The influence of small and medium-sized enterprises on economic growth

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The influence of small and medium-sized enterprises on economic growth

Ph.D. Dissertation

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The definition of symbols

Y : Total product in production sector (goods sector).

y : Average product per labor in production sector (goods sector).

L : The labor in the representative firm.

K : The total physical capital.

k : Physical capital per capita.

Q : The total human capital.

q : Average production per labor in education sector.

A : The basic technical level in goods sector.

B : The technical level in education sector.

k_a : Average economy wide capital stock.

θ_1 : The internal effect scale of firm.

θ_2 : The external effect scale of firm.

C : The total consumption.

c : Consumption per capita.

α : The coefficient of the representative firm's capital in production function.

β : The coefficient of average widely capital in production function.

σ : The inverse of the intertemporal elasticity of substitution.

ρ : Discount rate of time (constant subjective rate of time preference)

δ : Depreciation rate (it is same in goods section and education section)

u : The proportion of human capital, which be used in goods sector.

$1-\omega$:The ratio of the initial output that the small firm spends in education sector to improve its knowledge base or technology

ω : The ratio of the initial output that the small firm keeps in goods sector.

Chapter 1 Introduction

1.1 Motive

Small and medium-sized enterprises (SMEs [1]) have been making great contributions to the economic development of developing countries, because SMEs is easier to establish for reducing the unemployment and producing light industrial products. The SMEs also play an important role in the economics of Taiwan.

In 1945, Taiwan's economy suffered from severe damage caused by the Second World War. After the war, the government has dedicated itself to developing light labor-intensive industry. Technology required for production in this industry was relatively simple. Taiwan was able to earn foreign exchange reserves. Private enterprises were encouraged to import raw materials, semi-finished products and machinery to produce consumer goods which could replace imported merchandise in the domestic market; thereby it contributed to establishing a solid foundation for the development of those industries producing everyday necessities. The development of SMEs speeded up, and enterprises with ten or fewer employees came to account for over 90 percent of all enterprises in Taiwan. Most of these enterprises were producing commodities for the domestic market. If there were not so many SMEs, Taiwan 's economic miracle might not have happened at all.

From the "White Paper on Small and Medium Enterprises in Taiwan 2008 ", we know that SMEs account for 97% of Taiwanese enterprises. They alone generated 28% of the total corporate sales and 77% of the workforce in the whole country were

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[1] The definitions of SMEs in Taiwan are:
a. In the manufacturing, construction, mining and quarrying industries, a paid-in capital is less than NT$80 million (US$2.42 million) or the number of regular employees less than 200.
b. In the agriculture, forestry and fisheries, water, electricity and gas, commercial, transportation, warehousing and communications, finance, insurance and real estate, industrial and commercial services or social and personal services industries, the sales revenue is less than NT$100 million (US$3.03 million) or less than 50 regular employees.

employed in SMEs in 2007. It is widely acknowledged that they are contributive to the reduction of the unemployment rate. Besides, most of large firms started off as small firms, such as Acer, one of the most famous computer manufacturers in Taiwan.

Acer is a Taiwan-based multinational computer technology and electronics corporation that manufactures personal computer, personal digital assistants, servers, monitors, etc. As of the fourth quarter of 2009, Acer was the world's second largest personal computer manufacturer and one of the most well-known brands in Europe. Acer was founded by Stan Shih and his wife Carolyn Yeh in 1976 in Taiwan. It began only with 11 employees and US$25,000 in capital. By 2005, Acer employed 7,800 people and its revenues soared to US$11.31 billion in 2006.

Another economic miracle in the history that attracted most researchers' attention was Germany's swift economic recovery from the depression after World War II. This result could be explained on several accounts, such as the trend of international trade and economic liberation, the change of German political and economic conditions. The restructured industrial system in Germany was one of the most important factors in terms of improving the development of the German economy.

It is known that German enterprises were and are still very competitive in the global market. Roy Rothwell and Walter Zwgveld (1982) stated SMEs have played a key role in the post-war development of the West German manufacturing industry. They created a stable, social and economic environment and were central to the post-war economic recovery plans of the West German government. In fact, almost 95% of the German enterprises were classified as SME and 85% of German workers were hired by SMEs in Germany between the 1950s and 1960s. Thus, the success of SMEs had much to do with the prosperity of the German economy.

Like Taiwan's example, the famous supermarket Aldi which actually transformed itself from a very small firm is another example. The mother of brothers Albrecht opened a small store in a suburb of Essen in 1913. After the end of World War II, the brothers took over their mother's business in 1946, but it was simply a very small grocery store back then. With the brothers' efforts, now, the Aldi group became the largest chain supermarket in Germany. In the beginning, Aldi could only hire two people, but now it hires over thousands of employees and buys lots of material from its upstream firms.

Rothwell (1981) pointed out the role of SMEs in industry development after World War II. He thought small firms are the seed of tomorrow's large firms and new industries. If a small firm grows into a large firm successfully, it has the potential to hire more workers and demand more resources from other firms. In other words, it creates greater external effects. When SMEs have a higher probability to grow into large firms, it could lead to a higher growth rate.

For example, in Table 1-1, we distinguish these countries into four groups. The countries in group 1 are USA, Germany and Canada. In 1960 these countries were already richer countries than the other groups. In 2007, they were still richer countries and have over 40% of global large firms.

In group 3, we compare Kenya, Taiwan and South Korea. In 1960, the GDP of Kenya was higher than Taiwan and South Korea and they all had nigh on zero international large firms. But between 1960's and 1990's Taiwan and South Korea experienced a high economic growth process; meanwhile, there were many small firms growing into large firms. For example, Samsung and LG in South Korea, whereas Acer and Foxconn in Taiwan. There were similar cases in other countries, like Germany, Japan, etc.

Table 1-1 The global firms and growth rate between countries

	GDP(1960) Per capita (US dollars)	GDP(2007) Per capita (US dollars)	Average growth rate (1960-2007)	The number of 500 global firms in 2007
Group 1				
Canada	10558.75	36168.29	2.33	16
USA	14766.36	42886.92	2.28	162
Germany	15490*	31306.26	1.95	37
Group 2				
Chile	5813.99	18381.16	2.76	0
Japan	5471.59	30585.38	3.99	67
Mexico	4456.54	11203.82	2.12	0
Group 3				
Kenya	1817.59	2025.18	0.41	0
South Korea	1764.73	23849.62	5.64	14
Taiwan	1591.98	27004.98	6.20	6
Group 4				
China	508.09	8511.33	6.17	24
India	961.6	3826.32	3.04	6
Indonesia	1027.81	5185.52	3.58	0

* The year of GDP is 1970

Date source: Penn world table 6.2, http://pwt.econ.upenn.edu/.

Fortune: http://money.cnn.com/magazines/fortune/

The situations were similar between Germany and Taiwan. We know that the SMEs play an important role in economy. Nevertheless, what is the real influence of the SME on economic growth? Aw, B. Y. and Batra G. (1998) thought SMEs had less physical and human capital, but they subcontracted activities among large firms and learnt new technologies and disseminated that throughout the economy. Romer (1986) asserted that there exist positive spillover effects in industry and the positive spillover effects are beneficial to economic growth. Caballero and Lyons (1990) and Chan (1995) pointed out that there also exist external economies of scale in Germany and Taiwan. Greiner (2003) indicated that the external effect of investment could explain why countries may converge in terms of the growth paths they were taking in the long run.

Considering these papers, they assumed that the external effects of firms are homogeneous in the same industry and the firms are of the same scale. Meanwhile, the external effects are beneficial to economic growth. However, the external effects of firms may be different in each country and of variant firm scale. In some countries the external effect of SMEs was larger than others and perhaps it offered the SMEs more opportunities to transform themselves into large firms. When they become large firms, they can hire more employees and buy more production material from other firms and create higher economic growth. If we can establish the external effects of firms between the large enterprises and SMEs, and find out which factors could affect small firms to grow up into large firms, we could, perhaps, establish what factors led to the different paths that countries took in terms of their economic system.

In addition, the probability or process that transforms SMEs into large firms may yet be another key factor that accounts for the difference of growth rate in each individual country.

Almost all large firms practically started off small with a limited amount of capital or saving of the owner. The initial number of employees was also small, typically fewer than a dozen. Later, some firms would expand and operated on a medium or large scale, whereas others would shut down or close. If a small firm grows into a large firm successfully, it can hire more workers and need more resources from other firms. In other words, it spurs more external effects and brings about a higher growth rate.

We tried to use the empirical data from three Asian countries in Table 1-2 (South Korea, Taiwan and India) to look at the relationship between the growth rate of GDP and growth rate of large firms. Because our data is limited, we could not use long-term data to establish a regression model. Nonetheless, we still could observe the phenomenon on the trend. Except for India, the relationships between the GDP growth rate and the growth rate of large firms in Taiwan and South Korea are significant. The coefficient for South Korea is 0.664. It means if the numbers of large firms raise 1 percent, the growth rate will increase by 0.664 percent, and the effect in South Korea is larger than in Taiwan. The reason perhaps is that most large firms in South Korea are supported by its government and they could use more resources and hence take a more advantageous position than small firms from government.

Between the 1970's and 1990's Taiwan and South Korea experienced a high economic growth rate. Meanwhile, there were many small firms transforming into large firms. We could also find the similar cases in others countries.

Dennis Anderson (1982) claimed that predominance of large firms is due to (1) the economics of scale with respect to plant, (2) the economics of scale with respect to management and marketing, (3) the superior technical and management efficiency, and (4) the preferential access to supporting infrastructure service and external finance. Hence, we assume that there exist higher internal and external effects in large firms.

Table 1-2 The relationship between economic growth and the growth rate of large enterprises (LE)

Dependent variable: ($GDP_t - GDP_{t-1} / GDP_{t-1}$)			
Country	South Korea	Taiwan	India
Observation	13	21	14
	(1994-2007)	(1984-2004)	(1991-2004)
Constant	0.036*	0.075*	-1.928
	(0.011)	(0.006)	(4.018)
($LE_t - LE_{t-1} / LE_{t-1}$)	0.664**	0.063**	49.70
	(0.289)	(0.029)	(100.82)
$\overline{R^2}$ =	0.32	0.19	0.02
Durbin-Watson	1.608	0.955	2.23

Stand errors are in parentheses. The method to estimate equation is ordinary least square (OLS).* means significant level is 1 percent, ** 10 percent

Date source: Taiwan: The Small and Medium Business Administration of Economic Affair in Taiwan.
South Korea: The Small and Medium Business Administration of Korea.
India: Annual report (2004). Government of India Ministry of small scale industries.

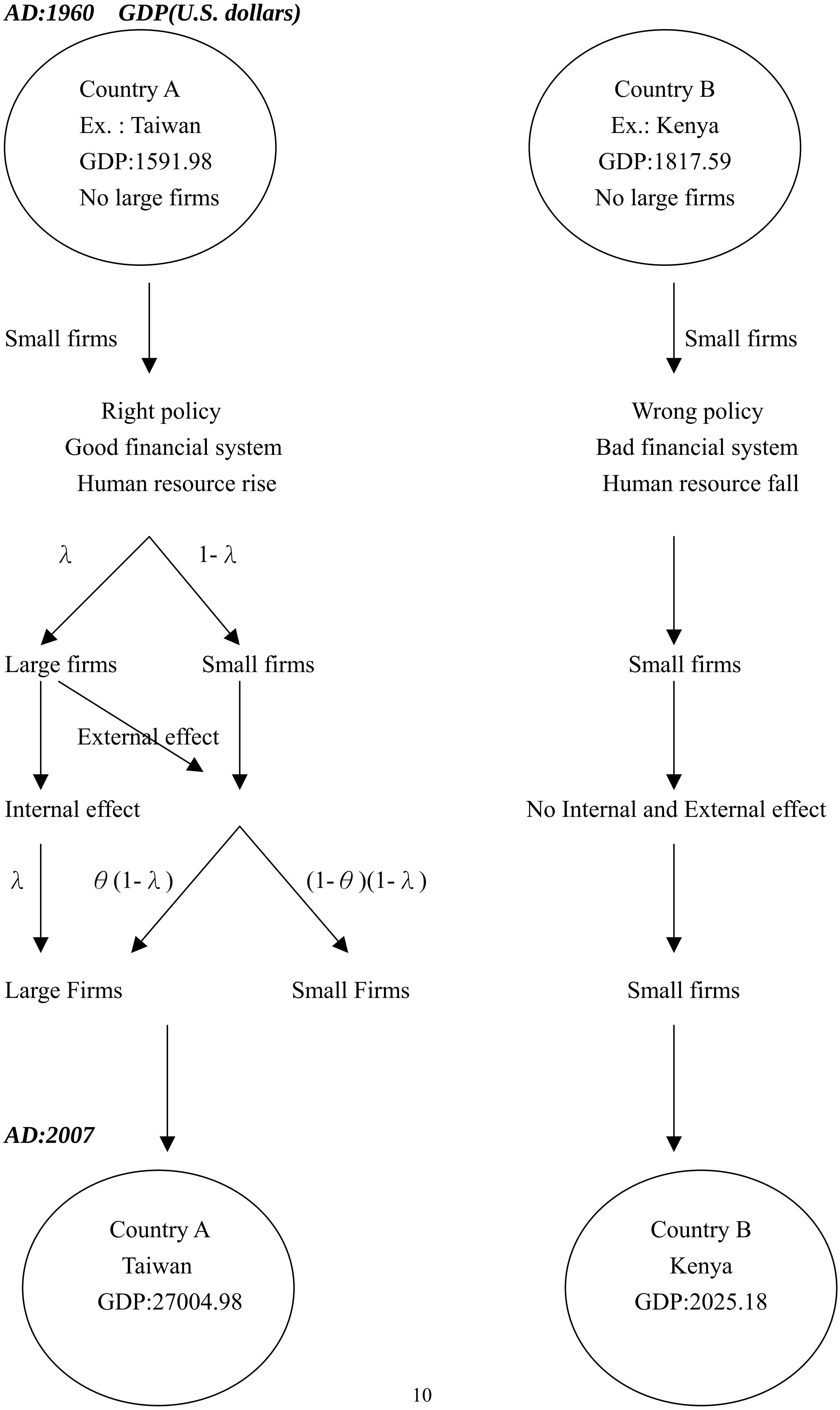
1.2 Research contents

According to these arguments, the process of small firms transforming into large firms seems to be an important factor on economic growth. We may suggest that there are some connections between the process of small firms' transformation and economic growth. The present dissertation seeks to fill this gap and detect the relationship and interplay among the influence of the scale of firm, the transformation process, and factors that encourage firms' transformation. The research questions addressed in the current study are as follows:

- In chapter 2 and chapter 3, we discuss the influence of SMEs' transformation on the economic growth and improve on the model concerning the process of firms transforming from small ones to large ones with different assumptions. To be exact, we assume an industry where only one-sector or two-sectors exist.
- In chapter 3, we discuss to what extent the share of large firms influences the speed of economic growth.
- In chapter 4, we also discuss to what extent the share of large firms influences the speed of economic growth, but we manipulate the share of large firms to be an endogenous variable. It means small firms could do something to improve the probability transforming into large firms.
- In chapter 5, we use Taiwan's data to calculate the share of external effect in firms' production function.
- In chapter 6, we use econometrics method to find out the important factors that make small firms transform to large firms.
- In chapter 7, we sum up the results and close up the whole study with conclusions.

This dissertation is expected to make a contribution to the research field of small firms' transformation contributive to economic growth. First, it models the production function which combines two kinds of firms, large and small, and calculates the influence of different parameters on the economic growth. It perhaps could help us to explain why two countries in the same economic condition at the start-up phase could take divergent paths years later. In Figure 1-1 we create a flow chart to help readers easily to understand what the important points are in this dissertation. Moreover, the investigation of the factors that affect small firms transforming into large firms successfully is hopefully to be beneficial for governments to come up with some viable policy.

Figure 1-1 The process of firms' transformation



Chapter 2 One-Sector Model

2.1 Introduction

Since the Industrial Revolution in the 18th century, the wealth gap between the poor countries and rich countries has greatly enlarged.

For the reason, many economists pay attention to varying growth rates across countries and try to find the reasons why each country has a different growth path.

The economic growth theories in the earlier period cannot explain the reason for the economic growth, nor why the economy can have a sustainable growth.

Past literature was based on Solow’s (1956) neoclassical growth model. However, the theory cannot properly explain why per capita income grows persistently and how government policy can influence economic growth in reality.

Since Romer (1986), the external effect of the manufacturer's output has become one of the important factors that motivate economic growth.

The recent development of endogenous growth theories emphasizes that the steady state growth is an endogenous outcome, for instance (Barro,1990; Rebelo,1991).

Endogenous growth theories propose that the momentum of economic growth comes from endogenous factors of the model, such as the accumulation of human capital, research and development. However, we do not know the influence on the scale of firms in endogenous growth theories.

It is very likely that the structures between large and small firms’ economies would be different. Furthermore, for a large firm’s economy, the stock of physical and human capital is larger than that of a small firm’s economy. That means there would be larger external effects in large firms’ economies.

We can look at the evolution of firms in history. At the early agricultural society, it was inconvenient to transport goods. The so-called large enterprises, strictly speaking, may be just small regional enterprises.

Thanks to the technological development, the large international enterprises' influence is not limited to its motherland, but also reaches other faraway nations.

By observing the data of global 500 large companies[2], most of the international large enterprises, are located in developed countries, like USA, Japan and Germany, or even some of the new developed countries, such as South Korea, Singapore and Taiwan.

The external effect of large enterprises, however, is not of the same value across countries in the world. For example, the external effect of a large enterprise between Philippines and USA may be totally different, because the scale of the largest firm in the Philippines may be just a medium firm in the USA.

Except for some large enterprises that are directly supported by governments, most of the large enterprises are transformed from the small enterprises. The process of a small enterprise transformed into a large enterprise and whether it will bring more external effects to other manufacturers is worth discussing.

The purpose of this chapter is to examine the role of a firm's scale and its effect on the economic growth. We also discuss whether a country with more large-scale enterprises has a higher economic growth rate than a country with fewer large-scale enterprises.

[2] The source of the global 500 large companies: http://money.cnn.com/magazines/fortune/global500/2009/

2.2 The basic model

The basic model will adopt the assumption of Romer (1986). Consider a closed economy consisting of two parts: a representative household and a productive firm that produces a single commodity.

There exist large enterprises[3], small enterprises and hybrid enterprises in three countries A, B and C. In the country A, all firms are large enterprises. In the country B, all firms are small enterprises and in the country C exists only hybrid enterprises. What is a hybrid enterprise? In this chapter, we define a hybrid enterprise as a new enterprise that couples a large enterprise with a small enterprise. The proportion of large firms is λ_1 and the proportion of small firms is $1-\lambda_1$, 0< λ_1 <1. There is no government in our model. That means that there is no tax and subsidy. International trade in our model is not allowed for.

2.2.1 The representative household

There exists a representative household in each economy. The representative household is assumed to have an infinite planning horizon in each country. Moreover, the labor supply of the representative household is fixed. It means that there is no birth and death in the representative household. Hence, the number of household is constant. The household is postulated to choose its private level of consumption to maximize the discounted sum of future instantaneous utilities:

$$\max \ U = \int_0^\infty e^{-\rho t} \mu(c(t)) L dt \quad \text{……………………….(2-1)}$$

and

3 We define an enterprise as a large firm if the internal and external effects are larger than 0. A small firm does not have internal and external effects in the production function.

$$\mu(c(t)) = \frac{c^{1-\sigma}(t) - 1}{1-\sigma} \quad \text{................................(2-2)}$$

subject to:

$$y(t) = c(t) + \delta k(t) + \dot{k}(t)$$

where U is the utility function and ρ is the subjective time preference rate. c is the consumption , L is the number of labor which is set equal to 1 and σ is the inverse of the elasticity of intertemporal substitution which measures the curvature of the utility function.

2.2.2 The production function

The production function is assumed to be

$$y(t) = f(k, k_a) = A(1+\theta_1)k(t)^{\alpha}[(1+\theta_2)k_a(t)]^{1-\alpha} \text{...............(2-3)}$$[4],

where y is output per unit of labor, k is the physical capital of the representative firm, and k_a is other firm's average capital (also called average economy wide capital stock), θ_1 is the internal effect scale of the representative firm; θ_2 is the external effect scale of the representative firm; α is the share of firm's capital in production function; 1-α is the share of other firm's average capital in production function. A is the basic technology level. Here we assume that the internal effect scale θ_1 of the representative can make greater contribution than the external effect scale θ_2, because the representative firm could adjust its optimal production process. Even if θ_1 and θ_2 is of the same value, the contribution of the parameter $(1+\theta_1)$ is larger than $(1+\theta_2)^{1-\alpha}$ in production function.

The production function exhibits diminishing returns to scale with *k*. If θ_1 and θ_2 are both larger than 0 and there exist constant returns to scale. Every firm has the same basic technology level A. However, with the different scale the firms have,

[4] In the following we ignore the time argument if no ambiguous results.

they have different internal effect $A(1+\theta_1) = A_1$.

2.2.3 The investment function

In the aspects of investment, the representative household did not save anything. The output deducts to consume and depreciate, which will be devoted to next period. The physical capital accumulation constraint of a representative firm is

$$\dot{k} = f(k) - \delta k - c \quad \text{…………………………..………..(2-4),}$$

where δ is the depreciation rate, and an over dot denotes the time derivative. In this chapter, we assume that k_a could make contribution to production process, it means there exists externality in production function. The decentralized solution will not be the Pareto optimal because it does not consider the spillover of physical capital across the firms. The social planner could internalize the spillover of physical capital across the firms into the production function and it will be the best solution. Therefore, we just consider trying to find " The social planner optimal solution".

2.3 The social planner optimal solution in one-sector

2.3.1 The small firm

In the beginning, the number of labor is L. For convenience, we assume L keeps constant to 1. u is the utility function of consumption and it is assumed to satisfy the Inada conditions.

$$\lim_{c\to 0} \mu'(c) = \infty, \lim_{c\to\infty} \mu'(c) = 0 \quad \text{……………………….(2-5)}$$

$$L(t) = L = 1$$

$$\mu(c(t)) = \frac{c^{1-\sigma}(t) - 1}{1-\sigma}$$

$$Y/L = y = f(k, k_a)$$

When all firms are small firms, it means that there are no internal and external effects in the production function. $\theta_1 = \theta_2 = 0$, equation (2-6) states the production function per labor.

$$y = Ak^{\alpha} k_a^{1-\alpha} \quad \text{.....................................(2-6)}$$

The present value Hamiltonian for the representative household 's optimization is given by:

$$H = \frac{c^{1-\sigma}(t) - 1}{1-\sigma} e^{-\rho t} + \lambda(\dot{k}) \quad \text{...............................(2-7)}$$

After differentiating equation (2-7) with respect to c (control variable), we obtain equation (2-8). It means that the present value marginal utility must equal the marginal value of physical capital.

$$\frac{\partial H}{\partial c} = c^{-\sigma} e^{-\rho t} - \lambda = 0 \rightarrow c^{-\sigma} e^{-\rho t} = \lambda \quad \text{....................(2-8)}$$

equation (2-7) differentiating with respect to k （state variable）, we obtain equation (2-9).

$$\frac{\partial H}{\partial k} \Rightarrow \lambda(f'(k) - \delta) = -\dot{\lambda} \quad \text{......................................(2-9)}$$

We need equation (2-10) as transversality condition; it means that there will be an optimal solution.

$$\lim_{t \to \infty} \lambda(t) k(t) = 0 \quad \text{...(2-10)}$$

From equation (2-9), we get equation (2-11)

$$(f'(k) - \delta) = -\frac{\dot{\lambda}}{\lambda} \quad \text{...(2-11)}$$

Logarithm equation (2-8) and differentiating with respect to time, we obtain equation (2-12). We can also put equation (2-11) into equation (2-12) and obtain the common growth rate from the expression for consumption.

$$\frac{\dot{c}}{c}=\frac{-\frac{\dot{\lambda}}{\lambda}-\rho}{\sigma}=\frac{f'(k)-\delta-\rho}{\sigma} \quad \text{..........................(2-12)}$$

From equation (2-4), we obtain equation (2-13)

$$\frac{\dot{k}}{k}=\frac{f(k)-\delta k-c}{k} \quad \text{......................................(2-13)}$$

We know all firms are identical. In the social planner optimal equilibrium, $k=k_a$.

