Inequality of Nations, Endogenous Fluctuations, and Financial Market Globalization

A Dynamic General Equilibrium Approach

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Preface

The whole is more than the sum of its parts. This holistic view of the economy is the cornerstone of this thesis. In particular, to address the issue of inequality of nations we do not treat economies individually, i.e. separately detached from the rest of the world, but we treat the world as one system in which economies are integrated. These economies interact in a single global market and therefore, it is to be expected that economic implications drawn from such a model will not be the same as if we had analyzed each country separately. It is an attempt not only to understand how an economy evolves over time within its own structural framework but also to understand economic development as an interactive process with other economies. This view was also expressed by Frank (1998) on the development of the world economy. In this book he surveys the economic history of the world during the last 500 years and reveals how historical development can be misinterpreted if we miss the economic interlinkages between regions, countries, and sectors. He emphasizes the importance of relating parallel developments in different regions of the world to understand historical events even in a particular country.

This thesis is a purely theoretical work without any substantial empirical analysis. However, the theoretical modeling is motivated mainly by two empirical observations. The first observation is the globalization of financial markets. There may be disagreements as to the extent of the globalization and how to measure it, but there is no doubt that the operation and transactions in financial markets have encompassed more and more of the world in the last decades. The second observation concerns the inequality of nations and may be more controversial. There are empirical studies which claim that the distribution of income per capita in the world has developed a higher level of inequality in the last decades.

The goal of the thesis is to analyze theoretically to how the financial globalization and the distribution of world income may be related. Does the financial market globalization exert an equalizing force? Or is it responsible for the economic stagnation of some countries in the world? These questions will be investigated in more detail with special attention on the interactions of economies. From a technical point of view we will treat the world economy as a dynamical system in which two countries interact in a single global financial market. The economic agents in the two countries are homogenous and each economy has an identical structure and an identical law of evolution. The purpose of this simplification is to identify the mechanism of the global financial market instead of searching for country specific reasons why a country might be relatively poor or rich. Policy implications drawn from such a model will have an impact on the whole system.

There are two main aspects of the mechanism in the global market today that we will study in detail. The first aspect is related to the size of population within countries. Once each data point is weighted by population there seems to be a reduction in global inequality in the past decades. This result is not surprising given the high growth trend of populous countries such as China and India. However, poor countries still comprise a large part of the world population while rich countries represent only a small fraction. We will consider a world economy, which consists of two economies with different population sizes to analyze the relationship between population size and the inequality of nations. The second aspect is the role of nominal assets in allocating savings for production. It is commonly assumed that stock markets provide funds for productive purposes. However, this is only true if firms issue new shares. On the other hand, shares are available for allocating risks and transferring wealth over time. This means that a large part of financial transactions does not necessarily provide funds for productive use but is merely a re-trading of existing financial assets. We will analyze the impact of such financial assets on the capital accumulation of countries.

This thesis is born out of my research activities at the Department of Business Administration and Economics at Bielefeld University. I have greatly benefited from the guidance and the numerous discussions I have had while being a member of the research team of Prof. Volker Böhm. Especially, I acknowledge the financial support of the German Research Foundation for the research projects "Stochastische dynamische Konjunkturtheorie" Bo. 635/9–1 and "Financial Markets and Development" Bo. 635/12–1.

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Tomoo Kikuchi

Bielefeld, September 6, 2006

Chapter 1

Introduction

> *If there indeed was a single globe-encompassing world economic system with its own structure of interlinkages among its regions and sectors, then it stands to reason that what happened in one of them should or at least may have had repercussions also in one or more others.... Indeed, not only can one part of the system affect another, but the interlinked structure and dynamic of the whole system may affect any and even all of its parts. Therefore to account for and understand any local or regional process, it may also be necessary to inquire into how those processes are affected by and respond to contemporary events elsewhere and/ or to simultaneous processes in the world economic system as a whole.*
>
> Frank (1998, p.227)

1.1 Motivation

Globalization has been a recognized economic force for at least three decades. The term is typically defined among economists by international integration of commodity markets, labor markets, and capital markets. While international integration of financial markets seems to advance, there are also pessimistic views about its implications. In particular, financial crises in Latin America in 1994-95, East Asia and Russia in 1997-98, and Argentina in 2001-02, raised concerns over international financial integration.

Regional financial crises seem to have become more frequent. Moreover, there is empirical evidence showing that a large discrepancy in the world income distribution has emerged. One would be tempted to ask whether the globalization of financial markets is responsible for the financial crises and the biased distribution of world income that we observe. Or is it possible that financial crisis and an unequal world income distribution are inherent in a global financial market? Surprisingly, we cannot find systematic answers to these questions in the theoretical literature. Therefore, to start with, it might be helpful to explore why these questions have not received the attention in economic theory that they deserve.

Firstly, the connection between the development of the financial sector and that of the real sector seems to be a well-established fact since the studies of Goldsmith (1969), McKinnon (1973), and Shaw (1973). The link that is usually emphasized is the role of financial markets in channeling savings towards more productive investment. Recently, there have been increasing numbers of theoretical models which examine the role of "financial deepening" or the relation between the evolution of credit markets and economic development in a general equilibrium framework. They show that "frictions" in financial markets such as fixed costs or investment indivisibilities can generate multiple equilibria. Typically, high income economies are better disposed to undertake a growth enhancing financial structure than low income ones. However, these models on "financial development and growth" are closed economy models. This means that they cannot predict what happens when then the economy is open to international financial markets.

Secondly, conventional economic wisdom seems to encourage financial integration. Obstfeld (1994) extends the model by Romer (1990) and shows in a continuous time stochastic model that the possibility of world portfolio diversification can raise steady state growth as individuals place a larger fraction of their wealth in risky but high-yielding capital investments. The idea behind such a model can be best summarized by the observation in Arrow (1971, p.137) "the mere trading of risks, taken as given, is only part of the story and in many respects the less interesting part. The possibility of shifting risks, of insurance in the broadest sense, permits individuals to engage in risky activities that they would not otherwise undertake". In the neoclassical framework with diminishing marginal productivity, the integration of financial markets has an even starker implication. Generally, perfect international financial markets, in which agents may borrow up to the present value of their life time income, imply an immediate adjustment of the per capita income across countries. These results suggest that financial integration induces a growth enhancing, as well as an income equalizing, force.

Thirdly, economic theory usually distinguishes between short run models and long run models. Short run is commonly understood as a period of time in which prices adjust themselves so that markets clear in equilibrium. On the other hand, long run is considered to be a period of time in which production takes place and adjusts to its steady state level. In other words, price fluctuations are regarded as short run phenomenon while growth is regarded to drive the long run development of the economy. To integrate short-term fluctuations into long-run growth analysis, the prevailing view is that of the real business cycles (RBC) model. There, the economy is perturbed by disturbances at random intervals, and those disturbances then propagate through the economy. The ups and downs in financial markets are interpreted as an adjustment process from a temporary shock to the steady state.

1.2 Approach

Let us make some remarks on the methodological approach we take. In a mathematical sense we treat an economy as a dynamical system in which time is a discrete set of integers from 0 to ∞. We define individual decisions, which include forming expectations about the unknown future, in every period explicitly. This allows us to derive the temporary equilibrium of the economy in every period. Instead of analyzing an equilibrium solution implicitly, an explicit description of the process leading to such an equilibrium is given. In this way we can define the evolution of the economy over time and analyze the endogenous propagation of differential features outside of steady state. In other words, we analyze how interactions of different economic variables affect the stability properties of the economy. This allows us to investigate the potential cause for endogenous fluctuations by means of known results from bifurcation theory (see for example Azariadis (1993) and Kuznetsov (1998)). We use specific functional forms which allow for closed form solutions. Therefore, the numerical techniques applied in this thesis are straightforward and do not involve any sophisticated algorithm.

Besides the RBC models, there have been extensive application of the theory of complex dynamical systems to models of economic growth in the past decades. Among them one finds the well known results (see Benhabib & Nishimura (1985) and Boldrin & Montrucchio (1986)) that any complex dynamic behavior can be generated within two sector models of optimal growth. This line of research has also been extended to various macroeconomic contexts. These models are typically characterized by a representative agent with perfect foresight maximizing utility over an infinite horizon (see Boldrin et al. (2001) and Matsuyama (1999, 2001)).

We adopt the overlapping generations framework. The representative infinite-lived agent model can be seen as a special case of the overlapping generations model where households are altruistic and care about their descendants. An important aspect of overlapping generations models is the interaction of heterogenous agents and the distributional consequences on the economy. Optimizing behavior of individuals affect the general equilibrium and thus the evolution of the economy. Therefore, the linkage and the feedback structure between individual behavior and the aggregate phenomena will be analyzed. The most popular framework with overlapping generations assumes that each person lives for only two periods. People work in the first period when they are young and retire in the second period when they are old. If we assume that there is no storage technology for the consumption commodities, they have to transfer a part of their wage income by investing in the financial markets to consume in the second period of their life. Within this framework the debt instrument allows people to trade within a generation while the equity instrument allows people to trade between generations as well. This is because old consumers cannot make a debt contract as they are dead in the next period and are not allowed to leave any negative bequest. In the asset market the ownership of firms is transferred between generations or within a generation if people are heterogenous. All models in this thesis are characterized by this basic overlapping generations structure.

1.3 The Convergence Hypotheses

Galor (1996) points out that the controversy around the convergence hypothesis has been largely empirical, focusing primarily on the validity of the three following competing hypotheses.

1. *The absolute convergence hypothesis* - per capita incomes of countries converge to a steady state in the long-run, independent of their initial conditions.
2. *The conditional convergence hypothesis* - per capita incomes of countries which are identical in their structural characteristics converge to a steady state in the long-run, independent of their initial conditions.
3. *The club convergence hypothesis* - per capita incomes of countries that are identical in their structural characteristics converge to a steady state in the long run, provided that their initial conditions are similar as well.

Since the steady state of an economy depends on its structural characteristics (e.g. technologies, preferences, population growth, government policy, factor market structure, etc.), theoretically absolute convergence requires the convergence in structural characteristics across countries. For example, the Solow model does not predict absolute convergence in general, as countries do not converge to the same level of output per worker unless countries have identical saving and population growth rates as well as identical technologies. Not surprisingly, the absolute convergence hypothesis has been refuted in empirical studies (see Romer (1986), Lucas (1988) and Barro (1991)). The initial claim for the absolute convergence hypothesis by Baumol (1986) has been rejected by De Long (1988). His objection is that the presence of convergence in a sample of countries, which are selected according to their proximity in initial or terminal conditions, does not provide empirical support for the absolute convergence hypothesis, but rather, for the conditional and club convergence hypotheses. More recently, by analyzing patterns of economic growth across countries from the perspective of distribution dynamics, Quah (1993, 1996a, 1997) revealed robust characterizations of a tendency towards an endogenous formation of convergence clubs and a polarization of the distribution of income across countries. However, this literature uses countries as the unit of analysis. Jones (1997) showed that the emergence of the so called "twin peaked" distribution disappears once each country data point is weighted by population. More recently Sala-i-Martin (2006) merged survey data about the income distribution within individual countries with national account data to estimate the world income distribution. He concludes that there has been a reduction in global inequality during the 1980s and 1990s.

One might expect to observe convergence of structural characteristics of economies in a world of fully integrated markets for goods, capital, and ideas. However, it is still ambiguous from a theoretical point of view whether conditional or club convergence prevails. Galor (1996) is an excellent survey on one sector growth models with a neoclassical framework. He argues that in contrast to the prevailing wisdom, the assumption on diminishing marginal productivity of factors of production does not necessarily lead to conditional convergence. In particular he argues that the inclusion of variables such as human capital, income distribution, and fertility in conventional growth models along with capital market imperfections, externalities, and non-convexities strengthens the viability of club convergence as a competing hypothesis. In other words, these extended neoclassical growth models are characterized by an economic system with multiple, locally stable steady states.

1.4 Financial Development and Growth

The section above reviewed the concept of convergence and discussed relevant empirical findings. The empirical studies center primarily on the testing of the neoclassical growth model which predicts conditional convergence. While the literature established the conditional convergence as a dominant hypothesis, extensions of the neoclassical models show that this theoretical conjecture is not robust. Therefore, we review the extensions of the neoclassical models which are related to financial markets in this section.

Goldsmith (1969), McKinnon (1973), and Shaw (1973) emphasized the role of financial markets that channel savings towards more productive investment. More recently, financial intermediaries are regarded as solving informational problems that would otherwise lead to inefficient outcomes. The influence of financial intermediaries on growth was first modeled in a general equilibrium framework by Greenwood & Jovanovic (1990) followed by Saint-Paul (1992), Bencivenga & Smith (1991), Zilibotti (1994), Greenwood & Smith (1997), and Acemoglu & Zilibotti (1997) among others.

The functions of the financial markets emphasized are different in each paper. Greenwood & Jovanovic (1990), Zilibotti (1994), and Acemoglu & Zilibotti (1997) emphasize the role of financial institutions to facilitate trade in the economy by internalizing the information externality that allows investors' resources to flow to their most profitable use. In addition, the pooling of risks by financial institutions across large numbers of investors is stressed in Greenwood & Jovanovic (1990).[1] In Bencivenga & Smith (1991) and Greenwood & Smith (1997) financial intermediaries arise to service the liquidity needs of agents. Along the lines of Diamond & Dybvig (1983), agents who face random future liquidity needs accumulate a liquid, but unproductive asset. Financial intermediaries permit an economy to reduce the fraction of its savings held in the form of unproductive liquid assets. Saint-Paul (1992) was the first to model risk diversification leading to more productive specialization. Financial markets have a positive impact on productivity because they allow a greater specialization of resources. Specialization of a particular resource (capital, labor) into a narrower range of tasks can occur without having a negative impact through the accompanying increase in risk, because agents can hedge the risk by holding a diversified portfolio in the financial market. Hence, financial markets contribute to growth by facilitating a greater division of labor.

Even though financial markets promote growth in different ways, there might be a cost involved in opening the financial market in the first place. The role of fixed costs when opening new markets is emphasized in Greenwood & Jovanovic (1990), Saint-Paul (1992), Zilibotti (1994), and Greenwood & Smith (1997). In these papers, there is a "threshold effect", i.e., if the economy is wealthy enough to open a financial market then it increases the equilibrium rate of growth.[2] Consequently, high-income economies are better disposed to undertake such financial superstructure building than ones with low income levels. This interaction between the development of financial markets and economic growth derives the dynamics of growth in each paper. As mentioned above, in Saint-Paul (1992), a more developed financial market allows for a more specialized technology and thus leads to higher growth. This implies that there is a strategic com-

[1] By investing through intermediated structures, individuals obtain both a higher and a safer return.

[2] By contrast, in Bencivenga & Smith (1991) the state of development of financial markets is taken as exogenously imposed. This analysis follows the suggestions of Cameron (1967), McKinnon (1973), and Shaw (1973) that differences in the extent of financial markets across countries seem to depend primarily on legislation and government regulation.

plementarity between financial markets and technology because both can be used for risk diversification. Because of the fixed cost, however, individuals may be reluctant to open a financial market. Consequently, a transition to a faster growth path with financial markets can happen at any time when income is between the two critical levels. Below the lower bound there is no incentive to open a financial market since the cost of opening a financial market is large relative to the cost of technological diversification. Above the upper bound the cost of opening a financial market is small relative to the cost of technological diversification and financial markets will be opened.[3] Multiple equilibria can arise between these two critical values. Financial intermediaries have to internalize the externality of the impact of individual actions on technological choice to achieve a financial equilibrium. Similarly, in Zilibotti (1994) economies endowed with capital below some threshold level might not attain self-sustained growth with the first-best allocation. The "thick market" externality can lock an economy into a stationary equilibrium under a laissez-faire regime. Each firm, when investing, adds to the total demand of intermediation services and causes a fall in the gap between the price of internal and external capital. In a laissez-faire economy, firms ignore such external effects and cannot coordinate their demand. Multiple equilibria exist between certain levels of initial capital and the equilibrium depends on the coordination of demand. In other words, there exists "strategic complementarity" in the investment demands of final producers in the presence of imperfect intermediation. In the bad equilibrium, because of the high intermediation costs, the equilibrium rate of interest is too low to warrant sustained capital accumulation, i.e., people consume more now than in the future and the economy converges to a zero-income equilibrium. In the good equilibrium the economy is in a balanced growth condition with a constant saving rate. Thus, in Saint-Paul (1992) and Zilibotti (1994) one can obtain non-convergence results in both levels and growth rates across countries identical in technology and preferences. In contrast, Greenwood & Jovanovic (1990) and Greenwood & Smith (1997) do not have multiple equilibria. In their model financial intermediaries internalize the complementarities between technological and financial diversification. The cost of market formation requires that endogenous

[3]Saint-Paul (1992) assumes that the cost of trading in financial markets is less than proportional to the volume traded whereas if technological diversification implies less productivity, its cost is proportional to output. Multiple equilibria arise in the range where the cost of trading and the cost of technological diversification are equal.

market development follow some period of real development, eventually however, there is convergence to the steady state.

Acemoglu & Zilibotti (1997) shows that capital accumulation is associated with an increase in the volume of intermediation and financial activities as a proportion of the gross domestic product.[4] However, while other papers derive their dynamics from the presence of the fixed costs of financial intermediation, Acemoglu & Zilibotti (1997) assumes no explicit costs of financial relations. Instead, there is investment indivisibility in some highly productive sectors. This micro-level indivisibility together with risk averse agents leads to a particular dynamic growth path. As an additional sector opens, all existing projects become more attractive relative to the safe asset because the amount of undiversified risk they carry is reduced. However, due to investment indivisibility only a limited number of imperfectly correlated sectors can be opened. Therefore, risk averse agents seek insurance by investing more in safe, but less productive assets. This characterizes the dynamics of the economy. Eventually, with sufficient capital accumulation the economy takes off and converges to its steady state where all the sectors are open and risk is completely diversified. However, at the earlier stage the typical development pattern consists of a lengthy period of primitive capital accumulation with highly variable output since the growth is subject to the realization of the state of the risky investment.

The literature on financial development and growth contributed to the understanding of the microstructure of financial markets which influences the path of development and generates multiple steady states. Especially, it gives a microeconomic foundation to "financial market imperfection" which is given exogenously in the model we will introduce in Section 2.3. The models with multiple steady states are referred to as "poverty trap" models where an economy, caught in a vicious cycle, suffers from persistent underdevelopment. Poverty trap is often interpreted as an explanation for the cross country income difference. This is a misinterpretation. The message of the poverty trap models is the self-perpetuating nature of poverty. It suggests that the long run performance of an economy could be much better if its initial condition were better.[5] To explain the cross country difference in the long run performance we need to model the link-

[4]See the empirical findings of Goldsmith (1969), Atje & Jovanovic (1993), and King & Levine (1993).

[5]For a more detailed discussion on poverty trap see Matsuyama (2006).

age between the countries. This is the main conceptual difference between poverty and inequality. We can only discuss inequality when we treat more than one country in relation to other countries. Therefore, the following chapters introduce open economy models where countries with identical structural characteristics are integrated through an international financial market. These models will analyze the international market forces which exert an impact on the development of individual economies. It is important to mention that the closed economy models and the open economy models have different policy implications. Policy implications obtained by the closed economy models would deal with the issue of development as isolated problems independently from the rest of the world. Hence, the policy will focus on the reduction of poverty and not on inequality.

1.5 Definition of Financial Markets

A financial market can take the form of a capital market, a credit market, or an asset market. The dividing line between these markets is however not so neat, as one market can sometimes play different roles. In both capital and credit markets, agents make a contract, a so called debt instrument, which specifies the obligation of the debtor to repay the sum of principal plus interests in the future. In this sense, the literature on financial development and growth is a sophisticated treatment of the capital market. This is done by giving financial intermediaries roles to play in markets with 'frictions'. In this thesis we abstract from this sophistication of the mechanism of the financial market. This simplification helps to focus on the effect of integration of financial markets. We show that even in these models with simple financial markets, the dynamic behavior can be very rich. We distinguish between capital markets (credit markets) and asset markets. In an asset market (stock market), agents trade a share of stock, a so called equity instrument which is a claim to a firm's profit. The difference between the debt instrument and the equity instrument leads to different trading structure. While the capital markets serves for trading of savings only within a generation, the asset market allows for intertemporal trading of savings.

1.6 Outline

The thesis is organized as follows. Chapter 2 discusses the implications of the neoclassical growth model once it is open to international financial trade. First, we introduce the notion of a perfect international capital market and analyze its implications. The perfect international capital market implies that capital stocks across countries converge to one another immediately. The intuition behind this is that the capital flows freely in the perfect international capital market from rich countries, where the return on capital is lower, to poor countries, where the return on capital is higher. This process continues until the marginal product of capital is equalized in all countries. This result led to the paradox "Why doesn't capital flow from rich to poor countries?" by Lucas (1990). Boyd & Smith (1997) and Matsuyama (2004) give us one possible solution to the Lucas' paradox. We introduce the model by Matsuyama (2004) to discuss his results on the autarky case and the small open economy.

Chapter 3 extends the small open economy model by Matsuyama (2004) to a two country model. This extension is a very important step since it allows us to analyze the feedback effects between capital stocks in both countries through the world interest rate. The two country model also allows for an analysis of interactions of two countries with different population sizes. Besides the inequality of the world income distribution, the more obvious inequality in the world economy is the population distribution. Even though there is a large bias towards huge populations in the poorer regions of the world, there seems to be little attention paid to the relationship between population size and income. The two country model will investigate how a change in the relative population size of the two countries affects the stability property of the dynamical system and thus the allocation of incomes.

Chapter 4 develops an overlapping generations model with random production, capital accumulation, and nominal assets to analyze the interaction between the real sector and the financial sector of the economy. It is commonly assumed that financial trade typically goes hand in hand with the trade of real commodities. In particular, higher risk is assumed to be associated with not just higher return, but also with higher productivity in the real sector. In other words, the role of financial markets is confined to supplying real capital for production directly. However, financial market integration has increased opportunities for allocating risks where assets play a major role of transferring wealth over time. Hence, a large part of financial trade comprises of the re-trading of assets which are already in the market. We shall investigate the role of such assets within markets.

Chapter 5 generalizes the model in Chapter 4 to a two country case. Two identical economies which only differ in their levels of capital stock are now linked through an international asset market. Firms in both countries form international mutual funds and pay their stochastic profits as dividends. Agents in both countries can transfer their wealth by investing capital for production and by investing in the international mutual funds. The asset market creates a feedback mechanism between the asset demand and capital investment.

Chapter 6 summarizes the findings and gives a possible outlook for the future research.

The proofs of lemmas and propositions which are not provided in the main text are collected at the end of each chapter. Following the convention in each field, mathematical notations are specific to each chapter. For example, α denotes the production elasticity in Chapter 3 while it denotes the measure of risk aversion in Chapters 4 and 5.

Chapter 2

Capital Accumulation of Open Economies

2.1 Introduction

Open economy models can be categorized into small open economy models and multiple country models. The small open economy models predict how a small economy behaves when it is integrated into the world economy. The small open economy has no influence at all on the rest of the world and the influence from the rest of the world on the small open economy is assumed to be constant over time. In other words, there are no feedback effects between the small open economy and the rest of the world. In multiple country models, however, interactions of agents in different economies generate feedback effects on the rest of the world and vice versa. This induces an endogenous evolution of the world economy as a whole. In the following sections identical economies, which differ only in their levels of capital stock are integrated into the world economy through an international financial market. In other words, each identical economy is open to the international financial market. The purpose of this setup is to examine whether interaction mechanisms in the international financial market change the development pattern of economies. Each economy produces an identical commodity using capital and labor. To focus on the role of the international financial market it is assumed that labor is not mobile across countries. The financial market serves to allocate savings for production. In this sense capital is mobile across countries. Section 2.2 presents a multiple country model with a perfect international capital market. In contrast to the empirical observation this model predicts that incomes across countries converge immediately. Therefore, we introduce a model with financial imperfections in Section 2.3.[1]

2.2 A Perfect International Capital Market

The economy consists of markets for labor, capital and output. Consider an overlapping generations model with two period's life time. Young agents in country $i \in (1, 2, 3, ..., T)$ maximize the utility function in an arbitrary period t

$$u_t^i = \ln(c_t^i) + \beta \ln(c_{t+1}^i), \tag{2.2.1}$$

where $\beta \in (0, 1)$ is the time discount factor. There exists a single infinitely lived firm in each country, which produces aggregate consumption goods Y_t in each period t using the total amount of labor L_t and capital K_t. The production function $F(K_t, L_t)$ is assumed to be linear homogeneous. Then the output per capita is given by

$$y_t = f(k_t) \tag{2.2.2}$$

where $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$ and $f(k_t) := F(1, k_t)$. Let us assume a standard neoclassical production function with diminishing marginal product. The factor markets in the economy are competitive meaning that the firm pays wage income and capital income according to the marginal product rule $w_t^i = f(k_t^i) - k_t^i f'(k_t^i)$ and $r_t^i = f'(k_t^i)$. We assume that there exists a perfect international capital market in which 1) agents can lend or borrow an amount of commodities b^i without incurring any operating costs, 2) agents can borrow up to the present value of their life time income, and 3) agents are price takers. Young consumers with a given wage income w_t^i at time t decide first how much to save and then makes their international portfolio decision. If we assume that capital depreciates fully after one period, the capital accumulation law and next period

[1] The assumption of financial market imperfections is well founded in the models which are presented in Section 1.4.

consumption are given by

$$
\begin{aligned}
k^i_{t+1} &= w^i_t - b^i_t - c^i_t && (2.2.3)\\
c^i_{t+1} &= (1+r^i_{t+1})k^i_{t+1} + (1+r_{t+1})b^i_t. && (2.2.4)
\end{aligned}
$$

Substituting equation (2.2.3) into equation (2.2.4), next period consumption can be rewritten as

$$c^i_{t+1} = (1+r^i_{t+1})(w^i_t - b^i_t - c^i_t) + (1+r_{t+1})b^i_t. \tag{2.2.5}$$

The young consumer's objective is

$$\max_{b^i,c^i} \left\{ \ln(c^i_t) + \beta \ln((1+r^i_{t+1})(w^i_t - b^i_t - c^i_t) + (1+r_{t+1})b^i_t) \middle| w^i_t - b^i_t - c^i_t \geq 0 \right\}. \tag{2.2.6}$$

To solve the constraint maximization problem, the Lagrangian can be written as

$$\mathcal{L} = \ln(c^i_t) + \beta \ln((1+r^i_{t+1})(w^i_t - b^i_t - c^i_t) + (1+r_{t+1})b^i_t) + \lambda(w^i_t - b^i_t - c^i_t). \tag{2.2.7}$$

The associated Kuhn-Tucker necessary conditions are

$$
\begin{aligned}
\frac{\partial \mathcal{L}}{\partial c^i} &= \frac{\partial u^i}{\partial c^i_t} - (1+r^i_{t+1})\frac{\partial u}{\partial c^i_{t+1}} - \lambda \\
&= \frac{1}{c^i_t} - \beta \frac{1+r^i_{t+1}}{(1+r^i_{t+1})(w^i_t - b^i_t - c^i_t) + (1+r_{t+1})b^i_t} - \lambda = 0, && (2.2.8)\\
\frac{\partial \mathcal{L}}{\partial b^i} &= (r_{t+1} - r^i_{t+1})\frac{\partial u^i}{\partial c^i_{t+1}} - \lambda \\
&= \beta \frac{1+r_{t+1} - (1+r^i_{t+1})}{(1+r^i_{t+1})(w^i_t - b^i_t - c^i_t) + (1+r_{t+1})b^i_t} - \lambda = 0, && (2.2.9)
\end{aligned}
$$

the budget constraint given by equation (2.2.5), and the complementary slackness condition

$$\lambda^i(w^i_t - b^i_t - c^i_t) = 0. \tag{2.2.10}$$

If $w^i_t - b^i_t - c^i_t = 0$, then $b^i_t = \beta c^i_t$ from equations (2.2.8) and (2.2.9). Then, $c^i_t = \frac{w^i_t}{1+\beta}$ implies $b^i_t = \frac{\beta}{1+\beta} w^i_t$. Market clearing in the international capital market requires $\sum_{i=1}^T b^i_t = \frac{\beta}{1+\beta}\sum_{i=1}^T w^i_t$=0. This is only possible when at least one w^i_t is negative and therefore we rule out this solution. If $w^i_t - b^i_t - c^i_t > 0$, then $\lambda^i = 0$ from equation (2.2.10). Then equation (2.2.9) implies that $r^i_{t+1} = r_{t+1}$ at equilibrium which in turn implies $k_{t+1}^i = k_{t+1}^j$ at equilibrium as $f'(k_{t+1}^i) = f'(k_{t+1}^j)$ for $i, j \in (1, 2, 3..., T)$. This implies that any economies with identical characteristics which differ only in the stock of capital immediately adjust to an equal capital stock if they are integrated through a perfect international capital market. Intuitively the capital will flow from the economy with higher capital stock to the economy with lower capital stock.

