilibrium.

Kato and Nishiyama (2001) analytically prove and numerically show that the optimal monetary policy in the presence of the zero bound is highly nonlinear even in a linear-quadratic model. Eggertsson and Woodford (2003) simulate an economy with zero bound on the interest rate and argue that

monetary policy will be effective only if interest rates can be expected to persistently stay low in the future. Coenen and Wieland (2003) explore the effect of a zero-interest-rate bound on the inflation and output in Japan in the context of an open economy. Ullersma (2001) surveys several researchers' views on the zero lower bound.

Most of the recent research on the Liquidity Trap has been concerned with deflation, namely the decrease of the price level in the product markets. Yet most literature has ignored the depression in the financial markets. The depression of the financial markets can also be a problem in practice, if the financial markets can influence the output and, as a result, affect the inflation rate. Take Japan as an example, the share price index was about 200 in 1990 and decreased to something below 80 in 2001. The Industrial Production Index was about 108 in 1990 and fluctuated between 107 and 92 afterwards. The inflation rate (changes in the CPI), IPI and share price index of Japan are shown in Figure 6.7A-C (Data source: International Statistical Yearbook). The depression in the share markets seems to be as serious as the deflation. One finds that the correlation coefficient between the IPI and share price index was as high as 0.72 from 1980-2001 and the correlation coefficient between the IPI and the two-quarter lagged share price index was even as high as 0.80. Moreover, the estimates of  $c_2$  in Eq. (6.8) have enough significant T-Statistics (3.505 for the sample from 1980.1-1999.1 and 3.220 for the sample from 1990.1-1999.1). This seems to suggest that the influence of the financial markets on the output should not be overlooked.

Let us now return to the Liquidity Trap problem. The main difference of my research from that of others is that I will explore the zero bound on the nominal interest rate with depression in the financial markets as well as in the product markets (namely deflation).

Let's define  $r_t = R_t - \bar{R}$ , with  $R_t$  being the nominal rate and  $\bar{R}$  denoting the long-run level of  $R_t$ . In the research below I assume  $\bar{R} = 0$  for simplicity.

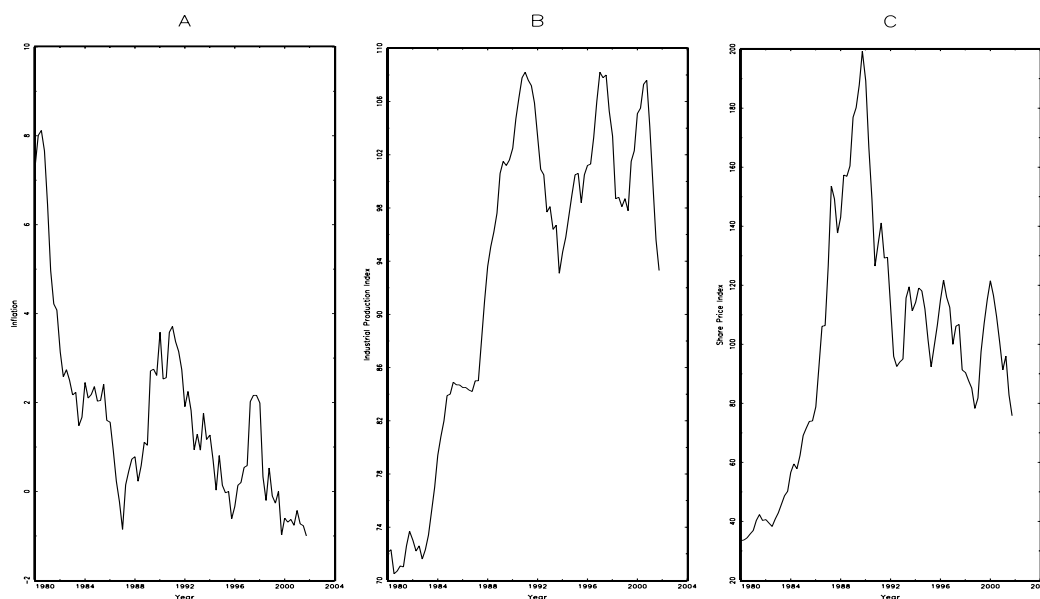


Figure 6.7: The Inflation Rate, IPI and Share Price Index of Japan, 1980.1-2001.4

In the presence of the zero bound on the nominal rate, I then assume<sup>7</sup>

$$r_t = \begin{cases} r_o, & \text{if } r_o \geq 0; \\ 0, & \text{if } r_o < 0; \end{cases} \quad (6.31)$$

where  $r_o$  denotes the optimal monetary policy rule derived from the models in the previous sections. The equation above implies that if the optimal monetary policy rule is nonnegative, the central bank will adopt the optimal rule, if the optimal rule is negative, however, the nominal rate is set to zero, since it cannot be negative.<sup>8</sup>

I will first undertake some simulations without asset prices considered, as the simple model (6.1)-(6.2). The parameters are set as follows:<sup>9</sup>  $\alpha_1 = 0.8$ ,

<sup>7</sup>This is similar to the assumption of Coenen and Wieland (2003) who analyze the effect of a zero-interest-rate bound on inflation and output in Japan in the context of an open economy.

<sup>8</sup>There are some exceptional cases with negative nominal rates, see Cecchetti (1988), for example, but I will ignore these exceptional cases here.

<sup>9</sup>In order for consistent expectations to exist,  $\alpha_1$  is usually assumed to be 1. The simulations with  $\alpha_1 = 1$  are found essentially unchanged from those with  $\alpha_1 = 0.8$ .