Together with equations (2-13) and (2-3),

$$\frac{\dot{k}}{k}=\frac{f(k)-\delta k-c}{k}=Ak^{\alpha-1}k_a^{1-\alpha}-\delta-\frac{c}{k}\Rightarrow-\frac{c}{k}=\frac{\dot{k}}{k}-A+\delta \quad \text{........(2-14)}$$

In the steady state, all variables grow at constant rates. The growth rate of per person capital is constant. A and δ are exogenous variables and also constant. Therefore, the right side of equation (2-14) for $\frac{c}{k}$ is constant. With logarithm equation (2-14) and differentiating with respect to time, we get equation (2-15). The rate of change in the consumption equals the rate of change in physical capital.

$$\frac{\dot{c}}{c}=\frac{\dot{k}}{k} \quad \text{.....................................(2-15)}$$

We know $y=Ak^{\alpha}k_a^{1-\alpha}=Ak$, and it also follows $\frac{\dot{k}}{k}=\frac{\dot{y}}{y}$. Because of the rate of change in output is equal to the rate of change in physical capital, we know the rate of change in output is equal to the rate of change in physical capital and consumption.

Then $\frac{\dot{c}}{c}=\frac{\dot{k}}{k}=\frac{\dot{y}}{y}$ …………………….…………(2-16)

So we can induce the growth rate in a country with small firms as equation (2-17)

$$\frac{\dot{y}}{y}=\frac{f'(k)-\delta-\rho}{\sigma}=\frac{A-\delta-\rho}{\sigma} \text{……..(2-17)}$$

If $A>\delta+\rho$, the growth rate will be positive, the result is similar to AK model.

2.3.2 The large firm

The other assumptions stay the same as those of small firms, except for the value of the internal effect and external effect, which are both larger than 0. The production function is equation (2-18).

$$y=f(k,k_a)=A(1+\theta_1)k^{\alpha}[(1+\theta_2)k_a]^{1-\alpha} \text{………..(2-18)}$$

The present value Hamiltonian for the representative household 's optimization is illustrated by:

$$H=\frac{c^{1-\sigma}(t)-1}{1-\sigma}e^{-\rho t}+\lambda(\dot{k}) \text{…………………..….(2-19)}$$

After differentiating equation (2-19) with respect to c (control variable), we obtain equation (2-20). It means that the present value marginal utility must equal the marginal value of the physical capital.

$$\frac{\partial H}{\partial c}=c^{-\sigma}e^{-\rho t}-\lambda=0\rightarrow c^{-\sigma}e^{-\rho t}=\lambda \text{…………..(2-20)}$$

By differentiating equation (2-19) with respect to k (state variable), we obtain equation (2-21).

$$\frac{\partial H}{\partial k}\Rightarrow\lambda(f'(k)-\delta)=-\dot{\lambda} \text{…………….………(2-21)}$$

We need equation (2-22) as a transversality condition; it means that there will exist an optimal solution.

$$\lim_{t\to\infty} \lambda(t)k(t) = 0 \quad \text{(2-22)}$$

From equation (2-21), we get equation (2-23)

$$(f'(k)-\delta) = -\frac{\dot{\lambda}}{\lambda} \quad \text{(2-23)}$$

Logarithm equation (2-20) and differentiating with respect to time, we obtain equation (2-24). We can also put equation (2-23) into equation (2-24) and get the common growth rate from the expression for consumption.

$$\frac{\dot{c}}{c} = \frac{-\frac{\dot{\lambda}}{\lambda}-\rho}{\sigma} = \frac{f'(k)-\delta-\rho}{\sigma} \quad \text{(2-24)}$$

From equation (2-4), we obtain equation (2-25)

$$\frac{\dot{k}}{k} = \frac{f(k)-\delta k - c}{k} \quad \text{(2-25)}$$

Together with equation (2-18) and equation (2-25), and in social planner optimal equilibrium, $k = k_a$. We obtain equation (2-26)

$$-\frac{c}{k} = \frac{\dot{k}}{k} - A(1+\theta_1)(1+\theta_2)^{1-\alpha}(\frac{k_a}{k})^{1-\alpha} + \delta = \frac{\dot{k}}{k} - A(1+\theta_1)(1+\theta_2)^{1-\alpha} + \delta \quad \text{(2-26)}$$

In the steady state, the growth rate of per person capital $\frac{\dot{k}}{k}$ is a constant value. A, δ, θ_1 and θ_2 are all exogenous variables and also constant. Therefore, the right side of equation (2-26) for $\frac{c}{k}$ is constant. With logarithm equation (2-26) and differentiating with respect to time, we get equation (2-27). The rate of change in consumption equals the rate of change in physical capital.

$$\frac{\dot{c}}{c} = \frac{\dot{k}}{k} \quad \text{(2-27)}$$

The production function $y = A(1+\theta_1)(1+\theta_2)^{1-\alpha}k$, θ_1, θ_2 and α are all exogenous variables, thus, $\frac{\dot{k}}{k} = \frac{\dot{y}}{y}$. Because the rate of change in output equals the rate of change in physical capital, we know that the rate of change in output is equal to the rate of change in physical capital and consumption.

$$\frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{y}}{y} \text{ ………………………………………….(2-28)}$$

Hence, we can obtain the growth rate in a country with large firms as equation (2-29)

$$\frac{\dot{y}}{y} = \frac{f'(k) - \delta - \rho}{\sigma} = \frac{A(1+\theta_1)(1+\theta_2)^{1-\alpha} - \delta - \rho}{\sigma} \text{ …….(2-29)}$$

If $A\alpha(1+\theta_1)(1+\theta_2)^{1-\alpha} > \delta + \rho$, we obtain a positive growth rate in this model.

2.3.3 The hybrid firm

In this chapter, we assume the production function in large firms is

$$y = f(k, k_a) = A(1+\theta_1)k^{\alpha}[(1+\theta_2)k_a]^{1-\alpha}$$

and production function in small firms is

$$y = f(k, k_a) = Ak^{\alpha}k_a^{1-\alpha}$$

In this model, we assume that physical capital is homogeneous, no matter what the firm's scale is. It means one unit physical capital in both production functions has the same productivity.

Suppose we combine a large firm and a small firm into a new firm. The proportion of large firms is λ_1. The proportion of small firms is $1-\lambda_1$, 0< λ_1 <1.

According to Felipe (2006), we can set up the new production function such as

$$y = \lambda_1 A(1+\theta_1)k^{\alpha}[(1+\theta_2)k_a]^{1-\alpha} + (1-\lambda_1)Ak^{\alpha}k_a{}^{1-\alpha}$$
$$= Ak^{\alpha}k_a{}^{1-\alpha}\ [\lambda_1(1+\theta_1)(1+\theta_2)^{1-\alpha} + (1-\lambda_1)] \quad \text{....(2-30)}$$

Now that we have the production function of a hybrid firm, we can use the previous assumption to calculate the growth rate of a country with a representative hybrid firm. The other assumptions are the same as in the previous subsections:

The present value Hamiltonian for the representative household 's optimization is illustrated by:

$$H = \frac{c^{1-\sigma}(t)-1}{1-\sigma}e^{-\rho t} + \lambda(\dot{k}) \text{ ....(2-31)}$$

After differentiating equation (2-31) with respect to c , we get equation (2-32). It means that the present value marginal utility must equal the marginal value of the physical capital.

$$\frac{\partial H}{\partial c} = c^{-\sigma}e^{-\rho t} - \lambda = 0 \rightarrow c^{-\sigma}e^{-\rho t} = \lambda \text{ ....(2-32)}$$

Differentiating equation (2-31) with respect to k , we obtain equation (2-33).

$$\frac{\partial H}{\partial k} \Rightarrow \lambda(f'(k)-\delta) = -\dot{\lambda} \text{ ....(2-33)}$$

We also need equation (2-34) as a transversality condition; it means that there will exist an optimal solution.

$$\lim_{t\to\infty} \lambda(t)k(t) = 0 \text{ ....(2-34)}$$

From equation (2-33), we get equation (2-35)

$$(f'(k)-\delta) = -\frac{\dot{\lambda}}{\lambda} \text{ ....(2-35)}$$

Logarithm equation (2-32) and differentiating with respect to time, we get equation (2-36). We can also put equation (2-35) into equation (2-36) and get the common growth rate from the expression for consumption.

$$\frac{\dot{c}}{c} = \frac{-\frac{\dot{\lambda}}{\lambda}-\rho}{\sigma} = \frac{f'(k)-\delta-\rho}{\sigma} \text{ ....(2-36)}$$

From previous equation (2-4), we obtain equation (2-37)

$$\frac{\dot{k}}{k} = \frac{f(k) - \delta k - c}{k} \quad \text{................................(2-37)}$$

We know all firms are identical. In the social planner optimal equilibrium, $k = k_a$.

Together with equation (2-30) and (2-37),

$$-\frac{c}{k} = \frac{\dot{k}}{k} - A[\lambda_1(1+\theta_1)(1+\theta_2)^{1-\alpha} + (1-\lambda_1)] + \delta \quad \text{.....(2-38)}$$

In the steady state, all variables grow at constant rates. The growth rate of capital per person is constant. A , δ , θ_1 , θ_2 and λ_1 are all exogenous variables and also constant. Therefore, the right side of equation (2-38) for $\frac{c}{k}$ is constant. With logarithm equation (2-38) and differentiating with respect to time, we obtain equation (2-39). The rate of change in consumption equals the rate of change in the physical capital.

$$\frac{\dot{c}}{c} = \frac{\dot{k}}{k} \quad \text{......................(2-39)}$$

Given $y = A\ [\lambda_1(1+\theta_1)(1+\theta_2)^{1-\alpha} + (1-\lambda_1)]k$, it also follows $\frac{\dot{k}}{k} = \frac{\dot{y}}{y}$. Because the rate of change in output equals to the rate of change in physical capital, we know that the rate of change in output equals the rate of change in physical capital and consumption.

$$\text{Then } \frac{\dot{c}}{c} = \frac{\dot{k}}{k} = \frac{\dot{y}}{y} \quad \text{.....................(2-40)}$$

Hence, we can induce the growth rate in a country with small and large firms as equation (2-41)

$$\frac{\dot{y}}{y}=\frac{A\ [\lambda_1(1+\theta_1)(1+\theta_2)^{1-\alpha}+(1-\lambda_1)]-\delta-\rho}{\sigma}\ \ldots.(2\text{-}41)$$

Because the parameters are exogenous variables and they are the same among these countries, we can compare the growth rate between these three countries

Country A (large firms): $\frac{\dot{y}}{y}=\gamma_L=\frac{A(1+\theta_1)(1+\theta_2)^{1-\alpha}-\delta-\rho}{\sigma}$

Country B (small firms): $\frac{\dot{y}}{y}=\gamma_S=\frac{A-\delta-\rho}{\sigma}$

Country C (hybrid firms): $\frac{\dot{y}}{y}=\gamma_M=\frac{A[\lambda_1(1+\theta_1)(1+\theta_2)^{1-\alpha}+(1-\lambda_1)]-\delta-\rho}{\sigma}$

Because $(1+\theta_1)(1+\theta_2)^{1-\alpha}>1$, we know $\gamma_L>\gamma_M$ and $\gamma_M>\gamma_S$

Accordingly, we get the result that reveals the growth rate is positively correlated between the scale of the internal effect and the external effect. Although the internal effect and the external effect are exogenous variables, they could make a difference between those countries. If a country has more large firms, a higher growth rate will exist.

Second, the ratio of large firms λ_1 can determine the growth rate in this model. The larger λ_1 in a country is, the higher the growth rate it has.

2.3.4 Transitional dynamic in one-sector

Transitional dynamic is a process that every variable converge to its stable point. When one-sector model attains long-term equilibrium (i.e., balanced growth path), it means the rate of change in consumption per labor equals the rate of change in the physical capital per labor.

$$\frac{\dot{c}}{c} = \frac{-\frac{\dot{\lambda}}{\lambda} - \rho}{\sigma} = \frac{A - \delta - \rho}{\sigma} = \frac{\dot{k}}{k} \qquad \text{....(2-42)}$$

Here the inverse of the elasticity of intertemporal substitution σ must be large than 0. If $A > \delta + \rho$, then the economy will grow up continually until the parameters change. Therefore, the situation $\frac{\dot{C}}{C} = 0$ does not exist. If some parameters changed, the growth rate will change from one value to another one suddenly. In consequence, in one-sector model (i.e. AK Model[5]), there does not exist transitional dynamic. We will talk about and solve the problem in chapter 3.

In the social planner equilibrium solution, we take the economy wide stock (k_a) as one of the production factors. The representative firm's capital equals the economy wide stock ($k_a = k$). If a country has a higher ratio of large firms, the firms would enjoy the higher positive internal effect (θ_1) and the positive external effect (θ_2) in the production function. Although the internal effect (θ_1) and the external effect (θ_2) are exogenous variables, we do not know what the real values of the exogenous variables are. Nevertheless, we get an important result in this model. If a country has a larger percentage of large firms, it will create a higher economic growth rate.

In addition, the proportion of large firms λ_1 can also influence the growth rate in this model. It is beneficial to economic growth with a larger λ_1.

In this chapter, the variables that could affect economic growth are all exogenous variables, it means the firms could not do anything (e.g. innovation) or government could not formulate any policy to promote the economic growth. It seems to be a disadvantage to use this model, even the ratio of large firms and the external effect is beneficial to economic growth. We would solve this problem in chapter 3

[5] The first economist to use a production function of the AK type was von Neumann(1937)

Chapter 3　Two-Sector Models (exogenous variable)

3.1 Introduction

In chapter 2, the one-sector model has confirmed that a country with more large firms will have a higher growth rate. But in the one-sector model, there exists only one physical capital sector and we cannot find the transitional dynamics. Now we would like to modify the model by adding a human capital sector to the production function.

Uzawa (1964) and Lucas (1988) both stated that the accumulation of human capital could raise economic growth. For instance, the workers can enhance their skill through accumulated work experience. It is also called 'Learning by doing'. Lau (1994) stated that the physical capital and the human capital are substitutable. The human capital can slow down the effect of decreasing return to scale in the physical capital. Romer (1990) and Rebelo (1991) illustrated that the human capital is a reproduction factor in the production function, and most of the human capital does not show decreasing return to scale.

The basic model in this chapter will adopt the Uzawa (1965), Lucas(1988) and Rebelo (1991) approach. Aside from the physical capital in the goods sector, we add human capital in this model. It means that there are two-sectors in a country. One sector produces the physical capital and the other sector produces the human capital. We can image that the first sector is a factory and the second sector is the school.

If we add the human capital into the production function, the human capital should be beneficial to production process. For instance, the workers with higher levels of education can apply their knowledge in the production process. They can pass on their skill to other co-laborers (i.e., the spillover effect) and make other firms produce more efficiently.

3.2 The production function of firms

In this chapter, we assume the physical production function in the representative large firm is

$$y = f(k, k_a) = A(1+\theta_1)k^{\alpha}[(1+\theta_2)k_a]^{\beta}\,(uh)^{1-\alpha-\beta}$$

and physical production function in the representative small firm is

$$y = f(k, k_a) = Ak^{\alpha}k_a{}^{\beta}\,(uh)^{1-\alpha-\beta}$$

In this model, we also define that physical capital k have the same quality. No matter what the scale of firms is, the quality of physical capital is always identical. It means one unit physical capital in both production functions has the same utility.

Suppose there exist the large firms and small firms in a country in the meantime. The proportion of large firms is λ_1. The proportion of small firms is $1-\lambda_1$, 0< λ_1 <1.

In reference to Felipe (2006), we can suppose the new production function is as follows:

$$y = \lambda_1 A(1+\theta_1)k^{\alpha}[(1+\theta_2)k_a]^{\beta}(uh)^{1-\alpha-\beta} + (1-\lambda_1)Ak^{\alpha}k_a{}^{\beta}(uh)^{1-\alpha-\beta}$$

Now that we have the production function of representative firm, we can use the previous assumption to calculate the growth rate in a country.

3.3 The basic model

Consider a closed economy consisting of a representative household and a skill training unit (i.e., the education sector). The representative household is also the producer of a single commodity (i.e., the goods sector), and the skill training unit produces only human capital. For convenience's sake, there are merely two kinds of firms in this world. There exist large firms[6] and small firms in a country and both of

[6] In this chapter, we still assume only large firms have internal and external effects.

them will form a new company. The ratio of large firms is λ_1 and the ratio of small firms is $1-\lambda_1$. We can image that the large firm merge the small firm to form a new firm and they keep their own production function. The large firm has the share λ_1 of the new firm and the small firm has the share $1-\lambda_1$ of the new firm. Hence, they can combine the two different production functions to a single production function that depends on the number of shares they have. The other assumptions are the same as in chapter 2. There is no government in our model. It means that there is no tax and subsidy in this economic system and international trade is not allowed for.

The two-sectors will be referred to as the goods sector and education sector. There are two reproducible factors in production. One is the physical capital (K), and the other is the human capital (Q). Human capital is used in goods sector and education sector. The physical capital is only used in goods sector, because we assume the process of production in human capital only depends on the quantity of human capital the representative firm used, and it makes it easier to solve the equations below.

3.3.1 The representative household

The main assumption is the same as in chapter 2. There exists a representative household in each economy, in which the representative household is the producer of goods. A representative household with an infinite planning horizon is assumed in this model. Moreover, the labor supply in the representative household is fixed (i.e., the number of workers is constant). It means that there is no birth and death in the representative household and the number of household stays constant. The household is postulated to choose its private level of consumption to maximize the discounted sum of future instantaneous utilities:

$$\text{Max } U = \int_0^\infty e^{-\rho t} \mu(c(t)) L dt \quad \text{and} \quad \mu(c(t)) = \frac{c^{1-\sigma}(t) - 1}{1-\sigma}$$

In the function above, U is the utility function and ρ is the subjective time preference rate. c is the consumption, L is the number of labor and σ is the inverse of the elasticity of intertemporal substitution which measures the curvature of the utility function.

3.3.2 The production function in the goods sector

We follow the assumptions by Rebelo (1991) and modify some setup. The production function in the goods sector is assumed to be

$$y = f(k, k_a, h) = A[\lambda_1(1+\theta_1)(1+\theta_2)^\beta + (1-\lambda_1)](k)^\alpha \; k_a{}^\beta \; (uh)^{1-\alpha-\beta} \quad \text{......(3-1)}$$

where y is the output per unit of labor, k is the physical capital of representative firm, k_a is the other firm's average capital (also called average economy wide capital stock), and h is the human capital per unit of labor. θ_1 is the size of the internal effect of the firm[7]; θ_2 is the size of the external effect of the firm. A is the basic technology level. α is the share of a firm's capital in the production function , while β is the share of the average economy wide capital stock. $1-\alpha-\beta$ is the share of human capital in the production function. u is the proportion of total human capital input in the goods sector.

The production function exhibits diminishing returns to scale with k. However, when θ_1 and θ_2 is larger than zero, there exists constant or increasing returns to scale.

If there are only small firms, in social planner equilibrium solution the equation (3-1) will become

---

[7] Different internal effects exist with different θ_1. The value of θ_1 is related with the scale of firm positively.

$$y = f(k,h) = A(k)^{\alpha+\beta} \ (uh)^{1-\alpha-\beta}$$

and the result is just as Lucas (1988) has reported ,and we do not discuss the condition without internal and external effects here.

3.3.3 The production function in education sector

We follow the assumptions by Uzawa (1965) and Lucas (1988) and apply their setup in the human capital. The production function in the education sector is assumed to be

$$Q = B(1-u)h \text{ ……………………………(3-2)},$$

where h is the human capital per unit of labor and used in the physical sector and education sector, but k is not productive in the education sector. Q is the output in education sector. B is exogenous knowledge level. To simplify our calculation, we assume B is the same in each country. $1-u$ is the proportion of the total human capital used in the education sector.

3.3.4 The investment and education production function

In the aspects of physical investment, the representative household does not save. The output deducts to consumption and depreciation and the rest of the output will be devoted to the next period.

The physical capital accumulation constraint of firm is

$$\dot{k} = f(k) - c - \delta k \text{ ……………………(3-3)}$$

In the aspects of education production, new human capital for the next period equal to the average output in education sector deducts depreciation.

$$\dot{h} = Q - \delta h \text{ ……………….…………...(3-4)}$$

where δ is depreciation rate and an over dot denotes the time derivative.

In this chapter, in order to find the Pareto optimality, we try to find the solution in "social planner equilibrium solution".

3.4 The social planner optimal solution in two-sector models

As in chapter 2, the number of labor is L and keeps constantly equal to 1.

$$L(t) = L = 1$$

,where U is the overall utility of household and is supposed to be the constant intertemporal elasticity of substitution

$$\max \; U = \int_0^\infty e^{-\rho t} \mu(c(t)) L dt$$

$$\mu(c(t)) = \frac{c^{1-\sigma}(t) - 1}{1-\sigma}$$

μ is the utility function of consumption and it is assumed to satisfy the Inada conditions. $\lim_{C \to 0} \mu'(c) = \infty, \lim_{C \to \infty} \mu'(c) = 0$

The present value Hamilton equation for the representative household 's optimization is illustrated by

$$H = \frac{c^{1-\sigma}(t) - 1}{1-\sigma} e^{-\rho t} + \lambda(\dot{k}) + \phi(\dot{h}) \qquad \text{……………………(3-5)}$$

, where λ and ϕ are constant variables of the physical capital and human capital, and it also means the shadow price between k and h.

In the social planner optimal equilibrium solution[8], all firms are identical. Thus, the other firm's average capital (average economy wide capital stock) equals the capital of the representative firm ($k = k_a$)。

After differentiating with respect to c (control variable), we obtain equation (3-6). It means that the present value marginal utility must equal the marginal value of physical capital.