Equation (2.2.8) implies $c_{t+1}^i = \beta(1 + r^i)c_t^i$. Substituting this equation into equation (2.2.5) we obtain the first period consumption

$$c_t^i = \frac{w_t^i}{1+\beta}. \tag{2.2.11}$$

Equations (2.2.11) and (2.2.5) together with $r^i = r$ give us the second period consumption

$$c_{t+1}^i = \frac{1+r}{1+\beta}\beta w_t^i. \tag{2.2.12}$$

The world excess demand should be zero at equilibrium in the international capital market, i.e.

$$\sum_{i=1}^{T} b_t^i = 0. \tag{2.2.13}$$

Equations (2.2.3) and (2.2.13) give us the capital stock of each country in the second period

$$k_{t+1}^i = \frac{1}{T}\frac{\beta}{1+\beta}\sum_{i=1}^{T} w_t^i. \tag{2.2.14}$$

Equations (2.2.3) and (2.2.14) finally give us the net demand of each economy in the international capital market

$$b_t^i = \frac{\beta}{1+\beta}\left(w_t^i - \frac{1}{T}\sum_{i=1}^{T} w_t^i\right). \tag{2.2.15}$$

These equations verify the intuition that the capital will flow from the economy with higher capital stock to the economy with lower capital stock. In fact, the above result implies an immediate conditional convergence. This theoretical finding is obviously not satisfactory in many aspects. As Lucas (1990) points out we do not observe international financial flows of the magnitude the model predicts. Furthermore, empirical studies by De Long (1988), Barro (1991) and Quah (1996a, 1997) among others do not support the conditional convergence even in the long run. Therefore, the next section will discuss a possible feature in financial markets which can impede the conditional convergence.

2.3 A Model with Credit Market Imperfections

Lucas (1990) posed the paradoxical implication of the perfect international capital market in his paper titled "Why doesn't capital flow from rich to poor countries?". In the paper he discusses why capital does not flow from rich to poor countries to the extent that a standard neoclassical model would predict. As a remedy to this paradox he suggests inter alia aspects to incorporate imperfections in the financial market. The idea of imperfection in the financial market is based on a so called enforcement problem. The enforcement problem arises from restrictions on the range of financial contracts people can sign, as contracts are not always honored. The difficulties in enforcing contracts, ex post, limit the range of contracts agents will agree to ex ante. This enforcement problem is related to the problem of financial intermediary we discussed in Section 1.4. However, instead of modeling the decision making of financial intermediary explicitly, the imperfection in the financial market is assumed in the literature on convergence. It is assumed that the investor requires external finance and can only borrow up to a fraction of his life time wealth. This wealth dependent borrowing constraint relaxes as the economy accumulates more capital since the investor requires less external capital. Hence, models with financial market imperfections share with the models on financial development and growth the implication that richer economies are better equipped to fully exploit the production possibility of the economy. Several theoretical models incorporated this enforcement problem in order to solve the Lucas' paradox. Barro, Mankiw & Sala-i-Martin (1995) show that the neoclassical growth model accords with empirical evidence on convergence if physical capital can be used as collateral for international borrowing whereas human capital cannot. In an overlapping generations model Obstfeld & Rogoff (1999) demonstrate that conditional convergence need not take place if international borrowing is limited to a fraction of current output. These models illustrate how assumptions on the microeconomic level in the financial market can lead to different results on convergence. However, the analysis is confined to whether a small open economy converges to the world economy. In other words, there are no feedback structures between the world and the small open economy.

There are notable exceptions. In a one sector overlapping generations model modified to incorporate financial market imperfections Boyd & Smith (1997) and Matsuyama (2004) show that the interaction between competitive international financial trade can amplify the inequality of income across nations. In the two country model of Boyd & Smith (1997) the capital investment in both countries is subject to a costly state verification (CSV) problem. The country with higher capital stock provides more internal financing, hence mitigating the CSV problem. Higher internal financing in the rich country counteracts the higher marginal product in the poor country. As a consequence an initially poorer country remains poorer in the long rung by the operation of the international financial market. In Matsuyama (2004) the domestic investment requires borrowing in the credit market which is constrained by the domestic wealth. Poorer countries with higher marginal productivity face borrowing constraints which preclude countries from immediate global convergence. This borrowing constraint generates multiple steady states for the small open economy. Matsuyama (2004) assumes that the world consists of a continuum of small open economies. Hence, the capital stock of individual economies does not affect the world interest rate. This assumption simplifies the analysis and he obtains the complete characterization of the stable steady states. Symmetry breaking occurs in the presence of an international credit market. That is, the symmetric steady state loses its stability and stable asymmetric steady states arise. Section 2.3.1 introduces the basic structure of the model with credit market imperfection by Matsuyama (2004). Section 2.3.2 and 2.3.3 review the autarky case and the small open economy respectively.

2.3.1 The Model Structure

There are domestic markets for output and labor and an international credit market. It is assumed that production factors are non-tradable and agents cannot start an investment project abroad. In other words we rule out foreign direct investment. This is to focus on the effects of financial market globalization not on factor market globalization. All markets operate under perfect competition implying that the respective agents are price takers. We refer to the financial market here as credit market because as we will see later, the agents do not trade physical capital which is distinguished from the consumption goods they trade within a generation (intragenerational trade) in the credit market. We assume that the capital stock depreciates fully after one period and there is no population growth.

The Production Sector

There exists a single firm that lives infinitely long in each country, which produces aggregate consumption goods Y_t in each period t using the total amount of labor L_t and physical capital K_t by use of a linear homogeneous production function $F(L_t, K_t)$. Then output per capita is given by

$$y_t = f(k_t),$$

where $y_t = \frac{Y_t}{L_t}$, $k_t = \frac{K_t}{L_t}$ and $f(k_t) := F(1, k_t)$. We assume that factor markets are competitive meaning that the firm pays wages and returns on capital according to the marginal product rule, i.e., $W(k_t) := f(k_t) - k_t f'(k_t)$ and $r(k_t) := f'(k_t)$ respectively.

Assumption 2.3.1 *The production function in intensive form $f : \mathbb{R}_+ \to \mathbb{R}_+$ is C^2, and satisfies $f(0) = 0$, $f''(k) < 0 < f'(k)$, and the Inada conditions $\lim_{k\to\infty} f'(k) = 0$ and $\lim_{k\to 0} f'(k) = \infty$.*

To avoid multiple steady states that are not related to credit market imperfection we impose the following assumption.

Assumption 2.3.2 *$\lim_{k\to 0} W'(k) = \infty$ and $W''(k) < 0$.*

Many standard production functions satisfy Assumption 2.3.2. Especially, if we use the Cobb-Douglas production function $f(k) = Ak^\alpha$, $W(k) = (1-\alpha)Ak^\alpha$ which satisfies Assumption 2.3.2.

The Consumption Sector

There are overlapping generations of two-period lived consumers, who supply one unit of labor inelastically in the first period and consume only in the second period. There are a continuum of young consumers indexed by $j \in [0,1]$ with income $W(k_t)$ at the beginning of period t. They have two options to transfer their income to the next period. Firstly they may lend their income in the competitive credit market and receive $r_{t+1}W(k_t)$ in the next period. Secondly they may start an investment project which comes in discrete, nondivisible units. The project the investor can start is restricted to one which requires one unit of consumption goods. There exists a homogeneous linear technology to transform one unit of consumption goods into R units of physical capital.

Assumption 2.3.3 $W(R) < 1$.

Assumption 2.3.3 ensures that $W(k_t) < 1$ as we will see later. This assumption is crucial for the results later since it means that young consumers always have to borrow an amount $1 - W(k_t)$ to start an investment project. The R units of physical capital are used as an input for production. Then, the investor's return in the next period will be the rate of return on the capital investment minus the debt repayment, $Rf'(k_{t+1}) - r_{t+1}(1 - W(k_t))$.

The Credit Market

There are two major assumptions which characterize the credit market. Firstly, the investor has to be willing to start a project. We call this condition the profitability constraint meaning the return from starting a project hast to be at least equal to the return from saving. This requires

$$Rf'(k_{t+1}) \geq r_{t+1}. \tag{2.3.1}$$

Secondly, the borrower in the credit market cannot credibly commit to repay more than a fraction of the revenue of the investment project. Thus the borrowing constraint is written as

$$\lambda Rf'(k_{t+1}) \geq r_{t+1}(1 - W(k_t)), \tag{2.3.2}$$

where $\lambda \in (0,1)$ can be interpreted as a measure of imperfection in the credit market. Note if the credit market were perfect, i.e. $\lambda = 1$, this constraint would never be binding. These two constraints have to always hold in the credit market. In other words, agents must be willing and able to to start an investment project. The two constraints can be summarized as

$$r_{t+1} \leq R_t := \begin{cases} \dfrac{\lambda R f'(k_{t+1})}{1 - W(k_t)} & \text{if } k_t < K(\lambda) \\ \\ R f'(k_{t+1}) & \text{if } k_t \geq K(\lambda), \end{cases} \tag{2.3.3}$$

where R_t may be interpreted as the project productivity required in order for the project to be undertaken in period t, and $K(\lambda)$ is defined implicitly by $W(K(\lambda)) = 1 - \lambda$. If $k_t < K(\lambda)$, the borrowing constraint is the relevant constraint since the profitability constraint is always satisfied. If $k_t \geq K(\lambda)$, the profitability constraint is the relevant constraint since the borrowing constraint is always satisfied. Therefore it depends entirely on k_t as to which constraint has to be considered in the credit markets. The young consumers are price takers in the competitive credit markets and make investment decision to maximize their next period consumption. If $k_t \geq K(\lambda)$, the agents prefer starting the investment project to lending until the profitability constraint is binding. In other words at the market equilibrium where the profitability constraint is binding, young agents are indifferent between borrowing and lending. If $k_t < K(\lambda)$, the agents always prefer starting the investment project to lending. This implies that at the market equilibrium where the borrowing constraint is binding, some of young consumers will be denied to take credit. If $\bar{j}_t$ denotes the measure of investors among young agents at time t, the aggregate capital investment is given by

$$k_{t+1} = \int_0^{\bar{j}_t} R dj.$$

Obviously, the proportion of young agents who are rationed in equilibrium will be $(\bar{j}_t, 1]$. In order to analyze the equilibrium we have to know how the interest rate is determined.

2.3.2 The Autarky Case

Without international lending and borrowing, saving must be equal to investment in the economy in equilibrium. From equation (2.3.3), investment is equal to zero if $r_{t+1} > R_t$, and to one if $r_{t+1} > R_t$, and may take any value between zero and one if $r_{t+1} = R_t$. Since the young agents receive wage income $W(k_t)$ and consume only when they are old, the aggregate saving is equal to $W(k_t)$, which is less than one from Assumption 2.3.3. The equilibrium interest rate is determined so that the aggregate investment is made equal to the aggregate saving. This requires $r_{t+1} = R_t$ in equilibrium.

Since the investment project requires one unit of consumption goods and the aggregate saving is less than one, the fraction of young agents who become borrowers and start the project, $\bar{j}_t$ is equal to $W(k_t)$ while the rest become lenders.[2] If $k_t \geq K(\lambda)$, young agents are indifferent between borrowing and lending. When $k_t < K(\lambda)$, on the other hand, they strictly prefer borrowing to lending. Therefore, the equilibrium allocation necessarily involves credit rationing, where the fraction $1 - W(k_t)$ of young agents are denied credit when the borrowing constraint is binding. Since the measure of the young agents who start the project is equal to $W(k_t)$ and every one of them supplies R units of physical capital,

$$k_{t+1} = RW(k_t). \tag{2.3.4}$$

Equation (2.3.4) completely describes the dynamics of capital formation in autarky. Note that, if $k_t < R, k_{t+1} = RW(k_t) < RW(R) < R$ from Assumption 2.3.3. Therefore, $k_0 < R$ implies $k_t < R$ and $W(k_t) < 1$ for $t > 0$, as has been assumed. From equations (2.3.3), (2.3.4), and $r_{t+1} = R_t$, the equilibrium interest rate is given by

$$r_{t+1} = \begin{cases} \dfrac{\lambda R f'(RW(k_t))}{1 - W(k_t)} & \text{if } k_t < K(\lambda) \\ \\ Rf'(RW(k_t)) & \text{if } k_t \geq K(\lambda). \end{cases} \tag{2.3.5}$$

Assumptions 2.3.2 and 2.3.3 ensure that equation (2.3.4) has a unique steady state $k = K^*(R) \in (0, R)$ defined implicitly by $k = RW(k)$, and for $k_0 \in (0, R)$, k_t converges monotonically to $k = K^*(R)$. The function $K^*(R)$ is increasing and satisfies $K^*(0) = 0$ and $K^*(R^+) = R^+$ where R^+ is defined by $W(R^+) = 1$. It is worth mentioning that the dynamics of capital formation in autarky is unaffected by the degree of credit market imperfection λ. This is because domestic investment is made equal to domestic saving by the adjustment of the interest rate. Hence, the credit market imperfection has only an influence on whether the borrowing constraint will be binding or not in equilibrium.

[2]This follows from "fraction of young agents × one unit of consumption goods = own endowment + borrowing = investment"$=W(k_t) \times 1 = W(k_t) \times W(k_t) + (1 - W(k_t)) \times W(k_t) = W(k_t)$".

2.3.3 The Small Open Economy

The world interest rate r is constant in the small open economy. This means that the small open economy does not have any influence on the world economy and the influence of the world economy on the small open economy is constant throughout time. From Section 2.3 we know that both constraints are binding in equilibrium. Then we obtain from equation (2.3.3) the following proposition.

Proposition 2.3.1 *For any given $0 < R < R^+$ and $0 < \lambda < 1$, there exists k_{t+1} which clears the credit market of the small open economy defined by*

$$r = \begin{cases} \dfrac{\lambda R f'(k_{t+1})}{1 - W(k_t)} & \text{if } k_t < K(\lambda) \\ \\ R f'(k_{t+1}) & \text{if } k_t \geq K(\lambda). \end{cases} \tag{2.3.6}$$

if and only if $Rf'(R) \leq r$.

Proof: Note that $j \in [0, 1]$ implies $k_{t+1} \in [0, R]$. Then $Rf'(k_{t+1}) \in [Rf'(R), \infty)$. Therefore $Rf'(R) \leq r$ guarantees the existence for $k_t \geq K(\lambda)$. $Rf'(R) \leq r$ also guarantees the existence for $k_t < K(\lambda)$ as $1 - W(k_t) > \lambda$. This proves the proposition. $\square$

Figure 2.3.1 visualizes the idea of the temporary equilibrium. Recall that there exists a continuum of young consumers with altogether, unit mass. If all young consumers start an investment project, $k_{t+1} = R$. Given a fixed level of technology R, the lowest possible revenue from the investment project is $Rf'(R)$. If the interest rate in the credit market is lower than the lowest possible revenue, i.e., $r < Rf'(R)$, the profitability constraint is always violated. If $r \geq Rf'(R)$, there will be more and more young consumers who start the investment project until $r = Rf'(k_{t+1})$. The proportion of young consumers who start the investment project is denoted by $\bar{j} \in [0, 1]$ in equilibrium.

Solving equation (2.3.6) for k_{t+1}, the physical capital investment of any country subject to the two constraints is given by

$$k_{t+1} = \quad \Psi(k_t, r) := \begin{cases} \Phi\left[\dfrac{r(1 - W(k_t))}{\lambda R}\right] & \text{if } k_t < K(\lambda) \\ \\ \Phi\left(\dfrac{r}{R}\right) & \text{if } k_t \geq K(\lambda) \end{cases} \tag{2.3.7}$$

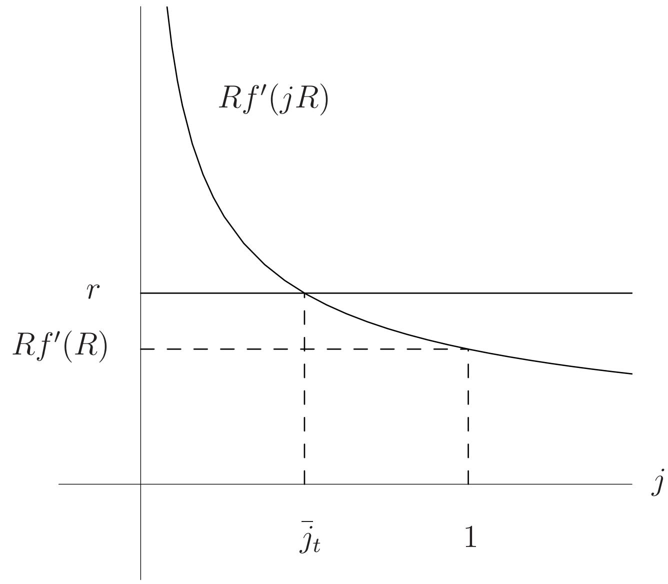


Figure 2.3.1: Temporary equilibrium in the international credit market

where $\Phi := (f')^{-1}$. The following lemma characterizes the steady states of the small open economy.

Lemma 2.3.1 (Lemma Matsuyama (2004))

(a) Equation (2.3.7) has at least one steady state.

(b) Equation (2.3.7) has at most one steady state above $K(\lambda)$. If it exists, it is stable and equal to $\Phi(r/R)$.

(c) Equation (2.3.7) has at most two steady states below $K(\lambda)$. If there is only one, k_L, either it satisfies $0 < k_L < \lambda R/r$ and is stable, or $k_L = \lambda R/r$ at which Φ is tangent to the 45° line. If there are two, k_L and k_M, they satisfy $0 < k_L < \lambda R/r < k_M < K(\lambda)$, and k_L is stable and k_M is unstable.

For the exact condition for each of the three cases see Matsuyama (2004), Proposition 2. Figure 2.3.2 shows the case where there exist three steady states of the small open economy.

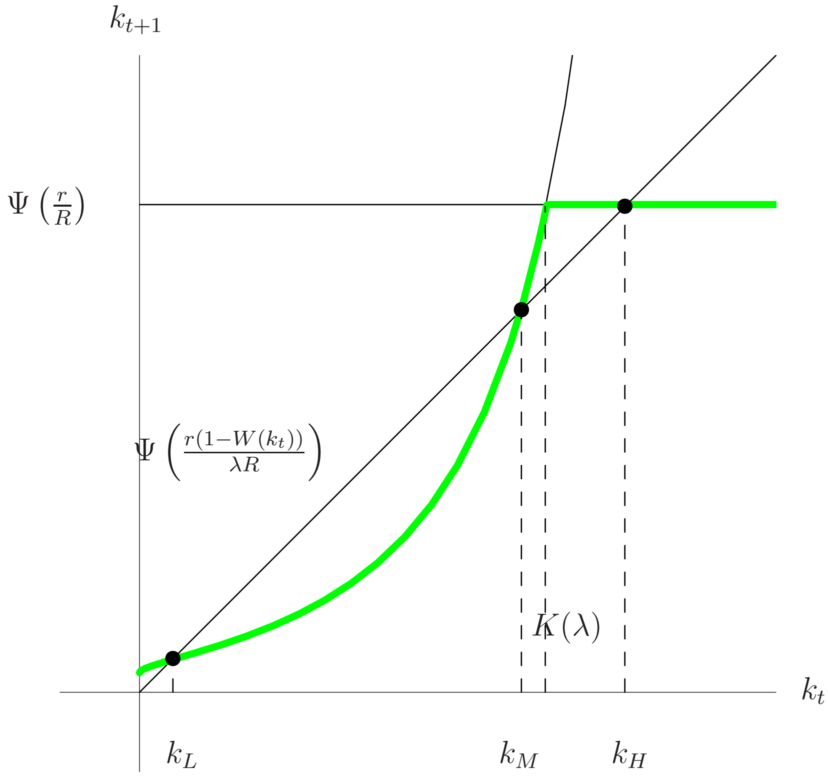


Figure 2.3.2: Time One Map of the Small Open Economy

Notice that there are two steady states below and one steady state above the critical value $K(\lambda)$. We denote these steady states by $k_L < k_M < k_H$. The steady states k_L and k_H are stable while the steady state k_M is unstable. In contrast to the autarky case, the credit market imperfection in the small open economy generates multiple steady states due to the borrowing constraint. This is because domestic saving is not necessarily equal to domestic investment in the small open economy. The world interest rate does not adjust to equate domestic saving to domestic investment in the small open economy. Instead, the fraction of young agents who start investment project changes so that either the borrowing constraint or the profitability constraint is binding in equilibrium.

2.4 Summary

Section 2.2 showed that capital stocks of countries, which have production technology with diminishing marginal product, converge to each other immediately in a perfect international capital market. As one way of reconciling this theoretical implication with empirical observation, Section 2.3 introduced the model by Matsuyama (2004) with credit market imperfection. Section 2.3.2 showed that if saving is equal to investment, the credit market imperfection has no influence on the dynamics of capital formation in autarky. Section 2.3.3 showed that the result in the autarky case no longer holds in the small open economy since the interest rate does not adjust to equalize the investment and the saving in the domestic economy. Instead, the fraction of people who become lenders in the international credit market is adjusted so that either the profitability constraint or the borrowing constraint is binding in equilibrium. This has consequences on the dynamics of the economy. While the autarky has a unique steady state, which is globally stable, the small open economy may have multiple steady state, which are locally stable. The small open economy model shows that access to international credit markets may be detrimental to economies with low initial capital stock. This result can be interpreted as a poverty trap where the small open economy, caught in a vicious cycle, suffers from persistent underdevelopment. Chapter 3 generalizes the small open economy to a two country case. Allowing for interactions between two identical economies enables us to draw implications for inequality between countries.

Chapter 3

A Two Country Model with Imperfect International Credits

3.1 Introduction

The debate on the "convergence controversy" does not seem to be resolved in the empirical literature. Some economists claim that a large discrepancy in the distribution of income per capita has emerged in the world while others claim the opposite. There is little doubt that the operation of and the transactions in any markets today are global. This being a commonly accepted fact, both sides claim a reinforcement of their own arguments. In other words, for one group globalization is responsible for the global inequality while for the other one it induces convergence of income per capita in the world.

Jones (1997) showed that the emergence of the so called "twin peaked" distribution, which was characterized in Quah (1997), disappears once each country data point is weighted by population. More recently Sala-i-Martin (2006) merged survey data about the income distribution within individual countries with national account data to estimate the world income distribution. He concluded that there has been a reduction in global inequality during the 1980s and 1990s. This result is not surprising given the high growth trend of populous countries such as China and India during that period. However, poor countries still comprise a large part of the world population while rich countries represent only a small fraction (see Milanovic 2002). These observations of course do not imply immediately that the global financial market is to be blamed for the unequal distribution of income across nations.

If countries are equipped with identical technologies and there are no operation costs in international financial market, the standard neoclassical theory would predict that per capita incomes are immediately equalized. This is because the international financial market would allocate the savings of the integrated economies to where it yields the highest return. This induces conditional convergence of per capita income across countries even without international mobility of labor forces. The solution the literature offers for this paradoxical result of the integrated economies is to incorporate some kind of imperfections in financial markets (see Boyd & Smith 1997, Matsuyama 2004).

This chapter explores the Matsuyama model under the alternative assumption that the world economy consists of two countries which are possibly different in population size, instead of a continuum of homogeneous small open economies. This confines the state space of the dynamical system to two dimensions. As each country has a positive measure, the capital stock in each country has an impact on the world interest rate and vice versa. These feedback effects are absent in the model by Matsuyama (2004) since the atomless economies do not influence the world interest rate. In a two country model suggested by Boyd & Smith (1997), they do not analyze these feedback effects in the international financial market explicitly.The present model shows that new stable steady states emerge in the presence of the spillover effects of capital stock via an endogenous determination of the world interest rate. Thus the model identifies additional forces of international financial markets. All sets of steady states of the two country model are characterized.

The present model also analyzes the dynamics when two countries have heterogeneous population sizes. This may be justified on the grounds that the relative population size might be one of the most persistent attributes in the world economy considering the immobile nature of population and the long time span needed for adjustment. It is shown that the heterogeneity in population sizes breaks the symmetric structure of the model. The model implies that, if the initial capital stocks of the two countries are sufficiently unequal, greater inequality in population size also induces greater inequality in income distribution. This result may be consistent with the situation in today's world, which consists for the large part of the world population of poor countries while rich countries represent only a small fraction.

Boyd & Smith (1997) motivate their paper by referring to cyclicality of credit allocation between developing and developed economies in empirical data (see United Nations 1992). However, their theoretical findings are confined to a dynamical equilibrium path displaying damped oscillations. The asymmetric steady state generated by heterogeneous population sizes of the two countries in the present model induces endogenous fluctuations. This implies fluctuations in capital stock as well as international capital flows in the long run in contrast to the transitory feature in Boyd & Smith (1997). As opposed to the real business cycles models where fluctuation is viewed as a propagation mechanism of exogenous shocks, the model implies that endogenous fluctuation is inherent in the international financial market.

The remainder of the chapter is organized as follows. Section 3.2 extends the small open economy model in Section 2.3.3 to a two country model. Section 3.2.1 shows how the spillover effect changes the stability property of the small open economy. Section 3.2.2 investigates the effect of a change in the relative population size on income distribution between the two countries. Section 3.2.3 shows that the spillover effect and the population size effect together induce fluctuations of international financial flows endogenously. Section 3.3 concludes.

3.2 Two Country Model

In Section 2.3.3 the world interest rate was assumed to be constant. In this section the world interest rate will be determined endogenously by the excess demand of both countries. Since we ruled out international factor movements, one country influences the other only through the world interest rate. From equation (2.3.7) the capital investment in country i is described by

$$k^i_{t+1} = \Psi(k^i_t, r_{t+1}). \tag{3.2.1}$$

In the present model the world economy consist of two countries $i = 1, 2$ with arbitrary initial conditions. Equating total credit demand and total credit supply, the equilibrium interest rate $r_{t+1} = \mathcal{R}(k_t^1, k_t^2)$ in the international financial market is implicitly defined by a solution of

$$L\Psi(k_t^1, r_{t+1}) + (1-L)\Psi(k_t^2, r_{t+1}) = R(LW(k_t^1) + (1-L)W(k_t^2)), \tag{3.2.2}$$

where $L \in (0,1)$ is defined to be the relative population size of country 1.[1] For a Cobb-Douglas production function of the form $f(k) := Ak^\alpha$, the equilibrium interest rate can be obtained explicitly (see the appendix). Equation (3.2.2) defines the temporary equilibrium of the two country model. By dropping the time index in (3.2.2) we obtain

$$L\Psi(k^1, r) + (1-L)\Psi(k^2, r) = R\left[LW(k^1) + (1-L)W(k^2)\right].$$

For any $k^1, k^2 > 0$ the right hand side of the above equation is a positive constant. The left hand side is monotonically decreasing in r since $\Psi(k^i, r)$ is monotonically decreasing in r, for $i = 1, 2$. Since $\lim_{r\to 0}\Psi(k^i, r) = \infty$ and $\lim_{r\to\infty}\Psi(k^i, r) = 0$ for $i = 1, 2$, there exists a unique solution $r = \mathcal{R}(k^1, k^2)$. Substituting the solution $r_{t+1} = \mathcal{R}(k_t^1, k_t^2)$ into equation (3.2.1) we can explicitly solve the two dimensional dynamical system

$$k_{t+1}^i = \Psi(k_t^i, \mathcal{R}(k_t^1, k_t^2)), \qquad i = 1, 2. \tag{3.2.3}$$

In general, the spillover effect $\partial\mathcal{R}(k_t^1, k_t^2)/\partial k_t^i, \forall i = 1, 2$ is non-zero.[2]

This section analyzes the dynamic behavior of the world economy with two homogeneous countries as the benchmark case. By homogeneous we mean that all characteristics of two economies are identical and they differ only in the stock of capital. In particular we set the relative population size L to one half. In Section 3.2.3 we relax this assumption and see how heterogeneous population sizes affect the existence and stability of the steady states.

