On the Intergenerational Formation and Evolution of Continuous Cultural Traits

Inauguraldissertation zur Erlangung des Grades eines Doktors der Wirtschaftswissenschaften (Dr. rer. pol.) an der Fakultät für Wirtschaftswissenschaften der Universität Bielefeld

vorgelegt von

Magister rer. soc. oec. Michael Markus Pichler

Bielefeld, April 2011

Erstgutachter

Professor Dr. Walter Trockel
Institut für Mathematische
Wirtschaftsforschung (IMW)
Universität Bielefeld

Zweitgutachter

Professor Dr. Herbert Dawid
Fakultät für Wirtschaftswis–
senschaften
Universität Bielefeld

Gedruckt auf alterungsbeständigem Papier ISO 9706

Acknowledgements

In fall 2007, when I decided to accept my offer as a PhD–student at the International Research Training Group EBIM at the Institute of Mathematical Economics (IMW), I was not sure whether my decision was the right one. More than three years later, in spring 2010, I have no doubts that I will throughout my career consider this as my best and most central academic decision. The mixture between academic excellence and supervision, inspiring since friendly and warm atmosphere, leeway to develop own strengths and scientific skills and paths, international exchange, available funds for participation in international conferences, workshops and summer schools, etc., is simply outstanding and unmatched (which I claim until proven else). Notably, the financial side of this package is largely due to the fundings of the German Research Foundation (DFG). I also appreciate the personal fundings obtained from the DFG.

I feel deeply indebted to my supervisors Walter Trockel and Herbert Dawid for offering me the opportunity to take part at EBIM. This even more since they have permanently guided me throughout my walk on academic paths. Their inter–disciplinary academic expertise has made it possible that what was once a vague idea on 'the formal representation of the socialization process of children' turned into my theory 'On the Intergenerational Formation and Evolution of Continuous Cultural Traits'.

In this respect, I want to mention that my thanks do also go to a considerable extent to University of Valencia. Especially, I am grateful to Gonzalo Olcina Vauteren, who invited me for a research stay in Valencia in the summer term 2010, and turned into a mentor and friend. Overall, my stay in Valencia was a great experience, both on academic and social grounds.

I do further want to appreciate the so many enjoyable hours and days that I could spend with the friends I made in Bielefeld. Out of these, I would only like to mention explicitly Berno Büchel and Tim Hellmann personally, since they also act as co–authors of one part of this dissertation thesis.

Last but by far not least, I am overly thankful to my parents, family and friends in Austria, who have supported and encouraged me throughout my stay 'so far away' from home.

Michael Markus Pichler

Research Declaration

This 'Inauguraldissertation' is being submitted in form of a cumulative dissertation. It consists of three scientific papers. The first is titled 'The Economics of Cultural Formation of Preferences' and has been published as IMW Working Paper 431 in April 2010 (Pichler [49]). The second is titled 'Cultural Formation of Preferences and Assimilation of Cultural Groups', published as IMW Working paper 438 in August 2010 (Pichler [48]). The third one is part of a larger joint research project with Berno Büchel from Saarland University and Tim Hellmann from Bielefeld University. It is titled 'The Evolution of Continuous Cultural Traits in Social Networks', and unpublished to date. These three scientific papers are organized as the three main parts of the present work.

The three individual scientific papers are related in an exceedingly natural way, since they share a uniform phenomenological umbrella. This is constituted by the question of the inter–generational formation and evolution of continuous cultural traits. These refer to those types of traits that (a) are subject to formation in the socialization process, and (b) can reflect different intensities, located in a convex subset of the real line.

In particular, the first scientific paper introduces a generalized representation of the formation of continuous cultural traits. Thereby, the intensity of the continuous cultural trait that a child adopts is being formed as the collective outcome of all role models for trait intensities that it socially learns from. These role models are constituted by the observable socioeconomic action patterns of adults. It is shown how the adopted trait intensities induce preference relations over socioeconomic action patterns. Finally, this cultural formation of preferences process is endogenized as resulting out of optimal parental socialization decisions. Thus, an endogenous determination of the intergenerational evolution of trait intensities and the induced preferences over socioeconomic action patterns is obtained.

Based on this framework, the second scientific paper analyzes the evolution of trait intensities and behavior in a two cultural groups setting. It is shown that the dynamic properties depend crucially on what parents perceive as the optimal trait intensities for their children to adopt. Under inter–temporarily fixed (and distinct) optimal trait intensities, the trait

intensities of the cultural groups will always stay distinct. If the optimal trait intensities coincide with those derived from the representative group behavior, then a multitude of convergence path types can realize. These contain an inter–generational assimilation process toward the same trait intensity point; an initial but incomplete assimilation, with steady state trait intensities that are less distinct than initially; as well as inter–generational dissimilation with steady state trait intensities that are more distinct than initially. Which of those patterns will realize depends (among others) on the initial distance of the trait intensities. Notably, these theoretical insights can add to the understanding of empirically observable processes of the integration and assimilation of cultural groups.

It shall also be noted, that the representations of the first two scientific papers are embedded in a continuum of agents framework. The third scientific paper replaces this with a finite population setting. More importantly, it incorporates a social network structure into the cultural formation of continuous cultural traits framework. The theoretical focus is then on analyzing the static and dynamic properties of the model when parents perceive their adopted intensity of the continuous trait as the 'socialization target', and when they are free to choose their behavior subject to an inter–generationally fixed social network. This model constitutes a significant generalization of the DeGroot [18] model, first since it is subject to any arbitrary continuous trait intensity type (including that of continuous opinions), and second in terms of the induced evolution of the continuous trait intensities.A particular condition on the social network structure is derived that ensures convergence such that all adopted trait intensities of the dynasties of a connected subset are identical ('consensus').

Scientific Work Share The first two scientific papers constitute my exclusive own work. As has been mentioned above, the third scientific paper is part of a larger scientific project with Berno Büchel and Tim Hellmann. To be precise, it constitutes the first of two main parts of the larger research project with equal overall contributions of all authors. However, for the first main part, as manifested in this dissertation thesis, my scientific work share well exceeded one third.

Contents

List of Figures

Part 1

The Economics of Cultural Formation of Preferences

CHAPTER 1

Introduction

The concept of preferences is one of the most important cornerstones of economic theory, since preferences provide economic agents with the necessary means to choose between different possible socio–economic actions. The question of how preferences are being formed is thus of central interest to economic theory. The aim of the present paper is to contribute to the resolution of this question in a two–step approach. In a first step, it provides a general framework that represents the *formation of continuous cultural traits* in the socialization period of individuals. In a second step, it shows how these can be interpreted such as to induce preference relations over the choice of socio–economic action patterns in the adult life period of the indivuals.

With continuous cultural traits, we mean those types of traits that (a) are subject to formation in the socialization process, and (b) can reflect different intensities (or magnitudes, valuations, strengths, importances...), located in a convex subset of the real line. Notably, this characterization is not particularly restrictive since most types of traits can be (re–)interpreted in a continuous way (e.g. instead of asking whether a person has a 'status preference', one can ask *how important* status is for the person). Specifically, it contains concepts that are in standard use in economic theory, like the degree of altruism, the intensity of preferences for leisure or for social status, the patience of a person, etc.; but notably, it also contains (sociological) concepts like the values, attitudes, (strength of) norms and 'continuous opinions' that a person adopts.

Contributions and Results A natural question that arises in the context of this characterization of continuous cultural traits is then which of the possible intensities a person adopts, and how a process that determines this can be described in formal terms. Our approach will be to let the trait intensities be formed in the socialization period of a person, out of social

learning from *role models* for trait intensities.[1] These role–models correspond to the observable socio–economic action patterns of the adults of the society.

Upon observation of the socio–economic action pattern of an adult, children also receive a *cognitive impulse*. The latter can be understood as the signal on the valuation (or importance, magnitude, etc.) of the continuous cultural trait that is embodied in the choice of the particular socio–economic action pattern over the other available choices. We even endow these sorts of cognitive impulses with a cardinal meaning and call them *displayed trait intensities*.

In the next step we then introduce the representation of the socialization process that leads to the children's adoption of a specific trait intensity. This is embedded in a framework of socialization inside the family and by the general adult social environment, or 'direct vertical and oblique socialization'.[2] Specifically, we let the children's adopted trait intensities result as a weighted average between the displayed trait intensity that is *chosen* by its family, and the representative displayed trait intensity that the child observes in its general adult social environment.

Given the trait intensity that a person has adopted at the beginning of its adult period, we show how this can be interpreted such as to induce preference relations over the choices over the role models for trait intensities, i.e. the socio–economic action patterns. The central importance of this step is that it closes the circle between the socio–economic action patterns taken by one adult generation and the preferences over these patterns by the succeeding adult generation. We thus obtain a fully consistent and closed representation of the *evolution* of the trait intensities and the induced preferences of a sequence of generations.

It follows that any model framework that determines the adult choices of socio–economic action patterns (i.e. also the choices of displayed trait intensities), together with the families' socialization weights, equally endogenizes the process of formation of trait intensities. In the present paper, we will introduce one possible approach to achieve this, based on purposeful socialization decisions of the family. Notably, we restrict the latter to consist of a single parent only (through the assumption of asexual reproduction).

[1]Our viewpoint will be primarily that of an economist, with references to findings in the socio–psychological literature on child socialization whenever needed. A thorough placement of the present paper within this literature is though far beyond scope. See e.g. Grusec and Hastings [31] and Grusec and Kuczynski [32] for related book long treatments.
[2]This terminology stems from Cavalli-Sforza and Feldman [16] and is distinguished from 'horizontal socialization', i.e. socialization by members of the same generation.

That parents are willing to engage into costs associated with active socialization stems from the fact that they obtain an inter–generational utility component. Thereby, we let this utility be negatively related to the distance between the adopted trait intensity of their adult children and a parentally perceived *optimal* trait intensity.

The parental decision problem is it then to choose their weight in the child's socialization process and their displayed trait intensity. These choices are subject to the perceived optimal trait intensity of the parents and the representative displayed trait intensity of the general social environment. Since the latter results from the individual parents' choices, this introduces strategic interaction.

The corresponding parental best reply choices have the following central characteristics. First, consider the case where the representative displayed trait intensity of the general social environment deviates from the parentally perceived optimal trait intensity. Then, generically, parents countervail this suboptimal socialization influence on their children by choosing strictly positive *socialization instruments*. This means on the one hand that they choose a displayed trait intensity that deviates from their (utility maximal) adopted trait intensity. Specifically, this deviation is into the opposite direction as the deviation of the representative displayed trait intensity from the optimal trait intensity. On the other hand, this behavioral countervailing is coupled with a strictly positive choice of their socialization weight.

Furthermore, we could show that under certain conditions, parents use their investments into their socialization instruments and the representative displayed trait intensity of the general social environment as *cultural substitutes*. This means that if the representative displayed trait intensity becomes more favorable (i.e. its distance to the optimal trait intensity becomes smaller), then parents would reduce investments into both socialization instruments.

In the final step of the model, we then show that a Nash equilibrium (of the 'socialization game') in pure strategies exists under weak conditions. These equilibrium choices govern the inter–generational *evolution* of the trait intensities (and with it the preferences over socio–economic action patterns) of the society. However, to derive substantial qualitative properties of these dynamics, the model has to be specified.

We introduce one such specification, based on the assumptions that all parents have 'imperfect empathy' (this concept is due to Bisin and Verdier [7] and is shortly discussed in chapter 1). The central feature is that under a certain condition, the trait intensities of the sequence of adult generations converge to a homogeneous steady state (where the trait intensities of all

adults are identical). This 'melting pot' property is global since it holds for any initial distribution of the trait intensities.

Related Literature By basing the formation of trait intensities and preferences process on the children's social learning, the approach of the present paper stands in a natural relation to the literature on the economics of cultural transmission.[3] This literature has been established by Bisin and Verdier [7, 8, 9] and Bisin et al. [6], and is based on the work of Cavalli-Sforza and Feldman [15, 16] and Boyd and Richerson [12] in evolutionary anthropology. It studies the population dynamics of the distribution of a discrete set of cultural traits under an endogenous intergenerational cultural transmission mechanism.

The endogeneity stems from the purposeful parental choice of socialization intensity, which effectively determines the probability that the child will directly adopt the trait(s) of the parents. Parents engage into the cost of purposeful socialization in order to avoid (decrease the probability) that their child will not adopt their trait(s) — in which case parents encounter subjective utility losses.

The properties of the model framework have been applied in several different contexts, such as e.g. preferences for social status (Bisin and Verdier [7]), voting and political ideology (Bisin and Verdier [8]), corruption (Hauk and Sáez-Martí [34]), hold up problems (Olcina and Penarrubia [45]), gender discrimination (Escriche et al. [21]), etc. For an exhaustive overview of the literature on cultural transmission see Bisin and Verdier [10].

Related to this strand of literature are the contributions of Cox and Stark [17] and Stark [60]. They argue that parents might choose altruistic behavior in front of their children even though they are themselves not altruistic. This comes in an attempt to instrument the 'demonstration (or preference shaping) effect', which means an increase of the probability that the child becomes altruistic. In this case, the parents benefit from their child's future care taking.

[3]As Bisin and Verdier [7, p. 299] point out, this approach is thus distinct from those based on evolutionary selection mechanisms (where preferences/traits are either genetically inherited or imitated, with the reproductive/'imitative' success being increasing in the material payoff of the different preferences/traits), like in Rogers [54], Bester and Güth [4], Fershtman and Weiss [22], Kockesen et al. [37], [27], and from those based on the agents' introspective self selection of preferences, as in e.g. Becker [2] and Becker and Mulligan [3].
Alternative approaches that deal with preference endogeneity in 'non–purposeful–socialization' frameworks are based on e.g. 'bandwagon' or 'snob' effects (Leibenstein [39]), 'keeping up with the Joneses' (Duesenberry [20]), 'emulation effects' (Veblen [61]) or 'interdependent preferences' (Pollak [51]).

However, the theories mentioned consider the probabilistic *transmission* of traits and do not approach the issue of formation of the latter. This restricts their applicability mainly to *discrete* (sets of) cultural traits. So far, little has been contributed to resolve the question of the cultural formation of continuous cultural traits. Important early treatments of the topic are Cavalli-Sforza and Feldman [16] in a theoretical, and Otto et al. [46] in an empirical context.

More recently Bisin and Topa [5] proposed a representation of the formation of the intensities of continuous cultural traits, while Panebianco [47] did so for the case of inter–ethnic attitudes. In the terminology of the present paper, both represented the adopted intensity of the cultural trait (attitude) as a weighted average between the displayed trait intensity of the family and the (weighted) average of the intensities of the cultural traits (attitudes) that the society has adopted.

In this respect, the major limitation of both contributions is, however, that they do consider only a degenerate behavioral choice. In particular, Bisin and Topa [5] assume that parents always choose socio–economic action patterns the displayed trait intensity of which exactly accords with their 'target intensity' (i.e. the optimal trait intensity in the terminology of the present paper); and Panebianco [47] assumes that the parents set a displayed trait intensity that exactly accords with their inter–ethnic attitudes. Given this degenerate view on the family's behavioral choices, its socialization decision is then restricted to choosing its weight in the formation of the trait intensity of their child.

Outline The further outline of this paper is as follows. Chapter 2 introduces the general representation of the cultural formation of preferences process, while as chapter 3 delivers a framework for its endogeneization. The proofs of the propositions in the latter chapter can be found in Appendix A 1. Chapter 4 discusses additional aspects that show routes how to apply the model, and chapter 5 concludes.

CHAPTER 2

Cultural Formation of Preferences

. . . or: We are all the sum total of our experiences.

In this chapter, we will show how children adopt intensities of any type of continuous cultural trait through social learning from role models for trait intensities, and how the adopted trait intensities induce preference relations over choices of the role models in the adult life period. This kind of closed circle is the motivation to label the representation of the socialization process that this paper proposes as *cultural formation of preferences.*

Consider an overlapping generations society populated by a continuum of adults,[1] $a \in A = [0, 1]$ endowed with Lebesgue measure λ, and their children. For simplicity, we will assume that reproduction is asexual and every adult has one offspring, so that we can denote with $\tilde{a} \in \tilde{A}$ the children of the parents $a \in A$.

Let us assume that all adults have available the same non–empty set of socio–economic action patterns, X. This set is endowed with a complete and transitive binary relation T. Thereby, for all $x, x' \in X$, $x\,T\,x'$ means that the socio–economic action pattern x is (weakly) 'more characteristic' for the continuous trait type under scrutiny than socio–economic action pattern x'.[2] This general formulation is owed to the fact that we consider any type of continuous cultural trait. Which socio–economic action patterns would be considered as 'more/less characteristic' in a particular case depends on the (formulation of) the continuous cultural trait under scrutiny.[3] In case of e.g. 'importance of religion', 'more or less characteristic' would correspond to more or less religious behavior patterns (since they reflect a higher or lower importance of religion). Given transitivity and completeness, we can

[1]The logic of the cultural formation of preferences process that is presented in the present paper would be preserved in the case where the set of adults is finite.

[2]Given the abstract set of socio–economic action patterns X, we could equally endow it with a full set T_i, $i = 1, \ldots, n$ of binary relations, each of which would correspond to one of n different continuous cultural traits. The rest of the exposition in this paper would then generalize analogously.

[3]Considering 'classes' of continuous cultural traits, in case of attitudes or opinions, 'more characteristic' would sensibly be replaced by 'more positive'; in case of values, 'more characteristic' would sensibly be replaced by 'higher'; etc.

represent the ordinal relation T by a cardinal function

$$\phi^d : X \mapsto \mathbb{R}\,.^4$$

Thus, to any socio–economic action pattern $x \in X$, ϕ^d assigns a number with cardinal meaning, $\phi^d(x)$. We will call this the *displayed trait intensity* (DTI) embodied in the choice of socio–economic action pattern x.[5] Thus, $\phi^d(X)$ is the set of possible DTIs.

Now, the role models of the children's social learning of trait intensities are the observable socio–economic action patterns $x \in X$ taken by the adults $a \in A$; and we assume that the *cognitive impulse* that any of the children obtains through such an observation is the corresponding DTI, $\phi^d(x)$. The exposition so far makes clear that in the present work we treat the function ϕ^d as an 'objective entity' in the sense that the cognitive processing of observed socio–economic action patterns of all children is in terms of this function (and also all adults assign to any socio–economic action pattern the same DTI).[6]

To simplify the subsequent exposition, we will denote the DTI of the socio–economic action pattern of adult $a \in A$, $x_a \in X$, as $\phi^d_a := \phi^d(x_a)$.

Example 1.1 (Patience). *For illustration, let us consider the formation of 'patience' in a very stylized way. Assume that the socio–economic action pattern for the social learning of patience is the share of adult period income that is saved for pension period consumption. Denoting as $y_a \in \mathbb{R}_{++}$ the adult period income, and as $s_a \in [0, y_a]$ the savings of adult $a \in A$ (there is no lending), we thus have that $x_a :\equiv \frac{s_a}{y_a} \in [0,1] \equiv: X$. Naturally, we want ϕ^d to be strictly increasing in the present case, so that we can simply choose $\phi^d(x) = x$ and then $\phi^d(X) = [0,1]$.*

We will now introduce the representation of the socialization process that this paper proposes. This will be established on grounds of the *tabula rasa* assumption, which means in the present context that children are born with unformed trait intensity (TI), and equally, with unformed preferences (a corresponding assumption is also taken in the literature on the economics of cultural transmission, see e.g. Bisin and Verdier [9]). This assumption

[4]Thus, the relation T and $\phi^d \sim b + d\phi^d$ together define an equivalence class with respect to $\sim$ on the set of real valued (cardinal) functions.

[5]This can be understood in the way that any adult who observes another adult $a \in A$ taking socio–economic action pattern $x \in X$ could reflect upon this observation by the statement that 'adult a behaves as if she would have a trait intensity of $\phi^d(x)$'.

[6]Indeed, there is room here for a generalization. In particular, the way how the children's cognitive processing of observed socio–economic patterns takes place could also be treated as being subject to an individual social learning process (thus, children would adopt individual functions ϕ^d_a, which they would eventually internalize and keep in their adult life–period).

implies that we restrict the analysis of the determination, respectively formation, of traits to cultural factors ('nurture'), while as the issue of the contribution of genetic inheritance ('nature') is left aside.[7]

On this basis, we then let the formation of the TI that a child adopts result out of social learning from the socio–economic action patterns of adults (only) that it is confronted with. Specifically, this is being embedded in a framework of socialization inside the family and by the general adult social environment, or 'direct vertical and oblique socialization'. In this context, we will let the TI that a child $\tilde{a} \in \tilde{A}$ adopts be formed according to a weighted average between the representative DTIs of both socialization sources (i.e. as a weighted average of all cognitive impulses obtained in the socialization process). In the case of the child's family, this coincides with the DTI of its single parent $a \in A$, $\phi^d_a \in \phi^d(X)$. The representative DTI of the child's general social environment, $A_a := A \backslash \{a\}$, will be denoted $\phi^d_{A_a}$. These result out of the children's social learning from the observed DTIs of (eventually) different subsets of adults that they are confronted with.

More precisely, we assume that there is a measurable partition of the adult set, $\{A_J\}_{J=1}^K$,[8] and that the children obtain as cognitive impulses the *average* DTIs of these subsets, $\phi^d_{A_J} := \frac{1}{\lambda(A_J)} \int_{A_J} \phi^d_{a'} \, d\lambda\,(a') \in \text{con}\ \phi^d(X)$, $\forall J = 1, \ldots, K$.[9] Specifically, for every child $\tilde{a} \in \tilde{A}$ there are *oblique socialization weights*, $\sigma_{\tilde{a}J}$, $J = 1, \ldots, K$, that represent the relative cognitive impacts of the child's social learning from the various subsets of adults. These weights satisfy $\sigma_{\tilde{a}J} \in [0, 1]$ and $\sum_{J=1}^K \sigma_{\tilde{a}J} = 1$, $\forall \tilde{a} \in \tilde{A}$, $\forall J = 1, \ldots, K$. They can, among others, result from the population shares of the subsets, or else from a local structure that determines the social(ization) interaction times with the members of the subsets, or from differing pre–dispositions for social learning from different groups (the members of which e.g. share the same

[7]An introduction to the cross–disciplinary 'nature–nurture' debate can be found in Rogers [54]; Sacerdote [55, 56, 57] provides for empirical investigations of the relative importances of both influences.

[8]In this paper, this partition is assumed to be exogenously given. It can, however, be motivated to result from a local structure (i.e. where the adults reside), or from a classification of the adults in different social and economic categories.

[9]We refrain here from a further generalization through distinguishing the children's social learning from all individual adults $a' \in A_a$. In this case, the Nash equilibrium existence result in Proposition 1.3 could not be maintained.
To see that the average choice of a continuum of players endowed with Lebesgue measure and with identical choice set (a subset of $\mathbb{R}^n$) is indeed located in the convex hull of the choice set, confer e.g. Rath [53, p. 430].

social and economic characteristics).[10] We obtain, $\forall \tilde{a} \in \tilde{A}$,

$$\phi^d_{A_a} := \sum_{J=1}^{K} \sigma_{\tilde{a}J} \phi^d_{A_J} \in \text{con } \phi^d(X).$$

The weight that the DTI of the parent of a child $\tilde{a} \in \tilde{A}$ has in the socialization process of the child will be called the *parental socialization success share*, $\hat{\sigma}_a \in [0,1]$. This corresponds to the cognitive impact of the parental DTI relative to the cognitive impact of the representative DTI of the child's general social environment. Factors that would determine this relative cognitive impact would include the social(ization) interaction time of the parent with its child, as well as the effort and devotion that the parent spends to socialize its child to the chosen DTI.[11] We thus assume that the parental socialization success share can be chosen by the parents (and in chapter 3, we will endogenize this choice).[12]

We now obtain the formation of the TI that a child $\tilde{a} \in \tilde{A}$ adopts through the 'direct vertical and oblique socialization' process, $\phi_{\tilde{a}}$, as

$$\phi_{\tilde{a}} = \hat{\sigma}_a \phi^d_a + (1 - \hat{\sigma}_a)\phi^d_{A_a}. \tag{1.1}$$

We will call this the *parental socialization technique.* It is a generalization of the representation of the formation of continuous cultural traits, respectively inter–ethnic attitudes, in Bisin and Topa [5] and Panebianco [47]. Equation (1.1) embodies the view that the parents set a TI benchmark, $\phi^d_a \in \phi^d(X)$, and can invest into their parental socialization success share, $\hat{\sigma}_a \in [0,1]$, to countervail the socialization influence that the child is exposed to in its general social environment, $\phi^d_{A_a}$.[13] Thus, for any $\phi^d_{A_a} \in \text{con } \phi^d(X)$, the parents could fully determine the adopted TIs of their children (whether or not they also have an incentive to do so will concern us in chapter 3). Hence the set of possible TIs that a child can adopt always coincides with the convex hull of the set of possible DTIs, con $\phi^d(X) \subseteq \mathbb{R}$ (a convex subset of the real line).

Example 1.2 (Discrete Choice Sets). *To illustrate the last point consider any discrete choice set of socio–economic action patterns, and let us take the simplest (non–degenerate) example where $X = \{0,1\}$, e.g. not buying or*

[10] In this respect, Panebianco [47] considers the effect that different schemes of oblique socialization weights have on the formation of inter–ethnic attitudes.

[11] See e.g. Grusec [30] for an introductory overview of theories on determinants of parental socialization success.

[12] That parents can choose their socialization success shares within the whole unit interval is a non–trivial assumption (which is though also taken in Bisin and Topa [5] and Panebianco [47]).

[13] This context can be interpreted as the generalized and continuous equivalent to the 'preference shaping demonstration effect' of Cox and Stark [17] and Stark [60].

buying a status good. Let again $\phi^d(x) = x$*, so that* $\phi^d(X) = \{0, 1\}$*. However, under the formation of TIs (1.1), we have that the set of possible TIs is* $\text{con } \phi^d(X) = [0, 1]$*. Thus, although adults can only display through their socio–economic action patterns that they either disfavor/not have (*$x = 0$*) or favor/have (*$x = 1$*) a certain trait (e.g. 'status'), the children can adopt also any intermediate TI through the socialization process.*

We will assume that the TI that a child adopts through the socialization process is being internalized and kept in its adult life–period. Notably, the concept of an adopted TI of an adult corresponds to a *cognitive element* in the cognitive dissonance theory of Festinger [23] — and so does the concept of a DTI. According to the cognitive dissonance theory, people dislike dissonance between cognitive elements, the strength of which depends on the degree of the dissonance. In the present context, it is immediate that this degree of dissonance could be described by the (Euclidean) distance between a DTI and the adopted TI. Thus, adults can compare and rank different DTIs based on their distance to the adopted TI. Obviously then, since socio–economic action patterns are pre–images of DTIs, the adopted TI of an adult does also constitute a 'filter' under which adults can evaluate different choices of socio–economic action patterns.

Assumption 1.1 (Preferences). $\forall a \in A$,

(a) the adopted TI, $\phi_a \in \text{con } \phi^d(X)$*, induces a complete and transitive preference relation* $\succ^{\phi_a}$ *over DTIs* $\phi_a^d \in \text{con } \phi^d(X)$*,*[14] *and*

(b) the preferences $\succ^{\phi_a}$ *are single–peaked with peak* ϕ_a*. This means that* $\forall \phi_a^d, \phi'^d_a \in \text{con } \phi^d(X)$*,* $\phi_a^d \succ^{\phi_a} \phi'^d_a \Leftarrow \phi'^d_a <> \phi_a^d \leq\geq \phi_a$*.*

Given their basic properties, we will represent the preferences $\succ^{\phi_a}$ by single–peaked utility functions with peak ϕ_a

$$u\left(\cdot\,|\phi_a\right) : \text{con } \phi^d(X) \mapsto \mathbb{R}$$

which are strictly increasing/decreasing at all $\phi_a^d \in \text{con } \phi^d(X)$ such that $\phi_a^d < / > \phi_a$.[15]

Example 1.3 ('Displayed Patience' Utility). *Continuing the first example, assume that adults earn interest on their savings and, thus, their pension period consumption is* $(1+r)s_a$*,* $r \in \mathbb{R}_+$ *(prices are constant and there is no other pension period income and also no bequests).*

[14]Equally, thus, $\phi_a \in \text{con } \phi^d(X)$, induces a complete and transitive preference relation $\succ^{\phi_a}$ over socio–economic action patterns $x_a \in X$, where $\forall x_a, x'_a \in X$, $x_a \succ^{\phi_a} x'_a \Leftrightarrow \phi^d\left(x_a\right) \succ^{\phi_a} \phi^d\left(x'_a\right)$.

[15]In an extension of the previous footnote, the induced preferencs over socio–economic action patterns $\succ^{\phi_a}$ can be represented by a utility function $w\left(\cdot\,|\phi_a\right) : X \mapsto \mathbb{R}$, where $\forall x_a, x'_a \in X$ $w\left(x_a\,|\phi_a\right) > w\left(x'_a\,|\phi_a\right) \Leftrightarrow u\left(\phi^d(x_a)\,|\phi_a\right) > u\left(\phi^d\left(x'_a\right)|\phi_a\right)$.

Assuming Cobb–Douglas utility, the life–time utility out of the adult savings decision can be represented as $u\left(s_a\,|\phi_a\right) = \left(y_a - s_a\right)^{1-\phi_a}\left((1+r)s_a\right)^{\phi_a}$, *i.e. consumptions in the first and second life period are weighted according to the 'impatience' and 'patience' (intensities). Dividing and multiplying the right hand side of the latter by* y_a, *we obtain* $u\left(\phi_a^d\,|\phi_a\right) = \left(1-\phi_a^d\right)^{1-\phi_a}\left(\phi_a^d\right)^{\phi_a}\cdot\left(y_a(1+r)^{\phi_a}\right)$. *Thus, we have transformed utility out of a socio–economic choice into utility out of the choice of 'displayed patience (intensity)',* ϕ_a^d. *It is immediate that* $\frac{\partial\, u\left(\phi_a^d|\phi_a\right)}{\partial\,\phi_a^d} >=< 0\ \forall\phi_a^d \in [0,1]$ *such that* $\phi_a^d <=> \phi_a$ *so that the single peak property is satisfied naturally (furthermore,* $u\left(\cdot\,|\phi_a\right)$ *is strictly concave).*

CHAPTER 3

Endogenous Cultural Formation of Preferences

. . . or: How far does the apple fall from the tree?

In the previous chapter, we have introduced a representation of the inter–generational formation of continuous cultural traits. One major innovation that this approach embodies is that it interconnects the choices of socio–economic action patterns (respectively of displayed trait intensities) of the adult generation with the preferences over the available choices that the next generation adults adopt. Thus, any model framework that determines the adult choices of socio–economic action patterns, together with the parental socialization success shares, equally endogenizes the cultural formation of preferences process (see chapter 4 for a more detailed discussion).

In the present chapter, we will lay down one specific way of achieving this endogeneization based on purposeful socialization decisions of parents. Thereby, we notably restrict the latter to consist of their choice of a displayed trait intensity (as determined through the choice of the underlying socio–economic action patterns) and of their parental socialization success share. This means that we leave the oblique socialization weights (that determine the children's relative social learning from the different adult subsets) exogenously fixed.

1. Motivation for Purposeful Socialization

In a first step, we have to clarify what motivation parents have to actively engage in their children's socialization process, i.e. what induces them to purposefully employ their socialization technique (the functioning of which we assume them to be fully aware of). Basically, we let this motivation stem from the fact that parents also obtain an inter–generational utility component. Thereby, this is either related to the adopted TI of their adult children and/or to the DTI (respectively the underlying socio–economic action patterns) that they expect their adult children to take.

As far as the latter expectations are concerned, we make here an assumption on a specific form of parental myopia: Although parents obtain an inter–generational utility component, which eventually induces them to choose a DTI that does not coincide with their adopted TI (see below), we

assume that they do not realize that this form of behavior changing impact will also be present in their adult children's decision problems. Thus, any parent $a \in A$ expects its adult child to choose a DTI that is in the set of maximizers of its 'own' utility function, $\arg\max_{\phi^d_{\tilde{a}} \in \phi^d(X)} u\left(\phi^d_{\tilde{a}} \,|\phi_{\tilde{a}}\right)$. Under the following assumption, $\phi^d(X)$ is convex (and compact, which will be needed in the propositions below), and thus $\phi^d(X) = \text{con}\ \phi^d(X)$. This then guarantees by the single–peakedness of the utility functions that $\arg\max_{\phi^d_{\tilde{a}} \in \phi^d(X)} u\left(\phi^d_{\tilde{a}} \,|\phi_{\tilde{a}}\right) = \phi_{\tilde{a}}$, $\forall a \in A$. Hence, the parental expectations of their adult children's DTIs are uniquely determined.[1]

Assumption 1.2 (Convexity and Compactness). *X is a convex and compact subset of a finite dimensional Euclidean space, and ϕ^d is continuous. It follows that $\phi^d(X)$ is non–empty, convex and compact.*

Given the parents' myopic expectations, it is independent of whether the inter–generational utility component of a parent is related to the adopted TI or expected DTI of its adult child, since they coincide. Under this property, we will now assume that any parent perceives an *optimal trait intensity* that it wants its child to adopt (i.e. if the child would adopt this optimal TI, then this would be strictly preferred by a parent over all other possible TIs that the child could adopt). These parent–specific optimal TIs are subject to what we call *perception rules*.

Thereby, the perception rule of the optimal TI of any parent is determined by two 'ingredients'. The first one specifies a (set of) subset(s) of adults, which can be understood as reference group(s). The second ingredient then specifies the construction of the optimal TI that a parent perceives out of characteristics of the adults in these reference group(s) that are either observable (notably the DTIs of adults) or known to an individual parent.

To formally introduce the concept of perception rules, it will be convenient to define $\mathcal{A}$ as a σ–algebra generated by the finite partition $\{A_J\}_{J=1}^K$.

Definition 1.1 (Perception Rule). *For every parent $a \in A$, the* perception rule *for the optimal trait intensity is a pair $\left(R_a, \hat{\phi}_{\tilde{a}}\right)$, where $\emptyset \neq R_a \in \{a\} \cup \mathcal{A}$ and where $\hat{\phi}_{\tilde{a}} : \{a\} \cup \mathcal{A} \mapsto \text{con}\ \phi^d(X)$, $\hat{\phi}_{\tilde{a}}\left(R_a\right) \in \text{con}\ \phi^d(X)$.*

To ease the interpretation of this conceptualization, we will briefly introduce three sensible types of perception rules for optimal TIs. Note that this list is not meant to be exhaustive (one could e.g. consider combinations of the three types mentioned).

[1]That parents are not aware of the inter–generational utility of their children does also have the simplifying consequence that they do not care about their whole dynasty (this point has already been made by Bisin and Verdier [9, p. 305] in the context of cultural transmission of preferences).

PR 1 The optimal TI of a parent $a \in A$ is identical to its adopted TI, $R_a = \{a\}$ and $\hat{\phi}_{\tilde{a}}(\{a\}) = \phi_a \in \text{con } \phi^d(X)$.

One justification to consider this perception rule is based on a special form of parental altruism called 'imperfect empathy'. This concept has been introduced into the economics literature by Bisin and Verdier [7]. Parents are altruistic and fully internalize the utility of their adult child's socio–economic action pattern (respectively DTI). Nevertheless, parents can not perfectly empathize with their child and can only evaluate their adult child's utility under their own (not the child's) utility function — which attains its maximum at the adopted TI of the parent.

PR 2 The optimal TI of a parent $a \in A$ is identical to a parent–specific (model–exogenous) TI, $R_a = \{a\}$ and $\hat{\phi}_{\tilde{a}}(\{a\}) = e_a \in \text{con } \phi^d(X)$.

One motivation for this perception rule could be that the trait under scrutiny is a 'good preference' where parents thus want to maximize the TI of their adult children. This would e.g. concern certain characteristics (traits) that are favorable on the labor market. Hence, higher intensities of such traits increase the future expected income of the adult child, which the parents would aim to maximize if they are altruistic (and if their own utility function is increasing in monetary payoff).

PR 3 The optimal TI of a parent $a \in A$ is identical to the average DTI of a subset (with strictly positive measure) of the adults, $R_a \subseteq A$, and $\hat{\phi}_{\tilde{a}}(R_a) = \frac{1}{\lambda(R_a)} \int_{R_a} \phi^d_{a'} \, d\lambda(a') \in \text{con } \phi^d(X)$.

One potential justification for this perception rule is the case of 'endogenous behavioral norms' that equate to the average DTI of the respective subset of the adults. Norms are typically maintained by members of a group (a subset of the adults) through a system of social rewards and punishments (see e.g. Arnett [1]). In the present context, these could be related to the parents' success or failure to guarantee that the child will behave according to the behavioral norm.

Given the perception rules and the resulting optimal TIs, we assume further that parents perceive utility losses for deviations of the adopted TI of their children from these optimal TIs (note the structural analogy to the before introduced preferences and utility that are induced by adopted TIs). Specifically, for any parent $a \in A$, we introduce the parameter $i_a \in \mathbb{R}_+$ that shall capture the strength of the perceived inter–generational utility losses. We will call this the parent's *inter–generational trait intensity.*

Notably, this latter type of TI could also be interpreted as being subject to a cultural formation of preferences process. Nevertheless, we choose here

for simplicity a degenerate representation of this process and assume that the inter–generational TIs are invariably passed over from an adult to its child, $i_{\tilde{a}} = i_a$, $\forall a \in A$.

Assumption 1.3 (Inter–generational Utility). $\forall a \in A$,

(a) the perception rule and inter–generational trait intensity induce an inter–generational utility function $v\left(\cdot \left| \hat{\phi}_{\tilde{a}}\left(R_a\right), i_a\right.\right) : \text{con } \phi^d(X) \mapsto \mathbb{R}$, $v\left(\phi_{\tilde{a}} \left| \hat{\phi}_{\tilde{a}}\left(R_a\right), i_a\right.\right) \in \mathbb{R}$, *where*

(b) $\forall i_a \in \mathbb{R}_{++}$, $v\left(\cdot \left| \hat{\phi}_{\tilde{a}}\left(R_a\right), i_a\right.\right)$ *is single–peaked with peak* $\hat{\phi}_{\tilde{a}}\left(R_a\right)$, *thus strictly increasing/decreasing at all* $\phi_{\tilde{a}} \in \text{con } \phi^d(X)$ *such that* $\phi_{\tilde{a}} < / > \hat{\phi}_{\tilde{a}}$.

