The Electroweak Phase Transition in Models with Gauge Singlets

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Amine AHRICHE
Aus Algerien

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Gutachter: Prof. Dr. Mikko Laine
Prof. Dr. Dietrich Bödeker

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Abstract

A strong first order phase transition is needed for generating the baryon asymmetry; and also to save it during the electroweak phase transition (EWPT). However this condition is not fulfilled within the Standard Model (SM), but in its extensions. It is widely believed that the existence of singlet scalars in some Standard Model extensions can easily make the EWPT strongly first order. In this work, we will examine the strength of the EWPT in the simplest extension of the SM with a real gauge singlet using the sphaleron energy at the critical temperature. We find that the phase transition is stronger by adding a singlet; and also that the criterion for a strong phase transition $\Omega(T_c)/T_c \gtrsim 1$, where $\Omega = (v^2 + (x - x_0)^2)^{\frac{1}{2}}$ and x (x_0) is the singlet vacuum expectation value in the broken (symmetric) phase, is not valid for models containing singlets, even though often used in the literature. The usual condition $v_c/T_c \gtrsim 1$ is more meaningful, and it is satisfied for the major part of the parameter space for physically allowed Higgs masses. Then it is convenient to study the EWPT in models with singlets that couple only to the Higgs doublets, by replacing the singlets by their vevs.

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Chapter 1

Introduction

Cosmology, the study of the Universe, was of major interest for human beings since very early history. But an elegant theory (The Big Bang) that describes the universe appeared only last century following the work of Friedmann in 1922 and Lemaitre. It was based on the assumption of a homogeneous and isotropic Universe, which started in a very hot and dense phase; then cooled with time. This scenario was much elaborated upon by Gamow and others in the forties who clarified the physical picture of the model. The particle physics development in the sixties and seventies allowed to work it out in greater details. The development of a new version within the extended framework of gauge theories had a significant impact on this model and led to the so called standard Big Bang model in the seventies and eighties. The inflationary paradigm was introduced by Guth [1], to cure some of the pathological problems of the hot Big Bang model (notably the horizon and flatness problems), but it succeeded also to cure other problems of this model.

In recent years, the tremendous progress in observational cosmology has led to a huge increase in data both in volume as well as in accuracy. Then a new dialogue was opened between cosmology and particle physics, as particle physics started investigating different problems in cosmology; using cosmology as a relatively cheap laboratory to probe physics beyond the ability of the existing and next-generation accelerators.

Ever since the introduction of the idea of antimatter [2], where for every particle there exists an anti-particle which has the same mass, decay width but opposite quantum charges, and since its effective observation [3]; was asked with insistence the question is asked about what happened to the antimatter in our universe. All of the observations favor an asymmetry between matter and antimatter rather than the scenario where antimatter is separated from matter and thus difficult to detect [4].

This asymmetry has to be explained within the known physics; this is what is called the baryogenesis problem. In 1967, Sakharov presented the famous three conditions for baryogenesis; which are B violation, C and CP violation; and a departure from thermal equilibrium [5].

It appears that the Standard Model (SM) contains all baryogenesis ingredients, but most of the computations fail to get the exact value of baryon density $n_b/n_\gamma \sim 10^{-10}$ [6], within the SM range. The basic reasons are that the CP violation is found to be too small; and the departure from equilibrium cannot be obtained in the minimal SM.

This being the case, it is only logical to go beyond the SM, especially after it became evident that the SM was not a fundamental theory notably with the observation of neutrino oscillations [7]. There are many models that give an acceptable value for the baryon density in the universe, but not all of them are testable by available or near-future experiments. Only models that operate at and below TeV scale will be testable.

The electroweak baryogenesis scenario [8] is one of these testable models. The baryon number is violated in the anomalous non-perturbative interactions at the quantum level [9], where some CP violation sources are assumed to exist; and the departure from thermal equilibrium is realized via a strong first order phase transition.

In the SM, the electroweak phase transition (EWPT) is so weak [10] unless the Higgs upper bound is less than 45 GeV [11]; which is in conflict with present data [12]. Therefore there is basically no departure from equilibrium in the SM. But when extending the SM with additional gauge singlets, the EWPT gets stronger with physically allowed Higgs masses [13, 14, 15].

The EWPT has been extensively studied in the literature, in the SM with a gauge singlet [13, 14], in the singlet Majoron model [15], in the SM with a sixdimension Higgs operator with a low cut-off [16], in the MSSM perturbatively [17] or on the lattice [18], in the MSSM through charge and color breaking [19]; and using many techniques like dimensional reduction [20].

In our work, we will focus on the SM extension with an additional singlet. The importance of singlets comes from the fact that many SM extensions at higher scales contain hidden sectors with elements that transform non-trivially under a hidden sector gauge group but as a singlet under the SM gauge group. It has recently been noticed that the SM Higgs ϕ field plays a very special role with respect to such hidden sector; because it can provide a window into this sector through renormalizable interactions like $\phi^{+}\phi S^{2}$ where S is a SM gauge singlet [21]. Such a coupling to the hidden sector can have important phenomenological implications [21, 22].

A strong first order EWPT is easily obtained in the SM with a gauge singlet when this singlet develops a vacuum expectation value (vev) during the transition. Therefore we will focus on models with a singlet where it seems that the singlet vev plays a significant role in making the EWPT strongly first order. We will consider the SM with a real gauge singlet S, and study the EWPT strength using the model-independent criterion [23], which is based on the Sphaleron solution [24], and then compare the results with those coming from the simpler one in [14]. We find that the last criterion is not valid for this kind of models; and we will give the valid one.

This work is organized as follow: in the second chapter, we give a brief review of the hot Big Bang model, then we discuss the problem of baryon number of the universe. This is followed by a review of the Sakharov conditions and their fulfillment in the SM. In the last section, a brief review is given about some popular scenarios for baryogenesis. In the third chapter, we give a general discussion on the electroweak phase transition, by first introducing the main tool used in that context, namely the effective potential technique. We then show the meaning of the phase transition strength, taking the EWPT in the SM as an example. After that, we show how a first order phase transition proceeds. The translation of the EWPT model-independent criterion in [23]; to the simple criterion for the SM case, which is widely used in the literature, is shown in the next section. Finally, we discuss briefly the EWPT in some SM extensions. In the fourth chapter, we study the EWPT in the SM with a gauge singlet, starting by introducing the model and imposing some physical constraints on it. After that, we find the sphaleron solution for this model followed by the study of the EWPT using the model-independent criterion. In the last section, we check the validity of the simpler criterion in [14] and give the correct simple criterion. Finally, we summarize our results. Some notes about the standard cosmology and a brief history of the universe are given in appendix A. In appendix B, we give the values of the thermal bosonic masses that are needed for the effective potential. In appendix C, we find the boundary conditions of the sphaleron solution of the field equations for the SM with a singlet. Finally, the numerical method used to solve the field equations in the sphaleron configuration is described in appendix D.

Chapter 2

The Baryon Number in the Universe

2.1 The Hot Big Bang

Modern cosmology started to develop when Einstein introduced his general theory of relativity in the beginning of the last century. However, the development of particle physics in the recent years, has also infused new important ideas into cosmology, which can be applied to the study of the birth (or creation) of the universe and its evolution in the earliest epochs until today. The hot Big Bang model is the accepted cosmological theory up to date, it is based upon general relativity and the Friedmann model for the expanding universe. According to this theory, the existence of the universe starts in extreme conditions: ultra-huge energy density and temperature. Today, we observe just the consequences of different processes during the universe expansion. The Standard Cosmological Model (or the hot Big-bang scenario), and its different stages and evolution, are well described in an elegant way by the authors of [25, 26]. A detailed discussion on the formulation of this theory is given by Weinberg [27], Kolb and Turner [28]; Mukhanov [29], and Linde [30], while brief reviews can be found in [31].

The beginning of the theory was with Friedmann in 1922, when he assumed that the universe is spatially homogeneous and isotropic, which is based on Einstein's cosmological principle; and created a model for an expanding universe. This theory became the cornerstone of modern cosmology only, when it was experimentally supported by Hubble's and Slipher's observations of the galactic redshift in 1929.

Up to the mid–sixties it was not clear whether the universe had started as a hot or a cold big-bang. The new era in cosmology was opened when Penzias and Wilson discovered the 2. 7^0K microwave background radiation in 1964–65 [32], which had been predicted by the hot universe theory. The radiation detected by Penzias and Wilson was the relic of the initially hot photon gas emitted at the decoupling time, which cooled down during the expansion of the universe. This was the second stage in the development of modern cosmology and was decisive in the establishment of the hot Big Bang model as the Standard Cosmological Model.

In Friedmann's model[1], the universe scale factor a, which represents somehow the universe radius, is a function of time; and its evolution obeys the equations of general relativity. These equations relate the geometry of the universe with its matter components. A qualitative and approximate solution indicates that the universe was at first dominated by the interactions of hot ultra-relativistic particles like photons and neutrinos, that is called the radiation-dominated era. After cooling, it became dominated by non-relativistic particles (also called sometimes dust); this era is called matter-dominated era. After this era until now, it is not clear what is the dominant component: curvature or cosmological constant?

The hot Big Bang Model is supported by a number of important observations: the expansion of the universe observed by Edwin Hubble in 1929. The second one is the abundance of the light elements H, He, Li, where the model predicts that these light elements should have been formed from protons and neutrons in the first few minutes after the Big Bang with certain ratios. The third important one is the cosmic microwave background (CMB) radiation which showed that the early universe should have been very hot. These three measurable signatures strongly support the notion that our universe evolved from a dense, nearly featureless hot gas, just as the Big Bang model predicts.

However there remain serious problems for the Standard Cosmological Model: What is the nature of Dark Matter? Why does the universe expand, or what is the

[1]See Appendix A, where a brief review of this model is given.

nature of Dark Energy? And also, how to explain the origin of the usual known matter (baryons)?

2.2 The Baryon Asymmetry in the Universe

The CPT theorem assures that for each particle X there exists an anti-particle $\bar{X}$ with exactly the same mass, $m_X = m_{\bar{X}}$, and decay width, $\Gamma_X = \Gamma_{\bar{X}}$, and opposite charges $Q_X = -Q_{\overline{X}}$ [2]. This symmetry easily leads us to conclude that the Universe contains an equal number density for particles and anti-particles $n_X = n_{\bar{X}}$, which is not true in the observed Universe. We do not observe any anti-matter within the solar system except some anti-protons $\overline{p}$ in the cosmic rays, which are believed to be produced as secondaries in collisions $pp \to 3p + \overline{p}$ with the observed ratio

$$\frac{n_{\bar{p}}}{n_p} \sim 3 \times 10^{-4}; \tag{2.1}$$

and also the experimental limit on $\bar{n}_{^4\mathrm{He}}/n_{^4\mathrm{He}}$ is of the order of 10^{-6} [33]. This means that our solar system is entirely made of matter.

However, we can presume that matter dominance over anti-matter is only local, and it is realized up to a certain length scale ℓ_B, beyond which the picture is different and islands of anti-matter may exist. This picture predicts that:

• The size of the matter domain, which we consider our domain, should be quite large, roughly speaking $\ell_B \geq 10\ Mpc$ [4].

• Robust gamma rays bursts must be detected as results of matter-antimatter co-annihilation at the domain's borders.

• The existence of domains of anti-matter will destroy the spectrum of the cosmic microwave background radiation.

Since no gamma rays and no inhomogeneities in the CMB radiation are observed, matter dominates over anti-matter everywhere or at least in a huge scale.

It is well known, that ordinary (baryonic) matter represents only 4% of the Universe budget, which includes galaxies, our Milky Way, our Solar System and of course ourselves. The rest of the Universe budget is about 70% of Dark Energy or vacuum energy, a component with negative pressure responsible of Universe expansion, about 25% of Dark Matter, a component that has no electromagnetic interactions; and therefore can not be easily detected; and less than 1% of primordial neutrinos [28].

The important question now is, whether the origin of this baryonic component is well understood within the known physics, i. e. particle physics theories and general relativity.

Since the Universe is expanding, it is not convenient to speak about the baryonic density; it will be better to rescale it by such a quantity like the photon number density n_γ or the entropy density s ($s = 7.04 n_\gamma$), in order that the resulting quantity remains nearly constant after the decoupling during the Universe expansion.

From the theoretical point of view, there is no justification to assume that the Universe started its evolution with a defined baryon asymmetry; $n_b\,(t=0) > n_{\bar{b}}\,(t=0)$. The natural assumption is the Universe was initially zero [2]. It has been shown that the universe contains no appreciable primordial anti-matter; by direct observations like the Big Bang nucleosynthesis which requires that the ratio of the effective baryon number $(n_b - n_{\bar{b}})$ to the entropy density should be between

$$2.6 \times 10^{-10} < \eta \equiv \frac{n_b - n_{\bar{b}}}{s} < 6.2 \times 10^{-10}. \tag{2.2}$$

This number has been independently determined to be $\eta = (8.7 \pm 0.3) \times 10^{-11}$ from precise measurements of the relative heights of the first two microwave background (CMB) acoustic peaks by the $WMAP$ satellite [6].

In the Standard Cosmological Model, there is no obvious explanation for such a small value of η, consistent with nucleosynthesis and CMB; and it has to be imposed by hand as an initial condition, which is not elegant theoretically. In addition to that, does this asymmetry survive during different eras in the Universe evolution?

Thus, many questions are naturally asked: At what time during the Universe evolution did this asymmetry emerge? What are the processes responsible for the generation of this specific baryon asymmetry? Are the scenarios considered in agreement with different particle physics constraints?

[2]Even if there exists primordial baryonic asymmetry, it will be diluted during the inflation period; then any surviving asymmetry should be generated after the inflation.

2.3 Sakharov Criteria

After the inflation end, the Universe was initially baryon symmetric ($n_b = n_{\bar{b}}$) although the baryon number versus the number of anti-baryons appears to be large today ($n_b \gtrsim 10^9 n_{\bar{b}}$). The problem to get the parameter η in the range (2.2) is called the Baryogenesis problem, which is one the fundamental open questions in modern cosmology.

However, it has been suggested by Sakharov long ago (in 1967) [5] that a tiny baryon asymmetry may have been produced in the early Universe. He forwarded his three famous necessary criteria for a successful baryogenesis:

• Violation of the baryon number (B) symmetry.

• Violation of the discrete symmetries C (charge conjugation) and CP (the combination of parity and C).

• A departure from thermal equilibrium.

Let us discuss each criterion on its own.

2.3.1 The Baryon Number Violation

This condition is obvious, since the baryon number is conserved in all processes, then a baryon asymmetry will be never generated. Then at a certain period, there must be interactions that violate the baryon number. The question is: in which theory does this possibility exist?

Grand Unified Theories (GUTs) [34] describe the fundamental interactions by a unique gauge group G which includes the Standard Model (SM) gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ as subgroup. The key idea of GUTs is that at energies higher than a certain scale M_{GUT}, the symmetry group is G and that; at lower energies this symmetry is broken down to the SM gauge symmetry. The symmetry breaking may also go through a chain of several steps. The main motivation for this scenario is that the running gauge couplings of the SM unify to a single gauge coupling, at the scale $M_{GUT} \simeq 2 \times 10^{16}$ GeV [35].

GUTs have an interesting property which is the representation of quarks and leptons in the same multiplet; therefore it is possible for scalar and gauge bosons to mediate gauge interactions among fermions having different baryon number. Then baryon number violation seems to be very natural in GUTs.

However the situation is different for the case of Standard Model. In the Standard Model, the Lagrangian contains only color singlet Higgs fields, and is automatically invariant under global abelian symmetries which may describe the baryonic and leptonic numbers, and therefore, B and L are called accidental symmetries which are never violated at tree-level or at any perturbative order. Nevertheless in 1976, 't Hooft found that instantons, which are non-perturbative effects, may give rise to processes which violate the combination $B+L$, but not its orthogonal combination $B-L$; but the probability of these processes at zero temperature is estimated to be strongly suppressed $\Gamma \sim \exp\left(-16\pi^2/g^2\right) \simeq 10^{-162}$ [9], where the pre-exponential factor in this case is not important.

Since baryon and lepton symmetries are anomalous at the quantum level [36], their respective Noether currents j^μ_B and j^μ_L are not conserved, but satisfy

$$\begin{aligned} \partial_\mu j^\mu_B &= \partial_\mu j^\mu_L = \tfrac{n_f}{32\pi^2}\left(g^2 F^a_{\mu\nu}\tilde{F}^{a\mu\nu} - g'^2 f_{\mu\nu}\tilde{f}^{\mu\nu}\right) = \tfrac{n_f}{32\pi^2}\left(g^2\partial_\mu K^\mu - g'^2\partial_\mu k^\mu\right) \quad (2.3) \\ K^\mu &= \epsilon^{\mu\nu\alpha\beta} F^a_{\nu\alpha} A^a_\beta - \frac{g}{3}\epsilon_{abc} A^a_\nu A^b_\alpha A^c_\beta,\ k^\mu = \epsilon^{\mu\nu\alpha\beta} f_{\nu\alpha} a_\beta, \end{aligned}$$

where n_f is the number of families, g (g') is the gauge coupling of $SU(2)_L$ ($U(1)_Y$), A^a_ν and a_ν are the gauge fields of $SU(2)_L$ and $U(1)_Y$ respectively; and $\tilde{F}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}F_{\alpha\beta}$ ($\tilde{f}^{\mu\nu} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}f_{\alpha\beta}$) is the dual of the $SU(2)_L$ ($U(1)_Y$) field strength tensor. From eq. (2.3), one can compute the variation in baryon number from the initial time t_i to some arbitrary final time $t = t_f$ as

$$B\left(t_f\right) - B\left(t_i\right) = n_f[N_{CS}(t_f) - N_{CS}(t_i)]; \quad (2.4)$$

and N_{CS} is the Chern-Simons number

$$N_{CS}(t) \equiv \tfrac{g^2}{32\pi^2}\int d^3x\, \epsilon^{ijk}\, Tr\left(A_i\partial_j A_k + \frac{2}{3} i g A_i A_j A_k\right). \quad (2.5)$$

This quantity is not gauge invariant, but the difference ΔN_{CS} is. For a transition between two neighboring vacua, N_{CS} is changed by unity, and therefore B; and simultaneously L, is changed by $n_f = 3$; and therefore 9 left-handed quarks (3 color states for each generation) and 3 left-handed leptons (one per generation) are created.

All the field configurations that have integer Chern-Simons number N_{CS} are equivalent; and the transition between these degenerate vacua of the gauge theory is suppressed due to existence of a barrier in between them (see Fig. 2.1). Then, in order to understand this transition in the electroweak theory, one needs to know the field configuration that interpolates between two nearest vacua and makes the transition possible. Klinkhamer and Manton found that this configuration exists, and it is a saddle point solution to the equations of motion with a single negative eigenvalue squared. Its Chern-Simons number is $1/2$, and they called it *Sphaleron*, a Greek word, which means: *'ready to fall'* [24]. In Fig. 2.1, the dependence of the static energy of the system on N_{CS} is shown, where there exist barriers between different vacua.

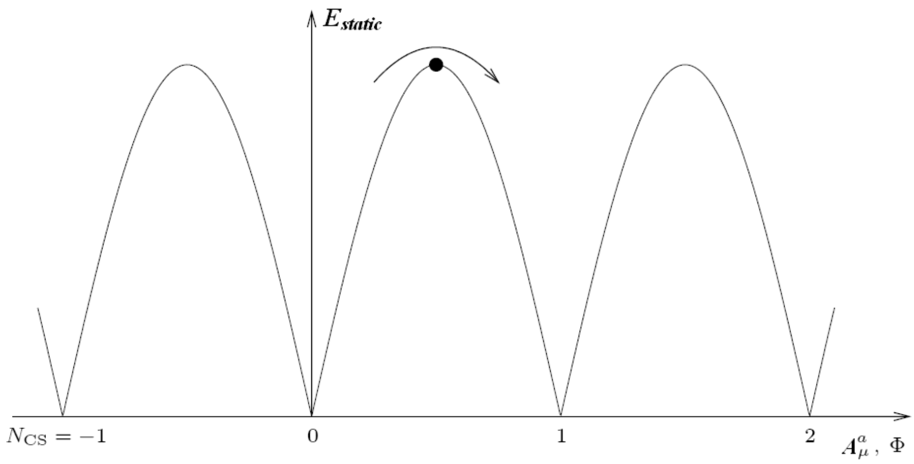

Figure 2.1: *Schematic plot of the static energy as function of the winding number that represent equivalent gauge and Higgs fields configuration. The minima correspond to the classical vacua.*

As mentioned before, the rate of baryon number violation at zero temperature suppressed by about 160 orders of magnitude. However, this is not the case at finite temperature. In the symmetric phase, sphalerons lead to very fast $B + L$ violating processes with the transition rate per unit volume [38]:

$$\Gamma_{sph} \sim \alpha_W^5 \ln(1/\alpha_W) T^4, \tag{2.6}$$

where $\alpha_W = g^2/4\pi$ is the fine structure constant. This estimate for the transition rate suggests that sphaleron processes are in thermal equilibrium for 100 $GeV \lesssim T < 10^{12}\ GeV$. In the broken phase, this rate is exponentially suppressed by a factor $exp\left(-E_{Sp}/T\right)$, where E_{Sp} is called sphaleron mass or energy, which is the static energy of the system at the top of the barrier in Fig. 2.1. In the next chapter, we will study this case in more detail.

2.3.2 C & CP Violation

It was mentioned before that B violating processes are needed for baryogenesis, however we know from particle physics that every process

$$Z \to X + Y \tag{2.7}$$

has a conjugate process:

$$\bar{Z} \to \bar{Y} + \bar{X} \tag{2.8}$$

where $\bar{Z}$, $\bar{Y}$ and $\bar{X}$ are the anti-particles of Z, X and Y, respectively; and if Z is a left-handed fermion then $\bar{Z}$ is a right-handed anti-fermion.

If the process (2.7) violates baryon number and a baryon density n_b will be generated after some time, then, within the same period, the conjugate process (2.8) generate with the same way a baryon density $n_{\bar{b}} = -n_b$. therefore the total generated baryon density is $n_b + n_{\bar{b}} = 0$. The only way to get a nonzero baryon number density is that the processes (2.7) and (2.8) have a property that $n_b \neq -n_{\bar{b}}$, or $n_b = -\left(1+\epsilon\right) n_{\bar{b}}$, where ϵ refers to such an asymmetry. This is exactly the meaning of the second Sakharov condition. This is one of the potential properties of the CP violation.