[1] In the Matsuyama model, there is a continuum of small open homogeneous economies, hence the world interest rate is determined by the condition

$$\int_0^1 \Psi(k_t^i, r_{t+1})di = R\int_0^1 W(k_t^i)di.$$

This equation with equation (3.2.1) defines the dynamical system which is infinite-dimensional.

[2] The spillover effect is zero when there is a continuum of small open economies as no country has a positive measure.

3.2.1 Spillover Effects and Symmetry Breaking

The symmetric steady state is identical to the steady state of autarky. It can be easily confirmed that setting $L = 1$ in equation (3.2.2) induces the same interest rate as setting $k^1 = k^2$ in steady state when $L = 1/2$. In other words, the world interest rate is identical to that of the autarky at the symmetric steady state. This implies that there is no financial transaction across two countries in the symmetric steady state. Note also that the symmetric steady state always exists.

Proposition 3.2.1 *Suppose that Assumption 2.3.1 and 2.3.2 hold.*

1. *There exists a unique positive symmetric steady state $K^*(R)$ which coincides with the steady state of autarky defined implicitly by $k = RW(k)$.*
2. *The function $K^* : [0, R^+) \to \mathbb{R}_{++}, R \mapsto K^*(R)$ is increasing in R and satisfies $K^*(0) = 0$.*

Proof: The proof follows directly from equations (3.2.1) and (3.2.2). □

Let R_c be defined by $f(K^*(R_c)) = 1$ which is the critical level of R above which there exist asymmetric steady states if $L = 1/2$(see Proposition 3.4.1 in the appendix for a proof).

Proposition 3.2.2 *Suppose that Assumption 3.2.1 and $L = 1/2$ are satisfied. The two countries converge to the symmetric steady state* $(K^*(R), K^*(R))$

1. *for all $k_0^1, k_0^2 > 0$ if $R < R_c$*
2. *if $k_0^1 = k_0^2 > 0$ or $k_0^1, k_0^2 \geq K(\lambda)$ for all $R \in \mathbb{R}_+$*

See the appendix for a proof. □

Firstly, Proposition 3.2.2 says that if the symmetric steady state is the only steady state of the world, it is globally stable meaning that the two countries converge to the steady state regardless of their initial conditions. Secondly, it says if the initial capital stocks of the two countries are high enough so that they do not face the borrowing constraint, they will converge to a symmetric steady state. This will happen even if the symmetric steady state is unstable, i.e. $K^*(R) \leq K(\lambda)$. This is because if $k_0^1, k_0^2 \geq K(\lambda)$, the capital stocks in both countries adjust to the same level in the following period. Once the capital stocks in both countries are the same, there is no transaction between countries and both countries follow a convergence path of the autarky economy.

For the subsequent numerical analysis we use the Cobb-Douglas production function specified below. The existence of asymmetric steady states is shown analytically in the appendix also using the Cobb-Douglas production function. For a better exposition of the main results here we draw phase diagrams numerically to analyze the existence as well as the stability of asymmetric steady states.

Assumption 3.2.1 *The production function is of the Cobb-Douglas form* $f(k) := Ak^{\alpha}$.

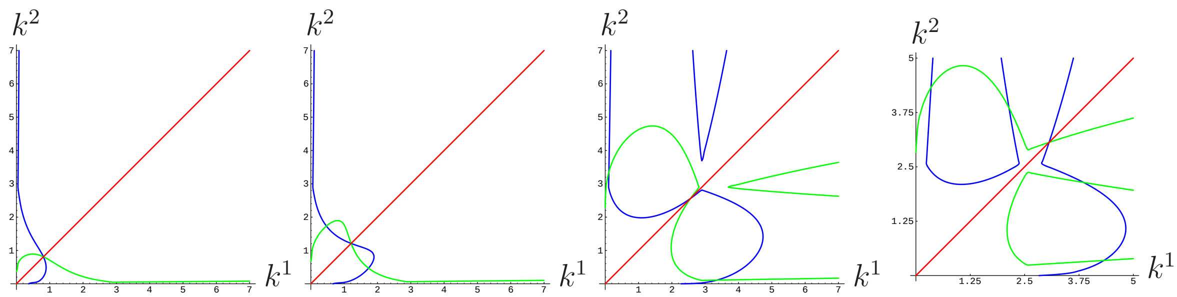
Figure 3.2.1 numerically shows the steady states of the two country model for the parameter set given in Table 3.1. Depicted are the zero contours of the functions $\Delta k^1(k^1, k^2){:=}k^1 - \Psi^1(k^1, k^2)$ and $\Delta k^2(k^1, k^2){:=}k^2 - \Psi^2(k^1, k^2)$.

A	α	λ	L	k_0^1	k_0^2	$K(\lambda)$	R_c
1	0.5	0.15	0.5	5	2	2.89	2

Table 3.1: Standard parameter set

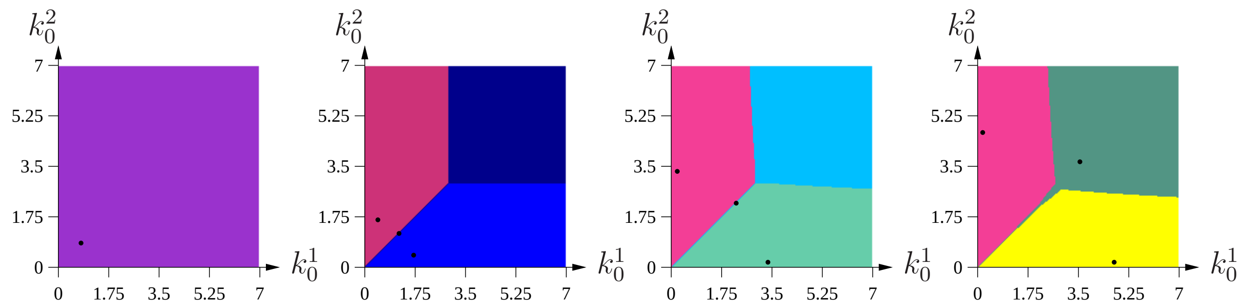
The intersections of the two curves are the steady states of the model. For $0 < R \leq 2$ the steady state is unique and symmetric. However, for $R > 2$, additional asymmetric steady states emerge. The system is symmetric for $L = 0.5$ and therefore the asymmetric steady state appear pairwise along the diagonal.

One finds that the system has one fixed point for $0 < R \leq 2$ as in (a), three fixed points for $2 < R \leq 3.4$ as in (b) and (c) and five fixed points for $R > 3.4$ as in (d). To analyze the stability of the steady states globally, basins of attraction are calculated and shown by different colors in Figure 3.2.2 for different values of R. Figure 3.2.2 (b), (c) and (d) show that the asymmetric steady states which emerge for $R > 2$ are stable. The additional asymmetric steady states which emerge for $R > 3.4$ are unstable (Compare Figure 3.2.1 (d) and Figure 3.2.2 (d)).



(a) R=1.8 (b) R=2.2 (c) R=3.2 (d) R=3.5

Figure 3.2.1: Existence of steady states: $K(\lambda) = 2.89$

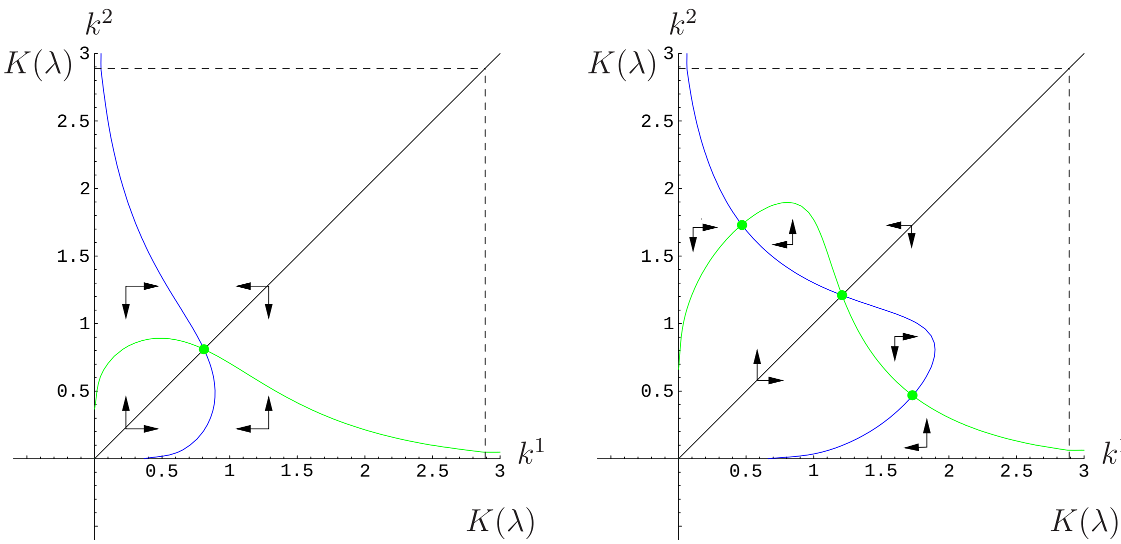


(a) R=1.8,$K^*(R)$=0.81 (b) R=2.2, $K^*(R)$=1.21, (k_L,k_M)=(0.47,1.73) (c) R=3.2, $K^*(R)$=2.56, (k_L,k_H)=(0.26,3.58) (d) R=3.5, $K^*(R)$=3.61, (k_L,k_H)=(0.35,4.32)

Figure 3.2.2: Stability of steady states

Figure 3.2.1 (a) and (b) are reproduced in Figure 3.2.3 as phase diagrams. Inside the dotted line $(k^1, k^2) < (K(\lambda), K(\lambda)))$ holds so that the borrowing constraint is binding. While the unique symmetric steady state is stable in 3.2.3 (a), it loses its stability with the emergence of the asymmetric steady states which are stable in 3.2.3 (b).

The transition from Figure 3.2.3 (a) to (b) is the symmetry breaking in the sense of Matsuyama (2004). While the symmetric steady state is globally stable in (a), it is a saddle point in (b). From Proposition 3.2.2 we know that only if $k_0^1 = k_0^2 > 0$ or $k_0^1, k_0^2 \geq K(\lambda)$ the world economy converges to the saddle point in (b) along the the saddle path. However, the asymmetric steady states are stable. This symmetry breaking can not be observed in Matsuyama (2004). Equation (3.2.1) implies that at any steady state of the world economy each economy must be at a steady state of the small open economy. Suppose that the two economies were two small open economies. Then one economy would have k_L and the other would have k_M in the asymmetric steady states



(a) $R = 1.8, L = 0.5$ (b) $R = 2.2, L = 0.5$

Figure 3.2.3: Phase diagrams $\Delta k^1(k^1, k^2) = 0$ and $\Delta k^2(k^1, k^2) = 0$

since they lie within the dotted lines. Now from Figure 2.3.2 we know that k_L is stable but k_M is unstable. Therefore, the asymmetric steady state (k_L, k_M) can not be stable if the two countries were small open economies. The difference lies in the assumption that in the present model the two economies are two "large" economies and have a size of positive measure. This implies that each economy influences not only its own capital stock but also that of the other through the world interest rate. In other words, it is the spillover effects, which is absent in the Matsuyama model that causes the symmetry breaking in this case.

Let us conclude this section with a brief description of the implication of the asymmetric steady state on the inequality of the two economies. The rich country is always better off at an asymmetric steady state than at a symmetric steady state and the poor country worse off. Suppose $k^1 > k^2$ at the asymmetric steady state. Then,

$$\begin{aligned} k^1 - RW(k^1) &> 0 \\ k^2 - RW(k^2) &< 0. \end{aligned} \tag{3.2.4}$$

This implies that at the asymmetric steady state the country with the higher capital stock has an excess demand of physical capital and the country with lower capital stock an excess supply. Therefore, at the asymmetric steady state the net financial flows are from the poor country to the rich country.

3.2.2 Population Size Effects

The analysis of the two country model with identical population sizes showed that the spillover effect causes symmetry breaking in the two country model by changing the stability property of the small open economy. By allowing for a different population size of the two country this section investigates how robust the results with an identical population size are with respect to a change in the relative population size of two countries. This means that we treat the relative population size as an exogenous parameter and investigate its influence on the dynamic behavior of the system.

It is obvious from equation (3.2.2) that the relative population size of an economy affects the other economy only through the world interest rate by affecting the aggregate demand and the aggregate supply in the international financial market. The population size does not play any role if two countries have the same capital stock. Thus, the symmetric steady state always exists as in the case with an identical population size.

Definition 3.2.1 *From equations (3.2.1) and (3.2.2), the steady state in the two country model is defined by a pair (k^1, k^2) satisfying for $i = 1, 2$*

$$r = \mathcal{R}(k^1, k^2) = \begin{cases} \dfrac{f'(k^i)\lambda R}{1 - W(k^i)} & \text{if } k^i < K(\lambda) \\ \\ f'(k^i)R & \text{if } k^i \geq K(\lambda), \end{cases} \tag{3.2.5}$$

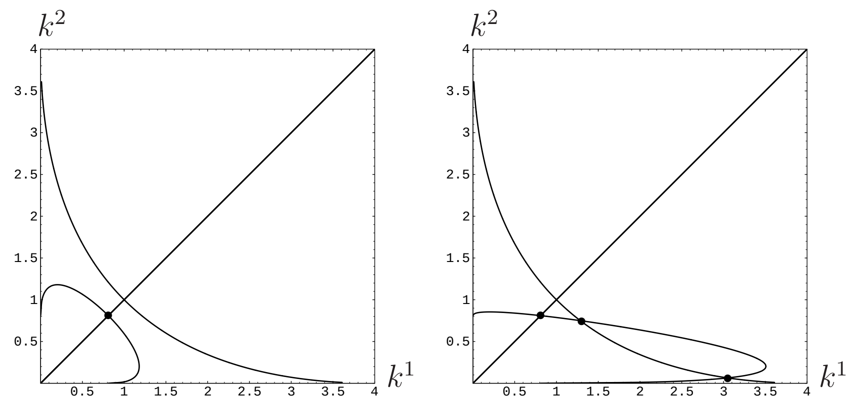
and

$$G(k^1, k^2) := L(k^1 - RW(k^1)) + (1 - L)(k^2 - RW(k^2)) = 0. \tag{3.2.6}$$

Proposition 3.2.3 *Suppose that Assumption 3.2.1 is satisfied. There exists a critical value of relative population size L_c below which two asymmetric steady states emerge for $R < R_c$. The country with the smaller population is richer at these steady states.*

See the appendix for a proof. □

For economies with an identical population size Proposition 3.4.1 showed that there exists a unique symmetric steady state for $R < R_c$. In contrast, Proposition 3.2.3 says that if we change the relative population size of the two countries, two asymmetric steady states emerge even for $R < R_c$. The reason behind the emergence of the two coexisting asymmetric steady states is the broken symmetric structure of the economy. Figure 3.2.4 shows the effect of this broken symmetry. On the 45° line and on the convex curve (k^1, k^2) satisfies equation (3.2.5). Satisfying equation (3.2.5) the two small open economies would be in a steady state for a given r. However (k^1, k^2) hast to be on the concave curve satisfying equation (3.2.6) too for the two economies to be in steady state if the world interest rate is determined endogenously. While there exists only a symmetric steady state in Figure 3.2.4 (a), Figure 3.2.4 (b) shows that by changing the relative population size the zero contour of $G(k^1, k^2)$ loses its symmetric structure in (k^1, k^2) space and two asymmetric steady states emerge even for $R > R_c = 2$.

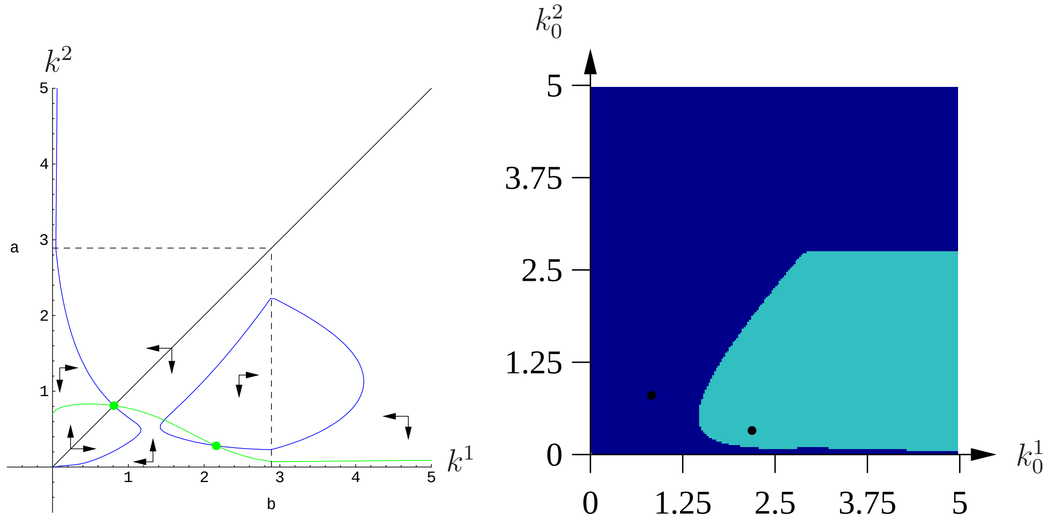


(a) $L = 0.5$ (b) $L = 0.2$

Figure 3.2.4: Broken symmetric structure : $R = 1.8 < R_c = 0.2$.

Figure 3.2.5 (a) shows the zero contours of the functions $\Delta k^1(k^1, k^2)$ and $\Delta k^2(k^1, k^2)$ for $L = 0.19$ indicating that the zero contours are no longer symmetric along the diagonal. The sensitivity of the behavior of the system on initial conditions is shown in Figure 3.2.5 (b). It shows that the asymmetric steady state which lies closer to the diagonal is unstable.

The stability property of the two steady states has an important implication for the inequality of the two economies. The arrows in the figure show that the asymmetric



(a) Zero Contour Plots (b) Basin of Attraction

Figure 3.2.5: Broken symmetric structure and inequality: $L = 0.19, R = 1.8$.

steady state which is more remote from the 45° line is stable.

From Proposition 3.2.2 we know that the two economies with an identical population size converge to a unique symmetric steady states for any given initial conditions when $R < R_c$. In contrast, two economies may converge to an asymmetric steady state for a large set of initial values if they have unequal population sizes. In particular, the world converges to an asymmetric steady state in which the economy with a smaller population size has a higher capital stock.

If the two countries have an identical population size, we know from equation (3.2.4) that the poor country is a net supplier of credit in the international financial market in any asymmetric steady state. Typically more people are denied credit in the poor country than in the rich country. Now in the asymmetric steady state with unequal population sizes the relative population size in the poor country is higher than in the rich country. This means that the number of people who are denied credit in the poor country increases. In other words while the supply of credits increases, the demand of credits decreases leading to an increased net supply of credits from the poor to the rich country at a lower world interest rate. It is the world interest rate that forces both countries to move together to adjust to the new situation. Notice that the poor country is a net suppler of credit in spite of the higher marginal productivity. This is because the income in poor country is so low that the borrowing constraint allows only a small fraction of people to start an investment project while the rest is forced to become a lender to the rich country in the international credit market.

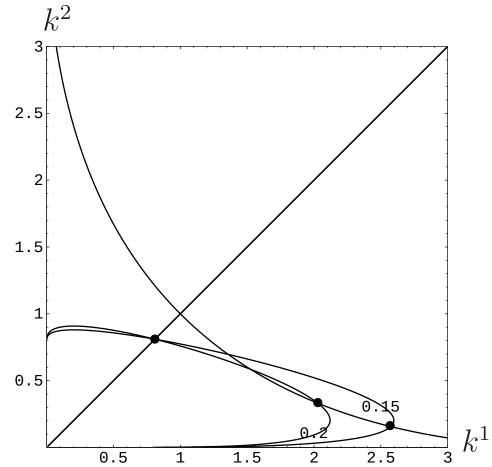


Figure 3.2.6: Population size effects on inequality: $R = 1.8, K(\lambda) = 2.89$.

Even if the two economies have identical structural characteristics a country with a relatively large population fails to catch up with a country with a relatively small population if the initial conditions are sufficiently unequal. Figure 3.2.6 also shows that a greater inequality in the population size is associated with a greater inequality in income.

Let us turn to empirical evidence. Milanovic (2002) found that the richest 25 percent of the world's population receives 75 percent of the world's income even when adjusting for Purchasing Power Parity. The poorest 75 percent of the world's population share just 25 percent. This occurs because a large proportion of the world's population lives in the poorest countries, and within the poorest regions of those countries, particularly in the rural areas of China, rural and urban India and Africa. It is beyond the scope of this thesis, however, to conduct numerical calibration. Nevertheless, it is instructive to show a simple numerical example to think about the implications of the model. The results by Milanovic (2002) imply that the income per capita of the richest 25 percent of the world's population is approximately 9.1 times more than the the poorest 75 percent. Figure 3.2.7 shows time series of the present model where two countries with very close initial conditions diverge in the long run. The income per capita of country 1 converges to 1,118 while that of country 2 converges to 0.123. This means that the income per capita of country 1, which composes 25 percent of the world's population, is approximately 9.1 time more than that of country 2, which composes 75 percent of the world's population. Thus, the model replicates the findings of Milanovic (2002). Of course, we have to be cautious to interpret the implications of the model. First of all, we separated the world into two units. This is, if at all, a very rough approximation Secondly, we assumed that the two countries have identical structural characteristics. We would typically expect the production elasticity α, the technology to produce physical capital R, and the degree of imperfection in financial markets λ to be different across countries. However, the identical structure of the model rather strengthens the implications of the model. That is, even if identical technology were available to all the countries, the model predicts that a rich country would diverge from a populous poor country.

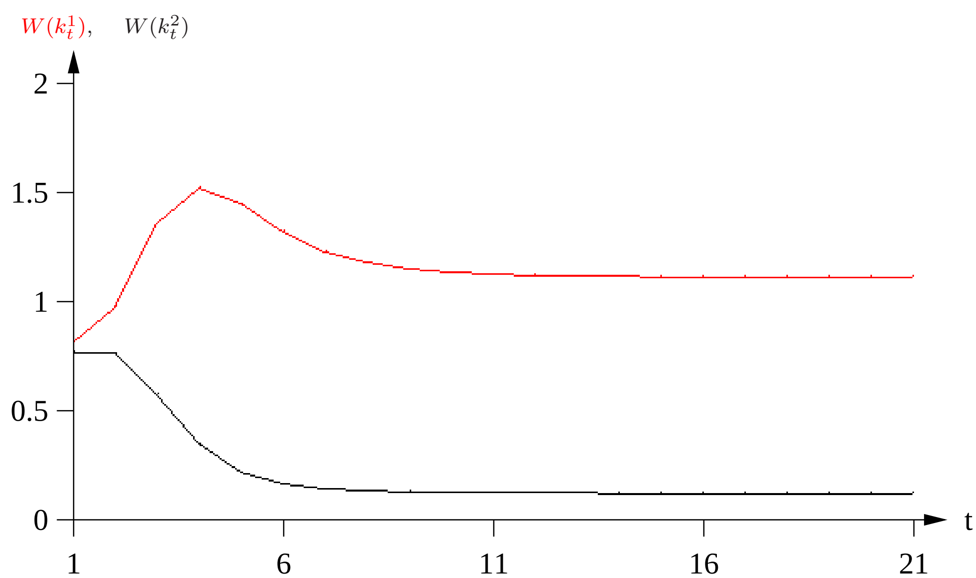


Figure 3.2.7: Divergence: $L = 0.25$, $k_0^1 = 2.1$, $k_0^2 = 2$, $\alpha = 0.504$, $\lambda = 0.1$, $R = 3.5$.

3.2.3 Endogenous Fluctuations

This section investigates the effect of a change in the relative population size on the long run behavior of the system more globally with help of numerical simulation. Figure 3.2.8 shows a bifurcation diagram with respect to L for k_t^1 and k_t^2 where $k_0^1 > k_0^2$ depicting the limiting behavior of the state variables, k_t^1 and k_t^2.

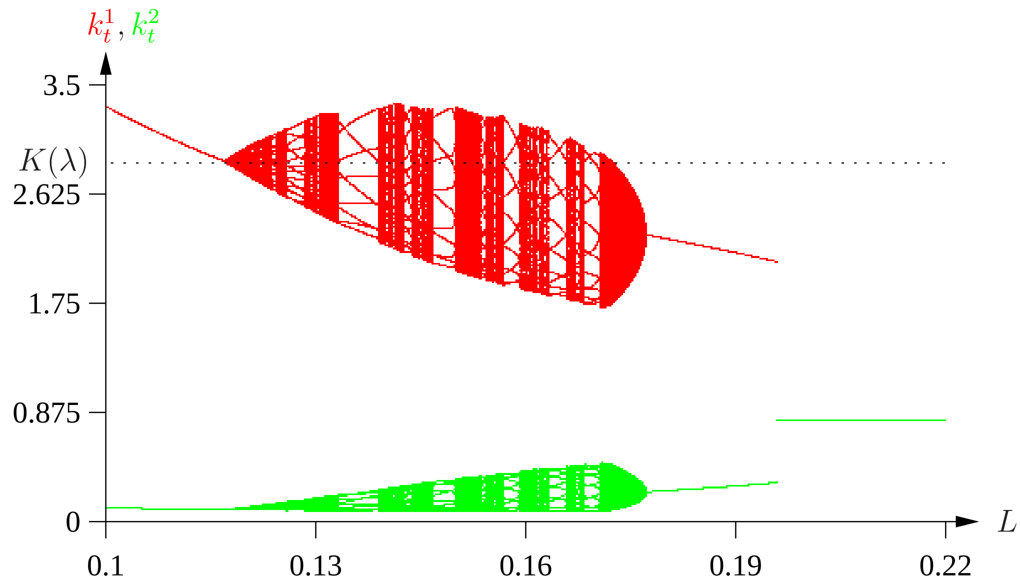


Figure 3.2.8: Bifurcation diagram: $R = 1.8, K(\lambda) = 2.89$.

Firstly, it can be confirmed that a greater inequality in the population size is associated with a greater inequality in incomes. This means that if the initial conditions of the two countries are sufficiently different, they converge to the asymmetric steady state that is associated with increasing inequalities for a smaller population size of the rich country. Secondly, we observe that the asymmetric steady state $(k^1, k^2) = (k_M, k_L)$ loses its stability for sufficiently low L, undergoing a bifurcation.

Proposition 3.2.4 *Suppose that Assumption 3.2.1 is satisfied. Consider the dynamics of the world economy by changing the bifurcation parameter L. The asymmetric steady state $(k^1, k^2) = (k_M, k_L)$ undergoes a supercritical Neimark-Sacker bifurcation.*

See the appendix for a proof. □

Now notice that at the asymmetric steady state the borrowing constraint is binding in both economies meaning $(k^1, k^2) < (K(\lambda), K(\lambda))$. From the previous discussion in Section 3.2.1 we know that the spillover effect in the two country model is essential for the stability of steady states where the borrowing constraint is binding in both economies. In addition, we know from Proposition 3.2.1 that the asymmetric steady states arise for $R < R_c$ because of the unequal population size. This means that the unequal population size and the spillover effect jointly cause the bifurcation we observe in Proposition 3.2.4.

In other words, the spillover effect caused by two "large" economies with an unequal population size, which is absent in the world with a continuum of small open economies, generates fluctuations in the international financial flows endogenously. Let us make a tentative explanation of the inherent structure of the model which induces endogenous cycles. Just as in a predator prey model think of the two economies as two rivals, which are struggling for survival. There are a poor and a rich country. On one hand, the poor country would sustain a constant capital stock without the rich country. However, since capital flows from the poor to the rich country, the spillover effect has a negative impact on the capital stock of the poor country. In other words, the rich country exploits the poor country. The richer is the rich country the stronger is the negative spillover effect. If the capital stock of the rich country is above a critical level A, the negative spillover effect dominates and impoverishes the poor country. In contrast, below the critical level A, the capital stock of the poor country grows. On the other hand, the enrichment of the rich country is dependent on the poor country. If the capital stock of the poor country is above a critical level B, the capital inflow from the poor country enriches the rich country. This spillover effect is not dominant if the capital stock of the poor country is below B. Figure 3.2.9 illustrates the above argument showing the direction of the movement of the capital stocks in both countries. Following the directions of the arrow the capital stock of the two countries would cycle around the unstable steady state S.