2. Best Reply Problems

In the last step toward the construction of the parental best reply problems, let us finally discuss the cost associated with investments into controlling the parental socialization success share. These would concern e.g. the opportunity cost of the time parents spend for the active socialization of a child, as well as the (psychological) cost of the effort and devotion invested. We will represent these costs by an indirect cost function of choices of socialization success shares, $c : [0,1] \mapsto \mathbb{R}_+$, $c\left(\hat{\sigma}_a\right) \in \mathbb{R}_+$.

The parental (optimization) problem is it then to choose a DTI and its socialization success share in a best reply to the child–specific representative DTI of the general social environment such as to maximize utility net of the cost of achieving the chosen socialization success share. We obtain, $\forall a \in A$,

$$\max_{\left(\phi_a^d, \hat{\sigma}_a\right) \in \phi^d(X) \times [0,1]} u\left(\phi_a^d \left| \phi_a\right.\right) + v\left(\phi_{\tilde{a}} \left| \hat{\phi}_{\tilde{a}}\left(R_a\right), i_a\right.\right) - c\left(\hat{\sigma}_a\right) \tag{1.2}$$

$$\text{s.t. } \phi_{\tilde{a}} = \hat{\sigma}_a \phi_a^d + (1 - \hat{\sigma}_a)\phi_{A_a}^d.$$

The best reply problems of the parents hence basically consist of trading off the cost and benefits of their socialization choices. The cost (and disutilities) are constituted by 'own' utility losses that parents experience when choosing a DTI that deviates from their adopted TI, together with the cost of a choice of their socialization success share. The benefits accrue in form of resulting inter–generational utility gains through reductions in the distance between the child's adopted TI and the optimal TI.

As mentioned above, the parents choose best reply pairs of a DTI and a socialization success share against the representative DTI. But notably, this choice is subject to the optimal TI, the adopted TI and the inter–generational TI. Therefore, for any $a \in A$, we will denote any pair of best reply choices as $\left(\phi_a^d\left(\phi_{A_a}^d, \hat{\phi}_{\tilde{a}}\left(R_a\right), \phi_a, i_a\right), \hat{\sigma}_a\left(\phi_{A_a}^d, \hat{\phi}_{\tilde{a}}\left(R_a\right), \phi_a, i_a\right)\right)$, which

we will abbreviate subsequently as $\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)\right)$. Furthermore, together with the representative DTI of the general social environment, any of the parental best replies also determines a best reply location of the adult child's adopted TI (through the formation of TIs (1.1)), $\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_{A_a}^d\right)$.

The following assumption specifies additional properties of the (inter–generational) utility and cost functions. These will allow for a significant characterization of the pairs of parental best reply choices, as well as of the resulting best reply locations of the adopted TIs of the adult children.

Assumption 1.4 (Slope).

(a) $u(\cdot|e)$ *and* $v(\cdot|f,g)$ *are continuous, and differentiable at their peaks,*

(b) c *is continuous, and differentiable with respect to the first argument at the origin, with zero slope, strictly increasing in the first argument on* $(0,1]$*, and decreasing in the second argument.*

Since both the utility and inter–generational utility function are single peaked, it follows by Assumption 1.4 (a) that both functions have zero slopes at their peaks. Thus, parents perceive zero (inter–generational) utility losses for marginal deviations of their chosen DTI from their adopted TI, respectively of their adult child's adopted TI from the optimal TI.

For the following two propositions, we will assume that the perception rules for the optimal TIs of all parents are as such that the individual parents' decisions have (at most) a negligible impact on the location of their own optimal TI.

Proposition 1.1 (Characterization of Best Replies). *Let Assumptions 1.1–1.4 hold. Then, if*

(a) $\phi_{A_a}^d \neq \hat{\phi}_{\tilde{a}}(R_a)$*, generically*[2] $\operatorname{sign}\left(\phi_a^d(\cdot) - \phi_a\right) = -\operatorname{sign}\left(\phi_{A_a}^d - \hat{\phi}_{\tilde{a}}(R_a)\right)$ *and* $\hat{\sigma}_a(\cdot) > 0$*, while always* $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_{A_a}^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = \operatorname{sign}\left(\phi_{A_a}^d - \hat{\phi}_{\tilde{a}}(R_a)\right)$.

(b) $\phi_{A_a}^d = \hat{\phi}_{\tilde{a}}(R_a)$*, it holds that* $\phi_a^d(\cdot) - \phi_a = 0$ *and* $\hat{\sigma}_a(\cdot) = 0$*, hence* $\phi_{\tilde{a}}\left(\phi_a, 0, \hat{\phi}_{\tilde{a}}(R_a)\right) - \hat{\phi}_{\tilde{a}}(R_a) = 0$.

Proof. In Appendix A 1.1.

[2]There are two kinds of exceptions to the generic characterization. The first is that if the deviation of the best reply DTI from the adopted TI into the characterized direction is not possible, i.e. if the adopted TI of a parent coincides with (the relevant) one of the boundaries of $\phi^d(X)$, then the best reply DTI will coincide with that boundary (while still $\hat{\sigma}_a(\cdot) > 0$). The second is that in the cases where $\hat{\phi}_{\tilde{a}}(R_a) > \phi_a$ and $\phi_{A_a}^d \in \left(\phi_a, \hat{\phi}_{\tilde{a}}(R_a)\right)$, respectively where $\hat{\phi}_{\tilde{a}}(R_a) < \phi_a$ and $\phi_{A_a}^d \in \left(\hat{\phi}_{\tilde{a}}(R_a), \phi_a\right)$, it can also hold that $\operatorname{sign}\left(\phi_a^d(\cdot) - \phi_a\right) = 0$ and $\hat{\sigma}_a(\cdot) = 0$, hence $\phi_{\tilde{a}}\left(\phi_a, 0, \phi_{A_a}^d\right) = \phi_{A_a}^d$.

The (generic) results of this proposition are illustrated in Figure 1.1. The left pair of graphs stylizes case (a) of Proposition 1.1, and the right pair the case (b). In both pairs of graphs, in the left interval (all intervals correspond to the set of possible DTIs) the context of the adult's decision problem is depicted. In the right interval a corresponding best reply choice is stylized. As can be seen both from Proposition 1.1 directly, as well as from the graphical illustration, the results feature two dominant characteristics.

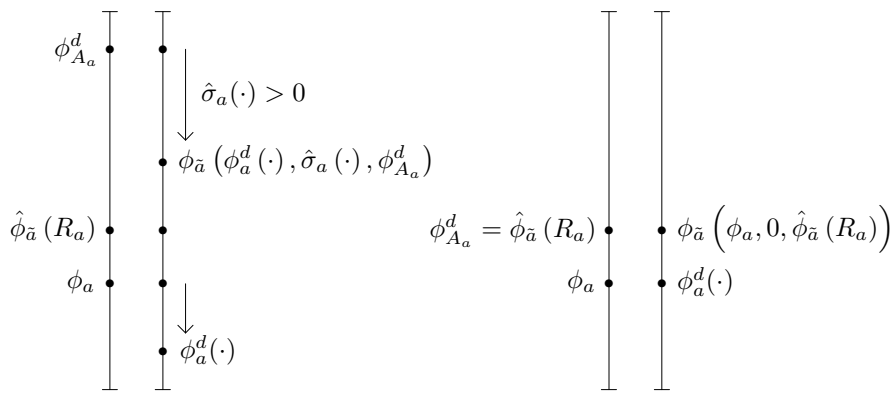

FIGURE 1.1. Characterization of Best Replies

The first concerns the generic location of the best reply choices. If the representative DTI does not coincide with the optimal TI, then parents countervail the respective socialization influence on their children by choosing strictly positive *socialization instruments*. This means first that they choose a DTI that deviates from their adopted TI. Notably, this deviation is always into the opposite direction as the deviation of the representative DTI from the optimal TI (if such a choice is available). Second, this behavioral countervailing is coupled with a strictly positive choice of their parental socialization success share (since otherwise, their chosen DTI would be fully ineffective in the child's socialization process).

This generic result means that parents choose strictly positive socialization instruments even for very small deviations of the representative DTI from the optimal TI. That this holds is due to the fact that marginal investments into the socialization instruments are (utility) costless (while as the resulting strictly positive decrease in the distance of the adult child's adopted TI from the optimal TI yields a strictly positive inter–generational utility gain). Obviously, if the representative DTI exactly coincides with the optimal TI, then parents have no incentives to actively employ their socialization technique.

The second dominant characteristic concerns the location of the adult children's adopted TIs that would result out of the parental best reply choices. Despite the parental countervailing in the case of suboptimal socialization influences of the general social environment, the investments into their socialization instruments would never be intense enough such as to guarantee that their adult children's adopted TIs would exactly coincide with the optimal TIs. Hence, there is always a strictly positive deviation of the adopted TI of an adult child from the optimal TI. Thereby, the direction of this deviation always accords with the direction of deviation of the representative DTI from the optimal DTI.

Again, this result holds for even very small deviations of the representative DTI from the optimal DTI. Analogously to before, this stems from the fact that parents do not perceive inter–generational utility losses for an only marginal deviation of the adult child's adopted TI from the optimal TI (while at any strictly positive choice of the socialization instruments, the marginal cost of additional investments to further reduce the distance between the adult child's adopted TI and the optimal TI would be strictly positive). Again obviously, in the case of an optimal representative DTI, the adopted TI of an adult child will also coincide with the optimal TI.

The following list of assumptions will be prerequisite for a further characterization of the parental best reply choices in terms of comparative statics.

Assumption 1.5 (Curvature). *$\forall a \in A$,*

(a) $u(\cdot\,|e)$ and $v\,(\cdot\,|f,g)$ are $\mathcal{C}^2$ and strictly concave, c is $\mathcal{C}^2$ and convex, and
(b) $\operatorname{sign}(f-f')\,\frac{\partial^2 v(f'|f,g)}{\partial f'\,\partial g} > 0$, $\forall\,(f',g) \in \operatorname{con}\phi^d(X) \times \mathbb{R}_{++}$.

Assumption 1.5 (b) means that the marginal cost of a deviation of the adopted TI of the adult child from the optimal TI is strictly increasing in the inter–generational TI. Notably, this is only necessary for the results related to the second column of the comparative statics matrix below to hold.

Proposition 1.2 (Comparative Statics of Best Replies). *Let Assumptions 1.1–1.5 be satisfied. Then, if $\phi^d_{A_a} \neq \hat{\phi}_{\tilde{a}}\,(R_a)$ and the optimization problem of parent $a \in A$ is strictly concave at its best reply choice, and if the two* socialization instruments $\left|\phi^d_a\,(\cdot) - \phi_a\right|$ *and $\hat{\sigma}_a\,(\cdot)$ are 'not too strong*

substitutes', then[3]

$$\begin{pmatrix} \frac{\partial\left|\phi_a^d\left(\phi_{A_a}^d,\hat{\phi}_{\tilde{a}}(R_a),\phi_a,i_a\right)-\phi_a\right|}{\partial\left|\phi_{A_a}^d-\hat{\phi}_{\tilde{a}}(R_a)\right|} & \frac{\partial\left|\phi_a^d\left(\phi_{A_a}^d,\hat{\phi}_{\tilde{a}}(R_a),\phi_a,i_a\right)-\phi_a\right|}{\partial\, i_a} \\ \frac{\partial\,\hat{\sigma}_a\left(\phi_{A_a}^d,\hat{\phi}_{\tilde{a}}(R_a),\phi_a,i_a\right)}{\partial\left|\phi_{A_a}^d-\hat{\phi}_{\tilde{a}}(R_a)\right|} & \frac{\partial\,\hat{\sigma}_a\left(\phi_{A_a}^d,\hat{\phi}_{\tilde{a}}(R_a),\phi_a,i_a\right)}{\partial\, i_a} \end{pmatrix} \gg 0.$$

Proof. In Appendix A 1.2.

The first column of the comparative statics matrix shows that (under the relevant conditions), parents use their investments into their socialization instruments and the representative DTI of the general social environment as *cultural substitutes*. This means that if the representative DTI becomes more favorable (i.e. its distance to the optimal TI becomes smaller), then parents would reduce investments into both socialization instruments.

The second column sheds light on the role that the inter–generational TI plays in determining the parental socialization decisions. Under the conditions of Proposition 1.2, parents with a higher inter–generational TI would choose more intense investments into their socialization instruments for any given strictly positive distance between the representative DTI and the optimal TI. This follows since the socialization TI basically determines the weight that parents put on their inter–generational utility. Thus, given a higher inter–generational TI, parents are willing to engage more 'own' utility losses and socialization success share cost such as to reduce their comparatively larger inter–generational utility losses.

3. Nash Equilibrium

In the previous section, we have characterized the individual best reply choices of a displayed trait intensity and a parental socialization success share. The next step is to discuss the existence of a (pure strategy) Nash equilibrium of the game that is induced by the strategic interdependence of the individual parental choices. To do this, it will be important to clarify the nature of the possible forms of the strategic interdependences.

First of all, as has already been discussed, the net life–time utility of an individual parent, i.e. the object of its optimization problem (1.2), depends on the location of the representative DTI of the general social environment. This is constructed out of the oblique socialization weights and the average DTIs of the adult subsets. Second, the decisions of the other adults could

[3]A technical version of the latter condition can be found in the proof of this proposition. Note that these comparative statics are subject to a fixed location of the parental TI. Furthermore, we assume here that none of the constraints of the decision variables is binding at the best reply choices. This assumption rules out both kinds of 'non–generic' cases in Proposition 1.1 (in case of the second kind, the lower bound for the parental socialization success shares would be binding).

influence the net life–time utility of an individual parent via the perception rule for its optimal TI (as e.g. in the third type of perception rule introduced in chapter 1). In this respect, for the Nash equilibrium existence result below to hold, we will require the following additional normalization: If the perception rule of a parent is based on the DTIs and/or socialization success shares of other adults, then this may only be in terms of the average DTIs or socialization success shares of the adult subsets $\{A_J\}_{J=1}^K$.

Let us now introduce a general representation that accounts for *all* of these possible forms of strategic interdependences. This is based on representing the payoff, i.e. the net expected life–time utility (this context is explicitly addressed below), of all individual parents as being dependent on the tuple of pairs of representative DTIs and average parental socialization success shares, $\left\{\phi^d_{A_J}, \hat{\sigma}_{A_J}\right\}_{J=1}^K$, where $\forall J = 1, \ldots, K$, $\hat{\sigma}_{A_J} := \frac{1}{\lambda(A_J)} \int_{A_J} \hat{\sigma}_{a'} \, d\lambda\,(a')$.

More precisely, the payoff that any parent gains out of its own decision pair and any given profile of pairs of average decisions of the subsets of adults is determined by the parent's adopted TI and inter–generational TI, the perception rule for its optimal TI, as well as the child–specific oblique socialization weights, $\{\sigma_{\tilde{a}J}\}_{J=1}^K =: \sigma_{\tilde{a}}$. We will call these quadruples *parent–child profiles*, $P_a := \left(\phi_a, i_a, \left(R_a, \hat{\phi}_{\tilde{a}}\right), \sigma_{\tilde{a}}\right)$, $\forall a \in A$. Given these, we will denote the payoff function of an individual adult $a \in A$ as $\mathcal{P}\left(\cdot, \cdot \,|P_a\right) : \left(\phi^d(X) \times [0,1]\right)^{K+1} \mapsto \mathbb{R}$, where

$$\mathcal{P}\left(\left(\phi^d_a, \hat{\sigma}_a\right), \left\{\phi^d_{A_J}, \hat{\sigma}_{A_J}\right\}_{J=1}^K |P_a\right) \equiv u\left(\phi^d_a \,|\phi_a\right) + v\left(\phi_{\tilde{a}} \left|\hat{\phi}_{\tilde{a}}\left(R_a\right), i_a\right.\right) - c\left(\hat{\sigma}_a\right)$$

and where $\phi_{\tilde{a}} = \hat{\sigma}_a \phi^d_a + (1 - \hat{\sigma}_a)\phi^d_{A_a}$ and $\phi^d_{A_a} := \sum_{J=1}^K \sigma_{\tilde{a}J} \phi^d_{A_J}$.

We hence obtain a family of games, parametrized by the tuple of parent–child profiles,

$$\left(\Gamma_{P_a}\right)_{a \in A} = \left(A, \left(\phi^d(X) \times [0,1]\right)^A, \left\{\mathcal{P}\left(\cdot, \cdot \,|P_a\right)\right\}_{a \in A}\right).^4$$

The definition below follows Schmeidler [59] and Rath [53].

Definition 1.2 (Nash Equilibrium). *Call a tuple* $\left\{\phi^{d^*}_a, \hat{\sigma}^*_a\right\}_{a \in A}$ *a* Nash equilibrium *of* $\left(\Gamma_{P_a}\right)_{a \in A}$*, if for almost all* $a \in A$*, for all* $\left(\phi^d_a, \hat{\sigma}_a\right) \in \phi^d(X) \times [0,1]$, $\mathcal{P}\left(\left(\phi^{d^*}_a, \hat{\sigma}^*_a\right), \left\{\phi^{d^*}_{A_J}, \hat{\sigma}^*_{A_J}\right\}_{J=1}^K |P_a\right) \geq \mathcal{P}\left(\left(\phi^d_a, \hat{\sigma}_a\right), \left\{\phi^{d^*}_{A_J}, \hat{\sigma}^*_{A_J}\right\}_{J=1}^K |P_a\right)$.

[4]Note here for clarification that the individual strategy sets could equally be defined as $X \times [0,1]$ since $\phi^d_a := \phi^d(x_a)$, $x_a \in X$. But since the parental payoffs, i.e. utilities, depend only on the own and observed (average) DTIs, we directly consider here the strategy sets $\phi^d(X) \times [0,1]$.

Proposition 1.3 (Nash Equilibrium Existence). *If Assumptions 1.1—1.3 hold and if the functions $\hat{\phi}a$ are continuous for all $a \in A$, then a Nash equilibrium exists for any parametrized game.*

Proof. In Appendix A 1.3.

The existence result above means that in any given period, we can use the Nash equilibrium choices for substitution in the formation of TIs equation (1.1). By doing so, we obtain an endogenous representation of the inter–generational formation of TIs, i.e. we have endogenized the cultural formation of preferences process.[5]

4. Evolution and Imperfect Empathy

In a dynamic context, the model framework of the present chapter determines the evolution of all endogenous quantities. These contain the displayed trait intensities, respectively the underlying socio–economic choices, the parental socialization success shares, as well as the the trait intensities and the induced preferences of the society.

Notably, these dynamics will be subject to a specification of the (initial) tuple of adult–child profiles. This means to specify (a) the initial tuple of TIs, which are the state variables of the model, (b) the fixed tuple of inter–generational TIs, (c) the tuple of perception rules for optimal TIs, and (d) the exogenously fixed tuple of child–specific oblique socialization weights.

Lacking a theory of the formation of the perception rules, it is sensible to assume for simplicity that they are (like the inter–generational TIs) invariantly passed over from a parent to its child, hence inter–temporarily fixed. Furthermore, to impose a minimum level of structure on the analysis, it would in any case be sensible to consider only assignments of equal types of perception rules to all parents (e.g. one of the three types of perception rules introduced in chapter 1).

A similar reasoning applies for the case of the child–specific oblique socialization weights. Unless the model is extended such as to allow for their endogenous determination, it is a sensible simplification to fix them inter–temporarily. One approach could be to consider *unbiased* oblique socialization where the socialization weights coincide with the population shares

[5]It shall be noted that the generality of the model allows not only for the existence of multiple Nash Equilibria in any given period, but also for the existence of Nash Equilibria with qualitatively different properties. In deriving qualitative (static or dynamic) properties of (a specification of) the model, it will thus be of central importance to point out whether these properties hold for *all* elements in the set of Nash Equilibria of a period, or eventually only for a sensibly defined subset. The section below shows a global convergence result which is indeed subject to all elements in the (evenutally non–singleton) sets of Nash Equilibria. To the contrary, the second part of this thesis contains an example where we considered only Nash Equilibria with particular properties.

(which are inter–temporarily fixed in the present model) of the subsets.[6] This approach would also have the consequence, that all children of the society are confronted with the same representative DTI of the general social environment. This then even coincides with the *average* DTI of the adults.

Notably, among the four types of (initial) adult–child profile tuples, it is the specification of the tuple of perception rules and the oblique socialization weights that can be supposed to most centrally govern the qualitative properties of the dynamics of any specified model.

Roughly spoken, the reasoning for this is as follows. The optimal TIs determine the direction of the purposeful socialization efforts of the parents; and the oblique socialization weights determine the intensities of 'socialization exchange' between the subsets of adults. Thus, the latter also determine how much the directional socialization efforts of the members of the different subsets impact the socialization decisions of the other parents. As a consequence, these two types of 'socialization effects' also govern the directions of the evolutions of the 'contextual ('own' utility) effects' that are induced by the adopted TIs of the parents. Finally, in any given period, the fixed inter–generational TIs determine the relative strength of the two types of 'socialization effects' versus the 'contextual effects'.

Let us illustrate this 'power' of the tuple of perception rules and oblique socialization weights by means of an example. We will show below the qualitative properties of the evolution of the TIs for the case where all parents have 'imperfect empathy' (respectively the first type of perception rule in chapter 1). This is coupled with the assumption that all oblique socialization weights are identical for all children, which holds e.g. in the case of unbiased oblique socialization. This example might be of special interest, since it accords with standard assumptions in the literature on the economics of cultural transmission of preferences.

Before showing the dynamic properties of this specification, let us first introduce a collection of useful definitions.

Definition 1.3 (TI Assimilation, Symmetric TI Point, Steady State).

(a) Consider any two succeeding periods and let $\phi^m := \max_{a\in A} \phi_a$, $\phi_m := \min_{a\in A} \phi_a$, *and* $\tilde{\phi}^m := \max_{\tilde{a}\in\tilde{A}} \phi_{\tilde{a}}$, $\tilde{\phi}_m := \min_{\tilde{a}\in\tilde{A}} \phi_{\tilde{a}}$. *Then, we speak of* (weak) TI assimilation *if* $\phi_m \leq \tilde{\phi}_m < \tilde{\phi}^m < \phi^m$ *(or) and* $\phi_m < \tilde{\phi}_m < \tilde{\phi}^m \leq \phi^m$.

(b) Call a tuple $\{\phi_a\}_{a\in A}$ *a* symmetric TI point *if for almost all* $a, a' \in A$ $\phi_a = \phi_{a'}$.

(c) Call a tuple $\{\phi_a, \phi_{\tilde{a}}\}_{a\in A}$ *a* steady state *if for almost all* $a \in A$ $\phi_{\tilde{a}} = \phi_a$.

[6] In the cultural transmission of preferences framework, Sáez-Martí and Sjögren [58] consider different forms of biases in the determination of oblique socialization weights.

Finally, let $\left\{\phi_a^0\right\}_{a\in A}$ denote the tuple of initial TIs of the adults.

PROPOSITION 1.4 (Evolution under Imperfect Empathy). *Let Assumptions 1.1—1.4 hold and let $R_a = \{a\}$ and $\hat{\phi}_{\tilde{a}}(\{a\}) = \phi_a$ hold in any period, for every $a \in A$. Consider any $\left\{\phi_a^0, i_a\right\}_{a\in A} \in \left(\text{con } \phi^d(X) \times \mathbb{R}_+\right)^A$.*

(a) Then, if in any period $\{\sigma_{\tilde{a}J}\}_{J=1}^K$ is identical for all $\tilde{a} \in \tilde{A}$, it holds that 1. for every two succeeding periods, the TIs weakly assimilate almost surely, thus 2. the TIs converge to a symmetric TI point, and 3. any symmetric TI point is a steady state.

(b) If additionally, $\sigma_{\tilde{a}J} > 0$, $\forall J = 1, \ldots, K$ in any given period, then it even holds that for every two succeeding periods, the TIs assimilate almost surely (with the rest of the results unchanged).

Proof. In Appendix A 1.4.

There are two driving forces for the global 'melting pot' property of Proposition 1.4 (the result is global also in the sense that it holds for any element in the possibly non–singleton set of Nash Equilibria of a period). The first is that in the case where all children have identical oblique socialization weights, they also face the same representative DTI of the general social environment. This, by itself, induces a tendency toward inter–generational TI homogenization. Even more, since all parents have imperfect empathy, the Nash equilibrium representative DTI can not lie above/below the boundaries that are constituted by the maximum/minimum TI of a given adult generation. This follows since otherwise, by Proposition 1.1 (a), the DTI best replies of all parents would be lower/larger than their adopted TI. This would contradict the representative DTI being supported by Nash equilibrium choices. This property strengthens the tendency toward inter–generational TI homogenization such that even the TIs (weakly) assimilate over generations (by Proposition 1.1 (a)).

Of course, even in the imperfect empathy case, there would be specifications of the tuple of oblique socialization weights where the global 'melting pot' property would not hold generically. To see this easily, consider e.g. the extreme case of two segregated subsets of adults and children (where the 'cross' oblique socialization weights are zero). In this case, the tuple of TIs of the two subsets would generically converge to different steady states.

Finally, it shall be noted that the dynamic properties of the model are particularly easy to characterize under global imperfect empathy. This follows since in this case the adopted TI ('contextual effect') and optimal TI ('socialization effect') coincide. This is not the case for all other possible types of perception rules, which would make the task of characterizing the dynamic properties more complex (in most of the cases).

In any case, it shall have become clear from the above discussion that any significant qualitative characterization of dynamic properties of the model will have to be based on a sensible specification of the tuple of (initial) adult–child profiles.

CHAPTER 4

Applications

In the preceding two sections, we have laid down a general framework to determine the inter–generational formation of continuous cultural traits. Given its generality, this framework can be specified for applications in a large variety of different settings and socio–economic questions. In what follows, we will briefly outline four different dimensions along the lines of which any application, respectively specification, of the model could be oriented.

Level of the Analysis Any analysis of the properties of a specified model can be pursued on two different levels. The first, 'meta–level analysis', takes place at the level of the intensities of the trait under scrutiny, and concerns the evolution of the TIs and DTIs, as discussed already above. Interesting issues in this context would then typically be to characterize the dynamics of the model under different specifications of the tuple of (initial) adult–child profiles. Specifically, it would be of interest to identify specifications of tuples of perception rules and oblique socialization weights under which (stable) heterogeneous and/or homogeneous steady state distributions of the TIs exist. One specification, based on 'imperfect empathy', for global convergence to a homogeneous (symmetric) steady state distribution has already been shown in section 4 of chapter 3 above.

The second, 'empirical analysis', would take place at the level of the observable socio–economic choices of the adults. For this end, it would be necessary to clarify (a) which socio–economic choices are supposed to serve as the role models for the social learning of the intensities of the trait under scrutiny, and (b) how the relationship between the socio–economic choices and the DTIs can be represented in terms of the DTI function. Given this, the 'meta–level analysis' would additionally answer the question of the evolution of the underlying socio–economic choices.

Complexity of the Adult Problem The *purposeful socialization* framework of chapter 3 embeds parents with inter–generational concern in a strategic socialization interaction environment, in which they choose optimal DTIs and socialization success shares. This structure entails a certain degree of complexity. This could, however, be decreased by employing alternative (less

'rich') designs of the parental optimization problems. These would either feature a lower dimensionality and/or would eliminate the strategic socialization interaction. Notably, it depends on the specific application, which of these alternatives (as introduced below) would eventually be suitable.

One alternative that reduces the dimensionality of the parental optimization problem would be to assign (strictly positive) exogenous socialization success shares, but to leave endogenous the choices of DTIs. Even, by setting the socialization success shares equal to one so that the children are exclusively socialized by their parents, one could additionally eliminate the strategic socialization interaction in the choices of DTIs. Still, one could then introduce other forms of strategic interaction into the model (as e.g. being induced by endowing the parents also with a utility component derived from inter–adult social interactions).

Another alternative would obviously be to exogenously fix the chosen DTIs of the parents while as the decision of their socialization success shares is left endogenous (as in Bisin and Topa [5] and Panebianco [47]). This approach would also additionally eliminate the strategic socialization interaction.

The double effect of reducing the dimensionality of the parents' decision problems as well as doing away with the strategic socialization interaction could furthermore be achieved by considering a *naive socialization framework*. This means that the adults (parents) fully neglect the children's preference formation process or are not aware of it — while this process is still taking place. In such a setting, one would again have to assign (exogenous) parental socialization success shares.[1] Notably, in the competitive socio–economy version of such a model, all adults would always choose to behave exactly in accordance with their adopted TI. This follows since the parents would lack the behavior shifting incentives that would be created by the presence of a (non–constantly zero) inter–generational utility component. Thus, one would typically aim at giving additional substance to such a framework, e.g. by introducing alternative forms of strategic interaction, or by considering a social planner problem (as discussed below).

Finally, one could eliminate the strategic interaction in the decision problems by basing these on the parents' *expectations* of the representative DTI of the general social environment. These expectations would sensibly be based on the representative DTI that the adults have observed in their own child period. The drawback of this approach would be that one could not allow

[1]In the simplest possible way, one could even assign to the parental socialization success shares the value zero so that effectively, there is oblique socialization only.

for the alteration of the parents' decisions upon observations of representative DTIs that do deviate from the expectations. Thus, on the transitory path, parents would generically not choose best reply choices against the true realized representative DTI of the general social environment.

Social Planner Problem The cultural formation of preferences frameworks opens routes toward new kinds of social planner problems. These routes basically follow the closed circle between the adopted TIs of the adults, their chosen DTIs (and underlying socio–economic action patterns) and the induced adopted TIs and preferences of the next adult generation.

In a first step, let us clarify possible ways how a social planner could intervene in the cultural formation of preferences process. The first way would be targeted directly at the 'meta–level' of the TIs, and would primarily concern the social planner serving for an additional source of child socialization. This could e.g. be in the form of the influence that the designs of the legal system and the institutions (including schools and media) of a society have in the socialization process of a child; see Bowles [11] for an overview of related issues. Within the terminology of the present paper, the social planner could thus effectively set a DTI coupled with (investments into) its socialization success *relative* to the socialization successes of the family and the general social environment.

The second possible way of social planner intervention is only indirectly targeted at the level of the TIs. This would concern 'standard' socio–economic incentive shifting policies, like e.g. a consumption tax or pension schemes in the context of the first and third example in chapter 2. Since these measures are designed such to influence the adults' socio–economic decisions, the same is being achieved in terms of the corresponding adults' choices of DTIs. This then in turn influences the formation of the TIs of the children.

Let us now discuss the possible motivations of a social planner to actively employ its 'socialization technique'. The first motivation can result out of a benevolent social planner's aim of maximizing the weighted sum of the life–time utilities of a sequence of generations. Notably, since the social planner would be assumed to be aware of the inter–temporal externalities that are inherent in the cultural formation of preferences process, she has, via her two ways of intervention, access to a new level of efficiency: She can inter–connect the question of the optimal inter–generational distribution of utilities with the question of the optimal inter–generational distribution of utility functions (since they are determined by the cultural formation of preferences process).

The second motivation can be in terms of the social planner perceiving, respectively having information about, a socially optimal (distribution of) the TIs and/or DTIs within the society, which it aims at instilling in a paternalistic way; see e.g. Qizilbash [52] for a discussion of related issues. The typical question would then be whether the social planner can design a transitory policy regime such as to achieve this form of social optimum in the steady state.

Structure of the (initial) Adult–Child Profiles In chapter 4, we have already shortly discussed basic issues concerning potential ways of specifying the tuple of (initial) adult–child profiles. Additionally to what has already been said there, it could be of interest to characterize the properties of a specified model for different degrees of symmetry embodied in the distribution of these profiles on the adult set. Obviously, the maximum symmetry would be achieved in the case of a representative agent model, while as the minimum symmetry would correspond to assigning any arbitrary distribution.

As an intermediate step, one could partition the adult set into (possibly a continuum of) subsets of adults that have identical (initial) adult–child profiles. Thus, one would obtain a set of adult types, which could be interpreted as *cultural groups*. Under suitable conditions that guarantee the inter–temporal TI symmetry of the members of the groups, one could then answer the question of behavioral (DTI) and cultural (TI) assimilation of the groups. Within the present continuous cultural traits framework, if the set of adult types is discrete, this would constitute the analogue to the analysis on the dynamics of the population distribution of discrete traits in the economics of cultural transmission of preferences literature.

CHAPTER 5

Conclusions

This paper has introduced a general representation of the formation of continuous cultural traits. We showed in the first main part of this paper (chapter 2) how children adopt trait intensities through social learning from observed socio–economic action patterns of the adults. Upon such an observation, children receive a cognitive impulse, which we called a *displayed trait intensity*. The trait intensity that a child adopts in the socialization process (and keeps in its adult period) is then represented as a weighted average between all such cognitive impulses obtained. We then showed how to interpret the trait intensities that adults have adopted such as to construct and characterize preferences over displayed trait intensities, thus also the underlying socio–economic action patterns. The representation of the socialization process that this paper proposes thus constitutes a consistent and closed circle between the socio–economic action patterns taken by one adult generation and the preferences over these patterns by the succeeding adult generation.

In the second main part of the paper (chapter 3), we proposed one possible way to endogenize the cultural formation of preference process as resulting out of purposeful parental socialization decisions. These are twofold. One is the choice of a displayed trait intensity. The second consists of investments into the weight that this role model has in the socialization process of the child, relative to the weight that the observed representative displayed trait intensity of the general social environment has. Thus, basically, the parents decision problem is to choose best replies against this representative role model of the general social environment. Notably, this is subject to the location of the optimal trait intensity that they would like their children to adopt. We showed conditions under which a pure strategy Nash equilibrium of the induced 'strategic socialization interaction game' of the parents exists. These equilibrium choices govern the inter–generational *evolution* of the trait intensities and the preferences of the society.

The strength of the framework presented in the present paper arguably lies in its generality. This allows for a large number of possible forms of adoptions and specifications such as to apply it to an accordingly large

variety of different socio–economic questions. In chapter 4, we also outlined lines along which any such application could be oriented.

Despite the generality of the model, there is however still considerable room for further generalizations. Among other possible directions, this would concern (a) considering an n–dimensional representation of the formation of continuous cultural traits with an optional endogeneization of the formation of the inter–generational trait intensities, (b) endogenously determining the formation of the perception rules of parents, (c) endogenizing the determination of the oblique socialization weights (in the form of parental decision problems), (d) consistently introducing 'horizontal socialization' and the socialization influence of institutions (like the legal system, schools, media, etc.), (e) changing the population structure of the model by dropping the assumption of asexual reproduction and potentially endogenizing the reproduction decision, and/or considering a finite population setting, (f) allowing for a pro–active role of the children in the formation process of their preferences, and (g) considering a representation of displayed trait intensities subject to heterogeneous choice sets of socio–economic action patterns.

Finally, remember that the subject of the present paper was the formation of continuous cultural traits in the *socialization period* of a person. However, socialization is without doubt a life–long process. It would therefore be of central interest to extend and suitably adopt the logic of the processes described to the formation/adoption of continuous cultural traits in the adult life period of individuals.[1]

[1]Existing related analyses contain, among others, Friedkin and Johnson [25], DeMarzo et al. [19], Brueckner and Smirnov [13, 14] and Golub and Jackson [28, 29]. These contributions are embedded in a social network structure.

Part 2

Cultural Formation of Preferences and Assimilation of Cultural Groups

CHAPTER 1

Introduction

When different cultural groups live together, then there is always cultural exchange through the social(ization) interactions between the members of the groups. While this can well concern the mutual dissemination of the customs of the groups, it notably consists to a large extent of a mutual (inter–generational) influencing of the preferences, values, norms, attitudes and beliefs of the groups' members.

This context raises interest both on empirical and theoretical grounds. In the empirical context, the question of assimilation and integration of immigrants with different cultural backgrounds into hosting societies has attained increasing attention in recent years, both in media and on the political agenda. This calls for a framework that allows for a theoretical representation and analysis, optimally leading into a leveraged understanding of the empirical processes at work.

The present paper presents such a theoretical framework, based on a recent theory of Pichler [49] on the inter–generational formation of continuous cultural traits.[1] We will show a static and dynamic analysis of the evolution of behavior and the trait intensities in a two cultural groups setting, subject to one type of continuous cultural traits. Thereby, one of the focus points will be to derive conclusions about the underlying assimilation process between the two cultural groups, both in terms of their adopted trait intensities, as well as in terms of their behavioral decisions.

Contributions and Results The first part of this paper is devoted to a recapitulation of the cultural formation of preferences framework of Pichler [49]. In doing so, we will show in a first step how children come to adopt intensities of any arbitrary continuous cultural trait type. We let this be based on the children's social learning from the observed socio–economic action patterns of the adults. Upon observation of the socio–economic action pattern of an adult, children also receive a *cognitive impulse*. The latter can be understood as the signal on the valuation (or importance, magnitude,

[1]The latter are meant to contain all types of traits that (a) are subject to formation in the socialization process, and (b) can reflect different intensities, located in a convex subset of the real line.

etc.) of the continuous cultural trait that is embodied in the choice of the particular socio–economic action pattern over the other available choices. We even endow these sorts of cognitive impulses with a cardinal meaning and call them *displayed trait intensities*. The final adopted trait intensity of a child then results as a weighted average between the displayed trait intensity that is *chosen* by its family, and the representative displayed trait intensity that the child observes in its general adult social environment.

In a second step, we introduce one possible way to endogenize the cultural formation of trait intensities process as resulting out of purposeful parental socialization decisions. These are twofold. The first is the choice of a displayed trait intensity. The second consists of investments into the weight that this displayed trait intensity has in the socialization process of the child relative to the weight that the observed representative displayed trait intensity of the general social environment has. We will call this weight the *parental socialization success share*. Thus, basically, the parental decision problem is to choose best replies against the representative displayed trait intensity of the general social environment. Notably, this is subject to the perception that the parents have of the optimal trait intensity for their children to adopt (and different perceptions can have a remarkable impact on the qualitative static and dynamic properties, as will be discussed below).

In the second and main part of this paper, we then embed the endogenous cultural formation of trait intensities process in a society that is populated by two distinct cultural groups. With these, we basically refer to a collection of families, for which it holds that the parental (adult) members have identical adopted trait intensities and form identical perceptions of the optimal trait intensities for their children. We introduce conditions under which all parents choose the same behavior and socialization success share in a Nash Equilibrium. Under such group–symmetric choices, all children of the same cultural group do adopt the same trait intensities.