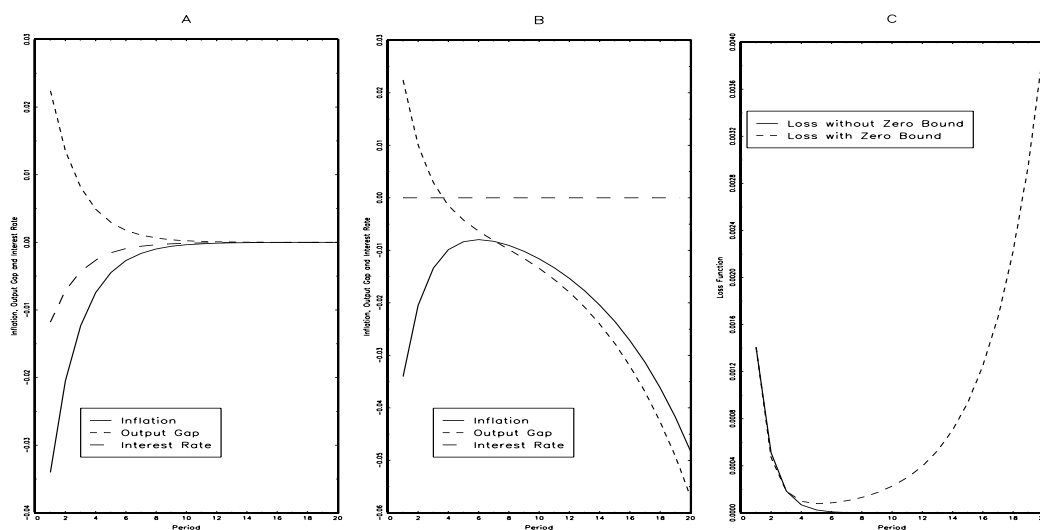


Figure 6.8: Simulation without Asset Price

$\alpha_2 = 0.3$ ,  $\beta_1 = 0.9$ ,  $\beta_2 = 0.3$ ,  $\lambda = 0.5$  and  $\rho = 0.97$ . In order to explore the effect of the zero bound of the nominal rate on the economy, I assume there exists deflation. The starting values of  $\pi_t$  and  $y_t$  are set as  $-0.08$  and  $0.1$  respectively. The optimal monetary policy rule from the basic model is given by Eq. (6.3). The simulations with and without the zero-interest-rate bound are shown in Figure 6.8. In Figure 6.8A I show the simulation of the inflation, output gap and  $r_t$  without the zero bound on the nominal rate. Therefore  $r_t$  is always set in line with (6.3). It is clear that all three variables converge to zero over time. The loss function can, as a result, be minimized to zero. Figure 6.8B shows the simulation with a zero-interest-rate bound. One finds that the optimal nominal rate, which is negative as shown in Figure 6.8A, cannot be reached and has to be set to zero. The inflation and output gaps, as a result, do not converge to zero, but instead evolve into a recession. The deflation becomes more and more severe and the output gap changes from positive to negative and continues to go down over time. Figure 6.8C shows the loss function  $\pi^2 + \lambda y^2$  with and without a zero-interest-rate bound. One observes that in the case of no zero-interest-rate bound the loss function converges to zero as  $\pi_t$  and  $y_t$  goes to zero. In the presence of a zero-interest-rate bound, however, the loss function increases rapidly over time because of

the recession.

The simulation undertaken above does not consider the effects of asset prices on the inflation and output. The simulation below assumes that the asset prices can influence the output as Eq. (6.9) and the asset-price bubble has the path (6.7). In order to simplify the simulation I just take  $b_{t+1} = Eb_{t+1|t}$ , therefore with an initial value of the bubble one can obtain a series of  $b_t$ . With other parameters assigned the same values as above, the remainder of the parameters are assigned the following values:  $g_1 = 0.1$ ,  $g_2 = 0.2$ ,  $p = 0.5$  and  $\beta_3 = 0.5$ . The initial values of  $\pi_t$  and  $y_t$  are the same as above. The initial value of  $b_t$  is  $-0.02$ , indicating a depression in the financial markets. The optimal rate  $r_o$  is given by Eq. (6.12). The simulations with and without a zero-interest-rate bound are shown in Figure 6.9A-C. In Figure 6.9A I show the simulation without a zero bound on  $r_t$ , this is similar to the case in Figure 6.8A where all three variables converge to zero except that  $r_t$  in Figure 6.9A is lower and converges more slowly than in Figure 6.8A. Figure 6.9B shows the simulation with a zero bound on  $r_t$ . Again one finds that the optimal rate cannot be reached and  $r_t$  has to be set to zero. The economy experiences a recession. This is similar to the case in Figure 6.8B, but the recession in Figure 6.9B is more severe than that in Figure 6.8B. In Figure 6.8B  $\pi_t$  and  $y_t$  decrease to about  $-0.06$  with  $t = 20$ , but in Figure 6.9B, however,  $\pi_t$  and  $y_t$  experience larger and faster decreases and go down to about  $-0.8$  in the same period. This is because the output is affected by the depression in the financial markets (negative  $b_t$ ) which also accelerates the deflation through the output. In Figure 6.9C I show the loss function with and without a zero bound on  $r_t$ . The loss function when no zero-interest-rate bound exists converges to zero over time but increases rapidly when there exists a zero-interest-rate bound. But the loss function with a zero-interest-rate bound in Figure 6.9C is higher than that in 6.8C because of the more severe recession in Figure 6.9B caused by the financial market depression.

Next, I assume that the financial market is not in depression but instead in a boom, that is, the asset-price bubble is positive. I set  $b_0 = 0.02$  and obtain a series of positive bubbles. The simulation with the same parameters as above is shown in Figure 6.9D-F. In Figure 6.9D all three variables converge

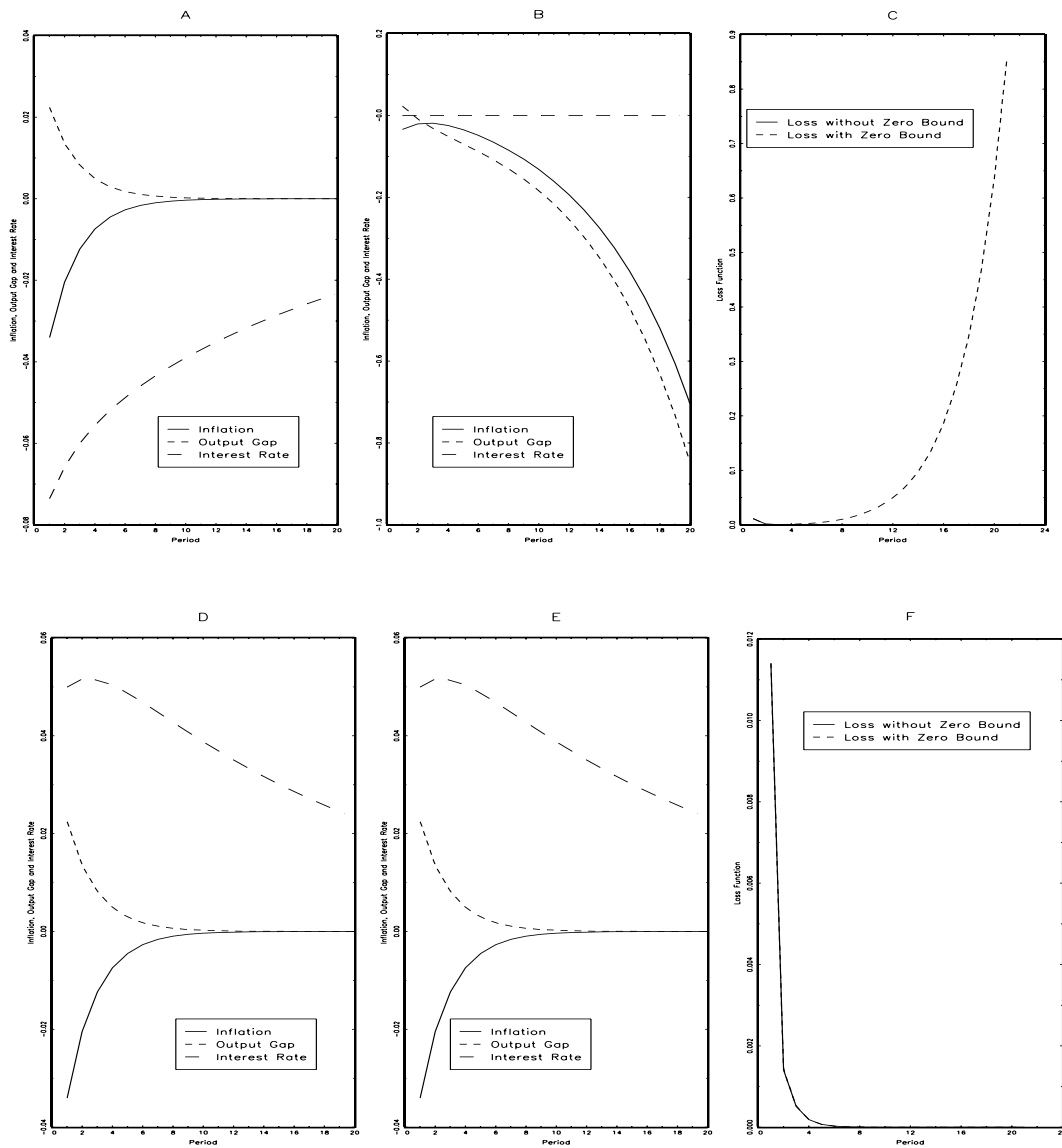


Figure 6.9: Simulation with Asset Price

to zero when no zero bound on  $r_t$  is implemented. In Figure 6.9E, however, all three variables also converge to zero over time even if there exists a zero bound on the nominal rate. This is different from the cases in Figure 6.8B and 6.9B where a severe recession occurs. The reason is that in Figure 6.9E the asset-price bubble is positive and the optimal interest rate turns out to be positive. The zero-interest-rate bound is therefore not binding. As a result, Figure 6.9E is exactly the same as Figure 6.9D. The two loss functions with and without a zero-interest-rate bound are therefore also the same, as shown in Figure 6.9F.