The above argument can be confirmed by looking at time series and attractor plots in the state space showing the limiting behavior of the system just after the bifurcation point at $L = 0.175$ in Figure 3.2.10 (a) and (b). Figure 3.2.10 (b) shows the loss of the stability of the asymmetric steady state and the emergence of a closed invariant curve (see Kuznetsov (1998) for details).

Note that bifurcations occur at two points. At the bifurcation point with lower L the steady state value of k^1 reaches $K(\lambda) = 2.89$. At this point the dynamical system is not differentiable, as it switches from the case in which one country faces the borrowing constraint to the case in which both countries face the borrowing constraint. As shown in the proof of Proposition 3.2.4, the asymmetric steady state $(k^1, k^2) = (k_H, k_M)$ is stable while $(k^1, k^2) = (k_M, k_L)$ is unstable after this bifurcation point. This implies that there are two forces which pull the world economy in opposite directions. One

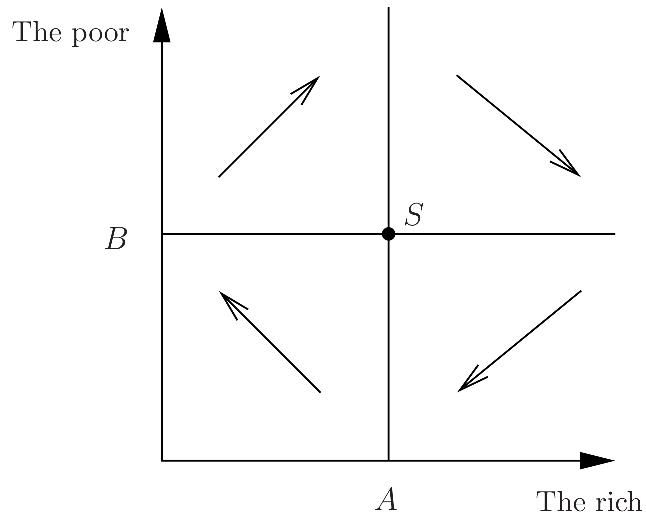


Figure 3.2.9: The change of the capital stock in the rich and the poor country

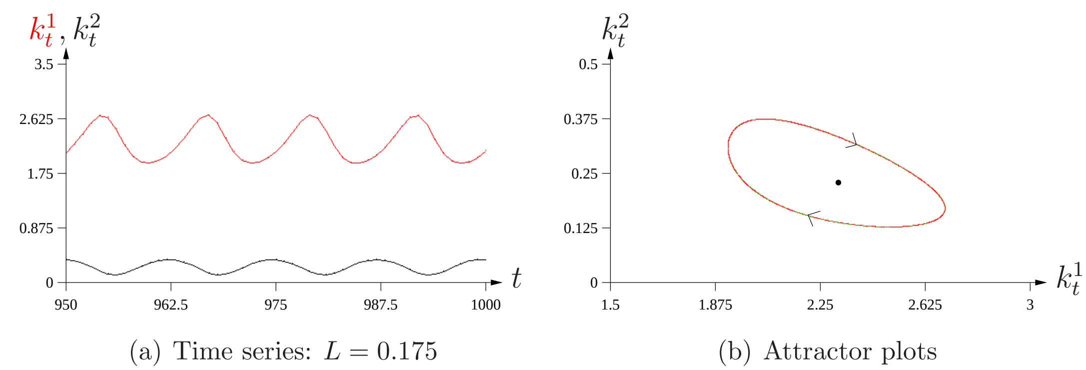


Figure 3.2.10: Endogenous Cycles: $R = 1.8, L = 0.175$

pulls the economy close to $k^2 < K(\lambda) < k^1$ and the other pulls the economy away from $k^2 < K(\lambda) < k^1$. These forces generate non-stationary orbits of k_t^1 around $K(\lambda)$.

3.3 Concluding Remarks

We have examined how the stability of the two country model is influenced by the spillover effect through the world interest rate. We singled out the spillover effect by comparing the results of the world economy model with small open economies to the results of the two country model. The symmetry breaking results in Boyd & Smith

(1997) and Matsuyama (2004) hold in the present paper. There are some additional common features in Boyd & Smith (1997), Matsuyama (2004) and the present paper. Firstly, an initially poor country remain relatively poor if it does not converge to a symmetric steady state. Secondly, the poor country is better off in a symmetric steady state than in an asymmetric steady state while the rich country is worse off. Thirdly, the aggregate wealth of the world economy is higher in a symmetric steady state than in any asymmetric steady states. These results are the consequence of the borrowing constraint which limits the amount of domestic investment in the poor country and forces it to supply credits in the international financial market. In addition, the present paper shows that a change in the relative population size of the two countries generates new asymmetric steady states. They induce endogenous fluctuations of international financial flows from the poor to the rich country in the presence of the spillover effect. It is surprising that endogenous cycles can arise by basically changing the interactive structure of an relatively simple model. So it is natural to suspect that there are many other models which embody a possibility of endogenous fluctuations if we just modify the structure of interaction. Further investigations into the financial structures and the interaction mechanisms which lead to endogenous fluctuations may provide an alternative explanation for patterns of international economic development.

3.4 Appendix

The Equilibrium Interest Rate: The Cobb-Douglas Case

Let the production function be of the Cobb-Douglas form, $f(k) = Ak^{\alpha}$. Then, the equilibrium interest rate, which is defined by equation (3.2.2), is given by

$$r_{t+1} = \mathcal{R}(k_t^1, k_t^2) := \begin{cases} \frac{\alpha A\lambda R\left[L(1-W(k_t^1))^{\frac{1}{\alpha-1}}+(1-L)(1-W(k_t^2))^{\frac{1}{\alpha-1}}\right]^{1-\alpha}}{\left[R[LW(k_t^1)+(1-L)W(k_t^2)]\right]^{1-\alpha}} & \text{if } k_t^1, k_t^2 < K(\lambda) \\ \frac{\alpha A\lambda R\left[L[1-W(k_t^1)]^{\frac{1}{\alpha-1}}+(1-L)\lambda^{\frac{1}{\alpha-1}}\right]^{1-\alpha}}{[[LW(k_t^1)+(1-L)W(k_t^2)]R]^{1-\alpha}} & \text{if } k_t^1 < K(\lambda) \leq k_t^2 \\ \frac{\alpha A\lambda R\left[L\lambda^{\frac{1}{\alpha-1}}+(1-L)[1-W(k_t^2)]^{\frac{1}{\alpha-1}}\right]^{1-\alpha}}{[[LW(k_t^1)+(1-L)W(k_t^2)]R]^{1-\alpha}} & \text{if } k_t^2 < K(\lambda) \leq k_t^1 \\ \frac{\alpha AR}{[[LW(k_t^1)+(1-L)W(k_t^2)]R]^{1-\alpha}} & \text{if } k_t^1, k_t^2 \geq K(\lambda). \end{cases}$$

Substituting the equilibrium interest rate into the capital accumulation law (3.2.1), we obtain the dynamical system of the two country model in a closed form given by

$$k_{t+1}^1 = \tilde{\Psi}^1(k_t^1, k_t^2) := \begin{cases} \frac{R[LW(k_t^1)+(1-L^1)W(k_t^2)]}{L+(1-L)\left[\frac{1-W(k_t^1)}{1-W(k_t^2)}\right]^{\frac{1}{1-\alpha}}} & \text{if } k_t^1, k_t^2 < K(\lambda) \\ \frac{R[LW(k_t^1)+(1-L^1)W(k_t^2)]}{L+(1-L)\left[\frac{1-W(k_t^1)}{\lambda}\right]^{\frac{1}{1-\alpha}}} & \text{if } k_t^1 < K(\lambda) \leq k_t^2 \\ \frac{R[LW(k_t^1)+(1-L^1)W(k_t^2)]}{L+(1-L)\left[\frac{\lambda}{1-W(k_t^2)}\right]^{\frac{1}{1-\alpha}}} & \text{if } k_t^2 < K(\lambda) \leq k_t^1 \\ R[LW(k_t^1) + (1-L)W(k_t^2)] & \text{if } k_t^1, k_t^2 \geq K(\lambda). \end{cases}$$

and

$$k_{t+1}^2 = \tilde{\Psi}^2(k_t^1, k_t^2) := \begin{cases} \frac{R[LW(k_t^1)+(1-L^1)W(k_t^2)]}{L\left[\frac{1-W(k_t^2)}{1-W(k_t^1)}\right]^{\frac{1}{1-\alpha}}+(1-L)} & \text{if } k_t^1, k_t^2 < K(\lambda) \\ \frac{R[LW(k_t^1)+(1-L)W(k_t^2)]}{L\left[\frac{\lambda}{1-W(k_t^1)}\right]^{\frac{1}{1-\alpha}}+(1-L)} & \text{if } k_t^1 < K(\lambda) \leq k_t^2 \\ \frac{R[LW(k_t^1)+(1-L)W(k_t^2)]}{L\left[\frac{1-W(k_t^2)}{\lambda}\right]^{\frac{1}{1-\alpha}}+(1-L)} & \text{if } k_t^2 < K(\lambda) \leq k_t^1 \\ R[LW(k_t^1)+(1-L)W(k_t^2)] & \text{if } k_t^1, k_t^2 \geq K(\lambda). \end{cases}$$

Proof of the Existence of Steady States

Let us consider only asymmetric steady states. Since both countries face the same world interest rate, equation (3.2.5) for $k^1 \neq k^2$ can be rewritten as

$$H(k^1) = H(k^2) \quad \text{if } k^1, k^2 < K(\lambda) \tag{3.4.1}$$

$$H(k^1) = f'(k^2) \quad \text{if } k^1 < K(\lambda) < k^2 \tag{3.4.2}$$

$$H(k^2) = f'(k^1) \quad \text{if } k^2 < K(\lambda) < k^1, \tag{3.4.3}$$

where $H(k) := \frac{\lambda f'(k)}{1-W(k)}, \forall k \in [0, R^+)$. If $k^1, k^2 > K(\lambda)$, the steady state is a symmetric steady state. It is obvious that each country must be at a steady state of the small open economy to satisfy equation (3.4.1), (3.4.2) or (3.4.3). This implies that $(k^1, k^2) \in \{(k_L, k_M), (k_M, k_L), (k_L, k_H), (k_H, k_L), (k_M, k_H), (k_H, k_M)\}$ at any asymmetric steady state. Due to the symmetric structure of the model, these asymmetric steady states emerge pairwise along the diagonal in the (k^1, k^2) space. Before analyzing the exact condition for each asymmetric steady state to exist let us redefine the zero contour $G(k^1, k^2) = 0$ in (k^1, k^2) space to help the technical exposition later on.

Lemma 3.4.1 *Let $k \in [0, K^*(R)]$, $R > 0$ and $L = 1/2$.*

1. *There exists an implicit function*

$$g : [0, K^*(R)] \times \mathbb{R}_+ \to \mathbb{R}_{++}, (k; R) \longmapsto g(k; R)$$

   satisfying $G(k, g(k; R)) = 0$.

2. *Due to the symmetry of G, $G(g(k; R), k) = 0$ holds. The zero contour of $G(k^1, k^2)$ is defined by the union of the graphs $g(k^1; R)$ and $g(k^2; R)$ in (k^1, k^2) space.*

3. *The map $k \mapsto g(k; R)$ is increasing if and only if $k \in [0, (W')^{-1}(\frac{1}{R})]$. By the implicit function theorem $\frac{dg(K^*(R);R)}{dk} = -1$.*

Proof: The function $k - RW(k)$ is decreasing for $k \in [0, (W')^{-1}(\frac{1}{R})]$ and increasing afterwards. Also $0 - RW(0) = 0$ and $K^*(R) - RW(K^*(R)) = 0$. The zero contour $G(k^1, k^2) = 0$ can be written as $\{(k^1, k^2) \in \mathbb{R}^2_+ : k^1 - RW(k^1) = -(k^2 - RW(k^2))\}$. The property of function g follows directly. □

Figure 3.4.1 shows that the zero contour of $G(k^1, k^2)$ is the union of graphs $g(k^i)$ for $i = 1, 2$ defined on $[0, K^*(R)]$. More formally,

$$\begin{aligned} \mathcal{G}(R) &:= \{(k^1, k^2) \in \mathbb{R}^2_+ | G(k^1, k^2) = 0\} \\ &= \{(k, g(k; R)) | k \in [0, K^*(R)]\} \cup \{(g(k; R), k) | k \in [0, K^*(R)]\}. \end{aligned}$$

To prove the existence of asymmetric steady states we have to show that the zero contour of $G(k^1, k^2)$ has an intersection with the set defined by equations (3.4.1), (3.4.2) or (3.4.3). Lemma 3.4.2 characterizes the property of equation (3.4.1).

Lemma 3.4.2

1. *There exists an implicit function*

$$h : [0, f^{-1}(1)] \to [f^{-1}(1), W^{-1}(1)), k \mapsto h(k)$$

   such that $H(k) - H(h(k)) = 0$.

2. *The function h is decreasing in k and satisfies $h(f^{-1}(1)) = f^{-1}(1)$.*

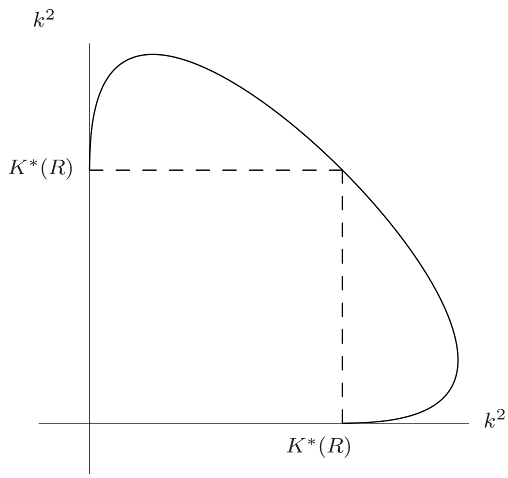


Figure 3.4.1: The zero contour of $G(k^1, k^2)$

Proof: We first investigate the properties of the function $H(k)$ by looking at the first and second derivative. One has that

$$H'(k) = \frac{f''(k)(1 - f(k))}{(1 - W(k))^2} \gtreqless 0 \Leftrightarrow f(k) \gtreqless 1. \tag{3.4.4}$$

The function H has its global minimum at $f^{-1}(1)$. In addition, $H(0) = \infty$ and $H(W^{-1}(1)) = \infty$. Moreover, for all $k > 0$

$$H''(k) = \frac{f'''(k)(1 - f(k)) - f''(k)f'(k)}{(1 - W(k))^2} + \frac{2f''(k)(1 - f(k))W'(k)}{(1 - W(k))^3} > 0. \tag{3.4.5}$$

Hence, the function H is strictly convex and has a configuration as depicted in Figure 3.4.2.

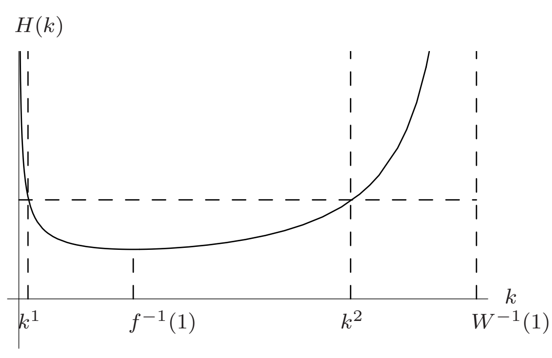


Figure 3.4.2: The graph of $H(k)$

It follows immediately that $(k^1, k^2) = (f^{-1}(1), f^{-1}(1))$ is the unique pair which solves equation (3.4.1). Then for $k^2 < f^{-1}(1)$, there exists a unique $k^1 > f^{-1}(1)$ which solves equation (3.4.1) (see Figure 3.4.2). Suppose that $k \in [0, f^{-1}(1)]$, then we obtain an implicit function $h : [0, f^{-1}] \to [f^{-1}, W^{-1}), k \mapsto h(k)$ such that $H(k) - H(h(k)) = 0$. From the implicit function theorem,

$$h'(k) = \frac{H'(k)}{H'(h(k))} < 0. \tag{3.4.6}$$

Hence, the function h is decreasing and satisfies $h(f^{-1}(1)) = f^{-1}(1)$. □

Given Lemma 3.4.2 we obtain the set

$$\begin{aligned} \mathcal{H} &:= \{(k^1, k^2) \in \mathbb{R}_+^2 | H(k^1) - H(k^2) = 0, k^1, k^2 < K(\lambda)\} \\ &= \{(k, h(k)) | k \in [0, f^{-1}(1)]\} \cup \{(h(k), k) | k \in [0, f^{-1}(1)]\} \end{aligned}$$

Figure 3.4.3 shows the graph of $h(k)$.

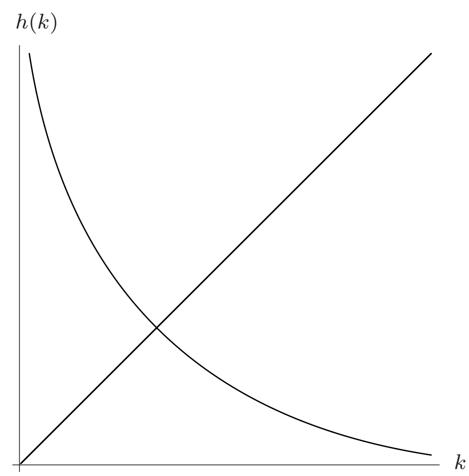


Figure 3.4.3: The graph of $h(k)$

In the following we consider only the asymmetric steady states which lie above the diagonal in the (k^1, k^2) space. In other words, we only consider points in the set $\mathcal{U} := \{(k^1, k^2) \in \mathbb{R}_+^2 \big| k^1 \geq k^2\}$. Due to the symmetric structure of the system, the asymmetric steady states in the set $\mathbb{R}_+^2 \setminus \mathcal{U}$ can be obtained analogously.

Proposition 3.4.1 shows the existence of the steady state where $k^1, k^2 < K(\lambda)$, i.e. $(k^1, k^2) = (k_M, k_L) =\in \mathcal{U} \cap \mathcal{G}(R) \cap \mathcal{H}$ by the intersection of the graphs $h(k^2)$ and $g(k^2; R)$. For the following analysis we will use the Cobb-Douglas production function.

Proposition 3.4.1 *Suppose that Assumption 3.2.1 and* $L = 1/2$ *are satisfied. There exists the asymmetric steady state* $(k^1, k^2) = (k_M, k_L)$ *if* $R > R_c$.

Proof: The asymmetric steady state $(k^1, k^2) = (k_M, k_L)$ is defined by following equations

$$k^1 = h(k^2) \tag{3.4.7}$$

$$k^1 = g(k^2) \tag{3.4.8}$$

$$k^2 < k^1 < K(\lambda). \tag{3.4.9}$$

The graph of $g(k^2; R)$ defined on $[0, f^{-1}(1)]$ has a unique intersection with the graph of $h(k^2)$ if and only if $K^*(R) > f^{-1}(1)$ (see Figure 3.4.4). Due to the symmetric structure, the asymmetric steady state $(k^1, k^2) = (k_L, k_M)$ can be obtained analogously. □

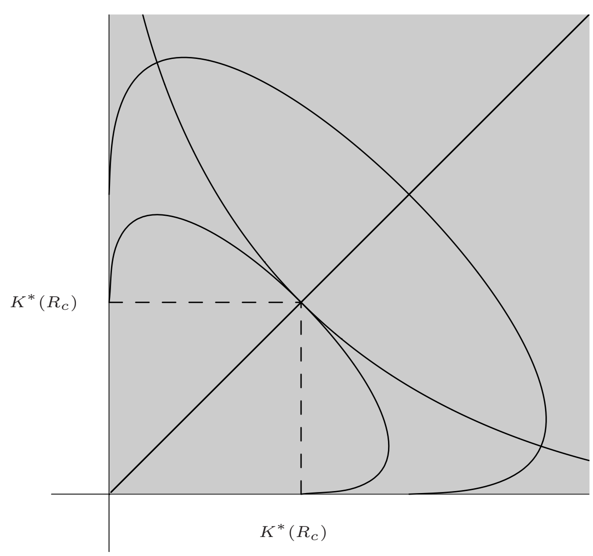


Figure 3.4.4: Existence of the asymmetric steady state $(k^1, k^2) = (k_M, k_L)$

To prove the existence of the steady state where $k^2 < K(\lambda) < k^1$, i.e., $(k^1, k^2) = (k_H, k_L)$, we have to show that the graph $g(k^2; R)$ has an intersection with the set defined by equation (3.4.3). Equation (3.4.3) defines k^1 as a function of k^2. This function $\phi : k^2 \mapsto \phi(k^2) := (f')^{-1}\left(\frac{\lambda f'(k^2)}{1-W(k^2)}\right)$ is increasing if and only if $k_2 < f^{-1}(1)$ and satisfies $\phi(0) = 0$

and $\phi(K(\lambda)) = K(\lambda)$. More formally,

$$\begin{aligned}\mathcal{F} &:= \{(k^1,k^2) \in \mathbb{R}^2_+ | H(k^1) - f'(k^2) = 0, k^2 < K(\lambda) < k^1\} \\ &= \{(k, \phi(k)) | k \in [0, K(\lambda)]\}.\end{aligned}$$

Hence, $(k^1, k^2) = (k_H, k_L) \in \mathcal{G}(R) \cap \mathcal{F}$. Figure 3.4.5 shows the graph of $\phi(k)$.

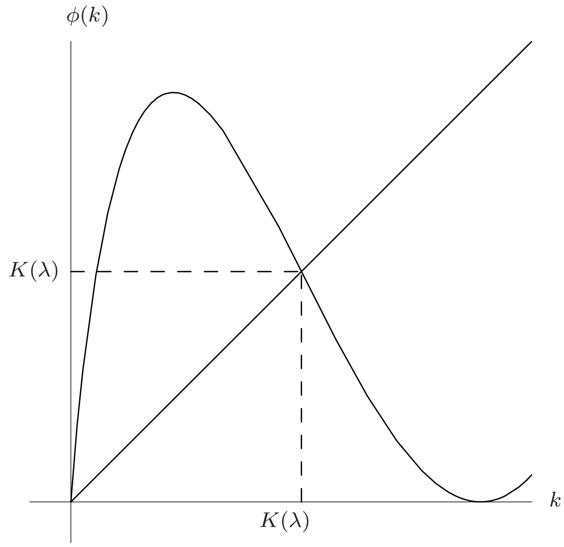


Figure 3.4.5: The graph of $\phi(k)$

Let $\mathcal{R}_{LM} = \{R \in [0, R^+) | \exists k \in (0, K^*(R)) : \text{graph } g(k^2) \cap \text{graph } \phi(k^2) \cap \text{graph } K(\lambda) \neq \emptyset, k^2 \neq K(\lambda)\}$.

Proposition 3.4.2 *Suppose that Assumption 3.2.1 and $L = 1/2$ are satisfied.*

1. *There exists no asymmetric steady state where $k^1, k^2 > K(\lambda)$ if $\phi'(K(\lambda)) > 0$, which is equivalent to $\lambda > \alpha$.*

2. *$\mathcal{R}_{LM} = R_{LM}$*

3. *For $-1 < \phi'(K(\lambda)) < 0$, the asymmetric steady state $(k^1, k^2) = (k_H, k_L)$ exists if and only if $R \in [R_{LM}, (K^*)^{-1}(K(\lambda)))$.*

4. *The transition from $(k^1, k^2) = (k_M, k_L)$ to $(k^1, k^2) = (k_H, k_L)$ is continuous in R.*

Proof: The asymmetric steady state $(k^1, k^2) = (k_H, k_L)$ is defined by the following equations

$$k^1 = g(k^2) \tag{3.4.10}$$

$$k^1 = \phi(k^2) \tag{3.4.11}$$

$$k^2 < K(\lambda) < k^1. \tag{3.4.12}$$

Let us first introduce the expressions $\phi'(k)$, $\phi''(k)$, and $\phi'(K(\lambda))$ for the Cobb-Douglas function, which will be used below. One obtains that

$$\phi'(k) = \frac{1}{1-\alpha} C^{\frac{1}{1-\alpha}} f''(k) \left(\frac{k}{1-W(k)} - \frac{1}{f'(k)} \right) \tag{3.4.13}$$

and

$$\begin{aligned} \phi''(k) = \left(\frac{1}{1-\alpha}\right)^2 C^{\frac{1}{1-\alpha}} (f''(k))^2 \left(\frac{k}{1-W(k)} - \frac{1}{f'(k)} \right)^2 \\ - \frac{1}{1-\alpha} C^{\frac{1}{1-\alpha}} f''(k) \frac{(2-\alpha)}{k} \left(\frac{k}{1-W(k)} - \frac{1}{f'(k)} \right) \\ + \frac{1}{1-\alpha} C^{\frac{1}{1-\alpha}} f''(k) \left(\frac{1-(1-\alpha)W(k)}{(1-W(k))^2} - \frac{1-\alpha}{\alpha A k^\alpha} \right) \end{aligned} \tag{3.4.14}$$

where $C = \left(\frac{\alpha A(1-W(k))}{\lambda f'(k)}\right)$. We can show that $\phi''(k) = 0$ is equivalent to $k = \left(\frac{1+\alpha}{A}\right)^2$. This implies that the function ϕ has a unique inflection point.

Moreover,

$$\phi'(K(\lambda)) = -f'(k) \left(\frac{k}{\lambda} - \frac{1}{f'(k)} \right). \tag{3.4.15}$$

1) We show that there exists no asymmetric steady state for $\phi'(K(\lambda)) > 0$. Using equation (3.4.15) we can show that $\phi'(K(\lambda)) > 0$ is equivalent to $\lambda > \alpha$. This is also equivalent to $f^{-1}(1) > K(\lambda)$. Hence, the graph of $\phi(k)$ lies below $K(\lambda)$ for $k \in [0, K(\lambda))$. This proves that there exists no asymmetric steady state where $k^1, k^2 > K(\lambda)$.

2), 3) We show that $\phi''(k) < 0, \forall k \in [0, K(\lambda)]$ if $\phi'(K(\lambda)) > -1$ by contradiction. Suppose that $\phi''(k) > 0$. Then, the Cobb-Douglas production function implies that

$$k > \left(\frac{1+\alpha}{A}\right)^{\frac{1}{\alpha}}. \tag{3.4.16}$$

Equation (3.4.16) and $k < K(\lambda)$ imply that

$$\left(\frac{1+\alpha}{A}\right)^{\frac{1}{\alpha}} < K(\lambda) := \left(\frac{1-\lambda}{A(1-\alpha)}\right)^{\frac{1}{\alpha}} \Longleftrightarrow \lambda < \alpha^2. \tag{3.4.17}$$

This means that the inflection point of $\phi(k)$ lies below $K(\lambda)$ if and only if $\lambda < \alpha^2$.

Our assumption was $\phi'(K(\lambda)) > -1$, which equivalent to

$$\lambda > \frac{\alpha}{2-\alpha}. \tag{3.4.18}$$

However, equations (3.4.17) and (3.4.18) together imply

$$\alpha - \frac{1}{2-\alpha} > 0, \tag{3.4.19}$$

which is never satisfied for $\alpha \in (0,1)$. Hence, $\phi''(k) < 0, \forall k \in [0, K(\lambda)]$ if $\phi'(K(\lambda)) > -1$. This means that $\phi(k)$ is concave for all $k \in [0, K(\lambda)]$ if $\phi'(K(\lambda)) > -1$.