The central task pursued in this paper is the analysis of the group–symmetric Nash equilibrium choices and the resulting dynamic evolution of the adopted trait intensities under two different benchmark perception rules for the optimal trait intensities. In the main part of the paper, we consider first exogenously fixed (and distinct) optimal trait intensities, and second the case where the parents of a group perceive the average displayed trait intensity of their own group members as the reference value ('endogenous norms'). Finally, in Appendix B 2, we also discuss the case where all parents have 'imperfect empathy'.[2]

[2]The concept of 'imperfect empathy' has been introduced in the economics literature by Bisin and Verdier [7]. It basically means in the present context that parents perceive

Under any possible perception rule for optimal trait intensities, the direction of the socialization efforts of the parents of both groups is always toward the perceived optimum. In the case of exogenously *fixed optimal trait intensities*, this leads to an inter–generational coordination toward a situation where the positions of the adopted trait intensities can be considered 'consistent' with the relative location of the fixed optimal trait intensities (if this situation has not been given initially). With this we mean that (a) the group with the strictly larger fixed optimal trait intensity does also have a strictly larger adopted trait intensity, and (b) the trait intensities of both groups do lie strictly between the two optima.

Within this 'generic state space', the socialization efforts of the members of the two cultural groups are in the opposite directions. This yields the result that the parents of both cultural groups dis–integrate behaviorally (i.e. the parents with the strictly larger/lower adopted trait intensity choose to display a strictly larger/lower than adopted trait intensity) and choose strictly positive parental socialization success shares. This has the consequence that the relative positions of the two cultural groups are inter–generationally preserved and the 'generic state space' can not be left.

Since we were not able to obtain analytic results on the dynamic properties of this model specification, we resorted to numerical methods. The central outcome was that for any considered pair of initial trait intensities (in the generic state space) we obtained convergence of the trait intensity paths, subject to any combination of the strenghts of the groups' norms on behavior that we considered. Furthermore, if the norms were high enough for both groups, then we obtained a unique globally asymptotically stable steady state. However, if the norms were comparatively weak, then this gave rise to the existence of multiple steady states, which typically featured only very low distances between the two steady state trait intensities.

The qualitative (numerical) results of the fixed optimal trait intensity case do thus feature the opposite extreme to the 'imperfect empathy' case: While as in the latter case, the preferences of the cultural groups do always converge to the same point, this will never happen under fixed optimal trait intensities.

Compared to these sorts of uniqueness of the qualitative asymptotic properties, the case of *endogenous norms* features a larger variety of possible convergence path types. First of all, we could show that generically, any sequence of adopted trait intensities of the two groups converges to a steady

---

their own adopted trait intensity as optimal for their children. As has already been shown by Pichler [49], this case features a global 'melting pot' property, i.e. the adopted trait intensities of (almost) all dynasties converge to the same point.

state. Even, there is a basin in terms of a maximum distance of the adopted trait intensities, such that all pairs of adopted trait intensities that enter (or start in) this basin converge to a point where all adults have the same trait intensities. However, for a large enough initial trait intensity distance, it is possible that the cultural groups dissimilate on the transitory path and a steady state with a larger than initial trait intensity distance is reached.

Related Literature The present analysis stands in a close relation to few existing contributions on the question of the cultural formation of continuous cultural traits. Important early treatments of the topic are Cavalli-Sforza and Feldman [16] in a theoretical, and Otto et al. [46] in an empirical context. More recently Bisin and Topa [5] proposed a representation of the formation of the intensities of continuous cultural traits. In the terminology of the present paper, they represented the adopted intensity of the cultural trait as a weighted average between the displayed trait intensity of the family and the (weighted) average of the intensities of the cultural traits that the society has adopted.

The major limitation of this contribution is, however, that it features a degenerate representation of the parental choices of socio–economic action patterns, and the associated displayed trait intensities. In this respect, Bisin and Topa [5] assume that parents always choose a socio–economic action pattern that displays their 'target intensity' (i.e. the optimal trait intensity in the terminology of the present paper). Given this restricted view on the family's behavioral choices, its socialization decision is then reduced to the choice of its weight in the formation of the trait intensity of their child.[3]

A second, and well established, related strand is the literature on the economics of cultural transmission. It has been introduced by Bisin and Verdier [7, 8, 9] and Bisin et al. [6], and is based on the work of Cavalli-Sforza and Feldman [15, 16] and Boyd and Richerson [12] in evolutionary anthropology. The focus is on the analysis of the population dynamics of the distribution of a discrete set of cultural traits under an endogenous intergenerational cultural transmission mechanism.

The endogeneity stems from the purposeful parental choice of socialization intensity, which effectively determines the probability that the child will directly adopt the trait(s) of the parents. Parents engage into the cost of purposeful socialization in order to avoid (decrease the probability) that their child will not adopt their trait(s) — in which case parents encounter subjective utility losses. For an exhaustive overview of foundations of and

[3]The same sort of critique applies to the approach of Panebianco [47], who considers the formation of inter–ethnic attitudes.

contributions to this literature, see Bisin and Verdier [10].[4]

Outline The further setup of this paper is as follows. The succeeding chapter 2 recapitulates the general framework on the (endogenous) cultural formation of continuous cultural traits of Pichler [49]. This is followed by the analysis of static and dynamic properties of the model in a two cultural groups setting in chapter 3. We consider both fixed optimal trait intensities in section 1, as well as endogenous norms in section 2. The proofs of the propositions of the latter two subsections can be found in Appendix B 1. Finally, Appendix B 2 contains a short treatment of the case where all parents have 'imperfect empathy', and chapter 4 concludes.

[4]Related to this strand of literature are the contributions of Cox and Stark [17] and Stark [60] on the 'demonstration (or preference shaping) effect' of parental altruism choices in front of their children.

CHAPTER 2

Cultural Formation of Preferences

This chapter discusses a general model of the formation of continuous cultural traits through the socialization process (in section 1). In section 2, we will also show how this cultural formation of preferences process can be derived out of optimal parental socialization decisions. Notably, the framework developed here constitutes a shortened representation of the one introduced in Pichler [49]. For the details, please confer the original source directly. The reader who is familiar with the latter can read the present chapter as a refresher, but can well directly proceed to chapter 3.

1. Cultural Formation of Preferences

Consider an overlapping generations society. In the present and next section, we will restrict our glance on the cultural formation of preferences process between two succeeding generations. This makes it possible to drop all time indexes (for ease of exposition).

In any given period, let our society be populated by a continuum of adults, $a \in A = [0, 1]$ endowed with Lebesgue measure λ, and their children. For simplicity, we will assume that reproduction is asexual and every adult has one offspring, so that we can denote with $\tilde{a} \in \tilde{A}$ the children of the parents $a \in A$ (and the population size is constant).

Let us assume that all adults have available the same non–empty set of socio–economic action patterns, X. This set is endowed with a complete and transitive binary relation T. Thereby, for all $x, x' \in X$, $x\,T\,x'$ means that the socio–economic action pattern x is (weakly) 'more characteristic' for the continuous trait type under scrutiny than socio–economic action pattern x'. This general formulation is owed to the fact that we consider any type of continuous cultural trait. Given transitivity and completeness, we can represent the ordinal relation T by a cardinal function

$$\phi^d : X \mapsto \mathbb{R}\,.$$

Thus, to any socio–economic action pattern $x \in X$, ϕ^d assigns a number with cardinal meaning, $\phi^d(x)$. We will call this the *displayed trait intensity* (DTI) embodied in the choice of socio–economic action pattern x. Thus, $\phi^d(X)$ is the set of possible DTIs.

Now, the role models of the children's social learning of trait intensities are the observable socio–economic action patterns $x \in X$ taken by the adults $a \in A$; and we assume that the *cognitive impulse* that any of the children obtains through such an observation is the corresponding DTI, $\phi^d(x)$. To simplify the subsequent exposition, we will denote the DTI of the socio–economic action pattern of adult $a \in A$, $x_a \in X$, as $\phi^d_a := \phi^d(x_a)$.

We will now introduce the representation of the socialization process that this paper proposes. This will be established on grounds of the *tabula rasa* assumption, which means in the present context that children are born with unformed trait intensity (TI), and equally, with unformed preferences. On this basis, we then let the formation of the TI that a child adopts result out of social learning from the socio–economic action patterns of adults (only) that it is confronted with. Specifically, this is being embedded in a framework of socialization inside the family and by the general adult social environment, or 'direct vertical and oblique socialization'.

In this context, we will let the TI that a child $\tilde{a} \in \tilde{A}$ adopts be formed according to a weighted average between the representative DTIs of both socialization sources (i.e. as a weighted average of all cognitive impulses obtained in the socialization process). In the case of the child's family, this coincides with the DTI of its single parent $a \in A$, $\phi^d_a \in \phi^d(X)$. The representative DTI of the child's general social environment, $A_a := A \backslash \{a\}$, will be denoted $\phi^d_{A_a}$. These result out of the children's social learning from the observed DTIs of (eventually) different subsets of adults that they are confronted with.

More precisely, we assume that there is a measurable partition of the adult set, $\{A_J\}_{J=1}^K$, and that the children obtain as cognitive impulses the *average* DTIs of these subsets, $\phi^d_{A_J} := \frac{1}{\lambda(A_J)} \int_{A_J} \phi^d_{a'} \, d\lambda(a') \in \text{con } \phi^d(X)$, $\forall J = 1, \ldots, K$. Specifically, for every child $\tilde{a} \in \tilde{A}$ there are *oblique socialization weights*, $\sigma_{\tilde{a}J}$, $J = 1, \ldots, K$, that represent the relative cognitive impacts of the child's social learning from the various subsets of adults. These weights satisfy $\sigma_{\tilde{a}J} \in [0, 1]$ and $\sum_{J=1}^K \sigma_{\tilde{a}J} = 1$, $\forall \tilde{a} \in \tilde{A}$, $\forall J = 1, \ldots, K$. We obtain, $\forall \tilde{a} \in \tilde{A}$,

$$\phi^d_{A_a} := \sum_{J=1}^K \sigma_{\tilde{a}J} \phi^d_{A_J} \in \text{con } \phi^d(X).$$

The weight that the DTI of the parent of a child $\tilde{a} \in \tilde{A}$ has in the socialization process of the child will be called the *parental socialization success share*, $\hat{\sigma}_a \in [0, 1]$. This corresponds to the cognitive impact of the parental DTI relative to the cognitive impact of the representative DTI of the child's general social environment. Factors that would determine this relative cognitive impact would include the social(ization) interaction time

of the parent with its child, as well as the effort and devotion that the parent spends to socialize its child to the chosen DTI. We thus assume that the parental socialization success share can be chosen by the parents.

We now obtain the formation of the TI that a child $\tilde{a} \in \tilde{A}$ adopts through the 'direct vertical and oblique socialization' process, $\phi_{\tilde{a}}$, as

$$\phi_{\tilde{a}} = \hat{\sigma}_a \phi_a^d + (1 - \hat{\sigma}_a)\phi_{A_a}^d. \tag{2.1}$$

We will call this the *parental socialization technique*. It embodies the view that the parents set a TI benchmark, $\phi_a^d \in \phi^d(X)$, and can invest into their parental socialization success share, $\hat{\sigma}_a \in [0, 1]$, to countervail the socialization influence that the child is exposed to in its general social environment, $\phi_{A_a}^d$. Thus, for any $\phi_{A_a}^d \in \text{con } \phi^d(X)$, the parents could fully determine the adopted TIs of their children. Hence the set of possible TIs that a child can adopt always coincides with the convex hull of the set of possible DTIs, $\text{con } \phi^d(X) \subseteq \mathbb{R}$.

We assume next that, in their adult life period, all individuals keep the TI that they have adopted in their childhood in an unchanged way. These adopted TIs of the adults can be interpreted to induce 'filters' under which adults can compare and rank different choices of socio–economic action patterns. This form of evaluation takes place in terms of comparing the DTIs of the socio–economic action patterns to the own adopted TIs.[1] Specifically, we assume that the adopted TIs induce complete and transitive preference relations over choices of DTIs (respectively the underlying socio–economic action patterns).

Assumption 2.1 ('Own' Utility). *For every $a \in A$,*

(a) the adopted trait intensity induces an 'own' utility function $u(\cdot|\phi_a) : \text{con } \phi^d(X) \mapsto \mathbb{R}$, $u\left(\phi_a^d|\phi_a\right) \in \mathbb{R}$, *where*

(b) $u(\cdot|\phi_a)$ is single–peaked with peak ϕ_a, thus strictly increasing/decreasing at all $\phi_a^d \in \text{con } \phi^d(X)$ such that $\phi_a^d < / > \phi_a$.

Intuitively, the single–peakedness property means that we assume adults to prefer choosing behaviors (DTIs) that are as close as possible in line with their adopted TIs.

2. Endogenous Cultural Formation of Preferences

In the present chapter, we will lay down one specific way of achieving an endogeneization of the cultural formation of preferences process. This will be based on purposeful socialization decisions of parents. Thereby, we notably restrict the latter to consist of their choice of a displayed trait intensity

[1] This is in line with the cognitive dissonance theory of Festinger [23].

(as determined through the choice of the underlying socio–economic action patterns) and of their parental socialization success share. This means that we leave the oblique socialization weights (that determine the children's relative social learning from the different adult subsets) exogenously fixed.

Motivation for Purposeful Socialization. In a first step, we have to clarify what motivation parents have to actively engage in their children's socialization process, i.e. what induces them to purposefully employ their socialization technique (the functioning of which we assume them to be fully aware of). Basically, we let this motivation stem from the fact that parents also obtain an inter–generational utility component. Thereby, this is either related to the adopted TI of their adult children and/or to the DTI (respectively the underlying socio–economic action patterns) that they expect their adult children to take.

As far as the latter expectations are concerned, we make here an assumption on a specific form of parental myopia: Although parents obtain an inter–generational utility component, which eventually induces them to choose a DTI that does not coincide with their adopted TI (see below), we assume that they do not realize that this form of behavior changing impact will also be present in their adult children's decision problems. Thus, any parent $a \in A$ expects its adult child to choose a DTI that is in the set of maximizers of its 'own' utility function, $\arg\max_{\phi^d_{\tilde{a}} \in \phi^d(X)} u\left(\phi^d_{\tilde{a}} \,|\phi_{\tilde{a}}\right)$. Under the following assumption, $\phi^d(X)$ is convex (and compact, which will be needed in the propositions below), and thus $\phi^d(X) = \text{con}\ \phi^d(X)$. This then guarantees by the single–peakedness of the utility functions that $\arg\max_{\phi^d_{\tilde{a}} \in \phi^d(X)} u\left(\phi^d_{\tilde{a}} \,|\phi_{\tilde{a}}\right) = \phi_{\tilde{a}}$, $\forall a \in A$. Hence, the parental expectations of their adult children's DTIs are uniquely determined.

Assumption 2.2 (Convexity and Compactness). *X is a convex and compact subset of a finite dimensional Euclidean space, and ϕ^d is continuous. It follows that $\phi^d(X)$ is non–empty, convex and compact.*

Given the parents' myopic expectations, it is independent of whether the inter–generational utility component of a parent is related to the adopted TI or expected DTI of its adult child, since they coincide. Under this property, we will now assume that any parent perceives an *optimal trait intensity* that it wants its child to adopt (i.e. if the child would adopt this optimal TI, then this would be strictly preferred by a parent over all other possible TIs that the child could adopt). These parent–specific optimal TIs are subject to what we call *perception rules*.

Thereby, the perception rule of the optimal TI of any parent is determined by two 'ingredients'. The first one specifies a (set of) subset(s) of

adults, which can be understood as reference group(s). The second ingredient then specifies the construction of the optimal TI that a parent perceives out of characteristics of the adults in these reference group(s) that are either observable (notably the DTIs of adults) or known to an individual parent.

To formally introduce the concept of perception rules, it will be convenient to define $\mathcal{A}$ as a σ–algebra generated by the finite partition $\{A_J\}_{J=1}^{K}$.

Definition 2.1 (Perception Rule). *For every parent $a \in A$, the* perception rule *for the optimal trait intensity is a pair $\left(R_a, \hat{\phi}_{\tilde{a}}\right)$, where $\emptyset \neq R_a \in \{a\} \cup \mathcal{A}$ and where $\hat{\phi}_{\tilde{a}} : \{a\} \cup \mathcal{A} \mapsto \text{con } \phi^d(X)$, $\hat{\phi}_{\tilde{a}}(R_a) \in \text{con } \phi^d(X)$.*

To ease the interpretation of this conceptualization, we will list here three sensible types of perception rules for optimal TIs. In chapter 3, we will, in a two cultural groups setting, be concerned with analyzing evolutionary processes subject to the second and third type of perception rules mentioned here.[2] Note also that the list below is not meant to be exhaustive (one could e.g. consider combinations of the three types mentioned).

PR 1 The optimal TI of a parent $a \in A$ is identical to its adopted TI, $R_a = \{a\}$ and $\hat{\phi}_{\tilde{a}}(\{a\}) = \phi_a \in \text{con } \phi^d(X)$.

PR 2 The optimal TI of a parent $a \in A$ is identical to a parent–specific (model–exogenous) TI, $R_a = \{a\}$ and $\hat{\phi}_{\tilde{a}}(\{a\}) = e_a \in \text{con } \phi^d(X)$.

PR 3 The optimal TI of a parent $a \in A$ is identical to the average DTI of one of the adult subsets, $R_a = A_M$, $M \in \{1, \ldots, K\}$, and $\hat{\phi}_{\tilde{a}}(A_M) = \phi^d_{A_M} \in \text{con } \phi^d(X)$.

Given the perception rule rules and the resulting optimal TIs, we assume further that parents perceive utility losses for deviations of the adopted TI of their children from these optimal TIs. Specifically, for any parent $a \in A$, we introduce the parameter $i_a \in \mathbb{R}_+$ that shall capture the strength of the perceived inter–generational utility losses. We will call this the parent's *inter–generational trait intensity*. For simplicity, we assume that these are invariably passed over from an adult to its child, $i_{\tilde{a}} = i_a$, $\forall a \in A$.

Assumption 2.3 (Inter–generational Utility). $\forall a \in A$,

(a) the perception rule and inter–generational trait intensity induce an inter–generational utility function $v\left(\cdot \middle| \hat{\phi}_{\tilde{a}}(R_a), i_a\right) : \text{con } \phi^d(X) \mapsto \mathbb{R}$, $v\left(\phi_{\tilde{a}} \middle| \hat{\phi}_{\tilde{a}}(R_a), i_a\right) \in \mathbb{R}$, where

(b) $\forall i_a \in \mathbb{R}_{++}$, $v\left(\cdot \middle| \hat{\phi}_{\tilde{a}}(R_a), i_a\right)$ is single–peaked with peak $\hat{\phi}_{\tilde{a}}(R_a)$, thus strictly increasing/decreasing at all $\phi_{\tilde{a}} \in \text{con } \phi^d(X)$ such that $\phi_{\tilde{a}} < / > \hat{\phi}_{\tilde{a}}$.

[2]The first, 'imperfect empathy', type has already been discussed in Pichler [49]. In Appendix B 2, the respective results for the two cultural groups setting are shortly discussed.

Best Reply Problems. In the last step toward the construction of the parental best reply problems, let us finally discuss the cost associated with investments into controlling the parental socialization success share. These would concern e.g. the opportunity cost of the time parents spend for the active socialization of a child, as well as the (psychological) cost of the effort and devotion invested. We will represent these cost by an indirect cost function of choices of socialization success shares. This function is assumed to be identical for all adults $a \in A$ and will be denoted $c : [0,1] \mapsto \mathbb{R}_+$, $c(\hat{\sigma}_a) \in \mathbb{R}_+$.

For every $a \in A$, the parental best reply problem (against the representative DTI, and subject to the adopted TI, the perceived optimal TI and the inter–generational TI) of a choice of its DTI and its socialization success share is then represented by

$$\max_{\left(\phi_a^d, \hat{\sigma}_a\right) \in \phi^d(X) \times [0,1]} u\left(\phi_a^d \,|\phi_a\right) + v\left(\phi_{\tilde{a}} \left|\hat{\phi}_{\tilde{a}}\left(R_a\right), i_a\right.\right) - c\left(\hat{\sigma}_a\right) \tag{2.2}$$

$$\text{s.t. } \phi_{\tilde{a}} = \hat{\sigma}_a \phi_a^d + (1 - \hat{\sigma}_a)\phi_{A_a}^d.$$

The best reply problems of the parents hence basically consist of trading off the cost and benefits of their socialization choices. The cost (and disutilities) are constituted by 'own' utility losses that parents experience when choosing a DTI that deviates from their adopted TI, together with the cost of a choice of their socialization success share. The benefits accrue in form of resulting inter–generational utility gains through reductions in the distance between the child's adopted TI and the optimal TI. For a detailed discussion of the properties of the best reply solutions, confer Pichler [49].

CHAPTER 3

Assimilation of Cultural Groups

In this chapter, we will embed the endogenous cultural formation of preferences framework in an environment where the society is populated by two distinct cultural groups. The focus of the subsequent subsections will be on the analysis of the evolution of the adopted trait intensities and induced preferences subject to the Nash equilibrium socialization decisions of the parents of both cultural groups. This will be done by imposing two distinct types of perception rules. In section 1 we will consider the second type of perception rule discussed above, while as section 2 is based on the third type. Finally, the results for the first, 'imperfect empathy', type of perception rule in the present setting are shortly discussed in Appendix B 2.

Consider the case where the adult set is partitioned into two groups, $A = H \cup L$. Let us index the groups $G \in \{L, H\}$, and denote their population shares $q_G := \lambda(G)$. In the present setting, it will be convenient to index the members of the groups as $g \in G$, and to denote $-G := A \backslash \{G\}$. We will below introduce normalizations that will guarantee that the *adult–child profiles* (i.e. all model–relevant variables and parameters) of all members of a group are identical in any period. This will allow us to speak of $\{L, H\}$ as the *cultural groups* of the society.

First, we assume that in any period $t \in \mathbb{N}$, all adult members of a group have identical inter–generational TIs and identical perception rules for optimal TIs. Assume that both are fixed inter–generationally and denote them i_G, respectively $\left(R_G, \hat{\phi}_G\right)$. Second, we assume *unbiased* oblique socialization (with the adult subsets from which the children socially learn from coinciding with the cultural groups), so that $\forall t \in \mathbb{N}$, $\forall a \in A$, $\phi^d_{A_a}(t) = \phi^d_A(t) := \int_{a' \in A} \phi^d_{a'}(t)\ d\lambda(a') = \phi^d_L(t)(1 - q_H) + \phi^d_H(t) q_H$ (remember that $\phi^d_G(t) := \frac{1}{q_G} \int_{g \in G} \phi^d_g(t)\ d\lambda(g)$, $G = L, H$), i.e. the society's *average DTI*. We finally need to establish that also the adopted TIs of all members of a group are identical in any period.

Assumption 2.4 (Compactness, Convexity, Concavity).

(a) X is compact and convex and ϕ^d is continuous. If $n > 1$ then ϕ^d is additionally concave. Thus, $\phi^d(X)$ is compact and convex.

(b) *The target functions of the best reply problems (2.2) are continuous and strictly concave.*[1]

Subsequently, we will call a *symmetric Nash equilibrium (SNE)* a Nash equilibrium where all parents of the same cultural group choose identical strategies.

PROPOSITION 2.1 (Symmetric Nash Equilibrium Path). *Let Assumption 2.4 hold and let the adopted TIs be identical within groups in the initial period. Then, a path of symmetric Nash equilibria exists.*

Proof. In Appendix B 1.1.

The logic for the existence of a SNE path is straightforward. In the initial period, since all parents of the same cultural group do have the same adopted TI (and by the other symmetry assumptions), they do also have identical and unique best reply pairs to a given average DTI. It is then straightforward to see that the necessary conditions to apply Brouwer's Fixed Point Theorem hold, such that the existence of a SNE in the initial period is guaranteed. Under SNE choices in the initial period, it further follows that all children of the same cultural group adopt the same TI. Thus, in the second period, all adults of the same cultural group have identical adopted TIs, and a SNE must exist again, and so forth.

Within the set–up of the present chapter, the *set of Symmetric Nash Equilbiria* of any period depends on the adopted and inter–generational TIs, the perception rules, as well as on the population shares of the two cultural groups, $P(t) := \{\phi_L(t), \phi_H(t), i_L, i_H, \left(R_L, \hat{\phi}_L\right), \left(R_H, \hat{\phi}_H\right), q_H\} \in \phi^d(X)^2 \times \mathbb{R}^2_+ \times \left(\mathcal{A} \times \mathcal{C}^0\right)^2 \times [0,1]$. We will thus denote the set of SNEs of a period $E(P(t)) \subseteq \left(\phi^d(X) \times [0,1]\right)^2$, and their typical elements

$$\left\{\phi_G^{d^*}(t), \hat{\sigma}_G^*(t)\right\}_{G=L,H} \in E(P(t)).$$

Using any of these for substitution in the parental socialization techniques (2.1),[2] we obtain the rule for the inter–generational evolution of the

[1]The latter assumption is stronger than it might appear on first glance. To see this note that concavity of the own and inter–generational utility functions together with convexity of the cost function is not in general sufficient to guarantee concavity of the target functions of the optimization problems. This follows since the Hessian matrices of the parental socialization techniques with respect to the two decision variables are indefinite (the determinants of these Hessian matrices are -1). Thus the inter–generational utility functions are not in general concave with respect to the two decision variables. To cure this, it is thus necessary that the own utility functions together with the cost functions are jointly concave and convex enough compared to the concavity of the inter–generational utility functions.

[2]In the subsequent analyses, we will always point out, whether the derived properties hold indeed *for all* ellements in the SNE–sets of a given period, or whether these are subject to a particular selection.

adopted TIs of the cultural groups $G = L, H$ under SNE choices as

$$\phi_G(t+1) = \phi_G^{d^*}(t) - \left(\phi_G^{d^*}(t) - \phi_{-G}^{d^*}(t)\right)(1 - \hat{\sigma}_G^*(t))(1 - q_G), \tag{2.3}$$

where $\phi_G(t)$ obviously denotes the identical adopted TI of the adults of the cultural group $G = L, H$ of period t.

Integration and Assimilation The analysis in the succeeding two subsections will always be initiated by a discussion of the SNE choices of any given period under the different types of perception rules. In this context, we will speak of *behavioral dis–integration* of the adult members of a cultural group $G \in \{L, H\}$ in period t whenever it holds that $\left|\phi_G^{d^*}(t) - \phi_{-G}^{d^*}(t)\right| > \left|\phi_G(t) - \phi_{-G}^{d^*}(t)\right|$. This means that these adults choose a more 'radical' DTI relative to the DTI of the other group's adults than the choice of their adopted TI would mean.

In an inter–temporal context, it will be crucial to determine the endogenous evolution of the SNE choices — and with it the endogenous evolution of the adopted TIs. Specifically, we will also want to answer the question of the inter–temporal assimilation (or dissimilation) process between the two cultural groups. In a slight variation of the terminology introduced in Pichler [49], we will speak of *(TI) assimilation* whenever the TI–distance $\Delta^\phi(t) := |\phi_L(t) - \phi_H(t)|$ strictly declines over generations, i.e. $\Delta^\phi(t+1) < \Delta^\phi(t)$. From equation (2.3), we obtain the TI–distances under SNE choices as

$$\Delta^\phi(t+1) = \left|\left(\phi_L^{d^*}(t) - \phi_H^{d^*}(t)\right)\right| \left(\hat{\sigma}_L^*(t) q_H + \hat{\sigma}_H^*(t)(1 - q_H)\right). \tag{2.4}$$

Furthermore, if the assimilation is such that the adopted TIs of the members of the cultural group with the contemporaneously smaller TI strictly increase over generations, while the opposite holds vice verso, we will speak of *strict assimilation*.

Finally, with *behavioral assimilation*, we will call a situation where the absolute distance between the SNE choices of DTIs of the two groups strictly declines between two generations.

1. Fixed Optimal Preference Intensities

In the present section, we consider a situation where the parents of both cultural groups perceive (exogenously given) inter–generationally fixed optimal trait intensities. Thus, in any given period and for both $G \in \{L, H\}$, $\hat{\phi}_G(R_G) = e_G \in \operatorname{con} \phi^d(X)$. This structure corresponds to the second type of perception rule. Without loss of generality, consider subsequently the (non–degenerate) case where $e_H > e_L$.

The following assumption will be prerequisite for a meaningful characterization of the (set of) symmetric Nash equilibrium choices.

Assumption 2.5 (Slope).

(a) u^b *and* $v^d(\cdot|h)$ *are differentiable at their peaks, and*

(b) c *is differentiable at the origin with slope zero, and strictly increasing in the interval* $(0,1]$.

Since both the utility and inter–generational utility function are single peaked, it follows by Assumption 2.5 (a) that both functions have zero slope at their peaks. Thus, parents perceive no (inter–generational) utility losses for marginal deviations of their chosen DTI from their adopted TI, respectively of their adult child's adopted TI from the optimal TI.

In the rest of the analytical part of this section, we will be concerned with characterizing the (set of) SNE choices of the parents as well as the resulting evolutions of the TIs of the two cultural groups. To do this, we will focus our attention on what we call the *generic state space*.

Proposition 2.2 (Generic State Space). *Let Assumptions 2.1–2.5 hold. Then,* $\forall P(0) \in \phi^d(X)^2 \times \mathbb{R}^2_{++} \times \left(\mathcal{A} \times \mathcal{C}^0\right)^2 \times (0,1)$ *such that* $\hat{\phi}_H(R_H) = e_H > e_L = \hat{\phi}_L(R_L)$, $\exists \infty > T(P(0)) \geq 0$ *such that* $e_H > \phi_H(T(P(0))) > \phi_L(T(P(0))) > e_L$.

Proof. In Appendix B 1.2.

This latter proposition states the following. Independent of the initial TIs of the two cultural groups (and subject to any of the elements in the sets of SNEs of the periods), the TIs will enter a basin in the state space where the positions of the two TIs can be considered 'consistent' with the relative location of the fixed optimal TIs. With this we mean that (a) the group with the strictly larger fixed optimal TI does also have a strictly larger adopted TI, and (b) the TIs of both groups do lie in the interior of the 'TI–space' that is formed by the two fixed optimal TIs.

That any path that starts outside this generic state space must lead into it is illustrated in the phase diagram 2.1. In any of the fields in this diagram, the dotted lines indicate the boundaries of the range of the angles that the phase vectors can take (notably, the boundaries themselves are not included in this range) under *any* element in the set of SNEs of any period.[3] Also, one of these possible phase vectors is always depicted. Furthermore, the phase vectors on the boundaries between the various fields share (the combination of) the properties of those in their neighboring fields. This also implies that all phase vectors on the boundary of the generic state space point into it.

[3]The phase vectors are $(\Delta\phi_L(t), \Delta\phi_H(t))$, where $\Delta\phi_G(t) := \phi_G(t+1) - \phi_G(t)$, $G = L, H$.

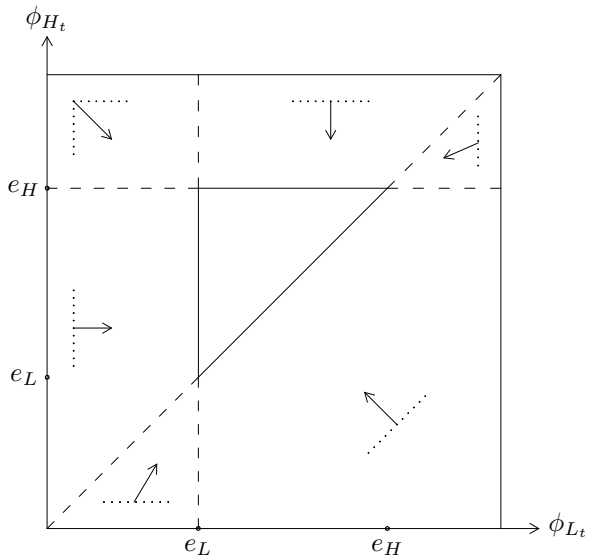

FIGURE 2.1. Phase Diagram (Non–generic State Space)

Let us briefly discuss the basic intuition to understand this phase diagram. We start with the two ('non–generic') fields in the upper triangle of the state space where the TI of group L is smaller than optimal. This implies that the direction of the socialization efforts of the members of this group is 'upwards' (i.e. they tend to choose a DTI that is larger than their adopted TI, jointly with a strictly positive parental socialization success share). Since also both the adopted TI and the optimal TI of group H are strictly larger than the adopted TI of group L, their chosen DTI tends to be strictly larger than the adopted TI of group L. This combination leads to a strict inter–generational increase of the adopted TI of group L under SNE choices. The analogous logic shows that, within the fields in the upper triangle of the state space where the TI of group H is larger than optimal, the adopted TI of group H must strictly decrease.

Consider now the lower left triangle in the state space. In this, the TI of group H is smaller than that of group L, and both are smaller than the optimal TI of group L. In this case, the directional socialization efforts of both groups are (strictly) 'upwards'. This implies that at least the adopted TI of group H must strictly increase inter–generationally. Again, the analogous logic shows that in the upper right triangle, the adopted TI of group L must strictly decrease.

Finally, consider the lower right field in the state space. In this, the TI of the members of cultural group L is larger than both their optimal TI and the adopted TI of cultural group H. Furthermore, the latter is smaller than

optimal. This implies that the directional socialization effort of the members of group L is 'downwards' while that of group H is 'upwards'. This combination then yields the effect that under SNE choices, the inter–generational increase in the adopted TI of group H must be strictly larger (respectively strictly less negative) than that of group L.

We will now turn to the characterization of SNE choices within the generic state space. Note that the results below do again hold for all elements in the sets of SNE choices of any period.

PROPOSITION 2.3 (SNE Characterization). *Let Assumptions 2.1–2.5 hold and let* $e_H > \phi_H(t) > \phi_L(t) > e_H$. *Then,* $\forall\{i_L, i_H, q_H\} \in \mathbb{R}^2_{++} \times (0,1)$, $\forall E(P(t)) \ni \left\{\phi_G^{d^*}(t), \hat{\sigma}_G^*(t)\right\}_{G=L,H}$

(a) $\phi_H^{d^*}(t) > \phi_H(t) > \phi_L(t) > \phi_L^{d^*}(t)$,
(b) $\hat{\sigma}_G^*(t) \in (0,1]$, $\forall G \in \{L, H\}$,
(c) $e_H > \phi_H(t+1) > \phi_L(t+1) > e_L$.

Proof. In Appendix B 1.3.

Within the generic state space, the socialization efforts of the members of the two cultural groups are in the opposite directions. This yields the result that in *any* SNE, the parents of both cultural groups dis–integrate behaviorally and choose strictly positive socialization success shares. Nevertheless, their socialization investments would never be intense enough such that the next generation's adopted TIs would exactly coincide with the optimal one (the logic of this sort of result is being discussed in Pichler [49]). This means that once the TIs of the two groups have entered the generic state space, they will never leave it again. Thus, in an extension of Proposition 2.2, it follows that for every $t' \geq T(P(0))$, $e_H > \phi_H(t') > \phi_L(t') > e_L$.

1.1. Numerical Dynamic Analysis. From the analysis above, it is obvious that any steady state must be located within the generic state space. However, we were neither able to analytically characterize the dynamic behavior of the model under fixed norms on behavior, nor the stability properties of the steady states. In short words, the central barriers for such an analysis were (a) the high generality and nonlinearity of the model, and (b) that no explicit solutions for the SNE choices can be obtained, so that all convergence and stability criteria have to be calculated with results from the Implicit Function Theorem (which necessites the inverse of the $4x4$–matrix of second partial derivatives of the parental best reply problems evaluated at an 'anonymous' steady state). Thus, to illustrate the dynamic properties of the model, we resorted to numerical simulation methods. Thereby, the

following quadratic (dis–)utility and cost functions were used.

$$u\,(b\,|d\,) = -(b-d)^2$$
$$v\,(e\,|f,h\,) = -h(e-f)^2$$
$$c(j) = j^2,$$

and we set $(e_L, e_H) = (-1, 1)$, so that the generic state space corresponds to the (interior of the) triangle $(-1,-1)$–$(-1,1)$–$(1,1)$.[4]

In the numerical simulation, we then proceeded in the following way: First, we fixed a value for q_H. Second, we considered ten linearly spaced values for i_G, $G = L, H$, between 0.2 and 2.0 (thus, the resulting matrix of combinations of i_L and i_H had 100 entries). Third, we considered 10 linearly spaced points between -0.95 and 0.95 in both dimensions of the state space. The combinations of these (that were contained in the generic state space) yielded the initial TIs.[5] For any of the combination of inter–generational TIs, we then calculated the resulting path of TIs for any of the initial TIs. The following summarizing statistics were collected (subject to a fixed q_H):

(1) Did all paths of TIs converge?
(2) If yes, was there a unique steady state?
(3) Have all steady states attained been locally asymptotically stable?

These statistics were calculated for $q_H = 0.5, 0.7, 0.9$ (there is no need for considering more values of q_H, since already enough asymmetries are embodied in the variations of the inter–generational TIs). These summarizing statistics are collected in Table B.1 at the end of Appendix B 1.

As can be seen from this table, the question of convergence could be globally answered with 'Yes'. It can therefore be claimed that the existence of cycles or chaotic behavior in the model under fixed norms on behavior is at best highly nongeneric.

As far as the other criteria are concerned, note as a basic illustrating rule that the intensities of socialization investments (i.e. choice of behavioral (DTI) deviation and parental socialization success share) of the parents of both groups tend to be decreasing in the direction 'north–west' (movements in the generic state space with decreasing distance to the 'optimum point' (e_L, e_H)); and they tend to be increasing/decreasing for group L/H in the direction 'north–east' (movements in the generic state space that preserve the TI–distance but decrease the distance to the point (e_H, e_H)).

[4]Note that all that counts here is the distance between the two norms, not their exact location. However, in the graphical analysis below, the particular choice will simplify the interpretation.

[5]Knowing that the boundaries of the state space are rejecting, we could well disregard them with respect to initial TIs.

The question of (local) stability of a steady state is thus the question of whether these rules do hold locally around a steady state, and whether the associated 'socialization forces' are not too unbalanced in magnitude for the parents of both groups; and obviously, the question of uniqueness of a steady state is that of whether the socialization forces exactly even out at only one point. Now, as can be seen from Table B.1, neither of these questions can be globally answered with 'Yes'. But there are clear cut regularities in the dynamical patterns that we will discuss next.