Let us recall briefly how C, P and CP operate on scalar fields, spinors, and vector fields. For complex scalars

$$\begin{aligned} C &: \phi \to \phi^* \\ P &: \phi(t,\vec{r}) \to \pm\phi(t,-\vec{r}) \\ CP &: \phi(t,\vec{r}) \to \pm\phi^*(t,-\vec{r}), \end{aligned} \tag{2.9}$$

for vector bosons

$$\begin{aligned}
C &: A^\mu \to -A^\mu \\
P &: A^\mu(t,\vec{r}) \to (A^0, -\vec{A})(t, -\vec{r}) \\
CP &: A^\mu(t,\vec{r}) \to (-A^0, \vec{A})(t, -\vec{r});
\end{aligned} \tag{2.10}$$

and for fermions

$$\begin{aligned}
C &: \psi_L \to i\sigma_2\psi_R^*, \quad \psi_R \to -i\sigma_2\psi_L^*, \quad \psi \to i\gamma^2\psi^* \\
P &: \psi_L \to \psi_R(t,-\vec{r}), \quad \psi_R \to \psi_L(t,-\vec{r}), \quad \psi \to \gamma^0\psi(t,-\vec{r}) \\
CP &: \psi_L \to i\sigma_2\psi_R^*(t,-\vec{r}), \quad \psi_R \to -i\sigma_2\psi_L^*(t,-\vec{r}), \quad \psi \to i\gamma^2\psi^*(t,-\vec{r})
\end{aligned} \tag{2.11}$$

In the electroweak theory, only left-handed fermions are $SU(2)_L$ gauge coupled; and are represented in chiral doublets, therefore C is maximally broken in the SM. CP violation is seen experimentally in the neutral kaon system through K_0, $\bar{K}_0$ mixing. Thus, CP violation is a natural feature of the standard electroweak model.

Moreover, it is well known that CP violation exists due to the phase $e^{i\delta}$; in the so-called Cabibbo-Kobayashi-Maskawa (CKM) matrix, a matrix transforming weak interaction eigenstates to mass squared eigenstates:

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} c_1 & -s_1c_3 & -s_1s_3 \\ s_1c_2 & c_1c_2c_3 - s_2s_3e^{i\delta} & c_1c_2s_3 + s_2c_3e^{i\delta} \\ s_1s_2 & c_1s_2c_3 + c_2s_3e^{i\delta} & c_2s_2s_3 - c_2c_3e^{i\delta} \end{pmatrix}, \tag{2.12}$$

where c_i (s_i) is a shortcut for $cos\theta_i$ ($sin\theta_i$). However there is no unique way to express CP violation in an invariant quantity. Jarlskog introduced the determinant of the commutator of mass matrices of up and down quarks [39]

$$\begin{aligned}
J &= det\left[M_u^2, M_d^2\right] = (m_t^2 - m_c^2)(m_t^2 - m_u^2)(m_c^2 - m_u^2) \\
&\quad \times (m_b^2 - m_s^2)(m_b^2 - m_d^2)(m_s^2 - m_d^2)\,K \\
K &= s_1^2 s_2 s_3 c_1 c_2 c_3 \sin\delta = Im\, V_{ii}V_{jj}V_{ij}^*V_{ji}^* \text{ for } i \neq j.
\end{aligned} \tag{2.13}$$

Forming a dimensionless quantity out of J by dividing by the critical temperature where the sphaleron processes are efficient, gives:

$$J/T_c^{12} \sim J/\,(100\;GeV)^{12} \sim 10^{-20}, \tag{2.14}$$

which is much too small[3] to account for the required value of $\eta \sim 10^{-10}$.

Since CP violation amount is not large enough in the Standard Model, it is natural to extend the SM in some fashion that increases the amount of CP violation in the theory while does not lead to results in a conflict with current experimental data, especially bounds on the electron and neutron electric dipole moment [42]. In some models like two Higgs Doublet (2HD), we can have CP violation at tree level, in the quadratic or quartic terms. In a well-motivated SM extension, the minimal supersymmetric standard model (MSSM), CP violation effect can exists explicitly in the μ-term in the superpotential or in the soft gauginos mass terms.

In GUTs, CP violation arises in loop-diagram corrections to baryon number violating bosonic decays.

2.3.3 The Departure from Equilibrium

One can easily understand this criterion by considering again the B violating process (2.7), which is supposed to be in thermal equilibrium. Then the number of transitions for (2.7) is by definition the same as that of the inverse process:

$$X + Y \rightarrow Z, \tag{2.15}$$

therefore any produced B number by (2.7) will be destroyed by (2.15). If the mass of Z is larger that temperature ($M_Z > T$), then the decay (2.7) is out of thermal equilibrium. In this case the process (2.15) is kinematically forbidden and its rate is Boltzmann suppressed:

$$\Gamma\left(X + Y \rightarrow Z\right) \sim e^{-M_Z/T}, \tag{2.16}$$

then the thermal equilibrium should be lost in order to get a net baryon number density if the two preceding conditions are satisfied.

[3] Farrar and Shaposhnikov [40], tried to correct this argumentation: CP violation effect that comes from the box diagram in $K\bar{K}$ system is not proportional to J, and the relevant scale is much smaller than 100 GeV, which is the mass of K^0. Then the fact that J/T^{12} measures CP violation, is meaningful only if all the ratios of mass to temperature can be treated as perturbatively small, which is not the case for the ratio of the top quark mass to the K^0 mass [41].

One can see the importance of this condition from another corner: if B and CP violating processes are in thermal equilibrium, the mean value of the baryon number at temperature T is:

$$\begin{aligned} \langle B \rangle_T &= Tr\left\{e^{-H/T}\, B\right\} = Tr\left\{(CPT)^{-1}\, CPT\, e^{-H/T}\, B\right\} \\ &= Tr\left\{e^{-H/T}\, CPT\, B\, (CPT)^{-1}\right\} = -Tr\left\{e^{-H/T}\, B\right\} = -\langle B \rangle_T = 0, \end{aligned} \tag{2.17}$$

the later step because B is odd under CPT (see eqs. (2.9-2.11)).

If CPT is violated, even only during a very early time in the universe evolution, the third condition is not required for successful baryogenesis [43].

In most of the successful baryogenesis scenarios, this condition is fulfilled in one of two ways: the first one via an out-of-equilibrium decay of a (super)heavy particle like Leptogenesis and GUTs scenarios. In general, these scenarios operate at higher scales. The second one is during a rapid (strongly first order) phase transition like the case of electroweak baryogenesis.

2.4 Different Scenarios for Baryogenesis

Baryogenesis is a very attractive subject for physicists. A huge number of papers discussing this subject exists; some of them are giving different mechanisms, and the rest are discussing the realization of these mechanisms within some models of particle physics or imposing on them constraints using the available data from cosmological observation or particle physics experiments. The evolution of the publications number on baryogenesis as a function of time is shown in Fig. 2.2.

In this section we will describe briefly some of the scenarios which are: the oldest one, GUTs baryogenesis [5, 44], Baryogenesis via Leptogenesis [45], which is the most popular these days, and Electroweak baryogenesis [8], which was extensively studied last years due the possibility of testing with the next generation of accelerators like LHC.

Of course these are not the only popular models. One of the most favored models is Affleck-Dine mechanism [46], which is based on the out-of-equilibrium decay of a scalar condensate that carries a baryon number. There are also, as mentioned

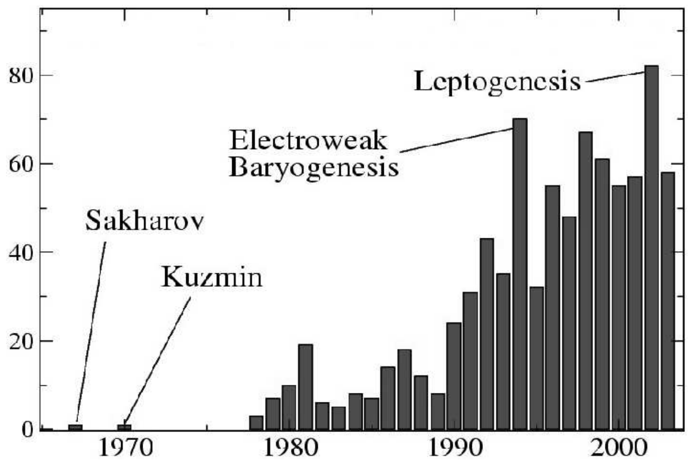

Figure 2.2: *The Number of publications on Baryogenesis with the time. This figure is taken from [41].*

above, mechanisms based on CPT violation at early stages of the Universe [43]; and also models based on Lorentz violation [47].

2.4.1 GUT Baryogenesis

As noted before, Sakharov conditions are satisfied in GUTs if there exist particles whose decay is out-of-equilibrium. Indeed in $SU(5)$, and also in $SO(10)$, the Higgs and Gauge bosons mediate transitions between the elements of the same irreducible representation; and therefore B is naturally violated; and also C is violated. It was shown that CP violation occurs naturally at two-loop level [48].

For baryogenesis scenarios that operate at very high scales ($\gg 100\ GeV$), the combination $B - L$ should be violated to avoid the washout processes above the electroweak scale, which is the case for the $SO(10)$ model.

In general, GUTs models contain super-heavy Higgs and Gauge bosons with B-violating and CP-violating Yukawa couplings to quarks and leptons. To show a simple realization of GUTs baryogenesis, let us consider a super-heavy leptoquark gauge boson X, which is supposed to have the quark-quark and anti-quark-lepton decay channels:

$$
\begin{aligned}
X &\rightarrow qq \\
X &\rightarrow \ell\bar{q}.
\end{aligned}
\tag{2.18}
$$

As it is clear, the first decay breaks B by 2/3 units, while the second by -1/3. If r ($\bar{r}$) is the branching ratio of the first (second) decay in (2.18), then the baryon number produced in the decays of X and $\bar{X}$ is

$$
\begin{aligned}
B_X &= \frac{2}{3}r - \frac{1}{3}(1-r) \\
B_{\bar{X}} &= -\frac{2}{3}\bar{r} + \frac{1}{3}(1-\bar{r});
\end{aligned}
\tag{2.19}
$$

and the net baryon number produced is

$$
\begin{aligned}
\Delta B_X &\equiv B_X + B_{\bar{X}} = r - \bar{r} \\
&= \frac{\Gamma(X \rightarrow qq)}{\Gamma_{tot}(X)} - \frac{\Gamma(\bar{X} \rightarrow \bar{q}\bar{q})}{\Gamma_{tot}(\bar{X})} \\
&= \frac{\Gamma(X \rightarrow qq) - \Gamma(\bar{X} \rightarrow \bar{q}\bar{q})}{\Gamma_{tot}},
\end{aligned}
\tag{2.20}
$$

where $\Gamma_{tot}(X) = \Gamma_{tot}(\bar{X})$ due to CPT invariance. It is obvious that if C or CP are conserved then ΔB_X will be relaxed to 0.

However GUT baryogenesis seems to be in conflict with inflation in its old picture and it suffers from some problems; it should happen after inflation, where all particles were created from the decay of the inflaton[4], at a temperature $T_R = 10^{13}$ GeV in a generic inflation scenario. Therefore the creation of X bosons from the decay of the inflaton is kinematically forbidden because $M_X \sim M_{GUT} \sim 10^{16}$ $GeV \gg T_R$. If one tries to make $M_X \sim T_R$, then gravitinos[5] would be abundance during nucleosynthesis and destroy the good agreement of the theory with the observations [51].

[4]The inflaton is the scalar field which is responsible of the inflation; and this mechanism is called *reheating* [49].

[5]The gravitino is a spin-(3/2) particle which appears in the extension of supersymmetry to supergravity [50], which is the fermionic superpartner of the graviton. Its mass is of the order of TeV and it couples very weakly to the SM particles and their superpartners, $\alpha \sim 1/M_{pl}$. And therefore it has a slow decay rate.

Recently, GUTs baryogenesis has re-emerged with the realization that reheating may differ significantly from the old picture where in the first stage of reheating, called *preheating* [52], nonlinear effects may lead to an extremely effective dissipating dynamics and explosive particle production even when single particle decay is kinematically forbidden, and particles can be produced in the regime of a broad parametric resonance. Indeed, it was shown in [53] that the baryon asymmetry can be produced efficiently just after the preheating era, thus many problems of GUTs baryogenesis are solved.

2.4.2 Baryogenesis via Leptogenesis

A very popular mechanism for baryogenesis today is leptogenesis, invented by Fukugita and Yanagida in 1986 [45] (for a review see [54]). Leptogenesis is a very natural mechanism, which ties in with currently observed properties of neutrinos; hence its popularity. The simplest versions of leptogenesis occur at untestably high energies, similar to GUT baryogenesis, but it is possible to bring leptogenesis down to the TeV scale [55].

In the leptogenesis scenario, a lepton asymmetry is created at a scale above the electroweak one, while this asymmetry should reside only in neutrinos in order to save the electric charge neutrality of the universe. Since sphaleron processes, that involve all left handed fermions, violate $B + L$, and at the same time conserve $B - L$, an excess in L will be transformed to produce a baryon number asymmetry. If a lepton asymmetry L_i, is created before the sphaleron processes, then the final asymmetry in B results after the $SU(2)_L \otimes U(1)_Y$ breaking, which is given by [56]

$$B_f = \left\{ \begin{array}{c} -\frac{28}{79} \text{ in the SM} \\ -\frac{8}{23} \text{ in the MSSM} \end{array} \right\} L_i \sim -\frac{1}{3} L_i. \tag{2.21}$$

The lepton asymmetry is produced via the out-of-equilibrium decay of a heavy right-handed neutrino ν_R (sometimes denoted in the literature by N or N_R). The discovery of neutrino oscillations [7] gave a strong evidence that neutrinos are massive, their masses should be $\max(m_\nu) \lesssim \sqrt{\triangle m^2_{Atm}} \sim 4.\ 8 \times 10^{-2}$ eV. The only natural mechanism which makes the left-handed neutrino mass very small is the so-called seesaw mechanism [50, 57], which is, in its simplest version, based on the neutrino mass terms

$$y_{ij}\bar{\nu}_{R,i}HL_j + \text{h. c.} - \frac{1}{2}\left(M_{ij}\bar{\nu}^c_{R,i}\nu_{R,j} + \text{h. c.}\right), \tag{2.22}$$

here y is the Yukawa coupling matrix for left handed neutrinos; and M is the right handed neutrino mass before the symmetry breaking. After the Higgs develops its vev, the mass matrix in the basis ν_L and ν_R is

$$\begin{pmatrix} 0 & m_D \\ m_D^T & M \end{pmatrix}, \tag{2.23}$$

where the Dirac mass matrix is $(m_D)^{ij} = vy^{ij}$. Then the matrix (2.23) can be partially diagonalized to *diag*$(-m_D m_D^T/M,\ M)$. This means that the light neutrino masses are very small $\sim y^2v^2/M$ and the heavy neutrino masses are of order M. If we assume the largest mass of the light neutrinos to be $m_\nu = 0.05$ eV, then the size of the ν_R mass scale should about $M \sim y^2 \times 10^{14}$ GeV.

The CP violation sources arise from the interference of tree-level and one loop diagrams of the decay and can be parameterized by the dimensionless quantity

$$\epsilon_1 = \frac{\left|\mathcal{M}_{\nu_{R,1}\to lH}\right|^2 - \left|\mathcal{M}_{\nu_{R,1}\to \bar{l}\bar{H}}\right|^2}{\left|\mathcal{M}_{\nu_{R,1}\to all}\right|^2} = -\frac{3}{16\pi}\sum_i \frac{Im(y^\dagger y)^2_{i1}}{(y^\dagger y)_{11}}\frac{M_1}{M_i}, \tag{2.24}$$

where $\nu_{R,1}$ is the lightest right handed neutrino. The baryon asymmetry can be expressed as

$$\eta = 10^{-2}\epsilon_1\kappa, \tag{2.25}$$

where κ is the efficiency factor, which takes into account the washout processes: inverse decay, LH scattering and $L\nu_R$ scattering. In general, all leptogenesis models put constraints on neutrino mass values, mixing and/or the form of the mass matrix . . . etc.

2.4.3 The Electroweak Baryogenesis

This scenario was extensively studied last years, because it works at a relatively low scale i. e, the electroweak scale. The physics at this scale will be explored in much detail next years within the new generation of accelerators like Large Hadronic Collider (*LHC*), that start giving results next year.

At scales much below the GUTs scale, the universe expansion rate ($\sim T^2/M_{pl}$) is much slower than the particle's interaction rate ($\sim T$), then the third Sakharov condition will be never fulfilled via the universe expansion.

The possibility to get a net baryon asymmetry at the electroweak scale was first discussed by Kuzmin et. al [8], where Sakharov conditions seemed to be satisfied within the Standard Model. As noted above, B is violated via the anomalous processes, CP violation effects arises in the CKM matrix phase; and the third condition could not be reached by the universe expansion; but via a strong first order phase transition[6]. For reviews see [59].

Here, we will not discuss how each one of Sakharov conditions is satisfied in the SM or in its extensions. We will just describe how this mechanism works.

In the case of a first order phase transition, the Higgs field gets its vev through tunneling via bubble nucleation due to the existing barrier between the stable (true) and the metastable (false) minima, these bubbles expand until the space is filled by the true vacuum. As we will see later in detail, the rate of processes that violate B and CP is very huge at the symmetric phase ($\langle h \rangle = 0$) $\Gamma \sim T^4$, but exponentially suppressed in the broken phase ($\langle h \rangle \neq 0$) $\Gamma \sim 0$. When the bubble wall passes through a region in space in which B and CP are violated, an asymmetry occurs in the broken phase and persists due to the suddenly suppressed back-reaction.

Historically, this mechanism has been separated into two categories:

a) *Nonlocal baryogenesis* [60]: when the bubble wall passes through a region where CP and another quantum number than B, are violated, an asymmetry is generated for this quantum number. Then this asymmetry is converted to a baryon asymmetry. This mechanism is called sometimes in the literature 'charge transport mechanism'.

b) *Local baryogenesis* [61]: in this case, CP and B together are violated in a region that the bubble wall passes through, and a resulting baryon asymmetry resides in

[6]The departure from thermal equilibrium could be caused also by TeV scale topological defects that can arise in SM extensions [58].

the broken phase.

In general, both local and nonlocal baryogenesis will occur and the baryon asymmetry will be the sum of the asymmetries generated by the two processes. However, if the speed of the wall is greater than the sound speed in the plasma, then local baryogenesis dominates. In other cases, nonlocal baryogenesis is usually more efficient.

The electroweak baryogenesis scenario seems to be well working in some SM extensions like the two-Higgs doublets model (2HDM) [62], the MSSM [63]; and also for some MSSM extensions [64, 65].

In practice, the calculation of the resulting baryon asymmetry depends on many parameters like the thickness of the bubble wall (thin or thick wall regime), the speed of the bubble wall, the nature of processes that violate the discrete symmetry CP; and the diffusion dynamics of the asymmetry into the broken phase.

Chapter 3

The Electroweak Phase Transition

One of the most interesting scenarios for baryogenesis is the electroweak baryogenesis. A successful electroweak baryogenesis scenario requires a strong first order phase transition at the electroweak scale. In order to investigate how the electroweak phase transition (EWPT) could have occurred at an early stage of the Universe evolution, we need to work within the framework of quantum field theory at high temperatures. The basic tool here is called the effective potential, that means the potential free energy density of the system under consideration.

In the first section, we discuss in detail the notion of the effective potential. The idea of the phase transition strength is discussed in the second section taking the standard electroweak theory as a clarifying example. In the third section, the dynamics of the first order phase transition in terms of bubble nucleation is shown. After that, we review how the condition for a strong first transition order is derived starting from the saddle point solution of the field equations. Finally, we present the status of phase transition in some of the Standard Model extensions.

3.1 The Effective Potential

The effective potential for quantum field theories was originally introduced by Euler, Heisenberg and Schwinger [66], and applied to studies of spontaneous symmetry breaking by Goldstone, Salam, Weinberg and Jona-Lasinio [67]. Calculations of the effective potential were initially performed at one-loop by Coleman and E. Weinberg [68] and at higher-loop by Jackiw [69] and Iliopoulos, Itzykson and Martin [70].

Quantum field theories involve virtual particles, that affect the field energy density through emission and reabsorbing processes. The generalization of the classical theory by including quantum corrections is known as the effective theory; and its corresponding potential density is called the effective potential. The minimization of the effective potential gives the field configuration with the minimal energy, i. e. the vacuum of the theory.

Proceeding further, analysis of matter behavior at non zero temperatures involves thermal fluctuations of the fields that one should take into account. Thus, a generalization of the effective potential at finite temperature is also needed, for the inclusion of temperature dependent quantum effects. As will be clear in what follows from the mathematical definition the effective action has the meaning of the free energy of the quantum system under consideration. The finite temperature effective potential $V(\phi, T)$ as Linde [30] states at its extreme coincides with the free energy density.

The effective potential has been studied extensively in the literature. An elegant discussion on the physical meaning of the effective potential and its calculation is presented by Coleman [71]. A detailed analysis of the theory of the effective potential at zero and finite temperature with applications to cosmological models is given by Brandenberger [72]. The electroweak Higgs potential for the Standard Model and its extensions has been investigated by Sher [73]. Generalization of the effective potential to finite temperature is given by Dolan and Jackiw [74] and Linde [30,75]. In what follows we give a formal discussion on the notion of the effective potential as it appears in the framework of quantum field theory at zero and non zero temperature.