Now, suppose that $-1 < \phi'(K(\lambda)) < 0$. Then, the graph of $\phi(k)$ has a unique intersection with the graph of $g(k)$ for $k < K(\lambda) < \phi(k)$ if and only if $R \in [R_{LM}, (K^*)^{-1}(K(\lambda))]$. R_{LM} is the value of R for which the graph of $g(k)$ has a unique intersection point with the graphs of $\phi(k)$ and $K(\lambda)$ for $k \neq K(\lambda)$. Note that if $\phi'(K(\lambda)) > 0$, $(K(\lambda), K(\lambda))$ is the only point which satisfies $(K(\lambda), \phi^{-1}(K(\lambda))$. Due to the symmetric structure, the asymmetric steady state $(k^1, k^2) = (k_L, k_H)$ can be obtained analogously.

4) To prove that the transition from $(k^1, k^2) = (k_M, k_L)$ to $(k^1, k^2) = (k_H, k_L)$ is continuous in R, observe that when $k = K(\lambda)$, $h(k) = \phi(k)$. The claim follows since the steady states $(k^1, k^2) = (k_M, k_L)$ and $(k^1, k^2) = (k_H, k_L)$ are continuous functions in R. □

Let $\mathcal{R}_{LH} = \{R \in [0, R^+) | \exists k \in (0, K^*(R)) : g'(k; R) = \phi'(k), g(k; R) = \phi(k)\}$.

Proposition 3.4.3 *Suppose that Assumption 3.2.1 and $L = 1/2$ are satisfied.*

1. $\mathcal{R}_{LH} = R_{LH}$
2. *For $\phi'(K(\lambda)) < -1$, the asymmetric steady state $(k^1, k^2) = (k_H, k_L)$*
   (a) *exists if $R \in [R_{LM}, R_{LH})$*

(b) coexists with $(k^1, k^2) = (k_H, k_M)$ *if and only if* $R \in [(K^*)^{-1}(K(\lambda)), R_{LH})$.

Proof: Suppose that $\phi'(K(\lambda)) < -1$. Then, the graph of $g(k)$ has a unique intersection with the graph of $\phi(k)$ if $R \in [R_{LM}, (K^*)^{-1}(K(\lambda)))$ and two intersections if and only if $R \in [(K^*)^{-1}(K(\lambda)), R_{LH})$. R_{LH} is the value of R for which the graph of $\phi(k)$ is tangent to the graph of $g(k)$. For $R > R_{LH}$ there is no intersection of the graphs $\phi(k)$ and $g(k)$ (see Figure 3.4.6).

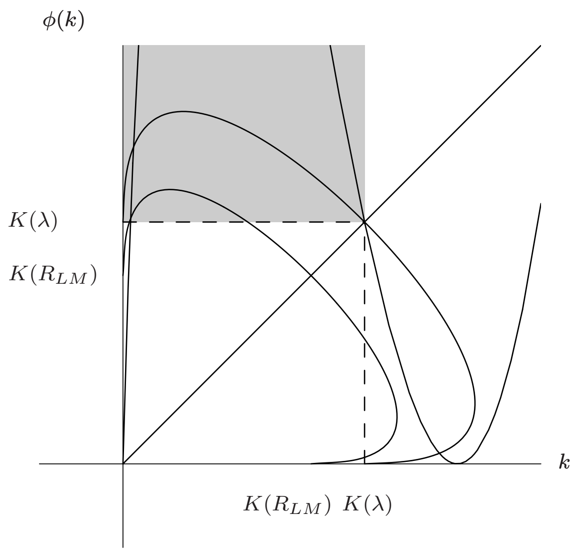


Figure 3.4.6: Existence of the asymmetric steady states $(k^1, k^2) \in \{(k_H, k_L), (k_H, k_M)\}$

For a third intersection to exist, the graph $\phi(k^2)$ has to cut the graph $g(k^2)$ from inside at the third intersection. This would imply that $\phi''(k^2)$ has to change its sign two times in $[0, K(\lambda)]$. This is a contradiction because we know for the Cobb-Douglas function that there exist a unique inflection point, $k = (\frac{1+\alpha}{A})^{\frac{1}{\alpha}}$ where $\phi''(k)$ changes its sign. Hence, $\phi'(K(\lambda)) < -1$ guarantees that there are no more than two intersection points of the graphs $\phi(k)$ and $g(k)$. Due to the symmetric structure, the asymmetric steady state $(k^1, k^2) \in \{(k_L, k_H), (k_M, k_H)\}$ can be obtained analogously. □

Proof of Proposition 3.2.2

Let us first prove that the symmetric steady state is stable if $R < R_c$. For $k^1 < K(\lambda), k^2 < K(\lambda)$, we have

$$k^1 = \tilde{\Psi}^1(k^1, k^2) = \frac{R\left[W(k^1) + W(k^2)\right]}{1 + \left[\frac{1-W(k^2)}{1-W(k^1)}\right]^{\frac{1}{\alpha-1}}}$$
$$k^2 = \tilde{\Psi}^2(k^1, k^2) = \frac{R\left[W(k^1) + W(k^2)\right]}{1 + \left[\frac{1-W(k^1)}{1-W(k^2)}\right]^{\frac{1}{\alpha-1}}}.$$

Let

$$J(k,k) = \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} \frac{\partial \tilde{\Psi}^1(k,k)}{\partial k^1} & \frac{\partial \tilde{\Psi}^1(k,k)}{\partial k^2} \\ \frac{\partial \tilde{\Psi}^2(k,k)}{\partial k^1} & \frac{\partial \tilde{\Psi}^2(k,k)}{\partial k^2} \end{pmatrix}.$$

Observe that $a = d, b = c$. The characteristic polynomial reads $p(\mu) = \mu^2 - 2a\mu + a^2 - b^2$. The eigenvalues of the system are

$$\mu_1 = a + b = \alpha$$
$$\mu_2 = a - b = \alpha\left(\frac{Ak^\alpha}{1 - (1-\alpha)Ak^\alpha}\right)$$

where $k = K^*(R) := \left(\frac{1}{(1-\alpha)AR}\right)^{\frac{1}{\alpha-1}}$. It follows that $0 < \mu_1 < 1$ and $0 < \mu_2 < 1$ if and only if

$$R < R_c = \frac{1}{(1-\alpha)A^{\frac{1}{\alpha}}}.$$

Now, let us prove that countries converge to the symmetric steady state if $k_0^1 = k_0^2 =: k_0 > 0$ or $k_0^1, k_0^2 \geq K(\lambda)$. Let $\Psi^i(k_t^1, k_t^2) := \Psi(k_t^i, \mathcal{R}(k_t^1, k_t^2))$ for $i = 1, 2$. If $k_0^1 = k_0^2 =: k_0 > 0$, then $\Psi^1(k_0, k_0) = \Psi^2(k_0, k_0) = RW(k_0) = k_1^1 = k_1^2 =: k_1$. By induction, $(\Psi^1)^n(k_0, k_0) = (\Psi^2)^n(k_0, k_0) = \underbrace{RW \circ RW \cdots \circ RW}_{n-times}(k_0) = k_n^1 = k_n^2 =: k_n, \forall n \in \mathbb{N}$. Given Assumption 2.3.1, the orbit $\lim_{n\to\infty} \underbrace{RW \circ RW \cdots \circ RW}_{n-times}(k_0)$ converges to $K^*(R)$ and hence (k_n^1, k_n^2) converges to the symmetric steady states $(K^*(R), K^*(R))$. If $k_0^1, k_0^2 \geq K(\lambda)$, $\Psi^1(k_0^1, k_0^2) = \Psi^2(k_0^1, k_0^2) \Longrightarrow k_1^1 = k_1^2 =: k_1 > 0$. Convergence to $(K^*(R), K^*(R))$ follows from the first part of this proof. □

Proof of Proposition 3.2.3

The asymmetric steady state $(k^1, k^2) = (k_M, k_L)$ requires

$$k^1 = h(k^2) \tag{3.4.20}$$

$$k^1 = g(k^2) \tag{3.4.21}$$

Let us first investigate how the change in L affects $k^1 = g(k^2)$, which is implicitly defined by $G(k^1, k^2) := L(k^1 - RW(k^1)) + (1 - L)(k^2 - RW(k^2))$. Figure 3.4.7 depicts how the function $L(k - RW(k))$ depends on L. The function $L(k - RW(k))$ implies

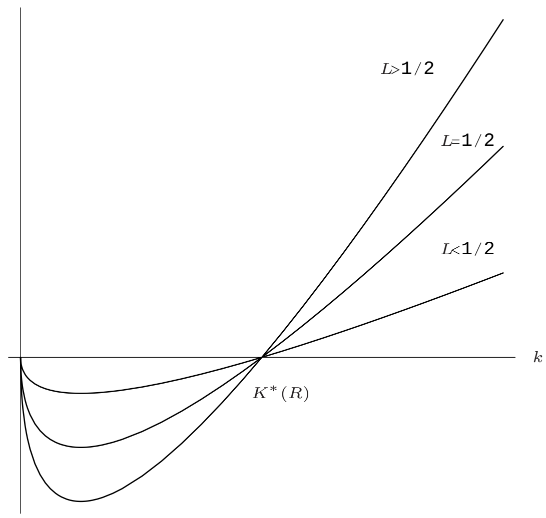


Figure 3.4.7: The graph of $L(k - RW(k))$

that the graph $g(k^1)$ which satisfies $G(k^1, k^2) = 0$ shifts downwards for lower L. On the other hand, $g(k^2)$ shifts upwards for lower L. Furthermore, $\frac{dg(K^*(R);R,L)}{dk^1} = -\frac{L}{1-L}$ by the implicit function theorem. If $R < R_c$, there exists always a unique $L_c < 1/2$ defined by $\frac{\partial g(k^2)}{\partial k^2} = \frac{\partial h(k^2)}{\partial k^2}$ and $g(k^2) = h(k^2)$. Analogously we can proof the existence of the asymmetric steady state $(k^1, k^2) = (k_M, k_L)$ for $L_c > 1/2$. □

Proof of Proposition 3.2.4

We will prove that the asymmetric steady state $(k^1, k^2) = (k_M, k_L)$ undergoes a supercritical Neimark-Sacker bifurcation by calculating the determinant and trace numerically. The determinant and the trace of the Jacobian matrix of the system (3.2.3) when $k^1, k^2 < K(\lambda)$ can be written as

$$\det = \frac{W'(k^1)k^1W'(k^2)k^2}{(LW(k^1)+(1-L)W(k^2))^2}\frac{1}{R(1-\alpha)}\left(k^2L^2\frac{(1-W(k^1))^{\frac{1}{\alpha-1}}}{(1-W(k^2))^{\frac{\alpha}{\alpha-1}}}\right.$$
$$+k^1(1-L)^2\frac{(1-W(k^2))^{\frac{1}{\alpha-1}}}{(1-W(k^1))^{\frac{\alpha}{\alpha-1}}}$$
$$\left.+k^2(1-L)L\frac{(1-W(k^1))^{\frac{2-\alpha}{\alpha-1}}}{(1-W(k^2))^{\frac{1}{\alpha-1}}}+k^1(1-L)L\frac{(1-W(k^2))^{\frac{2-\alpha}{\alpha-1}}}{(1-W(k^1))^{\frac{1}{\alpha-1}}}\right) \quad (3.4.22)$$

and

$$\mathrm{tr} = \frac{1}{LW(k^1)+(1-L)W(k^2)}\left(W'(k^1)k^1\left(L+\frac{k^1(1-L)}{R(1-\alpha)}\frac{(1-W(k^2))^{\frac{1}{\alpha-1}}}{(1-W(k^1))^{\frac{\alpha}{\alpha-1}}}\right)\right.$$
$$\left.+W'(k^2)k^2\left(1-L+\frac{k^2L}{R(1-\alpha)}\frac{(1-W(k^1))^{\frac{1}{\alpha-1}}}{(1-W(k^2))^{\frac{\alpha}{\alpha-1}}}\right)\right). \quad (3.4.23)$$

The determinant and the trace when $k^2 < K(\lambda) < k^1$ can be written as

$$\det = \frac{W'(k^1)k^1W'(k^2)k^2}{(LW(k^1)+(1-L)W(k^2))^2}\frac{1}{R(1-\alpha)}\left(k^2L^2\frac{\lambda^{\frac{1}{\alpha-1}}}{(1-W(k^2))^{\frac{\alpha}{\alpha-1}}}\right.$$
$$\left.+k^1(1-L)L\frac{(1-W(k^2))^{\frac{2-\alpha}{\alpha-1}}}{\lambda^{\frac{1}{\alpha-1}}}\right) \quad (3.4.24)$$

and

$$\mathrm{tr} = \frac{W'(k^1)k^1L+W'(k^2)k^2}{LW(k^1)+(1-L)W(k^2)}\left(1-L+\frac{k^2L}{R(1-\alpha)}\frac{\lambda^{\frac{1}{\alpha-1}}}{(1-W(k^2))^{\frac{\alpha}{\alpha-1}}}\right). \quad (3.4.25)$$

If $k^1 = K(\lambda)$, equations (3.4.24) and (3.4.25) can be written as

$$\det = \frac{W'(k^1)k^1W'(k^2)k^2}{(LW(k^1)+(1-L)W(k^2))^2}\frac{1}{R(1-\alpha)}\left(k^2L^2\frac{(1-W(k^1))^{\frac{1}{\alpha-1}}}{(1-W(k^2))^{\frac{\alpha}{\alpha-1}}}\right.$$
$$\left.+k^1(1-L)L\frac{(1-W(k^2))^{\frac{2-\alpha}{\alpha-1}}}{(1-W(k^1))^{\frac{1}{\alpha-1}}}\right) \quad (3.4.26)$$

and

$$\text{tr} = \frac{W'(k^1)k^1 L + W'(k^2)k^2}{LW(k^1) + (1-L)W(k^2)} \left(1 - L + \frac{k^2 L}{R(1-\alpha)} \frac{\lambda^{\frac{1}{\alpha-1}}}{(1-W(k^2))^{\frac{\alpha}{\alpha-1}}} \right). \tag{3.4.27}$$

Comparing equations (3.4.22) and (3.4.23) with equations (3.4.24) and (3.4.25), we can see that the determinant and the trace are not equal respectively at $k^1 = K(\lambda)$. The dynamical system is not differentiable at this point. Figure 3.4.8 shows how the determinant and the trace of the system moves as we change the bifurcation parameter L. The points (a),(b),(c),(d),(e),(f) correspond to $L = (0.117, 0.117, 0.13, 0.16, 0.177, 0.19)$. The points (a) and (b) are defined by equations (3.4.22) and (3.4.23), and equations (3.4.26) and (3.4.27) respectively. We can observe that at $L = 0.117$ when $k^1 = K(\lambda)$ the determinant and the trace jump from (a) to (b). As the value of L increases, the determinant crosses 1 at (e) which proves that the bifurcation is a Neimark-Sacker bifurcation.

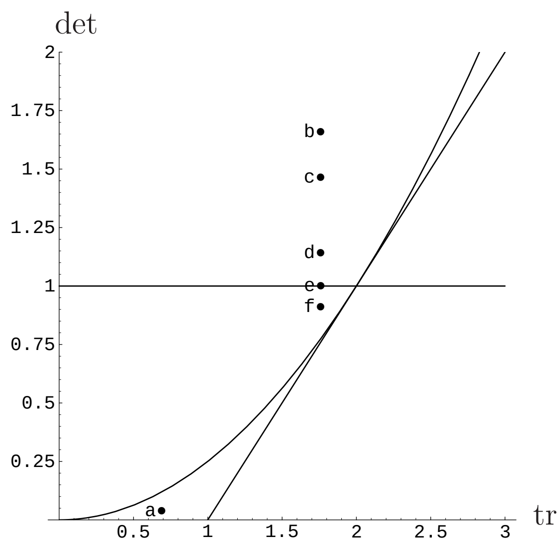


Figure 3.4.8: Stability triangle: $L = (0.117, 0.117, 0.13, 0.16, 0.177, 0.19)$

□

Chapter 4

Uncertainty, Assets, and Capital Accumulation

4.1 Introduction

Chapter 2 and 3 analyzed models in which there were no risks associated with economic activities. It has been a tradition in economic theory to conduct separate analysis of the activities of the real and financial sectors of the economy. However, when financial markets are incomplete, the two sectors cannot be treated independently. Production and consumption decisions depend on the risk sharing possibilities offered by the financial sector, while agents' financial decisions in turn depend on the consumption needs and investment opportunities created by the real sector. Therefore, the framework in Chapter 2 and 3 ignores two important aspects. Firstly, it prevents us from studying the nature of a wide array of assets, which are subject to uncertainty. Secondly, it obscures the role of asset trade in reaction to uncertain events.

The role of financial markets in an uncertain world is well established in the literature. The theory of general equilibrium with incomplete markets suggests how to overcome the effects of uncertainty and how to allocate the risk optimally. Section 1.4 reviewed general equilibrium models which address the issue of financial intermediation and development. There are two main aspects which characterize the literature. Firstly, the financial intermediary facilitates the trading of risk thereby allowing individuals to engage in risky activities that yield higher return on the aggregate level. Thus, higher risk is assumed to be associated with not just higher return but also with higher productivity in the real sector. Therefore, the efficiency in the financial market is linked to the productivity in the real sector. Secondly, it is assumed that the activities in the real sector go hand in hand with the activities in the financial sector. In other words, capital accumulation is associated with an increase in the volume of intermediation. Therefore, financial activities grow as a proportion of gross domestic product. Goldsmith (1969) provides empirical evidence for this argument.

However, the development of financial markets today is typically accompanied by a disproportionate increase in the trade volume of financial capital and not of real capital. For example, firms can raise capital by issuing new shares in stock markets. However, it is known that a large part of financial trading in the stock markets is trading of existing shares in the markets. Therefore, transactions in financial markets need not be related to productivity in the real sector. So what is it that creates the deviation we observe between the trade volume between financial capital and real capital? Typically, trading of existing shares is influenced by price expectations, which may be influenced by various factors. To analyze the nature of such a financial market and its implication on capital accumulation we have to develop a model in which an asset price process and an endogenous income process are integrated.

There are a number of works, which embed the analysis of income flows on financial markets into a structure of real markets. Donaldson & Mehra (1984) were the first to provide the link between asset prices, the profit maximizing firms, and utility maximizing representative agents in a general equilibrium model. They analyzed the quantitative effects of how underlying preferences and technologies are related to asset prices. Huffman (1986) employed a two period overlapping generations model, which allows for heterogenous participation in the asset market. However, the underlying economy is modeled as an exogenous process leaving the question of general equilibrium out of the analysis. Donaldson & Mehra (1984) and Huffman (1986) derive the asset price from the stochastic intertemporal Euler equation, while the dividend is defined as the difference between the value of capital before and after production. Thus, the asset price and the dividend are intimately related to real capital reflecting the fundamentals of the firm. The asset price is interpreted as a shadow price which supports the intertemporal consumption decision and therefore trading does not actually take place in the financial market.

This chapter modifies the standard overlapping generations model with two period life time in two ways. Firstly, it introduces and an additive shock to production. Secondly, it introduces an additional commodity, a nominal asset, that can be traded by agents to transfer their wealth over time. The asset market is modeled as in Böhm & Chiarella (2005) in which asset prices are determined endogenously by the interaction of utility maximizing agents. Since agents consume only in the second period, a young agent's objective is to choose a portfolio of assets and capital investment to maximize the utility of next period consumption. The model by Böhm & Chiarella (2005) is extended so that the income stream is endogenous and the factor prices are determined by their respective marginal products.[1] The return of the capital investment is the marginal product of capital, while the price of the assets is not linked to production. We abstract from the issuing of new shares. The firm pays out the random profit as dividends to shareholders. The asset price is determined by the trading of the existing shares between young and old agents in the market. This allows us to examine the interplay between the capital investment and the trading of existing shares.

The role of a nominal asset, which can be traded in an uncertain world can be twofold in an overlapping generations model. Firstly, it can be used by the firm to transfer the random part of the production to the consumption of the old. This shift of the randomness between generations induces a deterministic law of capital accumulation, making the consumption of the old the only stochastic variable. Secondly, young consumers can hold the asset to transfer wealth to the next period. This serves to smooth their consumption plan given their preferences. In contrast to the credit market in which only intergenerational trade takes place this means that intertemporal trade between generations takes place in the asset market.

The remainder of the chapter is organized as follows. Section 4.2 introduces the basic structure of the model. Section 4.3 defines the temporary equilibrium of the closed economy and Section 4.4 analyzes its dynamics.

[1]This basic framework was first presented by Böhm (2002b).

4.2 The Model Structure

We consider an overlapping generations economy evolving in discrete time. In addition to the markets for output, labor, and capital, there is a market for paper assets which can be re-traded. Purchase of the re-tradable paper assets is distinguished from investment in capital in two ways. Firstly, paper assets are not linked to production. Secondly, while capital is reproduced every period, the number of assets is exogenously given in the model. Each generation consisting of homogeneous consumers lives for two periods and there is no population growth. All markets operate under perfect competition implying that agents are price takers.

4.2.1 The Production Sector

There is a single firm, which lives infinitely long in the economy and uses one unit of labor L and capital K to produce consumption goods. The aggregate production function is given by

$$F(K, 1) + \varepsilon,$$

where F is homogeneous of degree one, ε is an additive shock to production. Then the intensive form can be written as

$$f(k) + \varepsilon.$$

where $k := \frac{K}{L}$. The labor and capital markets are assumed to be competitive such that the profit maximizing firm pays the wage $w(k) := f(k) - kf'(k)$ and the return on capital investment $r(k) := f'(k)$ according to the marginal product rule. The stochastic output is paid to shareholders as a dividend per share. In the overlapping generations structure the young agents are the shareholders of the firm and receives the dividend payment when they are old. This time structure is particulary important since the source of the randomness is completely absorbed by the asset market. The firm transfers the random component of production to the consumption of the old thereby leaving all the other variables deterministic.

Assumption 4.2.1 *The production function in the intensive form* $f : \mathbb{R}_+ \to \mathbb{R}_+$ *is* C^2 *and* $f''(k) < 0 < f'(k)$ *and satisfies the Inada conditions* $\lim_{k\to\infty} f'(k) = 0$ *and* $\lim_{k\to 0} f'(k) = \infty$ *for* $k > 0$.

4.2.2 The Consumption Sector

The typical young consumer in period $t = 0$ supplies one unit of labor inelastically in the first period of his life time and receives labor income w in units of the consumption good which is the numéraire good.[2] His lifetime utility depends on old age consumption only. There is no storage possibility for the consumption goods. He can transfer his wage income to the next period either by investing in capital or by purchasing assets. The young agent cannot take credit in the capital market. In the second period of his life time when he is old, the agent receives the rate of return R_1 on his capital investment y and a random dividend ε_1 on his share holdings x, which he resells in the market. We assume that consumers have risk preferences over the mean μ and the standard deviation σ of future consumption/wealth described by a utility function

$$U : \begin{cases} \mathbb{R}_+ \times \mathbb{R}_+ \to \mathbb{R} \\ (\mu, \sigma) \mapsto U(\mu, \sigma) \end{cases}$$

which is increasing in the mean μ and decreasing in the standard deviation σ.

Let $(x, y) \in \mathbb{R} \times \mathbb{R}_+$ denote a portfolio of assets and capital investment and let $p \in \mathbb{R}_+$ denote the current price of assets in units of the consumption good. The budget constraint takes the form

$$w = px + y.$$

Then, the investor's wealth in the following period $t = 1$ is given by

$$W(w, p, x, R_1, p_1, \varepsilon_1) = R_1(w - px) + (p_1 + \varepsilon_1)x.$$

When making the portfolio decision, the next period's return on capital, equity price, and dividend $(R_1, p_1, \varepsilon_1)$ are uncertain for young agents. It is assumed that they make point forecasts (R^e, p^e) for the return on capital and the asset price. We separate expectations for the asset price from expectations for the dividend payment, which is the only source of randomness. The following assumption is made about the expectation for the next period's dividend payment ε_1.

[2] For ease of notation the time index t will be suppressed unless necessary. Variables without time subscript refer to an arbitrary period t while subscript 1 refers to period $t + 1$ and -1 to period $t - 1$.

Assumption 4.2.2 *Consumers are endowed with a subjective probability distribution $\nu \in P(\mathbb{R}_+)$ for the next period's dividend payment parameterized by a pair $(\mathbb{E}_\nu[\varepsilon], \mathbb{V}_\nu[\varepsilon]) \in \mathbb{R}_+ \times \mathbb{R}_{++}$ of an expected value and a variance.*

Then, for any asset portfolio $x \in \mathbb{R}$ the subjectively expected value of the future wealth can be expressed as

$$\begin{aligned} \mathbb{E}_\nu[W(w,p,x,R^e,p^e,\cdot)] &= \int_{\mathbb{R}_+} (R^e w + (p^e + \varepsilon - R^e p)x)\nu(d\varepsilon) \\ &= R^e w + (p^e + \mathbb{E}_\nu[\varepsilon] - R^e p)x \end{aligned} \tag{4.2.1}$$

with the associated subjective variance

$$\begin{aligned} \mathbb{V}_\nu[W(w,p,x,R^e,p^e,\cdot)] &= \int_{\mathbb{R}_+} (W(w,p,x,R^e,p^e,\varepsilon) - \mathbb{E}_\nu(W(w,p,x,R^e,p^e,\cdot)))^2\nu(d\varepsilon) \\ &= x^2\mathbb{V}_\nu[\varepsilon] \end{aligned} \tag{4.2.2}$$

where $p^e + \mathbb{E}_\nu[\varepsilon] - R^e p$ is the expected risk premium. The young agent's objective is to maximize the utility of next period consumption defined by

$$\max_{x\in\mathbb{R}} \left\{ U\left(\mathbb{E}_\nu[W(w,p,x,R^e,p^e,\cdot)], \mathbb{V}_\nu[W(w,p,x,R^e,p^e,\cdot)]^{\frac{1}{2}}\right) \Big| x \leq \frac{w}{p} \right\}$$

which by Equation (4.2.1) and (4.2.2) is identical to

$$\max_{x\in\mathbb{R}} \left\{ U\Big(R^e w + (p^e + \mathbb{E}_\nu[\varepsilon] - R^e p)x, x\sqrt{\mathbb{V}_\nu[\varepsilon]}\Big) \Big| x \leq \frac{w}{p} \right\}.$$

The following assumption characterizes the rational expectations of young consumers.[3]

Assumption 4.2.3 *$\{\varepsilon_t\}_{t\geq 0}$ is an i.i.d sequence of random variables with finite first and second moments. We assume that the agents have correct knowledge of these moments such that*

$$\mathbb{E}_\nu[\varepsilon_t] = \mathbb{E}[\varepsilon_t],$$

where $\mathbb{E}[\varepsilon_t]$ is the mean value of the random variable ε_t and

$$\mathbb{V}_\nu[\varepsilon_t] = \mathbb{V}[\varepsilon_t],$$

where $\mathbb{V}[\varepsilon_t]$ is the variance of the random variable ε_t.

[3]More specifically by rational expectations we mean an unbiased prediction and/or a perfect prediction whenever available.

4.3 The Closed Economy Model

We assume that the amount of assets is constant and normalized to be one in the economy.[4] There is no imperfection associated with the asset market. In the overlapping generation structure all the assets sold by old consumers are bought by young investors at equilibrium.

4.3.1 Temporary Equilibrium

Assumption 4.3.1 *Let the preference of an investor be given by the linear mean variance function of future wealth*

$$U(\mu,\sigma) = \mu - \frac{\alpha}{2}\sigma^2,$$

where α is usually interpreted as a measure of risk aversion.

This assumption assures that there is no direct income effect on the asset demand and thus simplifies our analysis (see Böhm (2002a) for more general mean variance functions). Then, the asset demand of the young investor is given by

$$x = \varphi(p, p^e, R^e, k) := \mathrm{Min}\left(\frac{p^e + \mathbb{E}[\varepsilon] - R^e p}{\alpha \mathbb{V}[\varepsilon]}, \frac{w(k)}{p}\right).$$

The price law $p = S(p^e, R^e, k)$ is implicitly defined by the solution of

$$\varphi(p, p^e, R^e, k) = 1. \tag{4.3.1}$$

Notice that the asset demand has an expectational lead and consumer's preferences are parameterized by the first two moments of the random variable ε. This means that the asset price is a deterministic function of expectations. Let $c := \mathbb{E}[\varepsilon] - \alpha\mathbb{V}[\varepsilon]$, which can be interpreted as risk adjusted dividend payment. Then, the risk adjusted expected cum-dividend price is given by $p^e + c$.