[8] Here we assume the social planner is a decision maker who maximizes the social welfare and achieve the best result for all conditions involved. The result will be on Pareto optimality.

$$\frac{\partial H}{\partial c} = c^{-\sigma}e^{-\rho t} - \lambda = 0 \rightarrow c^{-\sigma}e^{-\rho t} = \lambda \text{ ………………..(3-6)}$$

Equation (3-5) differentiating with respect to u, we obtain equation (3-7)

$$\frac{\partial H}{\partial u} = \lambda A(1-\alpha-\beta)[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]k^{\alpha+\beta}h^{1-\alpha-\beta}u^{-\alpha-\beta} - \phi Bh = 0 \text{ …….(3-7)}$$

Equation (3-5) differentiating with respect to k and h (state variable), we obtain equations (3-8) and (3-9).

$$\frac{\partial H}{\partial k} \Rightarrow \lambda A(\alpha+\beta)[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]u^{1-\alpha-\beta}h^{1-\alpha-\beta}k^{\alpha+\beta-1} - \delta) = -\dot{\lambda}$$

……...(3-8)

$$\frac{\partial H}{\partial h} \Rightarrow \lambda A(1-\alpha-\beta)[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]k^{\alpha+\beta}u^{1-\alpha-\beta}h^{-\alpha-\beta} + \phi[B(1-u) \; -\delta] = -\dot{\phi}$$

…….(3-9)

From equation (3-8), we obtain equation (3-10)

$$\frac{\dot{\lambda}}{\lambda} = -(A(\alpha+\beta)[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]u^{1-\alpha-\beta}h^{1-\alpha-\beta}k^{\alpha+\beta-1} - \delta) \text{ ….(3-10)}$$

From equations (3-3) and (3-4), we obtain equations (3-11)(3-12)

$$\frac{\dot{k}}{k} = A[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]k^{\alpha+\beta-1}(uh)^{1-\alpha-\beta} - \frac{c}{k} - \delta \text{ …………(3-11)}$$

$$\frac{\dot{h}}{h} = B(1-u) - \delta \text{ ……………………(3-12)}$$

We need equations (3-12.1) and (3-12.2) as a transversality condition, and it means that there will exist an optimal solution.

$$\lim_{t\to\infty} \lambda(t)k(t) = 0 \quad \text{……………………(3-12.1)}$$

$$\lim_{t\to\infty} \phi(t)h(t) = 0 \quad \text{……………………(3-12.2)}$$

Together with equations (3-6) and (3-10), we get equation (3-13).

$$\frac{\dot{c}}{c}=\frac{A(\alpha+\beta)[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta}+(1-\lambda_1)]u^{1-\alpha-\beta}h^{1-\alpha-\beta}k^{\alpha+\beta-1}-\delta-\rho}{\sigma} \quad \text{……….(3-13)}$$

In the balanced growth path, all variables grow at the same speed. The growth rate of consumption is equal to the growth rate of physical capital.

3.5 Transitional dynamics and numerical methods

A steady state is the state where all variables grow at a constant rate. When all variables grow up at a constant rate (Balanced growth path), the growth rate of variable c equals that of variable k and variable h.

$$\frac{\dot{c}}{c}=\frac{\dot{k}}{k}=\frac{\dot{h}}{h}$$

Then we define $z\equiv\frac{h}{k}, x\equiv\frac{c}{k}$,

where z is the proportion of human capital and physical human, and x is the proportion of consumption and physical human. Because h, c and k grow at a common speed, the growth rate of z and x is equal to zero. Accordingly, we can find the transitional dynamics in this model.

Equations (3-11), (3-12) and (3-13) taken together, we obtain equations (3-14) and (3-15)

$$\begin{aligned}\frac{\dot{z}}{z}=\frac{\dot{h}}{h}-\frac{\dot{k}}{k}&=B(1-u)-\delta-[A\,[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta}+(1-\lambda_1)]k^{\alpha+\beta-1}(uh)^{1-\alpha-\beta}-\frac{c}{k}-\delta]\\&=B(1-u)-A\,[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta}+(1-\lambda_1)]k^{\alpha+\beta-1}(uh)^{1-\alpha-\beta}+\frac{c}{k}\\&=B(1-u)-A\,[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta}+(1-\lambda_1)]z^{1-\alpha-\beta}u^{1-\alpha-\beta}+x\end{aligned}$$

……….(3-14)

$$
\frac{\dot{x}}{x}=\frac{\dot{c}}{c}-\frac{\dot{k}}{k}=\frac{A(\alpha+\beta)[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta}+(1-\lambda_1)]u^{1-\alpha-\beta}h^{1-\alpha-\beta}k^{\alpha+\beta-1}-\delta-\rho}{\sigma}-[A[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta}+(1-\lambda_1)]k^{\alpha+\beta-1}(uh)^{1-\alpha-\beta}-\frac{c}{k}-\delta]
$$

$$
=\frac{A[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta}+(1-\lambda_1)](\alpha+\beta)u^{1-\alpha-\beta}z^{1-\alpha-\beta}-\delta-\rho}{\sigma}-[A[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta}+(1-\lambda_1)]z^{1-\alpha-\beta}u^{1-\alpha-\beta}-x-\delta]
$$

$$
=\frac{\alpha+\beta-\sigma}{\sigma}A[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta}+(1-\lambda_1)]z^{1-\alpha-\beta}u^{1-\alpha-\beta}+x+\frac{\delta(\sigma-1)-\rho}{\sigma}
$$

……….(3-15)

Equations (3-7) and (3-9), we obtain equation (3-16) and (3-17)

$$
\frac{A}{B}[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta}+(1-\lambda_1)]k^{\alpha+\beta}h^{-\alpha-\beta}u^{-\alpha-\beta}(1-\alpha-\beta)=\frac{\phi}{\lambda}
$$

$$
\Rightarrow\frac{A}{B}[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta}+(1-\lambda_1)]z^{-\alpha-\beta}u^{-\alpha-\beta}(1-\alpha-\beta)=\frac{\phi}{\lambda}
$$

……………..(3-16)

$$
\frac{\lambda}{\phi}A\,[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta}+(1-\lambda_1)]k^{\alpha+\beta}(1-\alpha-\beta)u^{1-\alpha-\beta}h^{-\alpha-\beta}+[B(1-u)-\delta]=-\frac{\dot{\phi}}{\phi}
$$

….(3-17)

Equations (3-16) and (3-17) put together is equation (3-18)

$$
-B+\delta=\frac{\dot{\phi}}{\phi}
$$

…………..(3-18)

By taking logarithms in equation (3-16) and differentiating with respect to time, we obtain equation (3-19)

$$
\frac{\dot{\phi}}{\phi}-\frac{\dot{\lambda}}{\lambda}=-(\alpha+\beta)\frac{\dot{u}}{u}-(\alpha+\beta)\frac{\dot{z}}{z}
$$

…………………………….(3-19)

Finally,

$$
\frac{\dot{u}}{u}=\frac{-1}{(\alpha+\beta)}[\frac{\dot{\phi}}{\phi}-\frac{\dot{\lambda}}{\lambda}]-\frac{\dot{z}}{z}
$$

$$
=\frac{-1}{(\alpha+\beta)}[-B+\delta+A\,(\alpha+\beta)[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta}+(1-\lambda_1)]u^{1-\alpha-\beta}z^{1-\alpha-\beta}-\delta]
$$

….(3-20)

$$
-B(1-u)+A\,[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta}+(1-\lambda_1)]z^{1-\alpha-\beta}u^{1-\alpha-\beta}-x
$$

$$
=\frac{B}{(\alpha+\beta)}-B(1-u)-x
$$

Now we have the change rate of z, x and u. When all variables are in a steady state, the change rates of those variables are equal to zero. The steady state of this system can be found by setting the three time derivatives equations ($\frac{\dot{x}}{x} = \frac{\dot{u}}{u} = \frac{\dot{z}}{z} = 0$) to zero.

The results are as follows:

$$\mu^* = \frac{\rho - (1-\sigma)(B-\delta)}{B\sigma}$$

$$z^* = \frac{A(\alpha+\beta)^2 g\{B[1+(\alpha+\beta-1)\sigma] - \rho + \delta(\sigma-1)\} - \sigma B^2}{gA(\alpha+\beta)\{\delta(\sigma-1) + [(\alpha+\beta-1)\sigma+1]B - \rho\}}$$

$$x^* = \frac{B}{\alpha+\beta} + \frac{\rho+\delta-B}{\sigma} - \delta$$

here we denote $g = \lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)$.

From these equations above, we would like to estimate the influence of internal and external effects on the growth rate in physical capital and consumption and calculate the real values of them.

In order to explain the influence of internal and external effects on the growth rate, we use some numerical methods to calculate it. In terms of the value of other parameters, we adapt the assumptions from Lucas (1988) and Benhabib (1994). ρ is the discount rate of time at 0.025. δ is the depreciation rate equal to 0.05. The basic technical levels in both sectors are the same and equal to 1($A = B = 1$). α is the coefficient of a firm's capital in the production function at 0.4, whereas β is the coefficient of average economy wide capital stock at 0.2. σ is the inverse of the constant intertemporal elasticity of substitution at 5. We take the real value from Taiwan about the ratio of large firms λ_1, and it is about 3 percent.

Table 3-1 The relationship between internal, external effects and growth rate (B=1)

α	β	σ	ρ	δ	θ_1	θ_2	$\frac{\dot{c}^*}{c}$
0.4	0.2	5	0.025	0.05	0.1	0.1	0.2638
0.4	0.2	5	0.025	0.05	1	1	0.2699
0.4	0.2	5	0.025	0.05	0.1	0.5	0.2642
0.4	0.2	5	0.025	0.05	0.5	0.1	0.2659
0.4	0.2	5	0.025	0.05	0.1	1	0.2646
0.4	0.2	5	0.025	0.05	1	0.1	0.2686
0.4	0.2	5	0.025	0.05	0.01	0.01	0.2632

Note: $A = 1, B = 1, \lambda_1$=0.03

From Table 3-1, we observe the influence of the internal effect θ_1 and external effect θ_2 on economic growth. The internal effect θ_1 has a slightly stronger effect on the economic growth than the external effect θ_2. If the value of the internal effect θ_1 increases by 0.1, the growth rate will rise by 0.00052. If the value of the external effect θ_2 increases by 0.1, the growth rate will grow by 0.0001.

If we assume $B = 0.5$, we obtain the same result. The internal effect θ_1 also has a weak stronger effect on the economic growth than the external effect θ_2. However, the scale of influence of internal effect will be different. If the value of the internal effect θ_1 increases by 0.1, the growth rate will rise by 0.0007. If the value of the external effect θ_2 increases by 0.1, the growth rate will grow by 0.0001. Thus, the internal effect will have larger effects on growth rate, when a country has a higher knowledge level. The internal effect has almost 7 time stronger effect on the growth rate than the external effect

Table 3-2 The relationship between internal, external effects and growth rate (B=0.5)

α	β	σ	ρ	δ	θ_1	θ_2	$\frac{\dot{c}^*}{c}$
0.4	0.2	5	0.025	0.05	0.1	0.1	0.2161
0.4	0.2	5	0.025	0.05	1	1	0.2242
0.4	0.2	5	0.025	0.05	0.1	0.5	0.2165
0.4	0.2	5	0.025	0.05	0.5	0.1	0.2189
0.4	0.2	5	0.025	0.05	0.1	1	0.2170
0.4	0.2	5	0.025	0.05	1	0.1	0.2224
0.4	0.2	5	0.025	0.05	0.01	0.01	0.2153

Note: $A=1, B=0.5, \lambda_1$=0.03.

By the simulation process, we know higher external and internal effects will lead to a higher growth rate. If we keep the external and internal effect constant, we can discuss the influence of the ratio of large firm in economic growth.

From Table 3-3, we observe the influence of the ratio of large firms λ_1 on economic growth. If a country has a higher proportion of large firms, the country will have higher economic growth. When the ratio of large firms λ_1 decreases by 0.1, the growth rate will decline by 0.0018.

Table 3-3 The relationship between the ratio of large firms and growth rate (B=1)

α	β	σ	ρ	δ	λ_1	$1-\lambda_1$	$\frac{\dot{c}^*}{c}$
0.4	0.2	5	0.025	0.05	1	0	0.2838
0.4	0.2	5	0.025	0.05	0.9	0.1	0.2817
0.4	0.2	5	0.025	0.05	0.7	0.3	0.2772
0.4	0.2	5	0.025	0.05	0.5	0.5	0.2736
0.4	0.2	5	0.025	0.05	0.3	0.7	0.2694
0.4	0.2	5	0.025	0.05	0.1	0.9	0.2653
0.4	0.2	5	0.025	0.05	0	1	0.2632

Note: $A=1, B=1, \theta_1$=0.1, θ_2=0.1.

In Table 3-4, we assume $B=0.5$, it means that now this country has a lower knowledge level and we can compare the result with Table 3-3. We just change the value of B from 1 to 0.5 and keep other parameters at the same value.

We get the similar results as in Table 3-4. A higher proportion of large firms will lead to higher economic growth. When the ratio of large firms λ_1 increases by 0.1, the growth rate will rise by 0.0017.

The economic growth rates in Table 3-4 are smaller than Table 3-3, because the knowledge level in Table 3-4 is low. Therefore, even if the higher ratio of large firms can be beneficial to economic growth, the knowledge level still plays an important role.

Table 3-4 The relationship between the ratio of large firms and growth rate (B=0.5)

α	β	σ	ρ	δ	λ_1	$1-\lambda_1$	$\frac{\dot{c}^*}{c}$
0.4	0.2	5	0.025	0.05	1	0	0.2329
0.4	0.2	5	0.025	0.05	0.9	0.1	0.2312
0.4	0.2	5	0.025	0.05	0.7	0.3	0.2278
0.4	0.2	5	0.025	0.05	0.5	0.5	0.2243
0.4	0.2	5	0.025	0.05	0.3	0.7	0.2208
0.4	0.2	5	0.025	0.05	0.1	0.9	0.2173
0.4	0.2	5	0.025	0.05	0	1	0.2155

Note: $A=1, B=0.5, \theta_1$=0.1, θ_2=0.1.

Now we will investigate the stability properties of the balanced growth paths (BGPs) and describe the region in the parameter space, which yields unique and indeterminate equilibrium.

Benhabib and Perli (1994) stated a special way to judge how many roots with positive real parts in a matrix. We adopt this method to calculate the number of roots with positive real parts and analyze the situation. For example, if

$$J^* = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$$

, the characteristic values of J^* are the solutions of its characteristic equation

$$-\lambda^3 + TrJ^*\lambda^2 - BJ^*\lambda + DetJ^* = 0,$$

where

$$BJ^* = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} + \begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} + \begin{bmatrix} a_{11} & a_{13} \\ a_{31} & a_{33} \end{bmatrix}$$

$$TrJ^* = a_{11} + a_{22} + a_{33}$$

$$DetJ^* = a_{11}\begin{bmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{bmatrix} - a_{12}\begin{bmatrix} a_{21} & a_{23} \\ a_{31} & a_{33} \end{bmatrix} + a_{13}\begin{bmatrix} a_{21} & a_{22} \\ a_{31} & a_{32} \end{bmatrix}$$

The number of roots of the polynomial in the characteristic equation with positive real parts is equal to the number of variations of signs in the scheme

–1, Trace J, -BJ+(Det J/Tr J) and Det J

If we calculate how many times the value change from positive to negative between –1, Trace J, -BJ+(Det J/Tr J) and Det J, we know the number of positive real roots in the matrix.

For example, the characteristic equation has three roots with positive real parts if Trace J＞0, -BJ+(Det J/Tr J)＜0 and Det J＞0, because the sign from the positive to the negative has changed three times.

We know z is the proportion of human capital and physical human, and that x is the proportion of consumption and physical human. It means z and x are both positive and larger than 0.

From Eq. (3-14), (3-15) and (3-20), we obtain

$$\dot{z} = B(1-u)z - A\,[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]z^{2-\alpha-\beta}u^{1-\alpha-\beta} + xz$$

$$\dot{x} = \frac{\alpha+\beta-\sigma}{\sigma}A\,[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]z^{1-\alpha-\beta}u^{1-\alpha-\beta}x + x^2 + \frac{\delta(\sigma-1)-\rho}{\sigma}x$$

$$\dot{u} = \frac{B}{(\alpha+\beta)}u - B(1-u)u - xu$$

We use the Jacobian matrix to calculate the characteristic values. First, we compute the Jacobian matrix of the system and obtain

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} \\ J_{21} & J_{22} & J_{23} \\ J_{31} & J_{32} & J_{33} \end{bmatrix}$$

$$J_{11} = \frac{\partial \dot{z}}{\partial z} = B(1-u) - A(2-\alpha-\beta)[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]z^{1-\alpha-\beta}u^{1-\alpha-\beta} + x$$

$$J_{12} = \frac{\partial \dot{z}}{\partial x} = z$$

$$J_{13} = \frac{\partial \dot{z}}{\partial u} = -Bz - A(1-\alpha-\beta)[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]z^{2-\alpha-\beta}u^{-\alpha-\beta}$$

$$J_{21} = \frac{\partial \dot{x}}{\partial z} = \frac{\alpha+\beta-\sigma}{\sigma} A(1-\alpha-\beta)[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]z^{-\alpha-\beta}u^{1-\alpha-\beta}x$$

$$J_{22} = \frac{\partial \dot{x}}{\partial x} = \frac{\alpha+\beta-\sigma}{\sigma} A[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]z^{1-\alpha-\beta}u^{1-\alpha-\beta} + 2x + \frac{\delta(\sigma-1)-\rho}{\sigma}$$

$$J_{23} = \frac{\partial \dot{x}}{\partial u} = \frac{\alpha+\beta-\sigma}{\sigma} A(1-\alpha-\beta)[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]z^{1-\alpha-\beta}u^{-\alpha-\beta}x$$

$$J_{31} = \frac{\partial \dot{u}}{\partial z} = 0$$

$$J_{32} = \frac{\partial \dot{u}}{\partial x} = u$$

$$J_{33} = \frac{\partial \dot{u}}{\partial u} = \frac{B}{(\alpha+\beta)} + B(2u-1) - x$$

The Jacobian matrix is huge, and it is very difficult to calculate the result. Since we are interested in the Jacobian matrix evaluated at the equilibrium, we can substitute the steady state values of x^*, u^* and z^* in the above matrix, which would lead to an easier method to obtain the result. The matrix can be rewritten as

$$J_{11} = B(1-u) - A[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]z^{1-\alpha-\beta}u^{1-\alpha-\beta} + x - A(1-\alpha-\beta)[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]z^{1-\alpha-\beta}u^{1-\alpha-\beta}$$

$$= \frac{\dot{z}}{z} - A(1-\alpha-\beta)[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]z^{1-\alpha-\beta}u^{1-\alpha-\beta}$$

Because $\frac{\dot{z}}{z} = 0$, on the BGP we have

$$J^*_{11} \equiv \frac{\partial \dot{z}}{\partial z}|_{BGP} = -A(1-\alpha-\beta)[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]z^{*1-\alpha-\beta}u^{*1-\alpha-\beta}$$

$$J_{22} = \frac{\alpha+\beta-\sigma}{\sigma}A[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]z^{1-\alpha-\beta}u^{1-\alpha-\beta} + x \ + \frac{\delta(\sigma-1)-\rho}{\sigma} + x$$

$$= \frac{\dot{x}}{x} + x$$

Because $\frac{\dot{x}}{x} = 0$, on the BGP we have

$$J^*_{22} \equiv \frac{\partial \dot{x}}{\partial x}|_{BGP} = x^*$$

$$J_{33} = \frac{B}{(\alpha+\beta)} - B(1-u) - x + Bu = \frac{\dot{u}}{u} + Bu$$

Because $\frac{\dot{u}}{u} = 0$, on the BGP we have

$$J^*_{33} \equiv \frac{\partial \dot{u}}{\partial u}|_{BGP} = Bu^*$$

If we express J^*_{13}, J^*_{21}, and J^*_{23} as functions of J^*_{11}, as well as relate J^*_{22} to $\frac{\dot{x}}{x}$ and J^*_{33} to $\frac{\dot{u}}{u}$, we can rewrite the Jacobian matrix evaluated at the BGP as

$$J^* = \begin{bmatrix} J^*_{11} & z^* & -Bz^* + J^*_{11}\frac{z^*}{u^*} \\ -\psi J^*_{11}\frac{x^*}{z^*} & x^* & -\psi J^*_{11}\frac{x^*}{u^*} \\ 0 & u^* & Bu^* \end{bmatrix}$$

The characteristic values of J^* are the solutions of its characteristic equation

$$-\lambda^3 + TrJ^*\lambda^2 - BJ^*\lambda + DetJ^* = 0,$$

where

$$BJ^* = \begin{bmatrix} J^*_{11} & z^* \\ -\psi J^*_{11}\frac{x^*}{z^*} & x^* \end{bmatrix} + \begin{bmatrix} x^* & -\psi J^*_{11}\frac{x^*}{u^*} \\ u^* & Bu^* \end{bmatrix} + \begin{bmatrix} J^*_{11} & -Bz^* + J^*_{11}\frac{z^*}{u^*} \\ 0 & Bu^* \end{bmatrix}$$

$$= J^*_{11}x^* + 2\psi J^*_{11}x^* + Bx^*u^* + BJ^*_{11}u^*$$

if $\frac{\alpha+\beta-\sigma}{\sigma} = \psi$

$$TrJ^* = J^*_{11} + x^* + Bu^*$$

$$DetJ^* = J^*_{11}Bx^*u^*(1+2\psi) + J^{*2}_{11}x^*(\psi - 1)$$

The number of roots of the polynomial in the characteristic equation with positive real parts is equal to the number of signs changed in the scheme

–1, Trace J, -BJ+(Det J/Tr J) and Det J

With this method, we can determine the signs of the real parts of the roots of the above characteristic equation.

Condition 1.

$DetJ^*$ is negative if $\sigma < \alpha + \beta < 2\sigma$, while $DetJ^*$ is indeterminate if $\alpha + \beta > 2\sigma$ or $\alpha + \beta < \sigma$.

Consider the $DetJ^* = J_{11}^* Bx^* u^* (1 + 2\psi) + J_{11}^{*2} x^* (\psi - 1)$.

Here J^*_{11} is negative, $(1 + 2\psi)$ is positive and $(\psi - 1)$ is negative while $\sigma < \alpha + \beta < 2\sigma$.

Condition 2.

BJ^* is negative if $\left|x^*\right| < \left|J_{11}^*\right|$; otherwise BJ^* is positive.

Consider the $BJ^* = J_{11}^* x^* (1 + 2\psi) + Bu^* (x^* + J_{11}^*)$.

We assume $\dfrac{\alpha + \beta - \sigma}{\sigma} = \psi$ and $\alpha + \beta$ is larger than 0. If $0.5\sigma < \alpha + \beta < \sigma$, then $\psi > -0.5$; $\psi > 0$ if $\sigma < \alpha + \beta$; If $2\sigma < \alpha + \beta$, $\psi > 1$.

That means $J_{11}^* x^* (1 + 2\psi) > 0$.

If $\left|x^*\right| > \left|J_{11}^*\right|$, then $Bu^* (x^* + J_{11}^*) > 0$

$BJ^* = J_{11}^* x^* (1 + 2\psi) + Bu^* (x^* + J_{11}^*) > 0$

Condition 3.

TrJ^* is positive if $\left|x^* + Bu^*\right| > \left|J_{11}^*\right|$, while TrJ^* is negative if $\left|x^* + Bu^*\right| < \left|J_{11}^*\right|$

We know $x^* > 0$ and $Bu^* > 0$, hence the sign of TrJ^* is determined by the absolute value between $x^* + Bu^*$ and J^*_{11}

If we can confirm the range of those parameters, we are able to calculate the number of positive characteristic value in this model. Form the above results, we derive proposition 1, proposition 2 and proposition 3.