3.1.1 At Zero Temperature

Any quantum field theory is described by the Lagrangian density $\mathcal{L}$ or the action S, which is given by

$$S[\phi] = \int d^4x \mathcal{L}\{\phi, \partial_\mu \phi\}, \tag{3.1}$$

where ϕ denotes scalars, vectors and fermions, and the Lagrangian density contains kinetic terms for all fields, potential term for scalars and the Yukawa terms. Let us focus for the moment only on pure scalar theories, therefore in what follows ϕ denotes only scalars. Then the connected correlation functions are generated by the energy functional $W[J]$ in the presence of an external field J, which is given by

$$Z[J] = e^{iW[J]} = \int \mathcal{D}\phi e^{i\int d^4x(\mathcal{L}[\phi]+J\phi)}, \tag{3.2}$$

where $Z[J]$ is the Green functions generating functional. The functional derivative of $W[J]$ leads to the vacuum expectation value of the field in the presence of the external source J, which is defined as the classical field ϕ_{cl},

$$\phi_{cl}(x) = -\frac{\delta W[J]}{\delta J(x)} = \frac{\int \mathcal{D}\phi\phi(x)\, e^{i\int d^4x(\mathcal{L}[\phi]+J\phi)}}{\int \mathcal{D}\phi e^{i\int d^4x(\mathcal{L}[\phi]+J\phi)}} = \frac{\langle 0|\,\phi(x)\,|0\rangle_J}{\langle 0|\,0\rangle_J}. \tag{3.3}$$

The effective action is the Legendre transformation of $W[J]$, which is analogous to the Gibbs free energy in thermodynamics, it equals

$$\Gamma[\phi_{cl}] = W[J] - \int d^4y J(y)\phi_{cl}(y). \tag{3.4}$$

Since the derivative of $\Gamma[\phi_{cl}]$ with respect to ϕ_{cl} vanishes in the absence of the external source $J = 0$, then ϕ_{cl} can be a stable or metastable state of the theory. Then $W[J]$ can be expanded in a power series of J, to obtain its representation in terms of connected Green functions $G_{(n)}$ as

$$iW[j] = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int d^4x_1 \ldots d^4x_n J(x_1) \ldots J(x_n) G_{(n)}(x_1, \ldots, x_n). \tag{3.5}$$

Similarly the effective action can be expanded in powers of ϕ_{cl} as

$$\Gamma[\phi_{cl}] = \sum_{n=0}^{\infty} \frac{1}{n!} \int d^4x_1 \ldots d^4x_n \phi_{cl}(x_1) \ldots \phi_{cl}(x_n) \Gamma^{(n)}(x_1, \ldots, x_n), \tag{3.6}$$

where $\Gamma^{(n)}$ are the one-particle irreducible ($1PI$) Green functions. Usually one considers a theory which is invariant under space-time translation, and therefore the solution for ϕ_{cl} is constant and independent of x. Then, removing an overall factor of space-time volume, we define the effective potential $V_{\text{eff}}(\phi_{cl})$ as

$$\Gamma[\phi_{cl}] = -\int d^4x V_{\text{eff}}(\phi_{cl}). \tag{3.7}$$

The $1PI$ diagrams are evaluated with no propagators on the external lines, therefore the Fourier transform of (3.6) can be written as

$$\Gamma(\phi_{cl}) = \sum_{n=0}^{\infty} \frac{1}{n!}\phi_{cl}^n (2\pi)^4 \delta^{(4)}(0)\Gamma^{(n)}(p_i = 0) = \sum_{n=0}^{\infty} \frac{1}{n!}\phi_{cl}^n \Gamma^{(n)}(p_i = 0) \int d^4x; \quad (3.8)$$

and comparing it with (3.7) we obtain the final expression

$$V_{\text{eff}}(\phi_{cl}) = -\sum_{n=0}^{\infty} \frac{1}{n!}\phi_{cl}^n \Gamma^{(n)}(p_i = 0). \quad (3.9)$$

The calculation of the effective potential by summing infinite series of Feynman graphs at vanishing external momentum is simple at the one-loop level. However this not always the case at higher-loop level. A beautiful calculation has been carried out in Ref. [69], we try to give the interesting results.

Let us consider again a scalar theory with a Lagrangian $\mathcal{L}$. One can define another Lagrangian $\hat{\mathcal{L}}$ as

$$\int d^4x \hat{\mathcal{L}}\{\phi_{cl}; \phi(x)\} \equiv S\{\phi_{cl} + \phi\} - S\{\phi_{cl}\} - \phi \frac{\delta S\{\phi_{cl}\}}{\delta \phi_{cl}}. \quad (3.10)$$

The second term in (3.10) makes the vacuum energy equal to zero, and the third term is there to cancel the tadpole part of the shifted action. One denotes by $\mathcal{D}\{\phi_{cl}; x-y\}$ the propagator of the (new) shifted theory

$$i\mathcal{D}^{-1}\{\phi_{cl}; x-y\} = \left.\frac{\delta^2 S[\phi]}{\delta\phi(x)\delta\phi(y)}\right|_{\phi=\phi_{cl}}, \quad (3.11)$$

and $i\mathcal{D}^{-1}\{\phi_{cl}; p\}$ its Fourier transform, the effective potential is found to be given by [69]

$$\begin{aligned} V_{\text{eff}}(\phi_{cl}) &= V_0(\phi_{cl}) - \frac{i}{2}\int \frac{d^4p}{(2\pi)^4} \log\left\{\det\left\{i\mathcal{D}^{-1}\{\phi_{cl}; p\}\right\}\right\} \\ &\quad + i\left\langle \exp\left[i\int d^4x \hat{\mathcal{L}}_{Interaction}\{\phi_{cl}; \phi(x)\}\right]\right\rangle \end{aligned} \quad (3.12)$$

The first term in (3.12) is just the classical tree-level potential. The second term is the one-loop potential, where the determinant operates on any possible internal indices defining the propagator. The third term summarizes all higher order corrections starting from two-loop level. In a simple example of a theory of one real scalar field, the one-loop term (the second term in (3.12)), is computed to be

$$V_1(\phi_{cl}) = -\frac{i}{2}\int \frac{d^4p}{(2\pi)^4}\log[p^2 - m^2(\phi_{cl})]. \tag{3.13}$$

The mass $m^2(\phi_{cl})$ is computed in the shifted theory like

$$m^2(\phi_{cl}) = \left.\frac{\partial^2 V_0\left\{\phi_{cl}+\phi\right\}}{\partial\phi^2}\right|_{\phi=0}. \tag{3.14}$$

The two-loop effective potential is harder to compute than the one-loop term, but usually manageable. All the previous procedures can be applied to theories containing fermions and gauge bosons.

For the calculation of the integral (3.12), some simplifications are needed. One should make Wick rotation $p^0 = ip_E^0$ and therefore the integral can be computed in an Euclidian four dimensional space-time. Then eq. (3.12) becomes

$$V_1(\phi_{cl}) = \frac{1}{2}\int \frac{d^4p}{(2\pi)^4}\log[p^2 + m^2(\phi_{cl})]. \tag{3.15}$$

This formula (3.15) is valid only for a theory with single scalar field. For a theory with multiple scalars, the contribution of these scalars to the one-loop correction for the potential is given by

$$\begin{aligned} V_1(\phi_{cl}) &= \frac{1}{2}\int \frac{d^4p}{(2\pi)^4} Tr\log[p^2 + M^2(\phi^a,\phi_b)] \\ M^2(\phi^a,\phi_b) &= \frac{\partial^2 V_0(\phi)}{\partial\phi_a^\dagger\partial\phi^b}, \end{aligned} \tag{3.16}$$

where the trace acts on the field indices space. A similar formula exists for fermions,

$$V_1(\phi_{cl}) = -\lambda\int \frac{d^4p}{(2\pi)^4} Tr\log[p^2 + M_f^2(\phi_{cl})], \tag{3.17}$$

where the parameter λ equals 1 if the field is a Weyl fermion and 2 in the case of a Dirac one, and the trace acts on the field space. For gauge bosons, their contribution, in Landau gauge, is given by

$$V_1(\phi_{cl}) = \frac{3}{2}\int \frac{d^4p}{(2\pi)^4} Tr\log[p^2 + M_G^2(\phi_{cl})]. \tag{3.18}$$

Since we are concerned in our work only with the one-loop level, we will not discuss higher-order corrections. It is clear that the integrals (3.15), (3.16), (3.17) and (3.18) are ultraviolet-divergent.

As it is known in quantum field theories, the theory is meaningful only if the divergences can be absorbed by some counterterms in the Lagrangian, whereby the theory parameters get renormalized. However there is no unique way to make the theory finite, different procedures are called renormalization schemes. For example, to make the integral (3.15) finite, one can cut it by a scale mass Λ and then apply the renormalization conditions, so that one finally gets a finite theory where the Lagrangian contains explicitly the scale Λ. This scheme is called cut-off regularization. Within this scheme the gauge invariance can be broken, and therefore this scheme is not widely used. Another scheme is called Dimensional regularization where the integral (3.15) is computed in $4-2\varepsilon$ dimensions and the infinities are parameterized by $1/\varepsilon$. The result can be expressed as

$$V_1(\phi_{cl}) = -\left\{-\frac{1}{\varepsilon} - \gamma_E + \log 4\pi\right\}\frac{m^4(\phi_{cl})}{64\pi^2} + \frac{m^4(\phi_{cl})}{64\pi^2}\left\{\log\frac{m^2(\phi_{cl})}{\Lambda^2} - \frac{3}{2} + \mathcal{O}(\varepsilon)\right\}, \tag{3.19}$$

where $\gamma_E = 0.5772156649$ is called Euler-Mascheroni constant; and Λ is a mass scale. In the so-called $\overline{MS}$ renormalization scheme [76], the first term in (3.19) is absorbed by counterterms, and therefore the one-loop correction to the potential is given by

$$V_1(\phi_{cl}) = \frac{1}{64\pi^2}\sum_i n_i a_i m_i^4(\phi_{cl})\left[\log\frac{m_i^2(\phi_{cl})}{\Lambda^2} - C_i\right], \tag{3.20}$$

where n_i is the degree of freedom of the particle i; and (a_i, C_i) are $\left(1, \frac{3}{2}\right)$ for scalars, $\left(-2, \frac{3}{2}\right)$ for fermions; or $\left(3, \frac{5}{6}\right)$ for gauge bosons. The popular scheme is a variant of $\overline{MS}$ scheme, called $\overline{DR}$ where the calculations are done in Landau gauge, all C_i's have the same value $\frac{3}{2}$ and a_i's are absorbed by redefining the degrees freedom like

$$V_1(\phi_{cl}) = \frac{1}{64\pi^2}\sum_i n_i m_i^4(\phi_{cl})\left\{\log\frac{m_i^2(\phi_{cl})}{\Lambda^2} - \frac{3}{2}\right\}. \tag{3.21}$$

In what follows, we will use $\overline{DR}$ renormalization scheme.

3.1.2 At Finite Temperature

In order to investigate the electroweak phase transition, our theory should be described at finite temperature and therefore the one-loop corrections to the potential (3.21) should be generalized to be valid at finite temperature.

At finite temperature, the scalar field is in a thermal bath where the background can be matter or radiation with non-vanishing temperature and density, the Feynmann rules of the theory get modified.

There are two known methods to generalize a zero temperature quantum field theory to a finite temperature one; imaginary time formalism and real time formalism [77,78,79]. We just give here the general results according to the imaginary time formalism.

In the imaginary time formalism, the bosonic and fermionic fields are respectively written as

$$\phi\left(\tau,\vec{r}\right) = \sum_{n=-\infty}^{\infty} \tilde{\phi}_n\left(\vec{r}\right) e^{i\omega_n^b \tau} \tag{3.22}$$

$$\psi\left(\tau,\vec{r}\right) = \sum_{n=-\infty}^{\infty} \tilde{\psi}_n\left(\vec{r}\right) e^{i\omega_n^f \tau}, \tag{3.23}$$

where $\tau = it$ is the Euclidean time, n represents the Matsubara mode, and ω_n^b's (ω_n^f's) are bosonic (fermionic) Matsubara frequencies which are given by $\omega_n^b = 2\pi nT$ and $\omega_n^f = (2n+1)\,\pi T$. Feynman rules are summarized by

$$\text{Boson propagator} \;:\; \frac{i}{p^2 - m^2};\; p^\mu = [i\omega_n^b, \vec{p}\,]$$

$$\text{Fermion propagator} \;:\; \frac{i}{\gamma\cdot p - m};\; p^\mu = [i\omega_n^f, \vec{p}\,]$$

$$\text{Loop integral} \;:\; iT\sum_{n=-\infty}^{\infty}\int \frac{d^3p}{(2\pi)^3}$$

$$\text{Vertex} \;:\; -\frac{i(2\pi)^3}{T}\delta_{\sum\omega_i}\delta^{(3)}(\sum_i \vec{p}_i). \tag{3.24}$$

Then non-zero temperature scalar contribution to the effective potential are given by recomputing eq. (3.15) using (3.24), as [74]

$$\begin{aligned} V_1(\phi_{cl}, T) &= \frac{T}{2} \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \log[\left(\omega_n^{b,f}\right)^2 + \vec{p}^2 + m^2(\phi_{cl})] \\ &= \frac{m^4(\phi_{cl})}{64\pi^2} \left\{ \log \frac{m^2(\phi_{cl})}{\Lambda^2} - \frac{3}{2} \right\} + \frac{T^4}{2\pi^2} J_B \left(m^2(\phi_{cl})/T^2 \right), \end{aligned} \quad (3.25)$$

$$J_B(\theta) = \int_0^\infty dx\, x^2 \log \left\{ 1 - \exp\left[-\sqrt{x^2 + \theta} \right] \right\}. \quad (3.26)$$

The gauge boson contribution also gives similar results; however fermionic contribution is different. Recomputing (3.17), one gets

$$\begin{aligned} V_1(\phi_{cl}, T) &= -\lambda T \sum_{n=-\infty}^{\infty} \int \frac{d^3p}{(2\pi)^3} \log[(2n+1)^2 \pi^2 T^2 + \vec{p}^2 + M_f^2(\phi_{cl})] \\ &= -\lambda \frac{M_f^4(\phi_{cl})}{32\pi^2} \left\{ \log \frac{M_f^2(\phi_{cl})}{\Lambda^2} - \frac{3}{2} \right\} - \lambda \frac{T^4}{\pi^2} J_F \left(M_f^2(\phi_{cl})/T^2 \right), \end{aligned} \quad (3.27)$$

$$J_F(\theta) = \int_0^\infty dx\, x^2 \log \left\{ 1 + \exp\left[-\sqrt{x^2 + \theta} \right] \right\}. \quad (3.28)$$

These functions (3.26) and (3.28), can be expanded in the limit $\theta \ll 1$ i. e. at high temperature like

$$J_B(\theta) \simeq -\frac{\pi^4}{45} + \frac{\pi^2}{12}\theta - \frac{\pi}{6}\theta^{3/2} - \frac{\theta^2}{32} \log \frac{\theta}{a_b} - 2\pi^{7/2} \sum_{l=1}^{\infty} (-1)^l \frac{\zeta(2l+1)}{(l+1)!} \Gamma\left(l + \frac{1}{2}\right) \left(\frac{\theta}{4\pi^2}\right)^{l+2}, \quad (3.29)$$

$$J_F(\theta) \simeq \frac{7\pi^4}{360} - \frac{\pi^2}{24}\theta - \frac{\theta^2}{32} \log \frac{\theta}{a_f} - \frac{\pi^{7/2}}{4} \sum_{l=1}^{\infty} (-1)^l \frac{\zeta(2l+1)}{(l+1)!} \left(1 - 2^{-2l-1}\right) \Gamma\left(l + \frac{1}{2}\right) \left(\frac{\theta}{\pi^2}\right)^{l+2}, \quad (3.30)$$

where $a_b = 16\pi^2 \exp(3/2 - 2\gamma_E)$, $a_f = \pi^2 \exp(3/2 - 2\gamma_E)$ and ζ is the Riemann ζ-function.

Finally one can write the full one-loop effective potential at finite temperature like

$$\begin{aligned} V_{eff}(\phi_{cl}, T) &= V_0(\phi_{cl}) + \sum n_i \frac{m_i^4(\phi_{cl})}{64\pi^2} \left\{ \log \frac{m_i^2(\phi_{cl})}{\Lambda^2} - \frac{3}{2} \right\} \\ &\quad + \frac{T^4}{2\pi^2} \sum n_i J_{B,F} \left(m^2(\phi_{cl})/T^2 \right). \end{aligned} \quad (3.31)$$

where n_i are modified degrees of freedom mentioned before (3.21). One should mention that both imaginary and real time formalisms give the same result.

3.2 The First-Order Phase Transition

Phase transitions play a very important role in modern cosmology. They are described by the evolution of an order parameter with respect to the temperature, which in the case of the EWPT is the expectation value of the scalar Higgs. In particle physics, a phase transition occurs if the state of the theory at certain temperature does not correspond to the global minimum of the potential. Then the field changes its value to the true ground state (i. e. the minimum of the potential which is the stable one). This can happened gradually or by tunneling when a barrier exists between the two minima. The transition can be first order or second order. To understand well what the difference between these two types is, let us consider the phase transition in the Standard electroweak theory as an example.

Using the high temperature expansion in (3.29) and (3.30), the effective potential of the Standard Model in the background Higgs field configuration, $\langle \phi \rangle = h/\sqrt{2}$, is given by[1]

$$V_{eff}(h,T) = D(T^2 - T_0^2)h^2 - ETh^3 + \frac{\lambda(T)}{4}h^4, \tag{3.32}$$

where the coefficients are given by

$$\begin{aligned} D &= \frac{2m_W^2 + m_Z^2 + 2m_t^2}{8v^2},\ E = \frac{2m_W^3 + m_Z^3}{4\pi v^3},\ T_0^2 = \frac{m_h^2 - 8Bv^2}{4D},\ B = \frac{3\left(2m_W^4 + m_Z^4 - 4m_t^4\right)}{64\pi^2 v^4} \\ \lambda(T) &= \lambda - \frac{3}{16\pi^2 v^4}\left(2m_W^4 \log\frac{m_W^2}{A_B T^2} + m_Z^4 \log\frac{m_Z^2}{A_B T^2} - 4m_t^4 \log\frac{m_t^2}{A_F T^2}\right), \end{aligned}$$

here $\log A_B = \log a_b - 3/2$ and $\log A_F = \log a_F - 3/2$, and a_B, a_F are given in (3.29) and (3.30). The parameter $\lambda(T)$ is slowly changing with temperature, so we can consider it as a constant. At this special temperature value:

$$T_c^2 = \frac{\lambda D T_0^2}{\lambda D - E^2}, \tag{3.33}$$

the potential has two degenerate minima: $h = 0$ and

$$h_c = 2ET_c/\lambda. \tag{3.34}$$

[1]We assume that $m_h \ll m_W$, and therefore only the contributions of gauge bosons and top quark to the effective potential are considered.

T_c is called the critical temperature. At temperatures below this value the minimum $h = 0$ becomes unfavored energetically (false vacuum) and therefore the decay to the true ground state $h \neq 0$ is possible by tunneling or thermal fluctuations. At the temperature $T = T_0$, the barrier between the two vacua disappears and the field can move directly to the true vacuum without tunneling. The behavior of the effective potential is given in Fig. 3.1 for different temperature values.

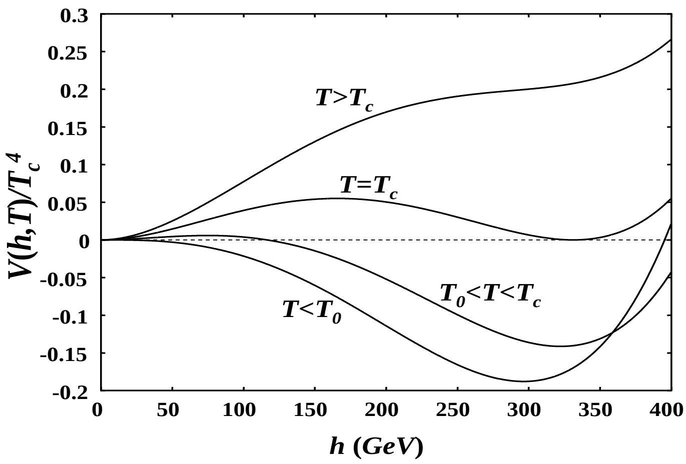

Figure 3.1: *The behavior of the effective potential for differen values of the temperature.*

In Fig. 3.2, we plot the value of the absolute minimum of the potential (vev) as a function of the temperature for two cases where we give by hand two different values for the parameter E in (3.32), we find that for the larger value of E, there is a discontinuity at $T = T_c$, and this is the character of a *first-order phase transition*. If the condition[2]

$$v_c/T_c > 1, \tag{3.35}$$

is fulfilled then we have a *strong first order* phase transition [80].

However for an extremely small (zero) E value, the absolute minimum of the potential changes continuously to zero, and then we have a *continuous phase transition*, and the quantity $v_c/T_c \sim 0$. Only in a strong first order phase transition, a (very) violent deviation from equilibrium does exist.

[2]We will see in Sec. 3.5 how this condition (3.35) is derived.

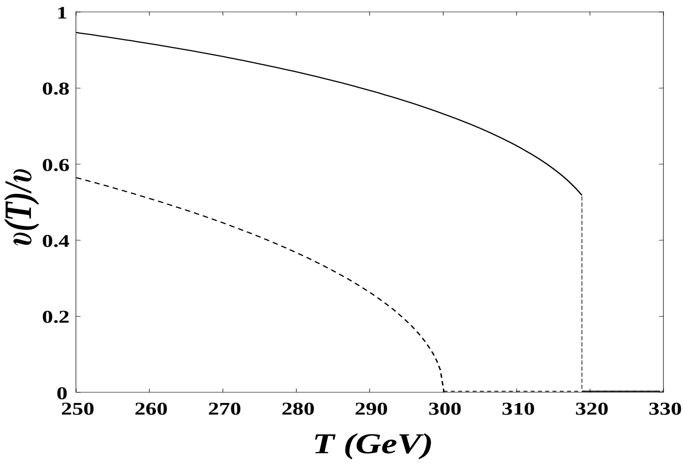

Figure 3.2: *The order parameter $v(T)$ as a function of the temperature T. The solid line corresponds to a first order phase transition, while the dashed one corresponds to a continuous order one. For the first order case we put by hand $E = 0.1$ in (3.32) where $T_c = 318.9$ GeV. For the second order one we put by hand also $E = 10^{-5}$, where $T_c = 300$ GeV.*

In the case of first order phase transition the transition occurs via bubble nucleation. Inside the bubbles the field acquires the true vacuum value. If the rate of bubble nucleation exceeds the universe expansion rate, the bubbles collide and therefore the space is filled by true vacuum.