[4]We do not address the issue of how firm's decision to raise capital influences the economy but focus on how consumption decision affects capital accumulation.

Proposition 4.3.1 *There exists a unique positive equilibrium price*

$$p = S(p^e, R^e, k) \tag{4.3.2}$$

if the risk adjusted expected cum-dividend price is greater than zero, i.e., $p^e + c > 0$.

Proof: The assertion in Proposition 4.3.1 is obvious as the asset demand function $\varphi(p, p^e, R^e, k)$ is decreasing in p, $\varphi(0, p^e, R^e, k) = \frac{p^e + \mathbb{E}[\varepsilon]}{\alpha \mathbb{V}[\varepsilon]}$ and $\lim_{p \to \infty} \varphi(p, p^e, R^e, k) = -\infty$. □

Note that in equilibrium short sale does not take place in the asset market as the young consumers are homogeneous. We assume that the capital investment is reversible. This means that depreciated capital is paid back as a part of the return on capital investment. Then, the capital investment, which is defined by wage minus purchases of assets gives the evolution of capital

$$k_1 = w(k) - p. \tag{4.3.3}$$

Equation (4.3.2) and (4.3.3) define the temporary equilibrium and the evolution of capital formation given expectations.

4.3.2 Expectations

Given Equation (4.3.3) for capital accumulation, the return on capital at $t = 1$ is given by

$$R_1 = R(k, p) := f'(w(k) - p) + 1 - \delta, \tag{4.3.4}$$

where δ is the depreciation rate of capital. The term $1 - \delta$ is there because we have assumed the capital investment to be reversible.[5] The perfect foresight at $t = 0$ for the return on capital at $t = 1$ requires the expected return to be equal to the actual return, i.e.

$$R^e = R(k, p). \tag{4.3.5}$$

[5] As we will see later, this assumption is essential when we use the quadratic production function for which $f'(k)$ is zero when k is above certain threshold. If the capital investment war not reversible, situation could occur where young agents hold a portfolio in the capital market even though they will lose their entire capital.

Substituting Equation (4.3.5) into Equation (4.3.2), the perfect predictor $p^e = \Psi(p^e_{-1}, k)$ at $t = 0$ for the asset price in $t = 1$, which is consistent with a perfect prediction for the return on capital, has to simultaneously satisfy

$$p = p^e_{-1} \tag{4.3.6}$$

$$p = S(p^e, R(k,p), k) \tag{4.3.7}$$

which is equivalent to satisfying

$$p = p^e_{-1} \tag{4.3.8}$$

$$\varphi(p, p^e, R(k,p), k) = 1. \tag{4.3.9}$$

Hence, the perfect predictor $p^e = \Psi(p^e_{-1}, k)$ is implicitly defined by the solution of

$$p^e_{-1} = S(p^e, R(k, p^e_{-1}), k) \tag{4.3.10}$$

or

$$\varphi(p^e_{-1}, p^e, R(k, p^e_{-1}), k) = 1. \tag{4.3.11}$$

The following proposition defines the existence of such perfect predictor. Note that given the perfect foresight for the return on capital, the asset demand now becomes dependent on wage income in general. This implies that the price law also depends on wage income in general.

Proposition 4.3.2 *Let $\mathcal{D} := \{(p^e_{-1}, k) | p^e_{-1} \in [0, w(k)), k \in \mathbb{R}_+\}$.*

1. *There exists a unique perfect predictor for the asset price consistent with the perfect forecasting rule in the capital market given by*

   $$\Psi : \mathcal{D} \to \mathbb{R}, (p^e_{-1}, k) \mapsto p^e_{-1}(f'\left(w(k) - p^e_{-1}\right) + 1 - \delta) - c$$

   if and only if $p^e_{-1} \in (0, w(k))$.

2. *The perfect predictor is positive if $c \leq 0$ or if $c > 0$ and $p^e_{-1} \in (h(k), w(k))$ where $h(k)$ is implicitly defined by $\Psi(h(k), k) = 0$.*

See the appendix for a proof. □

Proposition 4.3.2 defines a subset $\mathcal{P}(k) \subset \mathbb{R}_+$ for all $k \in \mathbb{R}_+$, such that for all $p^e_{-1} \in \mathcal{P}(k)$ there exists a positive perfect predictor for the next period's asset price which is given by

$$\mathcal{P}(k) := \begin{cases} (0, w(k)) & \text{if } c \le 0 \\ \\ (h(k), w(k)) & \text{if } c > 0. \end{cases} \tag{4.3.12}$$

Given the perfect predictor Ψ, there exists an equivalent price map along which a perfect point prediction is guaranteed. Then, the dynamical system for the closed economy under rational expectations is given by

$$\begin{aligned} k_1 &= G(p,k) := w(k) - p \\ p_1 &= \Psi(p,k) := p(f'(w(k)-p) + 1 - \delta) - c. \end{aligned} \tag{4.3.13}$$

4.4 Dynamics of the Economy with Rational Expectations

The dynamical system for the closed economy under rational expectations is defined by (4.3.13). It was shown in Section 4.3.2 that the perfect asset price predictor is not defined when the budget constraint is binding. We do not need to consider this case if the production function satisfies the Inada condition. Binding budget constraint means that the young agents invest all their income in the asset market. However, in this case the return on capital investment is infinite. Therefore, not to invest in the capital market contradicts the assumption on rational expectations. Even when the budget constraint is not binding, the perfect predictor can be negative if $c > 0$. This is a general feature of the CAPM models for a given positive return on riskless assets as in Böhm, Deutscher & Wenzelburger (2000) and Böhm & Chiarella (2005). Since the dynamical system is only defined for a subset of $\mathbb{R}^2_+$, the question arises whether there exists a forward invariant set of the system. To investigate the existence of steady states under rational expectation and their stability properties we have to specify the production function.

4.4.1 Cobb-Douglas Production Function Case

To ease the presentation, we assume a full depreciation of capital, i.e. $\delta = 1$. Relaxation of this assumption makes the analytical presentation complicated. For the Cobb-Douglas production function, $f(k) = Ak^{\theta}$ the steady states of the dynamical system (4.3.13) are given by the solution of the following system of equations

$$p = (1-\theta)Ak^{\theta} - k \quad (4.4.1)$$

$$p = \frac{c}{\theta A k^{\theta-1} - 1}. \quad (4.4.2)$$

The dynamical system can be rewritten in terms of the interest rate $r = \theta A k^{\theta-1}$. Notice that all the steady states with $r < 1$ are associated with a negative asset price. Combining equations (4.4.1) and (4.4.2), we obtain the condition for the steady states

$$\left(r - \frac{\theta}{1-\theta}\right)(r-1) = \frac{\theta c}{1-\theta}\left(\frac{r}{A\theta}\right)^{\frac{1}{1-\theta}}. \quad (4.4.3)$$

Let us rewrite equation (4.4.3) as $g(r) = h(r)$ for $r \in \mathbb{R}_+$. The function g is a quadratic function, which satisfies $g(0) = \frac{\theta}{1-\theta}$ and $\lim_{r\to\infty} g(r) = \infty$ and is tangent to zero if $\theta = 0.5$ and otherwise cuts zero at 1 and $\frac{\theta}{1-\theta}$. Figure 4.4.1 depicts the function $g(r)$ for three different value of $\theta = 0.25, 0.5, 0.75$.

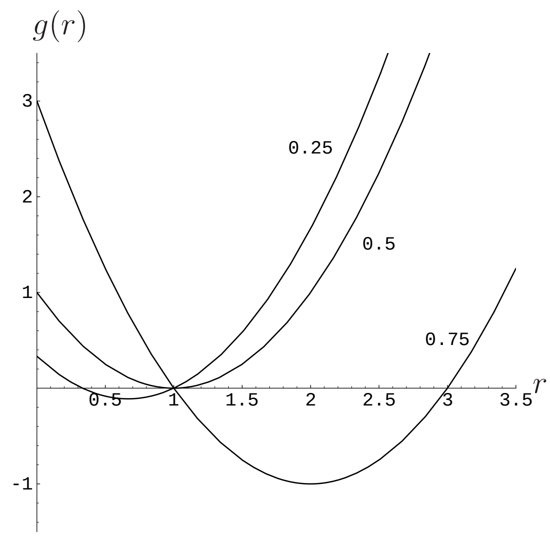
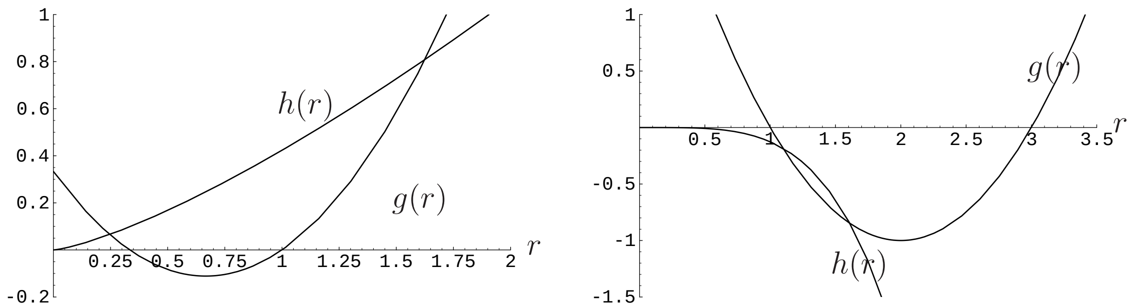


Figure 4.4.1: The function g

The function h satisfies $h(0) = 0$, $h'(r) > 0$ and $h''(r) > 0$ if $c > 0$ and $h'(r) < 0$ and $h''(r) < 0$ if $c < 0$. Let $c^*(A, \theta, \delta)$ denote the unique constant for which $g'(r) = h'(r)$ and $g(r) = h(r)$ hold. Figure 4.4.2 depicts two situations where the functions $g(r)$ and $h(r)$ intersect respectively for positive and negative c.



(a) $c > 0$ (b) $c < 0$

Figure 4.4.2: The steady states

The following proposition characterizes the full set of steady states.

Proposition 4.4.1 *Let the production function be of the Cobb-Douglas form* $f(k) = Ak^\theta$.

1. *Suppose that* $c = 0$, *there exists a unique positive steady state* $(k, p) = (((1 - \theta)A)^{\frac{1}{1-\theta}}, 0)$.
2. *Suppose that* $c < 0$.
   - (a) *There exists no negative steady state.*
   - (b) *If* $\theta > 0.5$, *there exists a unique positive steady state if* $c = c^*(A, \theta, \delta)$ *and two positive steady states if* $c > c^*(A, \theta)$.
3. *Suppose that* $c > 0$.
   - (a) *There always exists a unique negative steady state.*
   - (b) *There exists a unique positive steady state if* $\theta < 0.5$, *if* $\theta = 0.5$ *and* $c < \frac{A^2}{4}$, *or if* $\theta > 0.5$ *and* $c = c^*(A, \theta)$ *and two positive steady states if* $\theta > 0.5$ *and* $c < c^*(A, \theta)$.

See the appendix for a proof. □

Proposition 4.4.1 shows that multiple steady states arise depending on the risk adjusted expected dividends c and the production elasticity θ. The following proposition characterizes the stability property of these steady states.

Proposition 4.4.2 *Let the production function be of the Cobb-Douglas form $f(k) = Ak^{\theta}$.*

1. *The unique positive steady state is a saddle. If there are two positive steady states, both of them are unstable.*
2. *The negative steady state is a stable node.*

See the appendix for a proof. □

Proposition 4.4.1 and 4.4.3 show that multiple steady states may exist however all the steady states are unstable. This means that we do not obtain a forward invariant set of the dynamical system under rational expectation unless the economy is in a steady state initially or on a saddle path. In order to allow for a dynamic analysis of the closed economy for all possible initial conditions, we introduce another production function below.

4.4.2 Quadratic Production Function Case

Let the production function be of the following quadratic form

$$f(k) = \begin{cases} Ak(2d-k) & \text{if } k < d \\ Ad^2 & \text{if } k \geq d. \end{cases} \tag{4.4.4}$$

This quadratic production function has a technically convenient property that the first derivative is a linear function.[6]

$$f'(k) = \begin{cases} 2A(d-k) & \text{if } k < d \\ 0 & \text{if } k \geq d. \end{cases} \tag{4.4.5}$$

[6] Day (1983) was one of the first to exploit the property of this function.

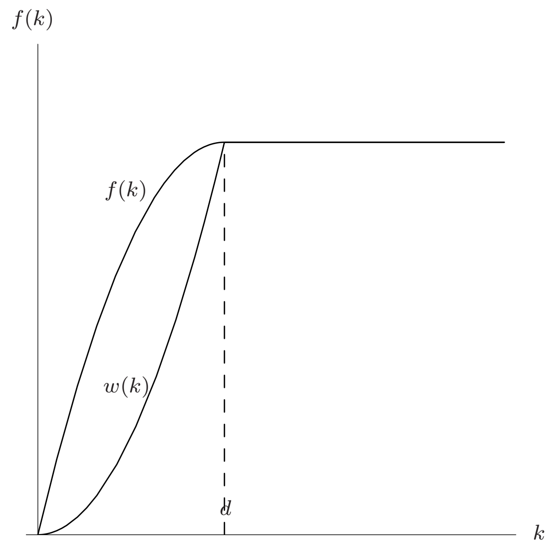


Figure 4.4.3: Quadratic Production Function

This allows us to derive the dynamical system for the closed economy model as well as for the two country model explicitly. However, notice that the first derivative of the quadratic function violates one of the Inada conditions since $\lim_{k \to 0} = 2Ad$. This properties have a decisive influence on the existence and stability properties of the dynamical system since the wage function is not globally concave. Figure 4.4.3 illustrates the quadratic production function with the associated wage function. The following proposition characterizes the existence and the stability property of all steady states.

Proposition 4.4.3 *Let the production function be given by equation (4.4.4).*

a) If $c > 0$, there exist at most two steady states. Both of them are positive, unstable and $k < d$.

b) If $c \leq 0$, there exist two positive steady states if and only if $(Ad^2 - d)\delta > -c$. One is unstable and $k < d$. The other is stable and $k \geq d$.

See the appendix for a proof. □

Proposition 4.4.3 shows that there exists a stable steady state with the quadratic production function under certain conditions. Let the steady states be defined by the zero of the following functions

$$\begin{pmatrix} \Delta p(p,k) \\ \Delta k(p,k) \end{pmatrix} := \begin{pmatrix} p - \Psi(p,k) \\ k - G(p,k) \end{pmatrix}. \tag{4.4.6}$$

Figure 4.4.4 shows the zero contour of the functions $\Delta p(p,k)$ and $\Delta k(p,k)$ given by

$$p = \begin{cases} \begin{pmatrix} -\frac{c}{\delta - 2A(d-k)} \\ Ak^2 - k \end{pmatrix} & \text{if } k < d \\ \begin{pmatrix} -\frac{c}{\delta} \\ Ad^2 - k \end{pmatrix} & \text{if } k \geq d. \end{cases} \tag{4.4.7}$$

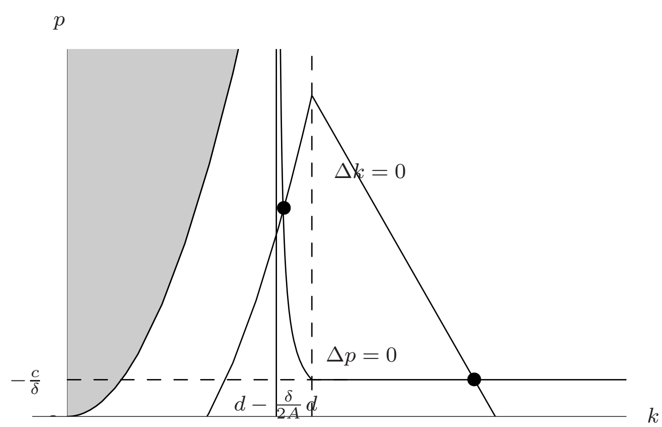


Figure 4.4.4: Phase diagram for the closed economy: $c < 0$

The steady states are given by the intersections of the graphs $\Delta p(p,k) = 0$ and $\Delta k(p,k) = 0$. The gray shaded area is defined by $p > w(k)$ and depicts the area where the budget constraint is binding. Remember from Proposition 4.3.2 that the perfect predictor Ψ is only defined on $p \in (0, w(k))$ for $c \leq 0$. Given the results from Proposition 4.4.3, it can be confirmed from the figure that there exists a forward invariant set of the dynamical system (4.3.13) around the stable steady state $(p,k) = (-\frac{c}{\delta}, Ad^2 + \frac{c}{\delta})$.

4.5 Summary

Section 4.2 introduced a standard OLG model, which was extended to include an additive shock to production and nominal assets, which serve to transfer wealth over time.

We described the atemporal profit maximization problem of a single firm and the intertemporal maximization problem of the young consumers in a general form. The firm pays the random component of output as a dividend to old consumers. The preference of young consumers is characterized by the first two moments of the random dividend payment. We assumed that the young consumers have rational expectations, i.e., they anticipate the correct moments of the random dividend when forming expectations. This implied that the asset market transfers the random component of output to the consumption of the old thereby leaving all other variables deterministic. Section 4.3 used a linear mean variance utility function to describe the temporary equilibrium of the closed economy. It was shown that the perfect asset price predictor can only be derived on a subset of $\mathbb{R}_+$. Section 4.4 showed that there exists a forward invariant set of the dynamical system for a quadratic production function but not for a Cobb-Douglas production function under rational expectations. For both production functions, the interaction between the real economy and the asset market generates multiple steady states.

4.6 Appendix

Proof of Proposition 4.3.2

If $\varphi(p, p^e, R(k,p), k) = \frac{w(k)}{p}$, $p = w(k)$. Then, the perfect predictor is not defined. If $\varphi(p, p^e, R(k,p), k) = \frac{p^e + \mathbb{E}[\varepsilon] - R(k,p)p}{\alpha \mathbb{V}[\varepsilon]}$, there exists a perfect predictor $\Psi(p^e_{-1}, k) := R(k, p^e_{-1})p^e_{-1} - c$. We obtain that $\frac{\partial}{\partial p^e_{-1}}\Psi(p^e_{-1}, k) > 0$ and $\Psi(0,k) = -c$. This implies that if $c > 0$, the perfect predictor Ψ is negative for $p^e_{-1} \in (0, h(k))$. □

Proof of Proposition 4.4.1

Let us prove the existence of the steady states by solving equation (4.4.3). We examine the solution for $\theta < 0.5$, $\theta = 0.5$, and $\theta > 0.5$ separately.

Suppose that $\theta < 0.5$. Then, $\frac{1}{1-\theta} < 2$. It follows that $\lim_{r\to\infty}\{g(r) - h(r)\} = \infty$. If $c > 0$, the function $g(r) - h(r)$ is negative on the interval $(\frac{\theta}{1-\theta}, 1)$. This implies that the function $g(r) - h(r)$ cuts the zero once on $[1, \infty)$. If $c < 0$, the function $g(r)$ and $h(r)$ are tangent to each other if $g'(r) = h'(r)$ and $g(r) = h(r)$. This is at $c = c^*(A, \theta)$. For $c < c^*(A, \theta)$, there are two intersection points on $(1, \infty)$.

Suppose that $\theta = 0.5$. The function $g(r) - h(r)$ can be reduced to $(r-1)^2 - \frac{4c}{A^2} r^2$. If $c > 0$, it has two roots if and only if $c \leq \frac{A^2}{4}$. The two real roots are on the oppositive side of 1. If $c < 0$, there is no solution for $g(r) - f(r) = 0$ for $r \in \mathbb{R}_+$.

Suppose that $\theta > 0.5$. If $c > 0$, the function $g(r)$ and $f(r)$ are tangent to each other if $g'(r) = h'(r)$ and $g(r) = h(r)$. This is at $c = c^*(A, \theta)$. For $0 < c < c^*(A, \theta)$, there are two intersection points on $(\frac{\theta}{1-\theta}, \infty)$. If $c < 0$, there is no solution for $g(r) - f(r) = 0$ for $r \in \mathbb{R}_+$.

□

Proof of Proposition 4.4.2

First we prove that 1) the eigenvalues of the Jacobian are real and positive and 2) det > 0 and tr > 0 in all steady states. Then to examine the stability properties of each steady state we have to evaluate 3) whether det > tr − 1 and det < 1.

1) Let the Jacobian matric of the dynamical system (4.3.13) be

$$J(k,p) = \begin{pmatrix} \dfrac{\partial G(k,p)}{\partial k} & \dfrac{\partial G(k,p)}{\partial p} \\ \dfrac{\partial \Psi(p,k)}{\partial k} & \dfrac{\partial \Psi(p,k)}{\partial p} \end{pmatrix} = \begin{pmatrix} w'(k) & -1 \\ pf''(k)w'(k) & f'(k) - pf''(k) \end{pmatrix}$$

Then, the determinant and the trace of $J(k,p)$ are $\det = w'(k)f'(k) > 0$ and $\mathrm{tr} = w'(k) + f'(k) - pf''(k) > 0$. It follows that

$$\begin{aligned} \mathrm{tr}^2 - 4\det &= (w'(k) + f'(k) - pf''(k))^2 - 4w'(k)f'(k) \\ &= (w'(k) - f'(k))^2 - 2(w'(k) + f'(k))f''(k)p + (pf''(k))^2 > 0. \end{aligned}$$

This implies that the eigenvalues are real and positive.

2) For the Cobb-Douglas production function notice that in steady state

$$\begin{aligned} f''(k)p &= (\theta-1)A\theta k^{\theta-2}p \\ &= (\theta-1)A\theta k^{\theta-1}k^{-1}p \\ &= (\theta-1)r\left(\frac{1-\theta}{\theta}r-1\right) \end{aligned}$$

where $\frac{p}{k}=\left(\frac{1-\theta}{\theta}r-1\right)$ in steady state. We obtain

$$\begin{aligned} \det &= (1-\theta)r^2>0 \\ \mathrm{tr} &= r+\frac{(1-\theta)^2}{\theta}r^2>0. \end{aligned}$$

It follows immediately that if $r<1$, $\det<1$. Since we know that $\det>0$, $\mathrm{tr}>0$ and the eigenvalues are real and positive, the stability depends on whether $\det-\mathrm{tr}+1 \lesseqgtr 0$ and $\det \lesseqgtr 1$. Figure 4.6.1 shows how the stability properties depend on the determinant and the trace. The following analysis identifies where each steady state lies on the non-shaded parts in the Figure 4.6.1.

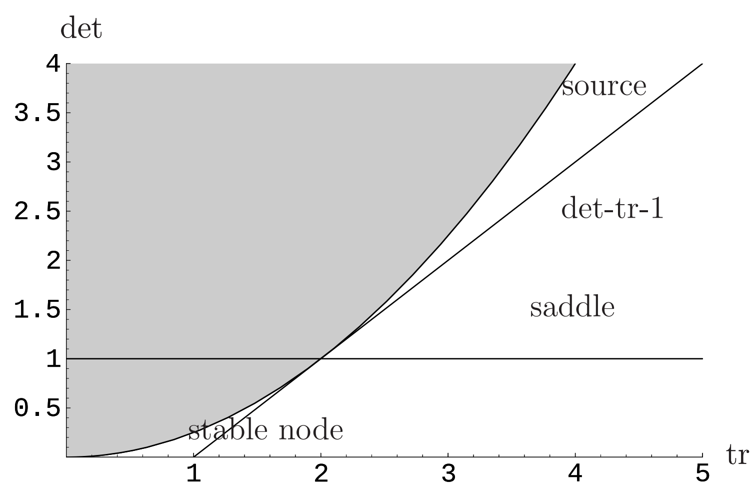


Figure 4.6.1: Stability properties

3) Let us define $\det-\mathrm{tr}+1$ as a function of the steady state interest rate,

$$D(r):=\det-\mathrm{tr}+1=1-r+(1-\theta)r^2\frac{(2\theta-1)}{\theta}. \tag{4.6.1}$$

Suppose that $\theta < 0.5$. Then, $D'(r) = -1 + 2(1-\theta)\left(\frac{2\theta-1}{\theta}\right) r < 0$, for $r \in \mathbb{R}_+$. If $r < 1$, we know that $r < \frac{\theta}{1-\theta}$, then

$$\begin{aligned} D(r) &= 1 - r - (1-\theta)r^2\left(\frac{1-2\theta}{\theta}\right) \\ &> 1 - \frac{\theta}{1-\theta} - (1-\theta)\left(\frac{\theta}{1-\theta}\right)^2\left(\frac{1-2\theta}{\theta}\right) \\ &> 1 - 2\theta > 0. \end{aligned}$$

Hence, the steady state where $r < 1$ for $\theta < 0.5$ is a stable node. If $r > 1$,

$$D(r) = 1 - r - (1-\theta)r^2\frac{(1-2\theta)}{\theta} < 0.$$

Hence, the steady state where $r > 1$ for $\theta < 0.5$ is a saddle.

Suppose that $\theta = 0.5$,

$$\begin{aligned} D(r) &= 1 - r + (1-\theta)r^2\frac{(2\theta-1)}{\theta} \\ &= 1 - r. \end{aligned}$$

Hence, $D(r) \lesseqgtr 0 \Leftrightarrow r \lesseqgtr 1$. Therefore, the steady state where $r < 1$ is a stable node and where $r > 1$ is a saddle for $\theta = 0.5$.

Suppose that $\theta > 0.5$. Then, $D'(r) > 0 \Leftrightarrow r > \frac{\theta}{2(1-\theta)(2\theta-1)}$ and $D''(r) = \frac{2(1-\theta)(2\theta-1)}{\theta} > 0$. If $r < 1$,

$$D(r) = 1 - r + (1-\theta)r^2\frac{(2\theta-1)}{\theta} > 0.$$

Hence, the steady state where $r < 1$ for $\theta > 0.5$ is a stable node. If $r > 1$, there exist two steady states for $c > c^*(A, \theta)$.

We know that at these steady states

$$r > \frac{\theta}{1-\theta}. \tag{4.6.2}$$

Suppose that the steady states are stable. Then, det < 1, which is equivalent to

$$r < \sqrt{\frac{1}{1-\theta}}. \tag{4.6.3}$$

Equations (4.6.2) and (4.6.3) imply that

$$\frac{\theta}{1-\theta} < \theta,$$

which is never satisfied for $\theta > 0.5$. Hence, the steady states where $r > 1$ and $\theta > 0.5$ are unstable.

The results from 1), 2), and 3) are summarized in Table 4.1.

	$\theta < 0.5$	$\theta = 0.5$	$\theta > 0.5$
$r < 1$	stable node	stable node	stable node
$r > 1$	saddle	saddle	saddle, source

Table 4.1: Stability properties with Cobb-Douglas function

□

Proof of Proposition 4.4.3

We prove the existence and stability of all positive steady states. We examine the case where 1) $k \geq d$ and then 2) $k < d$.

1) For $k \geq d$, the steady state is defined by

$$p = -\frac{c}{\delta} \tag{4.6.4}$$

$$p = Ad^2 - k. \tag{4.6.5}$$

This excludes any positive steady states (p, k) where $k > d$ and $p > 0$ for $c > 0$. If $c \leq 0$, there exists a unique positive steady state (p, k), if and only if $Ad^2 - d > -\frac{c}{\delta}$. The system in the neighborhood of the steady state is given by

$$p_1 = (1 - \delta)p - c \tag{4.6.6}$$

$$k_1 = Ad^2 - p. \tag{4.6.7}$$

The Jacobian is

$$J(p, k) = \begin{pmatrix} 1 - \delta & 0 \\ -1 & 0 \end{pmatrix}. \tag{4.6.8}$$

The determinant is zero and the trace is $1 - \delta$. The eigenvalues are 0 and $1 - \delta$. Thus the steady state where $k > d$ is stable.