Proposition 1:

If $\left|J_{11}^*\right| < \left|x^*\right|$, $\left|x^* + Bu^*\right| > \left|J_{11}^*\right|$ and $\sigma < \alpha + \beta < 2\sigma$, then TrJ^* >0,

BJ^*>0 and $DetJ^* < 0$

The signs of those variables are

−1,	Trace J,	-BJ+(Det J/Tr J),	Det J
(−)	(+)	(−)	(−)

Proposition 1 has two roots with positive real parts and one root with negative real parts. It means the equilibrium path in this model is a saddle path. There is only one single convergence path.

Proposition 2:

If $\left|x^* + Bu^*\right| < \left|J_{11}^*\right|$, $\left|x^*\right| > \left|J_{11}^*\right|$ and $\sigma < \alpha + \beta < 2\sigma$, then TrJ^* <0,

BJ^*>0 and $DetJ^* < 0$

The signs of those variables are

−1,	Trace J,	-BJ+(Det J/Tr J),	Det J
(−)	(−)	(uncertain)	(−)

If -BJ+(Det J/Tr J) is negative, there are three roots with negative real parts in position 2, and it means all paths are equilibrium paths. If -BJ+(Det J/Tr J) is positive, there is only one root with negative real parts in proposition 2, and it means there exists only one convergence path in this proposition. Hence, there are two possibilities in proposition2.

Proposition 3:

If $\left|x^* + Bu^*\right| < \left|J_{11}^*\right|$, $\left|x^*\right| < \left|J_{11}^*\right|$ and $\sigma < \alpha + \beta < 2\sigma$, then TrJ^* <0,

BJ^*>0 and $DetJ^* < 0$

The signs of those variables are

−1,	Trace J,	-BJ+(Det J/Tr J),	Det J
(−)	(−)	(uncertain)	(−)

If -BJ+(Det J/Tr J)>0, there are two roots with positive real parts and one root with negative real part in proposition 3. It means the equilibrium path in this model is also a saddle path. If -BJ+(Det J/Tr J)<0, there are not any roots with positive real parts and three roots with negative real parts in proposition 3. The situation is same as proposition 2. Hence, there are also two possibilities in proposition 3. One single convergence path or multiple convergence paths.

Proposition 4: ($DetJ^*$ is indeterminate, if $\alpha + \beta > 2\sigma$ or $\alpha + \beta < \sigma$)

In some conditions, the value of $DetJ^*$ is indeterminate. In order to explain the other conditions, we have to resort to numerical methods.

In terms of the value of parameters, we adapt the assumption from Lucas(1988) and Benhabib (1994). ρ is the discount rate of time at 0.025. δ is the depreciation rate equal to 0.05. The basic technical levels in both sectors are the same and equal to 1($A = B = 1$).

Table 3-5 The situation of character roots

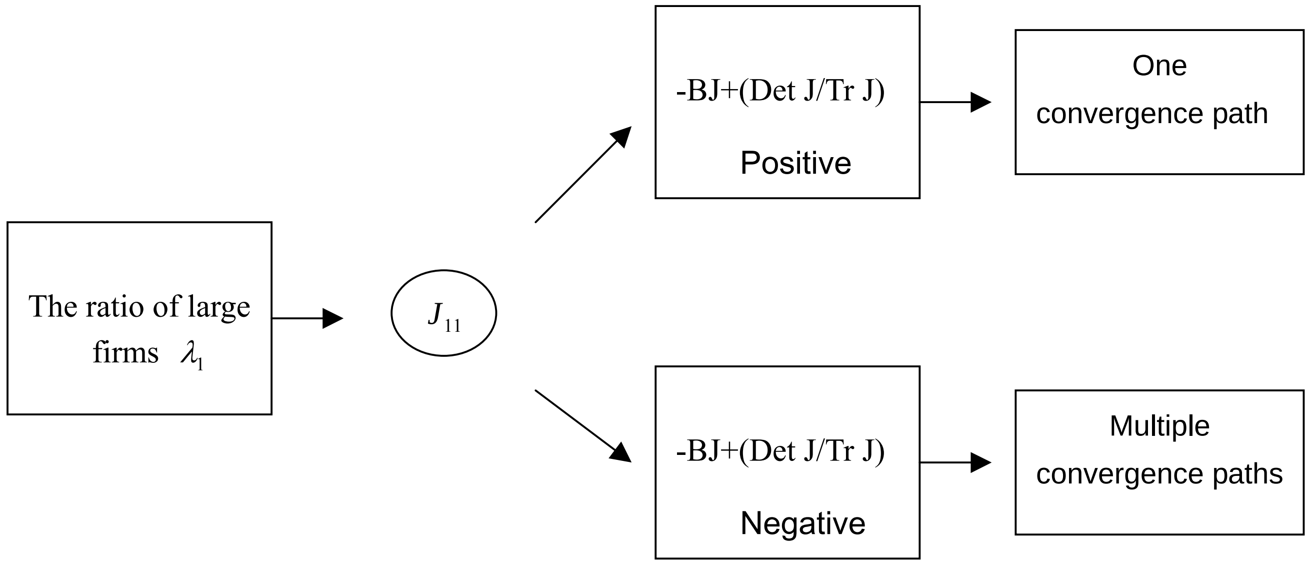
α	β	σ	ρ	δ	θ_1	θ_2	Roots
0.3	0.45	0.3	0.025	0.05	0.01	0.1	－－＋
0.3	0.45	3	0.025	0.05	0.01	0.1	－＋＋
0.4	0.35	0.3	0.025	0.05	0.01	0.1	－－＋
0.4	0.35	3	0.025	0.05	0.01	0.1	－＋＋
0.5	0.25	0.3	0.025	0.05	0.01	0.1	－－＋
0.5	0.25	3	0.025	0.05	0.01	0.1	－＋＋
0.6	0.15	0.3	0.025	0.05	0.01	0.1	－－＋
0.6	0.15	3	0.025	0.05	0.01	0.1	－＋＋

Note: A=1 and B=1

In the numerical examples of Table 3-5, we know the number of roots with positive real parts and negative real parts. If $\alpha + \beta > 2\sigma$, there always exist two roots with negative real parts and one root with positive real parts. It means there are multiple equilibrium paths. If $\alpha + \beta < \sigma$, there are only one root with negative real parts and two roots with positive real parts. There is a unique equilibrium path in this model.

How could the firms' scale affect the convergence paths? We could use Fig 3.1 to describe the situation.

Figure 3-1 The influence of firms' scale to convergence paths



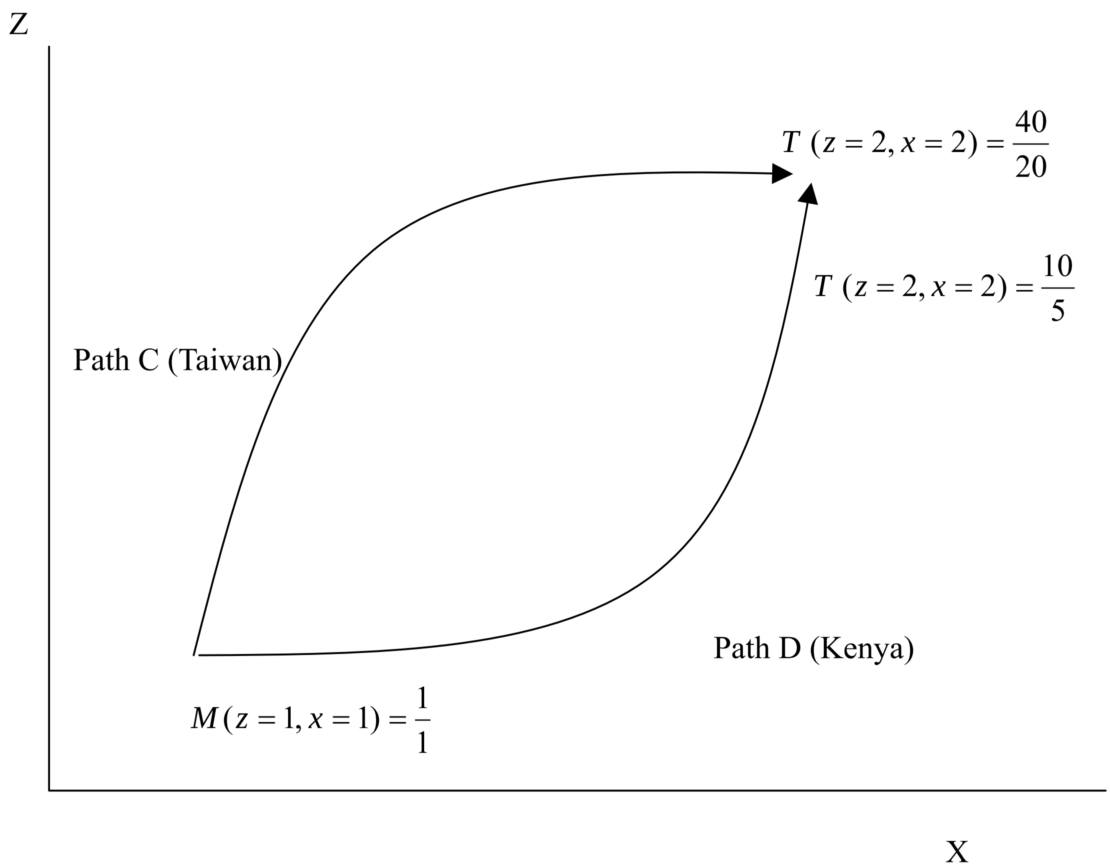
The ratio of large firms λ_1 could determine the value of J_{11}, and J_{11} could make the sign of -BJ+(Det J/Tr J) to be positive or negative, and finally determine how many convergence paths in this model.

If there are multiple convergence paths in that model, it could explain why two countries had the same economic situation in the beginning and converge to different way in the end.

In fig 3.2, we use Taiwan and Kenya as example. In the initial period, the two countries had same start point M and there are two convergence paths (Path C and path D) could make Taiwan and Kenya converge to their stable point T

The accumulation of human capital on path C is higher than Path D. For some reason, Taiwan chose the path C and Kenya decided the path D. In the process to the stable point T , the value of Z in path C is always higher than path D and it means Taiwan will accumulate more human capital than Kenya in its development process, and the human capital could be used to produce physical capital and more human capital. In the end, event the two countries all converge to their stable point T and the value of z and x are same, the real value of human capital and physical in Taiwan will be larger than Kenya. It perhaps could explain why two countries have different economic situation, even they have similar economic situation in the beginning.

Figure 3-2 The relationship of the multiple convergence paths and economic growth



In this chapter, we extend the one-sector model from chapter 2 to allow for two-sectors. One produces goods to consume and invest, and the other creates human capital. Meanwhile, we put human capital in the production function and observe the relationship between variables and the growth rate. The first result is that a large firm that has a stronger internal effect and external effect can create a higher growth rate, and the internal effect can lead to a higher growth rate than the external effect.

Second, a country with a larger proportion of large firms will create higher economic growth. Nevertheless, the effect is not very strong.

Finally, the stability properties of the transitional dynamics are not identical in each situation. The number of negative roots will be changed with different parameters. There is, at least one negative root in each proposition, and it means this model will converge in different situations. In some conditions, there would exist multiple convergence paths to the steady state and it could explain the differences of economic growth among countries.

Chapter 4 Two-Sector Models (endogenous variable)

4.1 Introduction and the model

In chapter 3, we assume the ratio of large firms λ_1 is an exogenous variable. It means the small firms cannot do anything to ensure their transformation. Some factors control the process of transformation, but we cannot observe that in the above model.

If we assume that a small firm can do some investment to improve the knowledge of its workforce or technology, it makes the stock of human capital increase and the probability of transforming into a large firm higher. We can image the means to this end is, for example, professional training for employees and employers or set up a research department in the company. However, with more investment in technology the firm's available resource in the goods sector will be lesser. The representative firm must decide on how many percentage of its resource it would devote to improving knowledge and transforming into a large firm. The more the firm spends on education sector, the higher the probability it has to become a big firm. Therefore, we could make λ_1 as an endogenous variable and calculate the optimal solution.

For the sake of convenience, we assume ω is the ratio ($0 \le \omega \le 1$) of the human capital that the small firm keeps in goods sector and $1-\omega$ used in education sector to improve its knowledge base or technology. The human capital in goods sector is ωh and in education sector is $(1-\omega)h$. It is like the firm that has to choose how much of human capital to be used in goods sector to produce physical capital and in education sector to improve human capital.

There exists a positive relationship between the probability of a small firm transforming into a large firm and the ratio of a small firm investing its human capital in education sector. The more human capital in education sector the firm invests, the higher probability it has for transformation. If the small firm uses all its human capital in education sector, it could become a large firm, but it will not have any human capital used in goods sector and it cannot produce any physical capital. Hence, the small firm has to allocate its human capital and make an optimal decision.

The expected value of a small firm that transforms into a large firm is equal to $1-\omega$. It means that the probability of small firm staying as ω.

$$0 \le E(\lambda_1) = 1-\omega \le 1 \text{.........(4-1)}$$

$$0 \le E(\lambda_2) = \omega \le 1 \text{............(4-2)}$$

Given $\lambda_1 + \lambda_2 = 1$, it means that when we control λ_1 and the value of λ_2 is decided, we just need to find the optimal value in one of them. There are many firms in this economic system and all of them are the same. It means the other firms also make the same decision that representative firm made.

And the two-sectors production function could be

$$y = A\{\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + \lambda_2\}k^{\alpha}k_a^{\beta}(\omega h)^{1-\alpha-\beta} \text{.........(4-3)}$$

$$q = B(1-\omega)h \text{...............................(4-4)}$$

y is the output in goods sector and q is the output in education sector

In the aspects of human capital production, new human capital for the next period equal to the average output in education sector deducts depreciation.

$$\dot{k} = f(k) - c - \delta k \text{.......................(4-5)}$$

$$\dot{h} = B(1-\omega)h - \delta h \text{.......................(4-6)}$$

As the assumption above, the number of labor is L and keeps constantly equal to 1.

$$L(t) = L = 1 \text{.....................(4-7)}$$

where U is the overall utility function of household and is supposed to be constant intertemporal elasticity of substitution

$$\max \ U = \int_0^\infty e^{-\rho t} \mu(c(t)) L dt$$

$$\mu(c(t)) = \frac{c^{1-\sigma}(t) - 1}{1-\sigma}$$

μ is the utility function of consumption and it is assumed to satisfy the Inada conditions.

$$\lim_{C \to 0} \mu'(c) = \infty, \lim_{C \to \infty} \mu'(c) = 0 \quad \text{…………………….(4-8)}$$

The present value Hamilton equation for the representative household 's optimization is illustrated by

$$H = \frac{c^{1-\sigma}(t) - 1}{1-\sigma} e^{-\rho t} + \lambda(\dot{k}) + \phi(\dot{h}) \quad \text{…………………….(4-9)}$$

where λ and ϕ are constant variables of the physical capital and human capital, and it also means the shadow price between k and h.

In the social planner equilibrium solution, all firms are identical. Thus, the other firm's average capital (average economy wide capital stock) equals the capital of the representative firm ($k = k_a$)。

After differentiating with respect to c (control variable), we obtain equation (4-10). It means that the present value marginal utility must equal the marginal value of physical capital.

$$\frac{\partial H}{\partial c} = c^{-\sigma} e^{-\rho t} - \lambda = 0 \to c^{-\sigma} e^{-\rho t} = \lambda \quad \text{……………………..(4-10)}$$

Equation (4-9) differentiating with respect to u, we obtain equation (4-11). In the beginning, all firms are small firms and λ_1 is equal to zero.

If we denote $\lambda_1(1+\theta_1)(1+\theta_2)^\beta + \lambda_2 = G$, the expected value of λ_1 is $1-\omega$

and the expected value of λ_2 is ω. There are many identical firms in this economic system. One firm knows there exist a positive relationship between the output in education sector and transforming into a large firm, but it knows just the expected value. Even two identical firms came to the same decision; it does not guarantee the process of transforming will be same. In the beginning, G is like an exogenous variable, because λ_1 and λ_2 cannot be decided. The firms make the decision without considering the λ_1 and λ_2. Later with all firms making the same decision, we could use the expected value to change the probability to be a certain value. $\mathrm{E}(\lambda_1) = 1-\omega$ and $\mathrm{E}(\lambda_2) = \omega$.

$$\frac{\partial H}{\partial \omega} = \lambda A(1-\alpha-\beta)Gk^{\alpha+\beta}h^{1-\alpha-\beta}\omega^{-\alpha-\beta} - \phi Bh = 0 \quad \text{……………(4-11)}$$

Equation (4-9) differentiating with respect to k and h (state variable), we obtain equations (4-13) and (4-14).

$$\frac{\partial H}{\partial k} \Rightarrow \lambda(A(\alpha+\beta)G\omega^{1-\alpha-\beta}h^{1-\alpha-\beta}k^{\alpha+\beta-1} - \delta) = -\dot{\lambda} \quad \text{…………...(4-13)}$$

$$\frac{\partial H}{\partial h} \Rightarrow \lambda A(1-\alpha-\beta)Gk^{\alpha+\beta}\omega^{1-\alpha-\beta}h^{-\alpha-\beta} + \phi[B(1-\omega)-\delta] = -\dot{\phi} \quad \text{..……..(4-14)}$$

From equation (4-13), we obtain equation (4-15)

$$\frac{\dot{\lambda}}{\lambda} = -(A(\alpha+\beta)G\omega^{1-\alpha-\beta}h^{1-\alpha-\beta}k^{\alpha+\beta-1} - \delta) \quad \text{……………….(4-15)}$$

From equations (4-3), (4-5) and (4-6), we obtain equations (4-16)(4-17)

$$\frac{\dot{k}}{k} = AGk^{\alpha+\beta-1}(\omega h)^{1-\alpha-\beta} - \frac{c}{k} - \delta \quad \text{…………………….………(4-16)}$$

$$\frac{\dot{h}}{h} = B(1-\omega) - \delta \quad \text{………….……………..……………(4-17)}$$

We need equations (4-18) and (4-19) as a transversality condition, it means that there will exist an optimal solution.

$$\lim_{t\to\infty}\lambda(t)k(t)=0 \text{ ....(4-18)}$$

$$\lim_{t\to\infty}\phi(t)h(t)=0 \text{ ....(4-19)}$$

Together with equations (4-10) and (4-15), we obtain equation (4-20).

$$\frac{\dot{c}}{c}=\frac{A(\alpha+\beta)G\omega^{1-\alpha-\beta}h^{1-\alpha-\beta}k^{\alpha+\beta-1}-\delta-\rho}{\sigma} \text{ ....(4-20)}$$

In the balanced growth path, all variables grow at the common speed. The growth rate of consumption is equal to physical capital. $\frac{\dot{c}^*}{c}=\frac{\dot{y}^*}{y}$.

$$\frac{\dot{c}^*}{c}=\frac{\dot{y}^*}{y}=\gamma=\frac{A(\alpha+\beta)G\omega^{*1-\alpha-\beta}h^{*1-\alpha-\beta}k^{*\alpha+\beta-1}-\delta-\rho}{\sigma} \text{ ....(4-20.1)}$$

4.2 The numerical methods

Here we use the same method as in chapter 3 to analyze the condition. If all variables grow at a constant rate (Balanced growth path), the growth rate of variable c equals variable k and variable h.

$$\frac{\dot{c}}{c}=\frac{\dot{k}}{k}=\frac{\dot{h}}{h}$$

Then we define $z\equiv\frac{h}{k}, x\equiv\frac{c}{k}$,

where z is the proportion of human capital and physical capital, x is the proportion of consumption and physical capital. Because h, c and k grow at a common speed, the growth rates of z and x are equal to zero. ω is constant and the growth rate of ω is also equal to zero. Accordingly, we could find the social planner optimal solution in this model.

Equations (4-16), (4-17) and (4-20) taken together, we obtain equations (4-21) and (4-22)

$$\frac{\dot{z}}{z}=\frac{\dot{h}}{h}-\frac{\dot{k}}{k}=B(1-\omega)-AGz^{1-\alpha-\beta}\omega^{1-\alpha-\beta}+x \qquad \text{……..……(4-21)}$$

$$\frac{\dot{x}}{x}=\frac{\dot{c}}{c}-\frac{\dot{k}}{k}=\frac{\alpha+\beta-\sigma}{\sigma}AGz^{1-\alpha-\beta}\omega^{1-\alpha-\beta}+x+\frac{\delta(\sigma-1)-\rho}{\sigma} \qquad \text{…………(4-22)}$$

Together with equations (4-11) and (4-14), we get equation (4-23) and (4-24)

$$\frac{A}{B}Gz^{-\alpha-\beta}\omega^{-\alpha-\beta}(1-\alpha-\beta)=\frac{\phi}{\lambda} \qquad \text{…………..(4-23)}$$

$$\frac{\lambda}{\phi}AGk^{\alpha+\beta}(1-\alpha-\beta)\omega^{1-\alpha-\beta}h^{-\alpha-\beta}+[B(1-\omega)-\delta]=-\frac{\dot{\phi}}{\phi} \qquad \text{…………(4-24)}$$

Put equations (4-23) and (4-24) together, we obtain equation (4-25)

$$-B+\delta=\frac{\dot{\phi}}{\phi} \text{…………..(4-25)}$$

By taking logarithm in equation (4-23) and differentiating with respect to time, we obtain equation (4-26)

$$\frac{\dot{\phi}}{\phi}-\frac{\dot{\lambda}}{\lambda}=(\alpha+\beta)\frac{\dot{\omega}}{\omega}-(\alpha+\beta)\frac{\dot{z}}{z} \qquad \text{…………………………….(4-26)}$$

Finally,

$$\frac{\dot{\omega}}{\omega}=\frac{B}{(\alpha+\beta)}-B(1-\omega)-x \qquad \text{…………..(4-27)}$$

Now we have the change rates of z, x and ω. When all variables are in a steady state, the change rates of those variables are all equal to zero. The steady state of this system can be found by setting the three time derivatives equations ($\frac{\dot{x}}{x}=\frac{\dot{z}}{z}=\frac{\dot{\omega}}{\omega}=0$)

to zero and find the optimal solution.