The condition (3.35) leads to an upper bound on the Higgs mass [11]

$$m_h \lesssim 42 \ GeV. \tag{3.36}$$

The constraint gets even stronger when the two-loops effects and a proper treatment of the top quark are included [81]. It is clear that this bound is in contradiction with the lower bound coming from LEP $m_h > 114$ GeV [12] and any generated baryon asymmetry within the Standard Model will be washed out. Therefore it is quite natural to look for extensions of Standard Model to increase this bound or to make it irrelevant.

3.3 The Bubble Formation

In the first order phase transition case, at temperatures just below the critical one T_c, there appears a new (true) vacuum, and the false vacuum starts to decay to the true one via thermal fluctuation. However another term is widely used in the literature: Thermal tunneling. This transition can be understood in terms of bubble nucleation of the broken phase in the sea of the symmetric one. As mentioned before, once this has happened, the bubbles spread throughout the universe converting false vacuum into the true one.

To understand the dynamics of this kind of transitions, an important value should be computed: the rate of bubble nucleation. The tunneling probability per unit time per unit volume was computed analogously to a simple quantum mechanical one-dimensional system of a particle trying to penetrate a potential barrier, the result is [82]

$$\Gamma \propto \exp\left\{-S_E\right\}, \tag{3.37}$$

where S_E is the four dimensional Euclidean action

$$S_E = \int d\tau d^3x \left[\frac{1}{2}\left(\tfrac{\partial}{\partial\tau}h\right)^2 + \tfrac{1}{2}\left(\vec{\bigtriangledown}h\right)^2 + V\left(h\right)\right]; \tag{3.38}$$

and the field obeys the equation of motion

$$\left(\tfrac{\partial^2}{\partial\tau^2} + \vec{\bigtriangledown}^2\right) h - \frac{\partial}{\partial h} V\left(h\right) = 0, \tag{3.39}$$

with boundary conditions $h \to 0$ for $\tau \to \pm\infty$, $r \to \infty$ and $\partial h/\partial\tau = \partial h/\partial r = 0$ for $\tau = r = 0$. The solution of (3.39) is $O(4)$ symmetric, i. e. $h = h(\sqrt{\tau^2 + r^2})$.

At finite temperature, to compute this quantity (3.37), one needs to take into account that the Euclidean time τ is periodic with the period T^{-1}, and the potential becomes an effective potential at finite T. The tunneling rate per unit time per unit volume is given by [83]

$$\Gamma \propto A\left(T\right) \exp\left\{-S_3/T\right\}, \tag{3.40}$$

where the prefactor A is of the order T^4. The three-dimensional action S_3 is:

$$\begin{aligned} S_3 &= \int d^3x \left[\frac{1}{2}\left(\vec{\nabla} h\right)^2 + V_{eff}\left(h,T\right)\right] \\ &= 4\pi \int dr\, r^2 \left[\frac{1}{2}\left(\frac{\partial}{\partial r} h\right)^2 + V_{eff}\left(h,T\right)\right], \end{aligned} \tag{3.41}$$

and the equation of motion is similar to (3.39) in the static (time-independent) case:

$$\frac{\partial^2}{\partial r^2} h + \frac{2}{r}\frac{\partial}{\partial r} h - \frac{\partial}{\partial h} V\left(h\right) = 0, \tag{3.42}$$

with the same boundary conditions $h \to 0$ for $r \to \infty$; and $\partial h / \partial r = 0$ for $r = 0$. The solution in this case is $O(3)$ symmetric. The three-dimensional action (3.41) can be rewritten as [10]

$$S_3 = 4\pi \int_0^R dr\, r^2 \left[\frac{1}{2}\left(\frac{\partial}{\partial r} h\right)^2 + V_{eff}\left(h,T\right)\right], \tag{3.43}$$

where R is the bubble radius. There are two contribution to S_3 in (3.43), which are

$$S_3 \sim 2\pi R^2 \left(\frac{\delta h}{\delta R}\right)^2 \delta R + \frac{4\pi R^3 \left\langle V \right\rangle}{3}, \tag{3.44}$$

where $\delta h \sim h_{true}$, δR is the thickness of the bubble wall, and $\langle V \rangle$ is the average of the potential inside the bubble. The first term is a surface term coming from kinetic contribution in (3.43), and the second one is a volume term coming from the potential contribution in (3.43).

The bubbles start to nucleate at a temperature called T_n below the critical one (see eq. (3.33)), which can be defined by the tunneling probability of one bubble inside a causal horizon volume, $\sim 2.\,16 \times 10^{-4}\, M_{Pl}^3 / T^6$, to be of the order $\mathcal{O}(1)$ [84]

$$\begin{aligned} \int_{T_n}^{\infty} \frac{dT}{T}\left(\frac{2\zeta M_{Pl}}{T}\right)^4 \exp\{-S_3(T)/T\} &= \mathcal{O}(1) \\ \zeta &= \frac{1}{4\pi}\sqrt{45/\pi g_*}, \end{aligned} \tag{3.45}$$

where g_* is the effective number of degrees of freedom. In the case where the barrier between the two minima is high enough, the volume term in (3.44) is dominant and therefore the amount $\delta R / R \ll 1$ is very small. This means that the nucleated bubbles in this case are thin-wall bubbles. In contrary, when the surface term is the dominant one, $\delta R/R \sim \mathcal{O}(1)$ which corresponds to thick-wall bubbles.

If the energy difference between the two potential minima, true and false one, is small compared to the height of the potential barrier, the radius of the bubble becomes much larger than the thickness of the bubble wall [82]. This leads to the so-called thin-wall regime.

At late time t where the bubble wall is already flat, it is more suitable to work in one dimension, then we use the variable z=x-$v_w t$ in order to be in the wall frame, where v_w is the wall velocity. Then the equation of motion becomes

$$\frac{\partial^2}{\partial z^2} h - \frac{\partial}{\partial h} V(h) = 0. \tag{3.46}$$

The wall profile in the case of the effective potential (3.32) is obtained by solving of (3.46), and the exact solution is:

$$h(r) = \frac{\alpha}{\cosh\{z/\zeta\} + \beta}; \tag{3.47}$$

with

$$\alpha = \frac{4D\left(T^2 - T_0^2\right)}{\sqrt{\lambda D - E^2}\sqrt{T_c^2 - T^2}}, \ \beta = 2\frac{T}{T_0}\sqrt{\frac{T_c^2 - T_0^2}{T_c^2 - T^2}} \text{ and } \zeta = \frac{\lambda v^2}{\sqrt{8D}\sqrt{T^2 - T_0^2}}.$$

In Fig. (3.3), we show the complete solution of (3.42) for different values of the temperature.

Inside the bubble, the field is in the true vacuum whereas outside it is still in the false one. In the SM case the term E in (3.32) is very small and therefore the barrier between the two minima is also small. This leads to the thick wall regime.

3.4 Sphalerons and the EWPT

We have seen in section 2.3 when we discussed the baryon number violation processes in the Standard Model, that their rate is suppressed by 160 orders of magnitude. However the situation looks different at high temperatures, where the field can move from a vacuum configuration to another classically over the barrier without tunneling.

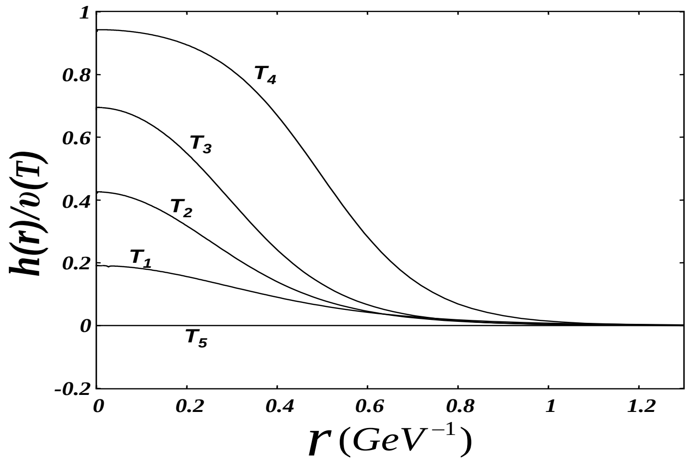

Figure 3.3: *The Bubble wall profile for different values of temperature* $T_{1,5}$*. The field configuration* $h(r)$ *is rescaled by its vacuum expectation value at temperature* T_i*;* $v(T_i)$*. The values of* $T_{1,5}$ *are* $T_0 + (T_c - T_0)/5$ *,* $T_0 + 2(T_c - T_0)/5$*,* $T_0 + 3(T_c - T_0)/5$*,* $T_0 + 4(T_c - T_0)/5$ *and* T_c*, respectively; and* T_c *and* T_0 *are given in (3.33) and (3.32).*

As noted before, the properties of the field configuration known as *Sphaleron* are needed to estimate the transition rate. Let us first define the Sphaleron configuration.

The sphaleron is identified to be the static (time independent: $A_0 = 0$, $\partial_0 A_i = 0$) configuration which is the barrier between two adjacent vacua ($\Delta N_{CS} = \pm 1$) whose minimal energy along a path among all the paths.

In the case of Standard Model, with the $U(1)_Y$ gauge is neglected[3] ($\theta_W \to 0$), and using the orthogonal gauge

$$x_i \cdot A_i = 0, \tag{3.48}$$

[3]The contribution of the $U_Y(1)$ field to the Sphaleron energy is very small, less than 0.2%, see ref. [24].

the Sphaleron solution has the following form [24]:

$$\begin{aligned}
\tfrac{i}{2}g\sigma^a A_i^a &= f(r)\ (\partial_i U^\infty)\,(U^\infty)^{-1} \\
\text{or}\quad A_i^a &= \frac{2\,f(r)}{gr^2}\epsilon_{aik}x_k \\
\phi &= \tfrac{v}{\sqrt{2}}h(r)\ U^\infty \begin{pmatrix} 0 \\ 1 \end{pmatrix} \\
U^\infty(\theta,\varphi) &= \begin{pmatrix} \cos\theta & e^{i\varphi}\sin\theta \\ -e^{-i\varphi}\sin\theta & \cos\theta \end{pmatrix}.
\end{aligned} \tag{3.49}$$

Here A_i is the $SU(2)_L$ gauge field, ϕ is the Higgs doublet; σ^a and are Pauli matrices. In particular, the gauge transformation [37]

$$U(\vec{r}) = \exp\left\{-i\frac{\pi}{2}\frac{\vec{\sigma}\cdot\vec{r}}{r}\right\}; \tag{3.50}$$

transforms the ansatz (3.49) into the following background field:

$$\begin{aligned}
A_i^a &= \frac{2\{1-f(r)\}}{gr^2}\epsilon_{aik}x_k \\
\phi &= \tfrac{v}{\sqrt{2}}h(r) \begin{pmatrix} 0 \\ 1 \end{pmatrix};
\end{aligned} \tag{3.51}$$

and the energy functional is:

$$E = \int d^3x \left[\frac{1}{4}F_{ij}^a F_{ij}^a + (D_i\phi)^\dagger (D_i\phi) + V(\phi)\right] \tag{3.52}$$

where $V(\phi)$ is the potential of the Higgs field. For the Sphaleron solution, the energy (3.52) can be rewritten as

$$\begin{aligned}
E_{sp} &= \tfrac{4\pi v}{g}\int_0^\infty d\zeta \left[4\left(\tfrac{df}{d\zeta}\right)^2 + \tfrac{8}{\zeta^2}\left[f(1-f)\right]^2 + \tfrac{\zeta^2}{2}\left(\tfrac{dh}{d\zeta}\right)^2 \right. \\
&\quad \left. + \left[h(1-f)\right]^2 + \tfrac{\zeta^2}{g^2}V\left(\tfrac{v}{\sqrt{2}}h(\zeta)\right)\right],
\end{aligned} \tag{3.53}$$

where $\zeta = gvr$ is a dimensionless coordinate.

The field equations can be obtained via inserting (3.49) into the Euler-Lagrange equations, or via minimizing (3.52), therefore the field equations are given by

$$\begin{aligned}
\zeta^2\frac{\partial^2}{\partial\zeta^2}f &= 2f\,(1-f)\,(1-2f) - \tfrac{1}{4}\zeta^2 h^2\,(1-f) \\
\frac{\partial}{\partial\zeta}\zeta^2\frac{\partial}{\partial\zeta}h &= 2h\,(1-f)^2 + \frac{\zeta^2}{g^2 v^3}\,\frac{\partial}{\partial\phi}V(\phi)\bigg|_{\phi=vh(\zeta)/\sqrt{2}};
\end{aligned} \tag{3.54}$$

taking into account that the energy must be finite, the functions f and h should have the boundary conditions

$$f(\zeta) \rightarrow \begin{cases} \sim \zeta^2, \ \zeta \rightarrow 0 \\ 1, \ \zeta \rightarrow \infty \end{cases} ; \quad h(\zeta) \rightarrow \begin{cases} \sim \zeta, \ \zeta \rightarrow 0 \\ 1, \ \zeta \rightarrow \infty. \end{cases} \tag{3.55}$$

The profiles of f and L are given in Fig. 3.4.

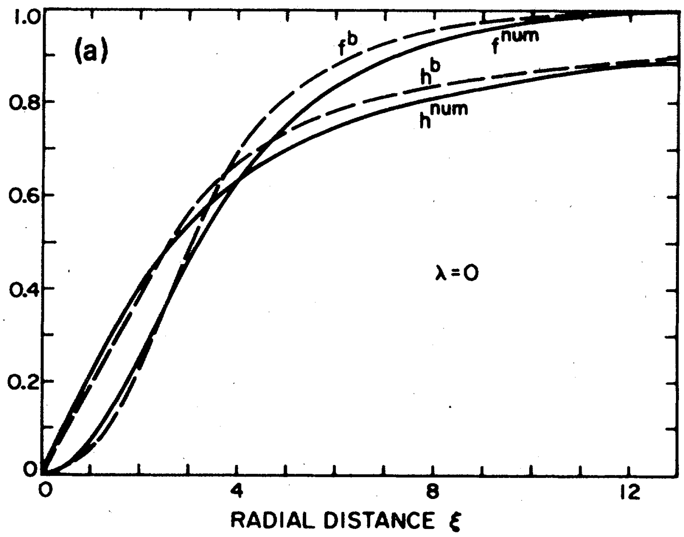

Figure 3.4: *The profiles of the functions f and L for the SU(2)$_L$ Higgs-gauge system. This figure is taken from in Klinkhamer & Manton in [24]. The results are were obtained for λ=0 and ξ =gvr is the dimensionless radial distance.*

If the theory is in the vacuum characterized by $N_{CS} = 0$, then the transition to the next vacuum $N_{CS} = 1$, proceeds with the help of Sphaleron configuration; which interpolates between $N_{CS} = 0$ and $N_{CS} = 1$. The Sphaleron energy is basically the contribution of the Higgs field and of the gauge configuration. The latter is more important than that of the Higgs field.

A more accurate result may be found by means of variational methods [24]

$$E_{sp} = \frac{4\pi v}{g} B\left(\frac{\lambda}{g^2}\right), \tag{3.56}$$

where λ is the self-coupling of the Higgs field; and B is a function which depends very weakly on λ/g^2: $B(0) \simeq 1.52$ and $B(\infty) \simeq 2.72$. The previous computation of the Sphaleron energy was performed at zero temperature. The Sphaleron at finite temperature in the broken phase was computed in [85], it was shown that its energy follows approximately the scaling law[4]

$$\frac{E_{sp}(T)}{v(T)} = \frac{E_{sp}(T=0)}{v(T=0)}, \tag{3.57}$$

here $v(T)$ is the *vev* of the Higgs field at finite temperature. Then (3.56) can be rewritten at finite temperature as:

$$E_{sp}(T) = \frac{4\pi v\,(T)}{g} B\left(\frac{\lambda}{g^2}\right). \tag{3.58}$$

In case of Standard Model, the Sphaleron transition rate per unit time per unit volume was computed in a semi-classical way to be a Boltzmann suppression factor $\exp(-E_{sp}(T)/T)$ up to a prefactor. This prefactor contains the determinant of all zero and non-zero modes, and near the critical temperature T_c, it can play a more important role, and the rate obtains the form [87]

$$\Gamma = \kappa \frac{T^4 \omega_-}{m_W} \left(\frac{\alpha_W}{4\pi}\right)^4 N_{tr} N_{rot} \left(\frac{2m_W}{\alpha_W T}\right)^7 \exp\left(-\frac{E_{sp}}{T}\right). \tag{3.59}$$

Here the factors N_{tr} and N_{rot} come from the zero mode normalization, which are given by $N_{tr} \simeq 26$, $N_{rot} \simeq 5.\,3 \times 10^3$ for the case of SM [88], ω_- is the eigenvalue of the negative mode [89, 37]; and κ is the functional determinant associated with fluctuations around the sphaleron [90]. It has been estimated to be in the range: $10^{-4} \lesssim \kappa \lesssim 10^{-1}$ [87,91].

The dilution of the baryon asymmetry in the anomalous processes can be described by the equation [87]

$$\frac{\partial}{\partial t} S = -R(t) S, \tag{3.60}$$

where $R(t)$ is the rate of B violating processes. If one supposes that the electroweak transition proceeds in a constant temperature, then the solution is [80]

$$S = \exp\left\{-\frac{13}{2}\zeta n_f \frac{M_{pl}\Gamma}{T^5}\right\}, \tag{3.61}$$

[4]This scaling law has been checked to be satisfied for the Minimal Supersymmetric Standard Model (MSSM) by Moreno et al. in ref [86].

where ζ is given in (3.45). If we take $T = T_c \sim 100\ GeV$, $n_f = 3$ and $B = 1.87$ (B from eq. (3.56)); and we impose the condition $S \gtrsim 10^{-5}$, then the ratio $E_{sp}(T_c)/T_c$ must satisfy the inequality

$$E_{sp}(T_c)/T_c \gtrsim 7\log(E_{sp}(T_c)/T_c) + 9\log 10 + \log\kappa \qquad (3.62)$$

which is translated for the upper bound of $\kappa = 10^{-1}$ to [23]

$$E_{sp}(T_c)/T_c \gtrsim 45 \qquad (3.63)$$

and for the lower bound of $\kappa = 10^{-4}$, 37 instead of 45. Since $E_{sp}(T)$ scales like $v(T)$ (see eq. (3.57)), then (3.63) transforms into $v(T_c)/T_c \gtrsim 1.3$ or $v(T_c)/T_c \gtrsim 1$ for the case of $\kappa = 10^{-4}$, which is the famous condition of a strongly first order phase transition (see eq. (3.35)). If this condition is not satisfied, any existing baryon number will be washed out.

3.5 The EWPT within some SM Extensions

It was shown in Section 3. 2 that the EWPT can not be strongly first order due to the severe bound (3.36) on the Higgs mass. In addition to the observation of neutrino oscillations [7], which represents a strong evidence that the SM is not a fundamental theory, it is quit natural to go beyond the SM looking for a strong first order EWPT.

There are many SM extensions, in many directions, to get a strong first order EWPT. In this section, we will review the EWPT in some of these models.

The EWPT within the SM with scalar Singlets

This is the simplest SM extension, that has been investigated to get a strong first order EWPT. In Ref. [10, 13], the authors added a gauge complex singlet to the SM spectrum, where this complex singlet couples only to the Higgs doublet and does not develop a vev. In this case, the cubic term in (3.32) gets larger due to the singlet contribution to the effective potential and the Higgs upper bound (3.36) is weakened up to ~100 GeV.

In Ref. [14], the authors added to the SM a real singlet S, where the symmetry allows explicit cubic terms[5], S^3 and $\phi^+\phi S$, in the potential at tree level; and this singlet develops a vev x. The transition occurs from the false vacuum $(0, x_0)$ to the true one (v, x). Therefore it is believed that the EWPT gets stronger without the need of the thermal induced cubic term.

In Ref. [14], the authors modified the criterion of a strong first order EWPT (3.35); by replacing v with the distance between the true and false minima in the singlet-Higgs space, i. e. , $\Omega = \left\{v^2 + (x - x_0)^2\right\}^{\frac{1}{2}}$. Since we have always $\Omega \geq v$, the modified condition is more easily fulfilled than the SM one for a large part of the physically allowed parameter space. Then unlike in the works done by [10, 13], the strong first order EWPT is not, in reality, obtained due to the increased cubic term in (3.32); but the reason is due to the singlet vev contribution to $\Omega(T_c)$. Until now, it is not proven that the last condition is viable for these kinds of models.

Similar works were carried out in [15], with a difference in defining the gauge singlet; it was considered as the singlet Majoron[6], but it plays the same role in the EWPT as in [14].

The EWPT within Supersymmetric Extensions

The most physically motivated SM extension is the Minimal Supersymmetric Standard Model (MSSM). Supersymmetry requires that each boson (fermion) have a fermionic (bosonic) superpartner. Therefore, the MSSM contains the particle content of the SM and their superpartner plus an extra Higgs doublet with its superpartner. In this case, new free parameters are introduced into the theory; but many of them are constrained by supersymmetry and by existing accelerator measurements.

In the context of baryogenesis, a strong electroweak phase transition is not the only motivation for the MSSM, but also new sources of CP violation can exist explicitly or spontaneously.

In the existence of bosons that strongly couple to the Higgs, the cubic term is increased and therefore the EWPT gets stronger. In the MSSM, both left- and right-handed top quark superpartners $\tilde{t}_{L,R}$ (stops) couple to the Higgs field background

[5]The cubic terms can exist at tree-level potential even if the singlet is complex like $\{S^3$+h.c$\}$ and $\{\phi^+\phi S$+h.c$\}$.

[6]The Majoron is a hypothetical type of Goldstone boson that helps to generate the neutrino mass, but violates the $U(1)_{B-L}$ symmetry.

configuration with strength of the order of top Yukawa coupling; this makes the EWPT strong better than the minimal SM case but with condition that the right-handed stop should be lighter than the top quark itself [17]. The EWPT was extensively studied using different techniques [20]. In some choices of parameters, the stop can condense and develop a vacuum expectation value and both color and charge are broken, but the absolute minimum is that of the scalar Higgs. The color charge breaking (CCB) should occur before the electroweak symmetry breaking; this can increase the strength of the EWPT [19].

However the MSSM is not a perfect SM extension; it suffers from some problems. One of them is the so-called $\mu-$problem, which is the mass mixing term between the two Higgs doublets in the superpotential. This term must be of the order of the electroweak scale for the consistency of the theory, which is much smaller than the GUT scale or Planck scale; and it is also stable under perturbative corrections. However, there is no a priori reason for μ to have such a small value.