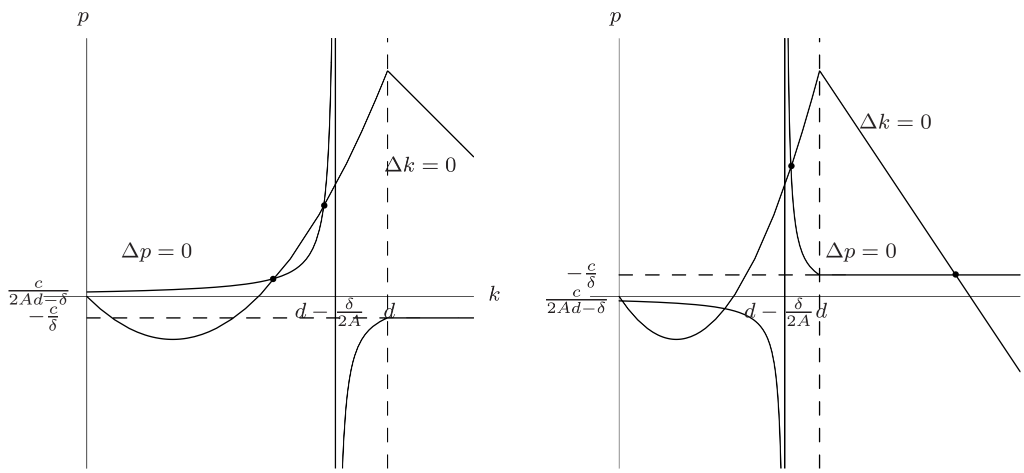
2) For $k < d$, the steady state is defined by

$$p = p\,(2A(d-k)+1-\delta) - c \tag{4.6.9}$$
$$p = Ak^2 - k. \tag{4.6.10}$$

The system in the neighborhood of the steady state is given by

$$p_1 = p\left(f'(Ak^2-p)+1-\delta\right) - c \tag{4.6.11}$$
$$k_1 = Ak^2 - p. \tag{4.6.12}$$

Figure 4.6.2 shows there exist at most two steady states if $c > 0$ and there exists always one steady state if $c \leq 0$ and $\delta(Ad^2 - d) > -c$.



(a) $c > 0$ (b) $c \leq 0$

Figure 4.6.2: Phase diagram for the closed economy

The Jacobian is

$$J(p,k) = \begin{pmatrix} 2A(d-k+p)+1-\delta & -4A^2pk \\ -1 & 2Ak \end{pmatrix}. \tag{4.6.13}$$

The determinant is $4A^2k(d-k)+2Ak(1-\delta) > 0$ and the trace is $2A(d+p)+1-\delta > 0$. Substituting Equation (4.6.10), the trace can be rewritten as $2A(d-k)+2A^2k^2+1-\delta$. From Equation (4.6.10) we know that at positive steady states $Ak > 1$. Thus, the trace is always greater than 2 at any positive steady states. Hence, all the steady states where $k < d$ are unstable. □

Chapter 5

A Two Country Model with International Mutual Funds

5.1 Introduction

How does international trading of assets influence capital accumulation of countries in an uncertain world? There are few models which provide us with an answer to this question. Acemoglu & Zilibotti (1997) augment the neoclassical growth model with the assumption that investment in risky projects is indivisible. They show that risk averse agents avoid risky investment which slows down capital accumulation. In addition, the inability to diversify idiosyncratic risk initially introduces a large amount of uncertainty in the growth process. The more the economy accumulates capital, the better it diversifies risk. Eventually, it converges to its steady state, in which all investment sectors are open and risk is completely diversified. Thus, they offer a theory of development that links the degree of market incompleteness to capital accumulation. Their results generalize to economies with international capital flows. Obstfeld (1994) extends the endogenous growth model by Romer (1990) and shows in a continuous time stochastic model that the possibility of world portfolio diversification can raise steady state growth, as individuals place a larger fraction of their wealth in risky but high-yielding capital investments. The implication of these two models is rather simple. That is, the possibility of the risk trading permits individuals to engage in risky activities that are more productive at the aggregate level. This naturally induces an equalizing force, which leads to convergence of incomes across countries.

This chapter examines the role of the nominal asset, which we introduced in Chapter 4 in a two country model. In contrast to the works mentioned above, the investment in the asset market diverts savings away from capital investment. The question is whether the portfolio decisions of utility maximizing agents bring about convergence of capital stocks between the two countries. The world consists of two homogeneous countries, which differ only in their initial levels of capital. International mutual funds are introduced where stochastic profits of firms in both countries constitute the dividends. Since young agents in both countries have different incomes in general, short selling is possible in the international asset market. This means that the poor country may take credits by short selling of assets, which induces trading of assets between generations as well as within a generation. International asset market brings about convergence of incomes between the two countries only if the risk adjusted dividend is negative and the initial conditions of the two countries are sufficiently high. If the risk adjusted dividend is positive and the initial condition of one country is sufficiently low while that of the other is sufficiently high, the two country diverge in the long run.

Boyd & Smith (1997) motivate their paper by referring to cyclicality of credit allocation between developing and developed economies in empirical data. However, their theoretical findings are confined to a dynamical equilibrium path displaying damped oscillation. In contrast in the present model fluctuations of international capital flows between the rich and the poor country occur endogenously in the long run. The closed economy model does not exhibit any fluctuations suggesting that the interactions in the international asset market generate endogenous fluctuations of international capital flows.

Perfect foresight models are often abandoned and real business cycles (RBC) models are adopted to integrate short term fluctuation into long rung growth analysis. The present model shows that it is a misconception that perfect foresight models can not explain short-term fluctuations. Fluctuations in RBC models are interpreted as propagation mechanism of exogenous shocks. This difference has different theoretical as well as political implications. While RBC models understand fluctuations as adjustment processes to a steady state, the present two country model suggests that fluctuations may be inherent in the structure of the international financial market.

The remainder of the paper is organized as follows. Section 5.2 then extends the closed economy model to a two country model. Section 5.3 analyzes the steady state properties of the two country model and compares the results with those from the closed economy model. Section 5.4 concludes.

5.2 Two country model

In this section we assume that the world economy consists of two countries inhabited by homogeneous consumers. The production technologies in both countries are assumed to be identical making the two countries distinguished only by the initial capital stock. The asset markets of the two countries are integrated into an international market, while there exists a capital market in each country. We assume that when young consumers buy assets in the international market, they do not distinguish between assets of the two countries. We also assume that consumers cannot invest in the capital market abroad. In other words, we rule out foreign direct investment. Therefore, agents affect the capital stock in the foreign country only through the international asset market. Different wage incomes in the two countries enable short selling of the international asset market. When the young agent sells assets short, he demands a negative amount of assets. This is as if he sold assets in the market by promising to buy the same amount of assets back in the next period. This trade can obviously take place only within a generation. Since there are homogenous agents in each country, short selling can only take place between agents from different countries. In this case, the international asset market serves as an international credit market inducing trading of consumption commodities across countries. The young agents with positive demand buy assets from the old agents.

5.2.1 Temporary Equilibrium in the International Asset Market

Suppose that there exist international assets composed of assets in the two countries which pay a dividend[1] of

$$d = \frac{\varepsilon^1 + \varepsilon^2}{2}. \tag{5.2.1}$$

Since the productivity shocks in two countries are both i.i.d. random variables drawn from the same distribution, the first and second moment of d will be

$$\mathbb{E}[d] = \mathbb{E}\left[\frac{\varepsilon^1 + \varepsilon^2}{2}\right] = \mathbb{E}[\varepsilon^1] = \mathbb{E}[\varepsilon^2] \tag{5.2.2}$$

and

$$\mathbb{V}[d] = \mathbb{V}\left[\frac{\varepsilon^1 + \varepsilon^2}{2}\right] = \frac{1}{2}\mathbb{V}[\varepsilon^1] = \frac{1}{2}\mathbb{V}[\varepsilon^2] \tag{5.2.3}$$

respectively. We assume rational expectations for the future dividend and the linear mean variance utility function as in the closed economy model. Then, we obtain the asset demand function of young consumers at $t = 0$ given by[2]

$$x^i = \varphi(p, p^e, R^{ie}, k^i) := \mathrm{Min}\left(\frac{p^e + \mathbb{E}[d] - R^{ie}p}{\alpha\mathbb{V}[d]}, \frac{w(k^i)}{p}\right), \text{for } i = 1, 2. \tag{5.2.4}$$

If we assume that the supply of assets is constant and normalized to two, the price law $p = S(p^e, R^{1e}, R^{2e}, k^1, k^2)$ is implicitly defined by the solution of

$$\varphi(p, p^e, R^{1e}, k^1) + \varphi(p, p^e, R^{2e}, k^2) = 2. \tag{5.2.5}$$

Proposition 5.2.1 *There exists a unique positive equilibrium price*

$$p = S(p^e, R^{1e}, R^{2e}, k^1, k^2) \tag{5.2.6}$$

if the risk adjusted expected cum-dividend price is greater than zero, i.e., $p^e + c > 0$.

---

[1]The random variable d should not be confused with the parameter d of the quadratic production function. In what follows the random variable d will appear only as $\mathbb{E}[d]$ and $\mathbb{V}[d]$.

[2]For ease of notation we suppress the superscript $i = 1, 2$ denoting the individual country whenever we only focus on the mathematical properties.

Proof: The assertion in Proposition 5.2.1 is obvious as the asset demand function $\varphi(p, p^e, R^e, k)$ is decreasing in p, $\varphi(0, p^e, R^e, k) = \frac{p^e + \mathbb{E}[\varepsilon]}{\alpha \mathbb{V}[\varepsilon]}$ and $\lim_{p \to \infty} \varphi(p, p^e, R^e, k) = -\infty$. □

In the overlapping generations structure, all assets in the market are purchased by young agents in the economy. In the two country model available assets in the market are purchased by young agents in both countries. Therefore, the amount of assets purchased by young agents in one country is no longer equal to the available assets in the market as it was the case in the closed economy model. Therefore, the next period capital in each country $i = 1, 2$ is now dependent on the asset demand in each country and is given by

$$k_1^i = w(k^i) - \varphi(p, p^e, R^{ie}, k^i)p. \tag{5.2.7}$$

5.2.2 Expectations and Dynamical System

To describe the complete dynamical system we have to define how the young agents form their expectations about the return on capital and the next period's asset price. Let us first define the perfect predictor for returns on capital R_1^i for $i = 1, 2$ and then we will see under what conditions there exists a perfect predictor for the next period asset price p_1, which is consistent with the perfect foresight on R_1^i. The return on capital in $t = 1$ in each country $i = 1, 2$ is given by

$$R_1^i = R(k^i, \varphi(p, p^e, R^{ie}, k^i), p) := f'(w(k^i) - \varphi(p, p^e, R^{ie}, k^i)p) + 1 - \delta. \tag{5.2.8}$$

The perfect foresight for the returns on capital requires that $R_1^i = R^{ie}$, which is equivalent to

$$R^{ie} = f'(w(k^i) - \varphi(p, p^e, R^{ie}, k^i)p) + 1 - \delta. \tag{5.2.9}$$

Notice that the perfect predictor $R^{ie} = \mathcal{R}(k^i, p^e, p)$ is only implicitly defined by the solution of equation (5.2.9). The following lemma proves the existence.

Lemma 5.2.1 *Suppose that $(k^i, p^e, p) \in \mathbb{R}_+^3$ and Assumptions 4.2.1 and 4.3.1 are satisfied.*

1. *There exists a unique perfect predictor* $R^{ie} = \mathcal{R}(k^i, p^e, p)$ *which solves the equation (5.2.9).*

2. *Given the perfect predictor* $\mathcal{R}$, *we always obtain an interior asset demand, i.e.,* $\varphi(p, p^e, R^{ie}, k^i) < \frac{w(k^i)}{p}$.

See the appendix for a proof. □

Lemma 5.2.1 ensures an interior equilibrium in the asset market, in which young agents do not invest their entire income. This is because the return on capital tends to infinity as the asset demand tends to $\frac{w(k^i)}{p}$. Given the perfect predictor $\mathcal{R}$, we can now write the asset demand which is consistent with the perfect foresight for the returns on capital investment as

$$\xi(p, k^i, p^e) := \varphi(p, p^e, \mathcal{R}(k^i, p^e, p), k^i) = \frac{p^e + \mathbb{E}[d] - \mathcal{R}(k^i, p^e, p)p}{\alpha \mathbb{V}[d]}. \tag{5.2.10}$$

The setting $p^e_{-1} = p$, the perfect predictor for the asset price $\Psi(p^e_{-1}, k^1, k^2)$, which is consistent with the perfect foresight for the return on capital, has to simultaneously satisfy

$$p = p^e_{-1} \tag{5.2.11}$$

$$\xi(p, k^1, p^e) + \xi(p, k^2, p^e) = 2. \tag{5.2.12}$$

Proposition 5.2.2 *Let* $\hat{\mathcal{D}} := \{(p^e_{-1}, k^1, k^2) | p^e_{-1} \in [0, \min\{w(k^1), w(k^2)\}), (k^1, k^2) \in \mathbb{R}^2_+\}$ *and Assumptions 4.2.1 and 4.3.1 be satisfied.*

1. *There exists a unique perfect predictor, which is consistent with the perfect foresight for the return on capital* $\mathcal{R}$ *given by*

$$\Psi : \hat{\mathcal{D}} \to \mathbb{R}, (p^e_{-1}, k^1, k^2) \mapsto \Psi(p^e_{-1}, k^1, k^2) \tag{5.2.13}$$

2. *The perfect predictor is positive if and only if* $\xi(p^e_{-1}, k^1, 0) + \xi(p^e_{-1}, k^2, 0) < 2$.

See the appendix for a proof. □

Given the existence of the perfect predictors $(\Psi, \mathcal{R})$ the dynamical system of the two country model under rational expectations is characterized by

$$\begin{aligned} k_1^1 &= \Phi^1(p,k^1,k^2) := w(k^1) - p\left(1 - \tfrac{\mathcal{R}(k^1,\Psi(p,k^1,k^2),p) - \mathcal{R}(k^2,\Psi(p,k^1,k^2),p)}{2\alpha\mathbb{V}[d]} \cdot p\right) \\ k_1^2 &= \Phi^2(p,k^1,k^2) := w(k^2) - p\left(1 - \tfrac{\mathcal{R}(k^2,\Psi(p,k^1,k^2),p) - \mathcal{R}(k^1,\Psi(p,k^1,k^2),p)}{2\alpha\mathbb{V}[d]} \cdot p\right) \quad (5.2.14) \\ p_1 &= \Psi(p,k^1,k^2) := \tfrac{\mathcal{R}(k^1,\Psi(p,k^1,k^2),p) + \mathcal{R}(k^2,\Psi(p,k^1,k^2),p)}{2} \cdot p - c. \end{aligned}$$

The dynamical system (5.2.14) shows the link between the international asset market and the capital accumulation in each country.

Suppose that $k^1 > k^2$, then $\mathcal{R}(k^1, \Psi(p, k^1, k^2), p) < \mathcal{R}(k^2, \Psi(p, k^1, k^2), p)$. This implies that the investment of country 1 in international mutual funds is greater than 1 and the investment of country 2 is less than 1. This means that country 1 accumulates less capital than country 2 inducing a convergence force.

Proposition 5.2.3 *There exist positive symmetric steady states under rational expectations which coincide with the positive steady states of the closed economy.*

The proof follows directly from the dynamical system (5.2.14). □

Whether the system converges to the symmetric steady state will depend on the interaction between the capital stock in each country and the asset price. In other words, how the total spending px on international mutual funds evolves with the capital stock is essential for the dynamics of the two countries.

5.3 Dynamics of the Two Country Model

Section 5.2.2 showed that the perfect predictor for the asset price is only defined on a subset of $\mathbb{R}^3_+$. The question arises whether there exists a forward invariant set of the dynamical system (5.2.14). Section 4.4 showed that the dynamical system of the closed economy has a forward invariant set for a large set of initial condition if we use the quadratic production function. Moreover, it was shown that the stable steady state is unique in the closed economy. The perfect predictor $\mathcal{R}$ for the return on capital is only implicitly defined in the dynamical system (5.2.14). The existence and the stability property of asymmetric steady states would obviously depend on the specification of the production function. Even for the Cobb-Douglas case, it is too difficult to characterize the steady states of the three dimensional system. However, the property of the quadratic production function not only allows us to obtain a closed from solution of the model but also reduces the dimension of the system to two in some cases which enable us to investigate the existence of the asymmetric steady state and the stability property of the system numerically. Hence, we use the quadratic production function in the following analysis. The quadratic function violates one of the Inada conditions has a consequence on the model structure. Remember that the asset demand was never constrained by income in Section 5.2. This was because the return on capital investment tends to infinity as agents invest more and more in the asset market. This result rests on the assumption that $\lim_{k \to 0} f'(k) = \infty$. Without this assumption, we need to consider the following three cases 1) the budget constraints are binding in both countries, 2) the budget constraint is binding only in one country, 3) the budget constraints are not binding in either countries. The derivation of the asset demand function φ and the perfect predictors $(\Psi, \mathcal{R})$ can be found in appendix.

5.3.1 Multiple Steady States

From Proposition 5.2.3 we know that the symmetric steady state of the two country model is identical to the steady state of the closed economy model. Therefore, the existence of the symmetric steady state is already shown by Proposition 4.4.3. The following proposition gives the condition when the two countries converge to the symmetric steady state.

Proposition 5.3.1

1. *There exists a positive symmetric steady state* $k^1 = k^2 = Ad^2 + \frac{c}{\delta}$ *if* $c \leq 0$ *and* $\delta(Ad^2 - d) > -c$.

2. *If* $k^1 = k^2$ *or* $k^1, k^2 > d$*, the two countries converge to this symmetric steady state for initial values in its neighborhood.*

See the appendix for a proof. □

If the two countries have identical initial conditions, there are no financial transactions between them and the economy follows the path of the closed economy. If the two countries have initial capital stocks which exceed the critical value d, they will have an identical law of accumulation. Therefore, the dynamics becomes that of the closed economy.

Definition 5.3.1 *We call an asymmetric steady state an interior steady state if the budget constraints are not binding for the asset demand in either rich and poor country and if there are financial transactions between two countries.*

Proposition 5.3.2 *There exists an interior asymmetric steady state in which* $k^2 < d < k^1$ *and* $x^2 < 0 < x^1$.

See the appendix for a proof. □

Generally, the asset demand of the poor country is always lower than that of the rich country since the asset demand function is increasing in k. Proposition 5.3.2 implies that $w(k^1) > I(k^1) > I(k^2) > w(k^2)$ at the asymmetric steady state where $I(k^i) := w(k^i) - px^i, \forall i = 1, 2$ denotes the capital investment in each country. This means that the poor country requires external finance from the rich country in form of short selling in the international asset market for its capital investment.

5.3.2 Nonconvergence and Inequality of Nations

To analyze the stability properties of the steady states we rely on numerical simulation in this section. The quadratic production function is used throughout the numerical analysis. To obtain rational expectations for the next period dividend, the following assumption is made about the random variable d.

Assumption 5.3.1 *We assume that the random variable* ε *has a uniform distribution on the interval* $[a, b]$. *The probability density function for a continuous uniform distri-*

bution on the interval $[a, b]$ *is*

$$P(\varepsilon) = \begin{cases} 0 & \text{if } d < a \\ \frac{1}{b-a} & \text{if } a \leq d \leq b \\ 0 & \text{if } d > b \end{cases} \tag{5.3.1}$$

with mean $\frac{a+b}{2}$ *and variance* $\frac{(b-a)^2}{12}$.

The standard parameter set in Table 5.1 will be used unless it is otherwise indicated.

A	d	$\mathbb{E}[d]$	$\mathbb{V}[d]$	α	c	k_0^1	k_0^2	p_0
0.5	3.2	0.5	0.25	1	0.25	3	1	0.4

Table 5.1: Standard parameter set

To analyze the sensitivity of the dynamical system with respect to initial conditions Figure 5.3.1 shows the typical basin of attraction for the asymmetric steady states for a negative and a positive c. The shaded area color depicts initial conditions for which two countries converge to the respective steady state and the white area those for which the dynamical system explodes.

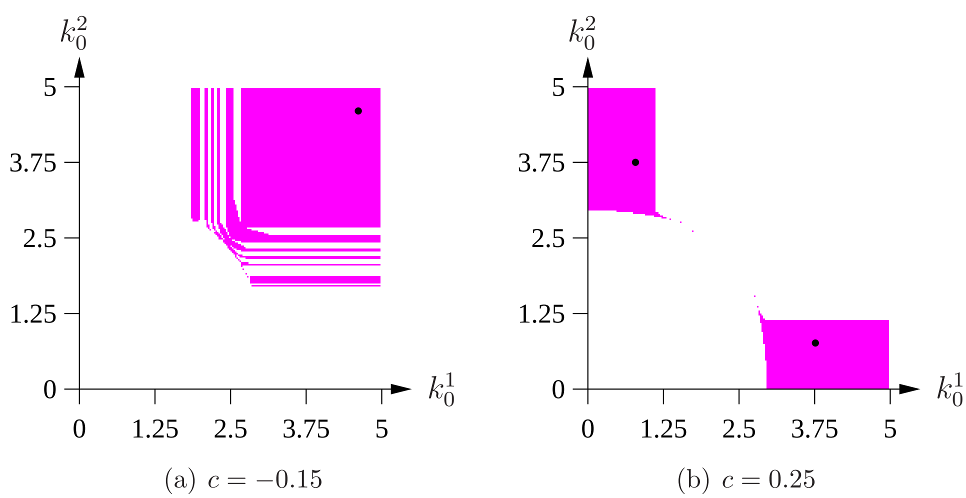


Figure 5.3.1: Basin of attraction for the asymmetric steady states: $\delta = 0.625$

Figure 5.3.1 shows that the asymmetric steady state $k^1 < d < k^2$ described in Proposition 5.3.2 is stable for $c > 0$.[3] The stability of the interior steady state suggests that unconstrained optimal behavior at individual level under rational expectations does not necessarily lead to convergence of income between the two countries even in absence of any imperfections in the markets. We know from Proposition 5.3.1 that there exists no positive symmetric steady state where $k > d$ if $c > 0$ and the steady state of the closed economy is stable only if $k > d$ and $c < 0$. Even in the two country model this stability property seems to hold. This means that the feedback mechanism between the capital stocks of the two countries through the asset price does not alter the stability properties of the steady states of the closed economy. In particular, the steady states of the closed economy where $k < d$ remain unstable. Moreover, Figure 5.3.1 suggests that stable asymmetric steady states do not coexist with a stable symmetric steady state. Put it differently, this suggests that the risk adjusted expected dividend c plays a crucial role on whether we observe convergence or divergence of the two countries. To summarize we observe that there exists a forward invariant set of dynamical system (5.2.14) which is consistent with rational expectations where initially poor and rich countries diverge if $c > 0$ and converge if $c \leq 0$. This statement should be treated with caveat. Especially, it does not mean that whether the two countries converge or diverge depends on c. Notice that there is no overlap of the basins of attraction for a positive and a negative c. Whether we obtain a forward invariant set depends on the initial conditions in each case. Only if the initial conditions of the two countries are sufficiently high and the risk adjusted dividend is negative, the two countries converge to each other. If the initial condition of one country is sufficiently low and that of the other sufficiently high, the two countries diverge in the long run.

Let us look at how the risk adjusted dividend c influences the equilibrium asset price. The equilibrium asset price in the steady state of the closed economy is negative for $c > 0$ and $k > d$ since

$$p = -\frac{c}{\delta - f'(k)}.$$

where $f'(k) = 0$ if $k > d$. This is not necessarily the case in the two country case since the equilibrium asset price is dependent on the return on capital investment in both

[3] The numerical simulation shows that there exists an open parameter set for which this interior steady state is stable.

countries so that

$$p = -\frac{2c}{2\delta - f'(k^1) - f'(k^2)}.$$

Notice that even if $k^1 > d$ and therefore $f'(k^1) = 0$, the equilibrium asset price is not necessarily negative for $c > 0$ if $k^2 < d$. This is in particular the case at the asymmetric interior steady state. In contrast, for a positive steady state to exist for a negative c,

$$f'(k^1) + f'(k^2) < 2\delta.$$

This means that both countries need to have high capital stock. For a positive c, Figure 5.3.1 shows that the two countries converge to a symmetric steady state if the initial conditions of the two countries are sufficiently high. The following proposition states the implication of the asymmetric steady state for the inequality of the two countries.

Proposition 5.3.3 *At the interior steady state the poor country has a higher capital stock while the rich country has a lower capital stock than at the low and the high steady state in the economy without the asset market respectively.*

Proof: Suppose that $k^2 < k^1$. From Proposition 5.3.2, we know $x^1 > 0 > x^2$ at the interior asymmetric steady state. The capital accumulation laws in both poor and rich countries at the asymmetric steady state are given by

$$k^1 = Ad^2 - px^1 \tag{5.3.2}$$
$$k^2 = A(k^2)^2 - px^2. \tag{5.3.3}$$

Equation (5.3.2) and (5.3.3) imply that $0 < k^2 < k^1 < Ad^2$. Suppose that there exists no asset market. Then the evolution of capital in the economy is given by

$$k_1 = w(k) = \begin{cases} Ak^2 & \text{if } k < d \\ Ad^2 & \text{if } k \geq d. \end{cases} \tag{5.3.4}$$

If $Ad > 1$, the economy without an asset market has three steady states, $0, 1/A$, and Ad^2. The steady state $1/A$ is unstable since the function $w(k)$ cuts the 45 degree line from below. Hence, the economy with $k_0 < \frac{1}{A}$ converges to zero while the economy with $k_0 > \frac{1}{A}$ converges to Ad^2. □

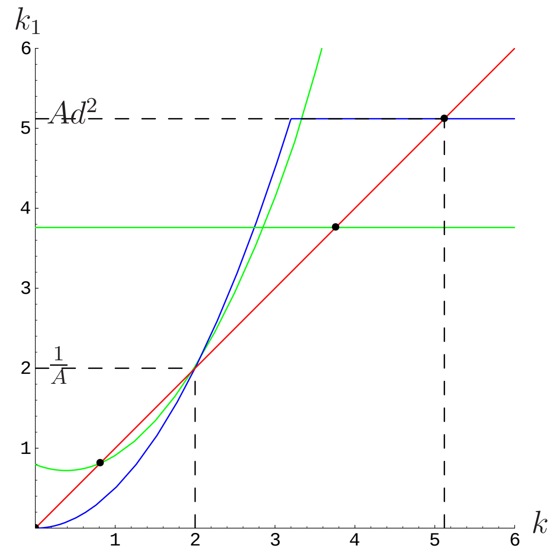


Figure 5.3.2: Time one maps given steady state asset demands: $\alpha = 1$.

Let us examine the result of Proposition 5.3.3 by comparing equation (5.3.2), (5.3.3), and (5.3.4). Figure 5.3.2 depicts the map (5.3.2) by the light colored horizontal line, the map (5.3.3) by the light colored curve, and the map (5.3.4) by the dark colored curve.

Figure 5.3.2 shows that the high steady state of the economy without an asset market $k = Ad^2$ shifts down while the low steady state 0 shifts up. The mechanism behind Proposition 5.3.3 is build on two aspects of the model. Firstly, the map (5.3.3) has a positive intercept at $k^2 = 0$ because $x^2 < 0$. Secondly, the multiple steady states arise from the convexity of the wage function in the map (5.3.3). On one hand, the poor country takes credits to invest capital in domestic production through short selling of assets in the international asset market, which constitutes an equalizing force. On the other hand, the non concavity of the wage function induces an unequalizing force since the initial difference in capital stocks between two countries leads to an even larger difference in their wages. The interaction of these two mechanisms supports the existence and the stability of the interior asymmetric steady state.

5.3.3 Endogenous Fluctuations of International Capital Flows

Let us examine the stability of the interior steady state from a more global viewpoint. Figure 5.3.3 shows a bifurcation diagram with respect to the depreciation rate δ displaying the limiting behavior of both state variables k^1 and k^2.