The simulations in this section indicate that in the presence of a zero-interest-rate bound, a deflation can become more severe and the economy may go into a severe recession. Moreover, the recession can be worse if the financial market is also in a depression, because the asset price depression can then decrease the output and as a result makes the deflation more severe. Facing the zero-interest-rate bound and a Liquidity Trap, some researchers have proposed some policy actions, see Clouse et al. (2000), for example. The simulations above indicate that policy actions that aim at escaping a Liquidity Trap should not ignore the asset prices, since the financial market depression can make the real-economy recession worse.

On the other hand, a positive asset-price bubble can make the zero-interest-rate bound non-binding, since the optimal rate which takes the financial markets into account may be higher than zero even if there exists deflation. This case has been shown in Figure 6.9E.

Note that the simulations undertaken above are based on the simple model in which the probability ( $p$ ) that the asset-price bubbles will increase or decrease in the next period is assumed to be exogenous. If  $p$  is taken as an endogenous variable, however, the analysis can be more complicated. In the basic model one finds that the optimal monetary policy rule turns out to be a linear function of  $b_t$ , but in the model with an endogenous  $p$ , the monetary policy rule turns out to be nonlinear in the inflation rate, output gap and asset-price bubble. This has been shown in the simulations in Figure 6.6. In the case of a linear rule it is clear that a negative asset-price bubble

lowers the optimal policy rule and therefore enlarges the possibility of the zero-interest-rate bound being binding, while a positive asset-price bubble increases the optimal nominal rate and therefore reduces the possibility of the zero-interest-rate bound being binding. When the optimal policy rule is a nonlinear function of the asset price, however, a positive bubble may enlarge the possibility of the zero-interest-rate bound being binding, since the optimal rule can be lowered (or become negative) even if the bubble is positive. On the other hand, a negative bubble may reduce the possibility of the zero-interest-rate bound being binding because the optimal rule can be increased (or become positive) even if the bubble is negative. An example of the linear and nonlinear policy rules in the presence of a zero-interest-rate bound is shown in Figure 6.10A-B. Figure 6.10B looks similar to Figure 6.6-(1). In Figure 6.10 I set the optimal rule to be zero if it is negative. In some cases, an endogenous  $p$  can make the optimal policy rule very different from that with a constant  $p$ . Figure 6.6-(5) is a good example: unlike the linear rule which is an increasing function of the asset-price bubble,  $r_t$  in Figure 6.6-(5) is a decreasing function of  $b_t$  and the effect of the zero-interest-rate bound on the economy through the channel of financial markets can, therefore, be greatly changed.

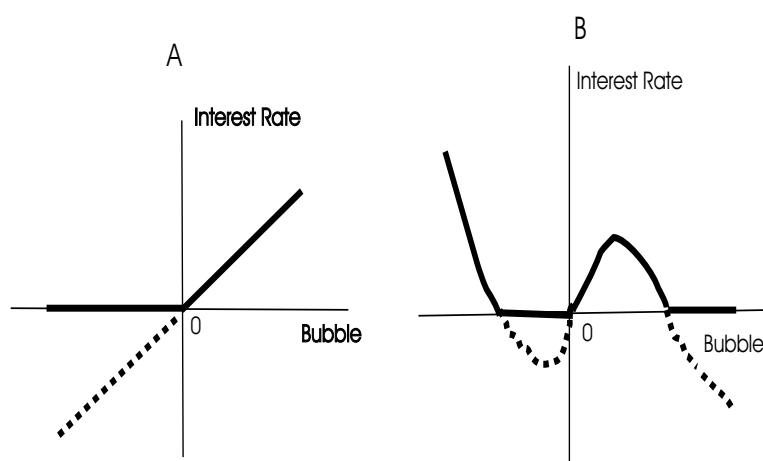


Figure 6.10: An Example of Linear and Nonlinear Policy Rules in the Presence of the Zero-Interest-Rate Bound

## 6.6 Conclusion

A dynamic model has been set up to explore monetary policy with asset prices in this chapter. If the probability for the asset-price bubble to increase or decrease in the next period is assumed to be a constant, the monetary policy turns out to be a linear function of the state variables. However, if such a probability is endogenized as a function of the asset-price bubble and interest rate, the policy reaction function becomes nonlinear in the inflation rate, output gap and asset-price bubble. Some empirical evidence has shown that the monetary policy rule in the Euro-area has, to some extent, taken into account the financial markets in the past two decades. I have also explored the effect of a zero-interest-rate bound on the real economy with financial markets considered. The simulations indicate that a depression of the financial markets can make a recession economy worse in the presence of a lower bound on the nominal rate. Therefore policy actions which aim at escaping a Liquidity Trap should not ignore the financial markets. I have also shown that the effect of the zero-interest-rate bound on the economy can be greatly changed if the probability for the asset price to increase or decrease in the next period is an endogenous variable rather than an exogenous one.

# Chapter 7

## Concluding Remarks

This dissertation is mainly concerned with monetary policy rules (to be precise, the interest-rate rules) with time-varying behaviors, uncertainty and financial markets at both theoretical and empirical levels. Empirical evidence and numerical studies have been undertaken using the data of some OECD countries.

Because the IS and Phillips curves have become the baseline model of monetary policy, I have shown some empirical evidence of the two curves with both backward- and forward-looking behaviors. The estimation for several OECD countries indicates some significant relations between the inflation rate and output gap, and between the output gap and real interest rate. I have also estimated a time-varying Phillips curve with the Kalman filter and find that the response coefficient of the unemployment gap has experienced some structural changes, which imply regime changes in the economy.

Based on the empirical evidence of the IS and Phillips curves I have then discussed briefly the advantages and potential problems of the Taylor rule, and derived an interest-rate rule from a dynamic macroeconomic model with a quadratic loss function of the central bank. One observes that this interest-rate rule is akin to the simple Taylor rule in that they both are linear functions of the inflation rate and output gap. Moreover, the interest-rate rule can be greatly affected by the parameters in the macroeconomic model which consists of the IS and Phillips curves and the central bank's

loss function. The empirical evidence of a time-varying Phillips curve, as a result, implies that the monetary policy rule may be time-varying rather than invariant. Therefore, I have estimated a time-varying interest-rate rule and found some empirical evidence of state-dependence. That is, the monetary policy rule is, to some extent, sensitive to the economic environment.

Employing the estimated time-varying US monetary policy rule, I have then undertaken some simulation of the IS and Phillips curves of the Euro-area, assuming that the Euro-area had followed the US monetary policy rule in the 1990s. The simulation results indicate that the monetary policy of the Euro-area was too tight in the 1990s.

What may complicate the monetary policy more than time-varying behaviors is uncertainty. Besides parameter uncertainty in economic models, there exist still other kinds of uncertainties such as data uncertainty and shock uncertainty. Employing a State-Space model with Markov-Switching I have explored some empirical evidence of uncertainties in the IS and Phillips curves—not only parameter uncertainty but also shock uncertainty. To be precise, the parameters are time-varying and, at the same time, they may have more than one state. The shocks in the model may also have state-dependent variances. Based on this empirical evidence, I have then explored monetary policy rules under uncertainty with two approaches: (a) the adaptive learning algorithm, and (b) robust control. While the former assumes that the central bank improves its knowledge of economic models by learning in a certain mechanism, the latter assumes that the central bank seeks an optimal policy rule from the “worst case”.