An elegant way to solve this problem is to introduce a gauge Singlet into the spectrum. The μ-term should be replaced by SH_1H_2 in the superpotential ; and the mass term is obtained dynamically when the singlet develops its vev. This is what is called the Next-to-Minimal Supersymmetric Standard Model (NMSSM)[7] [92].

Within the N(n)MSSM, the EWPT was studied by many authors [65, 93, 94], the authors found that the EWPT gets stronger easier than in the case of MSSM, in addition to that there are additional CP violating sources.

In Ref. [93], The authors studied the EWPT in the minimal nonminimal supersymmetric standard model (MNMSSM), where they took the criterion of the first order EWPT to be $\Omega/T = \left\{v_1^2 + v_2^2 + (x - x_0)^2\right\}/T > 1$ at $T = T_c$, instead that used in [65,94], where x is the singlet vev. Then the reason to get a strong first order EWPT may be the large singlet vev contribution to $\Omega(T_c)$.

In Ref. [65], the authors took the simplest form of the singlet superpotential. They minimized the scalar potential and computed the singlet minimum as a func-

[7]Sometimes it is called nMSSM, but the main difference is how to choose the pure singlet sector in the superpotential. The singlet couples only to the Higgs doublets, and the only constrains come from the stability of the theory and that the singlet vev should be of the order of the electroweak scale.

tion of the two Higgs vevs, then the problem is reduced to an MSSM-like model but a with different potential which leads to a strong first order EWPT easier than the MSSM case. Similar work was carried out in [94], where the authors look whether or not the condition (3.35) is satisfied along the path $\partial V_{eff}/\partial S\big|_{S=x} = 0$.

Chapter 4

The EWPT in the Standard Model with a Singlet

It is well known that the Standard Model (SM) of electroweak and strong interactions can not be a fundamental theory, due its failure in answering many open questions (the hierarchy, neutrino masses, cosmological constant and flavor problems, the origin of baryons, the Dark Matter and Dark Energy of the Universe, . . .), but it can be rather an effective theory with a low physical cutoff that can be probed at the LHC experiment.

Many SM extensions contain hidden sectors with a matter content that is transformed non-trivially under a hidden sector gauge group but singlet with respect to the SM. It has been noticed that the SM Higgs field h plays a very special role with respect to such hidden sector since it can provide a window (a portal [21]) into it through the renormalizable interaction $H^2 S_i$ where the bosons S_i are SM singlets. This coupling to the hidden sector can have important implications both theoretically and for LHC phenomenology as has been discussed in recent literature [21,22].

Usually the electroweak phase transition (EWPT) is at most weakly first order due the temperature induced cubic term in the Lagrangian. However in models with singlet(s) the cubic term is present at tree-level; this is why it is widely believed that in these models, one can get easily a strong first order phase transition.

A model independent criterion for a strong first order phase transition is given in (3.63), which is translated into (3.35) for the of standard model. Indeed, the passage from (3.63) to (3.35) is true due to scaling law in (3.57). In this chapter, we check whether this passage holds for models involving a singlet. We will focus on the minimal standard model plus a real singlet ('SM+S').

First, we will briefly introduce the model, and give the effective potential. In the second section, we will find the Sphaleron solution in this model. In the third section, we will study the EWPT in this model. In the last section, we will discuss the criterion for a strong first order EWPT in this model; and give our conclusion. Most of the results are summarized in [95].

4.1 The 'SM+S' Model

Let us consider an extension of the Standard Model by a singlet real scalar S coupled only to the standard Higgs. We concentrate here on the scalar sector (SM Higgs and the added singlet) and the $SU(2)_L$ gauge sectors. [1]

4.1.1 The Effective Lagrangian

The Lagrangian is given by

$$\mathcal{L} = -\frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} + \left(D_\mu\phi\right)^\dagger\left(D^\mu\phi\right) + \frac{1}{2}\left(\partial_\mu S\right)\left(\partial^\mu S\right) - V_0\left(\phi, S\right), \tag{4.1}$$

where ϕ is the Higgs doublet

$$\phi^T = 1/\sqrt{2}\left(\begin{array}{cc} \chi_1 + i\chi_2, & h + i\chi_3 \end{array}\right), \tag{4.2}$$

and h is the scalar standard Higgs, χ's are the three Goldstone bosons; and $F^a_{\mu\nu}$ is the $SU(2)_L$ field strength

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\epsilon^{abc}A^b_\mu A^c_\nu, \tag{4.3}$$

D_μ is the covariant derivative. When neglecting the $U(1)_Y$ gauge, it is given by

$$D_\mu = \partial_\mu - \frac{i}{2}g\sigma^a A^a_\mu. \tag{4.4}$$

[1]Since we are interested here in the Sphaleron solution, we assume that the $U_Y(1)$ contribution to the sphaleron energy is negligible as in the case of the Standard Model [96].

Finally $V_0(\phi, S)$ is the tree-level effective potential, which is given by

$$V_0(\phi, S) = \lambda |\phi|^4 - \mu_h^2 |\phi|^2 + \omega |\phi|^2 S^2 + \rho |\phi|^2 S + \tfrac{\lambda_S}{4} S^4 - \tfrac{\alpha}{3} S^3 - \tfrac{\mu_S^2}{2} S^2. \quad (4.5)$$

We can eliminate μ_h^2 and μ_S^2 by making (v, x) as the absolute minimum of the one-loop effective potential at zero temperature, where $v = 246.22$ GeV is the standard Higgs vev.

Now we write the explicit formula of the one-loop effective potential. We will consider the contributions of the gauge bosons, the standard Higgs h, the singlet S, the Goldstone bosons $\chi_{1,2,3}$ and the top quark. The field-dependent masses at zero temperature are given by

$$m_t^2 = \tfrac{1}{2} y_t^2 h^2,\ m_Z^2 = \tfrac{\bar{g}^2 + g'^2}{4} h^2, m_W^2 = \tfrac{g^2}{4} h^2,\ m_\chi^2 = \lambda h^2 - \mu_h^2 + \omega S^2 + \rho S, \quad (4.6)$$

where y_t is the Yukawa coupling for the top quark, and $\bar{g}^2 = g^2 + g'^2$, however we have already neglected the $U(1)_Y$ group; and therefore $g' = 0$ and $m_Z = m_W$. The Higgs-Singlet mass matrix is

$$\begin{pmatrix} 3\lambda h^2 - \mu_h^2 + \omega S^2 + \rho S & 2\omega h S + \rho h \\ 2\omega h S + \rho h & \omega h^2 + 3\lambda_S S^2 - 2\alpha S - \mu_S^2 \end{pmatrix}, \quad (4.7)$$

which leads to the eigenmasses:

$$m_{1,2}^2 = \tfrac{1}{2} \Big\{ (3\lambda + \omega) h^2 + (3\lambda_S + \omega) S^2 + (\rho - 2\alpha) S - \mu_h^2 - \mu_S^2$$
$$\mp \sqrt{\left((3\lambda - \omega) h^2 - (3\lambda_S - \omega) S^2 + (\rho + 2\alpha) S - \mu_h^2 + \mu_S^2\right)^2 + 4 (2\omega S + \rho)^2 h^2} \Big\}. \quad (4.8)$$

Then the one loop correction to the effective potential at zero temperature is given by (see eq. (3.21))

$$V_1^{T=0}(h, S) = \sum_{i=W,Z,t,h,S,\chi} n_i G\left(m_i^2(h, S)\right) \quad (4.9)$$
$$G(x) = \frac{x^2}{64\pi^2} \left\{ \log\left(\frac{x}{Q^2}\right) - \frac{3}{2} \right\},$$

here Q is the renormalization scale, which we take to be the SM Higgs vev $Q = 246.22$ GeV, n_i is the $i-$particle degree of freedom; which are

$$n_W = 6, n_Z = 3, n_h = 1, n_\chi = 3, n_S = 1, n_t = -12.$$

The temperature-dependent part at one loop is given by

$$V_1^{T\neq 0}(h,S) = \frac{T^4}{2\pi^2} \sum_{i=W,Z,t,h,S,\chi} n_i J_{B,F}\left(m_i^2(h,S)/T^2\right), \tag{4.10}$$

where $J_{B,F}$ are given in (3.26) and (3.28). We will use the high temperature ($m_i \ll T$) expansions (3.29) and (3.30). Since the bosonic contribution (3.29) involves a term like $\left(m_i^2\right)^{\frac{3}{2}} T$, the probability of a negative m_i^2 is present for the case of Higgs-Singlet ($m_{1,2}^2 < 0$) and Goldstone bosons ($m_\chi^2 < 0$), which can make problems to the potential behavior in some regions in the plane (h,S). This problem can be solved the same way as the infrared divergences are solved by considering the so-called *ring* (or *daisy*) contribution to the effective potential [97], where the mass in the cubic term is replaced by the thermal mass. The ring contribution is given by

$$V_{ring}(h,S,T) = -\tfrac{T}{12\pi} \sum_{i=W,Z,h,S,\chi} n_i \left\{\left(M_i^2(h,S,T)\right)^{\frac{3}{2}} - \left(m_i^2(h,S)\right)^{\frac{3}{2}}\right\}, \tag{4.11}$$

where $M_i^2(h,S,T)$ is the thermal mass of the bosons i (All bosonic thermal masses are given in Appendix B). The full one-loop effective potential at finite temperature is the summation of (4.5), (4.9), (4.10) and (4.11)

$$\begin{aligned} V(h,S,T) &= V_0(h,S) + \sum_{i=W,Z,t,h,S,\chi} n_i G\left(m_i^2(h,S)\right) \\ &+ \frac{T^4}{2\pi^2} \sum_{i=W,Z,t,h,S,\chi} n_i J_{B,F}\left(m_i^2(h,S)/T^2\right) \\ &- \tfrac{T}{12\pi} \sum_{i=W,Z,h,S,\chi} n_i \left\{\left(M_i^2(h,S,T)\right)^{\frac{3}{2}} - \left(m_i^2(h,S)\right)^{\frac{3}{2}}\right\}. \end{aligned} \tag{4.12}$$

4.1.2 The Space of Parameters

In our theory we have quite a few parameters

$$\lambda,\ \lambda_S,\ \omega,\ \rho,\ \alpha,\ \mu_h^2,\ \mu_S^2;$$

in addition to the singlet vev x. As mentioned above, μ_h^2 and μ_S^2 can be eliminated as

$$\mu_h^2 = \lambda v^2 + \omega x^2 + \rho x + \frac{1}{v}\frac{\partial}{\partial h} V_1^{T=0}(h,S) \tag{4.13}$$

$$\mu_S^2 = \omega v^2 + \tfrac{\rho v^2}{2x} + \lambda_S x^2 - \alpha x + \frac{1}{x}\frac{\partial}{\partial S} V_1^{T=0}(h,S), \tag{4.14}$$

after which our free parameters are λ, λ_S, ω, ρ, α and x. Since the theory is invariant under the discrete symmetry (x,ρ,α)→(-x,-ρ,-α), we will assume only positive values for the singlet vev x. We want also to keep the perturbativity of theory by imposing $\lambda, \lambda_S, |\omega| \ll 1$. We choose the parameters λ, λ_S, ω, ρ, α and x lying in the ranges:

$$\begin{aligned} 0.001 &\leq \lambda, \lambda_S \leq 0.6 \\ -0.6 &\leq \omega \leq 0.6 \\ 100 &\leq x/GeV \leq 350 \\ -350 &\leq \alpha/GeV \leq 350 \\ -350 &\leq \rho/GeV \leq 350. \end{aligned} \tag{4.15}$$

The stability of the theory implies that the potential goes to infinity when the field goes to the infinity in any direction, which implies $\omega^2 < \lambda \times \lambda_S$. Moreover, we impose that any minimum or extremum of the potential should be in the range of the electroweak theory; let us say that all the minima and extrema must be inside the circle $h^2 + S^2 = \{600\ GeV\}^2$ in the $h - S$ plane; and therefore the potential is monotonically increasing outside this circle in any direction.

In the Standard Model the Higgs mass lower bound is given by $m_h^{SM} > 114$ GeV [12]. The mixing between the standard Higgs and the singlet makes both Singlet-Higgs eigenstates ($h_{1,2}$) coupled to the gauge bosons and leptons but with the standard couplings multiplied by $\cos\theta$ and $\sin\theta$ respectively, where

$$\cos 2\theta = \frac{M_{11}^2 - M_{22}^2}{\sqrt{\left(M_{11}^2 - M_{22}^2\right)^2 + 4M_{12}^4}}, \ \sin 2\theta = \frac{2M_{12}^2}{\sqrt{\left(M_{11}^2 - M_{22}^2\right)^2 + 4M_{12}^4}}, \tag{4.16}$$

and M_{ij} are given in (4.7). Therefore the standard bound is not valid. In our work, we will not derive the new lower bound for the Higgs mass, but we will restrict ourselves only to masses $m_{1,2}$ in the range 65 GeV to 450 GeV.

4.2 The Sphaleron Solution in the 'SM+S' Model

In order to find the sphaleron solution for this model [2], we should follow the same steps as in the $SU(2)_L$ Higgs-gauge model (see Sec 3.4). Applying Euler-Lagrange condition to the Lagrangian (4.1) where we replace the tree-level potential by the effective one $V_{eff}(h, S, T)$, one finds the field equations

$$\begin{aligned}\partial_\gamma F^{q\gamma\tau} - g\epsilon^{qab} A^b_\alpha F^{a\alpha\tau} + \tfrac{1}{4} g^2 h^2 A^{q\tau} = 0 \\ \partial^2 h - \tfrac{1}{4} g^2 h A^a_\mu A^{a\mu} + \tfrac{\partial}{\partial h} V_{eff}(h, S, T) = 0 \\ \partial^2 S + \tfrac{\partial}{\partial S} V_{eff}(h, S, T) = 0.\end{aligned} \tag{4.17}$$

We choose the orthogonal gauge where

$$A_0 = 0, \;\; x_i \cdot A_i = 0. \tag{4.18}$$

We will not use the spherically symmetric ansatz for $\{\phi, A_i\}$ in [24], but another equivalent one [37],

$$\begin{aligned} A^a_i(r) &= 2\left(1 - f(r)\right) \frac{\epsilon_{aij} x_j}{g r^2} \\ H(r) &= \frac{h}{\sqrt{2}} \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \;\; h = vL(r) \\ S(r) &= xR(r). \end{aligned} \tag{4.19}$$

where v and x are the Higgs and singlet vevs in the general case (zero and nonzero temperature). Then one can rewrite the field equations (4.17) as

$$\begin{aligned} \zeta^2 \frac{\partial^2}{\partial \zeta^2} f &= 2f(1-f)(1-2f) - \frac{1}{4}\frac{v^2}{\Omega^2}\zeta^2 L^2 (1-f) \\ \frac{\partial}{\partial \zeta}\zeta^2 \frac{\partial}{\partial \zeta} L &= 2L(1-f)^2 + \frac{\zeta^2}{g^2 v \Omega^2}\frac{\partial}{\partial h} V_{eff}(h,S,T)\Big|_{h=vL, S=xR} \\ \frac{\partial}{\partial \zeta}\zeta^2 \frac{\partial}{\partial \zeta} R &= \frac{\zeta^2}{g^2 x \Omega^2}\frac{\partial}{\partial S} V_{eff}(h,S,T)\Big|_{h=vL, S=xR}, \end{aligned} \tag{4.20}$$

[2] There is a similar work done in [98], however there is a difference in the definition in the theory parameters, and also there is an error in the r.h.s of the first equation in (19) in this paper, where the term u^2/v^2 should be corrected as u^2/V^2 according to his notation. In our notation it is the term v^2/Ω^2 in (4.20).

where $\zeta = g\Omega r$, where the parameter Ω can take any non-vanishing value of mass dimension one (for example v, x or $\sqrt{v^2 + x^2}$); and the energy functional is then given by

$$\begin{aligned} E_{Sp} &= \frac{4\pi\Omega}{g} \int_0^{+\infty} d\zeta \left\{ 4\left(\frac{\partial}{\partial\zeta} f\right)^2 + \frac{8}{\zeta^2} f^2 (1-f)^2 + \tfrac{1}{2}\frac{v^2}{\Omega^2}\zeta^2 \left(\frac{\partial}{\partial\zeta} L\right)^2 + \frac{v^2}{\Omega^2} L^2 (1-f)^2 \right. \\ &\left. + \tfrac{1}{2}\frac{x^2}{\Omega^2}\zeta^2 \left(\frac{\partial}{\partial\zeta} R\right)^2 + \frac{\zeta^2}{g^2\Omega^4} \left\{ V(vL, xR) - V(v,x) \right\} \right\}, \end{aligned} \tag{4.21}$$

with the boundary conditions (See Appendix C)

$$\begin{array}{llll} \text{for } \zeta \sim 0 & f(0) = 0 & \text{for } \zeta \to \infty & f \to 1 \\ & L(0) = 0 & & L \to 1 \\ & R'(0) = 0 & & R \to 1. \end{array} \tag{4.22}$$

Let us compare the energy functional (4.21) to that of the minimal Standard Model (Eq. (10) in Klinkhamer & Manton in [24]). The difference between these quantities is of course the contribution of the singlet, which contains the kinetic term, the mixing with the standard Higgs, and a contribution to the potential term. However if we compare (4.21) with the same quantity in the MSSM case (eq. (2. 22) in [86]), we find that in the MSSM both Higgs fields, h_1 and h_2, have similar contributions to the sphaleron energy, and its general form remains invariant under $h_1 \leftrightarrow h_2$. However this is not the case for (4.21) if $h \leftrightarrow S$, because of a missing term like $R^2 (1-f)^2$ [3].

For the MSSM sphaleron energy, its form is invariant under $h_1 \leftrightarrow h_2$, and it scales like $\{v_1^2 + v_2^2\}^{\frac{1}{2}}$. For our model ('SM+S'), a similar invariance is absent. Could it nevertheless be that $E_{Sp} \propto \{v^2 + (x - x_0)^2\}^{\frac{1}{2}}$? We will check this in section 4.4. But when comparing (4.21) with the same quantity for the Next-to-Supersymmetric Standard Model (NMSSM) (eq. (2.20) in [100]; after eliminating explicit CP phases), we find no large difference except for what comes from the fact that the NMSSM contains a doublet more than the 'SM+S'; and we remark that the equations of motion and boundary conditions are also similar.

[3] To be more precise, the absence of a coupling term of the singlet to the gauge field is not the only reason to spoil this invariance, but this invariance is absent also in the tree-level potential.

The analytic solution of the system (4.20) is not possible, this should be done numerically. To solve this system numerically, we need to transform it into a system of 6 first order differential equations, and therefore we have a two-point boundary problem. We use the so-called relaxation method to solve it. The details are given in Appendix D.

	A	B	C	D	E	F
λ	0.4000	0.4000	0.5000	0.4150	0.5000	0.5000
λ_S	0.4003	0.4200	0.4100	0.5500	0.6000	0.6000
ω	0.3818	0.2818	0.3818	0.3000	0.4909	0.1636
x/GeV	200	250	350	350	200	300
α/GeV	-38.89	38.89	38.89	194.44	38.89	116.67
ρ/GeV	-272.22	-194.44	-272.22	-300	-272.22	-194.44
m_1/GeV	178.00	204.00	244.74	203.05	222.76	219.81
m_2/GeV	311.92	269.80	333.96	318.80	302.61	323.43
T_c/GeV	141.55	241.34	389.94	270.08	223.54	294.07
$E_{Sp}(0)/GeV$	9618.6	9721.3	9883.3	9726.6	9845.1	9832.4
v_c/T_c	1.680	0.838	0.495	0.386	1.050	0.638
Ω_c/T_c	3.138	1.232	1.436	0.703	1.321	0.942
$E_{Sp}(T_c)/T_c$	64.851	32.980	20.459	13.577	41.540	25.667

Table 4.1: *Representative parameter values and the corresponding values of the scalar masses, critical temperature and different ratios needed for the criterion of a strong first order phase transition.*

As an example, we solve the system (4.20) for some chosen sets of parameters (A, B, C, D, E and F), in the ranges (4.15), and then we can compute the sphaleron energy at any temperature $T < T_c$. These chosen sets of parameter values give the results summarized in Table. 4.1.

It is clear from Table. 4.1 that the case A satisfies both conditions (3.35) and (3.63), the case D does not satisfy either of them, the case B satisfies (3.35) but not (3.63); and the case C satisfies (3.63) but not (3.35). The profiles of the functions f, L

and R for the sets A, B, C, D, E and F are shown in Fig. 4.1.

From Fig. 4.1, we remark that the profile forms of the functions f and L are not different from those in the case of $SU(2)_L$ model in (Fig. 3.4). However the function R behaves differently from f and L, it is not much different from unity; this is due to the Neuman type boundary at $r = 0$. Thus we expect that the contribution of the singlet S should not be as significant as the contribution of the Higgs scalar and the gauge field.

4.3 The EWPT Strength in the 'SM+S' Model

As shown before, in order that the electroweak baryogenesis scenario to be successful, a strong first order EWPT is required. This condition is satisfied in some SM extensions (see Section 3.4). In this section we are interested in the EWPT in the 'SM+S' model.

In Ref. [14], the authors studied the EWPT in this model with some differences in the parameter definitions. They found that the EWPT gets much stronger than in the SM. The reason according to the authors is the presence of cubic terms at tree level in the potential (4.5), which make the barrier between the two minima larger; and therefore distance between them (in the plane $h - S$) becomes larger. Then

$$\Omega\left(T\right)/T = \left\{v^2 + \left(x - x_0\right)^2\right\}^{\frac{1}{2}}/T \geq 1, \tag{4.23}$$

is satisfied easier than (3.35) at the critical temperature T_c. This is shown in Fig. 4.2.

Given that always $\Omega\left(T_c\right) \geq v\left(T_c\right)$, the condition (4.23) leads to a strong first order EWPT easier than without being in conflict with the Higgs mass bound (3.36).

As mentioned before in section 3.5, the condition (4.23) was used in [14], to describe a strong first order EWPT. We use here the model independent condition (3.63) to check whether the EWPT strength gets increased in the 'SM+S' or not, and compare the results against the conditions (3.63) and (4.23).