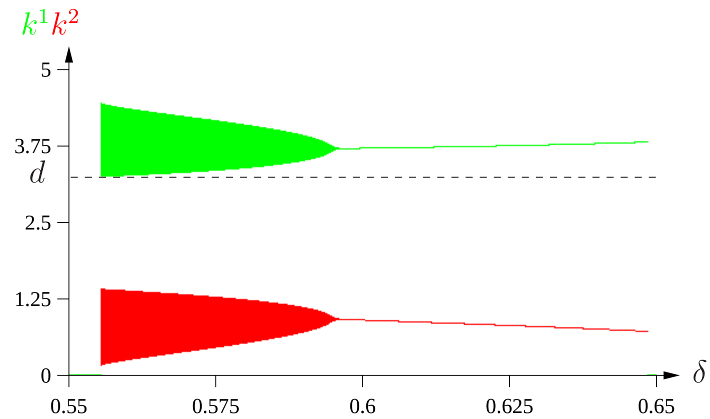


Figure 5.3.3: Bifurcation diagram

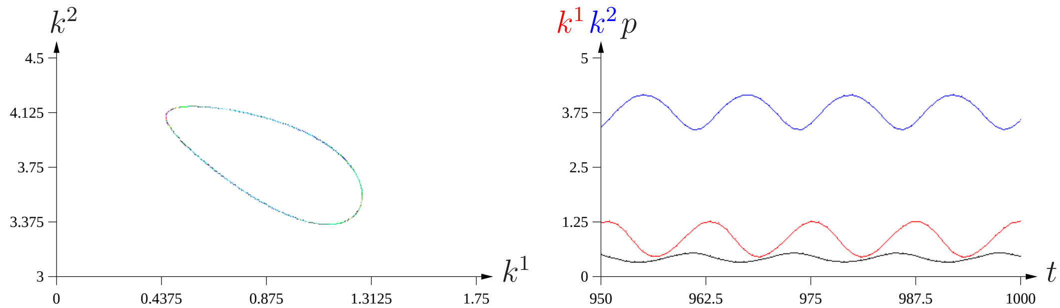
The figure confirms the existence of the stable asymmetric steady state where $k^2 < d < k^1$. One can observe that as the depreciation rate δ decreases, the steady state undergoes a bifurcation. The following proposition characterizes the bifurcation.

Proposition 5.3.4 *The interior asymmetric steady state $k^2 < d < k^1$ undergoes a supercritical Neimark-Sacker bifurcation.*

See the appendix for a proof. □

Figure 5.3.4 (a) shows a closed invariant curve which appears after the bifurcation point and Figure 5.3.4 (b) shows the corresponding time series. Figure 5.3.3 shows that the invariant curve around k^1 touches d if we further decrease δ. This means that the dynamical system switches from the case where $k^2 < d < k^1$ to the case where $k^1, k^2 < d$ causing the invariant curve to become unstable.

The closed economy model did not generate endogenous fluctuations. This suggests that the interaction between the two economies generates fluctuations in capital flows between the rich and the poor country endogenously. This result can be taken as an



(a) A closed invariant curve

(b) Time Series

Figure 5.3.4: Endogenous fluctuations: $\delta = 0.575$

evidence that fluctuations observed in international financial markets may occur under rational expectations even in absence of any exogenous shocks or imperfections in the economy.

5.4 Concluding Remarks

The conventional view of the implications of an international asset market in the presence of uncertainty is rather simple. With access to a larger market, countries can better diversify their risks and can be engaged in more efficient production. The two underlining aspects of this view is that 1) a larger market provides better opportunities for risk diversification 2) riskier projects are more productive. While we also kept the first aspect in our model, we diverted from the second aspect. We assumed that there exist nominal assets which are not productive but can be traded in the market. The firms pay stochastic profits as dividends and the young consumers choose an optimal portfolio to transfer their wealth over time. Since young agents in both countries have different incomes in general, short selling is possible in the international asset market. In other words, trading of assets takes place between generations as well as within a generation. Capital flows from rich to poor countries because the international asset market is more attractive to agents in the rich country where the rate of return in the domestic capital market is relatively low. However, the model shows that the optimal behavior at the individual level does not necessarily lead to convergence of incomes between the two countries. This result should be treated with caveat. We made a rather restrictive assumption that the asset market is the only market which allows for transactions between the two countries. This allows us to fucus on a particular aspect of the asset market that trading is subject to price expectations. The non-concavity of the wage function generated steady states, in which risk adjusted returns from the asset market while one country selling assets short are equalized for different capital stocks in two countries. The model showed that the associated risk in the asset market plays a decisive role on whether convergence or divergence prevails depending on the initial conditions of the two countries.

The result on divergence can be contrasted to the findings in Chapter 3. The asymmetric steady states do not emerge due to an enforcement problem in the financial market. In contrast, they arise due to the availability of trading of additional unproductive assets without any imperfection in the market. While consumers in the poor country in Chapter 3 face a borrowing constraint, they hold an optimal portfolio, which is an interior solution in the present paper. This induces capital flows from the rich to the poor country while the capital flows are reversed at the asymmetric steady state in the financial market with imperfections. The capital flows from the rich to the poor country is empirically more plausible. The deviation of the result in the present paper from that in Boyd & Smith (1997) and Matsuyama (2004) has different implications for the inequality of nations. While the poor country, trading with the rich country, is worse off in terms of income per capita in models with financial imperfections, the relationship is reversed in the present model with an additional asset market. The result on endogenous fluctuation offers a new insight into the nature of the integrated economies too. The closed economy model in Chapter 4 did not exhibit any fluctuations. This suggests that interactions in the international asset market generate fluctuations of economies. Financial market globalization may be accompanied by increasing volatility of the market and by periodic and cyclical reoccurrence of financial crisis without any exogenous shocks. This may provide an additional explanation to phenomena which can not be fully understood by a propagation mechanism of exogenous shocks.

5.5 Appendix

The Asset Demand: The Quadratic Production Function Case

Let $q^e := p^e + \mathbb{E}[d]$ be the expected cum-dividend price. The unconstrained asset demand function of each country which is consistent with the perfect foresight for the return on capital is implicitly defined by the solution $\bar{\xi}(p, k, p^e)$ of

$$x = \frac{q^e - (f'(w(k) - px) + 1 - \delta)p}{\alpha \mathbb{V}[d]}.$$

The asset demand function of each country can be defined as the following.

If $w(k) < d$,

$$x = \bar{\varphi}(p, p^e, k) := \begin{cases} \frac{q^e - R(w(k))p}{\alpha \mathbb{V}[d] + 2Ap^2} & \text{if } q^e < b(k, p) \\ \frac{w(k)}{p} & \text{if } q^e \geq b(k, p) \end{cases} \tag{5.5.1}$$

where $R(k) := 2A(d - k) + 1 - \delta$ with slight abuse of notation and $b(k, p) := \frac{w(k)\alpha \mathbb{V}[d]}{p} + 2Adp + (1 - \delta)p > 0$ denotes the expected cum dividend price above which the budget constraint is binding.

If $w(k) \geq d$,

$$x = \bar{\varphi}(p, p^e, k) := \begin{cases} \frac{q^e - (1-\delta)p}{\alpha \mathbb{V}[d]} & \text{if } q^e < z(k, p) \\ \frac{q^e - R(w(k))p}{\alpha \mathbb{V}[d] + 2Ap^2} & \text{if } z(k, p) \leq q^e < b(k, p) \\ \frac{w(k)}{p} & \text{if } b(k, p) \leq q^e \end{cases} \tag{5.5.2}$$

where $z(k, p) := \frac{\alpha \mathbb{V}[d](w(k) - d)}{p} + (1 - \delta)p.$ □

The Perfect Predictor: The Quadratic Function Case

Let the world capital investment be denoted by $I(k^2, k^2) := w(k^1) + w(k^2) - 2p$.

The Case $w(k^1) \leq w(k^2) < d$

If $q^e \geq b(k^i, p), \forall i = 1, 2.$, the budget constraint is binding and irrespective to expected future price the price law is

$$p = \frac{w(k^1) + w(k^2)}{2}.$$

From equation (5.5.1), we know there are two possible cases.

$$p_1 = \Psi(p, k^1, k^2) :=$$

$$\begin{cases} 2Ap^2 + \frac{R(w(k^1))+R(w(k^2))}{2} p - c & \text{if } q^e < b(k^1, p) \\ R(I(k^2, k^2))p + \alpha \mathbb{V}[d] \left(1 - \frac{w(k^1)}{p}\right) - c & \text{if } b(k^1, p) \leq q^e < b(k^2, p). \end{cases}$$

The case $w(k^2) \leq w(k^1) < d$ can be obtained analogously.

The Case $d \leq w(k^1) \leq w(k^2)$

If $b(k^i, p) \leq q^e, \forall i = 1, 2.$, the budget constraint is binding and irrespective to expected future price the price law is

$$p = \frac{w(k^1) + w(k^2)}{2}.$$

From equations (5.5.2), we know there are five possible cases.

$$p_1 = \Psi(p, k^1, k^2) :=$$

$$\begin{cases} (1-\delta)p - c & \text{if } q^e < z(k^1, p) \\ 2Ap^2 + \frac{R(w(k^1))+R(w(k^2))}{2} p - c & \text{if } z(k^2, p) \leq q^e < b(k^1, p) \\ \frac{\alpha \mathbb{V}[d] A(d - w(k^2) + p)p}{\alpha \mathbb{V}[d] + Ap^2} + (1-\delta)p - c & \text{if } q^e < z(k^2, p) \wedge z(k^1, p) \leq q^e < b(k^1, p) \\ \left(1 - \frac{w(k^1)}{p}\right) \alpha \mathbb{V}[d] + (1-\delta)p - c & \text{if } b(k^1, p) \leq q^e < z(k^2, p) \\ R(I(k^1, k^2))p + \alpha \mathbb{V}[d] \left(1 - \frac{w(k^1)}{p}\right) - c & \text{if } b(k^1, p) \leq q^e \wedge z(k^2, p) \leq q^e < b(k^2, p). \end{cases}$$

The case $d \leq w(k^2) \leq w(k^1)$ can be obtained analogously.

The Case $w(k^1) < d \leq w(k^2)$

If $b(k^i, p) \leq q^e, \forall i = 1, 2.$ the budget constraint is binding and irrespective to expected future price the price law is

$$p = \frac{w(k^1) + w(k^2)}{2}.$$

We do not have to consider the case $b(k^2, p) \leq q^e < b(k^1, p)$. $q^e < b(k^1, p)$ can be rewritten as $q^e < \frac{w(k^1)\alpha\mathbb{V}[d]}{p} + 2Adp + (1-\delta)p \leq b(k^2, p)$ which is a contradiction.

From equation (5.5.1) and (5.5.2), we know there are four possible cases.

$p_1 = \Psi(p, k^1, k^2) :=$

$$\begin{cases} \frac{\alpha\mathbb{V}[d]A(d-w(k^1)+p)p}{\alpha\mathbb{V}[d]+Ap^2} + (1-\delta)p - c & \text{if } q^e < b(k^1, p) \wedge q^e < z(k^2, p) \\ 2Ap^2 + \frac{R(w(k^1))+R(w(k^2))}{2}p - c & \text{if } q^e < b(k^1, p) \wedge z(k^2, p) \leq q^e \\ \left(1 - \frac{w(k^1)}{p}\right)\alpha\mathbb{V}[d] + (1-\delta)p - c & \text{if } b(k^1, p) \leq q^e \wedge q^e < z(k^2, p) \\ R(I(k^1, k^2))p + \alpha\mathbb{V}[d]\left(1 - \frac{w(k^1)}{p}\right) - c & \text{if } b(k^1, p) \leq q^e \wedge z(k^2, p) \leq q^e < b(k^2, p). \end{cases}$$

The case $w(k^2) < d \leq w(k^1)$ can be obtained analogously. □

Proof of Lemma 5.2.1

The left hand side of the equation (5.2.9) is the identity. The right hand side is a decreasing function in R^{ie} since $\frac{\partial}{\partial R^{ie}}\varphi(p, p^e, R^{ie}, k^i) < 0$ and $\lim_{R^{ie}\to\infty} f'(w(k^i) - \varphi(p, p^e, R^{ie}, k^i)p) = 0$. In addition, $f'(w(k^i) - \varphi(p, p^e, 0, k^i)p) + 1 - \delta > 0$. This proves the unique existence. Given the perfect predictor $\mathcal{R}$, the perfect foresight for the return on capital R^{ie} tends to infinity as the asset demand tends to $\frac{w(k^i)}{p}$. The utility function, which is increasing in future wealth, guarantees that the young agent will not invest the entire income in the asset market. □

Proof of Proposition 5.2.2

The prefect predictor is defined by equation (5.2.11). The right hand side is a positive constant. We show that the left hand side is an increasing function in p^e, which ensures a unique solution.

$$\frac{\partial}{\partial p^e}\xi(p^e_{-1},k,\cdot)=\frac{1}{\alpha\mathbb{V}[d]}\left(1-p^e_{-1}\frac{\partial}{\partial p^e}\mathcal{R}(k,\cdot,p^e_{-1})\right)$$

Thus, $\frac{\partial}{\partial p^e}\xi(p^e_{-1},k,\cdot)>0$ if and only if $p^e_{-1}\frac{\partial}{\partial p^e}\mathcal{R}(k,\cdot,p^e_{-1})<1$. By the implicit function theorem,

$$\begin{aligned}\frac{\partial}{\partial p^e}\mathcal{R}(k,\cdot,p^e_{-1}) &= -\frac{\frac{\partial}{\partial p^e}G(R^e,\cdot,k,p^e_{-1})}{\frac{\partial}{\partial R^e}G(\cdot,p^e,k,p^e_{-1})}\\ &= -\frac{f''(w(k)-\varphi(p^e_{-1},p^e,R^{ie},k^i)p^e_{-1})\frac{p^e_{-1}}{\alpha\mathbb{V}[d]}}{1-f''(w(k)-\varphi(p^e_{-1},p^e,R^{ie},k^i)p^e_{-1})\frac{(p^e_{-1})^2}{\alpha\mathbb{V}[d]}}\end{aligned}$$

where $G(R^e,p^e,k,p^e_{-1}):=R^e-f'(w(k)-\varphi(p^e_{-1},p^e,R^e,k)p^e_{-1})-1+\delta$.

Thus, $p^e_{-1}\frac{\partial}{\partial p^e}\mathcal{R}(k,\cdot,p^e_{-1})<1$ implies that $0<1$. Therefore, $\frac{\partial}{\partial p^e}\xi(p^e_{-1},k,\cdot)>0$. This ensures a unique solution $p^e=\Psi(p^e_{-1},k^1,k^2)$ defined by the solution of equation (5.2.11). If ξ is increasing in p^e and $\xi(p^e_{-1},k^1,0)+\xi(p^e_{-1},k^2,0)>2$, the solution p^e is obviously negative. □

Proof of Proposition 5.3.1

We show that the two countries have an identical law of accumulation if $k^1=k^2<d$ or $k^1,k^2\geq d$. Then, the dynamics follows that of the closed economy. If $k^1=k^2<d$, the dynamical system reduces to a two dimensional system given by

$$\begin{aligned}k_1 &= Ak^2-p\\ p_1 &= f'(Ak^2-p)p+(1-\delta)p-c.\end{aligned}$$

If $k^1,k^2\geq d$, the dynamical system reduces to a two dimensional system given by

$$\begin{aligned}k_1 &= Ad^2-p\\ p_1 &= (1-\delta)p-c.\end{aligned}$$

From Proposition 4.4.3 we know that there exists the stable steady state $Ad^2 - \frac{c}{\delta}$ if and only if $c \leq 0$ and $\delta(Ad^2 - d) > -c$. □

Proof of Proposition 5.3.2

Suppose that $k^2 < d < k^1$ in steady state. Then the steady state is defined by

$$k^1 = Ad^2 - p\left(1 + \frac{pA(d-k^2)}{\alpha\mathbb{V}[d]}\right) \quad (5.5.3)$$

$$k^2 = A(k^2)^2 - p\left(1 - \frac{pA(d-k^2)}{\alpha\mathbb{V}[d]}\right) \quad (5.5.4)$$

$$p = p(A(d-k^2) + 1 - \delta) - c \quad (5.5.5)$$

First we show the existence of the steady state for $c > 0$ and $Ad > \delta$. Then, we show that in the steady state, $x^2 < 0 < x^1$, i.e., the poor country sells assets short while the rich country demands a positive number.

Equation (5.5.4) and (5.5.5) can be rewritten as

$$k^2 = d - \frac{\delta}{A} - \frac{c}{Ap} \quad (5.5.6)$$

$$k^2 = \frac{\alpha\mathbb{V}[d] + p^2A}{2A\alpha\mathbb{V}[d]} \pm \sqrt{\left(\frac{\alpha\mathbb{V}[d] + p^2A}{2A\alpha\mathbb{V}[d]}\right)^2 + \frac{p(\alpha\mathbb{V}[d] - pAd)}{A\alpha\mathbb{V}[d]}}. \quad (5.5.7)$$

Substituting equation (5.5.5) into (5.5.3) we obtain

$$k^1 = Ad^2 - p\left(1 + \frac{c}{\alpha\mathbb{V}[d]} + \frac{p\delta}{\alpha\mathbb{V}[d]}\right). \quad (5.5.8)$$

Figure 5.5.1 shows the sets defined by equation (5.5.6), (5.5.7), and (5.5.8) for $c > 0$ and $Ad > \delta$ where the intersections of sets defined by equation (5.5.6) and (5.5.7) depict the steady state values for k^2 and p. The corresponding steady state value of k^1 is depicted on the set defined by equation (5.5.7). Notice that for the steady state value $\bar{p}$, there exist corresponding steady state values for k^1 and k^2 where $k^2 < d < k^1$.

Now we prove that $x^2 < 0 < x^1$ in the steady state by contradiction. Notice that in the steady state in Figure 5.5.1,

$$\frac{\alpha\mathbb{V}[d]}{Ad} < \frac{c}{Ad - \delta}.$$

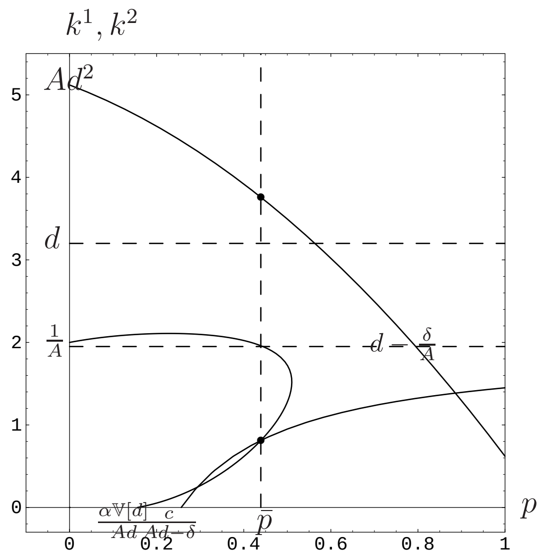


Figure 5.5.1: Existence of asymmetric steady states

Suppose that $x^2 > 0$ in the steady state. From equation (5.5.4) this means that

$$1 - \frac{p\delta + c}{\alpha\mathbb{V}[d]} > 0 \Longrightarrow p < \frac{\alpha\mathbb{V}[d] - c}{\delta}.$$

From Figure 5.5.1 a necessary condition for the existence of the steady state is that

$$p > \frac{c}{Ad - \delta}.$$

This means that $\frac{\alpha\mathbb{V}[d]-c}{\delta} > \frac{c}{Ad-\delta}$ has to hold so that $x^2 > 0$ in the steady state, which is equivalent to

$$\frac{\alpha\mathbb{V}[d]}{Ad} > \frac{c}{Ad - \delta}.$$

This is a contradiction. Hence, $x^2 < 0$ in the steady state. Since $p(x^1 + x^2) = 2$ in any steady state, $x^2 < 0 < x^1$ follows. □

Proof of Proposition 5.3.4

The dynamical system in the neighborhood of the steady state where $k^2 < d < k^1$ is defined by

$$\begin{aligned} k_1^1 &= \Phi^1(k^1,k^2,p) = Ad^2 - p\left(1 - \frac{p((1-\delta) - R^2(k^1,k^2,p))}{2\alpha\mathbb{V}[d]}\right) \\ k_1^2 &= \Phi^2(k^1,k^2,p) = A(k^2)^2 - p\left(1 - \frac{p(R^2(k^1,k^2,p) - (1-\delta))}{2\alpha\mathbb{V}[d]}\right) \\ p_1 &= \tilde{\Psi}(k^1,k^2,p) = \frac{p}{2}\left(1-\delta+R^2(k^1,k^2,p)\right) - c. \end{aligned}$$

where $R^2(k^1,k^2,p) = \frac{\alpha\mathbb{V}[d]\left(2A\left(d-A(k^2)^2+p+\frac{p^2(1-\delta)}{2\alpha\mathbb{V}[d]}\right)+(1-\delta)\right)}{\alpha\mathbb{V}[d]+Ap^2}$.

The Jacobian matrix of the dynamical system is

$$J(k^1,k^2,p) = \begin{pmatrix} \frac{\partial\Phi^1(\cdot)}{\partial k^1} & \frac{\partial\Phi^1(\cdot)}{\partial k^2} & \frac{\partial\Phi^1(\cdot)}{\partial p} \\ \frac{\partial\Phi^2(\cdot)}{\partial k^1} & \frac{\partial\Phi^2(\cdot)}{\partial k^2} & \frac{\partial\Phi^2(\cdot)}{\partial p} \\ \frac{\partial\Psi(\cdot)}{\partial k^1} & \frac{\partial\Psi(\cdot)}{\partial k^2} & \frac{\partial\Psi(\cdot)}{\partial p} \end{pmatrix}.$$

Since the first column of the above matrix has only zero entry, we can consider the sub-matrix

$$\begin{pmatrix} \frac{\partial\Phi^2(\cdot)}{\partial k^2} & \frac{\partial\Phi^2(\cdot)}{\partial p} \\ \frac{\partial\Psi(\cdot)}{\partial k^2} & \frac{\partial\Psi(\cdot)}{\partial p} \end{pmatrix} = \begin{pmatrix} 2Ak^2 + \frac{p^2}{2\alpha\mathbb{V}[d]}\frac{\partial R^2(\cdot)}{\partial k^2} & \frac{p(R^2(\cdot)-(1-\delta))}{\alpha\mathbb{V}[d]} - \bar{x} + \frac{p^2}{2\alpha\mathbb{V}[d]}\frac{\partial R^2(\cdot)}{\partial p} \\ \frac{p}{2}\frac{\partial R^2(\cdot)}{\partial k^2} & \frac{1-\delta+R^2(\cdot)}{2} + \frac{p}{2}\frac{\partial R^2(\cdot)}{\partial p} \end{pmatrix}.$$

The determinant and the trace of the above 2×2 matrix is

$$\det = \frac{2A^2k^2(d-A(k^2)^2+p)\alpha^2\mathbb{V}[d]^4}{(Ap^2+\alpha\mathbb{V}[d])^2} + \frac{2Ak^2\alpha\mathbb{V}[d](1-\delta)}{Ap^2+\alpha\mathbb{V}[d]}$$

$$\text{tr} = A\alpha\mathbb{V}[d]\left(\frac{A^2(k^2)^2p^2+(d+2(k^2+p))\alpha\mathbb{V}[d]}{(A(p^e)^2+\alpha\mathbb{V}[d])^2} - \frac{A(dp^2+k^2(-2p^2+k^2\alpha\mathbb{V}[d]))}{(Ap^2+\alpha\mathbb{V}[d])^2}\right) + \frac{(1-\delta)(\alpha^2\mathbb{V}[d]^4+2A\alpha\mathbb{V}[d]p^2+A^2p^2)}{(Ap^2+\alpha\mathbb{V}[d])^2}$$

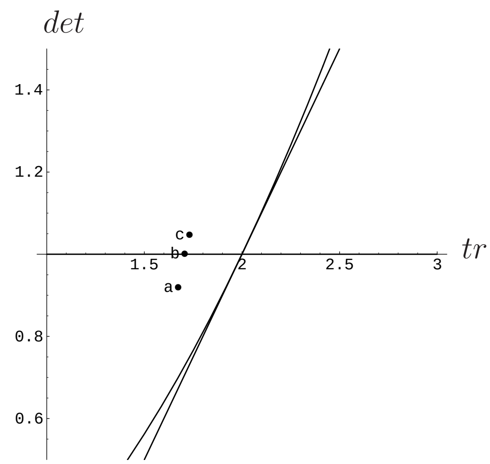


Figure 5.5.2: Stability triangle: $\delta = (0.625, 0.594719, 0.575)$

The points (a), (b) and (c) in Figure 5.5.2 corresponds to $\delta = (0.625, 0.594719, 0.575)$ in Figure 5.3.3. As the value of δ decreases from 0.625 to 0.575 the determinant crosses 1 at $\delta = 0.594719$ which proves that the system undergoes a Neimark-Sacker bifurcation. □

6

Conclusion

Chapters 3 and 5 are the main theoretical contributions to the dynamic nature of economies with capital accumulation, which are integrated through financial markets. The two models in those chapters are based on an overlapping generations framework with consumers who live two periods. Trading in financial markets is supported by the maximizing behavior of individuals and is modeled explicitly. This enabled us to analyze the impact of individual behavior at micro level on the development of the aggregate economy. Both models showed that the optimal behavior at the agent's level need not lead to an optimal social outcome even less so to equal welfare in all economies. It is this externality that a model should identify to analyze the mechanism of international financial markets.

The two models in Chapter 3 and 5 have a different structure of the financial market and therefore different implications for inequality. The model in Chapter 3 introduces a credit market in which agents can make a debt contract which specifies the obligation of the debtor to repay the sum of the principal and interest in the next period. Given the overlapping generations framework, this implies that financial trade takes place only among young consumers. In the presence of the enforcement problem, some agents may face credit rationing. This is the case in the asymmetric steady state where the poor country supplies credits to the rich country. Poor countries are constrained by their wealth to start up investment projects and therefore are net lenders to rich countries. As a result, the poor country is worse off trading with the rich country and the rich country better off than in autarky. The model identifies the relatively low population size of the rich country as one of the factors that induces the divergence of incomes between the two countries. In particular, the smaller the relative population size of the rich country, the greater is the gap between the rich and the poor country.

The model in Chapter 5 introduces a market in which agents can trade nominal assets which pay random dividends. The asset is a claim to the firm's profit and can be traded among young consumers as well as between young and old consumers. The equilibrium price is determined by interactions of old and young agents who construct an optimal portfolio. The construction of the optimal portfolio always brings about a net capital flow from the rich to the poor country since the marginal rate of return is higher in the poor country. However, this mechanism does not necessarily lead to convergence of the two countries. Since the two countries are only linked to each other through the asset market, the flow of capital is subject to expectations in the asset market. This is an important difference to the model in Chapter 3 where expectations did not play a role. The asset market creates a feedback mechanism between the asset demand and capital investment. The associated risk in the asset market decides whether the mechanism induces convergence of incomes between the two countries. The initially poor country is better off trading with the rich country while the rich country is worse off than without the asset market at the asymmetric steady state. This difference between the two models in Chapter 3 and Chapter 5 in their implication for the inequality arises from the fact that the consumers in the poor country are rationed in the credit market while they hold an optimal portfolio in the asset market. The results obtained in the both models are path dependent. In other words, the initial condition of the system is the decisive factor for the allocation in the long run. The initially poor country can at best catch up with the rich country.

The two models in Chapter 3 and 5 generate endogenous fluctuations. The credit market model undergoes a Neimark Sacker bifurcation only if the relative population sizes are sufficiently disparate. On the other hand, the asset market model which consists of two identical countries, also undergoes a Neimark Sacker bifurcation. It is technically beyond the scope of this thesis to fully identify the economic structure which causes this bifurcation. However, it is shown that the endogenous cycles arise from interactions of two economies in the presence of feedback effects. In the case of the credit market model it is the world interest rate and in the case of the asset market model it is the price in the international asset market that functions as a channel between the two countries. The results on endogenous inequality and fluctuation do not suggest that the financial market globalization is necessarily an unequalizing and a destabilizing force. Rather it shows that the issue of economic development is much more about the interlinkages between economies, generations, and sectors than a structural characteristic of an economy.

The two underlying assumptions in each model 1) the wealth dependent borrowing constraint and 2) the asset market as a only channel of capital flows, are essential to the obtained results on endogenous fluctuations and inequality of nations. They are restrictive assumptions but help us to focus on the particular role of the financial markets through which two economies interact. It was shown that the results also depend on factors such as the technology, preference, and population sizes. The interplay of these factors created feedback mechanisms between real and financial sectors of the two countries. Beside these aspects, the role of financial markets in real life is manifold. Therefore, outlook for the future research may be categorized into three directions. 1) to generalize the underlying assumptions of the models, 2) to identify other structural features of the economy to examine the robustness of the results, and 3) to examine the robustness of the results in an economy with more than one type of financial markets.

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