The research employing the RLS learning algorithm indicates that neither the state variables nor the control variable converge, even in a deterministic model. This is different from the conclusion of Sargent (1999) who employs an LQ framework and presumes that the central bank pretends that the time-varying parameter will remain invariant forever after it is updated. This is, in fact, inconsistent with the implication of the adaptive learning algorithm. In this dissertation, however, I have taken the time-varying parameter as an endogenous variable and employed a recently developed dynamic programming algorithm which can solve dynamic optimization problems with nonlinear



state equations using adaptive rather than uniform grids.

The robust control theory can, however, deal with more general uncertainties than the adaptive learning algorithm. The simulation with the US data suggests that uncertainty does not necessarily require caution. This is consistent with the conclusion of Gonzalez and Rodriguez (2003) and Giannoni (2000). The former analyze the effect of the robust parameter on the optimal feedback rule with one-state and one-control model, and the latter explores the robust optimal rule with forward-looking behaviors.

While most of the literature on monetary policy rules is concerned mainly with the real economy, some researchers argue that attention should also be given to the financial markets. This problem has arisen due to the stable and low inflation rate in the developed countries in the 1990s. The financial markets have, however, experienced some significant fluctuations. Therefore, I have explored monetary policy rules with the asset prices. That is, I have set up a dynamic model with both the real economy—the inflation rate and the output gap—and the financial markets taken into account. A monetary policy rule with the asset prices has been derived. The most important difference between my model and those of others, consists in the fact that I have endogenized the probability for the asset-price bubble to grow or decrease in the next period as a nonlinear function of the interest rate and the size of the asset-price bubble. Other researchers, such as Bernanke and Gertler (2000) and Smets (1997), either take such a probability as a constant or assume it to be a linear function of the policy instrument and the size of the bubble. The drawback of a linear probability function is that it is not bounded between zero and one, and it can only consider positive bubbles. The endogenization of such a probability in my model overcomes these problems. Moreover, such a probability function is found to lead to nonlinear monetary policy rules.

Another problem concerning the monetary policy rules and financial markets is the zero bound on the nominal interest rate. This problem has arisen mainly because of the Liquidity Trap, deflation and financial depression in Japan in the past decade. My simulation in the presence of a zero bound on the nominal rate suggests that policy actions that aim at escaping a Liquidity Trap should not ignore the effects of the asset prices, since the depression in

the financial markets can make the recession of the real economy worse.

Finally I want to note that this dissertation is mainly concerned with monetary policy rules in a closed economy. Monetary policy rules in open economies, as mentioned in Chapter 1, can be different from those in closed economies since exchange rate may play crucial roles in monetary policy-making. Svensson (1998), for example, points out that inflation targeting with exchange rate may have several important consequences:

First, the exchange rate allows additional channels for the transmission of monetary policy. ... Second, as an asset price, the exchange rate is inherently a forward-looking and expectations-determined variable. This contributes to making forward-looking behavior and the role of expectations essential in monetary policy. Third, some foreign disturbances will be transmitted through the exchange rate, for instance, changes in foreign inflation, foreign interest rates and foreign investors' foreign-exchange risk premium ... (Svensson, 1998, p.4).

Ball (1999) finds that the monetary policy rule in an open economy is different from that in a closed economy in two aspects: (a) the policy variable is a combination of the short-term interest rate and exchange rate, rather than the interest rate alone, and (b) the inflation rate in the Taylor rule is replaced by a combination of inflation and the lagged exchange rate. Benigno and Benigno (2000) explore different monetary policy rules under alternative exchange rate regimes and claim that a managed exchange rate is desirable. Using an open economy model under incomplete markets, Ghironi (2000) compares the performance of alternative monetary policy rules for Canada and concludes that flexible inflation targeting dominates strict inflation targeting rules and the Taylor rule. More research on monetary policy rules in open economies is surely expected to be forthcoming in the future.

# Bibliography

- [1] Adema, Yvonne, and Elmer Sterken (2003), “Monetary policy rules,” manuscript, Department of Economics, University of Groningen.
- [2] Aguiar, Alvaro, and Manuel M.F. Martins (2002), “Trend, cycle, and nonlinear trade-off in the Euro-area 1970-2001,” manuscript, CEM-  
PRE.
- [3] Akerlof, George A. (2002), “Behavior macroeconomics and macroeconomic behavior,” *The American Economic Review*, Vol. 92, pp.411-433.
- [4] Anderson, Evan W., Lars Peter Hansen, and Thomas J. Sargent (2000), “Robustness, detection and the price of risk,” manuscript.
- [5] Ball, L. (1997), “Efficient rules for monetary policy,” NBER Working Paper, No. 5952.
- [6] — (1999), “Policy rules for open economies,” in John B. Taylor (ed.), *Monetary Policy Rules*, Chicago, IL: University of Chicago Press, pp.127-144.
- [7] Batini Nicoletta, Richard Harrison, and Stephen P. Millard (2001), “Monetary policy rules for an open economy,” manuscript.
- [8] Baxter, Marianne, and Robert G. King (1995), “Measuring business cycles: approximate Band-Pass filters for economic time series,” NBER Working Paper, No. 5022.

- [9] Bean, Charles (2000), “The convex Phillips curve and macroeconomic policymaking under uncertainty,” manuscript, London School of Economics.
- [10] Beck, Günter W., and Volker Wieland (2002), “Learning and control in a changing economic environment,” *Journal of Economics Dynamics and Control*, Vol. 26, pp.1359-1377.
- [11] Benhabib, Jess, and Stephanie Schmitt-Grohé (2001), “The perils of Taylor rule,” *Journal of Economic Theory*, Vol. 96, pp.40-69.
- [12] Benhabib, Jess, Stephanie Schmitt-Grohé, and Martín Uribe (2001), “Monetary policy and multiple equilibria,” *The American Economic Review*, Vol. 91, No. 1, pp.167-186.
- [13] Benigno, Gianluca, and Pierpaolo Benigno (2000), “Monetary policy rules and the exchange rate,” manuscript, University of California at Berkeley, and Princeton University.
- [14] Bernanke, Ben, and Mark Gertler (2000), “Monetary policy and asset price volatility,” NBER Working Paper 7559.
- [15] Blanchard, Oliver (1981), “Output, the stock market, and interest rates,” *The American Economic Review*, Vol. 71, pp.410-425.
- [16] — (2003a), *Macroeconomics*, Prentice Hall.
- [17] — (2003b), “Monetary policy and unemployment,” remarks at the conference “Monetary policy and the labor market,” in honor of James Tobin, held at the New School University, NY.
- [18] Blanchard, Oliver, and Mark Watson (1982), “Bubbles, rational expectations, and financial markets,” in P. Wachtel, (ed.), *Crisis in the Economic and Financial Structure*, Lexington Books, pp.410-425.
- [19] Brainard, William (1967), “Uncertainty and the effectiveness of policy,” *The American Economic Review*, Vol. 52, No. 2, pp.411-425.