We take about 3000 randomly chosen sets of parameters λ, λ_S, ω, ρ, α and x in the ranges (4.15), and then compute the critical temperature T_c, the value of $\Omega\left(T_c\right)$; and the sphaleron energy (4.21) at the critical temperature $E_{Sp}(T_c)$. The results are

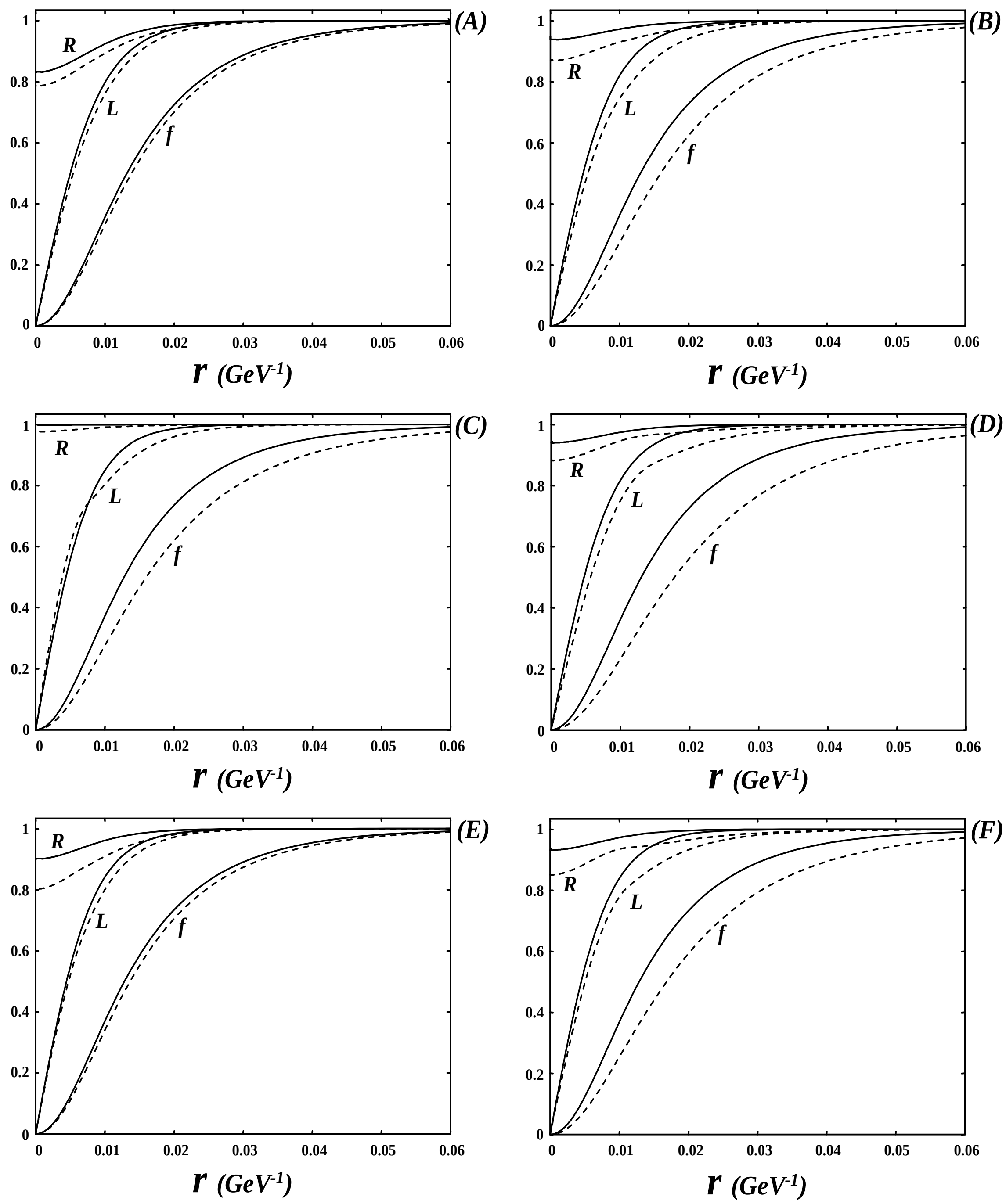

Figure 4.1: *A, B, C, D, E and F represent the profiles of the functions f, L and R for the sets of parameters A, B, C, D, E and F in Table- 4.1 respectively. The continuous lines represent the profiles at zero temperature and the dashed ones represent the profiles of the functions at finite temperature.*

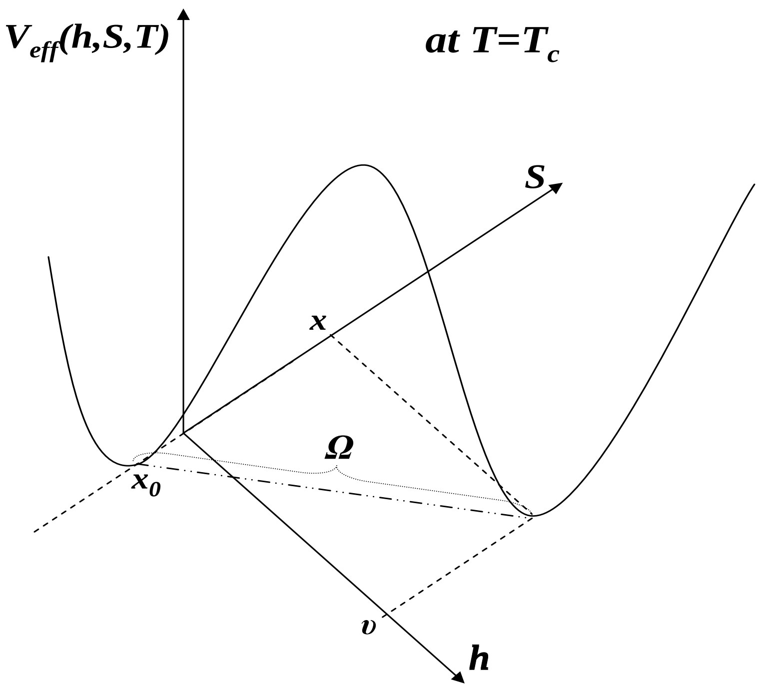

Figure 4.2: *A diagram showing the EWPT at the critical temperature in the 'SM+S' model in the h-S plane. The transition occurs from the false vacuum* $(0, x_0)$ *to the true one* (v, x).

plotted in Fig. 4.3 as functions of the lightest Higgs-Singlet mass m_1.

Comparing the points in Fig. 4.3-a with the curve which represents the Standard Model case, we remark that the addition of a singlet increases, in general, the quantity $E_{Sp}(T_c)/T_c$ which is relevant to the EWPT strength; and that there are even a large number of points above the line $E_{Sp}(T_c)/T_c = 45$.

When comparing the number of points above and below the dash-dotted line in both figures Fig. 4.3-a and Fig. 4.3-b, we find that the first order EWPT is stronger than that of the SM according to both conditions (3.63) and (4.23). However according to the large number of points below the dash-dotted line in *(a)*; and the relatively small number of point below the corresponding line in *(b)* , we conclude that there are a lot of points which satisfy (4.23) but they do not really give a strong first order EWPT, i. e. , they do not satisfy (3.63).

In our work, we have chosen the singlet vev to be of the electroweak scale order $x/v = \mathcal{O}(1)$, therefore the contribution of the singlet S to Ω in (4.23) is significant

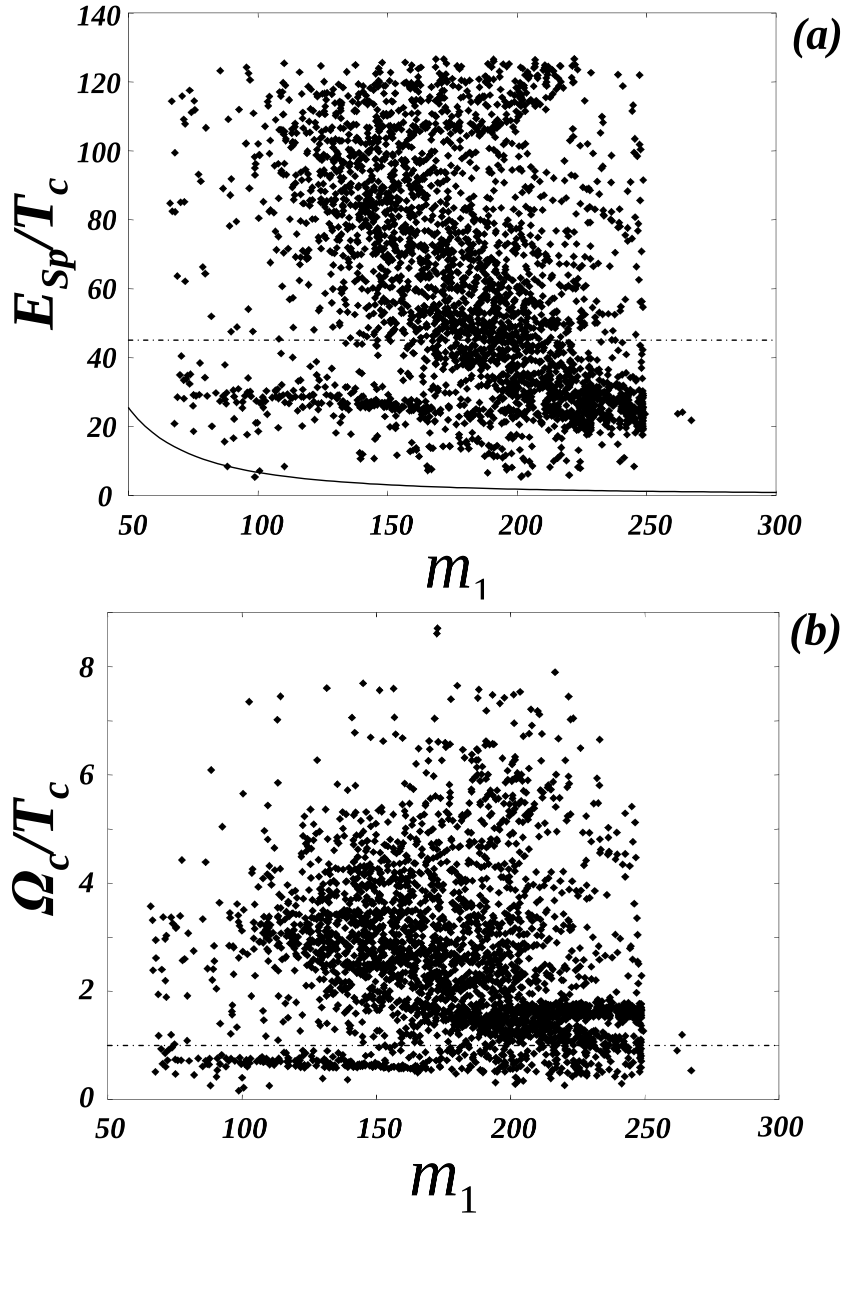

Figure 4.3: *The points above the dash-dotted lines in (a) and (b), the electroweak phase transition is strongly first order according to (3.63) and (4.23), respectively. In (a), the continuous curve represents $E_{SP}(T_c)/T_c$ as a function of the Higgs mass for the case of the Standard Model.*

and can be larger than that of the Higgs field h. However we claimed in the previous section that the contribution of the singlet S to the sphaleron energy (4.21) should be small. If this is true then there is a contradiction between (3.63) and (4.23); and the later condition does not describe a strong first order EWPT.

4.4 The Criterion for a Strong First Order EWPT

In this section we will discuss whether the passage from (3.63) to (4.23) for this model ('SM+S') works as in the SM case, i.e. , the passage from (3.63) to (3.35), (see section 3.4); or not?

The passage from the criterion (3.63), which is model-independent, to (3.35) for the SM case, was based on two assumptions [80]:

(I) The sphaleron energy at $T = 0$, is taken to be 1.87 in units of $4\pi v/g$.

(II) The sphaleron energy $E_{Sp}(T)$ scales like the vev $v(T)$. [4]

It is clear that if the two assumptions are satisfied for our model (i.e, $E_{Sp}(0) = 1.87 \times 4\pi\Omega(0)/g$ and $E_{Sp}(T) \propto \Omega(T)$), then (4.23) will be exactly the strong first order EWPT criterion. Let us now estimate the influence of the deviation from each assumption on spoiling the passage from (3.63) to (4.23) for the 'SM+S' model.

Supposing that the assumption *(II)* is satisfied in our model 'SM+S', i.e.

$$E_{Sp}(T) \propto \Omega(T); \tag{4.24}$$

we plot the ratio $E_{Sp}(0)/(4\pi\Omega(0)/g)$ for 3000 randomly chosen sets of parameters λ, λ_S, ω, ρ, α and x; in the ranges (4.15) as a function of the lightest scalar Higgs mass m_1. The results are shown in Fig. 4.4.

The ratio $E_{Sp}(0)/(4\pi\Omega(0)/g)$ in our model depends on many parameters unlike in the SM case where it depends only on one parameter (the Higgs self-coupling). We remark from Fig. 4.4, that this quantity is significantly different from the value 1.87; and therefore the criterion (4.23) should be relaxed to $\Omega_c/T_c \gtrsim 1 + \delta$, if the assumption *(II)* is still satisfied; and δ describes the deviation from the assumption *(I)*.

[4] As noted in section 3.4, this was verified for the SM [85]; and the MSSM [86].

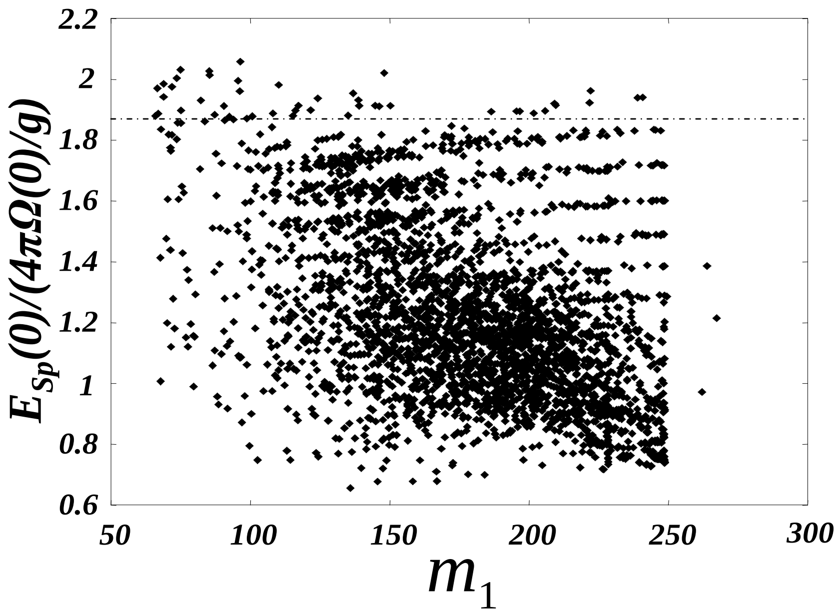

Figure 4.4: *The ratio* $E_{Sp}(0)/(\ 4\pi\Omega(0)/g)$ *for 3000 randomly chosen sets of parameters. The dash-dotted line represents the value* $E_{Sp}(0) = 1.87\times(4\pi\Omega(0)/g)$.

In order to probe the assumption *(II)* for our case, i.e. (4.24), we take the sets A, B, C, D, E and F that were used previously in Table-4.1; and plot the ratios $v\,(T)\,/v\,(0)$, $\Omega\,(T)\,/\Omega\,(0)$ and $E_{Sp}\,(T)\,/E_{Sp}\,(0)$; as functions of temperature, which lies between the critical temperature and another value. The results are shown in Fig. 4.5.

Let us here comment on each case in Fig. 4.5:

Case (A): the ratio $E_{Sp}(T)/E_{Sp}(0)$ is close to both $v(T)/v(0)$ and $\Omega(T)/\Omega(0)$, which is almost 1 at T_c.

Case (B): the ratio $E_{Sp}(T)/E_{Sp}(0)$ is very close to $v(T)/v(0)$; however it is different a little bit from 1 at T_c. At the temperature $T = 204.\ 5\ GeV$, there exist a secondary first order phase transition, it happens on the axis $h = 0$, where the false vacuum $(0, x_0)$ is changed suddenly.

Case (C): the ratio $E_{Sp}(T)/E_{Sp}(0)$ is closer to $v(T)/v(0)$ than to $\Omega(T)/\Omega(0)$; it is also different from 1 at T_c.

Case (D): there is also a secondary first order phase transition around $T \simeq 256$

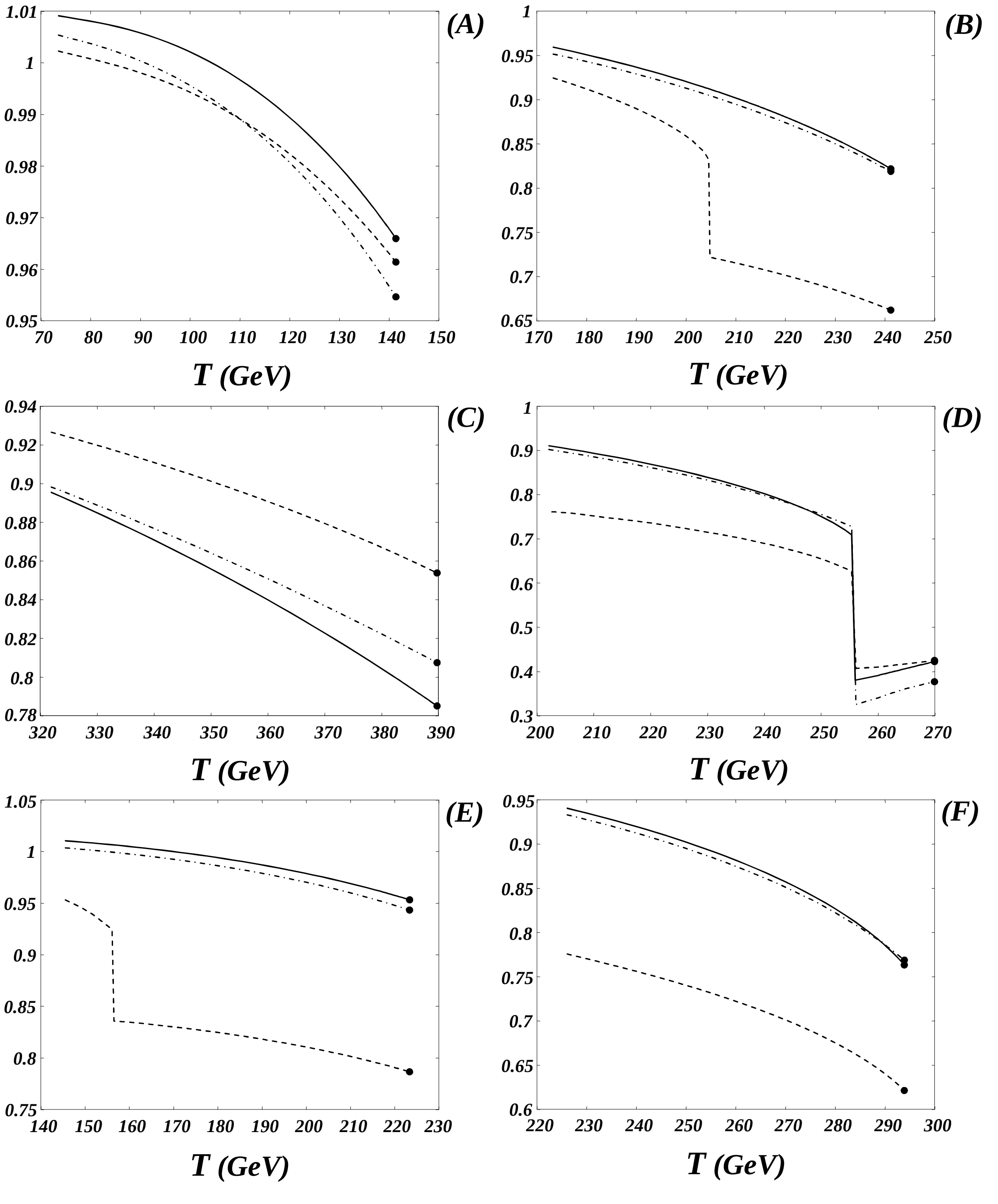

Figure 4.5: *The solid line denotes the ratio* $v(T)/v$*, the dashed one denotes* $\Omega(T)/\Omega$*; and the dot-dashed one denotes* $E_{Sp}(T)/E_{Sp}(0)$*. All the plots end at the critical temperature.*

GeV; where the true vacuum (v, x) changes discontinuously. We cannot call this an electroweak phase transition because the scalar h has already developed its vev. The ratio $E_{Sp}(T)/E_{Sp}(0)$ is still scaling like $v(T)/v(0)$, but significantly different from 1 at T_c.

Case (E): as in the case (B), there is a secondary phase transition in the axis $h = 0$; and the ratio $E_{Sp}(T)/E_{Sp}(0)$ is close to $v(T)/v(0)$; and also very close to 1 at T_c.

Case (F): the ratio $E_{Sp}(T)/E_{Sp}(0)$ is very close to $v(T)/v(0)$, but far from $\Omega(T)/\Omega(0)$; and different a bit from 1 at T_c.

It is clear that $E_{Sp}(T)$ does not scale like $\Omega(T)$, but roughly speaking it scales like $v(T)$; with a little deviation in some cases.

We claimed previously that the contribution of the singlet S to the sphaleron energy is small, and therefore may be this is the reason why $E_{Sp}(T)$ does not behave like $\Omega(T)$; and also does not behave exactly like $v(T)$. In order to estimate the effect of the Singlet field S on the sphaleron energy (4.21), we compute the sphaleron energy (4.21) with replacing the singlet S by its vev x, which we denote $\mathcal{E}_{Sp}(T)$. Then the fifth term in (4.21) disappears and the third equation in (4.17 and 4.20) disappears also; the problem is reduced to a $SU(2)_L$ Higgs-gauge like in section 3. 4 but with a modified potential $V(h)$=$V_{eff}(h, x, T)$. In Fig. 4.6, we show the behavior of the quantity $\{(E_{Sp}(T)$-$\mathcal{E}_{Sp}(T))/E_{Sp}(T)\}$ that can well describe this relative difference; as a function of temperature for the sets A, B, C, D, E and F.

We can remark that the singlet S gives a negative contribution to the value of the sphaleron energy (4.21); and its contribution is larger at higher temperatures. But the contribution size is, generally, negligible:

Case (A): the singlet contribution is less or equal 2.1 %.