The results are

$$\frac{\dot{z}}{z} = B(1-\omega) - AGz^{1-\alpha-\beta}\omega^{1-\alpha-\beta} + x = 0 \qquad \text{..............(4-28.1)}$$

$$\frac{\dot{x}}{x} = \frac{\alpha+\beta-\sigma}{\sigma}AGz^{1-\alpha-\beta}\omega^{1-\alpha-\beta} + x + \frac{\delta(\sigma-1)-\rho}{\sigma} = 0 \qquad \text{........(4-28.2)}$$

$$\frac{\dot{\omega}}{\omega} = -B(1-\omega) + \delta - x = 0 \qquad \text{..........................(4-28.3)}$$

$$E(G) = (1-\omega)(1+\theta_1)(1+\theta_2)^\beta + \omega \qquad \text{..........................(4-28.4)}$$

Taking expected value of equations (4-28.1)(4-28.2)(4-28.3) together with equation (4-28.4), the optimal solutions are

$$E(\frac{\dot{z}}{z}) = (B(1-\omega) - A((1-\omega)(1+\theta_1)(1+\theta_2)^\beta + \omega)z^{1-\alpha-\beta}\omega^{1-\alpha-\beta} + x) = 0$$

...(4-29.1)

$$E(\frac{\dot{x}}{x}) = (\frac{\alpha+\beta-\sigma}{\sigma}A((1-\omega)(1+\theta_1)(1+\theta_2)^\beta + \omega)z^{1-\alpha-\beta}\omega^{1-\alpha-\beta} + x + \frac{\delta(\sigma-1)-\rho}{\sigma}) = 0$$

...............(4-29.2)

$$E(\frac{\dot{\omega}}{\omega}) = -B(1-\omega) + \delta - x = 0 \qquad \text{..................(4-29.3)}$$

Here we denote $g = (1+\theta_1)(1+\theta_2)^\beta$

$$z^* =$$

$$\frac{(\alpha+\beta)\{[(1-g)(2\rho\sigma\delta - \sigma\delta^2 - \rho^2)] - (1-\alpha-\beta)\{(1-g)[(\rho-\sigma\delta)(2\delta+g\sigma)] + B^2\sigma^2\} + (1-\alpha-\beta)^2[\delta B\sigma g - \delta^2(1-g)] + [(1-\alpha-\beta)(\alpha+\beta-2)\delta B\sigma]\}}{\{(1-g)[(1-\alpha-\beta)\delta + \rho - \delta\sigma] + \sigma B\}\{(1-\alpha-\beta)(\delta+\sigma B) + \rho - \delta\sigma)\}^2 A + \{(1-g)[(1-\alpha-\beta)\delta + \rho - \delta\sigma] + \sigma B\}\{(1-\alpha-\beta)(\delta+\sigma B) + \rho - \delta\sigma)\}A\delta\sigma^2 B^2}$$

$$\omega^* = \frac{\delta(1-\alpha-\beta)+\sigma(B-\delta)+\rho}{B\sigma}$$

$$x^* = \frac{\delta(1-\alpha-\beta)+\rho}{\sigma}$$

If the range of ω^* would be limited between 0 and 1, the condition has to be equation (4-30), the discount rate of time cannot be too large.

$$B\sigma > \sigma\delta - \rho - \delta(1-\alpha-\beta) \quad \text{......................(4-30)}$$

In order to explain the influence resulting from the representative small firm that allocates the resource to improve its human capital, we try to use a numerical method to calculate its result.

In terms of the value of other parameters, we also adopt some assumptions from Lucas(1988) and Benhabib (1994). ρ is the discount rate of time at 0.025. δ is the depreciation rate equal at 0.05 . The basic technical levels in both sectors are the same and equal to 1($A = B = 1$). α is the coefficient of firm's capital in the production function at 0.4, whereas β is the coefficient of average economy wide capital stock at 0.2. σ is the inverse of the constant intertemporal elasticity of substitution at 5.

In Figure 4-1 we create a flow chart to help readers easily to understand what the process of the representative firm's optimal choice is. The representative small firm has to allocate its initial human capital into two sectors.

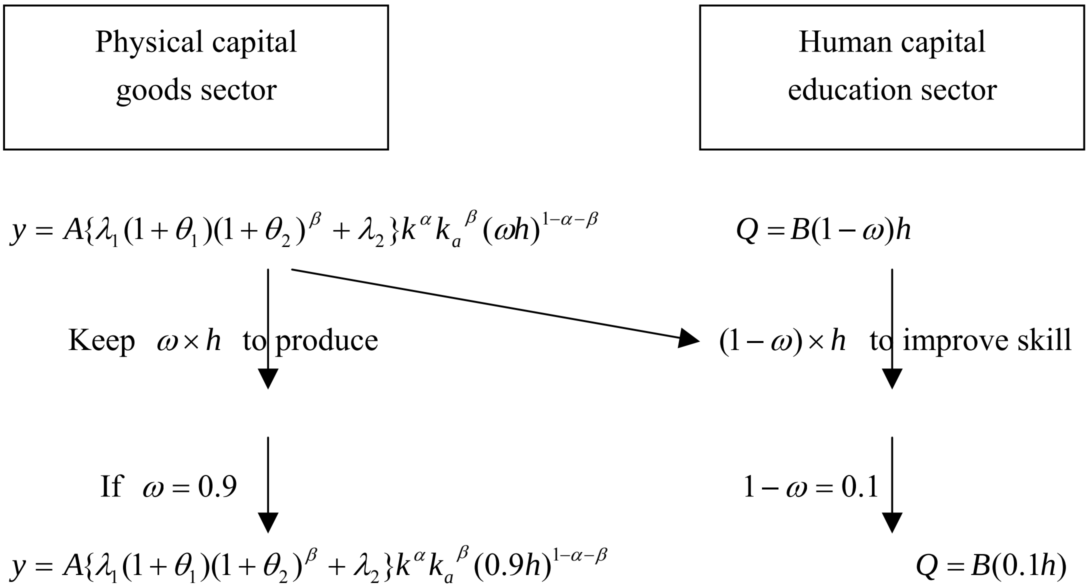
Table 4-1 The relationship between the investment in education sector and growth rate

A	B	α	β	ρ	δ	σ	θ_1	θ_2	ω^*	$\frac{\dot{c}^*}{c}$
1	1	0.4	0.2	0.025	0.05	5	0	0	0.9590	0.089
1	1	0.4	0.2	0.025	0.05	5	0.1	0.1	0.9590	0.102
1	1	0.4	0.2	0.025	0.05	10	0.1	0.1	0.9545	0.051
1	1	0.4	0.2	0.025	0.05	5	1	0	0.9590	0.197
1	1	0.4	0.2	0.025	0.05	5	0	1	0.9590	0.117
1	1	0.1	0.1	0.025	0.05	5	0.1	0.1	0.9630	0.00005
1.5	1	0.4	0.2	0.025	0.05	5	0.1	0.1	0.9590	0.156
1	1.5	0.4	0.2	0.025	0.05	5	0.1	0.1	0.9726	0.102

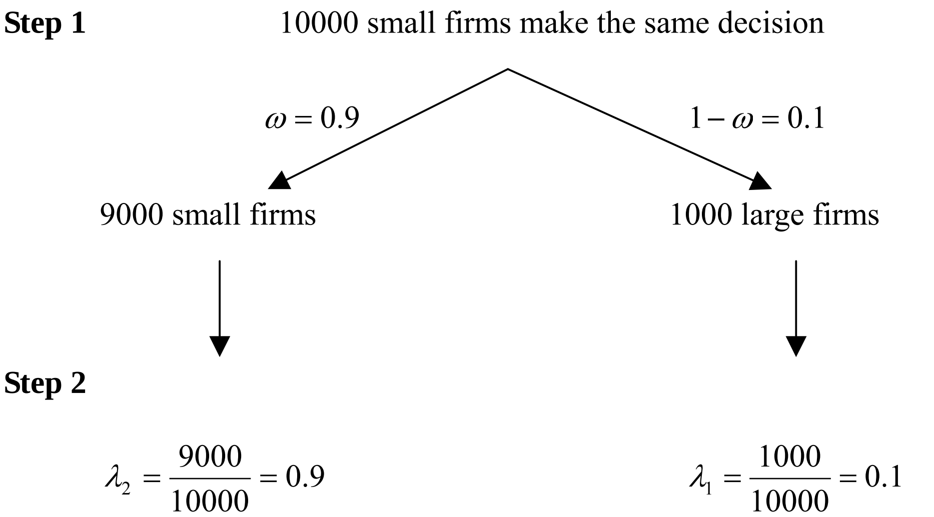
Note: All variables are in equilibrium situation.

Figure 4-1 The process of the representative firm's optimal choice

Two-sectors.



The probability of a small firm that transforms into a large one is positively related to the quantity inputted in education sector, but the firms do not know how the probability actually is. They consider λ_1 being an exogenous variable and just choose the optimal value of ω



Now we assume the expected value of a small firm that transforms into a large firm is equal to $1-\varpi$.

$$0 \le E(\lambda_1) = 1-\omega \le 1,\ \ 0 \le E(\lambda_2) = \omega \le 1$$

If the representative chooses the optimal ϖ , the ratio of small firms λ_2 is actually determined.

Figure 4-2 The relation between the discount rate of time (ρ)and the ratio of human capital in goods sector(ω)

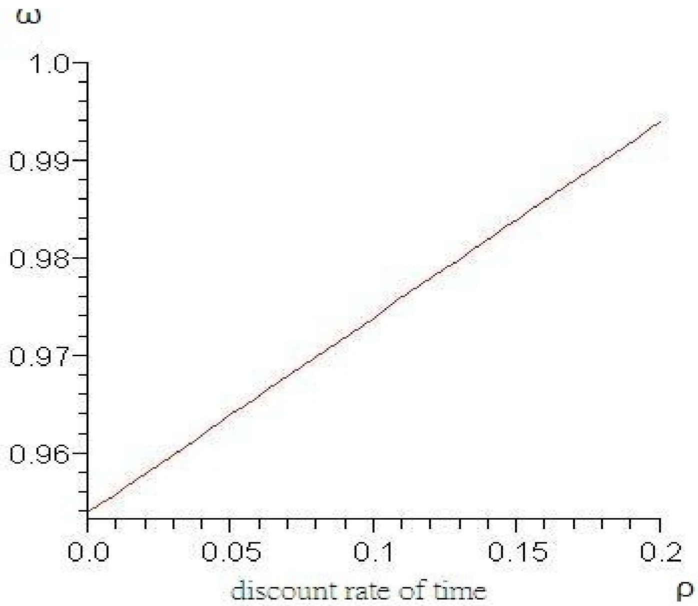


Figure 4-3 The relation between the discount rate of time (ρ)and the growth rate (γ)

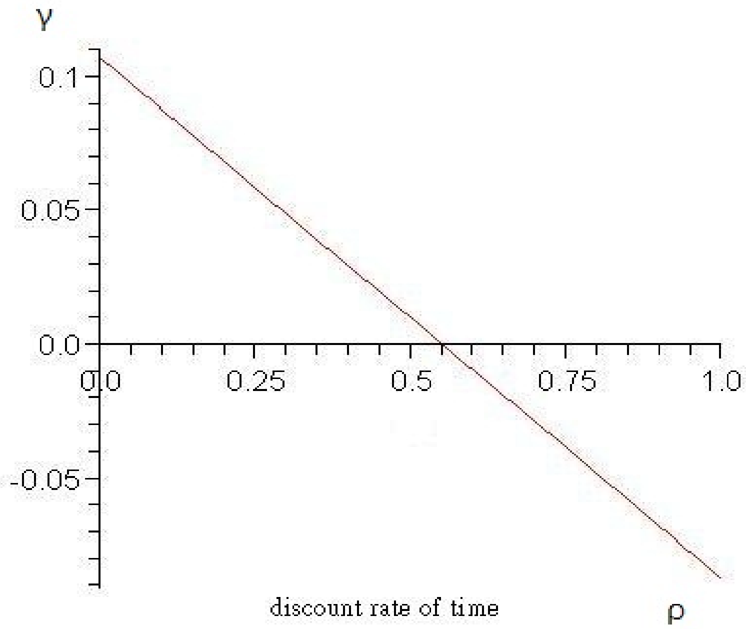


Figure 4-4 The relation between the inverse of the intertemporal elasticity of substitution (σ)and the ratio of human capital in goods sector (ω)

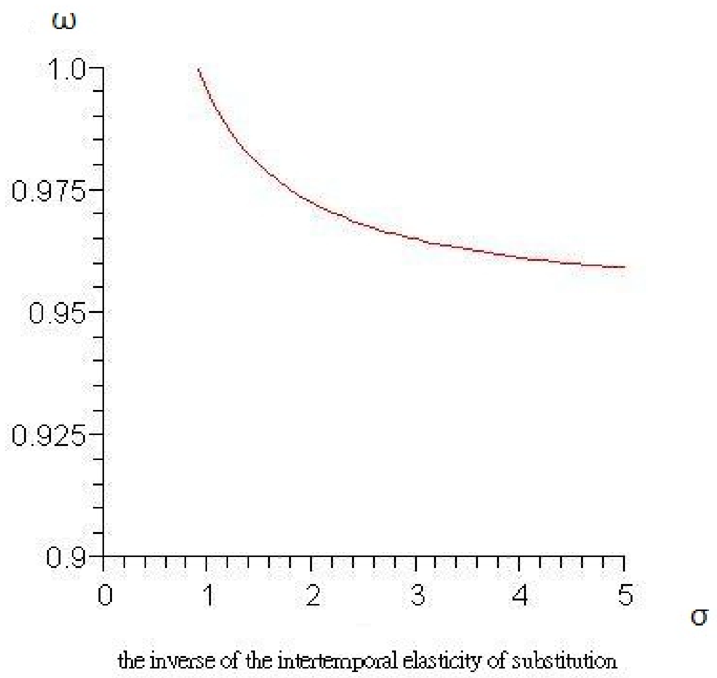


Figure 4-5 The relation between the inverse of the intertemporal elasticity of substitution (σ)and the growth rate (γ)

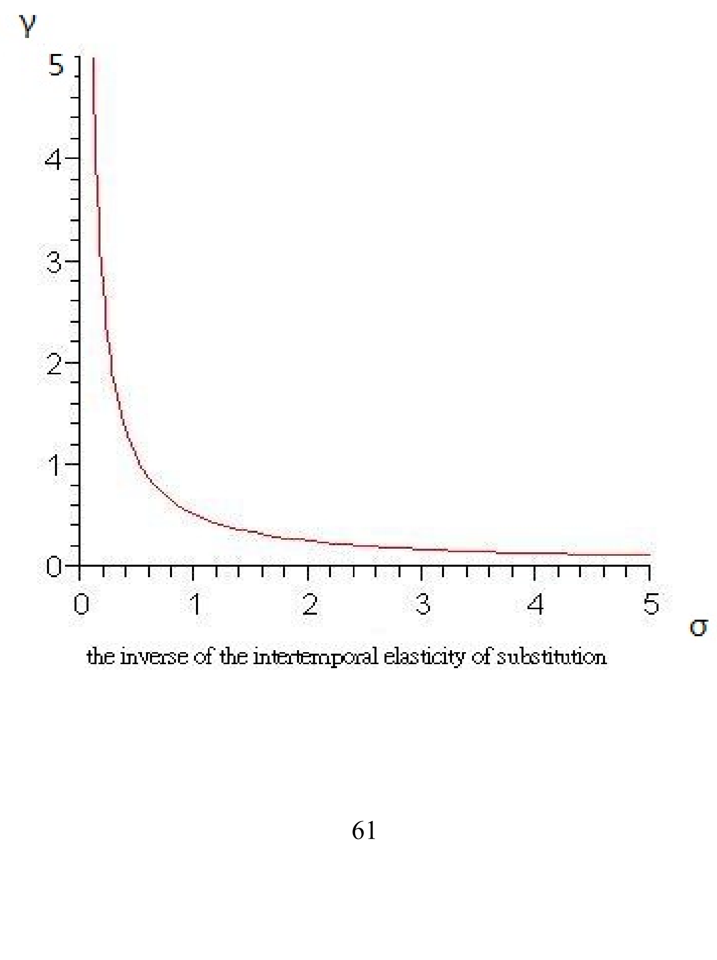


Figure 4-6 The relation between the depreciation rate (δ)and the ratio of human capital in goods sector (ω)

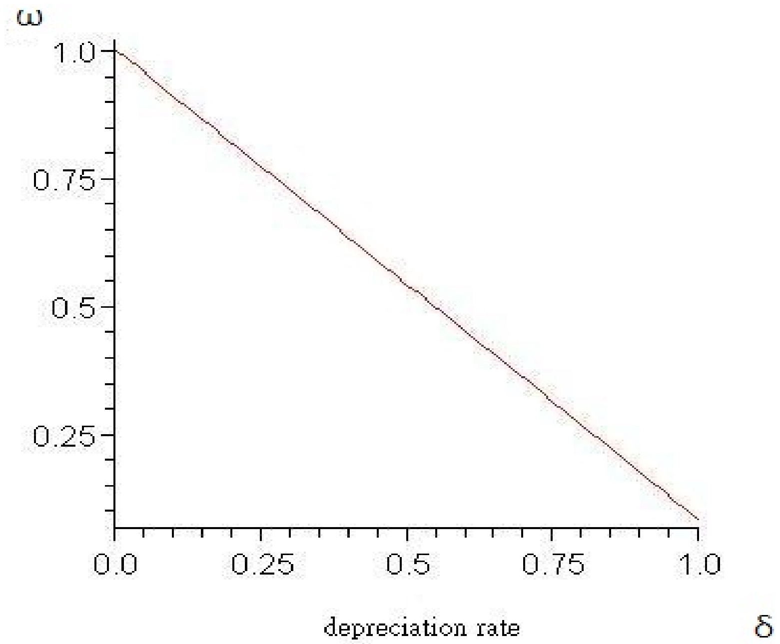


Figure 4-7 The relation between the depreciation rate (δ) and the growth rate (γ)

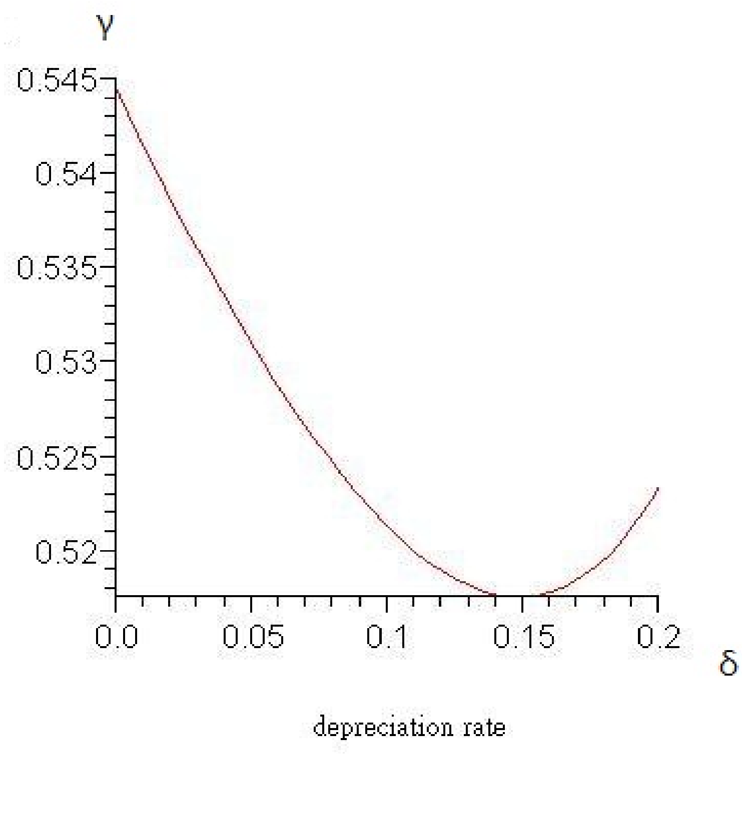


Figure 4-8 The relation between the average economy wide capital stock in production function (β)and the ratio of human capital in goods sector (ω)

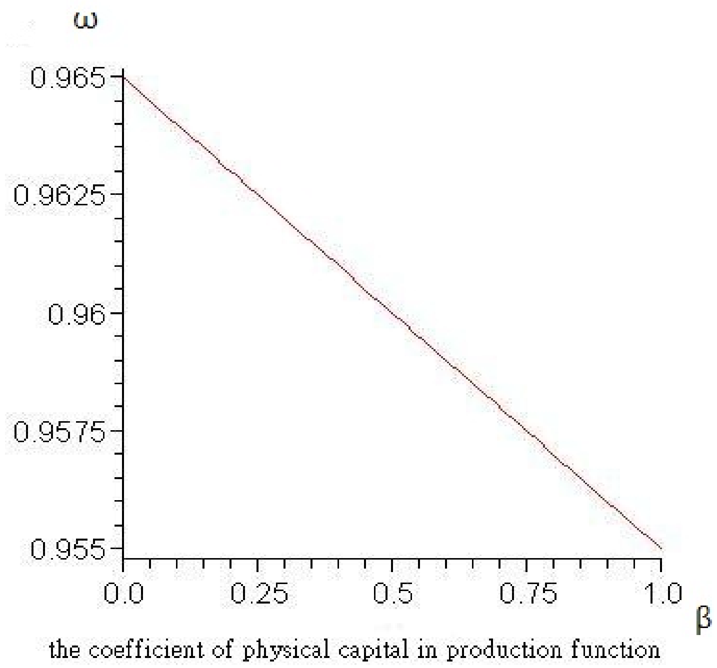


Figure 4-9 The relation between the average economy wide capital stock in production function (β)and the growth rate (γ)

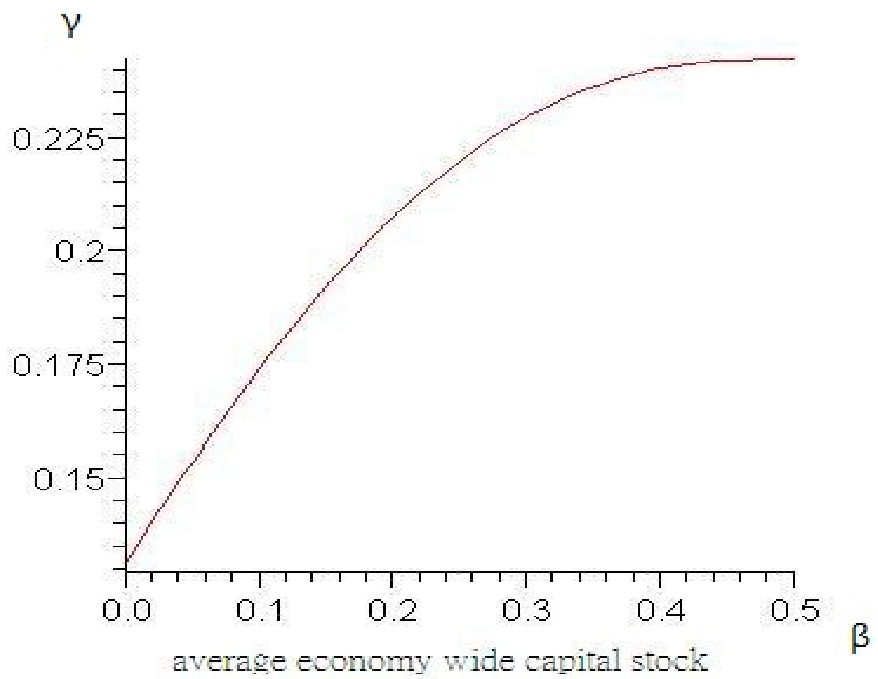


Table 4-2 The situation of SMEs among countries

Country	Industry sector			
	The number of small firms (ten thousand)	Ratio	Employees (ten thousand)	Ratio
Taiwan(2005)	121	97.8	706	75.54
Australasia(2004)	110	96.0	330	47.00
Canada(2004)	214	91.44	661	64.10
Japan(2004)	564	98.94	4,124	79.51
Malaysia(2001)	21	96.11	38	32.50
New Zealand(2004)	31	92.47	70	36.08
Philippics(2001)	80	99.60	410	70.00
Russia(2003)	873	97.57	3,996	60.86
Singapore(2003)	13.4	99.67	65.61	69.10
South Korea(2003)	295	99.81	1,038	86.66
Thailand(2003)	83	99.70	557	60.69
England(2004)	411	95.97	1,350	61.38
USA(2003)	2415	99.83	7,646	58.00

Source: The white book of SMEs (2008)

From Table 4-1 to Table 4-2 and Figure 4-2 to Figure 4-9, we could conclude several important results.

First, the internal (θ_1) and external effect (θ_2) could not affect the ratio ω^* that a firm wants to allocate its human capital in education sector or goods sector, because the representative small firm would not know the exact probability it could transform into a large firm in the beginning. It just chooses the optimal value of ω^* to invest in goods sector. But a higher value of the internal (θ_1) and external effect (θ_2) in production function will lead to a higher economic growth rate.