- [20] Brock, William A., and Anastasios Xepapadeas (2003), "Regulating nonlinear environmental systems under knightian uncertainty," in R. Arnott, B. Greenwald, R. Kanbur and B. Nalebuff, Eds, *Economics for an Imperfect World: Essays In Honor of Joseph Stiglitz*, The MIT Press.
- [21] Brock, William A., Steven N. Durlauf, and Kenneth D. West (2003), "Policy evaluation in uncertain economic environment," NBER Working Paper 10025.
- [22] Cagetti, Marco, Lars Peter Hansen, Thomas Sargent, and Noah Williams (2002), "Robustness and pricing with uncertain growth," *The Review of Financial Studies*, Special 2002, Vol.15, No. 2, pp.363-404.
- [23] Calvo, Guillermo (1983), "Staggered prices in a utility-maximizing framework," *Journal of Monetary Economics*, Vol. 12, pp.383-398.
- [24] Carlstrom, Charles T., and Timothy S. Fuerst (2000), "Money growth rules and price level determinacy," manuscript, Federal Reserve Bank of Cleveland, and Department of Economics, Bowling Green State University.
- [25] Cecchetti, Stephen (1988), "The case of the negative nominal interest rates: new estimates of the term structure of interest rates during the Great Depression," *Journal of Political Economy*, December 1988.
- [26] Chadha, Jagjit S., and Charles Nolan (2002), "Output, inflation and the New Keynesian Phillips curve," manuscript, University of Cambridge, and Durham University.
- [27] Chen, Z., and L.G. Epstein (2000), "Ambiguity, risk and asset returns in continuous time," manuscript.
- [28] Chen, Pu, and Peter Flaschel (2004), "Testing the dynamics of wages and prices for the US economy," CEM Working Paper 67, University of Bielefeld.

- [29] Chiarella, Carl, Rainer Franke, Peter Flaschel, and Willi Semmler (2001), "Output and interest rates: jump variable, relaxation oscillation and phase diagram switching methodologies," manuscript, University of Bielefeld.
- [30] Choi, Kyongwook, and Chulho Jung (2003), "Structural changes and the US money demand function," manuscript, Department of Economics, Ohio University.
- [31] Clarida, Richard, Jordi Galí, and Mark Gertler (1998), "Monetary policy rules in practice: some international evidence," *European Economic Review*, Vol. 42, pp.1033-1067.
- [32] — (1999), "The science of monetary policy: a new Keynesian perspective," *Journal of Economic Literature*, XXXVII, pp.1661-1707.
- [33] — (2000), "Monetary policy rules and macroeconomic stability: evidence and some theory," *Quarterly Journal of Economics*, Vol. CXV, Issue 1, pp.147-180.
- [34] — (2002), "A simple framework for international monetary policy analysis," *Journal of Monetary Economics*, 49, pp.879-904.
- [35] Clouse, J. Henderson, A. Orphanides, D. Small, and P. Tinsley (2000), "Monetary policy when the nominal short-term interest rate is zero" *International finance and Economics*, Discussion Paper, No. 2000-51.
- [36] Coenen, Günter, and Volker Wieland (2003), "The zero-interest-rate bound and the role of the exchange rate for monetary policy in Japan," ECB Working Paper, No. 218.
- [37] Cogley, Timothy, and Thomas J. Sargent (2001), "Evolving post-world war II US inflation dynamics," manuscript, June.
- [38] — (2002), "Drifts and volatilities: monetary policies and outcomes in the post WWII US," manuscript, March.

- [39] Corsetti, G., and P. Pesenti (1999), “Stability, asymmetry and discontinuity: the launch of European Monetary Union,” *Brookings Papers on Economic Activity*, No. 2, December, pp.295-372.
- [40] Deutsche Bundesbank Monthly Report (1999), April, pp.47-63.
- [41] Devereux, Michael B., and Charles Engel (2000), “Monetary policy in the open economy revisited: price setting and exchange rate flexibility,” NBER Working Paper 7665.
- [42] Dolado, Juan J., R. María-Dolores, and F.J. Ruge-Murcia, (2002), “Nonlinear monetary policy rules: some new evidence for the US,” Departamento de Economía, Universidad Carlos III de Madrid, Working Paper 02-29.
- [43] Dolado, Juan J., Ramón María-Dolores, and Manuel Naveira (2001), “Are central bank’s reaction functions asymmetric? Evidence for some central banks,” Department of Economics, Universidad Carlos III de Madrid.
- [44] Dupasquier, Chantal, and Nicholas Ricketts (1998a), “Nonlinearities in the output-inflation relationship,” manuscript.
- [45] — (1998b), “Nonlinearities in the output-inflation relationship: some empirical results for Canada,” Bank of Canada, Working Paper 98-14.
- [46] Dupor, William (2001), “Nominal prices versus asset price stabilization,” University of Pennsylvania, manuscript.
- [47] Eggertsson, Gauti, and Michael Woodford (2003), “The zero bound on interest rates and optimal monetary policy,” manuscript.
- [48] Eller, Jon W., and Robert J. Gordon (2003), “Nesting the New Keynesian Phillips curve within the Mainstream model of US inflation dynamics,” CEPR conference, the Phillips curve revisited.
- [49] Epstein, L., and M. Schneider (2001), “Recursive multiple priors,” manuscript.

- [50] Evans, George W., and Seppo Honkapohja (2001), *Learning and Expectations in Macroeconomics*, The Princeton University Press.
- [51] Fair, Ray C. (2000a), "Testing the NAIRU model for the United States," *The Review of Economics and Statistics*, Vol. 82, pp.64-71.
- [52] — (2000b), "Estimated, calibrated, and optimal interest rate rules," manuscript, Yale University.
- [53] Filardo, Andrew J. (1998), "New evidence on the output cost of fighting inflation," *Economic Review*, third quarter, pp.33-61.
- [54] — (2000), "Monetary policy and asset prices," Federal Reserve Bank of Kansas City.
- [55] Fisher, I. (1920), "Stabilizing the dollar," New York: Macmillan.
- [56] Flaschel, Peter, Göran Kauermann, and Willi Semmler (2004), "Testing wage and price Phillips curves for the United States," CEM Working Paper 68, University of Bielefeld.
- [57] FLaschel, Peter, and Hans-Martin Krolzig (2002), "Wage and price Phillips curve," CEM Working Paper 66, University of Bielefeld.
- [58] Flaschel, Peter, Willi Semmler, and Gang Gong (2001), "Nonlinear Phillips curve and monetary policy in a Keynesian macroeconomic model," Working Paper 18, CEM, University of Bielefeld.
- [59] Friedman, M. (1960), *A Program for Monetary Stability*, Fordham University Press: New York.
- [60] Fuhrer, Jeff, and George Moore (1995), "Inflation persistence," *Quarterly Journal of Economics* 110, pp.127-159.
- [61] Galí, Jordi, and Mark Gertler (1999), "Inflation dynamics: a structural econometric analysis," *Journal of Monetary Economics*, Vol. 44, pp.195-222.



- [62] Galí, Jordi, Mark Gertler, and J. David López-Salido (2001a), “European inflation dynamics,” *European Economics Review*, Vol. 45, pp.1237-1270.
- [63] — (2001b), “Notes on estimating the closed form of the hybrid new Phillips curve,” manuscript.
- [64] — (2003), “Robustness of the Estimates of the hybrid New Keynesian Phillips curve,” manuscript.
- [65] Gerlach, Stefan, and Franke Smets (2000), “MCIs and monetary policy,” *European Economic Review*, Vol. 44, pp.1677-1700.
- [66] Ghironi, Fabio (2000), “Alternative monetary policy rules for a small open economy: the case of Canada,” manuscript, Department of Economics, Boston College.
- [67] Giannoni, Marc P. (2002), “Does model uncertainty justify caution? Robust optimal monetary policy in a forward-looking model,” *Macroeconomic Dynamics*, No.6, pp.111-144.
- [68] Giordani, Paolo, and Paul Söderlind (2002), “Solution of macromodels with Hansen-Sargent robust policies: summary and some extensions,” manuscript, February.
- [69] Gonzalez, Fidel, and Arnulfo Rodriguez (2003), “Robust control: a note on the response of the control to changes in the free parameter,” manuscript, Department of Economics, University of Texas at Austin.
- [70] Gordon, R.J. (1997), “The time-varying NAIRU and its implications for economic policy,” *Journal of Economic Perspectives*, Vol. 11, pp.11-32.
- [71] Graham, Liam, and Dennis J. Snower (2002), “The return of the long-run Phillips curve,” Institute for the Study of Labor (IZA), discussion paper, No. 646.