Let us consider first the case $q_H = 0.5$, so that we can single out the regularities related to the combinations of i_L and i_H. First, we can see that if the socialization incentives embodied in the inter–generational TIs are not too unbalanced, then the steady states are all locally (or globally) asymptotically stable, which corresponds to the statements in the previous paragraph (and in case that the socialization incentives are too unbalanced, the steady states are typically not stable).

Even, if both inter–generational TIs are high relative to the size of the generic state space (determined by the difference $e_H - e_L$),[6] then the parents of both groups would choose comparatively intensive socialization investments for already comparatively low deviations of their children's TIs from the fixed norm on behavior — thus in a large area of the generic state space. Remembering the variations of the socialization investment intensities into the directions 'north–west' and 'north–east', this prevents the existence of multiple steady states. Even, any of the unique steady states appeared to be globally asymptotically stable (since they are subject to more or less balanced socialization incentives). One such case of (assumingly) global asymptotic stability is illustrated in Figure 2.2 below, but notably for the case of $q_H = 0.7$.[7]

Now, in the opposite case where both inter–generational TIs are low compared to the size of the generic state space, the parents of both groups choose small investments in their socialization instruments in a large area of the generic state space. This implies that the groups tend to assimilate inter–generationally, and that the TI–paths move into the direction of the

[6]The size of the generic state space determines the maximum deviation of the children's adopted TIs from the optimum. Thus, we always have to consider the strength of the inter–generational TIs relative to this size (since they then determine the maximum inter–generational disutilities that parents can encounter). As it turned out (and is discussed below), the choice of $e_H - e_L = 2$ featured all possible variations of dynamical behavior that we can expect the model to yield. Thus, it was not necessary to consider higher maximum values of inter–generational TIs than that of 2 (where the inter–generational 'concerns' are twice as important than own utility concerns).

[7]'Zooming' into this phase diagram and considering a small area around the steady state $(-0.3273, 0.3884)$, the phase vectors did still all point toward the steady state, confirming the asymptotic stability.

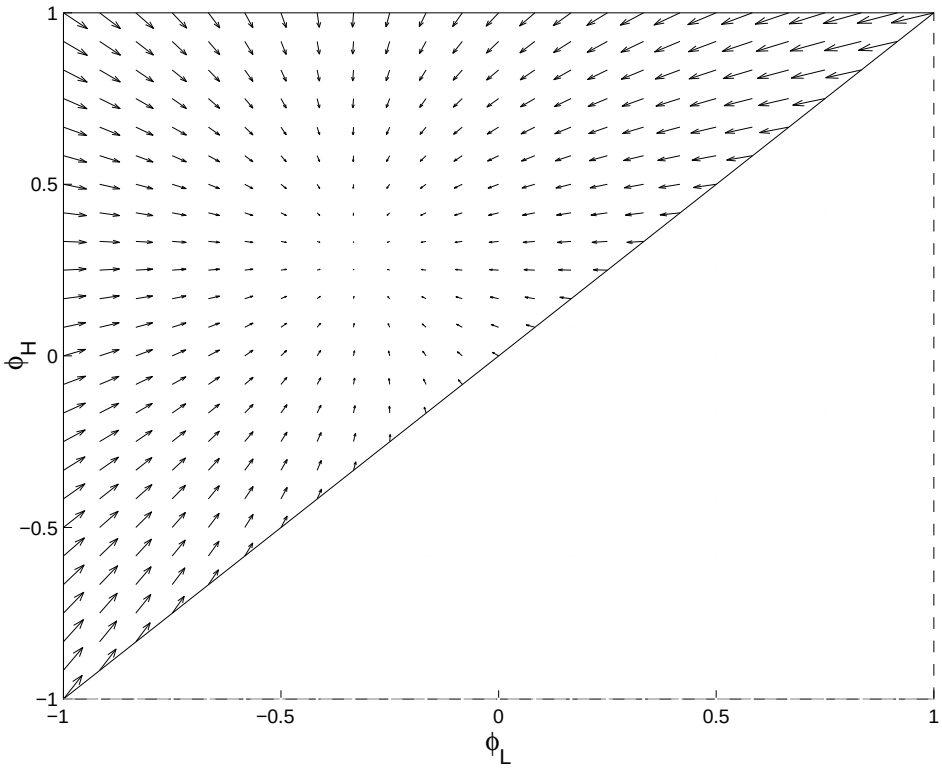

FIGURE 2.2. Phase Diagram for $q_H = 0.7$, $e_L = 1.2$, $e_H = 1.6$

main diagonal (at least after some periods), where the TIs of both groups are identical. Along a path of decreasing TI–distance, it furthermore holds that the DTI–choices of the parents of both groups tend to be closer, implying that the parents of both groups do have to invest less into countervailing the socialization influences of the other group's members. This furthers the assimilative tendencies. The combined result is the existence of multiple equilibria that typically feature a very small (in fact even marginal) deviation of the TIs with accordingly low steady state socialization investments of the parents of both groups. One such case is illustrated in Figure 2.3, here for the case $q_H = 0.9$ (compare also the left graph of Figure 2.5).[8]

Finally, there are also cases that feature a mixture between the two polar cases discussed above. This concerns cases where there is one locally asymptotically stable steady state with a significant basin of attraction together with finitely many (locally stable or unstable) steady states with marginal

[8]As can be seen in the third part of Table B.1, Figure 2.3 illustrates a case where all steady states are locally asymptotically stable (all TI–paths associated to the different initial TIs converged to different steady states, which were all located between $(-0.7323 - \epsilon, -0.7323 + \epsilon)$ and $(-0.0677 - \epsilon, -0.0677 + \epsilon)$). This fact is hard to verify graphically (even upon 'zooming' locally around a steady state), since the phase vectors on the main diagonal feature de facto zero length.

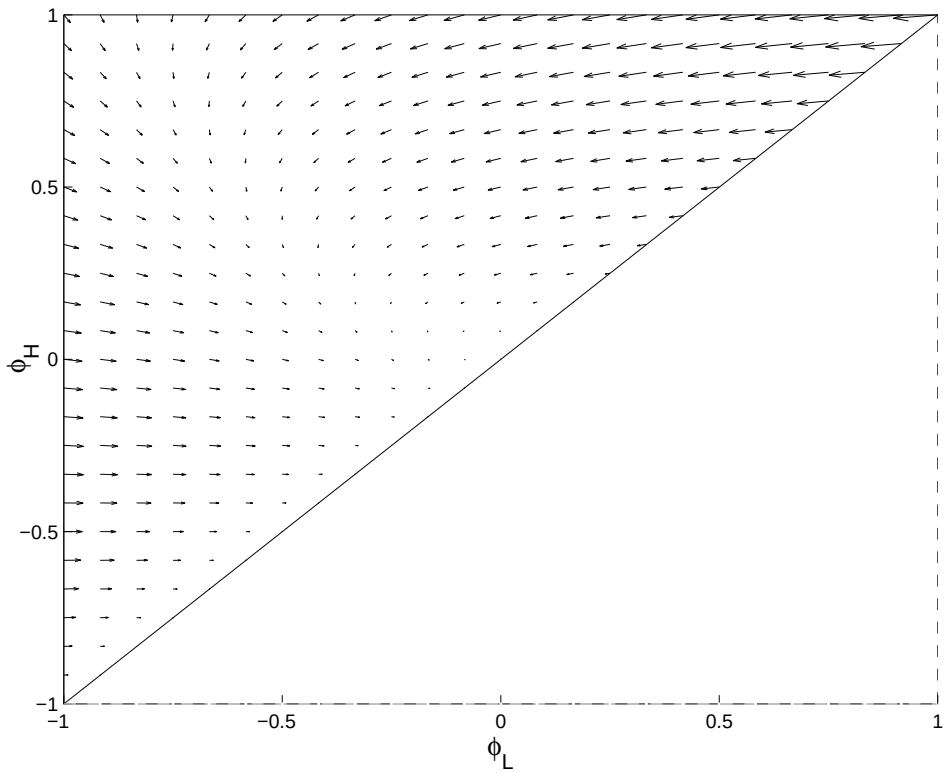

FIGURE 2.3. Phase Diagram for $q_H = 0.9$, $e_L = 0.8$, $e_H = 0.4$

TI–deviation. This necessites inter–generational TIs that are strong enough such as to induce comparatively high socialization investments in a large area of the generic state space, but which are also weak enough such that the groups assimilate if the (initial) TIs are close enough. This typically corresponds to inter–generational TI–combinations that are neighboring the 'upper right' (unique and stable) areas in the three sub–tables of Table B.1. One such case is illustrated in Figure 2.4 for the case $q_H = 0.7$.[9]

In the next step, let us discuss the role that the population shares of the groups play for the dynamical behavior. As can be seen from the comparison of the three individual tables in the global Table B.1, an increasing share q_H in a sense shifts the 'mass' of the three types of statistics–triples (YYY, YNY, YNN) counterclockwise with an outwards rotation. This can be explained as follows. First, associated to an increasing population share are less incentives of the individual parents to engage in active socialization (given any inter–generational TI). This holds since parents of a group with

[9]The TI–paths converged to three different steady states. The 'central' one, $(-0.2733, 0.398)$ which is locally asymptotically stable, and two close to the main diagonal, $(-0.3517 - \epsilon, -0.3517 + \epsilon)$ and $(-0.1738 - \epsilon, -0.1738 + \epsilon)$, both of which are unstable.

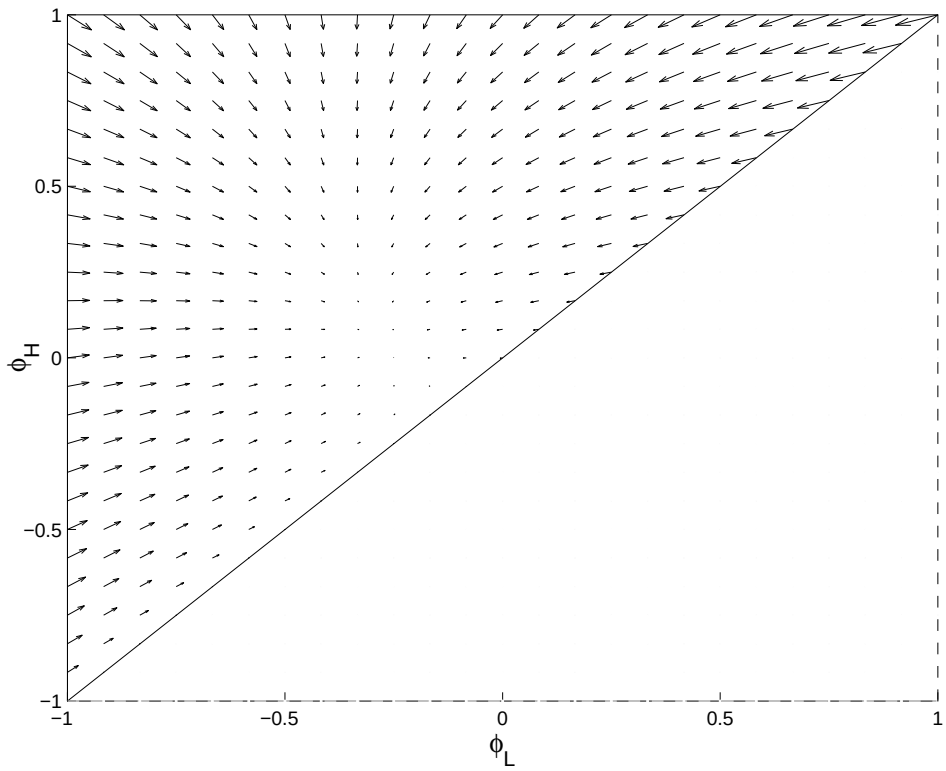

FIGURE 2.4. Phase Diagram for $q_H = 0.7$, $e_L = 1.2$, $e_H = 0.8$

a larger population share face a more favorable composition of the general society in the sense that more other parents orient their DTI–choice into the same direction (and all DTI–choices are even identical in a SNE). Thus, ceteris paribus, parents of a group with higher population share tend to choose less behavioral deviation and a smaller socialization success share.

Now, to maintain uniqueness and asymptotic stability, to even out the more unbalanced socialization incentives (compared to the case $q_H = 0.5$), a higher inter–generational TI of group H and/or a lower inter–generational TI of group L are required. This explains the counterclockwise outward rotation in the 'upper right' (unique and stable) areas of the tables. In the 'lower left' (multiple and stable) areas, the explanation needs to be more careful. Remember that the multiple steady states in these areas are such that the parents of both groups do hardly invest into active socialization at all. This stems from the fact that the inter–generational disutilities are very low even for comparatively high deviations of the children's adopted TIs from the norm. As a result, the individual substition effects between own socialization investements and those that are exerted on the children by the other adult members of the group have a significantly lower magnitude compared to the previous ('upper right' area) cases. Thus, in the

present cases, the (ceteris paribus) effect of an increasing population share is to increase the aggregate (population–share–weighted) socialization investments of a group. To even out the resulting inbalances, larger/lower inter–generational TIs of the group with the decreasing/increasing population share are required. This explains the counterclockwise outward rotation in the 'lower left' areas of the tables.

SNE– and TI–Path Illustration We will close this section with an illustration of two particular paths of SNEs and TIs that evolve if the initial TIs of both groups coincide with their fixed norms on behavior. Notably, we used here the same explicit functions, but chose $e_H - e_L = 4$ (so that, compared to above, e.g. lower inter–generational TIs are sufficient to obtain global asymptotic stability of a steady state). The lengths of the time–axes below corresponds to 100 periods.

In both cases of Figure 2.5, group L is always the minority with a population share of ten per cent. Such a constellation can e.g. be interpreted as having resulted from immigration (of group L), where initially the hosting and immigrant group have had adopted exactly their optimally perceived TI. The latter could e.g. be constantly indoctrinated by two distinct religious institutions which the two different cultural groups adhere to — and which thus constitute the respective norms on behavior of the two groups.

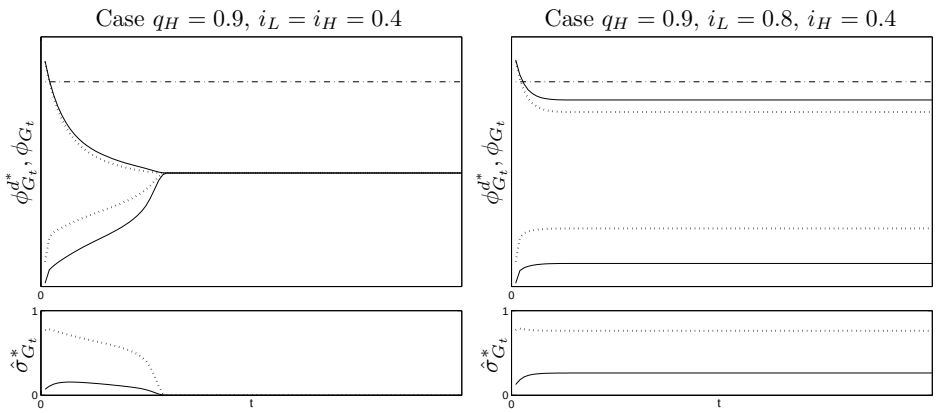

FIGURE 2.5. Evolution under Fixed Optimal TIs

In the upper graph of each case, the solid and dotted lines represent the DTIs and TIs of the two groups (and the dash–dotted line locates the population–share weighted convex combination of the initial TIs; this would equate to the steady state if parents of both cultural groups would not invest into their socialization instruments). The paths of the socialization success shares of the parents of group L are represented by the dotted lines in the lower graph of each case.

The left case of the figure stylizes immigration of a cultural group with a comparatively weak norm on behavior into a hosting society for which this also holds. To the contrary, the right case of the figure stylizes immigration of a cultural group with a strong norm on behavior into a hosting society with a comparatively weak norm.

Let us first collect the evolutionary regularities that can be seen in both cases. First, the members of both cultural groups dis–integrate behaviorally in every period. Second, there is an assimilative tendency until the steady state has been reached. Notably, this TI–assimilation of the groups is also accompanied by a behavioral assimilation.

Furthermore, in both cases the minority cultural group invests considerably more into both socialization instruments. As has been discussed shortly above, this is (partially) due to the fact that the minority group faces a much more unfavorable composition of the general social(ization) environment (in terms of the resulting location of the average DTI compared to the fixed optimal TI). The individual parents of that group thus aim to compensate this by increased investments into their socialization instruments. In the right case, the minority group does additionally have a much stronger norm on behavior, i.e. the social punishments from behavioral deviations from the norm are accordingly more intense. This additionally induces the parents of this group to invest more into socialization.

This latter effect has a remarkable impact on the dynamical evolution of the endogenous variables. In the left case, the norms of both cultural groups are low enough such as to allow for a substantial assimilation process. Even, the TIs of the two groups do nearly converge to a symmetric steady state (but stay distinct).[10] Compared to this case, the increased socialization investments of the minority group in the right case do trigger an according reaction of the majority group. Thus, both groups invest more into both socialization instruments. As a consequence, the TIs of the two groups are held back from assimilation already after very small deviations from the fixed optimal TIs. The resulting steady state TI–distance is accordingly larger.

2. Endogenous Norms on Behavior

The present section will be based on the third type of perception rule discussed in chapter 2. The latter lets all parents form their perception of the optimal trait intensity based on a the average DTI of a subset of the adults. In the present context, it is one immediate option to let the respective subsets

[10]We call a *symmetric steady state* a steady state where almost all adults have the same adopted TI.

coincide with the adult members of the own cultural group of a parent. We will do so and thus consider the case where $\hat{\phi}_G(G) = \phi_G^d(t)$, $\forall G = L, H$, $\forall t \in \mathbb{N}$ (and all subsequent Propositions of the present section are subject to this specification).

This structure opens routes to the existence of multiple, qualitatively different, SNEs. However, we can show that in any given period, SNE choices exist that preserve the relative positions of the adopted TIs of the two groups. To require this property can be considered sensible, since it assures a minimum sort of continuity of the inter–generational evolution of the TIs.

Proposition 2.4 (Relative–Position–Preserving SNE). *Let Assumption 2.1–2.5. Then* $\forall t \in \mathbb{N}$, $\forall \{i_L, i_H, q_H\} \in (\mathbb{R}_{++} \setminus \{\infty\})^2 \times (0,1)$, *there exixsts a* $\left\{\phi_G^{d^*}(t), \hat{\sigma}_G^*(t)\right\}_{G=L,H} \in E(P(t))$ *with the following characteristics.*

(a) Case $\phi_H(t) >< \phi_L(t)$
  (a) $\phi_H^{d^*}(t) >< \phi_H(t) >< \phi_L(t) >< \phi_L^{d^*}(t)$,[11]
  (b) $\hat{\sigma}_G^*(t) \in (0,1)$, $\forall G \in \{L, H\}$,
  (c) $\phi_H^{d^*}(t) >< \phi_H(t+1) >< \phi_L(t+1) >< \phi_L^{d^*}(t)$.

(b) Case $\phi_H(t) = \phi_L(t)$
  (a) $\phi_G^{d^*}(t) = \phi_G = \phi_G(t+1)$, $\forall G \in \{L, H\}$,
  (b) $\hat{\sigma}_G^*(t) = 0$, $\forall G \in \{L, H\}$.

Proof. In Appendix B 1.4.

The key to understanding these properties is the following. Consider a situation where the DTIs are such that both groups dis–integrate behaviorally. Then, the best reply directions of socialization efforts would coincide with this constellation, i.e. there would be best reply behavioral dis–integration of both groups. This is the basis for the existence of a SNE as characterized in the first part of Proposition 2.4.

Since both cultural groups dis–integrate behaviorally, together with a strictly positive socialization success share, it also follows that the relative TI positions of the two groups are preserved over generations. However, the parents of both cultural groups would never choose to exclusively socialize their children (choose a parental socialization success share of one). This follows since in this case, their adult children's adopted TI would coincide with the chosen DTI of the parents, thus with the optimal TI. This can though never be subject to best reply choices (as discussed in detail in Pichler [49]).

[11]The outer inequalities turn into equalities if the adopted TI of a cultural groups equals the relevant one of the boundaries of the set of possible DTIs.

Finally, in the case where the adopted TIs of both cultural groups are identical, the situation where the parents of both cultural groups do not actively socialize their children is possible under SNE choices. This is immediate since such a choice–constellation yields maximum possible utility for all parties involved. Notably, since the adopted TIs of all adult children then coincide with the adopted TIs of the contemporaneous adult generation, any such case constitutes a steady state (which is additionally relative position preserving).

Under these relative position preserving properties, it furthermore follows that no TI–trajectory that has its origin in the upper/lower triangle of the state space can enter the lower/upper triangle. We will next turn to a discussion of qualitative properties of the corresponding TI–dynamics.

Assumption 2.6 (Strict Concavity and Convexity). *The functions u and v are $\mathcal{C}^2$ and strictly concave, and the function c is $\mathcal{C}^2$ and strictly convex.*

Proposition 2.5 (Basin of Attraction). *Let Assumptions 2.1–2.6 be satisfied, and consider only relative–position–preserving SNEs. Then, for every $(i_L, i_H, q_H) \in \mathbb{R}_+^2 \times (0,1)$, $\exists \Delta\left(i_L, i_H, q_H\right) \in \left(0, \left|\operatorname{con} \phi^d(X)\right|\right]$ such that $\forall 0 < \Delta^\phi(t) < \Delta\left(i_L, i_H, q_H\right)$, $\Delta^\phi(t+1) < \Delta^\phi(t)$. This implies that $\forall \Delta^\phi(0) < \Delta\left(i_L, i_H, q_H\right)$, $\lim_{t\to\infty} \Delta^\phi(t, \Delta^\phi(0), i_L, i_H, q_H) = 0$.*

Proof. In Appendix B 1.5.

Indeed, there is a basin in terms of a maximum TI–distance such that for any pair of TIs that features a lower distance, the cultural groups assimilate inter–generationally. To show this property, we employed the Implicit Function Theorem, and the logic for the results is the following: From Proposition 2.4, we know that at any steady state $(\phi, \phi) \in \phi^d(X)^2$, the parents of both groups do not actively socialize their children (i.e. they choose DTIs that accord with their TIs and zero parental socialization success shares), since their adopted TI coincides with the optimal TI. Now, for marginal displacements from such a steady state, the adopted TIs of the children of both groups will also only marginally deviate from the optimal TI (given the local continuity of the SNE choices which follows since we can apply the Implicit Function Theorem). But such a deviation yields no inter–generational disutility, so that the parents will again not engage in active socialization. Thus, all children of the society adopt the society's average DTI, i.e. a(nother) steady state is immediately reached. This establishes the existence of a basin of attraction for the symmetric steady states.

Nevertheless, this basin of attraction does not in general coincide with the whole state space. We will next be concerned with establishing conditions that guarantee convergence to a steady state for any initial pair of

TIs. Notably, the latter property could not be attained in the discrete time OLG framework that we employed so far. Thus, we consider below a continuous time approximation of the model.[12] Notably, the general convergence result that we obtain for the continuous time approximation implies that, under the same conditions, we can expect the same convergence property to generically hold in the discrete time OLG model.

Additional to the continuous time approximation, we will require the following.

Assumption 2.7 (Symmetric Utility Functions). *$\forall b, b' \in \operatorname{con}\phi^d(X)$, $u(j|b) = u(j'|b')$ if $b - j = b' - j'$. Similarly, for every $d, d' \in \operatorname{con}\phi^d(X)$, and $h \in \mathbb{R}_+$, $v(k|d,h) = v(k'|d',h)$ if $d - k = d' - k'$.*

This assumption states that all 'own' and inter–generational utility functions yield identical felicity for identical 'directional' deviations from their peaks. On the one hand, this assumption appears to be quite natural. On the other hand, note that it implies that the dis–utilities that accrue due to deviations from the utility peaks are independent of the positions of the peaks relative to the boundaries of the set of possible DTIs (respectively 'adoptable' TIs).[13]

Proposition 2.6. *Let Assumptions 2.1–2.5 and 2.7 be satisfied. Then there is a SNE selection function such that $\forall(\phi_L(0), \phi_H(0)) \in \operatorname{con}\phi^d(X)^2$, and $\forall(i_L, i_H, q_H) \in \mathbb{R}^2_+ \times (0,1)$, the TIs converge to a steady state.*

This proposition states that (under the conditions imposed) there exists a SNE selection function such that even any initial pair of TIs that is located outside the basin of attraction of the symmetric TI points converges. Thereby, the relevant properties of the SNE selection function are achieved throuh normalizations of the phase vectors to rule out the existence of circles in the whole state space.

In very short words, these normalizations are such that the state space is being composed of a continuum of connected line segments. These consist of (a) a vertical line on which the lower/upper bound of the set of possible DTIs is binding in the DTI choice of the parents of group L (in the upper/lower

[12]Like Bisin and Verdier [8, p. 303] we could derive the continuous time approximation from an OLG society populated by "agents living Δ units of time and have children $1-h$ units of time after birth, by taking the limit for Δ, $h \to 0$, with $\frac{h}{\Delta} = 0$."

[13]To see that accounting for these relative positions might be sensible, consider a pair of unequal adopted TIs. Then, any identical DTI deviation from these utility peaks in the same direction would imply that always one chosen DTI can be considered more 'radical' relative to the maximum or minimum possible DTI. Thus, if one would e.g. like to account for the adults' eventual 'preferences' for moderate behavior, Assumption 2.7 would not be appropriate. A similar line of thought applies in case that parents would e.g. prefer their adult children having moderate adopted TIs (respectively choosing more moderate DTIs).

triangle of the state space), and on which the TI–change of group H is constant; (b) a 45 °–line on which the TI–changes of both groups are constant (notably, these lines can 'melt down' to single points); and (c) a horizontal line on which the upper/lower bound of the set of possible DTIs is binding in the DTI choice of the parents of group H (in the upper/lower triangle of the state space), and on which the TI–change of group L is constant.

Since the state space is thus constructed as a continuum of (connected) line segments on which the TI–change(s) of group L and/or group H are constant (notably, on the 45 °–lines, the TI–changes of group L are identical to that of the connected vertical lines; and the TI–changes of group H are identical to that of the connected horizontal lines), it follows that no circles can exist. Thus, any path of TIs must converge to a steady state.

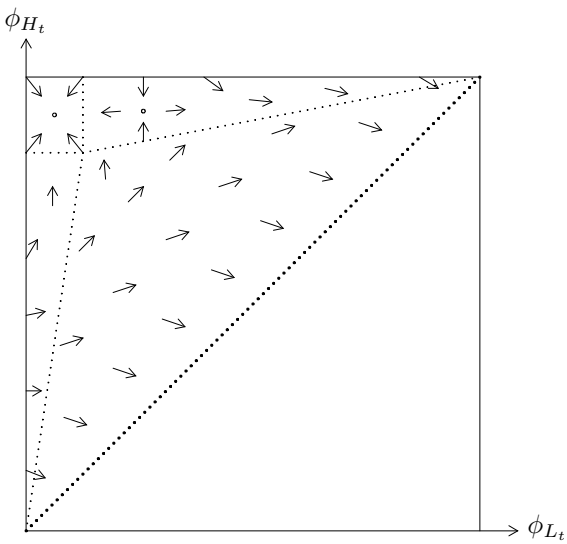

FIGURE 2.6. Phase Diagram (Upper Triangle of the State Space)

The results of Proposition 2.6 are illustrated in Figure 2.6, which stylizes possible qualitative properties of the phase vectors in the upper triangle of the state space (the phase diagram in the lower triangle would correspond to the mirror image). This upper triangle is partitioned into four distinct fields, indicated by the dotted lines. These stylize the regions where either the lower/upper bound of the set of possible DTIs is binding for group L/H (the leftmost/upper–rightmost triangle), respectively where both boundaries are binding (the rectangle), or where both boundaries are unbinding (the main triangle).

The central characteristic of this phase diagram is that with increasing TI–distance, the TI–assimilation of the cultural groups declines in magnitude. Specifically, in a neighborhood around the main diagonal (which consists of a continuum of steady states) the cultural groups do first strictly assimilate, followed by a neighborhood in which assimilation takes place. Furthermore, there is a 45 °–line in the main triangle where the TI–distance stays constant (but not the TIs themselves in this case).

For any point in the main triangle that features a larger TI–distance, the cultural groups do even (strictly) dissimilate. In the present illustration, where the socialization efforts of the parents of group H are always dominating (which can be due to e.g. a larger strength of the behavioral norm), this has the following consequence: Any TI–trajectory that starts in the according area of the state space must lead into a field where (at least) the upper bound of the set of possible DTIs is binding for group H.

In this field, there is then a separating vertical line with the following properties. If a trajectory enters (or starts in) the field 'to the left' (i.e. at a point with a lower adopted TI of group L) of this vertical line, then the TIs will converge to the asymptotically stable steady state in the rectangle. In the opposite case, the TIs will be subject to an assimilation process toward a symmetric steady state. Finally, if the trajectory should enter the field exactly at the vertical line (or starts thereon), then the depicted unstable steady steady state would be reached.

SNE– and TI–Path Illustration We again conclude this section with a numerical illustration of the evolutionary dynamics[14]. In both cases of Figure 2.7, group L is a again the minority with a population share of twenty per cent. Furthermore, it has a slightly lower intensity of the endogenous norm on behavior.

The only distinction between both cases is that the right case features a twice as high initial TI–distance as the left case. As can be seen, this has a crucial consequence on the evolutionary dynamics. The constellation in the left case is such that the initial TIs are located in the basin of attraction of the symmetric steady states. Even, both cultural groups do assimilate throughout the convergence path. This process is again accompanied by an assimilation of the chosen DTIs.

These results do not hold in the right case. To the contrary, the initial TI distance is large enough such as that even an inter–generational dissimilation

[14]Compared to the previous section, the total length of all time–axes is reduced to 30 units.

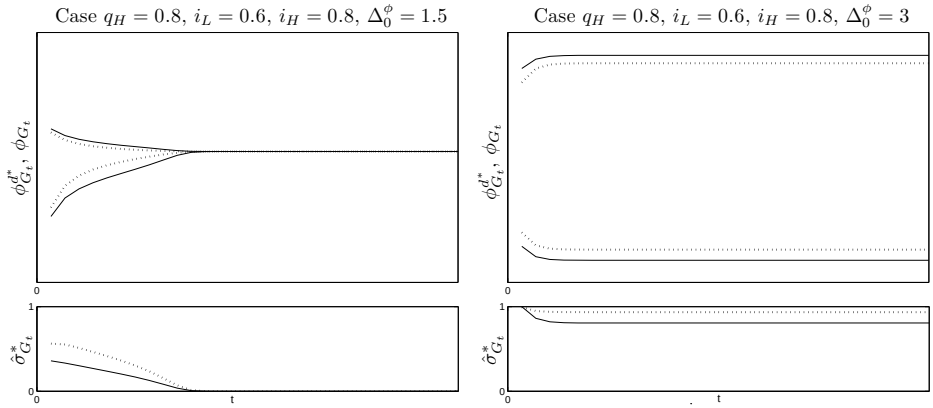

FIGURE 2.7. Evolution under Endogenous Behavioral Norms

process is triggered — both with respect to the TIs as well as to the chosen DTIs.

CHAPTER 4

Conclusions

This paper extended and specified a recent theory of Pichler [49] on the inter–generational formation of continuous cultural traits. Followed by a recapitulation of the latter theory, we analyzed the dynamic evolution of both the behavior and the intensities of the continuous cultural traits in a society populated by two distinct cultural groups.

We showed that the qualitative dynamic properties depend crucially on how parents form their perception of the optimal trait intensity that their children should adopt. As has already been shown in Pichler [49], if all parents have 'imperfect empathy', then the trait intensities of (almost) all dynasties converge to the same point. To the contrary, if all parents of a cultural group adhere to the same exogenously given and fixed optimal trait intensity, then this can never happen. Rather, the two cultural groups stay distinct forever.

The largest variety of possible qualitative properties of the convergence paths is being featured when the optimal trait intensities of all parents of a cultural group coincide with that derived from the average behavior of the group members. Given this, it is well possible that the trait intensities of (almost) all parents converge to the same point. However, it can occur that the cultural groups initially assimilate, but stay distinct in the long run. Even, an inter–generational dissimilation process that leads to a steady state with larger than initial trait intensity distance can realize.

This sort of analysis can also yield additional insights into empirically observable patterns of assimilation and integration of cultural groups. However, the present one–dimensional framework can only be considered the first step in a longer road toward a holistic representation of these processes. The next steps on this road could concern a general, n–dimensional analysis, both with respect to the number of continuous cultural trait types, as well as to the number of cultural groups. Furthermore, we considered here only benchmark cases of perception rules for optimal trait intensities. A more general approach would be sensible.

Note also that we restricted the parental decision problems to the socialization side only — and left other behavioral determinants (like general social interactions) unconsidered. Accounting for a richer 'adult world' could

yield qualitatively different results. Finally, the role of the children in their socialization process is so far that of passive receivers. Allowing for a pro–active role of children in the adoption process of trait intensities could also constitute a fruitful extension of the present baseline model (and that of Pichler [49]).

Part 3

The Evolution of Continuous Cultural Traits in Social Networks

CHAPTER 1

Introduction

Recently, Pichler [49] introduced a framework that determines the inter–generational formation and evolution of *continuous cultural traits*. These are meant to contain all types of traits that (a) are subject to formation in the socialization process, and (b) can reflect different intensities (or magnitudes, valuations, strengths, importances...), located in a convex subset of the real line. Specifically, this class contains concepts that are in standard use in economic theory, like the degree of altruism, the intensity of preferences for leisure or for social status, the patience (intensity), etc. Moreover, it also contains (sociological) concepts like the values, attitudes, (strength of) norms and *continuous opinions* that a person adopts.

The formation of the trait intensities is based on the children's social learning from the observed behavior of the adults in their social environment. In Pichler [49], this environment consists of a continuum of adults. To the contrary, in the present paper we consider a finite population setting in which the social learning of children takes place in a *social network*.

This change has remarkable consequences on the resulting evolution of the continuous cultural traits/opinions. In particular, we obtain a behaviorally induced transformation, respectively generalization, of the dynamics that would be obtained under the DeGroot model (see e.g. Jackson [36] for an overview over the properties of this model).

Contributions and Results The first part of this paper is devoted to an introduction of the cultural formation of continuous cultural traits framework of Pichler [49], given our finite population setting. In doing so, we will show in a first step how children come to adopt intensities of any arbitrary continuous cultural trait type. We let this be based on the children's social learning from observed socio–economic action patterns of the adults in their social environment. Upon observation of the socio–economic action pattern of an adult, children also receive a *cognitive impulse*. The latter can be understood as the signal on the valuation (or importance, magnitude, etc.) of the continuous cultural trait that is embodied in the choice of the particular socio–economic action pattern over the other available choices. We

even endow these sorts of cognitive impulses with a cardinal meaning and call them *displayed trait intensities*.

The trait intensity that a child adopts then results as a convex combination of the displayed trait intensities of its (single) parent on one hand, and the representative displayed trait intensity of the unrelated adults, on the other hand.[1] The latter is being determined by the relative social learning weighted average of the displayed trait intensities of the unrelated adults. The relative social learning weights can be interpreted as representing the children's *social learning networks*. Finally, the convex combination between both sources is then determined by the relative (overall) weight that the parents have in the socialization process of their children.

We then show how to interpret the trait intensities that adults have adopted such as to induce utility functions over the choice of displayed trait intensities, respectively the underlying socio–economic action patterns. Besides this utility component, parents do also obtain *inter–generational* utility, which is related to the adopted trait intensities of their children. Specifically, we assume that parents have a desire for their children to adopt the same trait intensity as they (the parents) have.[2] If the adopted trait intensities of the children deviate from these *socialization targets*, then parents perceive dis–utilities.

Given these two utility components, we then analyze static and dynamic properties of the model when all parents optimally choose their behavior (displayed trait intensities as determined by the socio–economic action patterns) subject to fixed parental socialization weights and subject to a fixed social learning network. The optimal choices of displayed trait intensities thereby result as best replies to the representative displayed trait intensities of the unrelated adults. These best replies are such that parents always countervail an eventually suboptimal representative displayed trait intensity of the unrelated adults. This means that (whenever the parental socialization weight is strictly positive and unequal one) parents do behaviorally deviate from their adopted trait intensity into the opposite direction as the deviation from the representative displayed trait intensity from their adopted trait intensity — which equals their socialization target — is.

The main focus of our present work is then the analysis of the dynamic evolution of the adopted trait intensities of the dynasties under Nash equilibrium behavioral choices. We start this analysis with a classification of possible steady states. These are such that in any steady state all adults

[1]We thus consider a framework of 'vertical and oblique socialization', the terminology of which stems from Cavalli-Sforza and Feldman [16].

[2]A more general representation of this context is introduced in Pichler [49].

behave as they are, i.e. they choose a displayed trait intensity that coincides with their adopted trait intensity. Moreover, the adopted trait intensities of all dynasties in certain parts[3] of the society must be identical. The central question is then under which conditions the sequence of adopted trait intensities converges to any such steady state.

To answer this, we introduce in a first step a representation of the non–linear discrete time dynamics of our model as the left product accumulation of matrices. Thereby, the respective matrix of any given period arises as a behaviorally induced transformation of the underlying (inter–generationally fixed) total social learning matrix. The latter is constituted by the parental socialization weights on the diagonal, and the normalized relative social learning weights of the unrelated adults as the off–diagonals. Thus, the social learning matrix is row–stochastic. However, this property is not in general sustained under the transformation which is induced by the (eventual) behavioral deviation of the adults from their adopted trait intensity. Notably, if this behaviorally induced transformation would not be present, our model would coincide with the DeGroot model. We thus obtain a transformation, respectively generalization, of the DeGroot model.

Given our behaviorally transformed (social learning) matrices, to answer the question of convergence would then coincide with deriving sufficient conditions for the convergence of the left product sequence of general — i.e. not necessarily (positive) row–stochastic — matrices. However, little results that were useful in our context were available on this issue. To the contrary, considerably more has been provided on conditions for the left product convergence of row–stochastic matrices, in particular by Lorenz [40, 41]. To apply these results, we thus had to guarantee in a first step that in any given period, our transformed matrices are positive, respectively row–stochastic. Applying a number of linear algebra results, we could then show that indeed if the social learning matrix is a so called *symmetric ultrametric matrix*, then the behaviorally transformed matrices of any given period are row–stochastic.

In our context, the major properties of symmetric ultrametric matrices are that (a) the social learning structure is symmetric, (b) parents are the 'primary socialization sources' of their children, i.e. no other adult has a larger relative socialization weight, as well as (c) a condition that basically guarantees that the socialization influence that any dynasty has on the other dynasties is too dominant. Even more, these properties do additionally guarantee that the conditions for convergence derived by Lorenz [40, 41] are satisfied. The most central step toward satisfying these is that we obtain (a

[3]We will formally introduce such 'communication classes' later.

special sort of) 'type symmetry' between the original social learning matrix and the transformed matrices.