Case (B): in this case, it is less than 1.1 %.

Case (C): here it almost zero, it is less than 0.08 %.

Case (D): there are here two different phases. At the first one before the secondary phase transition i.e, $T_c > T > 256$ GeV; the singlet contribution here is significant (between 8$\sim$9 %), this may be due to the smallness of the Higgs doublet vev at this range. While in the second phase $T < 256$ GeV; the singlet contribution,

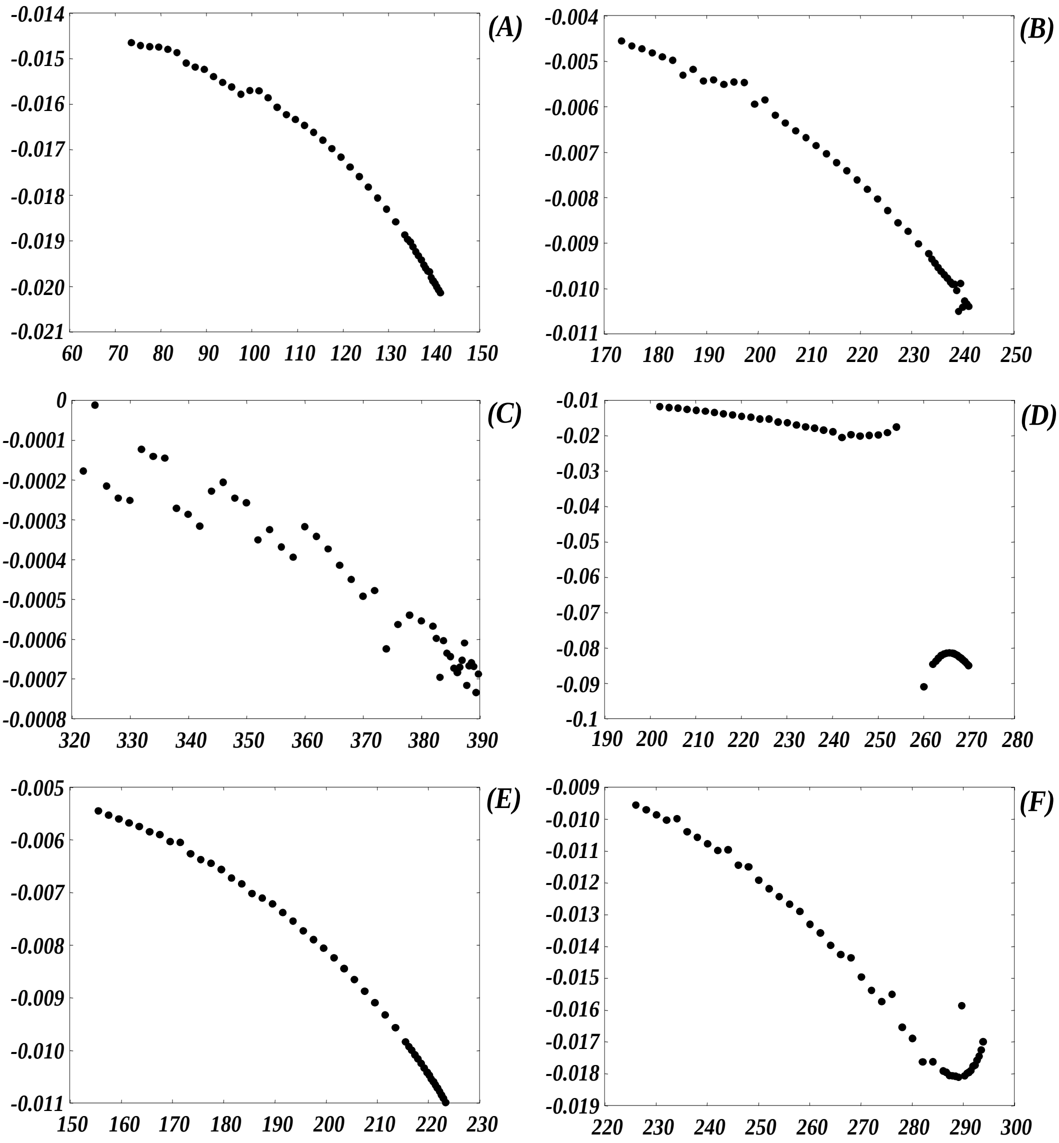

Figure 4.6: *The quantity* $\{E_{Sp}(T)$-$\mathcal{E}_{Sp}(T)\}/E_{Sp}(T)$ *as a function of temperature T for the sets A, B, C, D, E and F. The temperatures are in GeV; and all the plots end at the critical temperature and start from another temperature value.*

as in the other cases, is small and less than 2 %.

Case (E): in this case the singlet contribution is less than 1.1 %.

Case (F): the same remark here; it is less than 1.9 %.

Then in the absence of secondary first order phase transitions, we can neglect the singlet contribution, but in its presence the singlet contribution can be sizeable but not as large as the doublet or gauge field contributions.

To justify this picture, we take again 3000 random sets of parameters and plot $E_{Sp}(T_c)/T_c$ as a function of Ω_c/T_c in Fig. 4.7.

Since there exist too many points in the region $(E_{Sp}(T_c)/T_c \leq 45 \cap \Omega(T_c)/T_c \geq 1)$, $\Omega(T_c)/T_c \geq 1$ is not the definition of a strong first order EWPT. However (4.23) is satisfied for all points that give really a strong first order EWPT except for 2 points due to the existence of secondary first order phase transitions. Then we are now sure that (4.23) does not describe a strong first order EWPT.

In the sphaleron transitions, the singlet S has no relation to lepton or baryon number breaking phenomena. It does not couple to fermions or gauge bosons; it is just a compensating field in the field equations; (4.17) and (4.20); and its effect on the sphaleron transition is negligible as shown above. Then we claim that only the Higgs doublet vev is relevant for the phase transition strength.

We take 3000 random sets of parameters used previously, and plot $E_{Sp}(T_c)/T_c$ as a function of v_c/T_c in Fig. 4.8.

It is clear that $E_{Sp}(T_c)/T_c$ scales exactly[5] like v_c/T_c except for some points, and (3.35) can describe the strong first order EWPT criterion for most of the points. Then when studying the EWPT in models with a gauge singlet, one should treat the problem as in the SM case (in case of one doublet) with replacing the singlet by its vev; and look for the Higgs vev in the path $\partial V_{eff}(h,S)/\partial S\big|_{S=x} = 0$; whether it is larger than the critical temperature i.e, $v_c/T_c \geq 1$?

With this modified potential $\mathcal{V}_{eff}(h) = V_{eff}(h,S)\big|_{S=x}$; the EWPT can be obtained easily as done by the authors in [65, 94].

[5]Except some points due to the existence of secondary first order phase transitions; or due to the significant singlet contribution to the sphaleron energy; especially for smaller Higgs vev values.

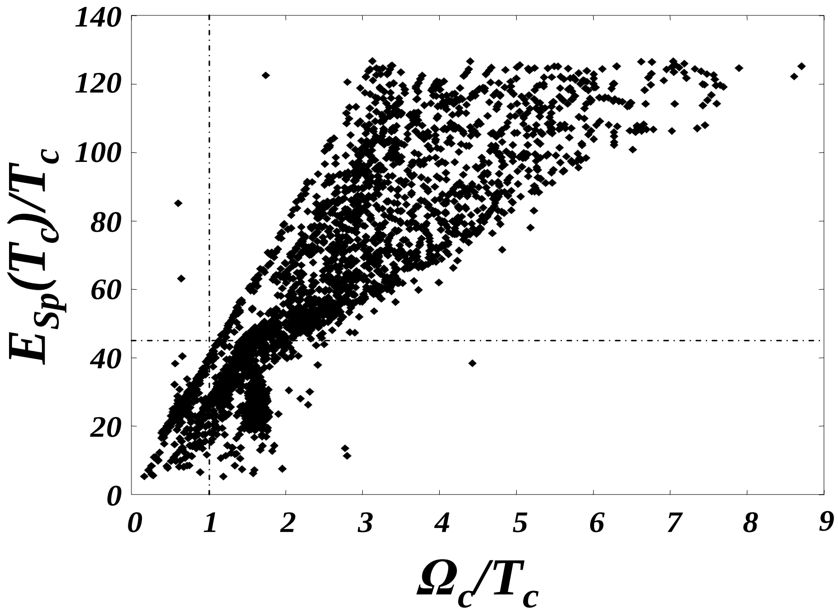

Figure 4.7: *$E_{Sp}(T_c)/T_c$ vs Ω_c/T_c for 3000 randomly chosen sets of parameters. The horizontal line represents the value $E_{Sp}(T_c)/T_c$=45; and the vertical one represents Ω_c/T_c=1.*

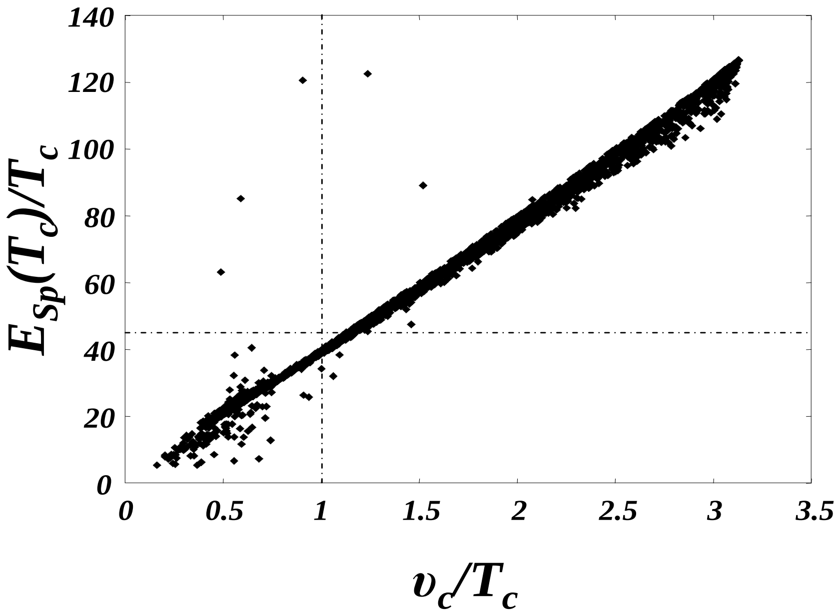

Figure 4.8: *$E_{Sp}(T_c)/T_c$ vs v_c/T_c for 3000 randomly chosen sets of parameters. The horizontal line represents the value $E_{Sp}(T_c)/T_c$=45; and the vertical one represents v_c/T_c=1.*

Chapter 5

Summary

The EWPT was investigated in the SM extended by a gauge singlet. We focused on the criterion of a strong first order phase transition in this model.

For a successful baryogenesis at the electroweak scale the Sakharov conditions have to be fulfilled. The third condition necessitates a rapid (strong) first order phase transition, since the expansion of the universe, $H \simeq T^2/M_{Pl}$, is very slow compared to the relevant interactions that violate B number, $\Gamma \sim T$, and is not sufficient to cause a deviation from equilibrium without a phase transition.

In the first order EWPT case, the field changes its value from the false vacuum to the true one through tunneling due to the existing barrier between the two minima; and the tunneling proceeds via bubble nucleation. The baryon asymmetry generation has to occur close to the moving bubble walls.

During the strong first order EWPT, the B violating processes are in thermal equilibrium outside the bubble, but exponentially suppressed inside in order to save any generated baryon asymmetry. Thus a strong first order EWPT is not only needed to create the baryon number asymmetry at the electroweak scale, but also to preserve it from any washout effect after the transition is completed.

A model independent criterion for a strong first EWPT was found in [23]:

$$E_{Sp}(T_c)/T_c \geq 45,$$

in order to save at least 10^{-5} of the violated B density number. This criterion is translated for the case of SM (and also MSSM) to a simpler condition $v_c/T_c \geq 1$.

This condition leads to an unacceptable upper bound on the Higgs scalar $m_h < 42$ GeV and therefore a departure from thermal equilibrium can not be obtained within the SM.

The natural way is to go beyond SM in order to have a departure from thermal equilibrium. One of the economical extensions is to add a gauge singlet to the SM. In this case the singlet can induce cubic terms in the tree-level potential, which enhances the EWPT strength. Depending on the theory parameters, this gauge singlet may develop a condensate. When this is the case, the criterion of a strong first order EWPT, $v_c/T_c \geq 1$, is modified in many references [14, 15], by replacing v with the distance between the two degenerate minima at the critical temperature in the field space, i.e $\Omega = \left\{v^2 + (x - x_0)^2\right\}^{1/2}$; where x and x_0 are the singlet values in the true and false vacua respectively.

The contribution of the singlet vev to Ω can be large with respect to the contribution of v if x is large enough or x_0 is shifted too much into a negative value; and therefore $\Omega(T) \geq T$ can be easily obtained at the critical temperature. However the singlet does not look relevant for the Sphaleron processes, so how could it enhance significantly the EWPT strength?

In this work, we examined this modified criterion for the Standard Model with a real singlet using the model-independent criterion.

Firstly, we introduced this model and we discussed the physical constraints on it. Then we found the effective potential at zero- and finite-temperature, which is the relevant tool to study the EWPT. In order to study the EWPT strength using the model-independent criterion, we found the sphaleron solution for this model.

We found that the EWPT gets stronger even for Higgs masses larger than 100 GeV and that this model does not suffer from the severe Higgs mass bound (3.36). The same conclusion was found by many authors for the same model [14], where they used the modified criterion. However, we remarked that a sizeable number of the parameters satisfy the modified criterion but do not really give a strong first order EWPT. This allowed us to conclude that the 'modified criterion' is not the criterion that describes a strong first order EWPT.

In order to understand why this modified criterion is not viable in this case, we returned back to the SM to see how the passage from the model-independent criterion to the simpler one proceeds. We found that the two assumptions needed for the passage to the simpler criterion are not fulfilled, in general, in our model 'SM+S':

∘ The sphaleron energy at zero temperature is different from the value1.87 in units of $(4\pi\Omega/g)$.

∘ The sphaleron energy at finite temperature does not scale like $\Omega\,(T)$.

We guess that the reason for this is that the singlet does not couple to the gauge field, and the missing of some contributions to the sphaleron energy like $R^2\,(1-f)^2$ in (4.21), can spoil the scaling law, $E_{Sp}\,(T) \propto \Omega\,(T)$. This can be compared with the case of the MSSM, where this scaling law does work; and the general form of the sphaleron energy is invariant under $h_1 \leftrightarrow h_2$. The fact that the singlet does couple only to the Higgs doublet leads to a singlet profile in the sphaleron configuration of a Neumann type at $r = 0$, which makes the singlet contribution too small. Another important remark is that the possibility of secondary first order phase transitions can, sometimes, spoil this scaling law.

As a conclusion, we can say that the condition $\Omega_c/T_c \geq 1$ is not valid as a strongly first order phase transition criterion. But the usual condition $v_c/T_c \geq 1$ is still the viable one, which can describe the strong first order phase transition for the majority of the physically allowed parameters as stated in Fig. 4.8. Moreover, this can be satisfied even for Higgs masses in excess of 100 GeV unlike in the Standard Model.

Then in such a model where the singlets couple only the Higgs doublets, it is convenient to study the EWPT within an effective model that contains only the doublets, while the singlets are replaced by their vevs.

We expect similar conclusions for models like the Next-to-Minimal Supersymmetric Standard Model (NMSSM), where in this model the singlet couples only to the two Higgs doublets; and its boundary conditions at $r = 0$ of Neumann type in the sphaleron configuration. Then the criterion for a strong first order EWPT is $\left\{v_1^2 + v_2^2\right\}^{\frac{1}{2}}/T \geq 1$ at the critical temperature, instead of $\left\{v_1^2 + v_2^2 + (x - x_0)^2\right\}^{\frac{1}{2}}/T \geq 1$.

Appendix A

Some Notes about Standard Cosmology

The standard cosmology is based on the following homogeneous and isotropic space-time metric, the so-called Robertson-Walker metric [1]

$$ds^2 = dt^2 - a(t)^2 \left\{ \frac{dr^2}{1-kr^2} + r^2 \left(d\theta^2 + \sin^2\theta \, d\phi^2 \right) \right\}, \tag{A.1}$$

where *a(t)* is the 'scale factor' of the expanding universe, t is the cosmic time, r is a dimensionless radius; and (t, r, θ, ϕ) are called the comoving coordinates. The Universe obeys the so-called Einstein equation that relates the energy-momentum tensor to the space-time curvature. For a perfect fluid with the above metric, the energy-momentum tensor is $T_{\mu\nu} = diag(\rho, p, p, p)$, where $\rho(t)$ and $p(t)$ denote the energy density and the pressure, respectively.

The Einstein field equations, in the general case, are:

$$G_{\mu\nu} = -\frac{8\pi T_{\mu\nu}}{M_{Pl}^2}, \tag{A.2}$$

[1] And also called the Friedmann-Lemaitre-Robertson-Walker (FLRW) metric. Historically, Friedmann presented the dynamical equation (A.4) only for the case of pressureless dust, while Lemaitre extended it to include the case of radiation and also wrote down the conservation equation (A.6).

with

$$\begin{aligned} G_{\mu\nu} &= R_{\mu\nu} - \tfrac{1}{2} g_{\mu\nu} R_c \\ R_{\mu\nu} &= g^{\alpha\beta} R_{\alpha\mu\beta\nu} \\ R_c &= g^{\mu\nu} R_{\mu\nu}, \end{aligned} \tag{A.3}$$

where $G_{\mu\nu}$ is the Einstein tensor, M_{Pl} is the Planck mass, $g_{\mu\nu}$ is the space-time metric, $R_{\mu\nu}$ is called Ricci tensor, R_c is the Ricci scalar; and $R_{\alpha\mu\beta\nu}$ is the Riemann tensor which represents the space-time curvature. In the case of FLRW Universe, the Einstein equations, can be expressed as:

$$\left(\frac{\dot{a}}{a}\right)^2 + \frac{k}{a^2} = \frac{8\pi G}{3}\rho \tag{A.4}$$

$$\ddot{a} = -\frac{4\pi\rho}{3M_{Pl}^2}\left(\rho + 3p\right) a. \tag{A.5}$$

The equation (A.4) is called Friedmann equation, it describes the expansion of the Universe, while the acceleration of the Universe expansion is described by (A.5). The energy-momentum conservation, $T^{\mu\nu}{}_{;\nu} = 0$, leads to the equation:

$$\frac{d\left(\rho a^3\right)}{dt} = -p\frac{d}{dt}\left(a^3\right), \tag{A.6}$$

which is nothing but the first law of thermodynamics; and it describes the dilution of the energy density due to the expansion.

Notice that the above relation (A.6) holds for each decoupled component of the energy density ρ_i due to the fact that the energy-momentum conservation $T_{i\ ;\nu}^{\mu\nu} = 0$ is satisfied separately for each component i.

In terms of Hubble parameter $H = \dot{a}/a$, reduced Planck mass $M_{Pl} \equiv (8\pi G)^{-1/2} = 2.\ 4 \times 10^{18}\ GeV$, critical energy density $\rho_{crit} \equiv 3M_{Pl}^2 H^2$ and density parameters $\Omega_i \equiv \rho_i/\rho_{crit}$, Friedmann equation (A.4) can be rewritten as

$$\Omega_R + \Omega_M + \Omega_V = \frac{\rho_R + \rho_M + \rho_V}{\rho_{crit}} = 1 + \frac{k}{a^2 H^2}, \tag{A.7}$$

where the subscripts R, M and V denote: radiation, matter and vacuum[2] respectively. During each era, the universe evolution was driven by a dominant compo-

[2]Using *'vacuum'* might not be correct, since the 'dark energy' component can be a dynamical value, like 'quintessence', instead of a cosmological constant.

nent; and the equation of state and the energy density behavior are summarized as

$$\begin{array}{ccc} \text{Radiation} & p = \rho/3 & \rho \propto a^{-4} \\ \text{Matter} & p \simeq 0 & \rho \propto a^{-3} \\ \text{Vacuum} & p = -\rho & \rho = const. \end{array}$$

The present[3] Hubble expansion rate is $H_0 = 100\,h\,[\mathrm{km\,s^{-1}\,Mpc^{-1}}]$, where $h \simeq 0.7$, and hence the critical energy density is $\rho_{crit}(t_0) = 3M_{Pl}^2 H_0^2 = 1.9\,h^2 \times 10^{-29}\,[\mathrm{g\,cm^{-3}}]$.

In terms of density parameters Ω_i, the total energy density is given by $\Omega(t_0) \simeq 1$, which includes:

• Radiation: $\Omega_R(t_0) = (2.47 + 0.56 N_\nu^{eff})\,h^{-2} \times 10^{-5}$, where N_ν^{eff} is the number of the generations of neutrinos which are still relativistic today.

• Matter: $\Omega_M(t_0) \simeq 0.3$, it contains baryons with the fraction $\Omega_B(t_0) \simeq 0.02h^{-2}$. Most of the matter is dominated by the so-called 'Cold Dark Matter', whose nature is still a big puzzle in cosmology today.

• Vacuum Energy: $\Omega_V(t_0) \simeq 0.7$. Its nature is also still a puzzle in cosmology, where there are many candidates: a cosmological constant, a slow-roll scalar field, or ...?

The cosmic evolution can be understood in a way that after a certain time in the very early universe, where the temperature was very huge, all particles are expected to be in thermal equilibrium because of their rapid interactions whose rates are faster than the universe expansion due to the increasing density and temperature. The species i decouple from this hot soup at the time when the expansion interaction rate H equals its rate Γ_i due to the universe expansion and the decreasing temperature.

Without going further in details about thermodynamics of the universe in different epochs, we give a brief history of the universe:

• $T \geq 10^{19}$ GeV. Nothing is known about this epoch except that the gravitational interactions are strong.

• $T \sim 10^{16}$ GeV. It is thought that at this scale, some GUTs ($SU(5)$, $SO(10)$, E_6 ...?) break down into the standard model gauge group, $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$;

[3]In general, the script 0 refers to the values of the quantities at the present time $t = t_0$.

may be via a chain of phase transitions. Little is known about this transition.

• $T \sim 10^2$ GeV. The Standard Model gauge symmetry breaks into $SU(3)_C \otimes U_{EM}(1)$.