- [72] Greiner, Alfred, and Willi Semmler (2002), “Monetary policy, non-uniqueness of steady states and hysteresis effects,” in Reinhard Neck (ed.), *Modelling and Control of Economic Systems*, Elsevier Science Ltd, Oxford.
- [73] Grüne, Lars (1997), “An adaptive grid scheme for the discrete Hamilton-Jacobi-Bellman equation,” *Numerische Mathematik*, Vol. 75, pp.319-337.
- [74] Grüne, Lars, and Willi Semmler (2002), “Using dynamic programming with adaptive grid scheme for optimal control problems in economics,” Working Paper 36, [www.wiwi.uni-bielefeld.de/~cem](http://www.wiwi.uni-bielefeld.de/~cem).
- [75] Hamilton, James D. (1994), *Time series analysis*, Princeton University Press.
- [76] Hansen, Lars, P. (1982), “Large sample properties of generalized method of moments estimators,” *Econometrica*, Vol. 50, pp.1029-1054.
- [77] Hansen, Lars Peter, and Thomas J. Sargent (1999), “Robustness and commitment: a monetary policy example,” manuscript, MarCh.
- [78] — (2001a), “Robust control and model uncertainty,” manuscript, January.
- [79] — (2001b), “Time inconsistency of robust control?,” manuscript, October.
- [80] — (2002), “Robust Control and Filtering for Macroeconomics,” book manuscript.
- [81] Harvey, A. C. (1989), *Time Series Models*, Phillip and Allan Publishers Limited.
- [82] — (1990), *Forecasting, Structural Time Series and the Kalman Filter*, the Cambridge University Press.

- [83] Isard, Peter, Douglas Laxton, and Ann-Charlotte Eliasson (1999), "Simple monetary policy rules under model uncertainty," *International Tax and Public Finance*, Vol. 6, pp.537-577.
- [84] Jenkins, Paul (2002), "Monetary policy and uncertainty," *Bank of Canada Review*, Summer 2002.
- [85] Judd, John P., and Glenn D. Rudebusch (1998), "Taylor's rule and the Fed: 1970-1997," *FRBSF Economic Review*, 1998, No. 3.
- [86] Kalaba, R., and L. Tesfatsion (1988), "The flexible least squares approach to time-varying linear regression," *Journal of Economic Dynamics and Control*, Vol. 12, pp.43-48.
- [87] Karanassou, Marika, Hector Sala, and Dennis J. Snower (2003), "A reappraisal of the inflation-unemployment tradeoff," manuscript.
- [88] Kato, Ryo, and Shinichi Nishiyama (2001), "Optimal monetary policy when the interest rates are bounded at zero," manuscript.
- [89] Kent, Christopher, and Phillip Lowe (1997), "Asset-price bubbles and monetary policy," manuscript, Research Bank of Australia, Dec. 1997.
- [90] Kim, Chang-Jin, and Charles R. Nelson (1999), *State-Space Models with Regime Switching*, The MIT Press.
- [91] Kim, Dong Heon, Denise R. Osborn, and Marianne Sensier (2002), "Nonlinearity in the Fed's monetary policy rule," University of Manchester, manuscript.
- [92] Kozicki, Sharon (1999), "How useful are Taylor rules for monetary policy?," *Economic Review*, second quarter, 1999.
- [93] Laxton, D., G. Rose, and D. Tambakis (1998), "The US Phillips curve: the case for asymmetry," manuscript.
- [94] Leeper, Eric M. (1991), "Equilibria under 'active' and 'passive' monetary and fiscal policies," *Journal of Monetary Economics*, Vol. 27, pp.129-147.

- [95] Leith, Campbell, and Simon Wren-Lewis (2002), “Taylor rules in the open economy,” manuscript, University of Glasgow, and University of Exeter.
- [96] Levin, Andrew, Volker Wieland, and John C. Williams (1999), “Robustness of simple monetary policy rules under model uncertainty,” in *Monetary Policy Rules*, John b. Taylor (ed.), The Chicago University Press.
- [97] Ludvigson, Sydney, and Charles Steindel (1999), “How important is the stock market effect on consumption?,” Federal Reserve Bank of New York, *Economic Policy Review*, July 1999, pp.29-52.
- [98] Luetkepohl, H., and H. Herwartz (1996), “Specification of varying coefficient time series models via generalized flexible least squares,” *Journal of Econometrics*, Vol. 70, pp.261-290.
- [99] Luetkepohl, H., and Jürgen Wolters (1998), “A money demand system for German M3,” *Empirical Economics*, Vol. 23, pp.371-386.
- [100] Mankiw, N. Gregory (2000), “The inexorable and mysterious tradeoff between inflation and unemployment,” manuscript, Harvard University.
- [101] Martin, Ben, and Chris Salmon (1999), “Should uncertain monetary policy-makers do less?,” manuscript, Bank of England.
- [102] McCallum, Bennett T. (1999), “Issues in the design of monetary policy rules,” in *Handbook of Macroeconomics*, John B. Taylor and Michael Woodford (eds.), Amsterdam: North-Holland.
- [103] — (2000), “Alternative monetary policy rules: a comparison with historical settings for the United States, the United Kingdom, and Japan,” manuscript, Carnegie, Mellon University and Federar Reserve Bank of Richmond.

- [104] McCallum, Bennett T., and Edward Nelson (2001), "Monetary policy for open economy: an alternative framework with optimizing agents and sticky prices," NBER Working Paper 8175.
- [105] McGough, Bruce (2003), "Statistical learning with time-varying parameters," *Macroeconomic Dynamics*, No. 7, pp.119-139.
- [106] Mehra, Rajnish (1998), "On the volatility of stock prices: an exercise in quantitative theory," *International Journal of Systems Science*, Vol. 29, No. 11, pp.1203-1211.
- [107] Mendizábal, Hugo Rodríguez (2004), "The behavior of money velocity in low and high inflation countries," manuscript, Universitat Autònoma de Barcelona Centre.
- [108] Meyer, Laurence H. (2000), "Structural change and monetary policy," FRB speech.
- [109] Meyer, Laurence H., Eric T. Swanson, and Volker W. Wieland (2001), "NAIRU uncertainty and nonlinear policy rules," manuscript, January.
- [110] Mishkin, Frederic S. (2003), *The Economics of Money, Banking, and Financial Markets*, Addison Wesley.
- [111] Nobay A., Robert, and David A. Peel (2003), "Optimal monetary policy in a model of asymmetric central bank preferences," *Economic Journal*, Vol. 113, No. 489, pp.657-665.
- [112] Okina, Kunio, Masaaki Shirakawa, and Shigenori Shiratsuka (2000), "The asset price bubble and monetary policy: Japan's experience in the late 1980s and the lessons," IMES discussion paper series, No. 2000-E-12, Bank of Japan.
- [113] Okina, Kunio, and Shigenori Shiratsuka (2002), "Asset price bubbles, price stability, and monetary policy: Japan's experience," *Monetary and Economic Studies*, October 2002, pp.35-76.