Thus, endowing the social learning matrix with sufficient structure, we obtain convergence to a steady state as classified above. However, the necessity to guarantee that the transformed matrices are positive, thus row–stochastic, in any given period, significantly reduces the types of possible (convergence) paths that we can address. Basically, we have to restrict our glance to dynamics that are analogous to that obtained in the DeGroot–model. However, the structure of our model, as induced by the behaviorally transformed matrices, is inherently more general. At the present point, we are though unfortunately limited in addressing the more general dynamic structure by non–existing results on the convergence of left–products of general (non–positive) matrices.

To underline, however, that more general convergence results could be obtainable, we introduce a specification of our general model based on explicit utility functions and unrestricted optimization. Most centrally, this yields the result that the transformed matrices are identical in any given period, so that we can then apply results on the convergence of the powers of matrices. In particular, we can then show that (a) if the social learning matrix is symmetric positive definite, or if (b) the 'strength' of the inter–generational utility is 'not too large' compared to that of the displayed trait intensity utility, then (in the first case generically) the sequence of the transformed matrices converges.

Related Literature Our present work stands in close relation to two distinct strands of literature. The first is the small existing literature on the question of the cultural formation of continuous cultural traits.[4] Important early treatments of the topic are Cavalli-Sforza and Feldman [16] in a theoretical, and Otto et al. [46] in an empirical context. More recently Bisin and Topa [5] proposed a representation of the formation of the intensities of continuous cultural traits (or preferences). Their approach is though restricted to the family's choice of its weight in the child's socialization process. The issue of the behavioral choice is left unconsidered.[5]

As has been discussed above, Pichler [49] introduced a more general approach to the cultural formation of continuous cultural traits. Applying this model, Pichler [48] analyzes the evolution of continuous cultural traits in a society which is populated by two distinct cultural groups. He shows

[4]Note that there exists a well established literature on the (probabilistic) transmission of a discrete set of preferences. See Bisin and Verdier [10] for an exhaustive overview.

[5]The same is true for the approach of Panebianco [47], who considers the evolution of inter–ethnic attitudes.

that the qualitative dynamic properties depend crucially on which type of socialization targets (called 'perception rules') the parents perceive. In particular, he contrasts the case of 'imperfect empathy' with the cases where the parents perceive exogenously given norms on behavior, respectively endogenously evolving norms on behavior.[6]

The second branch of literature related to our work is the literature on the evolution of continuous opinions (in social networks) introduced by DeGroot [18]. Here, the general assumption is that individuals do have an opinion or belief (measured by a parameter in $\mathbb{R}$) and update this opinion by interaction with other individuals according to their trust to others and self-trust. DeGroot [18] finds that if the interaction structure is strongly connected and aperiodic then the whole society will end up having the same opinion, i.e. the society reaches a consensus. A variation of this model is introduced by DeMarzo et al. [19] where the individuals' own beliefs can vary over time. The convergence result is similar to that of DeGroot [18] with additional assumptions on the self–trust weights. Moreover, DeMarzo et al. [19] study the speed of convergence. In Lorenz [40] and Lorenz [41] the whole interaction structure is allowed to change over time. Under some conditions, i.e. type-symmetry (if i puts some weight on the opinion of j the j also puts some weight on the opinion of i), positive self–belief, and non-convergence to zero of the positive entries, convergence to a consensus is obtained. Other studies on convergence of opinion dynamics include that of Krause [38], Hegselmann and Krause [35], Weisbuch et al. [62], and Golub and Jackson [29]. The additional objective of the latter paper is to show conditions under which a noisy opinion profile can converge to its mean.

Outline The further setup of this paper is as follows. The succeeding chapter 2 introduces the general framework on the cultural formation of continuous cultural traits of Pichler [49] within our setting. This is followed by the analysis of static and dynamic properties of the model when parents choose their behavior in chapter 3. The proofs of the more extensive propositions of the latter chapter can be found in the Appendix C. Finally, chapter 4 concludes.

[6]This concept has been introduced into the economics literature by Bisin and Verdier [7]. It basically implies that parents have a desire for the adopted trait intensity of their children to be close to their own adopted trait intensities (as we assume in the present paper).

CHAPTER 2

Cultural Formation of Preferences

Consider an overlapping generations society which is populated by the adults of a finite set of dynasties, $N = \{1, \ldots, n\}$. At the beginning of any given period $t \in \mathbb{N}$, adults reproduce asexually and have exactly one offspring, thus the population size is constant.

We will now lay down the framework that determines the cultural formation of one continuous trait type. Let us start with discussing how children come to adopt a certain *trait intensity (TI)* of the continuous trait type under scrutiny. To do so, let us assume that all adults have available the same non–empty set of socio–economic action patterns, X. This set is endowed with a complete and transitive binary relation T. Thereby, for all $x, x' \in X$, $x\,T\,x'$ means that the socio–economic action pattern x is (weakly) 'more characteristic' for the continuous trait type under scrutiny than socio–economic action pattern x'. This general formulation is owed to the fact that we consider any type of continuous cultural trait. Given transitivity and completeness, we can represent the ordinal relation T by a cardinal function

$$\phi^d : X \mapsto \mathbb{R}\,.$$

Thus, to any socio–economic action pattern $x \in X$, ϕ^d assigns a number with cardinal meaning, $\phi^d(x)$. We will call this the *displayed trait intensity* (DTI) embodied in the choice of socio–economic action pattern x. Thus, $\phi^d(X)$ is the set of possible DTIs.

Now, the role models of the children's social learning of trait intensities are the observable socio–economic action patterns $x \in X$ taken by the adults $a \in A$; and we assume that the *cognitive impulse* that any of the children obtains through such an observation is the corresponding DTI, $\phi^d(x)$. To simplify the subsequent exposition, we will denote the DTI of the socio–economic actions of the $t + 1^{th}$ generation adult member of dynasty $i \in N$, $x_i(t) \in X$, as $\phi^d_i(t) := \phi^d(x_i(t))$.

Example 3.1 (Articulated Opinions). *Consider the formation of continuous opinions. In this case, the children's social learning from role models can be interpreted as their listening to* articulated opinions. *The set of socio–economic action patterns X would then directly correspond to the*

(one–dimensional) 'set of possible articulable opinions'. Thus, in this setting, it is sensible to specify ϕ^d as the identity map.

We will now introduce the representation of the socialization process that this paper proposes. This will be established on grounds of the *tabula rasa* assumption, which means in the present context that children are born with unformed trait intensity (TI), and equally, with unformed preferences. On this basis, we then let the formation of the TI that a child adopts result out of social learning from the socio–economic action patterns of adults (only) that it is confronted with. In this context, we will let the TI that any child of $i \in N$ adopts in any period $t \in \mathbb{N}$ be formed according to a weighted average between the representative DTIs of both socialization sources. In case of the child's family, this coincides with the DTI of its single parent, $\phi_i^d(t) \in \phi^d(X)$. The representative DTI of the child's general social environment, $N_i := N \backslash \{i\}$, will be denoted $\phi_{N_i}^d(t)$. This results out of the child's social learning from the observed DTIs of the adults that it is confronted with.

Specifically, in any given period $t \in \mathbb{N}$ and for every child of $i \in N$ there are *oblique socialization weights*, $\sigma_{ij}(t)$, $j \in N_i$, that represent the relative cognitive impacts of the child's social learning from the different unrelated adults. For every $i \in N$, these weights satisfy $\sigma_{ij}(t) \in [0,1]$, $\forall j \in N_i$, and $\sum_{j \in N_i} \sigma_{ij}(t) = 1$. In general, these weights can be interpreted as the *social (learning) networks* that the children have with the unrelated adults. These networks are determined by different social interaction times, as well as by differing pre–dispositions of the children for the social learning from the unrelated adults. We obtain, $\forall i \in N$,

$$\phi_{N_i}^d(t) = \sum_{j \in N_i} \sigma_{ij}(t) \phi_j^d(t) \in \text{con } \phi^d(X).$$

The weight that the DTI of the parent of a child of $i \in N$ has in the socialization process of the child will be called the *parental socialization success share*, $\hat{\sigma}_i(t) \in [0,1]$. This corresponds to the cognitive impact of the parental DTI relative to the cognitive impact of the representative DTI of the child's general social environment. Factors that determine this relative cognitive impact could include the social(ization) interaction time of the parent with its child, as well as the effort and devotion that the parent spends to socialize its child to the chosen DTI.[1]

We assume that all individuals carry over the trait intensity that has been formed in their child period into their adult period (and keep them in

[1]See e.g. Grusec [30] for an introductory overview of theories on determinants of parental socialization success.

an unchanged way). We hence obtain for every $t \in \mathbb{N}$, for every $i \in N$

$$\phi_i(t+1) = \hat{\sigma}_i(t)\phi_i^d(t) + (1 - \hat{\sigma}_i(t))\phi_{N_i}^d(t). \tag{3.1}$$

Since the final adopted TI of an individual is by construction a convex combination of all DTIs that it observes, the adoptef TIs of all individual adults will be located in the convex hull of the set of possible DTIs, con $\phi^d(X) \subseteq \mathbb{R}$.

The adopted TIs of the adults can be interpreted to induce 'filters' under which adults can compare and rank different choices of socio–economic actions. This form of evaluation takes place in terms of comparing the DTIs of the socio–economic actions to the adopted TIs.[2] Specifically, we assume that the adopted TIs induce complete and transitive preference relations over choices of DTIs (respectively the underlying socio–economic actions).

Assumption 3.1 (DTI Utility). *For every $t \in \mathbb{N}$, for every $i \in N$,*

(a) the adopted trait intensities induce a DTI utility function $u\left(\cdot\,|\phi_i(t)\right) : \text{con } \phi^d(X) \mapsto \mathbb{R}$, $u\left(\phi_i^d(t)\,|\phi_i(t)\right)$,

(b) $u\left(\cdot\,|\phi_i(t)\right)$ is single–peaked with peak $\phi_i(t)$, i.e. strictly increasing / decreasing $\forall \phi_i^d(t) \in \text{con } \phi^d(X)$ such that $\phi_i^d(t) < / > \phi_i(t)$.

Intuitively, the single–peakedness property means that we assume adults to prefer choosing behaviors (DTIs) that are as close as possible in line with their adopted TIs.

Example 3.2 (Utility from Articulated Opinions). *In a continuation of the first example, consider the adults' choices of articulated opinions. If these do not coincide with the adults' adopted opinions, then the adults are* lying. *Lying can cause dis–utilities in terms of cognitive dissonance (see Festinger [23]) or in terms of the fear of being revealed. Intuitively, these dis–utilities are strictly increasing in the 'degree of the lies'.*

1. Intergenerational Utility

In the previous chapter, we showed how to interconnect the socio–economic action patterns that adults take in any period with the preferences over the very same action–patterns that next period's adults have. This interconnection is based on the children's social learning from the role models that the socio–economic choices constitute. It follows that any model framework that determines the adults' choices of socio–economic action patterns (respectively those of the corresponding displayed trait intensities), the parental socialization success shares, as well as the social (learning) network structure equally endogenizes the preference formation process.

[2]This is in line with the cognitive dissonance theory of Festinger [23].

In this paper we introduce a framework that (partially) achieves this endogeneization based on purposeful socialization decisions of the parents. The basis for doing so is to clarify what motivation parents have to actively engage in their children's socialization process (the functioning of which we assume them to be fully aware of), as represented by the trait intensity formation rule (3.1). Basically, we let this motivation stem from the fact that parents also obtain an inter–generational utility component. Thereby, we let this be related to the adopted TI of their adult children.

Specifically, in the present paper, we assume that all parents perceive their own adopted TI as the *optimal trait intensity* for their children to adopt.[3] Thus, the parents' adopted TIs constitute their socialization targets. There are two basic motivations to consider this case. The first is that parents simply have an intrinsic desire for their children to develop a personality (adopted TI) that is as similar as possible to their own personality.

The second motivation is based on a myopic form of parental altruism, called *imperfect empathy.*[4] Parents are altruistic and fully internalize the utility resulting from their expectations of their adult child's socio–economic action patterns (respectively DTI). Nevertheless, parents can not perfectly empathize with their child and can only evaluate their adult child's utility under their own (not the child's) utility function — which attains its maximum at the adopted TI of the parent.

As far as the expectations about their adult children's DTIs are concerned, parents are myopic: Although they obtain an inter–generational utility component, which eventually induces them to choose a DTI that does not coincide with their adopted TI (see below), we assume that they do not realize that this form of behavior changing impact will also be present in their adult children's decision problems. Thus, in any given period $t \in \mathbb{N}$, any parent of $i \in N$ expects its adult child to choose a DTI that is in the set of maximizers of its DTI utility function, $\arg\max_{\phi_i^d(t+1)\in\phi^d(X)} u\left(\phi_i^d(t+1)\,|\phi_i(t+1)\right)$. Under Assumption 3.2 below, $\phi^d(X)$ is convex and thus $\phi^d(X) = \text{con}\ \phi^d(X)$. This then guarantees by the single–peakedness of the utility functions that $\arg\max_{\phi_i^d(t+1)\in\phi^d(X)} u\left(\phi_i^d(t+1)\,|\phi_i(t+1)\right) = \phi_i(t+1)$, $\forall i \in N$. Hence, the parental expectations of their adult children's DTIs coincide with their adopted TIs.

Assumption 3.2 (Convexity). *X is a convex and compact subset of a finite dimensional Euclidean space, and ϕ^d is continuous. It follows that $\phi^d(X)$ is non–empty, convex and compact.*

[3]See Pichler [49, 48] for a more general representation of perceived optimal trait intensities.
[4]This concept has been introduced into the economics literature by Bisin and Verdier [7].

Given the perception that the socialization targets of the parents coincide with their adopted TIs, we assume further that parents perceive utility losses for deviations of the adopted TIs of their children from the socialization targets. Specifically, as far as the strengths of these perceived utility losses *relative* to the strength of the DTI utility losses are concerned, for every $i \in N$, we introduce the parameter $\beta_i \in \mathbb{R}_+$. Notably, we assume this parameter to be invariantly passed over from one generation to the next, thus we dropped the time indexes. We will call this parameter parent's *inter–generational trait intensity.* It will play a central role in the dynamical analysis of chapter 2.

Assumption 3.3 (Inter–generational Utility Function). *For every $t \in \mathbb{N}$, for every $i \in N$,*

(a) the inter–generational trait intensity and adopted trait intensity induce an inter–generational utility function $v\left(\cdot\,|\beta_i, \phi_i(t)\right) : \text{con}\ \phi^d(X) \mapsto \mathbb{R}$, $v\left(\phi_i(t+1)\,|\beta_i, \phi_i(t)\right)$, *where*

(b) $\forall \beta_i \in \mathbb{R}_{++}$, $v\left(\cdot\,|\beta_i, \phi_i(t)\right)$ is single–peaked with peak $\phi_i(t)$, thus strictly increasing / decreasing at all $\phi_i(t+1) \in \text{con}\ \phi^d(X)$ such that $\phi_i(t+1) < / > \phi_i(t)$.

CHAPTER 3

Choosing Behavior and the Evolution of Trait Intensities

In the present chapter, we will show static and dynamic properties of the model introduced above when parents can choose their behavior (displayed trait intensity) in the context of a fixed social learning structure of their children. This means that both the parental socialization success shares, as well as the social learning networks of the children (with their unrelated adults), are exogenously fixed in any given period. Such a setting can be motivated with, e.g., a fixed local and social structure that determines the adults' and children's social (learning) interactions.

1. Best Reply Problems and Nash Equilibrium

Given that we consider only the behavioral (DTI) choices of the parents, in any period $t \in \mathbb{N}$, the best reply problems of the parents $i \in N$ are

$$\max_{\phi_i^d(t) \in \phi^d(X)} u\left(\phi_i^d(t)\,|\phi_i(t)\right) + v\left(\phi_i(t+1)\,|\beta_i, \phi_i(t)\right) \tag{3.2}$$
$$\text{s.t. } \phi_i(t+1) = \hat{\sigma}_i(t)\phi_i^d(t) + (1-\hat{\sigma}_i(t))\phi_{N_i}^d(t).$$

The addition of the two types of utility functions embodies the trade off between DTI utility losses (by choosing DTIs that deviate from the adopted TIs) and eventual improvements in the location of the children's adopted TI relative to the optimal TI.

In choosing their best reply behavior, we assume that the parents do not internalize (or are not aware of) the effect that this has on the behavior decisions of the other parents. This *bounded rationality* assumption can be justified since full rationality would mean that all parents are aware of the best reply problems of all parents. This would imply that they know the social learning structure of all children, as well as the set of adopted and inter–generational TIs. This is without doubt more than we can expect from agents in a general (large) society.

The best reply problems of the parents $i \in N$ determine sets of best reply DTIs against the representative DTIs (which are subject to the fixed social learning structure of the children), subject to their fixed parental socialization success share, as well as their adopted TIs (which are also

their socialization targets), and their inter–generational trait intensities. For every adult $i \in N$, we will thus denote any of the elements in its best reply set as $\phi_i^d(t)\left(\phi_{N_i}^d(t), \hat{\sigma}_i(t), \phi_i(t), \beta_i\right)$, which we will abbreviate subsequently as $\phi_i^d(t)(\cdot)$. Furthermore, together with the representative DTI, any such best reply DTI also determines a best reply location of the adult children's adopted TIs (through the TI formation rule, see (3.1)), $\phi_i(t+1)\left(\phi_i^d(t)(\cdot), \hat{\sigma}_i(t), \phi_{N_i}^d(t)\right)$.

The following assumption assures non-emptiness of the adults' sets of best reply DTIs. Furthermore, it will allow for a significant characterization of these, as well as of the resulting best reply locations of the adults' subsequent TIs.

Assumption 3.4 (Compactness, Continuity).

(a) X is compact. If $m > 1$, then ϕ^d is (additionally) concave.

(b) The functions $u(\cdot|a)$ and $v(\cdot|b,d)$ are continuous and differentiable at their peaks.

Since both the utility and inter–generational utility function are single–peaked, Assumption 3.4 (b) implies that both functions have zero slope at their peaks. Thus, parents perceive no (inter–generational) utility losses for marginal deviations of their chosen DTI from their adopted TI, respectively of their adult child's adopted TI from the optimal TI.

Proposition 3.1 (Characterization of Best Replies). *Let Assumptions 3.1–3.4 hold. Then, $\forall t \in \mathbb{N}$, $\forall i \in N$, the sets of best reply DTIs are non–empty and satisfy the following characterization.*

(a) If $\hat{\sigma}_i(t) = 0$, then $\phi_i^d(t)(\cdot) = \phi_i(t)$, thus $\phi_i(t+1)\left(\phi_i(t), 0, \phi_{N_i}^d(t)\right) = \phi_{N_i}^d(t)$.

(b) If $\hat{\sigma}_i(t) = 1$, then $\phi_i^d(t)(\cdot) = \phi_i(t)$, thus $\phi_i(t+1)\left(\phi_i(t), 1, \phi_{N_i}^d(t)\right) = \phi_i(t)$.

(c) Let $\hat{\sigma}_i(t) \in (0,1)$. Then, it holds generically that $\operatorname{sign}\left(\phi_i^d(t)(\cdot) - \phi_i(t)\right) = -\operatorname{sign}\left(\phi_{N_i}^d(t) - \phi_i(t)\right)$,[1] while it always holds that $\operatorname{sign}\left(\phi_i(t+1)\left(\phi_i^d(t)(\cdot), \hat{\sigma}_i(t), \phi_{N_i}^d(t)\right) - \phi_i(t)\right) = \operatorname{sign}\left(\phi_{N_i}^d(t) - \phi_i(t)\right)$.

Proof. Non–emptiness as well as parts (a) and (b) are trivial. Part (c) follows as a straightforward corollary from the proof of Proposition 1 in Pichler [49]. □

The (generic) results of Proposition 3.1 (c) are illustrated in Figure 3.1 below. In the left interval (both intervals correspond to the set of possible

[1]The non–generic case holds if the deviation of the best reply DTI from the adopted TI into the 'desired' direction is not possible, i.e. if the adopted TI of an adult coincides with (the relevant) one of the boundaries of $\phi^d(X)$. Then, the best reply DTI will coincide with the adopted TI (i.e. with the boundary).

DTIs) the context of the adult's decision problem is depicted. In the right interval a corresponding best reply choice is stylized. As can be seen both from Proposition 3.1 (c) directly, as well as from the graphical illustration, the results feature two dominant characteristics.

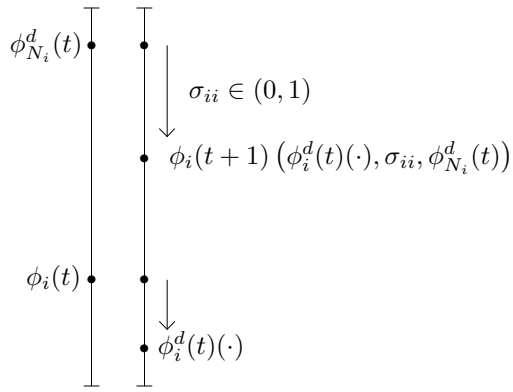

FIGURE 3.1. Characterization of Best Replies

The first concerns the generic location of the best reply choices. If the representative DTI does not coincide with the optimal TI, then parents countervail the respective socialization influence on their children by choosing DTI that deviates from their adopted TI.[2] This deviation is always into the opposite direction as the deviation of the representative DTI from the optimal TI (if such a choice is available). That this holds for very small deviations of the representative DTI from the optimal TI is due to the fact that marginal investments into the socialization instruments are (utility) costless (while as the resulting strictly positive decrease in the distance of the child's adopted TI from the optimal TI yields a strictly positive inter–generational utility gain).

The second dominant characteristic concerns the location of the adult children's adopted TIs that would result out of the parental best reply choices. Despite the parental countervailing in the case of suboptimal socialization influences of the general social environment, the behavioral deviation from the utility peak would never be intense enough such as to guarantee that their adult children's adopted TIs would exactly coincide with the optimal TIs. Hence, there is always a strictly positive deviation of the adopted

[2]Obviously, if the representative DTI exactly coincides with the optimal TI, then parents have no incentives to do so (since the adopted TI of an adult child will then anyhow coincide with the optimal TI).

TI of an adult child from the optimal TI. Thereby, the direction of this deviation always accords with the direction of deviation of the representative DTI from the optimal DTI.

Again, this result holds for even very small deviations of the representative DTI from the socialization target. Analogously to before, this stems from the fact that parents do not perceive inter–generational utility losses for an only marginal deviation of the adult child's adopted TI from the socialization target (while at any strictly positive DTI–deviation from the peak, the marginal cost of an increase in the deviation to further reduce the distance between the adult child's adopted TI and the socialization target would be strictly positive).

Assumption 3.5 (Concavity). *The functions $u(\cdot\,|a)$ and $v(\cdot\,|b,d)$ are concave (in the second argument).*[3]

Proposition 3.2 (Nash Equilibrium Existence). *Let Assumptions 3.1—3.5 hold. Then, for every $t \in \mathbb{N}$, a* Nash equilibrium *in DTI choices exists. Denote this $\Phi^{d^*}(t) := \left(\phi_1^{d^*}(t), \ldots, \phi_n^{d^*}(t)\right)'$.*

Proof. In Appendix C 1.1.

2. Evolution of Preferences

Given the static characterization of the previous section, we will now be concerned with a characterization of the long–run evolution of the adopted trait intensities of the dynasties. In particular, we will now (explicitly) assume that both the parental socialization success shares, as well as the relative socialization weights are inter–generationally fixed. We will thus drop the respective time–indexes of the social learning matrices below. Moreover, for every $i \in N$ and for every $j \in N_i$ we will denote the *aggregate* relative social learning weights of the unrelated adults $\hat{\sigma}_{ij} := (1-\hat{\sigma}_i)\,\sigma_{ij}$, and consider the total *social learning matrix* $\hat{\Sigma} = [\hat{\sigma}_{ij}]$ (with diagonal elements $(\hat{\sigma}_1, \ldots, \hat{\sigma}_n)$). Note that $\hat{\Sigma} \in \mathcal{S}(n)$, i.e. the social learning matrix belongs to the set of row stochastic square matrices of dimension n.

We will then be concerned with deriving conditions on $\hat{\Sigma} \in \mathcal{S}(n)$ and the vector of inter–generational trait intensities $\beta := (\beta_1, \ldots, \beta_n)' \in \mathbb{R}_+^n$, under which the tuple of adopted TIs converges to a steady state. To begin the analysis, we first introduce a characterization of the steady states of our model.

For this and for the analysis to follow, we introduce some additional useful terminology and notation related to any interaction matrix $A \in \mathcal{S}(n)$. We

[3]Note that under Assumptions 3.1 (b) and 3.3 (b), both utility functions are already strictly quasi–concave.

say that there exists a connection from i to j in A, denoted by $i \to j$, if there exists a $k \in \{0, ..., n\}$ such that $A^k_{ij} > 0$. Two dynasties communicate, denoted by $i \sim j$, if $i \to j$ and $j \to i$. A dynasty i is self-communcating if $i \to i$. Trivially, $\sim$ defines an equivalence relation on the set of self-communicating dynasties and, hence, this set can be partitioned into equivalence classes, called self-communicating classes. Denoting each non self-communicating dynasty as a single class, $\sim$ partitions the dynasty set into communication classes $\mathcal{P}(A) = \{N_1, ..., N_p\}$ such that for all $L \in \mathcal{P}(A)$, L is either a self-communicating class or a non self-communicating dynasty. A communication class $L \in \mathcal{P}(A)$ is called essential if for all $i \in L$ there does not exist a $j \notin L$ such that $i \to j$. A communication class is called inessential if it is not essential.

Proposition 3.3 (Steady States). *Let Assumptions 3.1–3.5 hold. Then, in any steady state*

(a) all adults behave as they are,
(b) all TIs of the dynasties in an essential communication class are identical, and
(c) the TIs of the dynasties in inessential communication classes $I \in \mathcal{P}(\hat{\Sigma})$ are convex combinations of the TIs of the communication classes $J \in \mathcal{P}(\hat{\Sigma})$ such that $I \to J$.

Proof. In Appendix C 1.2.

To see that part (a) must hold, note that per definition, in any steady state, the children adopt the same TIs as their parents have. From Proposition 3.1, we know that such a constellation can only be subject to (Nash equilibrium) individual best replies if the representative DTIs of all children coincide with the parents' adopted TIs. In such a case, all parents behave as they are. Parts (b) and (c) of the Proposition are then straightforward. If the TIs would differ within an essential communication class, then at least one of the parents with maximal (respectively minimal) TIs is facing a representative DTI that is lower (resp. larger) than its TI, contradicting the steady state. An analogous consideration holds if one of the parents in an inessential communication class has a TI that lies outside the interval of the TIs of the communication classes to which their communication class is connected. Note that within an inessential communication class the TIs of the adults may differ in a steady state.

Given this steady state classification it now remains to derive conditions under which the sequence of TIs actually converges to any such rest point. The following example shows that in case of two connected dynasties, such a condition is easy to obtain.

EXAMPLE 3.3 (Two Dynasties). *Consider the simplest case of a non–degenerate essential communication class, i.e. that of two parent–child pairs in any given period. Assume also that for all* $i = 1, 2$, $\hat{\sigma}_i \geq \frac{1}{2}$ *— so that the parents are the 'primary socialization sources' of their children.*

Then, it must hold that the distance between the adopted TIs of the adult members of both dynasties strictly declines over generations. Thus, the adopted TIs converge to the same point.

To see that this is true, consider the possible Nash Equilibrium DTI choices and the resulting locations of the children's adopted TIs in any given period. To do this, note first that under the primary parental socialization assumption ($\hat{\sigma}_i \geq \frac{1}{2}$)*, the child of the parent who chooses the larger DTI will also adopt the larger TI. Let us consider the case where the adopted TIs of the adults are unequal, and assume (by way of contradiction) that any child would be led to adopt a TI which is larger/lower than the maximum/minimum of the adopted TIs of the adults. Thus, this must (also) hold for the child of the parent who chose the larger/lower DTI.*

However, such a situation could never be subject to a best reply choice of at least the adult with the strictly larger/lower adopted TI. This follows from Proposition 3.1 since the DTI choice of this adult would have to be strictly larger/lower than its adopted TI. Thus, an at least marginal relaxation of its behavioral deviation would strictly increase its total utility.

Certainly, in the two dynasties case, the 'primary parental socialization' condition is stronger than necessary for obtaining convergence. However, if the socialization success shares deviate too far from this condition, i.e. if the unrelated adults have too large a socialization influence on the children, then cycling behavior can well arise.

This holds since both parents do always countervail the socialization influence exerted by the other adult on the own child. This (typically) leads to a situation where the adult with the lower/larger adopted TI chooses a strictly lower/larger DTI (than the adopted TI) in the Nash equilibrium. With large enough socialization success shares of the unrelated adults, it is possible that one child adopts a TI that is strictly larger than the larger one of the current adults' adopted TIs, and vice versa. Even, the relative TI–positions of the children would then be reversed compared to that of the parents. If this constellation continues to realize in subsequent periods, then the evolution of the trait intensities will be characterized by a limit cycle, with the relative positions of the adopted TIs changing between any two succeeding generations.

A sufficient and easy to derive condition for convergence in the two–dynasties case is that the parents of both dynasties are the 'primary socialization sources' of their children. However, in general, i.e. for an arbitrary number of connected dynasties, it was impossible for us to directly derive (analogous) conditions that would ensure convergence. For being nevertheless able to obtain such conditions, we will below embed the non–linear dynamical system into a tractable form.

Corollary 3.1 (Nash Equilibrium Map). *Let Assumptions 3.1–3.5 hold. Then, there exists a* Nash equilibrium map $E : \phi^d(X)^n \times \mathcal{S}(n) \times \mathbb{R}^n_+ \mapsto \mathbb{R}^n_+$, *such that for every* $i \in N$ *and for every* $t \in \mathbb{N}$, $E\left(\Phi(t), \hat{\Sigma}, \beta\right) = (e^*_1(t), \ldots, e^*_n(t))'$ *satisfies*

$$\phi_i^{d^*}(t) - \phi_i(t) = e_i^*(t)\left(\phi_i(t) - \phi_i^*(t+1)\right)$$

where $\phi_i^*(t+1) := \sum_{j\in N} \hat{\sigma}_{ij}\phi_j^{d^*}$.[4] *This map has the property that if for any* $i \in N$ $\hat{\sigma}_i\beta_i = 0$, *then* $e_i^*(t) = 0$, $\forall t \in \mathbb{N}$, *as well as that if* $\phi_i^*(t+1) = \phi_i(t)$ *then* $e_i^*(t) = 0$.

Proof. Follows immediately from the best reply characterization of Proposition 3.1 and the Nash equilibrium existence of Proposition 3.2. □

The Nash equilibrium map simply represents the Nash equilibrium DTI choices in terms of their deviations from the parent's adopted TIs relative to the deviation of the children's adopted TIs from the socialization targets. This representation can equivalently be written as $\phi_i^{d^*}(t) + e_i^*(t)\hat{\Sigma}_i\Phi^{d^*}(t) = (1 + e_i^*(t))\phi_i(t)$, for every $i \in N$. Defining $B(t) := \mathrm{diag}\left(e_1^*(t), \ldots, e_n^*(t)\right)$, we thus obtain[5]

$$\left(I + B(t)\hat{\Sigma}\right)\Phi^{d^*}(t) = (I + B(t))\Phi^*(t)$$

so that

$$\Phi^{d^*}(t) = \left(I + B(t)\hat{\Sigma}\right)^{-1}(I + B(t))\Phi^*(t)$$

and hence

$$\Phi^*(t+1) = \hat{\Sigma}\left(I + B(t)\hat{\Sigma}\right)^{-1}(I + B(t))\Phi^*(t).$$

For this representation to be well defined, it is e.g. sufficient that either $\hat{\Sigma}$ is diagonally dominant (since then $I + B(t)\hat{\Sigma}$ is strictly diagonally dominant, thus invertible) or symmetric positive semidefinite (below, we will restrict our glance to a class of matrices that even features symmetric positive definiteness of $\hat{\Sigma}$).[6]

---

[4]Actually, for all $i \in N$, $e_i^*(t) = e_i^*\left(\Phi(t), \hat{\Sigma}, \beta\right)$. We chose the representation in the text for brevity.

[5]$diag(y)$ denotes a diagonal matrix with diagonal entries specified by y.

[6]To see the latter, note that under symmetry, we can rewrite $I + B(t)\hat{\Sigma} = I + B(t)^{\frac{1}{2}}\hat{\Sigma}B(t)^{\frac{1}{2}}$. Now since $\hat{\Sigma}$ is positive semidefinite, it follows that $x'\hat{\Sigma}x \geq 0$, for all $x \in \mathbb{R}^n$, thus also

Finally, denoting $M(t) := \hat{\Sigma}\left(I + B(t)\hat{\Sigma}\right)^{-1}(I + B(t))$, it follows that

$$\Phi^*(t) = M(t-1)\dots M(0)\Phi(0) = M(t-1,0)\Phi(0),\ t \in \mathbb{N}\backslash\{0\} \tag{3.3}$$

where $M(t,0)$ denotes the *backward accumulation* of the sequence $\{M(t')\}_{t'=0}^{t}$.

The beauty of this representation is that we transformed the non–linear Nash Equilibrium solutions of our general model into an essentially linear form. This significantly increases the analytical tractability since it allows us to resort to linear algebra results on the convergence of left products of matrices. Specifically, Lorenz [40, 41] provided convergence results for left products of row stochastic matrices — while as (for our specific context) not sufficient results are available on the left product convergence of more general matrices (that have row sum one, but with possibly negative entries). However, to guarantee that the individual $M(t)$ are row stochastic in every period $t \in \mathbb{N}$, we have to endow the social learning matrix $\hat{\Sigma}$ with sufficient structure (we will discuss this context in more detail below).

Definition 3.1 (Symmetric Ultrametric Matrix). $\hat{\Sigma} \in \mathcal{S}(n)$ *is a* symmetric ultrametric matrix *if*

(i) $\hat{\Sigma}$ *is symmetric,*
(ii) $\hat{\sigma}_i \geq \max\{\hat{\sigma}_{ij} : j \in N_i\}$, $\forall i \in N$,
(iii) $\hat{\sigma}_{ij} \geq \min\{\hat{\sigma}_{ik}; \hat{\sigma}_{kj}\}$, $\forall i, j, k \in N$.

To motivate the symmetry property in our context, remember the basic determinants of the relative socialization successes that different unrelated adults have with the children. These determinants consist of the relative social interaction time on the one hand, and potentially differing social learning pre–dispositions on the other hand. Thus, for any pair of children, the required symmetry can be achieved by requiring the relative social interaction time that any one of the two children has with the parent of the other child to be identical, together with the assumption that all children have identical social learning pre–dispositions. Property (ii) is the generalized 'primary parental socialization' condition. It simply means that among all adults, the parents have the largest socialization influence on their children (respectively, among all adults, they spend the largest time share with their children). In general, the third property requires a sort of consistency of the socialization patterns. It states that for any triple $i, j, k \in N$, if the socialization influence of j on child i is strictly smaller than that of k on child i, then it must not hold that k has a strictly larger socialization influence on child j than on child i (since $\hat{\sigma}_{kj} = \hat{\sigma}_{jk}$). This requirement can be

for $x = B(t)^{\frac{1}{2}}y$, $y \in \mathbb{R}^n$. It follows that $I + B(t)\hat{\Sigma}$ is (symmetric) positive definite, thus invertible.

interpreted as ruling out the existence of dynasties that have a too dominant social learning influence on other dynasties.

Recall from Proposition 3.3 that we can only show convergence of trait intensities within communication classes. In the case of symmetric ultrametric social learning matrices $\hat{\Sigma}$, any communication class $L \in \mathcal{P}(\Sigma)$ is essential due to the symmetry of $\hat{\Sigma}$ and for $I, J \in \mathcal{P}(\hat{\Sigma})$ such that $I \neq J$ it holds that $i \not\to j$ for all $i \in I$, $j \in J$. For the following result, let $P_{\hat{\Sigma}}(i) \subseteq N$ be such that $P_{\hat{\Sigma}}(i) \in \mathcal{P}(\hat{\Sigma})$ and $i \in P_{\hat{\Sigma}}(i)$ (the element of the partition $\mathcal{P}\left(\hat{\Sigma}\right)$ which i belongs to). For a matrix $A \in \mathcal{S}(n)$ and some $J \subseteq N$ let A_J denote the matrix A restricted to the set of dynasties $J \subseteq N$. Finally, a consensus matrix is a row stochastic matrix where all rows are identical. We now get the following convergence result.

Proposition 3.4 (Convergence I). *Let Assumptions 3.1–3.5 hold, let the map E be continuous and let $\hat{\Sigma}$ be symmetric ultrametric. Then, $\forall \beta \in \mathbb{R}^n_+$, $\lim_{t\to\infty} M(t,0)$ exists. Moreover, $\lim_{t\to\infty} M(t,0)_L = K(L)$ for all $L \in \mathcal{P}$, such that $K(L)$ is a consensus matrix, and $\lim_{t\to\infty} M(t,0)_{ij} = 0$ if and only if $j \notin P_{\hat{\Sigma}}(i)$.*

Proof. In Appendix C 1.3.

Endowing the the total social learning matrix with sufficient structure, we thus arrive at a general result: In the long-run the communication classes of a society (these are the components of the social network) will end up with the same trait intensities. In the proof, we show first that each element $M(t)$ of the left product (3.3) is row stochastic. While it is straightforward to show that the rows of each $M(t)$ sum up to one (confer Lemma C.1), we make use of a number of linear algebra results on inverses of symmetric ultrametric matrices and inverse–positive matrices to show that $M(t)$ is positive.[7] Second, we can show that the entries of $M(t)$ corresponding to strictly positive entries of $\hat{\Sigma}$ can be bounded away from zero. This is due to the linearity of the determinants of the minors of $M(t)$ in all individual $e_i^*(t)$s, and the continuity and boundedness of E. In the last step, we construct a sequence of sub-accumulations of $M(t_{s+1}, t_s)_{s\in\mathbb{N}}$ such that for each element the minimal strictly positive entry can be uniformly bounded away from zero, which also implies type-symmetry and a strictly positive diagonal. For the sequence of sub-accumulations $M(t_{s+1}, t_s)_{s\in\mathbb{N}}$ we can then apply the convergence result by Lorenz [40], which implies that the adopted TIs of each connected subset converge to the same point, respectively the dynasties reach a consensus.