Second, if the share of a firm's physical capital ($\alpha + \beta$) in the production function is less than the share of human capital ($1 - \alpha + \beta$), the representative firm will choose a higher ratio ω^*. It means the physical capital in production function is not as important as human capital. Thus, the representative firm will spend more human capital in production function to get higher output. The growth rate is going to decrease with a lower share ($\alpha + \beta$) in the production function. It means the physical capital plays a very important role in production process. The firm cannot produce output just relying on human capital.

Third, the inverse of the elasticity of intertemporal substitution (σ) is a significant variable that determines the value of ratio ω^* and the growth rate $\frac{\dot{c}}{c}^*$. The elasticity of intertemporal substitution measures the extent to which consumers shift total expenditures across time in response to changes in the effective rate of return. Higher σ means the elasticity of intertemporal substitution is lower and it discourages consumers from sacrificing today's consumption to get more available consumption in the future. If σ is very high, that means people prefer today's consumption and it will make the saving decrease. It is more difficult for the firm to borrow physical capital and it has to use higher human capital in education sector to compensate for the lack of physical capital and the higher σ decreases the growth rate. Perhaps it could explain why a country has higher saving rate and its growth rate is higher than another country without high saving rate.

Fourth, the technological level of goods sector (A) and education sector (B) both could affect the economic growth rate and the effect in goods sector is larger than education sector. It means the production process in the physical capital sector is more efficiently if the technological level A is larger than B.

Fifth, the representative firm that chooses the ratio of human capital to use in a goods sector just depends on these parameters it faces. The value of ω^* always stays in a very high level even though these parameters are total different. We know ω^* represent the ratio of small firms, and we can compare the result with the empirical date for the globe (Table 4-2) to observe a very special phenomenon. We find the ratios of small firms in the most of countries are larger than 90 percent. In some countries, such as the Philippines, South Korea and Thailand, the ratios are larger than 99 percent. It could explain why there exist such a high percentage of small firms among these countries.

Sixth, the coefficient of the average widely capital (β) is positively related to the growth rate and negatively related to the ratio of small firms. It means the firm will have a higher probability to transform into a large one if the external effect is large, because the firm can obtain some benefits without paying for them. The higher external effect will lead to a higher growth rate.

Finally, even the representative firm chooses the same ratio ω^* to allocate its human capital in the goods sector, the growth rate could be still different with different parameters, such as the technological level, the elasticity of intertemporal substitution and the share of a firm's physical capital in the production function, etc. It could explain why two countries have the same proportion of large firms, but the growth rates between these countries diverge to different paths.

Chapter 5 Evidence and Statistics -(Taiwanese data)

5.1 Introduction

The neoclassical growth model of Solow (1956), which has been for a long time the central framework to account for economic growth, focuses on exogenous technological, population or saving rate factors that determine equilibrium capital per capita.

Despite the growth model of Solow being widely used in various applications, Lucas (1988) and Romer (1989) emphasized the major drawback was that in empirical applications over half of the growth in output went unexplained.

Schultz (1961) noted that the growth rate of output exceeded the growth rate of the relevant input measures (employment and physical capital). He suggested that investment in human capital is probably the main explanation for this difference.

Azariadis and Drazen (1990) found that no country was able to grow quickly during the postwar period without a highly literate workforce. Uzawa (1965) and Rosen (1976) also stressed the important role of human capital.

Romer (1986) stated external economics of scale (external effect) is the major factor that sustains economic growth.

In this chapter, we postulate that the external effect in production function is the locomotive of growth and argue for the theory; the relative scale of external effect plays a role in motivating and supporting physical capital accumulation.

At the same time, we focus on a single industrialized country. The main advantages of analyzing the economic growth in a single case are: (a) The use of a data set comprised of the most appropriate and the highest quality measures unconstrained by the need for measurement consistency, and (b) A more detailed account of the dynamic evolution of the economy. We first use Taiwan’s data, covering the period 1974 to 2002. If we get an acceptable result, then we might continue to explain another country's situation in the future research.

5.2 Recent history of Taiwan's economic development

Taiwan is a small island economy with an extremely high population density and with very few natural resources. However, the country in the past few decades has experienced rapid output growth with moderate inflation. Its real gross domestic product (GDP) per capita has increased by more than six-fold since 1954 with an average annual growth rate of over 6 percent, while its persona1 income distribution index measured by the ratio of household income share of the richest to the poorest quintile decreased from 5.33 to 4.94 over the period of 1964-89.

Taiwan's economy began to take off in the years from 1963 to 1965. Between these years, the per capita growth in real GDP increased from less than 3 percent to more than 6 percent; the investment to NNP (Net National Product) ratio increased from roughly 6 percent to above 10 percent; the net saving to NNP (Net National Product) ratio rose from below 8 percent to about 13 percent. The rate of domestic savings increased from 13 percent in the early 1960s to 20 percent by the late sixties, and reached 30 percent through the 1970s and 1980s. Furthermore, the output of industrial sectors began to exceed the output of agricultural sectors, and employees started moving from agriculture to other industries.

Beginning in the 1980s, the situation changes of social and political landscape took place in Taiwan. The economy became increasingly open and free from earlier restrictive and protectionist tendencies.

Due to the rapid growth of trade and a sharp increase in trade surplus, Taiwan's foreign exchange reserves reached the US$70 billion in 1987. The massive trade surpluses led to a rapid accrual of foreign exchange assets and a sharp appreciation of the New Taiwan Dollar against the US Dollar, which thus lowered Taiwan's competitiveness. Many small and medium sized enterprises with labor-intensive operations were unable to keep their foothold in Taiwan and relocated to Southeast Asia or the China, seeking new business opportunities and room to expand.

The greatest changes in the structure of Taiwan's industrial sector took place in the 1980s. Labor-intensive industry was no longer the mainstay of the industrial sector and was gradually being replaced by technology and capital intensive industry.

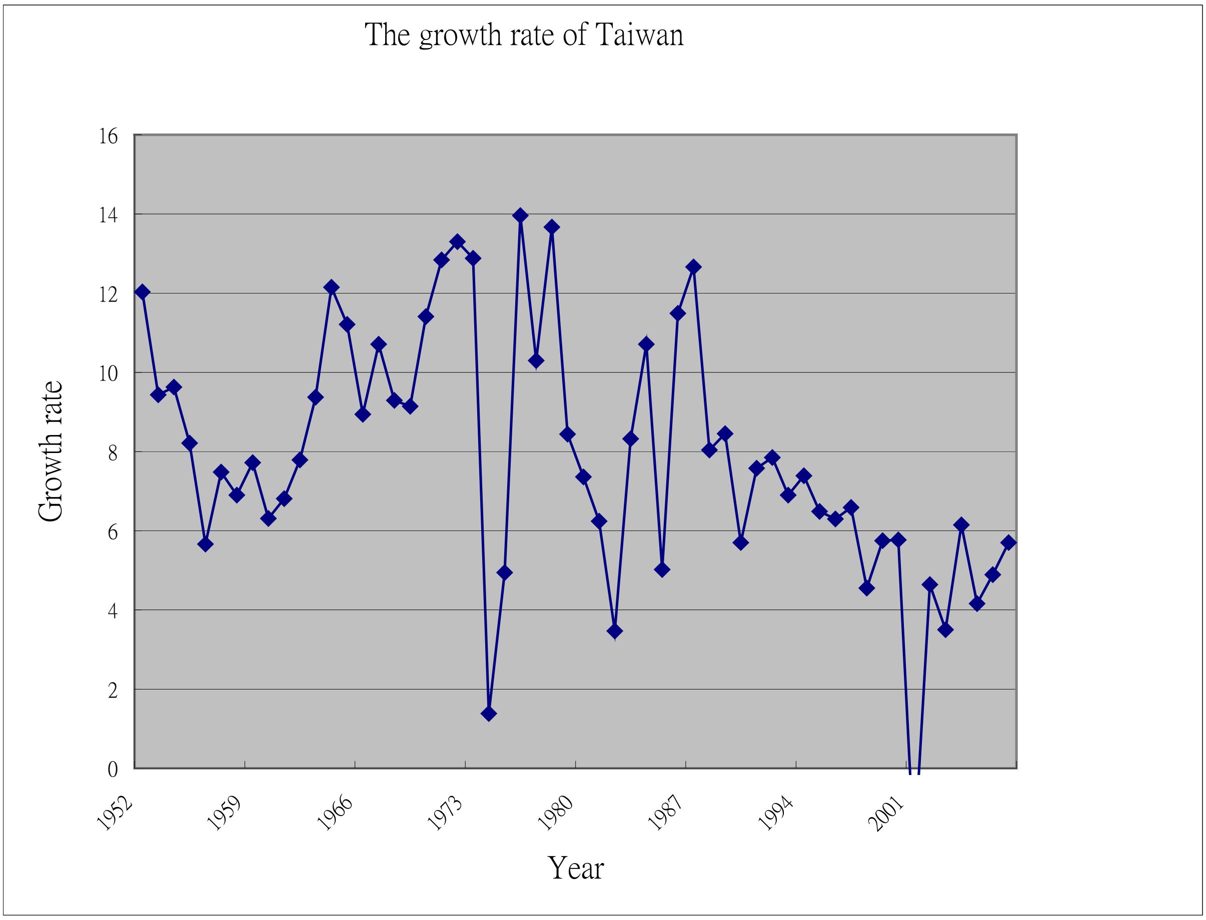
The transition of Taiwan's industrial sector from light to heavy industries is quite apparent. In terms of the production value, the ratio of light industries to heavy chemical industries dropped from 51.52 percent to 48.48 percent in 1986. In 1995, the output of the light industries dropped to only 33.63 percent, while the heavy chemical industries' share rose to 66.37 percent.

From 1981 to 1995 in Taiwan's economic development, the annual growth rate dropped to 7.52 percent from the near 10 percent average seen over the preceding 18 years (1963-1980). This mild slowdown was perhaps a natural consequence of structural changes in the nation's industrial sector. During this period of time, the agricultural sector had the lowest performance, with an average annual growth rate of a mere 1.24 percent and a GDP share of 4.74 percent. The average annual growth rate of the industrial sector was 6.46 percent, far below the 14 percent of the preceding 18 years; its GDP share dropped to 43.16 percent. The service sector, meanwhile, experienced the highest growth rate, with an average of 9 percent per annum. The sector's GDP share rose to 51.67 percent, far above that of the industrial sector.

Between 1952 and 1995, Taiwan set economic performance records in comparison to other countries around the world by attaining an average annual economic growth rate of 8.63 percent.

In recent years the structure of industry is stable. Although some firms relocated their factories to China or some Southeast Asia countries, the government of Taiwan made some policies to attract foreign capital (i.e., tax reduction and financial subsidy). The average annual economic growth rate can thus be maintained between 3 and 6 percent.

Figure 5-1 The economic growth rate in Taiwan between 1952 and 2006



Source: National Statistics, R.O.C. (Taiwan)

5.3 The model assumption

In the real world, there is no country, which has either large firms or small firms alone. All countries have large firms and small firms at the same time. So we adopt the production function of firms (chapter 3) to estimate the value of these parameters in Taiwan.

$$y = A[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)](k)^{\alpha}\ k_a^{\ \beta}\ (uh)^{1-\alpha-\beta} \text{........(5-1)},$$

where y is the output per capita, k is the capital of the representative firm, k_a is average economy wide capital stock, and h is the human capital. θ_1 is the internal effect scale of the large firm, θ_2 is the external effect scale of the large firm, A is the basic technology level, α is the share of the representative firm's capital in production function, and β is the share of the average economy wide capital stock in the production function. $1-\alpha-\beta$ is the share of human capital in the production function. u is the proportion of the total human capital input in the goods sector. The proportion of large firms is λ_1. The proportion of small firms is $1-\lambda_1$.

If we assume $R = A[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]$ ....(5-2),

from equation (5-1), we can take natural logarithm and derive the following estimable form.

$$\log(y) = \log(R) + \alpha\log(k) + \beta\log(k_a) + (1-\alpha-\beta)\log(uh) \text{...(5-3)}$$

On the condition of $k = k_a$, the estimate equation is,

$$\log(y) = \log(R) + (\alpha+\beta)\log(k) + (1-\alpha-\beta)\log(uh) \text{..........(5-4)}$$

How do we define these variables? For our output measure, we set the real gross domestic product (GDP) per capita as output (y). The gross capital stock per capita is k and the human capital stock (h) is measured by various education attainment measures

That here we use gross domestic product (GDP) as output (y) is uncontroversial, because Mankiw, Romer and Weil (1992), Caballero Ricardo J. and Ricardo K. Lyons (1990), and Chan (1995) all used the definition.

In fact, the measurement of human capital has great practical difficulties. We do not know the exact amount of human capital and, moreover, not all spending on education is intended to yield productive human capital. For example, family education is also important for human capital, but it is impossible to measure it. In this model, we try to use several proxy variables for the real value of human capital and compare the result.

The proportion of total human capital input in goods sector is also not easy to be estimated. Therefore, we use the ratio of education expenditure to GDP as a proxy variable and we get the proportion (u).

In this model, we assume the higher the education level of employees is, the higher the human capital they have. Here we distinguish three sub-parts of the labor: basic, prior and advanced labor.

Basic labor is a worker who completes only primary education or below(education years $\leqq 9$); prior labor is a worker, who finishes the secondary education($9 <$ education years $\leqq 12$); advanced labor means a worker completes the higher education(education years > 12)

We define the number of basic labor as h1; prior labor as h2 and advanced labor as h3, but how can we decide the influence degree between those labors? And how is the proportion a worker with a bachelor degree can create, as compared with a worker who only studied in a junior high school? Here we use the average salary of elementary school, senior high school and university of workers in Taiwan to define the difference between these people.

According to the labor data from the National Statistics, R.O.C. (Taiwan), we obtain the average salary for people who finished, respectively, elementary school, senior high school and higher than university in Taiwan and we could try to define the human capital in Taiwan.

First, because the wage rate is equal to the marginal products of labor, we suppose the difference between salaries is the divergence of human capital. For convenience, we define the average salary as the proxy variable of human capital (h). Second, the labor data from National Statistics, R.O.C.(Taiwan) shows the average salary of elemental school, senior high school and university in Taiwan (A.D. 2007) are N.T. 19000, N.T. 23000 and N.T. 45000.

In terms of physical capital, we adapt and modify the assumption from Mankiw, Romer and Weil (1992). We use the real net fix capital stock of industrial and service sectors per capita as the proxy of physical capital (k).

Here we first use the data from Taiwan to set up a regression equation and prove the theory.

The definitions of variables are:

y : real gross domestic product (GDP) per capita

h : human capital stock

$$h = \frac{(h1salary * h1) + (h2salary * h2) + (h3salary * h3)}{h1 + h2 + h3}$$ = human capital stock

The number of basic labor as $h1$; prior labor as $h2$ and advanced labor as $h3$

λ_1 :The proportion of large firms ($0 \le \lambda_1 \le 1$)

$1 - \lambda_1$:The proportion of small firms ($0 \le 1 - \lambda_1 \le 1$)

k : Net fix capital stock of industrial and service sectors per capita.

u : The proportion of total human capital input in goods sector

(We use the ratio of education expenditure to GDP as proxy variable)

Before we run the regression, we could check the correlation between the growth rate, the ratio of large firms and the change rate of large firms in Taiwan.

Table 5-1 The correlation coefficient between variables

	Growth rate	Ratio of large firms	The change rate of large firms
Growth rate	1	0.173	0.124
Ratio of large firms	0.173	1	0.286
The change rate of large firms	0.124	0.286	1

Data coverage: 1983-2006

Even though the value of correlation is not very large, it appears that there exists a positive correlation between the growth rate and the ratio of large firms.

After we run the regression, we could get the estimate variables R

$$R = A[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]$$

Now that we have the real value about the proportion of small and large firms (λ_1 and λ_2) in Taiwan, we could get the relationship between θ_1 , θ_2 , β and A.

Caballero and Lyons (1990) estimated the indexes of internal returns to scale and external economies for two-digit manufacturing industries in four European countries (West Germany, France, U.K and Belgium). They found very little evidence of increasing internal return to scale. In external economies, there existed for all four countries, and the effect was especially strong in France and Belgium.

Chen and Cheung (1995) estimated the indexes of internal returns to scale and external economies for two-digit manufacturing industries in Taiwan. The data strongly supported the presence of increasing external return to scale in all Taiwan's two-digit manufacturing industries. However, no observable evidence indicated that there existed an internal increasing return to scale in two-digit manufacturing industries.

Caballero and Lyons (1992) stated that estimates of degree of return to scale are larger for manufacturing than for two-digit industries in U.S.

From the aforementioned literature, we know that estimating the external effect and internal effect is very difficult, and with different estimation methods emerge different results regarding the amount of external effects and internal effects. Despite that, now we have just one equation and there are four variables. If we can define the value of the internal effect θ_1, external effect θ_2 and the share of average economy wide capital stock in the production function β, we can calculate the basic technology level A.

5.4 The result of the estimation and regression

Here we set up four models. The differences between those models are the estimation methods in the regression. In model 1 we use the OLS (Ordinary Least Square), in model 2 we use 2SLS[9](Two-Stage Least Squares), in model 3 we use WLS[10] (Weight Least Squares) and in model 4 we adopt feasible generalized least squares (FGLS).

Mankiw, Romer and Weil (1992) observed the data of United States. They expected the value of α (physical capital's share of income) is one-third and the

[9] In 2SLS, we use the 4 lagged independent variable as the instrumental variable.
[10] In WLS, we use the 1/ln(k) as the weighted parameter.

value of $1-\alpha-\beta$ (human capital's share) is between one-third and one half.

However, the value of α (physical capital's share of income) might be different in Taiwan. Tallman and Wang (1994) stated the value of α (physical capital's share of income) might be between 0.383 to 0.454. But they ignored the parameter β in the production function.

In our model, we assume α and β to be both larger than zero and smaller than one. The range of $\alpha+\beta$ and $1-\alpha-\beta$ will be between 0 and 1 if our model is correct.

In model 1, the coefficient of physical capital is positive (0.132), but it is not statistically significant. The coefficient of human capital is positive (0.88) and statistically significant. The sum of both coefficients closed to 1. In model 2, we try to use the 2SLS (Two-Stage Least Squares) to estimate these parameters. Both coefficient of physical capital and human capital are positive and statistically significant. The sum of both coefficients is a bit larger than 1. In model 3, only human capital is statistically significant, but there all exist serious problems in these models.

First, we make White heteroskedasticity test to check whether there are heteroskedasticity in residual term and it shows that the p-value is very small and significant from model 1 to model 3. The effect of heteroskedasticity in the residual term will make the standard errors underestimated and overestimated t-value. Second, we use Breusch-Godfrey Serial Correlation LM test to examine whether in these models there exists autocorrelated disturbance term. Meanwhile, we also use the Durbin-Watson value to check the result. The result is similar. It reveals that disturbance term is autocorrelated. The consequences of autocorrelation in the residual will also make the standard errors be underestimated and overestimated t-value. Besides, the higher values of R^2 are observed and lead to seriously misleading results.

Table 5-2 Estimation of physical capital and human model

Dependent variable: log GDP per person (1974-2002)				
Sample	Model 1(OLS)	Model 2(2SLS)	Model 3(WLS)	Model 4(FGLS)
Observation	29	29	29	29
Constant	0.986	-2.142	0.829	-0.87
	(1.74)	(2.39)	(3.54)	(0.70)
Ln(k)	0.132	0.471*	0.155	0.764**
	(0.193)	(0.259)	(0.223)	(0.297)
Ln(u*h)	0.88*	0.722***	0.861***	0.458***
	(0.099)	(0.132)	(0.11)	(0.146)
$\overline{R^2}$ =	0.98	0.967	0.89	0.76
Durbin-Watson	0.66	0.52	0.63	1.65
White Heteroskedasticity test	0.009#	0.0009#	0.014#	0.631
Breusch-Godfrey Serial Correlation LM Test:	0.0007#	0.0002#	0.0001#	0.90

Standard errors are in parentheses. We estimate the equation with ordinary least square (OLS) , two-stage least squares (2SLS) ,weighted least squares (WLS) and feasible generalized least squares(FGLS)

* means significance level is 10 percent.** 5 percent.*** 1 percent.

means there are autocorrelation and heteroskedasticity in disturbance term.

How can we know whether there exists a first-order autocorrelation in this regression? If the first-order autocorrelation coefficient is[11]

$$\varepsilon_t = \rho\varepsilon_{t-1} + \upsilon_t \quad \text{and} \quad \upsilon_t \sim iid(o, \sigma^2),$$

we can estimate the coefficient ρ by saving the residuals from the previous regression and running a least square regression of e_t on e_{t-1}.

If the p-value of ρ is significant, we could reject the null hypothesis of no serial correlation in this regression. We obtain the residuals from of OLS from model 1 and run a least square regression. The result is $e_t = 0.65e_{t-1} + 0.35e_{t-2} + \upsilon_t$, which means there exists the AR(2) in model 1, and then, we could use the FGLS[12] to estimate the parameters and the result in Table 5-2 shows that there does not exist heteroskedasticity and autocorrelation in residual term.

The coefficients of physical and human capital in model 4 conform to our theory and we can continue to estimate the parameters.

Although the sum of the coefficient of physical capital and human capital seems close to 1 and coincide with the Cobb-Douglas production function in model 1,2 and 3, the existence of heteroskedasticity and autocorrelation seems a serious problem for us to estimate and trust the parameters. For convenience' sake, we adopt the result from model 4 in Table 5-2.

In order to estimate the share of the average economy wide capital stock in Taiwan, we could adapt the assumption from Tallman and Wang (1994). They stated the value of α (physical capital's share of income) in Taiwan might be between 0.383 and 0.454. The coefficient of $\alpha + \beta$ in model 4 is 0.764. Therefore, we can calculate the value of β (the average economy wide capital stock in the production function)

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[11] e_t is sample disturbance term and ε_t is population disturbance term

[12] The disturbance term in OLS is an AR(2) process. So there exists an autocorrelated disturbance term to this model, and then the OLS estimates are biased, inconsistent and inefficient. Here we use feasible generalized least squares twice to solve the problem and obtain the suitable estimates.

might stand somewhere between 0.310 and 0.381.

Table 5-3 The difference of external effect in countries

Writer	Country	Estimate method	Internal effect	External effect
Caballero and Lyons (1990)	West Germany	Seemingly unrelated regression	Not significant	0.28
	France			1.19
	U.K.			0.36
	Belgium			1.15
Chen and Chung (1995)	Taiwan	Seemingly unrelated regression	1.26	1.28
		Three-stage least square	1.30-1.69	1.06-1.42

Table 5-4 The estimation value of internal effects and external effects

Country	Estimate method	Basic technology level A	The share of the average economy wide capital stock in the production function β
Taiwan	OLS	2.669	-0.322 ~ -0.251
Taiwan	2SLS	0.013	-0.063 ~ -0.008
Taiwan	WLS	3.532	-0.241 ~ -0.170
Taiwan	FGLS	0.133	0.310 ~ 0.411

Tallman and Wang (1994) also stated the value of human capital's share of income in Taiwan might be between 0.553 and 0.617. In model 4, the parameter of human capital we obtain is equal to 0.458. It is a little smaller but still very closed to the result Tallman and Wang (1994) had done. The reason for the different value in these parameters might be:

1.There exist both internal and external effects in this production function.

2.The definition of human capital is different.