- [114] Onatski, Alexei, and Noah Williams (2002), "Modeling model uncertainty," manuscript, Department of Economics, Columbia University, and Princeton University.
- [115] Orphanides, Athanasios, and Simon van Norden (2002), "The unreliability of output gap estimation in real time," *The Review of Economics and Statistics*, Vol. 84, No. 4, pp.569-583.
- [116] Orphanides, Athanasios, and John C. Williams (2002), "Imperfect knowledge, inflation expectations and monetary policy," manuscript, November.
- [117] Peersman, Gert, and Frank Smets (1998), "Uncertainty and the Taylor rule in a simple model of the Euro-area economy," manuscript.
- [118] Phillips, A.W. (1958), "The relation between unemployment and the rate of change of money wages in the United Kingdom, 1861-1957," *Economica*, Vol. 25.
- [119] Rigobon, Roberto, and Brian Sack (2001), "Measuring the reaction of monetary policy to the stock market," NBER Working Paper 8350.
- [120] Rudebusch, Glenn D. (2001), "Is the Fed too timid? Monetary policy in an uncertain world," *The Review of Economics and Statistics*, Vol. 83, pp.203-217.
- [121] Rudebusch, Glenn D., and Lars E. O. Svensson (1999), "Policy rules for inflation targeting," in John B. Taylor (ed.), *Monetary Policy Rules*, The University of Chicago Press.
- [122] Sack, Brian, and Volker Wieland (2000), "Interest-rate smoothing and optimal monetary policy: a review of recent empirical evidence," *Journal of Economics and Business*, Vol. 52, No. 1-2, pp.205-228.
- [123] Sargent, Thomas J. (1993), "Bounded Rationality in Macroeconomics," Clarendon Press, London.

- [124] — (1999), *The Conquest of American Inflation*, Princeton University Press.
- [125] Sargent, Thomas J., and Neil Wallace (1975), “ “Rational” expectations, the optimal monetary instrument, and the optimal money supply rule,” *Journal of Political Economy*, Vol. 83, No. 2, pp.241-254.
- [126] Schaling, Eric (1999), “The nonlinear Phillips curve and inflation forecast targeting,” Bank of England.
- [127] Scharnagl, Michael (1998), “The stability of German money demand: not just a myth,” *Empirical Economics*, Vol. 23, pp.355-370.
- [128] Schellekens, Phillip, and Jagjit Chadha (1998), “Utility functions for central bankers: the not so drastic quadratic,” LSE Financial Markets Group, discussion paper series, No. 308.
- [129] Semmler, Willi (2002), “Credit risk and sustainable debt: a model and estimations of why the Euro is stable in the long-run,” Working Paper 43, at [www.wiwi.uni-bielefeld.de/~cem](http://www.wiwi.uni-bielefeld.de/~cem).
- [130] — (2003), *Asset Prices, Booms and Recessions*, Springer-Verlag Heidelberg.
- [131] Semmler, Willi, and Wenlang Zhang (2003), “Monetary policy rules with nonlinear Phillips curve and endogenous NAIRU,” Working Paper 55, at [www.wiwi.uni-bielefeld.de/~cem](http://www.wiwi.uni-bielefeld.de/~cem).
- [132] Semmler, Willi, Alfred Greiner, and Wenlang Zhang (2003), *The Macroeconomy and Monetary and Fiscal Policies in the Euro-Area*, book manuscript, at [www.wiwi.uni-bielefeld.de/~cem](http://www.wiwi.uni-bielefeld.de/~cem).
- [133] Shiller, R. (1984), *Market volatility*, Cambridge: MIT-Press.
- [134] Sims, Christopher A. (2001a), “Pitfalls of a minimax approach to model uncertainty,” manuscript, Princeton university.

- [135] — (2001b), “Comment on Sargent and Cogley’s “Evolving US postwar inflation dynamics,” manuscript, Princeton University.
- [136] Sims, Christopher A., and Tao Zha (2002), “Macroeconomic switching,” manuscript, Princeton University.
- [137] Smets, Frank (1997), “Financial asset prices and monetary policy: theory and evidence,” Bank for international settlements, Working Paper No. 47, Sept. 1997.
- [138] Söderström, Ulf (1999), “Monetary policy with uncertain parameters,” manuscript, March.
- [139] Stiglitz, Joseph (1997), “Reflections on the natural rate hypothesis,” *Journal of Economic Perspectives*, Vol. 11, pp.3-10.
- [140] Svensson, Lars E.O. (1997), “Inflation forecasting targeting: implementing and monitoring inflation targets,” *European Economic Review*, Vol. 41, pp.1111-46.
- [141] — (1998), “Open-economy inflation targeting,” manuscript, Princeton University and Stockholm University; CEPR and NBER.
- [142] — (1999a), “Inflation targeting as a monetary policy rule,” *Journal of Monetary Economics*, Vol. 43, pp.607-654.
- [143] — (1999b), “Inflation targeting: some extensions,” *Scandinavian Journal of Economics*, Vol. 101, No. 3, pp.337-61.
- [144] — (2000), “Robust control made simple,” manuscript, October.
- [145] — (2002), “The inflation forecast and the loss function,” Discussion papers series, Center for Economic Policy Research London, No. 3365.
- [146] — (2003a), “The evolution of monetary policy rules and the Taylor rule,” lecture notes.



- [147] — (2003b), “What is wrong with Taylor rule? Using judgement in monetary policy through targeting rules,” *Journal of Economic Literature*, forthcoming.
- [148] Tambakis, Demosthenes N. (1998), “Monetary policy with a convex Phillips curve and asymmetric loss,” IMF Working Paper 98/21.
- [149] Taylor, John B. (1993), “Discretion versus policy rules in practice,” *Carnegie-Rochester Conference Series on Public Policy*, Vol. 39, 1993, pp.195-214.
- [150] — (1999), “A historical analysis of monetary policy rules,” in J.B. Taylor (ed.), *Monetary Policy Rules*, Chicago, IL: University of Chicago Press.
- [151] Tetlow, Robert J., and Peter von zur Muehlen (2001), “Simplicity versus optimality: the choice of monetary policy rules when agents must learn,” *Journal of Economic Dynamics and Control*, Vol. 25, pp.245-79.
- [152] — (2003), “Avoiding Nash inflation: Bayesian and robust responses to model uncertainty,” Board of Governors of the Federal Reserve System, manuscript.
- [153] Tucci, M.P. (1990), “A note on flexible least squares,” *Journal of Economic Dynamics and Control*, Vol. 14, pp.175-182.
- [154] — (1997), “Adaptive control in the presence of time-varying parameters,” *Journal of Economic Dynamics and Control*, Vol. 22, pp.39-47.
- [155] Tullio, G., E. de Souza, and P. Giucca (1996), “The demand for money functions in Europe and in Germany before and after the Berlin Wall,” in P. de Grauwe, S.Micossi and G. Tullio (eds.), *Inflation and Wage Behaviour in Europe*, Oxford: Clarendon.
- [156] Ullersma, C.A. (2001), “The zero lower bound on nominal interest rates and monetary policy effectiveness: a survey,” manuscript.