• $T \sim 10^1 - 10^3$ GeV. Weakly interacting dark matter candidates with GeV–TeV scale masses freeze-out [101].

• $T \sim 0.3$ GeV. The QCD phase transition occurs, which drives the confinement of quarks and gluons into hadrons.

• $T \sim 1$ MeV. Neutrino decoupling occurs.

• $T \sim 100$ keV. Nucleosynthesis: protons and neutrons fuse into light elements (D, 3He, 4He, Li). The standard Big Bang nucleosynthesis (BBN) provides stringent constraints on the Big Bang theory.

• $T \sim 1$ eV. The matter density becomes equal to that of the radiation, allowing for the formation of structure to begin.

• $T \sim 0.4$ eV. Photon decoupling produces the cosmic background radiation (CMB).

• $T = 2.7\ {}^0$K $\sim 10^{-4}$ eV. Today.

In the following diagram, we show the evolution of the universe and the corresponding effective theories of particle physics describing these epochs:

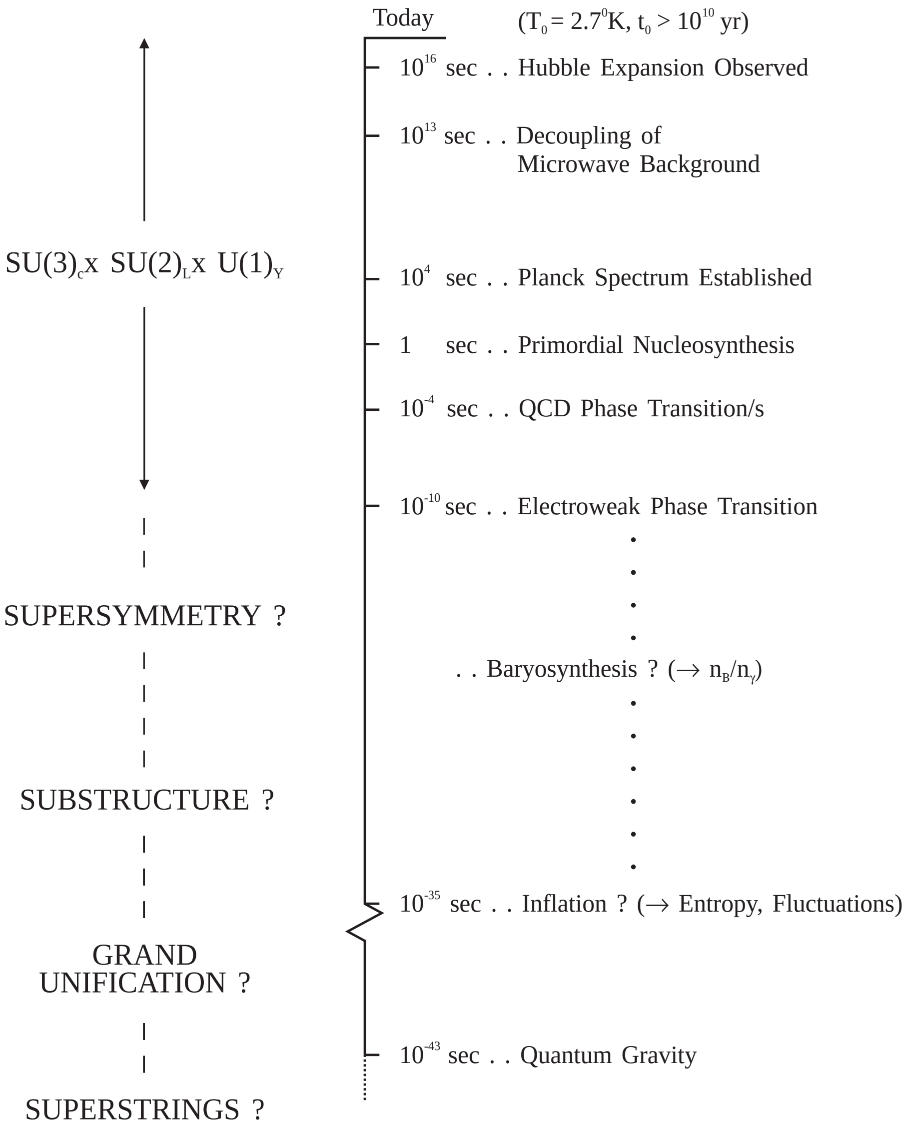

Figure A.1: *The History of the Universe. This figure is taken from S. Sarkar in [31].*

Appendix B

The Bosonic Thermal Masses

In this appendix, we will show how the thermal corrections to the bosonic masses are computed, and will give the results for our model. The thermal mass of a bosonic field is given by

$$M_i^2\left(h,S,T\right) = m_i^2\left(h,S\right) + \Pi_i\left(h,S,T\right), \tag{B.1}$$

where $\Pi_i\left(h,S,T\right)$ is the self-energy estimated at finite temperature. This quantity is, in general, the contributions of many diagrams. For our model, there are three kinds of one-loop contributions, which are shown if Fig. B.1. The diagram (B.1a) comes from the quartic interactions, (B.1-b) comes from the interaction with fermionic fields; and (B.1c) from the cubic interactions with scalars.

The contribution of the diagrams (B.1-a), (B.1-b) and (B.1-c) are proportional to the integrals $I_b(m^2)$, $I_f(m^2)$ and $K_b(m_1^2, m_2^2)$, respectively. Therefore one should know their values, which are given by [102]:

$$\begin{aligned} I_b\left(m^2\right) &= T\mu^{2\epsilon}\sum_{n=-\infty}^{\infty}\int\frac{d^{3-2\epsilon}k}{(2\pi)^{3-2\epsilon}}\frac{1}{k^2+(2\pi nT)^2+m^2} \\ &\simeq -\frac{mT}{4\pi}+\frac{T^2}{12}-\frac{m^2}{16\pi^2}\left[\frac{1}{\epsilon}+\gamma_E-\log\left(\frac{4\pi T^2}{\mu^2}\right)\right], \end{aligned} \tag{B.2}$$

$$\begin{aligned} I_f\left(m^2\right) &= T\mu^{2\epsilon}\sum_{n=-\infty}^{\infty}\int\frac{d^{3-2\epsilon}k}{(2\pi)^{3-2\epsilon}}\frac{1}{k^2+((2n+1)\,\pi T)^2+m^2} \\ &\simeq -\frac{T^2}{24}-\frac{m^2}{16\pi^2}\left[\frac{1}{\epsilon}+\gamma_E-\log\left(\frac{4\pi T^2}{\mu^2}\right)\right], \end{aligned} \tag{B.3}$$

$$\begin{aligned}
K_b\left(m_1^2, m_2^2\right) &= T\mu^{2\epsilon} \sum_{n=-\infty}^{\infty} \int \frac{d^{3-2\epsilon}k}{(2\pi)^{3-2\epsilon}} \frac{1}{\left\{k^2 + (2\pi n T)^2 + m_1^2\right\} \left\{\left(\vec{k} - \vec{p}\right)^2 + (2\pi n T)^2 + m_2^2\right\}} \\
&= \int_0^1 \left\{-\frac{\partial}{\partial \alpha(x)} I_b(\alpha(x))\right\} dx; \quad \alpha(x) = -x(x+1)p^2 + (1-x)m_1^2 + xm_2^2 \\
&\simeq -\frac{T}{8\pi} \int_0^1 \frac{dx}{\sqrt{\alpha(x)}} - \frac{1}{16\pi^2}\left[\frac{1}{\epsilon} + \gamma_E - \log\left(\frac{4\pi T^2}{\mu^2}\right)\right]. \qquad \text{(B.4)}
\end{aligned}$$

It is clear from the integrals (B.2) and (B.3), that the dominant contribution is T^2, therefore we will neglect the rest, and also any term containing (B.4) will be neglected. For example the correction to the pure Singlet mass m_{SS}^2, is given by:

$$\begin{aligned}
\Pi_{SS} &= 3\lambda_S I_b\left(m_{SS}^2\right) + 2\omega I_b\left(m_{hh}^2\right) + 3\cdot 2\omega I_b\left(m_\chi^2\right) + 2\left\{\alpha - 3\lambda_S x\right\}^2 K_b\left(m_{SS}^2, m_{SS}^2\right) \\
&\quad +2\left\{\omega x + \frac{\rho}{2}\right\}^2 K_b\left(m_{hh}^2, m_{hh}^2\right) + 3\cdot 2\left\{\omega x + \frac{\rho}{2}\right\}^2 K_b\left(m_\chi^2, m_\chi^2\right) \\
&\simeq 3\cdot\lambda_S\cdot\frac{T^2}{12} + 2\cdot\omega\cdot\frac{T^2}{12} + 3\cdot 2\cdot\omega\cdot\frac{T^2}{12}. \qquad \text{(B.5)}
\end{aligned}$$

Here, only the contributions of the first three terms are relevant. The situation is different for gauge bosons W and Z, because only the longitudinal components get a thermal mass correction and the transversal ones will not.

In our model, the bosonic self energies are given by:

$$\begin{aligned}
\Pi_W^L &= \frac{11}{6} g^2 T^2 \\
\Pi_Z^L &= \frac{11}{6} g^2 T^2 \\
\Pi_W^T &= \Pi_Z^T = 0 \\
\Pi_\chi &= \left(\frac{g^2}{4} + \frac{\lambda}{2} + \frac{y_t^2}{4} + \frac{\omega}{6}\right) T^2 \\
\Pi_{hh} &= \left(\frac{g^2}{4} + \frac{3\lambda}{4} + \frac{y_t^2}{4} + \frac{\omega}{6}\right) T^2 \\
\Pi_{SS} &= \left(\frac{\lambda_S}{4} + \frac{2\omega}{3}\right) T^2 \\
\Pi_{hS} &\simeq 0, \qquad \text{(B.6)}
\end{aligned}$$

and the thermal masses are given by:

$$M_{WL}^2(h,T) = m_W^2(h) + \Pi_W^L \tag{B.7}$$

$$M_{ZL}^2(h,T) = m_Z^2(h) + \Pi_Z^L \tag{B.8}$$

$$M_{WT}^2(h,T) = m_W^2(h) \tag{B.9}$$

$$M_{ZT}^2(h,T) = m_Z^2(h) \tag{B.10}$$

$$M_\chi^2(h,S,T) = m_\chi^2(h,S) + \Pi_\chi; \tag{B.11}$$

the masses of Higgs-Singlets are

$$\begin{aligned} M_{hh}^2(h,S,T) &= m_{hh}^2(h,S) + \Pi_{hh} \\ M_{SS}^2(h,S,T) &= m_{SS}^2(h,S) + \Pi_{SS} \\ M_{hS}^2(h,S,T) &\simeq m_{hS}^2(h,S), \end{aligned} \tag{B.12}$$

and the physical masses are

$$\begin{aligned} M_{1,2}^2(h,S,T) \simeq \frac{1}{2}\Big\{ & M_{hh}^2(h,S,T) + M_{SS}^2(h,S,T) \\ & \mp\sqrt{\left(M_{hh}^2(h,S,T) - M_{SS}^2(h,S,T)\right)^2 + 4m_{hS}^2(h,S)}\Big\} \end{aligned} \tag{B.13}$$

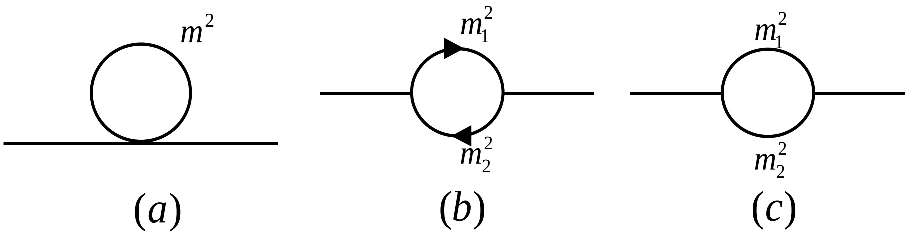

Figure B.1: *The one-loop contributions to the effective thermal masses.*

Appendix C

The Boundary conditions

To find the boundary conditions of (4.20), one should take into account that the energy functional (4.21) should be finite. It is clear that in order for the contributions of the second and fourth term in (4.21) to be finite, f must go to unity in the limit $\zeta \to \infty$. According to the sphaleron definition, scalars go to their vacuum at infinity, i. e. $L, R \to 1$ when $\zeta \to \infty$, which makes the last term contribution to (4.21) finite. Thus one can write all the functions as $1 - c_i \exp\{-d_i\zeta\}$, and find the values of c_i and d_i by inserting this behavior into the differential equations (4.20).

In the limit $\zeta \to 0$, let us assume that the functions f, L and R have the profiles

$$\begin{aligned} f(\zeta) &\sim \zeta^{n_f} \\ L(\zeta) &\sim c_1 + \zeta^{n_L} \\ R(\zeta) &\sim c_2 + \zeta^{n_R}, \end{aligned} \tag{C.1}$$

where n_f, n_L and n_R are some positive constants. In this limit (4.20) can be approximated as

$$\begin{aligned} \frac{\partial^2}{\partial\zeta^2} f &\simeq \frac{2}{\zeta^2} f - \tfrac{1}{4}\frac{v^2}{\Omega^2} L^2 \\ \frac{\partial^2}{\partial\zeta^2} L &\simeq -\frac{2}{\zeta}\frac{\partial}{\partial\zeta} L + \frac{2}{\zeta^2} L \\ \frac{\partial^2}{\partial\zeta^2} R &\simeq -\frac{2}{\zeta}\frac{\partial}{\partial\zeta} R + \frac{1}{g^2 x \Omega^2} \left.\frac{\partial V_{eff}\,(h, S, T)}{\partial S}\right|_{h=vL, S=xR}. \end{aligned} \tag{C.2}$$

From the second equation in (C.2) one can easily conclude that $L \sim \zeta$ or $\sim \zeta^{-2}$. However the second choice makes the energy functional integral (4.21) divergent, thus $L \sim \zeta$ or $\{c_1 = 0,\ n_L = 1\}$. Using this result, one can conclude from first equation in (C.2) that $f \sim \zeta^2$. However the situation is different for the last equation in (C.2), then one can make

$$\frac{1}{g^2 x \Omega^2} \frac{\partial V_{eff}(h, S, T)}{\partial S}\bigg|_{\substack{h=vL \\ S=xR}} \sim a\zeta^2 + \left\{A + b\zeta^2\right\} R(\zeta) + BR^2(\zeta) + CR^3(\zeta)\,, \quad \text{(C.3)}$$

then inserting (C.1) in (C.3), one finds that the only possibilities are $n_R = -1$ and $n_R = 2$, where the first choice is excluded in order that the energy functional integral (4.21) be convergent, thus $R \sim a + b\zeta^2$. Therefore at $\zeta = 0$, R satisfies the boundary condition of Neumann type, while f and L satisfy those of Dirichlet type. The boundary conditions are summarized in (4.22).

Appendix D

The Relaxation Method

The relaxation method is a good method to solve two-point boundary value problems numerically. In this appendix, I give a brief review for this technique. Let us suppose the following system of N first-order ordinary differential equations:

$$\frac{d}{dt}y_i = F_i(\mathbf{y}, t) \qquad i = 1, N, \tag{D.1}$$

with the conditions at the boundaries t=t_{in} and t=t_{fi} are:

$$E_i(\mathbf{y}(t_{in}), t_{in}) = 0, \qquad i = 1, n \tag{D.2}$$

$$E_i(\mathbf{y}(t_{fi}), t_{fi}) = 0, \qquad i = n+1, N. \tag{D.3}$$

We divide the interval $[t_{in}, t_{fi}]$ into M equal parts, and therefore we have $M + 1$ points (with $t_1 = t_{in}$ and $t_{M+1} = t_{fi}$). The length of every interval is $h = (t_{fi} - t_{in})/M$. Using the approximation between the two point k, $k - 1$:

$$\begin{aligned} \frac{dy_i(t_k)}{dt} &\simeq (y_{i,k} - y_{i,k-1})/h \\ y_i(t_k) &\simeq (y_{i,k} + y_{i,k-1})/2, \end{aligned}$$

the system (D.1) can be approximated into a finite difference equations system as

$$(y_{i,k} - y_{i,k-1})/h = F_i\left(\tfrac{1}{2}(\mathbf{y}_k + \mathbf{y}_{k-1}), \tfrac{1}{2}(t_k + t_{k-1})\right). \tag{D.4}$$

We call $E_{i,k}$ the quantities

$$E_{i,k} = y_{i,k} - y_{i,k-1} - h \times F_i\left(\tfrac{1}{2}(\mathbf{y}_k + \mathbf{y}_{k-1}), \tfrac{1}{2}(t_k + t_{k-1})\right) \qquad k = 2, M \tag{D.5}$$

At the first boundary we call (D.2) $E_{i,1} = 0$, $(i = 1, n)$; and at the last boundary, we call (D.3) $E_{i,M+1} = 0$, $(i = n+1, M)$, totally we have $N \times M$ E's elements. It is clear that $\mathbf{y}$ represent the solutions of (D.1) only when all the elements E's vanish at every point k.

As usually done, we will start from trial functions $y_{i,k}$ and look for some corrections $\Delta y_{i,k}$ where $y_{i,k} + \Delta y_{i,k}$ make all E_i's vanishing for all points k, i. e. $y_{i,k} + \Delta y_{i,k}$ are the needed solutions. Assuming that the trial functions are chosen to be close enough to the real solutions, then corrections $\Delta y_{i,k}$ are small enough to expand around the trials $y_{i,k}$, then we can write

$$\begin{aligned}
E_{i,k}(\mathbf{y}_k + \Delta\mathbf{y}_k) &\simeq E_{i,k}(\mathbf{y}_k) + \tfrac{\partial E_{i,k}}{\partial y_{j,k}}\Delta y_{j,k} && k = 1 \\
E_{i,k}(\mathbf{y}_k + \Delta\mathbf{y}_k, \mathbf{y}_{k-1} + \Delta\mathbf{y}_{k-1}) &\simeq E_{i,k}(\mathbf{y}_k, \mathbf{y}_{k-1}) + \tfrac{\partial E_{i,k}}{\partial y_{j,k}}\Delta y_{j,k} + \tfrac{\partial E_{i,k}}{\partial y_{j,k-1}}\Delta y_{j,k-1} && k = 2, M \\
E_{i,k}(\mathbf{y}_{k-1} + \Delta\mathbf{y}_{k-1}) &\simeq E_{i,k}(\mathbf{y}_{k-1}) + \tfrac{\partial E_{i,k}}{\partial y_{j,k-1}}\Delta y_{j,k-1} && k = M+1, \quad \text{(D.6)}
\end{aligned}$$

since $y_{i,k} + \Delta y_{i,k}$ is the solution, one can write

$$\begin{aligned}
\sum_{j=1,N} S_{i,j+N} \times \Delta y_{j,1} &= -E_{i,1} && k = 1 \\
\sum_{j=1,N} S_{i,j+N} \times \Delta y_{j,k} + \sum_{j=1,N} S_{i,j} \times \Delta y_{j,k-1} &= -E_{i,k} && k = 2, M \\
\sum_{j=1,N} S_{i,j} \times \Delta y_{j,M} &= -E_{i,M+1} && k = M+1, \quad \text{(D.7)}
\end{aligned}$$

where

$$\begin{aligned}
S_{i,j+N} &= \frac{\partial E_{i,k}}{\partial y_{j,k}} && i = 1, n;\ j = 1, N;\ k = 1 \\
S_{i,j} &= \frac{\partial E_{i,k}}{\partial y_{j,k-1}},\ S_{i,j+N} = \frac{\partial E_{i,k}}{\partial y_{j,k}} && i = 1, N;\ j = 1, N;\ k = 2, M \\
S_{i,j} &= \frac{\partial E_{i,k}}{\partial y_{j,k-1}} && i = n+1, N;\ j = 1, N;\ k = M+1. \quad \text{(D.8)}
\end{aligned}$$

Here the element $E_{i,k}$ depends only on $y_{i,k}$, $y_{i,k-1}$, t_k and t_{k-1}, therefore we do not need to solve the $(N \times M)$ system of equations, or to diagonalize the whole matrix $(N \times M)\times(N \times M)$, we need only to start with sub-blocks at each point k, and sub-block after sub-block (See Fig. D.1). This means that the first subsystem is of

```
X X X X X                                V   B
X X X X X                                V   B
X X X X X                                V   B
X X X X X X X X X X                      V   B
X X X X X X X X X X                      V   B
X X X X X X X X X X                      V   B
X X X X X X X X X X                      V   B
X X X X X X X X X X                      V   B
          X X X X X X X X X X            V   B
          X X X X X X X X X X            V   B
          X X X X X X X X X X            V   B
          X X X X X X X X X X            V   B
          X X X X X X X X X X            V   B
                    X X X X X X X X X X  V   B
                    X X X X X X X X X X  V   B
                    X X X X X X X X X X  V   B
                    X X X X X X X X X X  V   B
                    X X X X X X X X X X  V   B
                              X X X X X  V   B
                              X X X X X  V   B
```

Figure D.1: *Matrix structure of a set of linear finite-difference equations (D.5) with boundary conditions (D.2,D.3). Here X represents a coefficient of (D.8), V represents a component of the unknown solution vector* $\Delta y_{i,k}$*, and B is a component of the known right-hand side* $-E_{i,k}$*. Empty spaces represent zeros. In this example, we took* $n_1 = 3$ *and* $N = 5$.

n_1 linear equations, the last one is of $N - n_1$ linear equations, and in between all subsystems are of N linear equations.

Using the Gaussian elimination method, every submatrix can be rewritten in a way where all the diagonal elements equal to 1 and the lower part with respect the diagonal elements should be zero (See Fig. D.2). Then we make a backsubstitution to get the values $\Delta y_{j,k}$.

We redo the same procedure many times until we get the required precision, and our mean error can be parameterized by

$$\frac{\sum_{i,k} |E_{i,k}|}{N \times M} \tag{D.9}$$

For more details, this method is well described in Section 17.3 in *Numerical Recipes* [99]. The routine name doing this procedure is '*solvde*'.

Figure D.2: *Target structure of the Gaussian elimination. Once the matrix of Fig. D.1 has been reduced to this form, the solution follows quickly by backsubstitution.*

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Selbstständigkeitserklärung

Hermit erkäre ich, die vorliegende Arbeit selbstständig verfasst und nur die angegeben Literatur verwendet zu haben.

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Amine AHRICHE