3.The sample period[13] that we adopt is different.

In our model, we assume $R = A[\lambda_1(1+\theta_1)(1+\theta_2)^{\beta} + (1-\lambda_1)]$. Even if the value of R is not statistically significant, we still try to calculate the value of θ_1, θ_2 and A (the basic technical level). We do not know what the exact value of A is, but A represents the basic technical level, which is a constant in the production function. If we use the parameters from Chen and Chung (1995) who had estimated the internal effect θ_1=0.26, the external effect θ_2=0.28 and the ratio of large firms λ_1=0.022, then we can get the basic technical level A (See Table 5-4).

In this chapter, we first introduced the recent history of Taiwan's economic development and established a regression to estimate the coefficient of physical capital and human capital in the production function. In this regression, we did not set any restriction, but the sum of coefficients from human capital and physical capital are close to 1 and statistically significant.

In these regressions, we found the different relationship between internal effects, external effects, the ratio of large firms and the basic technical level. We also can estimate the physical and human capital's share in the production function with the

[13] Tallman and Wang (1994) used the data of Taiwan in period 1965-1989. In this chapter we use the data of Taiwan in period 1974-2002. In the aspect of human capital, Tallman and Wang (1994) used weighted labor as the proxy variable of human capital. Here we use the average salary of labor as the proxy variable of human capital.

data from Taiwan.

Meanwhile, we estimated the average economy wide capital stock share in production function β is between 0.310 and 0.381 and the human capital share in production function is 0.458. The result is similar to that of what other economists had done. However, we used a common and relatively easier method to get this result. The finding here were mainly empirical, but we just used Taiwan's data to validate the assumption. There might be different results that are country-specific.

Chapter 6 The Factors of Enterprise Growth

6.1 Introduction

How is it that a small firm transforms into a large firm? Firms practically always begin as very small entities, with low amounts of capital drawn from the saving of the owner or borrowed from friends and relatives. Initial levels of employment are low, typically less than a dozen. The social and occupational backgrounds of the owners vary greatly. Some firms would eventually expand into medium or large-scale activities and go on growing, while other would witness the slowdown of their growth.

Dennis Anderson (1982) stated the possible predominance of large firms results from economy of scale with respect to plant size, economy of scale with respect to management and marketing, superior technical and management efficiency, preferential access to infrastructure service and external finance.

Caballero and Leyons (1990) found that the external economies exist in four European countries (West Germany, France, U. K. and Belgium); the effect of external economies is especially strong in France and Belgium. Chan, Chen and Cheung (1995) stated in the industry in Taiwan also existed strong external effects. It means external effect seems beneficial to economic growth and we assume that large firms possess higher external effects in the production function.

In chapter 3, we proved the internal effect and external effect are beneficial to the economic growth. The internal effect θ_1 is positively correlated to the firm's physical capital. The more physical capital a firm owns, the higher θ_1 a firm has. The higher probability of transforming from a small one into a large one, the more physical capital and higher internal effect a firm will have in future. Therefore, we can assume the internal effect θ_1 is a function of probability. For convenience sake, in our model all firms are identified and have the same internal and external effects.

Many economists have different arguments in terms of growth theory. Arrow (1962) constructed models in which ideas were unbounded by products of production. In these models, each worker's new skill and discovery will spill over to the entire economy immediately. It is possible to spread technology and knowledge from person to other person, because knowledge is nonrival. The mechanism is described as learning by doing.

Romer (1986), Lucas (1988) and Rebelo (1991) introduced a theory built on the work of Arrow (1962) and Uzawa (1965). In these models, growth may go on indefinitely because the return to investment in a broad class of capital goods includes human capital. The spillover of knowledge across producers and external benefits from human capital are part of the process in economy development, because they help to avoid the tendency of diminishing returns to the accumulation of capital.

Aghion and Howitt (1992) supposed technological advance results from the purpose R&D activity, and the activity is rewarded by some form of monopoly power. The growth rate remains positive in the long run, if the new R&D for the economy is created continually.

In this dissertation, we assume a large firm has higher internal effect and external effect than a small firm. A country with higher percentage of large firms will see a higher economic growth.

We think the factors that make small firms transform into large firms are also the momentum of economics growth. In this chapter, we try to find out the factors which influence an enterprise's growth.

6.2 The regression approach (Probit and Logit model)

We presume a firm has higher internal and external effects with the larger scale of firm. External and internal effects may not be of the same value, but it could have a positive relationship between them.

$$\theta_1 = q\,\theta_2 \qquad q > 0 \; \ldots\ldots\ldots\ldots\ldots\ldots (6\text{-}1),$$

where θ_1 is the internal effect and θ_2 is the external effect.

If we assume a variable p^*, p^* is a value of a firm's ability or talent. With a higher p^* the firm will have a higher potential transforming into a large firm. We can imagine that ability p^* is normally distributed across all firms, i.e., $p^* \sim N(\mu, \sigma_p^2)$. If there exists a critical ability value p and a firm's ability is lesser than the value p, the firm will still stay as a small firm or close down. If a firm's ability is larger than the value p, the firm will turn into a large firm.

Unfortunately, we cannot observe the ability p^* of a given firm, we only observe whether the firm grows up or closes down. That means that we would observe y_i,

Hence,

y_i ={1 the *i-th* small firm transforms into a large firm}

y_i ={0 otherwise}…………………(6-2)

In our model, we assume the internal and external effects are the important factors for economic growth. When a small firm in a society or economic system has a higher probability of transforming into a larger firm, the growth rate in this economic system will be higher. But how can we measure the probability? It seems impossible, however, that we can use the probit or logit model to solve the problem.

In probit model, we do not know the value of firm's ability p^*, but we can observe the variable y that takes on only two values, 0 or 1. The value of y equal to 0 means a small firm stays in the same situation or closes down. The value of 1 means a small firm has transformed into a large one successfully.

Although we cannot observe the variable p^*, there exists a correlation between p^* and y.

$$p_i^* = \mathrm{X}_i \beta + \varepsilon_i = \beta_{i1} x_1 + \beta_{i2} x_2 ... + \beta_{in} x_n + \varepsilon_i \qquad \varepsilon_i \sim N(0, \sigma^2)$$

Large firm y_i ={1 if $p_i^* > 0$ }

Small firm y_i ={0 if $p_i^* \le 0$ }………………………...(6-3),

where p_i^* is the *i-th* firm's ability, and x_i is the *i-th* firm's independent variables. β_i is the parameter of regression. ε_i is the error term.

Now we can obtain the probability of *i-th* small firm which transforms into large firm (y_i =1).

Prob (y_i =1) = prob($p^* > 0$)

= prob($\mathrm{X}_i \beta + \varepsilon_i > 0$)=prob($\varepsilon_i > -\mathrm{X}_i \beta$)

= prob($\varepsilon_i / \sigma > -\mathrm{X}_i \beta / \sigma$)=prob($\varepsilon_i / \sigma < \mathrm{X}_i \beta / \sigma$)= $\phi(\mathrm{X}_i \beta / \sigma)$

Prob (y_i =0) = 1- $\phi(\mathrm{X}_i \beta / \sigma)$ …………………………………..…………(6-4)

In logit model, the development of the logit is identical to that of the probit model. The formulation of the model can be written as

Prob (y_i =1) = $\Lambda(-\mathrm{X}_i \beta)$

$$= \frac{\exp(X_i \beta)}{1 + \exp(X_i \beta)} \text{ ……………………………..…(6-5)}$$

With the formulation, the predicted probability will lie between 0 and 1.

If we take sampling, we can take the likelihood to estimate the parameters. The likelihood for the sample is the product of the probability of each observation.

We assume there are m observations (0,1,2… n , $n+1$,… m), from 0 to n as the n observations such as y_i =0, and $n+1$ to m as the $m-n$ observations such as y_i =1, yields

LL=prob(y_1 =0).prob(y_2 =0)…..prob(y_n =0)…

prob(y_{n+1} =1).prob(y_{n+2} =1)…..prob(y_m =1)

$$=\prod_{i=1}^{n}[1-\Phi(X_i \frac{\beta}{\sigma})]\prod_{i=n+1}^{m}\Phi(X_i \frac{\beta}{\sigma})$$

$$=\prod_{i=1}^{m}\Phi(X_i \frac{\beta}{\sigma})^{yi}[1-\Phi(X_i \frac{\beta}{\sigma})]^{1-yi} \quad \text{……..……………………………..…(6-6)}$$

We could image the equation (6-6) is like the several binary variables that combine together. Finally, we take logarithm LL and differentiate with respect to β and make the value equal to 0. The name of the model is log-likelihood function.

A standard procedure to calculate estimation from a linear probability model is to "guess" what to begin with in finding a solution. As each guess gets better and better, the value of a log-likelihood function rises at each step until no improvement is possible, and the solution is found.

Johnston (1997) illustrated a model in which he described several factors affecting a worker to affiliate to a union number (potential experience, the number of years of schooling completed, etc.). In this example, he did not know the characteristics that determine a person's propensity to join a union, but he could observe exactly whether a worker join a union. He used a simple linear probability model to prove that potential experience of workers and a highly unionized industry are significant to determine whether a worker would join a union. We also use the similar regression approach to calculate the value of parameters in our model and to figure out which factors are significant for an enterprise's growth.

6.3 The source of data and definition of variables

From the data of Taiwan Bureau of Labor Insurance, it is known that the average surviving years of small firms in Taiwan is about 13 years. It means that we can find a firm, which has survived over 13 years. At the beginning of its business it was a small firm, but it is a large firm now.

For example, suppose that we found 100 firms and all of them were SMEs in the 1980s, of which half of them had transformed into large firms in 1999. We are able to get some information about these firms, like the age and education level of employers, and the average education level of employees, whether they received subsidy from government, what kind of product the firms produce, how long the firms have been running in business, and how many percentage product are exported, etc.

So we can create a regression

$$p_i^* = X_i \beta + \varepsilon_i = \beta_{i1} x_1 + \beta_{i2} x_2 + ... + \beta_{i7} x_7 + \beta_{i8} x_8 + \varepsilon_i$$

$$y_i = 1 \{\text{large firm, if } p_i^* > 0 \}$$

$$y_i = 0 \{\text{small firm, if } p_i^* < 0 \}$$

x_1 = The type of company.

x_2 = Receiving any subsidy from the government or not.

x_3 = The education level of employer.

x_4 = The average education level of employees.

x_5 = The percentage of the product exported.

x_6 = Having a research department in the company or not.

x_7 = The company ever borrowing capital from a bank or not.

x_8 = The location of the company.

We can use a log-likelihood function to estimate these parameters and obtain a regression. With this regression, we might find some variables significant and prove it is useful to explain which factors are influential for small firms transforming into large firms and lead to economic growth.

With the help of Small and Medium Enterprise Administration at Ministry of Economic Affairs Taiwan, we have sent 300 questionnaires to 300 firms and the number of responses is 42. 41 of the responses are valid. 12 of them are large firms and the remaining 29 are small firms. All valid samples at the beginning of their business were small enterprises.

The types of firms in valid samples of large firms are: eleven in the manufacturing industry and one in the commercial industry. For small firms there are fifteen in the manufacturing industry, nine in the commercial, four hotel and restaurant operation and one leisure service. We design eight questions in the questionnaire and the details are in Appendix C.

Under the assumption, we think the quality of employers and employees are very important factors for a small firm's growth. The employers who have a higher education level can make intellectual decision to improve the enterprise and the employees who have a higher education level can perform tasks better.

Grossman and Helpman (1991) stated the increasing sorts of intermediate goods in the research department are the momentum of economic growth. We also think a small firm with a research unit perhaps has a higher probability of growing than other firms without a research unit. Besides, the location and the financial support for small firms should also be important factors. We put those variables in our regression model.

Band, Wang and Yip (1996) stated that each type of factor tax would reduce the rate of growth while the subsidy will increase it, regardless of the sector factor intensities. They also use the factor intensities in the transitional dynamic

(Stolper-samuelson effect)[14]. For the reason, we also assume the subsidy from the government plays an important role in a firm's growth in the regression above. When a small firm gets subsidy from government, the probability of this small firm transforming into a large firm is higher than other small firms without subsidy.

In this chapter, the mathematic software that we used to estimate the parameters is Eviews 5.

6.4 The statistical test models and estimation results

The coefficients from probit and logit model are not certain probability values of firms' transformation. The coefficients must be translated into a mathematic function, and then we can get the i-th firm's growth probability. In the probit model, the derivation of the probability with respect to a specific X_i in the set of variables X is

$$\frac{\partial E(y)}{\partial X_i} = \phi(X\beta)\beta_i;$$

we define $z = \frac{X\beta}{\sigma}$, here $\phi(z) = \frac{1}{\sqrt{2\pi}} \exp(-\frac{1}{2}z^2)$,which is the standard normal density.

However, we are interested in the dimension of the sign and significance of the coefficients, because finding out which factors can influence a firm's growth is the most important task in this chapter. In Table 6-1, we use probit and logit model and put all variables in the regression. Table 6-1 reports the results for all firms with all variables. The results show that the probability of a small firm's growth increases

[14] Stolper-samuelson effect means a rise in the relative price of a good will lead to a rise in the return to that factor which is used most intensively in the production of the good, and conversely, to a fall in the return to the other factor.

with the education level of employers, but it decreases with the average education level of employees. Apart from the two variables, the other variables are not significant.

Table 6-1 Estimation of a small firm's growth factors (8 variables)

Dependent variable: The scale of firm (Large=1, Small=0)				
	Probit		Logit	
Variable	Coefficient	Prob.>t	Coefficient	Prob.>t
Type	2.79	0.139	5.07	0.15
	(1.89)		(3.55)	
Subsidy	-0.57	0.447	-1.04	0.43
	(0.76)		(1.34)	
Year er	0.21	0.052**	0.36	0.06**
	(0.11)		(0.19)	
Year ee	-0.491	0.004*	-0.87	0.001**
	(0.17)		(0.33)	
Export	-1.79	0.210	-0.30	0.21
	(1.42)		(2.45)	
Rd	0.51	0.523	0.90	0.50
	(0.80)		(1.35)	
Loan	0.32	0.731	0.81	0.64
	(0.95)		(1.35)	
Area	0.84	0.205	1.41	0.20
	(0.66)		(1.12)	
Obs. with Dep.=0		29	Total observations	41
Obs. with Dep.=1		12		

* Significance level is 1 percent. ** Significance level is 10 percent.

Convergence achieved after 7 iterations

In Table 6-2, in order to observe and decrease the influence of multicollinearity in this regression, we eliminate three insignificant variables (Rd, Loan and Export) in the probit and logit model and get the results.

The results are slightly different. It shows that the probability of a small firm's growth increases with the education level of employers and the type of the firm, but decreases with the average education level of employees. Except for the three variables, the other variables are insignificant.

Table 6-2 Estimation of small firm's growth factors(5 variables)

Dependent variable: The scale of firm (Large=1, Small=0)				
	Probit		Logit	
Variable	Coefficient	Prob.>t	Coefficient	Prob.>t
Type	2.48	0.09**	4.21	0.10**
	(1.48)		(2.59)	
Subsidy	-0.51	0.41	-0.81	0.45
	(0.63)		(1.10)	
Year er	0.22	0.04**	0.38	0.05**
	(0.10)		(0.20)	
Year ee	-0.47	0.002*	-0.82	0.004*
	(0.15)		(0.29)	
Area	0.70	0.24	1.22	0.24
	(0.60)		(1.04)	
Obs. with Dep.=0		29	Total observations	41
Obs. with Dep.=1		12		

* Significance level is 1 percent. ** Significance level is 10 percent.

Convergence achieved after 7 iterations

In Table 6-3, we change the eliminated insignificant variables (Subsidy, Export and Area) in these models and get the results. The results are different from table 6-2. The coefficients of the education level of employer, the type of firm and the average education level of employee are still significant and the sign of the variables are the same as in Table 6-1. But the coefficient of the firm's type is significant and the sign of the type is positive. Aside from that, the other variables are insignificant.

Table 6-3 Estimation of small firm's growth factors (5 variables)

	Dependent variable: The scale of firm (Large=1, Small=0)			
	Probit		Logit	
Variable	Coefficient	Prob.>t	Coefficient	Prob.>t
Type	2.30	0.07**	3.87	0.09**
	(1.29)		(2.29)	
Rd	-0.11	0.82	-0.19	0.83
	(0.54)		(0.92)	
Year er	0.23	0.03**	0.42	0.04**
	(0.11)		(0.20)	
Year ee	-0.47	0.002*	-0.82	0.004*
	(0.15)		(0.29)	
Loan	0.13	0.86	0.37	0.78
	(0.78)		(1.35)	
Obs. with Dep.=0		29	Total observations	41
Obs. with Dep.=1		12		

* Significance level is 1 percent. ** 5 percent.*** 10 percent.

Convergence achieved after 6 iterations

Table 6-4 Estimation of small firm's growth factors (3 variables)

Dependent variable: The scale of firm (Large=1, Small=0)				
	Probit		Logit	
Variable	Coefficient	Prob.>t	Coefficient	Prob.>t
Type	2.29	0.06***	3.80	0.08***
	(1.25)		(2.20)	
Year er	0.23	0.02**	0.40	0.03**
	(0.10)		(0.19)	
Year ee	-0.46	0.001*	-0.79	0.003*
	(0.14)		(0.26)	
Obs. with Dep.=0		29	Total observations	41
Obs. with Dep.=1		12		

* Significance level is 1 percent. ** 5 percent.*** 10 percent.

Convergence achieved after 6 iterations

In Table 6-4, we just take three variables (the education level of employers and employees, the type of the companies) in the probit and the logit model. The results in the both models are satisfying. All variables (the education level of employers and employees, the type of the firm) are significant. The education level of employers and the type of the firms are positively correlated to a firm's growth, but the average education level of employees is negatively correlated to a firm's growth.

The goal of this chapter is to investigate several econometric explanations that have been assumed to find a correlation between a firm's growth and several other factors. The first key finding of this study is that the education level of employer is an important determinant of a firm's growth. The correlation between a firm's growth and the education level of employer is positive.

It may be due to the fact that a employer has a higher education level that helps make intellectual decision to improve the firm and the firm can have a higher probability to transform into a large firm. Employers who have a higher education level also can perform tasks better in they own company.

The second key finding is that the type of the firms is also an important factor for a firm's growth. If the firm is a corporation organization, there are more owners in a firm and they can make the policy decision together. Those policymakers can discuss and reach an optimal decision on the firm's policy. It can decrease the occurrence of making the wrong policy and the firm could be more capable of taking on risks.

The third key finding is that the average education level of employees is a negative factor in a firm's growth. If a firm has a higher academic level in the workforce at the start-up phase, it is not beneficial to the firm's growth. It seems to be against conventional wisdom. Considering the special economic condition of Taiwan, the reason might be that small firms usually have a smaller amount of capital and possess lowly technology in Taiwan. It is difficult for a small firm to hire workers with a high education background and keep the workers staying in the same firm.

Inversely, it is not easy for workers with low education levels to find and get a new job in a large firm. When workers with lower academic education get a job from a small firm, they usually stick to the job for a long time and stay loyal to the firm. They could have much experience and execute the production process with fewer mistakes. It could be beneficial to a small firm's growth.

The situation may be different in other countries, because we just use the data from Taiwan's firms. If we could collect the data from other countries, we may obtain different results and prove that different factors would also influence a firm's growth.

Chapter 7 Conclusion

The objective of this dissertation may shed light on the process and influence of the small firm's transformation on economic growth. We have extended Lucas (1988) and Rebelo (1991) model and added different features of the representative firm into the model. Meanwhile, we also added the process of transformation of a small firm into a large one as an endogenous variable and investigate how the representative firm allocated its resource.

The main purpose of this dissertation is to explore the contribution and influence of large firms to economic growth and which factors could help a small firm transform into a large one successfully.

In the first part of this dissertation, we only used one-sector (production sector or goods sector) to explain the role of large firms in economic growth. Under the assumption that large firms have higher external and internal effects, we used a simple model to prove a country which has higher proportion of large firms could create a higher growth rate. But ignoring the human capital in the production function seems a problem to explain the change of economic growth; hence, we added education sector in the chapter 3 and compared the results.

In order to make the model more complete and closer to the real-life economic situation, we first combined two different kinds of firms in goods sector and added another education sector in the meanwhile. By the process of simulations, we obtained several important conclusions about the converging paths.

We found there exist several different convergence paths in an economic system which depends on the value of the parameters it faces. The high percentage of large firms is beneficial for a country's growth, but the effect will be inconsistent with different parameters. In some conditions, there would exist multiple paths to the steady state. It means even though two countries have the same constitution of firms, the difference in parameters, like the depreciation rate or the inverse of the intertemporal elasticity of substitution would make each country move towards its specific path and equilibrium point.

Considering the disadvantage of a small firm could not control or promote the process of transformation, we changed some assumptions about the representative firm to make the process of transforming into a large one to be an endogenous variable. It gave us another view to investigate the representative effort and decision to transformation. We used the percentage of human capital spent in education sector as a variable which related to the probability of a small firm transforming into a large one and did some simulations about the influence from each kind of parameters on the growth rate. We also used the empirical data of the world to compare with the result that we found. It has shown that the simulation was very close to the data of the real world. The ratios of small firms in most countries were very high, and the optimal value of human capital kept in goods sector in our model was also very high (the optimal value of human capital kept in goods sector is positively related to the ratio of small firms). But it was similar to the result in chapter 3, with different economic parameters the two countries with the same structure will have different growth rate and convergence paths. It could explain the divergence between countries.

In order to find the real value of external effect in production function, we tried to use the data from Taiwan and some econometric models to calculate the value. The definition of human capital is difficult and controversial, and we tried to use the divergence of the average salary according to academic degrees to represent the differences in labors. We also used four different regression methods to estimate the parameters in production. By using feasible generalized least squares (FGLS) we solved the problem about autocorrelation in error term and obtained the best result about the ranges of external effect in Taiwan.

One of the most important things in this dissertation was to find which factors could affect the chance that a small firm transforms into a large one. For this purpose, we sent lots of questionnaires to firms in Taiwan to investigate this issue and obtained the information about them.

Considering the characteristics of the data that we have collected from the firms in Taiwan, using probit and logit models could be a way to help us find the important factors which are beneficial to the transformation of small firms. We found the education level of employer, the education level of employees and the type of the firms are important determinants for a firm's growth, but the direction and extent of their influence are different. Because the limitation of the corrected data, we could just use the firms' data from Taiwan to illustrate our theory. The results could be different in other countries because of the difference in labor culture and entrepreneurship.

In this dissertation, we always thought the process for small firms transforming into large ones is beneficial and positive for economic growth, but it is not the only factor to make a country continue to develop. In endogenous growth theory, there exist lots of variables that could affect economic growth, like government expenditure, the allocation of taxes, human capital and the freedom of finance, etc. In this dissertation, we hoped we could provide a different view and make some contributions to this field.

Many points in the dissertation can be studied further in the future research. First, we do not consider the role of government in this model. In fact, we found the government expenditure and the structure of taxation were related to economic growth from some liturature, like Nader and Ramirez (1997), Wang and Yip (1992). Meanwhile, we also ignored the international trade in this model. We just considered that the closed economy and the small firms grow up independently without the influence from foreign countries.

Second, the data we have collected from Taiwan can be enriched in the future. The quantity of the valid sample was not quite sufficient, and it perhaps could not thoroughly show the real factors which affect the process of a small firm's transformation into a large one. Furthermore, we wish to consider a more realistic structure of firms such as micro-firms and international firms in the model in our future research.