- [157] Varian, H.R.(1975), "A Bayesian approach to real estate assessment," in *Studies in Bayesian Econometrics and statistics in honour of Leonard J. Savage*, eds., S.E. Fienberg and Arnold Zellner, Amsterdam: North-Holland, pp.195-208.
- [158] Vázquez, J. (2003), "The role of the term-spread in an augmented Taylor rule," mimeo, Universidad del País Vasco.
- [159] Vega, Juan Luis (1998), "Money demand stability: evidence from Spain," *Empirical Economics*, Vol. 23, pp.387-400.
- [160] Vickers, John (1999), "Monetary policy and asset prices," manuscript, Bank of England, Sept. 1999.
- [161] Walsh, Carl E. (1999), "Monetary policy trade-offs in the open economy," University of California, Santa Cruz and Federal Reserve of San Francisco.
- [162] Wicksell, K. (1898), *Interest and Prices*, English translation by R.f. Kahn, London: Macmillan, 1936.
- [163] Wieland, Volker (2000), "Monetary policy, parameter uncertainty and optimal learning," *Journal of Monetary Economics*, Vol. 46, pp.199-228.
- [164] Wolters, Jürgen, Timo Teräsvirta, and Helmut Lütkepohl (1998), "Modeling the demand for M3 in the united Germany," *The Review of Economics and Statistics*, Vol. 80, No. 3, pp.399-409.
- [165] Woodford, Michael (1994), "Monetary policy and price level determinacy in a cash-in-advance economy," *Economic Theory*, 4, pp.345-380.
- [166] — (1996), "Control of the public debt: a requirement for price stability," NBER Working Paper, 5684.
- [167] — (2001), "The Taylor rule and optimal monetary policy," *The American Economic Review*, Vol. 91, No. 2, pp.232-237.

- [168] — (2003a), *Interest and Prices: Foundations of a Theory of Monetary Policy*, Princeton University Press, 2003.
- [169] — (2003b), “Optimal interest-rate smoothing,” forthcoming in *Review of Economic Studies*.

# List of Figures

2.1	Time-Varying $\alpha_{ut}$ . . . . .	29
2.2	Inflation Rate and Unemployment Rate of Japan . . . . .	31
2.3	Inflation Rates of Germany, France, Italy and the UK . . . . .	32
2.4	Unemployment Rates of Germany, France, Italy and the UK . . . . .	32
3.1	Simulation with $\lambda=0.1$ . . . . .	60
3.2	Simulation with $\lambda=10$ . . . . .	60
4.1	Inflation Rate, Output Gap and Short-Term Interest Rate of Germany . . . . .	67
4.2	Inflation Rate, Output Gap and Short-Term Interest Rate of Japan . . . . .	69
4.3	Inflation Rate, Output Gap and Short-Term Interest Rate of the US . . . . .	70
4.4	Inflation Rate, Output Gap and Short-Term Interest Rate of France . . . . .	72
4.5	Inflation Rate, Output Gap and Short-Term Interest Rate of the UK . . . . .	73
4.6	Inflation Rate, Output Gap and Short-Term Interest Rate of Italy . . . . .	75
4.7	Time-Varying $\beta_{\pi}$ and $\beta_y$ of Germany . . . . .	78
4.8	Time-Varying $\beta_{\pi}$ and $\beta_y$ of France . . . . .	79
4.9	Time-Varying $\beta_{\pi}$ and $\beta_y$ of the UK . . . . .	80
4.10	Time-Varying $\beta_{\pi}$ and $\beta_y$ of Italy . . . . .	81
4.11	Time-Varying $\beta_{\pi}$ and $\beta_y$ of Japan . . . . .	82

4.12	Time-Varying $\beta_\pi$ and $\beta_y$ of the US . . . . .	83
4.13	$\beta_\pi$ of E3 Countries . . . . .	86
4.14	Inflation Rates of E3 Countries . . . . .	86
4.15	$\beta_y$ of E3 Countries . . . . .	87
4.16	Output Gaps of E3 Countries . . . . .	88
4.17	Interest Rates of the US and Euro-Area . . . . .	89
4.18	Actual and Simulated Interest Rates of the Euro-Area . . . . .	90
4.19	Simulated Output Gap of the Euro-Area . . . . .	92
4.20	Actual and Simulated Output Gaps of the Euro-Area . . . . .	93
4.21	Actual and Simulated Inflation Rates of the Euro-Area . . . . .	93
5.1	Empirical Evidence of Uncertainty in the Phillips Curve . . . . .	104
5.2	Empirical Evidence of Uncertainty in the IS Curve . . . . .	106
5.3	An Example of $f(y_t)$ with the US Data from 1964-2003 . . . . .	108
5.4	Empirical Evidence of Uncertainty in the Convex Phillips Curve . . . . .	109
5.5	$\alpha_{2t}$ and $\alpha_{3t}$ in the Traditional and Convex Phillips Curves . . . . .	110
5.6	Simulations of RLS Learning (solid line) and Benchmark Model (dashed line) with Linear Phillips Curve . . . . .	117
5.7	Path of $\tilde{b}_t$ (solid line) in the Linear Phillips Curve . . . . .	118
5.8	RLS Learning with Linear (solid line) and Nonlinear (dashed line) Phillips Curves . . . . .	120
5.9	Paths of $\tilde{b}_t$ and $\bar{b}$ in Linear and Nonlinear Phillips Curves (NL stands for nonlinear) . . . . .	122
5.10	Detection Error Probability . . . . .	125
5.11	Simulation of the Robust Control with $\pi_0 = 0.02$ and $y_0 = 0.02$ . . . . .	127
5.12	Results of the Robust Control with Zero Shocks . . . . .	129
6.1	Actual and Simulated Interest Rates of EU3 (1978.1-98.4) . . . . .	151
6.2	$h(x)$ . . . . .	153
6.3	The LINEX Function . . . . .	154
6.4	An Example of $p_{t+1}$ with $r_t = 0$ . . . . .	156
6.5	$p_{t+1}$ with $\phi_1 = 0.4$ , $\phi_2 = 0.8$ and $\varphi = 10$ . . . . .	156
6.6	The Response of $r_t$ to $b_t$ with Alternative Values of Parameters . . . . .	159

6.7	The Inflation Rate, IPI and Share Price Index of Japan, 1980.1-2001.4 . . . . .	162
6.8	Simulation without Asset Price . . . . .	163
6.9	Simulation with Asset Price . . . . .	165
6.10	An Example of Linear and Nonlinear Policy Rules in the Presence of the Zero-Interest-Rate Bound . . . . .	167

# List of Tables

4.1	OLS Estimation of the Interest-Rate Rule of Germany . . . . .	67
4.2	OLS Estimation of the Interest-Rate Rule of Japan . . . . .	68
4.3	OLS Estimation of the Interest-Rate Rule of the US . . . . .	70
4.4	OLS Estimation of the Interest-Rate Rule of France . . . . .	72
4.5	OLS Estimation of the Interest-Rate Rule of the UK . . . . .	74
4.6	OLS Estimation of the Interest-Rate Rule of Italy . . . . .	75
4.7	State-Dependent Evidence of the US $\beta_\pi$ . . . . .	84
4.8	State-Dependent Evidence of the US $\beta_y$ . . . . .	85
5.1	Estimates of Hyperparameters in the Time-Varying Phillips and IS Curves . . . . .	103
5.2	Estimates of the Hyperparameters in the Convex Time-Varying Phillips Curve . . . . .	110
5.3	Mean and S.D. of State and Control Variables (L and NL stand for linear and nonlinear Phillips curves respectively). . . . .	121
5.4	Standard Deviations of the State and Control Variables with Different Values of $\theta$ . . . . .	126
6.1	Estimation of Eq. (6.8) . . . . .	145
6.2	Estimation of Eq. (6.15) . . . . .	147
6.3	GMM Estimation of Eq. (6.21) with Different n for $b_{t+n}$ . . . . .	150

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