[7]For literature on inverses of symmetric ultrametric matrices refer to Nabben and Varga [43, 44], Martinez et al. [42], and for results on inverse–positive matrices see e.g. Fujimoto and Ranade [26].

Note finally that the necessity to guarantee that all $M(t)$ are row stochastic significantly reduces the convergence path types that we can analytically address. Basically, we have to restrict our glance to dynamics that are analogous to that obtained in the DeGroot–model. This follows since $M(t)$ row stochastic implies that sequence of adopted TIs is such that all next–period adopted TIs lie in the interval formed by the minimum and maximum adopted TIs of the current period. However, the structure of our model is inherently more general — and also more general than existing models of opinion dynamics, which do not incorporate strategic interaction. In fact, the DeGroot model is a special case of our model (i.e. when $B(t) = diag(0, \ldots, 0)$, $\forall t$), as well as of the example that we discuss next. The example serves to underline the claim that convergence can be obtained under more general conditions.

Example 3.4 (Explicit Functions and Unrestricted Optimization). *Consider the case where* $u\left(a' \,|a\right) = -\left(a' - a\right)^2$ *and* $v\left(d' \,|b, d\right) = -b\left(d' - d\right)^2$. *Assume further (with loss of generality) that all parents can unrestrictedly choose their displayed trait intensities, or in other words that the set of possible DTIs would be unbounded. Then, in every period* $t \in \mathbb{N}$ *the parents* $i \in N$ *would face the* unrestricted *optimization problems*

$$\min_{\phi_i^d(t)} \left(\phi_i^d(t) - \phi_i(t)\right)^2 + \beta_i \left(\phi_i(t+1) - \phi_i(t)\right)^2. \tag{3.4}$$

From the first order conditions, it immediately follows that in this case $E\left(\cdot, \hat{\Sigma}, \beta\right) = (\beta_1\hat{\sigma}_1, \ldots, \beta_n\hat{\sigma}_n)'$. This has the consequence that $\forall t \in \mathbb{N}$, $B(t) = B = \operatorname{diag}\left(\beta_1\hat{\sigma}_1, \ldots, \beta_n\hat{\sigma}_n\right)$, thus $M(t) = M = \hat{\Sigma}\left(I + B\hat{\Sigma}\right)^{-1}(I + B)$, and finally

$$\Phi^*(t) = M^t\Phi(0).$$

Compared to our general representation, this form has a significant advantage: It transforms the problem of the convergence of the left–product of highly path–dependent matrices into one of the convergence of the powers of a time–invariant matrix.

Proposition 3.5 (a) and (b) give a (generically) sufficient and necessary condition on $\hat{\Sigma}$ to obtain convergence.

Proposition 3.5 (Convergence II). *Let the parental optimization problems be as in (3.4). Then, the following results are satisfied.*

(a) If $\hat{\Sigma}$ *is symmetric positive definite (henceforth: "PD"), then for every* $\beta \in \mathbb{R}_+^n$ *it holds that all eigenvalues of* M *are real and in the interval*

$(0,1]$ *(with at least one eigenvalue equal to* 1*). Thus, generically the sequence* $\left\{\Phi^*(t) = M^t\Phi(0)\right\}_{t\to\infty}$ *converges (for* $\Phi(0)$ *arbitrary).*[8]

(b) *Let* $\hat{\Sigma}$ *have a strictly positive diagonal. If for some eigenvalue* λ *of* $\hat{\Sigma}$ *we have* $Re(\lambda) < |\lambda|^2$,[9] *then there is a* $\beta \in \mathbb{R}^n_+$ *such that the spectral radius of* M *is strictly larger than* 1*. Thus, the sequence* $\left\{\Phi^*(t) = M^t\Phi(0)\right\}_{t\to\infty}$ *does not converge (for* $\Phi(0)$ *arbitrary).*

Proof. In Appendix C 1.4.

Proposition 3.5 (a) shows that PD is generically sufficient for convergence. PD and row stochasticity of a matrix imply in particular that $Re(\lambda) \geq |\lambda|^2$ for any eigenvalue λ of the matrix, i.e. the real part of each eigenvalue is larger than the squared absolute value of this eigenvalue.[10] $\hat{\Sigma}$ having this property is shown to be necessary for convergence (subject to any β) in part (b). In the proof of part (a) we show that if $\hat{\Sigma}$ PD then the eigenvalue property carries over to M. Even more, $\hat{\Sigma}$ PD guarantees that all eigenvalues of M are real and located in the interval $(0,1]$.

As has been mentioned above, the present special case of our general model is basically a transformation of the DeGroot model. Given that convergence is satisfied in the latter, it is intuitive that we also obtain convergence if the transformation (as induced by the parental socialization incentives, which are embodied in β) is small enough. This is confirmed as follows.

Proposition 3.6 (Convergence III). *Let the parental optimization problems be as in (3.4). Then, for every irreducible* $\hat{\Sigma} \in \mathcal{S}(n)$ *with strictly positive diagonal, there exists a nonempty neighborhood* $N\left(\mathbf{0}\middle|\hat{\Sigma}\right) \subset \mathbb{R}^n_+$,[11] *such that* $\forall\beta \in N\left(\mathbf{0}\middle|\hat{\Sigma}\right) \cup \mathbf{0}$*, the sequence* $\left\{\Phi^*(t) = M^t\Phi(0)\right\}_{t\to\infty}$ *converges (for* $\Phi(0)$ *arbitrary).*

Proof. In Appendix C 1.5.

In the proof of this Proposition, we show first that if $\hat{\Sigma}$ is has a strictly positive diagonal, then it has a simple Perron–Frobenius eigenvalue of 1 where the absolute value of all other eigenvalues is located in the interval $(0,1)$. Now, the eigenvalues are continuous in the underlying matrices. Thus, it must be possible to at least slightly perturb $\hat{\Sigma}$ such that the resulting matrices M do also have a unique eigenvalue 1 with the absolute value of

[8] "Generically" applies to all cases where the geometric multiplicity of the 1–eigenvalue equals its algebraic multiplicity; see also Lemma C.2.

[9] $Re(\lambda)$ means the real part of eigenvalue λ.

[10] Indeed an eigenvalue λ of a symmetric positive definite matrice is real and positive such that $\lambda \in (0,1]$ (because of row stochasticity), which implies that $Re(\lambda) \geq |\lambda|^2$.

[11] $N\left(\mathbf{0}\middle|\hat{\Sigma}\right)$ means that the size of the neighborhood around $\beta = \mathbf{0}$ depends on $\hat{\Sigma}$.

all other eigenvalues in the interval $(0, 1)$. Hence, M^t converges. Notably, this does hold even though M is not necessarily positive.

Analytically, we can not pin down the precise sizes of the neighborhoods that guarantee convergence. However, note that for $n = 3/10/50$ and $60,000/160,000/2,700,000$ random draws of uniformly distributed $\hat{\Sigma} \in \mathcal{S}(n)$ and β, the eigenvalue conditions for convergence were satisfied whenever $\beta \leq \mathbf{6/16/270}$.[12] Thus, the spread of socialization weights over a larger number of adults favors convergence — since eventually 'extreme' behaviors of individual adults tend to have a lower relative socialization weight, and thus tend to be evened out. To further illustrate the analytical results of the last Proposition 3.6, consider the following two examples of three dynasties. The first example is

$$\hat{\Sigma} = \begin{bmatrix} 0.40 & 0.30 & 0.30 \\ 0.25 & 0.45 & 0.30 \\ 0.35 & 0.30 & 0.35 \end{bmatrix}, \quad \beta = \begin{bmatrix} 100 \\ 200 \\ 400 \end{bmatrix}$$

with corresponding

$$M = \begin{bmatrix} +0.843 & +0.041 & +0.116 \\ -0.017 & +0.961 & +0.056 \\ +0.060 & +0.015 & +0.925 \end{bmatrix}, \quad Eig\,(M) = \begin{bmatrix} 1.00 \\ 0.93 \\ 0.80 \end{bmatrix},$$

where $Eig(M)$ stands for the eigenvalues of matrix M. The matrix $\hat{\Sigma}$ is comparatively close to a symmetric ultrametric matrix. Given this structure, although the β–vector is large and thus M is not positive (which would have been required in our general representation), the power–sequence of the resulting matrix M does converge (since the eigenvalue conditions are satisfied). The corresponding convergence path is illustrated in the left graph of Figure 3.2 below. In both graphs of the figure, the black/red/green paths correspond to $i = 1/2/3$, and the initial TIs are $\Phi(0) = (0.0, 0.5, 1.0)'$.

The second example is

$$\hat{\Sigma} = \begin{bmatrix} 0.10 & 0.00 & 0.90 \\ 0.90 & 0.10 & 0.00 \\ 0.00 & 0.90 & 0.10 \end{bmatrix}, \quad \beta = \begin{bmatrix} 0.8 \\ 0.9 \\ 1.0 \end{bmatrix}$$

with corresponding

$$M = \begin{bmatrix} +0.114 & -0.086 & +0.972 \\ +0.955 & +0.114 & -0.069 \\ -0.077 & +0.962 & +0.115 \end{bmatrix}, \quad Eig\,(M) = \begin{bmatrix} 1.00 \\ -0.33 + 0.90i \\ -0.33 - 0.90i \end{bmatrix}.$$

[12]More precisely, we sequentially increased the upper bound for β by 1, starting with upper bound 1, and did $10,000$ random draws per step. We stopped the iterations as soon as the eigenvalue–conditions were not satisfied in a given step.

As can be seen, even though the matrix $\hat{\Sigma}$ is 'very far' from a symmetric ultrametric matrix (and also from a symmetric positive definite matrix), the power series of the matrix M converges, since the β–vector is 'small enough' (and even though M is not positive).[13] The resulting convergence path is illustrated in the right graph of Figure 3.2.

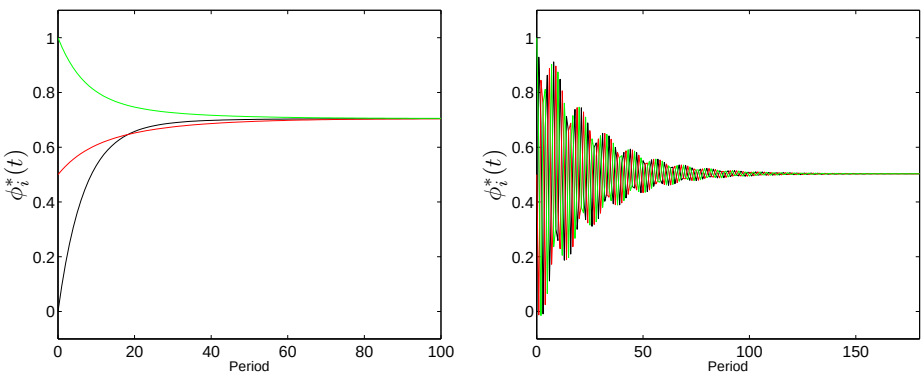

FIGURE 3.2. Adopted Preference Intensities

Obviously, the results for the unrestricted optimization problem of the example above can only be considered an approximation to the results that would be obtained in the restricted case. But note the following. First, subject to convergence in the sense of inter–generationally shrinking lengths of the intervals formed by the minimum and maximum adopted TIs of any generation, restricting the set of possible DTI–choices would basically be a binding matter for initial periods only (since 'closer' adopted TIs induce mutually 'closer' DTI Nash equilibrium choices). Second, the class of quadratic dis–utility functions obviously features the general properties that we require from our utility functions, namely single–peakedness, concavity and zero slopes at the peaks. Thus, the results obtained in the example can not be expected to be non–generic. It us though still an open issue to extend these to our general representation.

[13]The convergence result would not be sustained if one e.g. replaces the β–vector above with $\beta = (8, 15, 18)'$. The resulting eigenvalues would be $\text{Eig}(M) = (1.00, 1.07 + 1.37i, 1.07 - 1.37i)'$.

CHAPTER 4

Conclusions

This paper has incorporated a finite population social network structure into the cultural formation of continuous cultural traits framework of Pichler [49]. After introduction of the latter, we showed the static and dynamic properties of the model when parents perceive their adopted trait intensity as the 'socialization target', and when they are free to choose their behavior subject to an inter–generationally fixed social network.

Thereby, we obtained a behaviorally induced transformation, respectively generalization, of the DeGroot model. This has substantial implications for the evolution of the underlying continuous trait intensities (and opinions). Different to the DeGroot model, our framework does in general allow for the next–period adopted trait intensities to leave the frame that is constituted by the minimum and maximum of the adopted trait intensities of the contemporaneous period. However, to obtain a convergence result, we had to endow the social learning network with sufficient structure. This structure implies dynamics analogous to that in the DeGroot model. For addressing the more general convergence types of our model, we are limited by the insufficient availability of results on the convergence of the left–product of matrices that are not (in general) row–stochastic — and hope for more research on this issue in the future.

Despite this sort of analytical restriction, there is however substantial room for extensions and generalizations of our model. The first is to allow the parents to influence both their socialization weight, as well as the social learning network of their children. This is ongoing work of the authors, and will constitute the future second major part of the present paper. Second, the dynamic analysis could be extended to different types of what Pichler [48] called 'perception rules' (these determine the parental 'socialization targets'). Besides many other open issues in relation to the cultural formation of continuous traits framework (as discussed in Pichler [49, 48]), we emphasize that its logic could be fruitfully and straightforwardly extended to the formation/adoption of traits during the life period of an individual.

APPENDIX A

1. Proofs

1.1. Proof of Proposition 1.1. First note that since by Assumption 1.4, the target functions of the parental optimization problems (1.2) are continuous and since the choice sets are compact (Assumption 1.2), a non–empty set of maximizers, i.e. parental best reply choices, must exist. Consider below any $a \in A$.

Case $\phi^d_{A_a} \neq \hat{\phi}_{\tilde{a}}(R_a)$: It will be sensible to start the proof of this case by showing the second part first. Assume, by way of contradiction, that $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi^d_a(\cdot), \hat{\sigma}_a(\cdot), \phi^d_{A_a}\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = -\operatorname{sign}\left(\phi^d_{A_a} - \hat{\phi}_{\tilde{a}}(R_a)\right)$. For this to hold, it would necessarily have to hold that $\operatorname{sign}\left(\phi^d_a(\cdot) - \hat{\phi}_{\tilde{a}}(R_a)\right) = -\operatorname{sign}\left(\phi^d_{A_a} - \hat{\phi}_{\tilde{a}}(R_a)\right)$ together with $\hat{\sigma}_a(\cdot) > 0$. But this can never be subject to a best reply choice, since e.g. the choice of (the same) $\phi^d_a = \phi^d_a(\cdot)$ together with a $\hat{\sigma}_a < \hat{\sigma}_a(\cdot)$ such that $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi^d_a(\cdot), \hat{\sigma}_a, \phi^d_{A_a}\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = 0$ would yield the same 'own' utility, but strictly larger inter–generational utility as well as strictly lower socialization success share cost. Now assume that $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi^d_a(\cdot), \hat{\sigma}_a(\cdot), \phi^d_{A_a}\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = 0$, for which to hold it would be necessary that $\operatorname{sign}\left(\phi^d_a(\cdot) - \hat{\phi}_{\tilde{a}}(R_a)\right) \in \left\{0, -\operatorname{sign}\left(\phi^d_{A_a} - \hat{\phi}_{\tilde{a}}(R_a)\right)\right\}$ together with $\hat{\sigma}_a(\cdot) > 0$. In this case, the slope of the inter–generational utility function is zero, while the slope of the socialization success share cost function is strictly positive. From this, it follows that there is always an alternative choice pair where $\phi^d_a = \phi^d_a(\cdot)$ and $\hat{\sigma}_a < \hat{\sigma}_a(\cdot)$, thus $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi^d_a(\cdot), \hat{\sigma}_a, \phi^d_{A_a}\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = \operatorname{sign}\left(\phi^d_{A_a} - \hat{\phi}_{\tilde{a}}(R_a)\right)$, but for which it holds that the resulting reduction in the socialization success share cost strictly dominates the inter–generational utility loss. It thus must hold that $\operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi^d_a(\cdot), \hat{\sigma}_a(\cdot), \phi^d_{A_a}\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = \operatorname{sign}\left(\phi^d_{A_a} - \hat{\phi}_{\tilde{a}}(R_a)\right)$.

We will now show the first part of the proof for the present case. Assume, again by way of contradiction, that $\operatorname{sign}\left(\phi^d_a(\cdot) - \phi_a\right) = \operatorname{sign}\left(\phi^d_{A_a} - \hat{\phi}_{\tilde{a}}(R_a)\right)$ and $\hat{\sigma}_a(\cdot) \in [0,1]$. From above, we know that under the present assumption $\operatorname{sign}\left(\phi^d_a(\cdot) - \phi_a\right) = \operatorname{sign}\left(\phi_{\tilde{a}}\left(\phi^d_a(\cdot), \hat{\sigma}_a(\cdot), \phi^d_{A_a}\right) - \hat{\phi}_{\tilde{a}}(R_a)\right)$. It then follows that there always exists an alternative choice pair where $\hat{\sigma}_a = \hat{\sigma}_a(\cdot)$, and where $\operatorname{sign}\left(\phi^d_a - \phi_a\right) = \operatorname{sign}\left(\phi^d_a(\cdot) - \phi_a\right)$ but $\left|\phi^d_a - \phi_a\right| < \left|\phi^d_a(\cdot) - \phi_a\right|$, and

$\text{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d, \hat{\sigma}_a(\cdot), \phi_{A_a}^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = \text{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_{A_a}^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right)$ but $\left|\phi_{\tilde{a}}\left(\phi_a^d, \hat{\sigma}_a(\cdot), \phi_{A_a}^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right| \leq \left|\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_{A_a}^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right|$. Such a choice yields (a) strictly larger 'own' utility, (b) larger inter–generational utility and (c) less cost of achieving $\hat{\sigma}_a(\cdot)$ given (a). Thus, the best replies must satisfy $\text{sign}\left(\phi_a^d(\cdot) - \phi_a\right) \in \left\{0, -\text{sign}\left(\phi_{A_a}^d - \hat{\phi}_{\tilde{a}}(R_a)\right)\right\}$.

Assume next that $\text{sign}\left(\phi_a^d(\cdot) - \phi_a\right) = -\text{sign}\left(\phi_{A_a}^d - \hat{\phi}_{\tilde{a}}(R_a)\right)$ and $\hat{\sigma}_a(\cdot) = 0$. But this can not be a best reply since the choice $\phi_a^d = \phi_a$ and $\hat{\sigma}_a = \hat{\sigma}_a(\cdot) = 0$ would yield (a) strictly larger 'own' utility and (b) identical inter–generational utility and identical socialization success share cost. Hence $\text{sign}\left(\phi_a^d(\cdot) - \phi_a, \hat{\sigma}_a(\cdot)\right) \in \left\{(0,0), (0,+1), \left(-\text{sign}\left(\phi_{A_a}^d - \hat{\phi}_{\tilde{a}}(R_a)\right), +1\right)\right\}$.

Let us from now on consider the case where a choice pair that satisfies the third sign combination of above is available, i.e. the adopted TI does not coincide with the relevant boundary of $\phi^d(X)$.[1] We first rule out that nevertheless $\text{sign}\left(\phi_a^d(\cdot) - \phi_a, \hat{\sigma}_a(\cdot)\right) = (0,+1)$. To see that this can never be a best reply note that at such a choice, the slope of the 'own' utility function is zero. It then follows that there always exists a choice pair where $\hat{\sigma}_a = \hat{\sigma}_a(\cdot)$, and where $\text{sign}\left(\phi_a^d - \phi_a\right) = -\text{sign}\left(\phi_{A_a}^d - \hat{\phi}_{\tilde{a}}(R_a)\right)$, $\text{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d, \hat{\sigma}_a(\cdot), \phi_{A_a}^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = \text{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_{A_a}^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right)$ but $\left|\phi_{\tilde{a}}\left(\phi_a^d, \hat{\sigma}_a(\cdot), \phi_{A_a}^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right| < \left|\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_{A_a}^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right|$, such that the resulting strictly positive gain in inter–generational utility strictly dominates the combined loss in 'own' utility and the increase in the socialization success share cost.

Finally, consider the cases where $\hat{\phi}_{\tilde{a}}(R_a) \geq \phi_a$ and $\phi_{A_a}^d \notin \left(\phi_a, \hat{\phi}_{\tilde{a}}(R_a)\right)$, or $\hat{\phi}_{\tilde{a}}(R_a) \leq \phi_a$ and $\phi_{A_a}^d \notin \left(\hat{\phi}_{\tilde{a}}(R_a), \phi_a\right)$.[2] It rests to show that in these cases $\text{sign}\left(\hat{\sigma}_a(\cdot), \phi_a^d(\cdot) - \phi_a\right) = (0,0)$ can not be subject to a best reply.[3] To see this, note that at such a choice, both the slope of the socialization success share cost function and the slope of the 'own' utility function are zero. But this then again implies that there always exists an alternative choice where $\text{sign}\left(\phi_a^d - \phi_a, \hat{\sigma}_a\right) = \left(-\text{sign}\left(\phi_{A_a}^d - \hat{\phi}_{\tilde{a}}(R_a)\right), +1\right)$, $\text{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d, \hat{\sigma}_a, \phi_{A_a}^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right) = \text{sign}\left(\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_{A_a}^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right)$, but $\left|\phi_{\tilde{a}}\left(\phi_a^d, \hat{\sigma}_a, \phi_{A_a}^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right| < \left|\phi_{\tilde{a}}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot), \phi_{A_a}^d\right) - \hat{\phi}_{\tilde{a}}(R_a)\right|$, and

[1] In the other case, then the best replies satisfy $\text{sign}\left(\phi_a^d(\cdot) - \phi_a, \hat{\sigma}_a(\cdot)\right) \in \{(0,0), (0,+1)\}$. To see that if $\hat{\phi}_{\tilde{a}}(R_a) \geq \phi_a$ and $\phi_{A_a}^d \notin \left(\phi_a, \hat{\phi}_{\tilde{a}}(R_a)\right)$, or $\hat{\phi}_{\tilde{a}}(R_a) \leq \phi_a$ and $\phi_{A_a}^d \notin \left(\hat{\phi}_{\tilde{a}}(R_a), \phi_a\right)$, then the best replies must satisfy the second sign combination follows basically the same line of argumentation as in the rest of the proof below.

[2] In the other cases, no further restriction of the signs is possible, so that we have that $\text{sign}\left(\phi_a^d(\cdot) - \phi_a, \hat{\sigma}_a(\cdot)\right) \in \left\{\left(-\text{sign}\left(\phi_{A_a}^d - \hat{\phi}_{\tilde{a}}(R_a)\right), +1\right), (0,0)\right\}$.

[3] Except for the special case $\phi_{A_a}^d = \hat{\phi}_{\tilde{a}}(R_a) = \phi_a$, see below.

such that the resulting strictly positive gain in inter–generational utility strictly dominates the combined loss in 'own' utility and the increase in the socialization success share cost.

Case $\phi^d_{A_a} = \hat{\phi}_{\tilde{a}}(R_a)$: These best reply choices yield the maximum possible net life–time utility.

1.2. Proof of Proposition 1.2. Denote the Lagrangean of the optimization problem (1.2) of an adult $a \in \mathcal{A}$ as $\mathcal{L}\left(\phi^d_a, \hat{\sigma}_a \middle| \phi^d_{A_a}, \hat{\phi}_{\tilde{a}}(R_a), \phi_a, i_a\right)$, which we will abbreviate subsequently as $\mathcal{L}\left(\phi^d_a, \hat{\sigma}_a \middle| \cdot\right)$. Any pair of best replies, $\left(\phi^d_a(\cdot), \hat{\sigma}_a(\cdot)\right)$ must satisfy the first order conditions. Further, since we assume that the optimization problem is strictly concave at this best reply choice (so that the determinant of the Hessian matrix is strictly positive), all conditions for the Implicit Function Theorem are satisfied.

We will now show that $\exists\, |b_a| \in \mathbb{R}_{++}$, such that if $\frac{\partial^2 \mathcal{L}\left(\phi^d_a(\cdot), \hat{\sigma}_a(\cdot) | \cdot\right)}{\partial \left|\phi^d_a - \phi_a\right| \partial \hat{\sigma}_a} > -|b_a|$ i.e. the two socialization instruments are 'not too strong substitutes' at the parental best reply choice, then the desired signs of Proposition 1.2 hold.

To do this, we will transform the representation of the comparative statics matrix of Proposition 1.2 into a representation that involves only the sensitivities of the best reply choices to the relevant parameters. For this, it will be convenient to distinguish the cases where $\operatorname{sign}\left(\phi^d_{A_a} - \hat{\phi}_{\tilde{a}}(R_a)\right) = +1/-1$, so that by Proposition 1.1, it generically holds that $\operatorname{sign}\left(\phi^d_a(\cdot) - \phi_a\right) = -1/+1$ (the other, 'non–generic', cases are disregarded in Proposition 1.2). Thus, for the results in the first row of the matrix in Proposition 1.2 to hold, we require that

$$\operatorname{sign}\left(\frac{\partial \phi^d_a(\cdot)}{\partial \left|\phi^d_{A_a} - \hat{\phi}_{\tilde{a}}(R_a)\right|} \; \frac{\partial \phi^d_a(\cdot)}{\partial i_a}\right) = (-1/+1 \;\; -1/+1). \tag{A.1}$$

Next, note that $\left|\phi^d_{A_a} - \hat{\phi}_{\tilde{a}}(R_a)\right| = \operatorname{sign}\left(\phi^d_{A_a} - \hat{\phi}_{\tilde{a}}(R_a)\right)\left(\phi^d_{A_a} - \hat{\phi}_{\tilde{a}}(R_a)\right)$, so that the entries of the first column of the matrix of Proposition 1.2 could be decomposed accordingly. It is straightforward to show (by the Implicit Function Theorem) that

$$\operatorname{sign}\left(\frac{\partial \phi^d_a(\cdot)}{\partial \hat{\phi}_{\tilde{a}}(R_a)}, \frac{\partial \hat{\sigma}_a(\cdot)}{\partial \hat{\phi}_{\tilde{a}}(R_a)}\right)' = -\operatorname{sign}\left(\frac{\partial \phi^d_a(\cdot)}{\partial \phi^d_{A_a}}, \frac{\partial \hat{\sigma}_a(\cdot)}{\partial \phi^d_{A_a}}\right)'$$

and, thus, as far as the signs of the comparative statics are concerned, it is irrelevant, how a marginal change in the absolute distance between $\phi^d_{A_a}$ and $\hat{\phi}_{\tilde{a}}(R_a)$ is 'composed', and we can restrict our attention to marginal changes of $\phi^d_{A_a}$ only. Thus, for (A.1) to hold, it is necessary that

$$\operatorname{sign}\begin{pmatrix} \frac{\partial \phi^d_a(\cdot)}{\partial \phi^d_{A_a}} & \frac{\partial \phi^d_a(\cdot)}{\partial i_a} \\ \frac{\partial \hat{\sigma}_a(\cdot)}{\partial \phi^d_{A_a}} & \frac{\partial \hat{\sigma}_a(\cdot)}{\partial i_a} \end{pmatrix} = \begin{pmatrix} -1/-1 & -1/+1 \\ +1/-1 & +1/+1 \end{pmatrix}. \tag{A.2}$$

We can now use the Implicit Function Theorem to derive a necessary condition for these signs to hold. First note that since the Lagrangean is strictly concave at the best reply choice, the second partial derivatives with respect to the two decision variables are strictly negative, while as the cross second partial derivative

$$\frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)\,|\cdot\right)}{\partial \phi_a^d \, \partial \hat{\sigma}_a} = \frac{\partial^2 v\left(\phi_{\tilde{a}}(\cdot)\,\middle|\hat{\phi}_{\tilde{a}}(R_a), i_a\right)}{\partial \phi_{\tilde{a}}^2} \hat{\sigma}_a(\cdot)\left(\phi_a^d(\cdot) - \phi_{A_a}^d\right) +$$
$$+ \frac{\partial v\left(\phi_{\tilde{a}}(\cdot)\,\middle|\hat{\phi}_{\tilde{a}}(R_a), i_a\right)}{\partial \phi_{\tilde{a}}} - \frac{\partial u^{\phi_a}\left(\phi_a^d(\cdot)\right)}{\partial \phi_a^d} \frac{\partial^2 c\left(\hat{\sigma}_a(\cdot), \partial u\left(\phi_a^d(\cdot)\,|\phi_a\right)\right)}{\partial u\left(\phi_a^d(\cdot)\,|\phi_a\right) \partial \hat{\sigma}_a}$$

is ambiguous in sign. It is furthermore straightforward to show that

$$\operatorname{sign}\begin{pmatrix} \frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \phi_a^d \, \partial \phi_{A_a}^d} & \frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \phi_a^d \, \partial i_a} \\ \frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \hat{\sigma}_a \, \partial \phi_{A_a}^d} & \frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \hat{\sigma}_a \, \partial i_a} \end{pmatrix} = \begin{pmatrix} -1/-1 & -1/+1 \\ +1/-1 & +1/+1 \end{pmatrix}.$$

Given these signs, it follows from the Implicit Function Theorem that (A.2) is true if $\frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \phi_a^d \, \partial \hat{\sigma}_a} < / > b_a \in \mathbb{R}_{++} / \mathbb{R}_{--}$ where

$$b_a = \min / \max$$
$$\begin{pmatrix} \frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \hat{\sigma}_a^2} \frac{\frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \phi_a^d \, \partial \phi_{A_a}^d}}{\frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \hat{\sigma}_a \, \partial \phi_{A_a}^d}} & \frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \hat{\sigma}_a^2} \frac{\frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \phi_a^d \, \partial i_a}}{\frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \hat{\sigma}_a \, \partial i_a}} \\ \frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \phi_a^{d\,2}} \frac{\frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \hat{\sigma}_a \, \partial \phi_{A_a}^d}}{\frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \phi_a^d \, \partial \phi_{A_a}^d}} & \frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \phi_a^{d\,2}} \frac{\frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \hat{\sigma}_a \, \partial i_a}}{\frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \phi_a^d \, \partial i_a}} \end{pmatrix}$$

Remembering that $\operatorname{sign}\left(\phi_a^d(\cdot) - \phi_a\right) = -1/+1$, this condition is equivalent to requiring that $\frac{\partial^2 \mathcal{L}\left(\phi_a^d(\cdot), \hat{\sigma}_a(\cdot)|\cdot\right)}{\partial \left|\phi_a^d - \phi_a\right| \partial \hat{\sigma}_a} > -|b_a|$. □

1.3. Proof of Proposition 1.3. We will show here that all conditions to apply the Nash Equilibrium existence results of Rath [53] and Pichler [50] are satisfied.

First, our player set $A = [0,1]$ is endowed with Lebesgue measure λ, and $\{A_J\}_{J=1}^K$ is a measurable partition of A. Second, all players have an identical compact (and convex) action space $\phi^d(X) \times [0,1] \subset \mathbb{R}^2$. Further, $\forall a \in A$, $\mathcal{P}(\cdot, \cdot\,|P_a)$ is defined on $\left(\phi^d(X) \times [0,1]\right)^{K+1} =: Z$,[4] continuous since $u(\cdot\,|\phi_a)$, $v\left(\cdot\,\middle|\hat{\phi}_{\tilde{a}}(R_a), i_a\right)$, c and $\hat{\phi}_{\tilde{a}}$ are continuous,[5] and real valued. Since they are also bounded (as the functions are continuous and defined on a

[4]This representation is in line with that of Rath [53] and Pichler [50] since they show that the space where the average strategies of the player subsets can be located in coincides with the convex hull of the individual action spaces.

[5]Remember that the latter might depend on the own strategies of the players or on the average strategies of the player subsets.

compact set), thus their sup norm is well defined, it follows that $\mathcal{P}(\cdot,\cdot\,|P_a) \in \mathcal{U}_Z$, where $\mathcal{U}_Z$ denotes the set of all real valued continuous functions defined on Z endowed with sup norm topology.

Now, note that

$$\{P_a\}_{a\in A} : A \mapsto \left(\operatorname{con}\phi^d(X) \times \mathbb{R}_+ \times \left(\{a\} \cup \mathcal{A} \times \mathcal{C}^0\right) \times \Delta^{K-1}\right)^A,^6 \quad \text{(A.3)}$$

where Δ^{K-1} denotes the $K-1$–dimensional unit simplex, and that

$$\{\mathcal{P}(\cdot,\cdot\,|P_a)\}_{a\in A} : \left(\operatorname{con}\phi^d(X) \times \mathbb{R}_+ \times \left(\{a\} \cup \mathcal{A} \times \mathcal{C}^0\right) \times \Delta^{K-1}\right)^A \mapsto (\mathcal{U}_Z)^A.$$

It follows that we can (equivalently) represent the parametrized games

$$(\Gamma_{P_a})_{a\in A} = \left(A, \left(\phi^d(X) \times [0,1]\right)^A, \{\mathcal{P}(\cdot,\cdot\,|P_a)\}_{a\in A}\right)$$

as they are denoted in the main text by a function $g : A \mapsto g(A) \subset (\mathcal{U}_Z)^A$. To see that g is measurable, consider the σ–algebra of Lebesgue measurable sets of A, $\mathcal{L}(A)$, and the σ–algebra generated by $g(A)$, $\sigma(g(A))$ (or any other suitable σ–algebra over $g(A)$). Now since it must hold that $\forall s \in \sigma(g(A))$ $g^{-1}(s) \in \mathcal{L}(A)$ it follows that the function g is Lebesgue–measurable. □

1.4. Proof of Proposition 1.4. First note that all conditions for Proposition 1.3 to hold are satisfied. Second, let us denote the identical representative DTI of the general social environment of all children as ϕ^d_A.

Consider now any period and any $\{\phi_a, i_a\}_{a\in A} \in \left(\operatorname{con}\phi^d(X) \times \mathbb{R}_+\right)^A$. Let $a^m := \{a \in A\,|\phi_a = \phi^m\}$ and $a_m := \{a \in A\,|\phi_a = \phi_m\}$ (confer Definition 1.3 (a)). Assume that $\phi^m - \phi_m > 0$ and that $\lambda(A\backslash a^m) > 0$ and $\lambda(A\backslash a_m) > 0$ (otherwise, we have the case of a symmetric TI point).

(a) 1. First, we will show that in Nash equilibrium $\phi^{d^*}_A \in [\phi_m, \phi^m]$. To see this consider the parental best replies to $\phi^d_A > \phi^m$. From Proposition 1.1 (a), it follows that in this case $\forall a \in A$, $\phi^d_a(\cdot) < \phi^m$. Since in any Nash equilibrium, almost all adults choose best reply strategies (see Definition 1.2), it follows that $\phi^{d^*}_A \leq \phi^m$ must hold. Analogously, $\phi^{d^*}_A \geq \phi_m$ must hold.

For the next step, let us denote with A^N the set of adults that choose best reply strategies in the Nash equilibrium of a given period (where $\lambda\left(A^N\right) = 1$). Assume that $\phi^{d^*}_A = \phi^m$. Again by Proposition 1.1 (a), it then follows that for every $a \in a^m \cap A^N$ $\phi_{\tilde{a}}(\phi^m, 0, \phi^m) = \phi^m$, and for every $a' \in A^N\backslash a^m$ $\phi_{\tilde{a}}\left(\phi^{d^*}_{a'}, \hat{\sigma}^*_a, \phi^m\right) \in (\phi_{a'}, \phi^m)$. We can conclude that $\phi_m < \min_{a\in A^N} \phi_{\tilde{a}}\left(\phi^{d^*}_a, \hat{\sigma}^*_a, \phi^m\right) < \max_{a\in A^N} \phi_{\tilde{a}}\left(\phi^{d^*}_a, \hat{\sigma}^*_a, \phi^m\right) = \phi^m$. Analogously,

[6] Note that the notation in Rath [53] and Pichler [50] would here rather be $\{P_a\}_{a\in A} : A \mapsto \operatorname{con}\phi^d(X) \times \mathbb{R}_+ \times \left(\{a\} \cup \mathcal{A} \times \mathcal{C}^0\right) \times \Delta^{K-1}$. However, we will here and subsequently stick to the notation analogous to the main text of the present paper.

if $\phi_A^{d^*} = \phi_m$ then $\phi_m = \min_{a \in A^N} \phi_{\tilde{a}} \left(\phi_a^{d^*}, \hat{\sigma}_a^*, \phi_m\right) < \max_{a \in A^N} \phi_{\tilde{a}} \left(\phi_a^{d^*}, \hat{\sigma}_a^*, \phi_m\right) < \phi^m$.

Assume next that $\phi_A^{d^*} \in (\phi_m, \phi^m)$. In this case it follows by Proposition 1.1 (a) that for every $a \in A^N$ such that $\phi_a \in \left(\phi_A^{d^*}, \phi^m\right]$ it must hold that $\phi_{\tilde{a}} \left(\phi_a^{d^*}, \hat{\sigma}_a^*, \phi_A^{d^*}\right) \in \left(\phi_A^{d^*}, \phi_a\right)$, and for every $a \in A^N$ such that $\phi_a \in \left[\phi_m, \phi_A^{d^*}\right)$, we have $\phi_{\tilde{a}} \left(\phi_a^{d^*}, \hat{\sigma}_a^*, \phi_A^{d^*}\right) \in \left(\phi_a, \phi_A^{d^*}\right)$. It follows that $\phi_m < \min_{a \in A^N} \phi_{\tilde{a}} \left(\phi_a^{d^*}, \hat{\sigma}_a^*, \phi_A^{d^*}\right) < \max_{a \in A^N} \phi_{\tilde{a}} \left(\phi_a^{d^*}, \hat{\sigma}_a^*, \phi_A^{d^*}\right) < \phi^m$.

We can conclude that under the conditions of Proposition 1.4 (a), $\phi_m \leq \tilde{\phi}_m < \tilde{\phi}^m < \phi^m$ or $\phi_m < \tilde{\phi}_m < \tilde{\phi}^m \leq \phi^m$ almost surely.