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Appendix A: The definition of SMEs

In Taiwan

1. In the manufacturing, construction, mining and quarrying industries, the number of regular employees must be less than 200.
2. For enterprises in the following industries, those enterprises with less than 50 regular employees are classified as small and medium enterprises: agriculture, forestry, fisheries and animal husbandry, water, electricity and gas, wholesaling and retailing, hotel and restaurant operation, transportation, warehousing and communications, finance and insurance, real estate and leasing, professional, scientific and technical services, educational services, medical, healthcare and social welfare services, cultural, sporting and leisure services; other service industries.

Source: The Small and Medium Business Administration of Economic Affair, Taiwan

http://www.moeasmea.gov.tw

In European union

Enterprise category	Headcount	Turnover	(or)Balance sheet total
medium-sized	< 250	≤ € 50 million	≤ € 43 million
small	< 50	≤ € 10 million	≤ € 10 million
micro	< 10	≤ € 2 million	≤ € 2 million

Source: http://ec.europa.eu

In New Zealand

SMEs are defined as enterprises with 19 or fewer employees

Source: Ministry of Economic Development, New Zealand.

http://www.med.govt.nz

In U.S.A

- 500 employees for most manufacturing and mining industries.
- 100 employees for wholesale trade industries.
- $7 million of annual receipts for most retail and service industries.
- $33.5 million of annual receipts for most general & heavy construction industries.
- $14 million of receipts for all special trade contractors.
- $0.75 million of receipts for most agricultural industries.

Source: Small Business Administration (SBA) Size Standards Office.

In Australia

The numbers of employees

	Micro	**Small**	**Medium**	**Large**
Australia	X< 5	5-19	20-200	200+

Source: Department of Resources, Energy and Tourism, Australia.

http://www.ret.gov.au

In Singapore

Fixed assets	Employees
X< 8 million Singapore dollars	X< 50

Source: Ministry of Trade and Industry, Singapore

http://www.mti.gov.sg

In Japan

Sector	Employees	Turnover
Manufacturing	X< 300	X< 300 million Japanese yen
Construction		
Transportation		
Wholesale trade	X< 100	X< 100 million Japanese yen
Services industry:	X< 100	X< 50 million Japanese yen
Retailing	X< 50	X< 50 million Japanese yen

Source: Statistics Bureau, Japan

http://www.stat.go.jp/

In South Korea

Sector	Employees
Manufacturing	X< 300
Transportation	
Mining	
Construction	X< 200
Commerce	X< 20
Service industries	

Source: Small and Medium Business Administration, Korea

http://www.smba.go.kr

In China

Sector	Employees	Turnover
Industry	X< 2000	X< 300 million RMB
Construction	X< 3000	X< 300 million RMB
Retailing	X< 500	X< 150 million RMB
Transportation	X< 3000	X< 300 million RMB
Wholesale trade	X< 200	X< 300 million RMB
Hotel and Restaurant operation	X< 800	X< 150 million RMB

Source: National Bureau of Statistics of China

http://www.stats.gov.cn/

In OECD

	Employees
Large	500≤X
Medium	100-499
Small	20-99
Very small	X≤19

Source: Organisation for Economic Cooperation and Development (OECD)

Appendix B: The data of Taiwan and other countries

Table A-1 Correlative indictors of nation income in Taiwan

Year	Industrial Production Index(2001=100)	Percentage Distribution (%)			Unemploy-ment Rate	Annual Changes in CPI
		Agriculture	Industry	Services		
1951	0.73	56.69	16.31	27.00	4.52	...
1952	0.92	56.06	16.90	27.04	4.37	...
1953	1.15	55.57	17.61	26.82	4.20	...
1954	1.21	54.76	17.71	27.53	4.00	...
1955	1.38	53.63	18.02	28.35	3.81	...
1956	1.42	53.19	18.32	28.49	3.64	...
1957	1.60	52.31	18.95	28.74	3.73	...
1958	1.74	51.11	19.73	29.16	3.80	...
1959	1.95	50.32	20.31	29.37	3.88	...
1960	2.22	50.16	20.53	29.31	3.98	18.51
1961	2.57	49.84	20.89	29.27	4.10	7.78
1962	2.77	49.70	21.04	29.26	4.17	2.37
1963	3.03	49.42	21.27	29.31	4.26	2.19
1964	3.67	49.48	21.30	29.22	4.34	-0.17
1965	4.27	46.45	22.30	31.25	3.29	-0.06
1966	4.94	44.99	22.59	32.42	3.02	2.03
1967	5.76	42.54	24.57	32.89	2.29	3.36
1968	7.05	40.83	25.37	33.80	1.72	7.88
1969	8.45	39.32	26.31	34.37	1.88	5.06
1970	10.16	36.73	27.93	35.34	1.70	3.60
1971	12.54	35.14	29.91	34.95	1.66	2.77
1972	15.20	32.98	31.83	35.19	1.49	3.01
1973	17.67	30.49	33.70	35.81	1.26	8.16
1974	16.87	30.93	34.31	34.76	1.53	47.50
1975	18.46	30.45	34.90	34.65	2.40	5.22
1976	22.77	28.95	36.43	34.62	1.78	2.48
1977	25.81	26.71	37.63	35.66	1.76	7.06
1978	31.63	24.92	39.48	35.61	1.67	5.75

1979	33.64	21.46	41.60	36.92	1.27	9.76
1970	35.95	19.51	42.52	37.99	1.23	19.01
1981	37.21	18.84	42.39	38.77	1.36	16.32
1982	36.89	18.85	41.30	39.83	2.14	2.97
1983	41.56	18.63	41.15	40.23	2.71	1.35
1984	46.48	17.60	42.27	40.15	2.45	-0.03
1985	47.72	17.46	41.57	40.98	2.91	-0.16
1986	54.40	17.03	41.58	41.39	2.66	0.70
1987	60.18	15.28	42.77	41.96	1.97	0.51
1988	62.74	13.73	42.47	43.80	1.69	1.28
1989	65.10	12.91	42.09	45.01	1.57	4.42
1990	64.96	12.85	40.83	46.32	1.67	4.12
1991	69.76	12.95	39.93	47.13	1.51	3.62
1992	72.84	12.34	39.61	48.05	1.51	4.47
1993	75.65	11.49	39.09	49.43	1.45	2.94
1994	80.65	10.92	39.22	49.85	1.56	4.10
1995	84.51	10.55	38.74	50.71	1.79	3.67
1996	86.09	10.12	37.48	52.39	2.60	3.08
1997	91.39	9.57	38.16	52.26	2.72	0.89
1998	94.49	8.85	37.93	53.22	2.69	1.69
1999	101.45	8.25	37.21	54.53	2.92	0.17
2000	108.47	7.78	37.24	55.00	2.99	1.26
2001	100.00	7.52	35.99	56.47	4.57	-0.01
2002	107.92	7.50	35.24	57.26	5.17	-0.20
2003	115.61	7.27	34.83	57.90	4.99	-0.28
2004	126.96	6.56	35.21	58.23	4.44	1.62
2005	132.75	5.94	35.79	58.27	4.13	2.30
2006	139.38	5.49	36.02	58.49	3.91	0.60

Source: National Statistics, R.O.C.(Taiwan)

Table A-2 The employee's educational level in Taiwan

Year	The number of employee(Thousand people)			primary education or lower (h≦9 year)	secondary education (9＜h≦12 year)	higher education (h＞12 year
	All	Man	Woman			
1978	6231	4183	2048	4660	1041	530
1979	6432	4306	2126	4699	1157	576
1970	6547	4357	2191	4588	1279	681
1981	6672	4448	2224	4589	1358	726
1982	6811	4509	2301	4587	1465	758
1983	7070	4561	2509	4672	1584	814
1984	7308	4661	2647	4715	1711	882
1985	7428	4719	2709	4703	1797	928
1986	7733	4821	2912	4762	1971	1001
1987	8022	4966	3057	4779	2134	1109
1988	8107	5043	3064	4655	2260	1193
1989	8258	5149	3110	4603	2388	1267
1990	8283	5175	3108	4456	2473	1354
1991	8439	5274	3165	4460	2572	1407
1992	8632	5380	3252	4414	2691	1527
1993	8745	5422	3323	4265	2822	1658
1994	8939	5511	3428	4262	2925	1751
1995	9045	5558	3487	4180	2999	1866
1996	9068	5508	3560	3953	3095	2019
1997	9176	5562	3613	3877	3110	2189
1998	9289	5610	3679	3762	3214	2313
1999	9385	5624	3761	3609	3317	2459
2000	9491	5670	3821	3520	3375	2596
2001	9383	5553	3830	3318	3371	2694
2002	9454	5547	3907	3179	3424	2851
2003	9573	5579	3994	3063	3491	3019
2004	9786	5680	4106	2975	3592	3220
2005	9942	5753	4190	2880	3605	3458
2006	10111	5810	4301	2770	3631	3711
2007	10294	5868	4426	2689	3680	3925

Source: National Statistics, R.O.C.(Taiwan)

Table A-3 The proportion of large firms and SME in Taiwan

Year	All Firms	Large Firms		Small and Medium firms	
1983	706,526	10,088	0.014	696,438	0.986
1984	731,610	12,170	0.017	719,440	0.983
1985	727,230	11,006	0.015	716,224	0.985
1986	751,273	13,923	0.019	737,350	0.981
1987	761,553	18,279	0.024	743,274	0.976
1988	791,592	18,081	0.023	773,511	0.977
1989	798,865	20,823	0.026	778,042	0.974
1990	818,061	23,227	0.028	794,834	0.972
1991	850,679	25,123	0.030	825,556	0.970
1992	900,801	29,075	0.032	871,726	0.968
1993	934,588	32,820	0.035	901,768	0.965
1994	969,094	36,242	0.037	932,852	0.963
1995	1,012,212	20,597	0.020	991,615	0.980
1996	1,024,360	21,035	0.021	1,003,325	0.979
1997	1,043,286	22,851	0.022	1,020,435	0.978
1998	1,069,116	23,999	0.022	1,045,117	0.978
1999	1,085,430	24,692	0.023	1,060,738	0.977
2000	1,091,245	20,935	0.019	1,070,310	0.981
2001	1,098,185	20,023	0.018	1,078,162	0.982
2002	1,130,525	25,819	0.023	1,104,706	0.977
2003	1,171,780	24,580	0.021	1,147,200	0.979
2004	1,190,176	13,190	0.011	1,176,986	0.989
2005	1,253,604	27,509	0.022	1,226,095	0.978
2006	1,275,508	31,409	0.025	1,244,099	0.975

Source: Small and Medium Enterprise Administration.(Taiwan)

Table A-4 Series of real net fixed capital stock (excluded land) of industrial & service sectors (million NT$)

Year	Industrial Sector	Service Sector	Total Capital Stock	Capital Stock Per capita (NT.)
1971	664672	349407	1014079	67,276.88
1972	729766	395290	1125056	73,208.78
1973	808720	436985	1245705	79,636.10
1974	905934	474818	1380752	86,691.63
1975	1030681	514698	1545379	95,258.00
1976	1165377	567827	1733204	104,537.48
1977	1283065	637580	1920645	113,768.45
1978	1390551	730758	2121309	123,314.06
1979	1515329	850782	2366111	134,874.42
1980	1675693	994555	2670248	149,459.69
1981	1805727	1189811	2995538	164,644.69
1982	1962283	1314119	3276402	176,952.12
1983	2095272	1414077	3509349	186,761.50
1984	2223452	1509516	3732968	195,759.09
1985	2326259	1594972	3921231	203,027.16
1986	2416925	1673085	4090010	209,646.46
1987	2543114	1760222	4303336	218,166.48
1988	2701194	1868201	4569395	228,991.89
1989	2868750	1984401	4853151	240,772.46
1990	3039802	2102347	5142149	252,050.00
1991	3209033	2227791	5436824	263,848.81
1992	3388321	2373825	5762146	276,991.33
1993	3565493	2533074	6098567	290,471.36
1994	3749083	2688467	6437550	303,975.27
1995	3988205	2859611	6847816	320,629.20
1996	4271214	3048633	7319847	340,055.74
1997	4630338	3255567	7885905	362,690.16
1998	5083671	3518020	8601691	392,259.17
1999	5588544	3787192	9375736	424,387.64
2000	6206979	4019675	10226654	459,074.59
2001	6725050	4204975	10930025	487,826.28
2002	7221185	4364231	11585416	514,432.36

Source: National Statistics, R.O.C.(Taiwan)　　　Base period: 1996 year

Table A-5 The proportion of SME in South Korea

Year	GDP	All Firms	Large Firms	Small and Medium firms	
1994	13376.15	2,382,571	17,253	2,365,318	99.3
1995	14736.43	2,622,259	20,506	2,601,753	99.2
1996	15650.24	2,648,261	19,212	2,629,049	99.3
1997	15956.8	2,689,557	18,932	2,670,625	99.3
1998	14685.35	2,622,356	17,132	2,605,224	99.3
1999	15863.67	2,758,627	18,844	2,739,783	99.3
2000	16890.31	2,729,957	22,152	2,707,805	99.2
2001	17575.33	2,658,860	9,169	2,649,691	99.7
2002	18921.85	2,861,830	4,917	2,856,913	99.8
2003	19696.55	2,939,661	4,764	2,934,897	99.8
2004	21088.12	2,927,436	4,903	2,922,533	99.8
2005	22048.39	2,867,749	4,166	2,863,583	99.9
2006	23323.5	2,940,345	4,231	2,936,114	99.9
2007	24949.65	2,976,646	2,461	2,974,185	99.9

Source: The Small and Medium Business Administration of Korea.

Table A-6 The proportion of SME in India

Year	GDP	Small and Medium firms	Ratio
1992	1603.65	70630	0.959334
1993	1670.5	73510	0.959224
1994	1763.1	76490	0.959461
1995	1906.67	79600	0.959341
1996	1969.89	82840	0.959296
1997	2075.23	86210	0.959319
1998	2193.85	89710	0.959401
1999	2395.61	93360	0.959313
2000	2456.5	97150	0.959404
2001	2580.39	101100	0.959341
2002	2650.86	105210	0.959347
2003	2832.85	109490	0.959319
2004	3053.04	113950	0.959266

Source: Annual report (2004). Government of India ministry of small scale industries.

Table A-7 The size distribution of manufacturing industry among countries

		Percentage of Enterprises / Establishments				Percentage of Employment			
		Enterprise Size				Enterprise Size			
Country	**Year**	**1-19**	**20-99**	**100-499**	**500+**	**1-19**	**20-99**	**100-499**	**500+**
Australia	1994	82.0	14.1	3.4	0.4	22.3	27.5	32.7	17.5
Austria	1993	43.2	41.5	10.0	5.2	4.3	26.9	23.4	45.5
Canada	1994	50.6	37.8	10.2	1.4	7.6	27.8	39.4	25.2
Czech Republic	1995	94.9	2.9	1.6	0.5	18.0	10.3	24.6	47.1
Germany	1993	71.5	19.4	4.1	5.0	19.9	22.1	10.8	47.2
Greece	1992	59.0	34.3	6.0	0.7	20.4	35.0	27.5	17.2
Italy	1992	89.7	9.0	1.2	0.2	38.7	25.0	17.3	19.0
Japan	1994	74.3	21.6	3.6	0.5	22.4	30.9	25.0	21.6
Korea	1994	69.5	26.1	3.0	1.3	20.5	32.0	14.2	33.3
Luxembourg	1992	79.4	15.0	4.7	0.9	13.0	22.1	35.0	29.9
Mexico	1994	80.3	15.1	2.7	2.0	12.2	21.2	15.6	51.0
Netherlands	1993	78.0	17.2	4.3	0.6	15.7	24.8	27.8	31.7
New Zealand	1994	90.6	7.7	1.5	0.3	27.3	24.7	24.0	24.0
Norway	1994	40.2	47.4	7.5	4.9	9.3	34.9	18.2	37.6
Portugal	1994	85.8	11.8	2.2	0.2	23.5	32.3	27.8	16.5
Sweden	1993	44.4	40.8	12.4	2.4	6.9	23.1	35.3	34.7
Switzerland	1991	84.2	12.3	3.1	0.4	20.2	26.9	31.3	21.5
Turkey	1992	36.6	47.1	13.3	3.0	5.5	22.2	32.2	40.1
United Kingdom	1994	82.7	12.9	3.7	0.8	13.2	21.6	28.9	36.3
United States	1993	73.7	19.8	5.1	1.4	7.4	14.6	16.5	61.5
Average		70.5	22.7	5.2	1.6	16.4	25.3	25.4	32.9

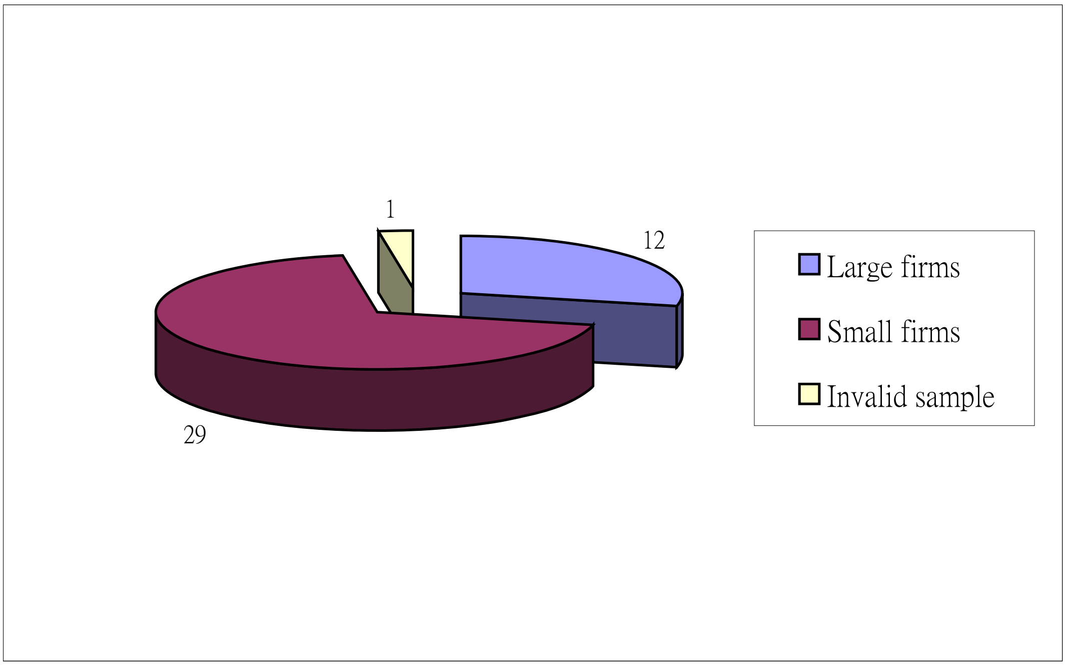
Source: OECD, Small Business, Job Creation and Growth: Facts, Obstacles and Best Practices (Paris: OECD, 1997), Table 1.1.

Appendix C: The result of questionnaire

The result of questionnaire

Total questionnaires	The number of responses	The response rate	The valid sample	Large firms	Small firms
300	42	14%	41	12	29

Type	Manufacturing	Commercial	Hotel and restaurant	Leisure service
Large firms	11	1		
Small firms	15	9	4	1



The questions I designed in the questionnaire

1.What is the age and educational level of boss in your company?

(Elementary school=6, Junior high school=9, Senior high school=12, Bachelor=15, Master=17, Doctor=21)

2. What kind of company do you have? (Single owner, corporation organization)

(Single owner=0, corporation organization=1)

3.What is the average educational level of employee in your company?

(Elementary school=6, Junior high school=9, Senior high school=12, Bachelor=15, Master=17, Doctor=21)

4.Have your company ever received any subsidy from government or any organization?

(Never receive subsidy=0 , ever receive subsidy=1)

5.How many percent product are exported to foreign countries?

(The value from 0~1, for example ,78 percent=0.78)

6.Is there any research department in your company? If there is, How many percent expenditure do the research department have?

(Don't have research department=0 , with research department=1)

7. Have your company ever borrowed capital from bank?

(Never=0 ,at least one=1)

8. Where is your company's location? Is it in industry area?

(Not in industry area=0, in industry area=1)

Appendix D: The data and the variables in regression

Type: The sort of company.

Subsidy: If the firm receive any subsidy from government

Year er: The academic years of employer.

Year ee: The average academic years of employees.

Export: The ratio of production are exported to foreign countries.

Rd: If the firm had research department.

Loan: If the firm borrowed money from bank.

Area: If the firm was in industry area.

Table A-8 The data in regression

	Large=L Small=S	Type	Subsidy	Year er	Year ee	Export	Rd	Loan	Area
Large firms									
Firm 1	L	1	0	16	12	0	0	0	1
Firm 2	L	1	0	16	12	0	0	0	1
Firm 3	L	1	0	16	12	0	1	0	0
Firm 4	L	1	0	9	12	0	0	0	0
Firm 5	L	1	0	16	12	0	0	0	0
Firm 6	L	1	1	12	6	0.8	1	0	1
Firm 7	L	1	0	12	12	0	0	0	1
Firm 8	L	1	0	16	12	0	1	0	0
Firm 9	L	1	1	16	14	0.49	1	1	1
Firm 10	L	1	1	14	14	0.4	1	0	1
Firm 11	L	1	0	18	12	0	1	0	0
Firm 12	L	1	1	14	9	0.05	0	0	0

	Large=L Small=S	Type	Subsidy	Year er	Year ee	Export	Rd	Loan	Area
Small firms									
Firm 13	S	1	0	6	12	0	1	0	0
Firm 14	S	1	1	12	12	0	0	1	0
Firm 15	S	1	0	12	12	0	0	0	0
Firm 16	S	1	0	12	12	0	0	0	0
Firm 17	S	0	0	9	12	0	0	1	1
Firm 18	S	1	0	9	12	0	0	0	0
Firm 19	S	0	0	12	12	0	0	1	0
Firm 20	S	0	0	6	12	0	0	0	1
Firm 21	S	1	0	16	12	0	0	0	0
Firm 22	S	0	0	9	12	0	0	0	0
Firm 23	S	1	1	9	12	0.25	0	1	1
Firm 24	S	0	0	6	6	0.5	0	0	0
Firm 25	S	0	0	6	6	0	0	0	0
Firm 26	S	0	0	6	6	0	0	1	1
Firm 27	S	1	0	12	12	0	0	0	0
Firm 28	S	0	1	11	11	0	1	0	0
Firm 29	S	1	1	11	11	0.85	1	0	0
Firm 30	S	0	1	16	12	0.1	1	0	0
Firm 31	S	1	1	16	16	0	1	0	0
Firm 32	S	1	1	12	12	0.05	1	0	1
Firm 33	S	0	0	14	14	0	0	0	0
Firm 34	S	1	0	14	12	0.8	1	0	0
Firm 35	S	0	0	14	14	0.3	0	0	1
Firm 36	S	1	0	14	12	0	0	0	0
Firm 37	S	0	0	14	14	0.1	1	0	1
Firm 38	S	0	0	16	12	1	1	0	0
Firm 39	S	1	1	14	11	0.2	1	0	1
Firm 40	S	1	1	14	14	0.4	1	0	1
Firm 41	S	1	0	14	12	0.75	1	0	1
Average		0.6829	0.3170	12.4634	11.6585	0.1717	0.4390	0.1463	0.3902

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