(b) 1. If additionally the identical oblique socialization weights are strictly positive for all subsets of adults, then it even holds that in Nash equilibrium $\phi_A^{d^*} \in (\phi_m, \phi^m)$. To see this consider the parental best replies to $\phi_A^d = \phi^m$. From Proposition 1.1 (a), it follows that in this case $\forall a \in a^m$, $\phi_a^d(\cdot) = \phi^m$ and $\forall a' \in A \backslash a^m$, $\phi_{a'}^d(\cdot) < \phi^m$. Since in any Nash equilibrium almost all adults choose best reply strategies, and since $\lambda\left(A \backslash a^m\right) > 0$, it then follows that $\phi_A^{d^*} < \phi^m$ must hold. By the same logic, $\phi_A^{d^*} > \phi_m$.

It follows (analogously to before) that $\phi_m < \tilde{\phi}_m < \tilde{\phi}^m < \phi^m$ almost surely.

(a+b) 2. Since for any two succeeding periods the TIs (weakly) assimilate almost surely for any tuple of pairs of (first period) TIs and inter–generational TIs, it follows that for any tuple of initial TIs coupled with any tuple of inter–generational TIs, the TIs converge to a symmetric TI point.

(a+b) 3. We will finally show that indeed any symmetric TI point is a steady state. Consider any symmetric TI point and denote the according TI as $\phi \in \text{con } \phi^d(X)$. Denote the set of adults that have this TI as A^s, where $\lambda\left(A^s\right) = 1$. We will show first that $\phi_A^{d^*} = \phi$. To see this, simply note that by Proposition 1.1 (a) the best replies to the cases where $\phi_A^d <> \phi$ must satisfy that $\forall a \in A^s$, $\phi_a^d(\cdot) >< \phi_a = \phi$. Thus, only the case $\phi_A^d = \phi$ can be supported by best replies of the adults of $A^s \cap A^N$, since $\lambda\left(A^s \cap A^N\right) = 1$. Given $\phi_A^{d^*} = \phi$ it then follows from Proposition 1.1 (b) that $\forall a \in A^s \cap A^N$, $\left(\phi_a^{d^*}, \hat{\sigma}_a^*\right) = (\phi, 0)$ and $\phi_{\tilde{a}}\left(\phi, 0, \phi\right) = \phi$. $\square$

APPENDIX B

1. Proofs

Many parts of the proofs below follow straightforwardly from the general characteristics of parental best reply choices shown in Proposition 1 in Pichler [49] (these characteristics must also hold for the individual best reply choices in a SNE). For ease of reference, we replicate this proposition here, which requires the following additional notation. For any $a \in A$, we will denote any pair of best reply choices (which are chosen against the representative DTI and subject to the optimally perceived TI, adopted and inter–generational TI) as

$$\left(\phi_a^d(t)\left(\phi_{A_a}^d(t), \hat{\phi}_a\left(R_a\right), \phi_a(t), i_a\right), \hat{\sigma}_a(t)\left(\phi_{A_a}^d(t), \hat{\phi}_a\left(R_a\right), \phi_a(t), i_a\right)\right)$$

which we will abbreviate below as $\left(\phi_a^d(t)\left(\cdot\right), \hat{\sigma}_a(t)\left(\cdot\right)\right)$. Furthermore, the resulting best reply location of the adult child's adopted TI will be denoted $\phi_a(t+1)\left(\phi_a^d(t)\left(\cdot\right), \hat{\sigma}_a(t)\left(\cdot\right), \phi_{A_a}^d(t)\right)$.

Proposition B.1 (Characterization of Best Replies). *Let Assumptions 2.1–1.3 hold. Then, $\forall t \in \mathbb{N}$, $\forall a \in A$, if*

(a) $\phi_{A_a}^d(t) \neq \hat{\phi}_a\left(R_a\right)$, it holds generically that $\operatorname{sign}\left(\phi_a^d(t)\left(\cdot\right) - \phi_a(t)\right) = -\operatorname{sign}\left(\phi_{A_a}^d(t) - \hat{\phi}_a\left(R_a\right)\right)$ *and $\hat{\sigma}_a(t)\left(\cdot\right) > 0$, while it always holds that*

$$\operatorname{sign}\left(\phi_a(t+1)\left(\phi_a^d(t)\left(\cdot\right), \hat{\sigma}_a(t)\left(\cdot\right), \phi_{A_a}^d(t)\right) - \hat{\phi}_a\left(R_a\right)\right) = \operatorname{sign}\left(\phi_{A_a}^d(t) - \hat{\phi}_a\left(R_a\right)\right).^1$$

(b) $\phi_{A_a}^d(t) = \hat{\phi}_a\left(R_a\right)$, it holds that $\phi_a^d(t)\left(\cdot\right) - \phi_a(t) = 0$ and $\hat{\sigma}_a\left(\cdot\right) = 0$, hence $\phi_a(t+1)\left(\phi_a(t), 0, \hat{\phi}_a\left(R_a\right)\right) - \hat{\phi}_a\left(R_a\right) = 0$.

Proof. Confer the proof of Proposition 1 in Pichler [49].

1.1. Proof of Proposition 2.1. First note that the sets of maximizers of the best reply problems (1.2) are nonempty and single–valued by the Weierstrass Theorem (the choice sets are compact and the target functions of the best reply problems are continuous) and strict concavity. Denote $\phi_A^d(t) = \phi_A^d(t)\left(\phi_L^d(t), \phi_H^d(t)\right) = (1-q_H)\phi_L^d(t) + q_H\phi_H^d(t)$, and $\hat{\sigma}_G(t) := \frac{1}{q_G}\int_{g\in G}\hat{\sigma}_g(t)\ d\lambda(g)$, $G = L, H$ (the average socialization success

shares of the groups). Let us now represent the parental best reply problems as those of choosing best replies against the tuple $\left\{\phi_G^d(t), \hat{\sigma}_G(t)\right\}_{G=L,H} \in \left(\phi^d(X) \times [0,1]\right)^2$.[2] Then, if in the given period $t \in \mathbb{N}$, the adopted TIs of the adults of the same group are identical, then the (unique) best reply pairs against such a tuple are identical for all adults of the same group. We will denote these identical pairs of best replies as

$$\left\{\phi_G^d(t)\left(\left\{\phi_{G'}^d(t), \hat{\sigma}_{G'}(t)\right\}_{G'=L,H}\right), \hat{\sigma}_G(t)\left(\left\{\phi_{G'}^d(t), \hat{\sigma}_{G'}(t)\right\}_{G'=L,H}\right)\right\}_{G=L,H}$$
$$\in \left(\phi^d(X) \times [0,1]\right)^2.$$

We thus obtained a continuous function (upper hemicontinuity follows by Berge's Theorem of the Maximum, which implies continuity since the best reply pairs are single valued) from a compact and convex set into itself, $\left(\phi^d(X) \times [0,1]\right)^2 \mapsto \left(\phi^d(X) \times [0,1]\right)^2$. Thus, by Brouwer's Fixed Point Theorem, a fixed point where for every $G = L, H$

$$\left(\phi_G^d(t)\left(\left\{\phi_{G'}^d(t), \hat{\sigma}_{G'}(t)\right\}_{G'=L,H}\right), \hat{\sigma}_G(t)\left(\left\{\phi_{G'}^d(t), \hat{\sigma}_{G'}(t)\right\}_{G'=L,H}\right)\right)$$
$$= \left(\phi_G^d(t), \hat{\sigma}_G(t)\right)$$

exists. This constitutes a SNE.

Now, since in the initial period, the adopted TIs of the members of the same cultural group are assumed to be identical, a SNE exists in the initial period. If all parents choose the SNE strategies, it follows immediately that all children of the same cultural group adopt the same TIs (using the SNE strategies for substitution in the TI formation rule (1.1)), i.e. the adults of the same cultural group have identical adopted TIs in the second period. Applying this process iteratedly, it follows that a path of SNEs exists.

1.2. Proof of Proposition 2.2. That Proposition 2.2 holds follows as an immediate consequence of the Lemma below. This shows the range of the phase vectors, $(\Delta\phi_L(t), \Delta\phi_H(t))$, where $\Delta\phi_G(t) := \phi_G(t+1) - \phi_G(t)$, $G = L, H$, depicted in Figure 2.1. Notably, these results hold $\forall \left\{\phi_G^{d*}(t), \hat{\sigma}_G^*(t)\right\}_{G=L,H} \in E(P(t))$.

Lemma B.1 (Phase Vectors).

(a) If $\phi_H(t) \geq e_H$ and $\phi_H(t) \geq \phi_L(t)$ then $\Delta\phi_H(t) < 0$; and if $\phi_L(t) \leq e_L$ and $\phi_L(t) \leq \phi_H(t)$ then $\Delta\phi_L(t) > 0$.

(b) If $\phi_L(t) \geq \phi_H(t) \geq e_H$ then $\Delta\phi_L(t) < 0$; and if $e_L \geq \phi_L(t) \geq \phi_H(t)$ then $\Delta\phi_H(t) > 0$.

[2]The parental best reply problems are actually independent of the average socialization success shares (thus, effectively 'constant' with respect to these).

(c) If $\phi_L(t) \geq e_L$, $\phi_L(t) \geq \phi_H(t)$ and $\phi_H(t) \leq e_H$ then $\Delta\phi_H(t) > \Delta\phi_L(t)$.

Proof. Before we start this proof, note that in the notation of the SNE quantities below, their dependence on $P(t)$ is not indicated for brevity. Also, we will denote with $\phi_G^d(t)\left(\phi_A^{d^*}(t)\right)$ the identical (and unique) best reply choices to the location of the average (SNE–)DTI of any individual member of the cultural groups $G \in \{L, H\}$ (again, the dependence on other parameters is not indicated). Note also that the indication (PB.1) will indicate that all claims that follow directly from Proposition B.1.

(a) Let $\phi_H(t) \geq e_H$ and $\phi_H(t) \geq \phi_L(t)$. Then $\phi_A^{d^*}(t) < \phi_H(t)$, thus $\Delta\phi_H(t) < 0$ (PB.1). Suppose, by ways of contradiction, that $\phi_A^{d^*}(t) \geq \phi_H(t)$. In this case, $\phi_H^d(t)\left(\phi_A^{d^*}(t)\right) \leq \phi_H(t)$ (PB.1). Furthermore, $\phi_L^d(t)\left(\phi_A^{d^*}(t)\right) \leq \phi_L(t)$ if $\phi_L(t)$ is larger than the lower bound of $\phi^d(X)$; and if $\phi_L(t)$ equals that lower bound, then $\phi_H(t) > \phi_L^d(t)\left(\phi_A^{d^*}(t)\right) = \phi_L(t)$ (PB.1). This contradicts $\phi_A^{d^*}(t) \geq \phi_H(t)$ being supported by best reply choices.

The proof for the 'opposite' case of $\phi_L(t) \leq e_L$ and $\phi_L(t) \leq \phi_H(t)$ is analogous.

(b) Let $\phi_L(t) \geq \phi_H(t) \geq e_H$. Then $\phi_A^{d^*}(t) < \phi_L(t)$, thus $\Delta\phi_L(t) < 0$ (PB.1). Suppose, again by ways of contradiction, that $\phi_A^{d^*}(t) \geq \phi_L(t)$. In this case, $\phi_L^d(t)\left(\phi_A^{d^*}(t)\right) < \phi_L(t)$ and $\phi_H^d(t)\left(\phi_A^{d^*}(t)\right) \leq \phi_H(t)$ (PB.1). This yields a contradiction again.

The proof for the 'opposite' case of $e_L \geq \phi_L(t) \geq \phi_H(t)$ is again analogous.

(c) Let $\phi_L(t) \geq e_L$, $\phi_L(t) \geq \phi_H(t)$ and $\phi_H(t) \leq e_H$ (note that at least one inequality must be strict). Then, if $\phi_A^{d^*}(t) \in [\phi_H(t), \phi_L(t)]$ it follows immediately that $\Delta\phi_H(t) \geq 0 \geq \Delta\phi_L(t)$, with at least one inequality strict (PB.1). The case $\phi_A^{d^*}(t) \notin [\phi_H(t), \phi_L(t)]$ can only be supported by best reply choices if $\phi_L^{d^*}(t) < \phi_A^{d^*}(t) < \phi_H^{d^*}(t)$. This must be true since by (PB.1) if $\phi_A^{d^*}(t) < \phi_H(t)$, then $\phi_H^d(t)\left(\phi_A^{d^*}(t)\right) \geq \phi_H(t)$; and if $\phi_A^{d^*}(t) > \phi_L(t)$, then $\phi_L^d(t)\left(\phi_A^{d^*}(t)\right) \leq \phi_L(t)$. Thus, $\phi_H(t+1) > \phi_L(t+1) \Rightarrow \Delta\phi_H(t) > \Delta\phi_L(t)$. □

1.3. Proof of Proposition 2.3. First, we show that $\phi_A^{d^*}(t) \in (e_L, e_H)$. Suppose, by ways of contradiction, that $\phi_A^{d^*}(t) \geq e_H$. But then, $\phi_H^d(t)\left(\phi_A^{d^*}(t)\right) \leq \phi_H(t)$ while $\phi_L^d(t)\left(\phi_A^{d^*}(t)\right) < \phi_L(t)$. This contradicts $\phi_A^{d^*}(t) \geq \phi_H(t)$ being supported by best reply choices. The analogous logic yields a contradiction for the case of $\phi_A^{d^*}(t) \leq \phi_L(t)$. Thus, by Proposition B.1, $\phi_H^d(t)\left(\phi_A^{d^*}(t)\right) \geq \phi_H(t)$, and $\phi_L^d(t)\left(\phi_A^{d^*}(t)\right) \leq \phi_L(t)$. We show next that the inequalities

must be strict. To see this, simply note that if $\phi_A^{d^*}(t) < \phi_H(t)$, then $\phi_H^d(t)\left(\phi_A^{d^*}(t)\right) > \phi_H(t)$, while if $\phi_A^{d^*}(t) \geq \phi_H(t)$, then this can only hold if $\phi_H^{d^*}(t) > \phi_H(t)$. The analogous logic yields the result that $\phi_L^{d^*}(t) < \phi_L(t)$. □

1.4. Proof of Proposition 2.4. Consider the case $\phi_H(t) > \phi_L(t)$. The first step of this proof is identical to the proof of Proposition 2 — only that we replace the compact and convex set $\left(\phi^d(X) \times [0,1]\right)^2$ by the set

$$\left\{\phi_L^d(t) \in \phi^d(X) \,\middle|\, \phi_L^d(t) \leq \phi_L(t)\right\} \times \left\{\phi_H^d(t) \in \phi^d(X) \,\middle|\, \phi_H^d(t) \geq \phi_H(t)\right\} \times [0,1]^2,$$

i.e. we show that a SNE must exist in this set. To see that a mapping from this set into itself can be constructed (with the same properties as in the proof of Proposition 2), assume that $\phi_H^d(t) \geq \phi_H(t) > \phi_L(t) \geq \phi_L^d(t)$, in which case $\phi_A^d(t) \in \left(\phi_L^d(t), \phi_H^d(t)\right)$. From Proposition B.1 we know that the best replies must then satisfy $\phi_H^d(t)\left(\left\{\phi_G^d(t), \hat{\sigma}_G(t)\right\}_{G=L,H}\right) \geq \phi_H(t)$ and $\phi_L^d(t)\left(\left\{\phi_G^d(t), \hat{\sigma}_G(t)\right\}_{G=L,H}\right) \leq \phi_L(t)$ (given the present specification of the perception rules).

From Proposition B.1, we know additionally that, under the conditions above, whenever $\phi_H(t)/\phi_L(t)$ is strictly smaller/larger than the upper/lower bound of the set of possible DTIs, then the latter equalities must be strict (so that these property must also hold in a SNE). That these SNE DTI choices can only be coupled with $\hat{\sigma}_G^*(t) > 0$ and $\hat{\sigma}_G^*(t) < 1$ holds immediately by Proposition B.1 (in the latter case since otherwise the adopted TIs of all children would coincide with the parents' optimal TIs). This finalizes the proof of part (a) of Proposition 2.4, since the proof for the case $\phi_H(t) < \phi_L(t)$ is analogous.

To see part (b) simply note that if $\phi_H(t) = \phi_L(t) = \phi$, then it must hold that $\left(\phi_G^d(t)\left(\{\phi, \hat{\sigma}_{G'}(t)\}_{G'=L,H}\right), \hat{\sigma}_G(t)\left(\{\phi, \hat{\sigma}_{G'}(t)\}_{G'=L,H}\right)\right) = (\phi, 0)$, $\forall G = L, H$. □

1.5. Proof of Proposition 2.5. Using the linearization of the phase vectors at any steady state where $(\phi_L, \phi_H) = (\phi, \phi)$, $\phi \in \phi^d(X)$, subject to any $(i_L, i_H, q_H) \in \mathbb{R}_+^2 \times (0,1)$ (we will drop these parameters below for brevity), it follows that locally around such a steady state

$$\begin{pmatrix} \Delta\phi_L\left(\phi_L, \phi_H\right) \\ \Delta\phi_H\left(\phi_L, \phi_H\right) \end{pmatrix} = \begin{pmatrix} \frac{\partial\,\Delta\phi_L(\phi,\phi)}{\partial\,\phi_L} & \frac{\partial\,\Delta\phi_L(\phi,\phi)}{\partial\,\phi_H} \\ \frac{\partial\,\Delta\phi_H(\phi,\phi)}{\partial\,\phi_L} & \frac{\partial\,\Delta\phi_H(\phi,\phi)}{\partial\,\phi_H} \end{pmatrix} \begin{pmatrix} \phi_L - \phi \\ \phi_H - \phi \end{pmatrix}.$$

Remember that we consider only relative position preserving SNE choices, which by Proposition 2.4 (b) implies that $\phi_L^{d^*}(\phi,\phi) = \phi_H^{d^*}(\phi,\phi) = \phi$ and $\hat{\sigma}_G^*(\phi,\phi) = 0$, $G = L, H$. From equation (2.3), it follows that we obtain for

$G = L, H$

$$\frac{\partial\,\dot{\phi}_G\,(\phi,\phi)}{\partial\,\phi_G} = \frac{\partial\,\phi_G^{d^*}\,(\phi,\phi)}{\partial\,\phi_G} - \left(\frac{\partial\,\phi_G^{d^*}\,(\phi,\phi)}{\partial\,\phi_G} - \frac{\partial\,\phi_{-G}^{d^*}\,(\phi,\phi)}{\partial\,\phi_G}\right)(1-q_G) - 1$$

$$\frac{\partial\,\dot{\phi}_G\,(\phi,\phi)}{\partial\,\phi_{-G}} = \frac{\partial\,\phi_G^{d^*}\,(\phi,\phi)}{\partial\,\phi_{-G}} - \left(\frac{\partial\,\phi_G^{d^*}\,(\phi,\phi)}{\partial\,\phi_{-G}} - \frac{\partial\,\phi_{-G}^{d^*}\,(\phi,\phi)}{\partial\,\phi_{-G}}\right)(1-q_G).$$

To obtain the necessary partial derivatives above, we will use the Implicit Function Theorem as follows. First, the sets of SNE are implicitly described by a system of four equations obtained by setting $\left(\phi_g^d, \hat{\sigma}_g\right) = \left(\phi_G^d, \hat{\sigma}_G\right)$, $G = L, H$, in any of the (identical) individual first order conditions of the best reply problems (1.2) of the parents of both groups. To relate this system of equations to the originating FOCs of the individual parents, we denote it $\left(\mathcal{L}_L^{\phi_L^d}\,(\phi_L,\phi_H)\,, \mathcal{L}_L^{\hat{\sigma}_L}\,(\phi_L,\phi_H)\,, \mathcal{L}_H^{\phi_H^d}\,(\phi_L,\phi_H)\,, \mathcal{L}_H^{\hat{\sigma}_H}\,(\phi_L,\phi_H)\right)' = (0,0,0,0)'$ (i.e. the first equation results from the partial derivative of the Lagrangeans of the parents of group L with respect to their DTI choice, etc.). Remembering that both utility functions have zero slopes at their peaks (and at any of the steady states we consider, the children's adopted TIs do indeed coincide with the optimal TIs), we obtain at any $(\phi,\phi) \in \phi^d(X)^2$

$$\begin{pmatrix} \frac{\partial\,\mathcal{L}_L^{\phi_L^d}(\phi,\phi)}{\partial\,\phi_L^d} & \frac{\partial\,\mathcal{L}_L^{\phi_L^d}(\phi,\phi)}{\partial\,\hat{\sigma}_L} & \frac{\partial\,\mathcal{L}_L^{\phi_L^d}(\phi,\phi)}{\partial\,\phi_H^d} & \frac{\partial\,\mathcal{L}_L^{\phi_L^d}(\phi,\phi)}{\partial\,\hat{\sigma}_H} \\ \frac{\partial\,\mathcal{L}_L^{\hat{\sigma}_L}(\phi,\phi)}{\partial\,\phi_L^d} & \frac{\partial\,\mathcal{L}_L^{\hat{\sigma}_L}(\phi,\phi)}{\partial\,\hat{\sigma}_L} & \frac{\partial\,\mathcal{L}_L^{\hat{\sigma}_L}(\phi,\phi)}{\partial\,\phi_H^d} & \frac{\partial\,\mathcal{L}_L^{\hat{\sigma}_L}(\phi,\phi)}{\partial\,\hat{\sigma}_H} \\ \frac{\partial\,\mathcal{L}_H^{\phi_L^d}(\phi,\phi)}{\partial\,\phi_L^d} & \frac{\partial\,\mathcal{L}_H^{\phi_L^d}(\phi,\phi)}{\partial\,\hat{\sigma}_L} & \frac{\partial\,\mathcal{L}_H^{\phi_L^d}(\phi,\phi)}{\partial\,\phi_H^d} & \frac{\partial\,\mathcal{L}_H^{\phi_L^d}(\phi,\phi)}{\partial\,\hat{\sigma}_H} \\ \frac{\partial\,\mathcal{L}_H^{\hat{\sigma}_H}(\phi,\phi)}{\partial\,\phi_L^d} & \frac{\partial\,\mathcal{L}_H^{\hat{\sigma}_H}(\phi,\phi)}{\partial\,\hat{\sigma}_L} & \frac{\partial\,\mathcal{L}_H^{\hat{\sigma}_H}(\phi,\phi)}{\partial\,\phi_H^d} & \frac{\partial\,\mathcal{L}_H^{\hat{\sigma}_H}(\phi,\phi)}{\partial\,\hat{\sigma}_H} \end{pmatrix} = diag \begin{pmatrix} \frac{\partial^2\,u(\phi|\phi)}{\partial\,\phi_L^{d\,2}} \\ -\frac{\partial^2\,c(0)}{\partial\,\hat{\sigma}_L^2} \\ \frac{\partial^2\,u(\phi|\phi)}{\partial\,\phi_H^{d\,2}} \\ -\frac{\partial^2\,c(0)}{\partial\,\hat{\sigma}_H^2} \end{pmatrix}$$

By the strict concavity of the own utility functions and the strict convexity of the cost function, it follows that the determinant of the latter matrix is strictly positive, and finally all conditions to apply the Implicit Function Theorem are satisfied. To do so, we have to use

$$\begin{pmatrix} \frac{\partial\,\mathcal{L}_L^{\phi_L^d}(\phi,\phi)}{\partial\,\phi_L} & \frac{\partial\,\mathcal{L}_L^{\phi_L^d}(\phi,\phi)}{\partial\,\phi_H} \\ \frac{\partial\,\mathcal{L}_L^{\hat{\sigma}_L}(\phi,\phi)}{\partial\,\phi_L} & \frac{\partial\,\mathcal{L}_L^{\hat{\sigma}_L}(\phi,\phi)}{\partial\,\phi_H} \\ \frac{\partial\,\mathcal{L}_H^{\phi_H^d}(\phi,\phi)}{\partial\,\phi_L} & \frac{\partial\,\mathcal{L}_H^{\phi_H^d}(\phi,\phi)}{\partial\,\phi_H} \\ \frac{\partial\,\mathcal{L}_H^{\hat{\sigma}_H}(\phi,\phi)}{\partial\,\phi_L} & \frac{\partial\,\mathcal{L}_H^{\hat{\sigma}_H}(\phi,\phi)}{\partial\,\phi_H} \end{pmatrix} = \begin{pmatrix} \frac{\partial^2\,u(\phi|\phi)}{\partial\,\phi_L^d\,\partial\,\phi_L} & 0 \\ 0 & 0 \\ 0 & \frac{\partial^2\,u(\phi|\phi)}{\partial\,\phi_H^d\,\partial\,\phi_H} \\ 0 & 0 \end{pmatrix}$$

Noting that for $G = L, H$, $\frac{\partial^2\,u(\phi|\phi)}{\partial\,\phi_G^d\,\partial\,\phi_G} = -\frac{\partial^2\,u(\phi|\phi)}{\partial\,\phi_G^{d\,2}}$, it follows from the Implicit Function Theorem results that for $G = L, H$, $\frac{\partial\,\phi_G^{d^*}\,(\phi,\phi)}{\partial\,\phi_G} = 1$ and $\frac{\partial\,\phi_G^{d^*}\,(\phi,\phi)}{\partial\,\phi_{-G}} =$

0, so that

$$\begin{pmatrix} \Delta\phi_L(\phi_L,\phi_H) \\ \Delta\phi_H(\phi_L,\phi_H) \end{pmatrix} = \begin{pmatrix} -q_H & q_H \\ 1-q_H & q_H-1 \end{pmatrix} \begin{pmatrix} \phi_L-\phi \\ \phi_H-\phi \end{pmatrix} = \begin{pmatrix} -q_H \\ 1-q_H \end{pmatrix} (\phi_L-\phi_H).$$

Thus, locally around any point $(\phi,\phi) \in \phi^d(X)^2$, $\Delta\phi_L(\phi_L,\phi_H) - \Delta\phi_H(\phi_L,\phi_H) = -(\phi_L-\phi_H)$, and there is an infinite speed of convergence to a(nother) steady state $(\phi',\phi') \in \phi^d(X)^2$.[3] In particular, this implies that for every $(i_L,i_H,q_H) \in \mathbb{R}^2_+ \times (0,1)$ $\exists\Delta(i_L,i_H,q_H) \in \left(0,|\operatorname{con}\phi^d(X)|\right]$ such that $\forall 0 < \Delta^\phi(t) < \Delta(i_L,i_H,q_H)$, $\Delta^\phi(t+1) < \Delta^\phi(t)$. □

1.6. Proof of Proposition 2.6. This proof will be based on the construction of sufficient properties of a SNE selection function in order to guarantee convergence. To discuss these properties, we will focus our attention to the upper triangle of the state space (i.e. where $\phi_H \geq \phi_L$). Note also that our dynamical system is time–autonomous, and we will drop the time–indexes. Furthermore, we consider below any $(i_L,i_H,q_H) \in \mathbb{R}^2_+ \times (0,1)$, but will also drop these parameters for brevity.

Consider a point where the adopted TI of the members of group L coincides with the lower bound of $\phi^d(X) = \left[\underline{\phi},\overline{\phi}\right]$. Denote this $\left(\underline{\phi},\phi^1_H\right)$, with $\phi^1_H > \underline{\phi}$. We know that at any such point $\phi^{d^*}_L\left(\underline{\phi},\phi^1_H\right) = \underline{\phi}$, and the lower DTI constraint is binding for the parents of group L. Then, there exists a SNE selection function for which it holds that there is a non–empty and right–open interval $\left[\left(\underline{\phi},\phi^1_H\right),\left(\phi^1_L,\phi^1_H\right)\right)$, where $\phi^1_L > \underline{\phi}$, such that (a) the lower DTI constraint stays binding for L; as well as that (b) for all points $\left(\phi_L,\phi^1_H\right)$ in this interval, $\phi^{d^*}_H\left(\phi_L,\phi^1_H\right) = \phi^{d^*}_H\left(\underline{\phi},\phi^1_H\right)$ and $\hat{\sigma}^*_H\left(\phi_L,\phi^1_H\right) = \hat{\sigma}^*_H\left(\underline{\phi},\phi^1_H\right)$.

Extending this sort of normalization to any point where $\phi_L = \underline{\phi}$ (and which is not located on the main diagonal of the state space where $\phi_L = \phi_H$), we obtain a continuum of right–open intervals on any of which it holds that $\dot{\phi}_H(\phi_L,\phi_H)$ is constant.

Analogously, consider a point where $\phi_H = \overline{\phi}$, and denote this point $\left(\phi^2_L,\overline{\phi}\right)$, with $\phi^2_L < \overline{\phi}$. We know that at this point $\phi^{d^*}_H\left(\phi^2_L,\overline{\phi}\right) = \overline{\phi}$, and the upper DTI constraint is binding for the parents of group H. Then, there exists a SNE selection function for which it holds that there is a non–empty and left–open interval $\left(\left(\phi^2_L,\phi^2_H\right),\left(\phi^2_L,\overline{\phi}\right)\right]$, where $\phi^2_H < \overline{\phi}$, such that (a) the upper DTI constraint stays binding for H; as well as that (b) for all points $\left(\phi^2_L,\phi_H\right)$ in this interval, $\phi^{d^*}_L\left(\phi^2_L,\phi_H\right) = \phi^{d^*}_L\left(\phi^2_L,\overline{\phi}\right)$ and $\hat{\sigma}^*_L\left(\phi^2_L,\phi_H\right) = \hat{\sigma}^*_L\left(\phi^2_L,\overline{\phi}\right)$.

[3]This infinite speed of convergence is due to the fact that locally around any steady state (ϕ,ϕ), the adopted TIs of the children of both groups will only marginally deviate from the optimal TI. Since parents perceive zero disutility in such a case, they will not engage in active socialization. Thus, all children of the society adopt the society's average DTI, $\phi^d_A(\phi_L,\phi_H) = \phi_L q_H + \phi_H(1-q_H)$.

Extending this sort of normalization to any point where $\phi_H = \overline{\phi}$ (and which is not located on the main diagonal), we obtain a continuum of left–open intervals on any of which it holds that $\dot{\phi}_L(\phi_L, \phi_H)$ is constant.

Consider now any pair of points that consists of a right–boundary point of the first type of intervals and a left–boundary point of the second type of intervals, which satisfies the following conditions: (a) At the first of these points, the upper DTI constraint is not binding for group H, and at the second of these points, the lower DTI constraint is not binding for group L (but the DTI–choices coincide with the upper bound, respectively lower bound of $\phi^d(X)$), and (b) these two points are connected through a $45\,^\circ$–line–segment (in state space).

We will now impose a normalization on the SNE selection on these sorts of $45\,^\circ$–line–segments. To introduce this, the following definition will be useful.

Definition B.1 (State–corrected SNE Choices). $\forall\,(\phi_L, \phi_H) \in \operatorname{con}\phi^d(X)^2$, *denote the tuple*

$$\left\{\phi_G^{d*}(\phi_L, \phi_H) - \phi_G, \hat{\sigma}_G^*(\phi_L, \phi_H)\right\}_{G\in\{L,H\}}$$

as state–corrected SNE choices.

We will now indeed require the SNE selection function to select identical state–corrected SNE choices for every point on any of the above constructed $45\,^\circ$–line–segments. Thus for all points on such $45\,^\circ$–line–segments, both $\dot{\phi}_L(\phi_L, \phi_H)$ and $\dot{\phi}_H(\phi_L, \phi_H)$ are constant (i.e. the $45\,^\circ$–line–segments are isoclines).

We can now give the following summarizing characterization of the phase vectors in the upper triangle of the state space. First, the main diagonal consists of a continuum of steady states (Proposition 2.4 (b)). This is neighbored by a continuum of line–segments consisting of a connection of (a) a horizontal line in state space where $\dot{\phi}_H(\phi_L, \phi_H)$ is constant, with (b) a $45\,^\circ$–isocline (which is eventually constituted by a single point), with (c) a vertical line where $\dot{\phi}_L(\phi_L, \phi_H)$ is constant.[4] Notably, by construction, on all $45\,^\circ$–isoclines $\dot{\phi}_L(\phi_L, \phi_H)$ is identical to that of the connected vertical lines; and $\dot{\phi}_H(\phi_L, \phi_H)$ is identical to that of the connected horizontal lines.

[4]Eventually, these line–segments 'melt down' in a point (ϕ_m, ϕ^m) where $\left(\phi_L^{d*}(\phi_m, \phi^m), \left(\phi_H^{d*}(\phi_m, \phi^m)\right)\right) = (\underline{\phi}, \overline{\phi})$, but where both DTI constraints are not binding. Generically, the vertical and horizontal lines connected to this point would constitute the borders of a rectangle in which it holds that the lower DTI constraint is binding for group L and the upper DTI constraint is binding for group H. This rectangle would thus be made of a continuum of horizontal lines with constant $\dot{\phi}_H(\phi_L, \phi_H)$ and a continuum of vertical lines with constant $\dot{\phi}_L(\phi_L, \phi_H)$.

We will show next that the connected line–segments can never be crossed twice by a (TI–)trajectory. To do so, we will consider the case where a trajectory crosses a connected line–segment from 'above' (respectively from the 'left') — the case for a crossing from 'below' (respectively from the 'right') can be shown analogously. Obviously, no trajectory can cross any of the three individual line–segments both from above and below.

Assume that a trajectory crosses a horizontal line–segment from above. Now, any point on the (closure of) the horizontal line has strictly lower ϕ_L and ϕ_H than any point on the connected (left–open) $45\,^\circ$–isocline and the connected vertical line. This implies that, for the trajectory to 'reach' these segments, it must ('initially') cross from above (respectively from the left) a continuum of connected line–segments on which $\dot{\phi}_L(\phi_L, \phi_H) \geq 0$ on the $45\,^\circ$–isocline and the connected vertical line. Now, these would have to be crossed again from below (respectively from the right) by the trajectory in order to actually 'return' to the original connected line–segment. But this is obviously impossible. By the analogous logic, a trajectory that crosses a $45\,^\circ$–isocline or a vertical line segment from above (respectively from the left) can not also cross the same connected line segment from below (respectively from the right).

Under this property, no cycles can exist in the upper triangle of the state space and any trajectory must end up in a steady state therein. Extending these properties (respectively the normalizations on the SNE selection function) to the lower triangle of the state space in an analogous way, we obtain the convergence property for the whole state space. □

i_H \ i_L	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
2.0	●○○	●○○	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
1.8	●○○	●○○	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
1.6	●○○	●○○	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
1.4	●○○	●○○	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
1.2	●○○	●○○	●○○	●●●	●●●	●●●	●●●	●●●	●●●	●●●
1.0	●○○	●○○	●○○	●●●	●●●	●●●	●●●	●●●	●●●	●●●
0.8	●○○	●○●	●○●	●○●	●●●	●●●	●●●	●●●	●●●	●●●
0.6	●○●	●○●	●○●	●○●	●○○	●○○	●●●	●●●	●●●	●●●
0.4	●○●	●○●	●○●	●○●	●○○	●○○	●○○	●○○	●○○	●○○
0.2	●○●	●○●	●○●	●○○	●○○	●○○	●○○	●○○	●○○	●○○

i_H \ i_L	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
2.0	●○○	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
1.8	●○○	●○○	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
1.6	●○○	●○○	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
1.4	●○○	●○○	●●●	●●●	●●●	●●●	●●●	●●●	●●●	●●●
1.2	●○○	●○○	●○○	●●●	●●●	●●●	●●●	●●●	●●●	●●●
1.0	●○○	●○○	●○○	●○○	●●●	●●●	●●●	●●●	●●●	●●●
0.8	●○○	●○○	●○○	●○○	●○○	●○○	●●●	●●●	●●●	●●●
0.6	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○
0.4	●○●	●○●	●○●	●○●	●○○	●○○	●○○	●○○	●○○	●○○
0.2	●○●	●○●	●○●	●○●	●○○	●○○	●○○	●○○	●○○	●○○

i_H \ i_L	0.2	0.4	0.6	0.8	1.0	1.2	1.4	1.6	1.8	2.0
2.0	●○○	●○○	●○○	●○○	●○○	●○○	●●●	●●●	●●●	●●●
1.8	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●●●	●●●
1.6	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○
1.4	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○
1.2	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○
1.0	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○
0.8	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○
0.6	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○	●○○
0.4	●○●	●○●	●○●	●○●	●○●	●○○	●○○	●○○	●○○	●○○
0.2	●○●	●○●	●○●	●○●	●○●	●○●	●○○	●○○	●○○	●○○

$1^{st}/2^{nd}/3^{rd}$ entry: Convergence/Uniqueness/Stability Yes/No: ●/○

FIGURE B.1. Dynamics Statistics for $q_H = (0.5, 0.7, 0.9)'$

2. Evolution under Imperfect Empathy

The central property of the evolution under global 'imperfect empathy' (respectively the first type of perception rule) is that if the oblique socialization is unbiased, then the TIs of (almost) all adults converge to the same point (confer Proposition 4 in Pichler [49]). Since in the present paper, we have assumed unbiased oblique socialization, this result must thus hold.

It rests to show the characterization of the SNE choices for the two cultural groups case under global imperfect empathy.

Proposition B.2 (Characterization of SNE choices). *Let Assumptions 2.1–1.4 hold. Then, the following properties are satisfied* $\forall\{i_L, i_H, q_H\} \in (\mathbb{R}_{++}\setminus\{\infty\})^2 \times (0,1)$, *and* $\forall\left\{\phi_G^{d^*}(t), \hat{\sigma}_G^*(t)\right\}_{G=L,H} \in E(P(t))$.

1. *Case* $\phi_H(t) <> \phi_L(t)$
   (a) $\phi_H^{d^*}(t) <> \phi_H(t) <> \phi_H(t+1) <> \phi_L(t+1) <> \phi_L(t) <> \phi_L^{d^*}(t)$[5], *and*
   (b) $\hat{\sigma}_G^*(t) \in (0,1)$, $\forall G \in \{L,H\}$.
2. *Case* $\phi_H(t) = \phi_L(t)$
   (a) $\phi_G^{d^*}(t) = \phi_G(t) = \phi_G(t+1)$, $\forall G \in \{L,H\}$, *and*
   (b) $\hat{\sigma}_G^*(t) = 0$, $\forall G \in \{L,H\}$.

Proof. Let $\phi_H(t) > \phi_L(t)$. By Proposition B.1, it suffices to show that $\phi_A^{d^*}(t) \in (\phi_L(t), \phi_H(t))$ (for all elements in the set of SNEs). Assume, by ways of contradiction, that $\phi_A^{d^*}(t) \geq \phi_H(t)$. In this case $\phi_H(t)\left(\phi_A^{d^*}(t)\right) \leq \phi_H(t)$ while even $\phi_L(t)\left(\phi_A^{d^*}(t)\right) < \phi_H(t)$. This yields a contradiction. Analogously, we obtain a contradiction for $\phi_A^{d^*}(t) \leq \phi_L(t)$. Also, the proof for the case $\phi_H(t) > \phi_L(t)$ is analogous.

Let $\phi_H(t) = \phi_L(t)$, and assume that $\phi_A^{d^*}(t) <> \phi_H(t) = \phi_L(t)$. But then, $\phi_G(t)\left(\phi_A^{d^*}(t)\right) \geq\leq \phi_H(t) = \phi_L(t)$, $\forall G \in \{L,H\}$, which yields a contradiction again. □

By the results of Proposition B.2, the cultural groups strictly assimilate inter–generationally and hence the TIs of the groups converge to the same point (confirming the result of Pichler [49]). This result can be interpreted to correspond to the 'melting pot' theory of assimilation of cultural groups (see e.g. Han [33]).

[5]Again, the outer inequalities would be strict if the respective adopted TI would coincide with the relevant boundary of $\operatorname{con}\phi^d(X)$. But this can only be the case in the initial period, given the results of the present Proposition.

APPENDIX C

1. Proofs

1.1. Proof of Proposition 3.2. From equation (3.1), it follows that $\forall i \in N$, $\phi_i(t+1)$ are concave in $\phi_i^d(t)$, thus also all $v\left(\phi_i(t+1)\,|\beta_i, \phi_i(t)\right)$ are concave in $\phi_i^d(t)$ (by Assumption 3.5). This implies that the target functions of the best reply problems of all parents are concave (and continuous). Since also the DTI choice set is compact and convex, a non–empty, upper hemi-continuous and convex set of DTI best replies exists for any parent (Berge's Theorem of the Maximum). Thus, a fixed point, i.e. a Nash equilibrium, exists (Kakutani's Fixed Point Theorem). □

1.2. Proof of Proposition 3.3. (a) That in any steady state, parents choose their adopted TI as DTI is directly implied by Proposition 3.1 (c).

(b) Given (a), it follows that the set of steady states given $\hat{\Sigma}$ coincides with the set $\left\{\Phi \in \phi^d(X)^n \,\middle|\, \hat{\Sigma}\Phi = \Phi\right\}$. Hence, it is immediate that if the TIs of all members of an essential communication class are identical, then $\hat{\Sigma}_L\Phi_L = \Phi_L$, where $\hat{\Sigma}_L$ is the restriction of $\hat{\Sigma}$ to some essential communication class L, and Φ_L is its vector of adopted TIs. We proceed by showing that steady state TIs cannot differ within an essential communication class. To show a contradiction, suppose that for an essential communication class $L \in \mathcal{P}(\hat{\Sigma})$, $|L| \geq 2$, there exists $i, j \in L$ with $\phi_i \neq \phi_j$. Denote by $\bar{\phi}_L := max\{\phi_i | i \in L\}$ the maximal TI in communication class L. Since L is a communication class it follows that there exists an $i \in \{l \in L : \phi_l = \bar{\phi}_L\}$ and a $j \in \{l \in L | \phi_l \neq \bar{\phi}_L\}$ such that $\hat{\sigma}_{ij} > 0$. Moreover, due to maximality of $\bar{\phi}_L$ and the fact that L is essential, $\hat{\sigma}_{ik} = 0$ for all $k \in N$ with $\phi_k > \bar{\phi}_L$. Thus, $\hat{\Sigma}_i\Phi_L \neq \phi_i$ implying that this cannot be a steady state.

(c) This is also straightforward. Suppose that for some inessential communication class $I \in \mathcal{P}(\hat{\Sigma})$ with connections to other dynasties $J := \{j \in N | i \to j,\ i \in I\}$ the set of TIs Φ_I is not included in $conv(\phi_j | j \in J)$. W.l.o.g. we have $\bar{\phi}_I := \max\{\phi_i | i \in I\} > \max\{\phi_j | j \in J\}$. Since I is a communication class and is inessential, with all outside connections being to dynasties with TIs strictly less than $\bar{\phi}_I$, we get (similarly to (b)) for some dynasty

$k \in \{i \in I | \phi_i = \bar{\phi}_I\}$ that there exists $j \in N$ and $\phi_j < \bar{\phi}_I$ such that $\hat{\sigma}_{kj} > 0$. Again, due to maximality of $\bar{\phi}_I$ and all other connections being to dynasties with TIs strictly less than $\bar{\phi}_I$, we get that $\hat{\Sigma}_k \Phi_I \neq \phi_k$, implying that this cannot be a steady state. Hence, all TIs of the dynasties in inessential communication classes $I \in \mathcal{P}(\hat{\Sigma})$ are convex combinations of the TIs of the communication classes $J \in P(\hat{\Sigma})$ such that $I \to J$.

1.3. Proof of Proposition 3.4. This proof is organized in three essential steps. In the first step, we will show that if $\hat{\Sigma}$ is symmetric ultrametric, then $M(t)$ is row stochastic for every $t \in \mathbb{N}$. In the second step we will show that subject to any $\beta \in \mathbb{R}^n_+$ it holds that for every $i, j \in N$ with $\hat{\sigma}_{ij} > 0$ there exists a $\delta_{ij} > 0$ such that for every $t \in \mathbb{N}$, $m_{ij}(t) \geq \delta_{ij}$. We use these results to show in the third step that the backward accumulation matrices are type symmetric and have a strictly positive diagonal. This allows us to apply Theorem 2 of Lorenz [40] to conclude that the desired convergence result holds. For the first step, we also need the following.

Lemma C.1 (Unit Eigenvectors). *Let $\hat{\Sigma}$ be positive definite. Then, $\forall x \in \mathbb{R}^n$, $\forall t \in \mathbb{N}$, $M(t)x = x$ iff $\hat{\Sigma}x = x$ (i.e. x is a unit–eigenvector of $M(t)$ if and only if x is a unit–eigenvector of $\hat{\Sigma}$).*

Proof. Note that

$$M(t) = \hat{\Sigma}\left(I + B(t)\hat{\Sigma}\right)^{-1}(I + B(t)) = \left(\hat{\Sigma}^{-1} + B(t)\right)^{-1}(I + B(t)).$$

That the latter representation is well defined if $\hat{\Sigma}$ is positive definite follows since $\hat{\Sigma}$ is then invertible and also its inverse is positive definite. Thus, also $\hat{\Sigma}^{-1} + B(t)$ is positive definite and invertible. Given this, both the 'if' and the 'only if' direction of the proof can be directly seen from the following sequence of transformations: $\hat{\Sigma}x = x \Leftrightarrow x = \hat{\Sigma}^{-1}x \Leftrightarrow (B(t) + I)x = (B(t) + \hat{\Sigma}^{-1})x \Leftrightarrow M(t)x = (B(t) + \hat{\Sigma}^{-1})^{-1}(B(t) + I)x = x$. □

1. In the first step of the (main) proof, we show that if $\hat{\Sigma}$ is symmetric ultrametric, then $M(t)$ is row stochastic for every $t \in \mathbb{N}$. To do so, note first that since $\hat{\Sigma}$ is symmetric ultrametric, it is also positive definite (see below). Hence, by Lemma C.1 (and setting $x = (1, 1, ..., 1)'$) the row entries of $M(t) = [m_{ij}(t)]$ sum up to one since the same holds for $\hat{\Sigma}$. Thus, $M(t)$ is row stochastic if and only if $M(t)$ is positive (that is $M(t) \geq 0$). Now, since $I + B(t)$ is a diagonal matrix with strictly positive entries, $M(t) = \hat{\Sigma}\left(I + B(t)\hat{\Sigma}\right)^{-1}(I + B(t))$ is positive if and only if

$$\hat{\Sigma}\left(I + B(t)\hat{\Sigma}\right)^{-1} = \left(\hat{\Sigma}^{-1} + B(t)\right)^{-1}$$

is positive (that this representation is well defined if $\hat{\Sigma}$ is positive definite has been discussed in the proof of Lemma C.1). In other words, we have to check whether $\hat{\Sigma}^{-1} + B(t)$ is inverse–positive.

Now, since $\hat{\Sigma}$ is symmetric ultrametric, it follows that its inverse is a diagonally dominant Stieltjes matrix (see Nabben and Varga [43, 44], Martinez et al. [42]), i.e. a real symmetric positive definite matrix with positive diagonal and negative off-diagonal entries. Thus, also $\hat{\Sigma}^{-1} + B(t)$ is a diagonally dominant Stieltjes matrix. In particular, it is an M–matrix, the class of which is inverse–positive (on this issue, see e.g. Fujimoto and Ranade [26]). Hence, $M(t)$ is positive.

2. In the second step we show that subject to any $\beta \in \mathbb{R}^n_+$ it holds that for every $i, j \in N$ with $\hat{\sigma}_{ij} > 0$ there exists a $\delta_{ij} > 0$ such that for every $t \in \mathbb{N}$, $m_{ij}(t) \geq \delta_{ij}$. Again, since $I + B(t)$ is a diagonal matrix with strictly positive entries, we can restrict our attention to the matrix $\left(\hat{\Sigma}^{-1} + B(t)\right)^{-1} =: \bar{B}(t) = \left[\bar{b}_{ij}(t)\right]$. Now, consider any $i, j \in N$ such that $\hat{\sigma}_{ij} > 0$. Since $\bar{B}(t)$ is positive, it follows that $\operatorname{sign}\left(\bar{b}_{ij}(t)\right) \in \{0, \operatorname{sign}(\hat{\sigma}_{ij})\}$.

Let us rule out the case $\operatorname{sign}\left(\bar{b}_{ij}(t)\right) = 0$. To do so, let us compare

$$\bar{b}_{ij}(t) = (-1)^{i+j} \frac{\left|\hat{\Sigma}^{-1} + B(t)\right|_{ij}}{\left|\hat{\Sigma}^{-1} + B(t)\right|} \quad \text{vs.} \quad (-1)^{i+j} \frac{\left|\hat{\Sigma}^{-1}\right|_{ij}}{\left|\hat{\Sigma}^{-1}\right|} = \hat{\sigma}_{ij}$$

where $|A|_{ij}$ denotes the determinant of the ij adjoint matrix of a square matrix A. Note next that since $\hat{\Sigma}$ is symmetric positive definite, the same holds for its inverse and $\hat{\Sigma}^{-1} + B(t)$. It follows that the determinants of the matrices $\hat{\Sigma}^{-1}$ and $\hat{\Sigma}^{-1} + B(t)$ are strictly positive. Hence, it rests to show that $\operatorname{sign}\left(\left|\hat{\Sigma}^{-1} + B(t)\right|_{ij}\right) = \operatorname{sign}\left(\left|\hat{\Sigma}^{-1}\right|_{ij}\right) \neq 0$.

To show this, note that for all $i, j \in N$, $\left|\hat{\Sigma}^{-1} + \operatorname{diag}\left(e_1^*(t), \ldots, e_n^*(t)\right)\right|_{ij}$ is linear in every individual element of $\{e_1^*(t), \ldots, e_n^*(t)\}$ (to verify this most easily, consider the Leibniz formula). In the following, let $\operatorname{abs}(\cdot)$ denote the absolute value of a real number. It is then immediate that for all $k \in N$

$$\frac{\partial \operatorname{abs}\left(\left|\hat{\Sigma}^{-1} + B(t)\right|_{ij}\right)}{\partial e_k^*(t)} \geq 0$$

since in the other case, the sign of $\left|\hat{\Sigma}^{-1} + B(t)\right|_{ij}$ would switch compared to the sign of $\left|\hat{\Sigma}^{-1}\right|_{ij}$ for $e_k^*(t)$ large enough. But this is ruled out since $\hat{\Sigma}^{-1} + \operatorname{diag}(x_1, \ldots, x_n)$ is an M-Matrix for arbitrary $(x_1, \ldots, x_n)' \in \mathbb{R}^n_+$ and hence $\left(\hat{\Sigma}^{-1} + \operatorname{diag}(x_1, \ldots, x_n)\right)^{-1}$ is positive for arbitrary $(x_1, \ldots, x_n)' \in \mathbb{R}^n_+$.

Hence, $\operatorname{sign}\left(\left|\hat{\Sigma}^{-1}+B(t)\right|_{ij}\right)=\operatorname{sign}\left(\left|\hat{\Sigma}^{-1}\right|_{ij}\right)$, so that $0\neq\operatorname{sign}\left(\bar{b}_{ij}(t)\right)=\operatorname{sign}\left(\hat{\sigma}_{ij}\right)=+1$.

Note next that the image–space of E is bounded. This follows by continuity of E and the fact that if $\phi(t)-\phi^*(t+1)=0$ then $e^*=0$, see Corollary 3.1. Moreover, $E\left(\Phi^d(X)^n,\hat{\Sigma},\beta\right)$ is closed since E is continuous in Φ^d and $\Phi^d(X)^n$ is closed. Since $(e_1^*(t),\ldots,e_n^*(t))'$ is thus bounded above and hence $\left|\hat{\Sigma}^{-1}+B(t)\right|$ is bounded above, it finally follows that $0\neq\operatorname{sign}\left(m_{ij}(t)\right)=\operatorname{sign}\left(\hat{\sigma}_{ij}\right)=+1$. Next, interpret our matrices $M(t)$ as functions of the elements $e\in E\left(\Phi^d(X)^n,\hat{\Sigma},\beta\right)$, $M(e)\in\mathcal{S}(n)$, and note that the set $\left\{M(e)\,\middle|\,e\in E\left(\Phi^d(X)^n,\hat{\Sigma},\beta\right)\right\}$ is compact. Thus, we can define $\min_{e\in E\left(\Phi^d(X)^n,\hat{\Sigma},\beta\right)}m_{ij}(e)=:\delta_{ij}>0$, for every $i,j\in N$ such that $\hat{\sigma}_{ij}>0$. It follows that for all $i,j\in N$ such that $\hat{\sigma}_{ij}>0$ and for all $\beta\in\mathbb{R}_+^n$ there exists a $\delta>0$ such that for all $m_{ij}(t)\geq\delta$, $\forall t\in\mathbb{N}$.

3. In the last step, we show that given the above, the left product of the matrices $M(t)M(t-1)\ldots M(0)$ converges such that the adopted TIs of all dynasties of a connected subset are identical (respectively, the communication classes in $\mathcal{P}(\hat{\Sigma})$ reach a consensus). Note that all communication classes of $\hat{\Sigma}$ are essential by symmetry of $\hat{\Sigma}$. By the definition of $\mathcal{P}\left(\hat{\Sigma}\right)$ we have that for all $L\in\mathcal{P}\left(\hat{\Sigma}\right)$ and for all $i,j\in L$ there exists a $k\in\{0,...,|L|\}$ such that $\hat{\Sigma}_{ij}^k>0$. Note that $\mathcal{P}\left(\hat{\Sigma}\right)=\mathcal{P}(M(t))$ for all $t\in\mathbb{N}$ since $\hat{\sigma}_{ij}>0$ implies $m_{ij}(t)\geq\delta$ for all $t\in\mathbb{N}$ as shown above and, since every communication class of $\hat{\Sigma}$ is essential, $m_{ij}(t)=0$ if $j\notin P_{\hat{\Sigma}}(i)$.[1] Hence, for all $L\in\mathcal{P}\left(\hat{\Sigma}\right)$ and for all $i,j\in L$ there exists a $k\in\{0,...,|L|\}$ such that $M(t+k,t)_{ij}>0$ for all $t\in\mathbb{N}$.[2]

Now, consider a sequence of time steps $(t_s)_{s\in\mathbb{N}}$ such that $t_0=0$ and $t_{s+1}=t_s+\bar{L}$, where $\bar{L}:=\max\{|L|:L\in P(M)\}$, and consider the sequence of accumulations $\left(M(t_{s+1},t_s)\right)_{s\in\mathbb{N}}$. By the rules of matrix multiplication, we get that for any two $A,B\in\mathcal{S}(n)$ with a positive diagonal, $(AB)_{ij}>0$ if and only if $A_{ij}>0$ or $B_{ij}>0$. Hence, for any $L\in\mathcal{P}\left(\hat{\Sigma}\right)$ and for all $i,j\in L$, $M(t+|L|,t)_{ij}>0$ for all $t\in\mathbb{N}$ since $M(t)$ is row stochastic with a positive diagonal. Moreover, $M(t+|L|,t)_{ij}=0$ if $j\notin P_{\hat{\Sigma}}(i)$ since $\mathcal{P}\left(\hat{\Sigma}\right)=\mathcal{P}(M(t))$ for all $t\in\mathbb{N}$. Thus, for the accumulations $M(t_{s+1},t_s)$ it holds

[1]Recall, $P_{\hat{\Sigma}}(i)\subseteq N$ is such that $P_{\hat{\Sigma}}(i)\in\mathcal{P}\left(\hat{\Sigma}\right)$ and $i\in P_{\hat{\Sigma}}(i)$ (the element of the partition $\mathcal{P}\left(\hat{\Sigma}\right)$ which i belongs to).

[2]Recall that $M(t',t)$ denotes the accumulation $M(t',t)=M(t')M(t'-1)\ldots M(t)$.

that $M(t_{s+1}, t_s)_{ij} > 0$ if and only if $j \in \mathcal{P}_{\hat{\Sigma}}(i)$. In particular, $M(t_{s+1}, t_s)$ is type-symmetric for all $s \in \mathbb{N}$.

For a non-negative matrix A let $\min^+(A)$ denote the lowest positive entry of A. We have shown above that there exists a $\delta > 0$ such that $\hat{\sigma}_{ij} > 0$ implies $m_{ij}(t) \geq \delta$ for all $t \in \mathbb{N}$. Note that for any $i, j \in L \in \mathcal{P}\left(\hat{\Sigma}\right)$ there exists a $k \leq |L|$ and a sequence of dynasties $(i_l)_{0 \leq l \leq k}$ with $i_0 = i$ and $i_k = j$ such that $\hat{\sigma}_{i_l, i_{l+1}} > 0$, implying $M(t+k, t)_{ij} \geq \prod_{l=0}^{k-1} m_{i_l, i_{l+1}}(t+l) \geq \delta^k$. Thus, for the accumulations $M(t_{s+1}, t_s)$ it holds that $M(t_{s+1}, t_s)_{ij} \geq \delta^{t_{s+1}-t_s}$ if $j \in P_{\hat{\Sigma}}(i)$ and $M(t_{s+1}, t_s)_{ij} = 0$ else. Hence, $\min^+\left(M(t_{s+1}, t_s)\right) \geq \delta^{t_{s+1}-t_s} = \delta^{|\bar{L}|}$.

In summary, we have shown that the backward accumulation matrices $\left(M(t_{s+1}, t_s)\right)_{s \in \mathbb{N}}$ have a uniform lower bound of the positive entries $min^+\left(M(t_{s+1}, t_s)\right) \geq \delta^{|\bar{L}|}$, are type symmetric and have a strictly positive diagonal. By Lorenz [40], Theorem 2, we get the desired result for the sequence $\left(M(t_{s+1}, t_s)\right)_{s \in \mathbb{N}}$. Since $\lim_{k \to \infty} \prod_{s=0}^{k} M(t_{s+1}, t_s) = \lim_{t \to \infty} M(t)$, we also establish the statement of the Proposition. □

1.4. Proof of Proposition 3.5. For both parts of the proposition, we will apply the following Lemma (see e.g. Friedberg and Insel [24]).

Lemma C.2 (Convergence). *Let A be a square matrix with complex or real entries. Then, the sequence $\left\{A^t\right\}_{t \to \infty}$ converges if and only if the following two conditions are satisfied.*

(i) If λ is an eigenvalue of A, then either $\lambda = 1$ or λ lies in the open unit disc of the complex plane, i.e. $|\lambda| \in (-1, 1)$.

(ii) If 1 is an eigenvalue of A, then its algebraic multiplicity equals its geometric multiplicity.

Let us denote by $Eig(A)$, the set of eigenvalues of a matrix A and let $eig(A) \in Eig(A)$. Moreover, if z is a complex number, then we denote by $Re(z)$ the real part and by $Im(z)$ the imaginary part of z.

Proof of part (a). We will show that condition (i) of Lemma C.2 is satisfied. To see this, note first that by definition $M = \hat{\Sigma}(I + B\hat{\Sigma})^{-1}(I + B) = (B + \hat{\Sigma}^{-1})^{-1}(I + B)$,[3] which implies that $M^{-1} = (I + B)^{-1}(B + \hat{\Sigma}^{-1})$. Let $\tilde{B} := (I + B)^{-1}$, i.e. for every $i \in N$, $\tilde{B}_{ii} = \frac{1}{1+\hat{\sigma}_i \beta_i}$, hence $\tilde{B}$ is a diagonal matrix with entries in $(0, 1)$. Thus, $\tilde{B}B = I - \tilde{B}$, and $M^{-1} = \tilde{B}(B + \hat{\Sigma}^{-1}) = I - \tilde{B} + \tilde{B}\hat{\Sigma}^{-1} = I + \tilde{B}(\hat{\Sigma}^{-1} - I)$.

Now let $\hat{\Sigma}$ be symmetric positive definite (henceforth: "PD"). Then $\hat{\Sigma}^{-1}$ is also PD and the eigenvalues of both matrices are real and positive.

[3]That this representation is well defined if $\hat{\Sigma}$ is positive definite can be seen in the proof of Lemma C.1.

Since $\hat{\Sigma}$ is row stochastic, we have $|eig(\hat{\Sigma})| \leq 1$ (here and in the rest of the present proof of part (a), the properties shown for one $eig(A)$ do hold for every $eig(A) \in Eig(A)$), which implies that $eig(\hat{\Sigma}^{-1}) \geq 1$. Thus, $eig(\hat{\Sigma}^{-1} - I) \geq 0$ (subtraction of I decreases all eigenvalues by 1). Note that both $\hat{\Sigma}^{-1} - I$ and $\tilde{B}$ are symmetric matrices with real non–negative eigenvalues. As a consequence, the product $\tilde{B}(\hat{\Sigma}^{-1} - I)$ has also only real non–negative eigenvalues.[4] Thus, we get $eig(\tilde{B}(\hat{\Sigma}^{-1} - I)) \geq 0$, which implies $eig(I + \tilde{B}(\hat{\Sigma}^{-1} - I)) \geq 1$, i.e. $eig(M^{-1}) \geq 1$, and hence all eigenvalues of M are real and located in the interval $(0, 1]$. Furthermore, since M has row sum one (see Lemma C.1, using $x = (1, 1, ..., 1)'$), at least one eigenvalue must be equal to 1.

Proof of Part (b). Let $\hat{\Sigma}$ have a strictly positive diagonal and let there be an eigenvalue λ that satisfies $Re(\lambda) < |\lambda|^2$. The second condition is equivalent to $Re(\lambda^{-1}) < 1$.[5] Note that λ^{-1} is an eigenvalue of $\hat{\Sigma}^{-1}$. Now let for each i $\beta_i = \frac{k}{\hat{\sigma}_i}$, $k \in \mathbb{R}$, so that $B = kI$. We show that if k is large enough, then M has an eigenvalue with absolute value larger than 1 and hence condition (i) of Lemma C.2 is violated.

To do so, we will use $M^{-1} = (I + B)^{-1}(B + \hat{\Sigma}^{-1}) = (I + kI)^{-1}(kI + \hat{\Sigma}^{-1}) = ((1+k)I)^{-1}(kI+\hat{\Sigma}^{-1}) = \frac{1}{1+k}(kI+\hat{\Sigma}^{-1})$. Now, since we have that we have $Re(\lambda^{-1}) = Re(eig(\hat{\Sigma}^{-1})) < 1$, this implies $Re(eig(kI + \hat{\Sigma}^{-1})) < 1 + k$, because $eig(kI + \hat{\Sigma}^{-1}) = k + eig(\hat{\Sigma}^{-1})$. For k large enough, we must have $|eig(kI+\hat{\Sigma}^{-1})| < 1+k$.[6] To see that this must hold, assume to the contrary $|eig(kI + \hat{\Sigma}^{-1})| \geq 1 + k$. Thus, we would get

$$\sqrt{Re^2(eig(kI + \hat{\Sigma}^{-1})) + Im^2(eig(kI + \hat{\Sigma}^{-1})} \geq 1 + k.$$

Denote $\epsilon := 1 - Re(\lambda^{-1}) > 0$. $Re(\lambda^{-1}) = 1 - \epsilon$ implies that $Re(eig(kI + \hat{\Sigma}^{-1})) = 1-\epsilon+k$. Since $Re(eig(kI+\hat{\Sigma}^{-1})) = k-\epsilon+1$ and $Im(eig(kI+\hat{\Sigma}^{-1}) = Im(\lambda^{-1})$, we have that $\sqrt{(1 - \epsilon + k)^2 + Im^2(\lambda^{-1})} \geq 1+k$, i.e. $(1-\epsilon+k)^2 + Im^2(\lambda^{-1}) \geq (1 + k)^2$. After simplifying, we have $k \leq \frac{Im^2(\lambda^{-1})-2\epsilon+\epsilon^2}{2\epsilon}$. For k large enough, this is not true because the right hand side is independent of k. A contradiction. Thus, we get

$$\frac{1}{1+k}|(eig(kI + \hat{\Sigma}^{-1}))| = \left|eig\left(\frac{1}{1+k}(kI + \hat{\Sigma}^{-1})\right)\right| = |eig(M^{-1})| < 1$$

and hence $|eig(M)| > 1$ so that condition (i) of Lemma C.2 is violated. □

[4]To see that this is true define $\Sigma^{-1} - I =: A$ and note first that since A is positive semidefinite we have $x^T Ax \geq 0$ for all vectors x. Next, let $\breve{B} = \tilde{B}^{\frac{1}{2}}$, i.e. $\breve{B}$ is the positive semidefinite square root of $\tilde{B}$. Then $x^T Ax \geq 0$ for all x implies that $(y^T \breve{B})A(\breve{B}y) \geq 0$ for any y. Thus $eig(\breve{B}A\breve{B}) \geq 0$. Finally, by symmetry, note that $\breve{B}A\breve{B} = (\breve{B}(\breve{B}A))^T = (\tilde{B}A)^T$ which has the same eigenvalues as $\tilde{B}A$.

[5]Simply because: $\lambda^{-1} = \frac{Re(\lambda)}{Re^2(\lambda)+Im^2(\lambda)} + \frac{-Im(\lambda)}{Re^2(\lambda)+Im^2(\lambda)}i$ and $|\lambda|^2 = Re^2(\lambda) + Im^2(\lambda)$.

[6]If λ^{-1} is a real number, then this holds trivially.

1.5. Proof of Proposition 3.6. As by Lemma C.2 above, for the convergence of the powers of a matrix A it is sufficient that 1 is exactly one eigenvalue of A and all other eigenvalues are in the interval $(-1, 1)$. To prove the proposition, we will in a first step apply the Perron-Frobenius Theorem (PFT) for a regular row–stochastic matrix A: (i) The spectral radius of A is 1 (the largest eigenvalue in absolute value). (ii) For all other eigenvalues λ it holds that $|\lambda| < 1$. (iii) The eigenvalue 1 is simple.

Consider any $\hat{\Sigma} \in \mathcal{S}(n)$ such that $\hat{\Sigma}$ is irreducible with strictly positive diagonal. This implies that $\hat{\Sigma}$ is regular, so that by the PFT for regular row stochastic matrices, $\hat{\Sigma}$ has simple eigenvalue 1 and all other eigenvalues are in $(-1, 1)$.

Let us now consider the transformations $M = \hat{\Sigma}\left(I + B\hat{\Sigma}\right)^{-1}(I+B)$. In a first step, we have to guarantee that $I+B\hat{\Sigma}$ is invertible, so that M exists. Note that strict diagonal dominance would be sufficient for non–singularity. For strict diagonal dominance, we require that $1 + \beta_i\left(\hat{\sigma}_i - \sum_{j\in N_i}\hat{\sigma}_{ij}\right) > 0$ holds for every $i \in N$. Since $\hat{\Sigma}$ has a strictly positive diagonal, this is always satisfied if e.g. $\beta \leq \mathbf{1}$.

Given above, it follows again by the continuity of the eigenvalues that there does exist a non–empty neighborhood $N\left(\mathbf{0}\middle|\hat{\Sigma}\right) \subset \mathbb{R}^n_+$ such that $\forall\beta \in N\left(\mathbf{0}\middle|\hat{\Sigma}\right) \cup \mathbf{0}$ both $I+B\hat{\Sigma}$ is strictly diagonally dominant and M has exactly one eigenvalue equal 1 and $n-1$ eigenvalues in the interval $(-1, 1)$. Thus, M^t converges. $\square$

Bibliography

[1] Arnett, J. J. (1995). Broad and narrow socialization: The family in the context of a cultural theory. *J. Marriage Fam.*, 57:617–628.

[2] Becker, G. (1996). *Accounting for Taste.* Harvard University Press, Cambridge MA.

[3] Becker, G. and Mulligan, C. B. (1993). The endogenous determination of time preference. *Quart. J. Econ.*, pages 729–758.

[4] Bester, H. and Güth, W. (1998). Is altruism evolutionarily stable? *J. Econ. Behav. Organ.*, 34(2):193–209.

[5] Bisin, A. and Topa, G. (2003). Empirical models of cultural transmission. *J. Europ. Econ. Ass.*, 1(2-3):363–375.

[6] Bisin, A., Topa, G., and Verdier, T. (2009). Cultural transmission, socialization and the population dynamics of multiple-trait distributions. *Int. J. Econ. Theory*, 5(1):139–154.

[7] Bisin, A. and Verdier, T. (1998). On the cultural transmission of preferences for social status. *J. Pub. Econ.*, 70(1):75–97.

[8] Bisin, A. and Verdier, T. (2000). A model of cultural transmission, voting and political ideology. *Europ. J. Polit. Economy*, 16(1):5–29.

[9] Bisin, A. and Verdier, T. (2001). The economics of cultural transmission and the dynamics of preferences. *J. Econ. Theory*, 97(2):298–319.

[10] Bisin, A. and Verdier, T. (2010). The economics of cultural transmission and socialization. In *Handbook of Social Economics (forthc.)*. Elsevier Science.

[11] Bowles, S. (1998). Endogenous preferences: The cultural consequences of markets and other economic institutions. *J. Econ. Lit.*, 36(1):75–111.

[12] Boyd, R. and Richerson, P. J. (1985). *Culture and the Evolutionary Process.* University of Chicago Press.

[13] Brueckner, J. K. and Smirnov, O. (2007). Workings of the melting pot: Social networks and the evolution of population attributes. *J. Reg. Sci.*, 47(2):209–228.

[14] Brueckner, J. K. and Smirnov, O. (2008). Social networks and the convergence of population attributes: A generalization. *J. Reg. Sci.*, 48(2):359–365.

[15] Cavalli-Sforza, L. L. and Feldman, M. (1973). Cultural versus biological inheritance: phenotypic transmission from parents to children. *Am. J. Hum. Genet.*, 25:618–637.

[16] Cavalli-Sforza, L. L. and Feldman, M. W. (1981). *Cultural Transmission and Evolution: A Quantitative Approach.* Princeton University Press, Princeton.

[17] Cox, D. and Stark, O. (1996). Intergenerational transfers and the demonstration effect. Boston College Working Papers in Economics 329., Boston College Department of Economics.

[18] DeGroot, M. H. (1974). Reaching a consensus. *Journal of the American Statistical Association*, 69(345):118–121.

[19] DeMarzo, P. M., Vayanos, D., and Zwiebel, J. (2003). Persuasion bias, social influence, and unidimensional opinions. *Quart. J. Econ.*, 118(3):909–968.

[20] Duesenberry, J. S. (1949). *Income, Saving and the Theory of Consumer Behavior.* Cambridge, Mass.: Harvard University Press.

[21] Escriche, L., Olcina, G., and Sanchez, R. (2004). Gender discrimination and intergenerational transmission of preferences. *Oxford Econ. Pap.*, 56(3):485–511.

[22] Fershtman, C. and Weiss, Y. (1998). Social rewards, externalities and stable preferences. *J. Pub. Econ.*, 70(1):53–73.

[23] Festinger, L. (1957). *A theory of cognitive dissonance.* Evanston, IL: Row, Peterson.

[24] Friedberg, S. H. and Insel, A. J. (1992). Convergence of matrix powers. *International Journal of Mathematical Education in Science and Technology*, 23(5):765–769.

[25] Friedkin, N. E. and Johnson, E. C. (1997). Social positions in influence networks. *Social Networks*, 19:209–222.

[26] Fujimoto, T. and Ranade, R. R. (2004). Two characterizations of inverse–positive matrices: The Hawkins–Simon Condition and the Le Chatelier–Braun Principle. *Electronic Journal of Linear Algebra*, 11:59–65.

[27] Galor, O. and Moav, O. (2002). Natural selection and the origin of economic growth. *Quart. J. Econ.*, 117(4):1133–1191.

[28] Golub, B. and Jackson, M. O. (2009). How homophily affects learning and diffusion in networks. Working Papers 2009.35, Fondazione Eni Enrico Mattei.

[29] Golub, B. and Jackson, M. O. (2010). Naïve learning in social networks and the wisdom of crowds. *Amer. Econ. J.: Microecon.*, 2(1):112–49.

[30] Grusec, J. E. (2002). *Handbook of Parenting: Practical Issues in Parenting*, chapter Parental Socialization and Children's Acquisition of Values, pages 153–167. Lawrence Erlbaum Associates.

[31] Grusec, J. E. and Hastings, P. D., editors (2007). *Handbook of Socialization: Theory and Research*. New York: The Guilford Press.

[32] Grusec, J. E. and Kuczynski, L., editors (1997). *Parenting and Children's Internalization of Values*. New York: John Wiley and Sons.

[33] Han, P. (2006). *Theorien zur internationalen Migration*. Lucius & Lucius.

[34] Hauk, E. and Sáez-Martí, M. (2002). On the cultural transmission of corruption. *J. Econ. Theory*, 107(2):311–335.

[35] Hegselmann and Krause (2002). Opinion dynamics and bounded confidence, models, analysis and simulation. *Journal of Artificial Societies and Social Simulation*, 5(3):1–33.

[36] Jackson, M. O. (2011). *Social and Economic Networks*. Princeton University Press.

[37] Kockesen, L., Ok, E. A., and Sethi, R. (2000). The strategic advantage of negatively interdependent preferences. *J. Econ. Theory*, 92(2):274–299.

[38] Krause, U. (2000). A discrete nonlinear and non-autonomous model of consensus formation. In Elyadi, S., Ladas, G., Popenda, J., and Rakowski, J., editors, *Communications in Difference Equations*, pages 227–236. Gordon and Breach Pub., Amsterdam.

[39] Leibenstein, H. (1950). Bandwagon, snot, and veblen effects in the theory of consumers' demand. *Quart. J. Econ.*, 64:183–207.

[40] Lorenz, J. (2005). A stabilization theorem for dynamics of continuous opinions. *Physica A: Statistical Mechanics and its Applications*, 355(1):217–223.

[41] Lorenz, J. (2006). Convergence of products of stochastic matrices with positive diagonals and the opinion dynamics background. In Commault, C. and Marchand, N., editors, *Positive Systems*, volume 341 of *Lecture Notes in Control and Information Sciences*, pages 209–216. Springer Berlin / Heidelberg.

[42] Martinez, S., Michon, G., and Martin, J. S. (1994). Inverse of strictly ultrametric matrices are of stieltjes type. *SIAM J. Matrix Anal. Appl.*, 15(1):98–106.

[43] Nabben, R. and Varga, R. S. (1993). On symmetric ultrametric matrices. In Reichel, L., Ruttan, A., and Varga, R. S., editors, *Numerical Linear Algebra*. Walter de Gruyter, New York.

[44] Nabben, R. and Varga, R. S. (1994). A linear algebra proof that the inverse of a strictly ultrametric matrix is a strictly diagonally dominant

stieltjes matrix. *SIAM J. Matrix Anal. Appl.*, 15(1):107–113.

[45] Olcina, G. and Penarrubia, C. (2004). Hold up and intergenerational transmission of preferences. *J. Econ. Behav. Organ.*, 54(1):111–132.

[46] Otto, S. P., Christiansen, F. B., and Feldman, M. W. (1994). Genetic and cultural inheritance of continuous traits. Morrison institute for population and resource studies working paper no. 0064, Stanford University.

[47] Panebianco, F. (2010). "driving while black": A theory for interethnic integration and evolution of prejudice. Eurodiv Paper 70.2010.

[48] Pichler, M. M. (2010a). Cultural formation of preferences and assimilation of cultural groups. Working Papers 438, Bielefeld University, Institute of Mathematical Economics.

[49] Pichler, M. M. (2010b). The economics of cultural formation of preferences. Working Papers 431, Bielefeld University, Institute of Mathematical Economics.

[50] Pichler, M. M. (2010c). A 'note' on rath (1992).

[51] Pollak, R. A. (1976). Interdependent preferences. *Amer. Econ. Rev.*, 66(3):309–20.

[52] Qizilbash, M. (2009). Well-being, preference formation and the danger of paternalism. Papers on Economics and Evolution 2009-18, Max Planck Institute of Economics, Evolutionary Economics Group.

[53] Rath, K. P. (1992). A direct proof of the existence of pure strategy equilibria in games with a continuum of players. *Econ. Theory*, 2(3):427–33.

[54] Rogers, A. R. (1988). Does biology constrain culture? *Am. Anthropol.*, 90(4):819–831.

[55] Sacerdote, B. (2002). The nature and nurture of economic outcomes. *Amer. Econ. Rev.*, 92(2):344–348.

[56] Sacerdote, B. (2007). How large are the effects from changes in family environment? a study of korean american adoptees. *Quart. J. Econ.*, 122(1):119–157.

[57] Sacerdote, B. (2008). Nature and nurture effects on children's outcomes: What have we learned from studies of twins and adoptees? In *Handbook of Social Economics* (forthcoming). Amsterdam: North Holland.

[58] Sáez-Martí, M. and Sjögren, A. (2008). Peers and culture. *Scandinavian Journal of Economics*, 110(1):73–92.

[59] Schmeidler, D. (1973). Equilibrium points in nonatomic games. *J. Stat. Phys.*, 7(4).

[60] Stark, O. (1995). *Altruism and Beyond*. Cambridge University Press.

[61] Veblen, T. (1934). *The theory of the leisure class*. New York: Modern Library.

[62] Weisbuch, G., Deffuant, G., Amblard, F., and Nadal, J.-P. (2002). Meet, discuss, and segregate! *Complexity*, 7(3):55–63.