Working Females: A Modern Statistical Approach

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Vorwort

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Torben Kuhlenkasper

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1 Motivation

"Economy is the art of making the most of life"

(George Bernard Shaw)

In the year 2004, the German Association for the German Language (*Gesellschaft für deutsche Sprache, GfdS*) elected "Human Capital" as the faux-pas word of the year in Germany. In its justification, the GfdS argued that the phrase degrades employees to an only economically relevant factor. "Human Capital" is assumed to abstract from important characteristics of the employee as a human being. At that time, the phrase has been heavily discussed in Germany and other developed countries in public debates concerning the discharging of employees and an assumed deletion of human capital as the one major resource for the country and the entire economy. However, 12 years earlier, Gary S. Becker has been awarded the Nobel Prize in economics for his advances in extending micro-economic theory to human behavior, especially to the behavior of employees and the time allocation within their families. "Human capital" is the central aspect of his work. After the prize has been awarded to Becker, almost every research publication in the wide field of labor economics takes human capital theory in account, despising the public discussions about its possible applications and conclusions. "Human Capital" has become an area of conflict for both, politics and research. Gary S. Becker combined many contributions published before to provide a consistent theoretical framework dealing with human capital. This dissertation is built upon the human capital theory, which traces back to Ben-Porath (1967), Mincer (1974) and Becker (1976) as one column. It is the aim of this work to enrich the controversy debate on human capital, carried out by its opponents and its advocates, by empirical findings in the field of labor economics. The second column of this dissertation consists of modern statistical regression techniques, tracing back to DeBoor (1978), Eubank (1988) and Hastie and Tibshirani (1990). This thesis employs modern statistical methods to analyze working females with respect to employment interruptions due to childcare. To be specific, the female-related withdrawal from the labor market due to childcare and the resulting consequences for their earnings compared to non-mothers are the topics of this work.

In contrast to men, women have forced a changing role allocation and are faced with the consequences: major educational advancements in the second half of the last century have lead to rising labor force participation of women in almost every developed country with severe consequences for both, the families and the labor market. However, due to biological constraints and existing social norms, childbirth and childcare are still domains of females. Due to the necessity of combining family related aspects with employment, females entered the focus of both, public discussions concerning national policy about maternity leave and maternity rights as well as research in the field of labor economics with the aim of analyzing female employment profiles. Making the most of life, according to the Nobel Prize winner in literature, George Bernard Shaw, can therefore be achieved by combining work and economical behavior in family planning, at least by analyzing life from the perspective of an economist.

In the past, most empirical findings in research publications used established classical statistical techniques as the well known linear model or the linear mixed model for longitudinal data. Although these findings contributed tremendously to an science-based understanding of female labor force participation, the conclusions were restricted not only to the database employed but also to the statistical methods used. In this work, I ease restrictions on the methods to allow for an advanced analysis and interpretation of the economic behavior of females, which is assumed to be relevant for the women themselves, relevant for their families and relevant for the labor market. With this work, I want to narrow the gap between recent statistical advances and recent socio-economic developments concerning the labor force participation of females. Recently developed modern statistical regression techniques have usually been tested and used with artificial, simulated data. This procedure is necessary to establish new, numerically challenging techniques, which are developed to provide reliable and valid enhancements for the tools empirical researchers can use. Applying these methods to empirical socio-economic data can lead to both: a further establishment of modern regression methods in empirical economic research and to an advanced understanding of working females, contributing to a science-based analysis of labor force participation.

As already noted, I address two central aspects concerning female labor force participation in this work: first, I model the duration of maternity leave as a major event in the work biographies of women in Germany in chapter 2. This chapter is retained from Kuhlenkasper and Kauermann (2009) and Kuhlenkasper and Kauermann (2010a). In chapter 3, I analyze the socio-economic consequences of giving birth by modeling the wage and the wage loss for mothers around childbirth and compare it to non-mothers. This chapter is retained from Kuhlenkasper and Kauermann (2010b) and gives an empirical investigation of one central aspect of the human capital theory with modern statistical techniques. I conclude and give an outlook in chapter 4. This inducing chapter 1 gives background information of one possible underlying economic theory, the general idea of P-spline smoothing as the employed method in the regression analysis and finally a discussion of the empirical database of the German Socio Economic Panel.

1.1 Economic Theory

In this work, two major economic theories can provide the framework for the analyses carried out in the chapters 2 and 3: the household production theory in combination with the above mentioned human capital theory. Both theories, consistently provided by Becker (1991) and Becker (1993) gain their importance from empirical validation in the last decades, from the award of the Nobel Prize in 1992 and from a micro-economic foundation in the context of labor supply. An introduction for the latter is subject to this chapter, new empirical findings however might be an result of this work. The human capital theory provides the implicit economic theory when analyzing the duration of maternity leave in chapter 2 and the explicit theory when modeling wages and wage losses in chapter 3. The household production theory motivates the importance of wages and earnings when having a closer look at the labor force participation of females.

The supply of labor by women and other family members with all of the socio-economic consequences for the household can be analyzed by employing a *household production approach.* Instead of maximizing the family's utility function with respect to the consumption of goods and the usage of time for leisure and work directly, which is common for traditional micro-economic theory, it is useful for this work to focus on the production of different "commodities" by the family. These commodities, consisting of goods and services, are produced by the members of the household and yield utility to the entire family. The definition of commodities is not restricted to usual consumption goods and can also consist e.g. of children and the care provided to them by members of the household or by purchased childcare in accordant facilities. This care is often provided by the mothers focused in this work but can also be subject to the time-allocation decision among the household members. It is worth noting, that the definition of households here includes single-mother households, too and is simplified for the purpose of this work. Following Becker (1991), the utility function of the household is defined as

$$U \; = \; U\left(C_1, ..., C_N\right), \tag{1.1}$$

being a function of N different goods and services. These commodities are produced and therefore

$$C_i \; = \; f_i\left(g_i, t_i^p; ENV_i\right) \;\; \forall\, i = 1, ..., N \tag{1.2}$$

with used input-goods g_i and the time t_i^p necessary to produce C_i. Other environmental variables are summarized by ENV_i, which include e.g. the human capital accumulated in the household and used for the specific commodity. Although these environmental variables contribute to the production of the commodities, they are assumed to be fix and not substitutable with other inputs, at least in the short run. Therefore ENV_i are not focused subjects in the maximization problem arising from (1.1) and they do not influence the decision concerning the allocation of g_i and t_i^p in (1.2) directly.

In the context of maternity leave and females' attachment to the labor force, let $C_k, k \in \{1, ..., N\}$ be the commodity *childcare*, which has to be provided or purchased by members of the household. Three scenarios are possible with respect to the household production function: first, professional childcare g_k is completely purchased at the market, e.g. by employing child minders. This however is not likely to occur in the extreme case since it implies a lack of any childcare by the members of the household. Secondly, the care is "produced" in the household exclusively with the usage of t_k^p, which can be observed in the time period around the childbirth itself: mothers withdraw completely but temporarily from their jobs outside of the household and provide time-intensive care to the babies. Third, a combination of the previous is likely when the mother (or other members of the household) decides to reenter the labor market with reduced working hours and therefore has to allocate the time between the household and the labor market. Although the commodities C_i can be produced with market-valued inputs g_i, the commodities itself lack a reliable market-price since they are not traded. Nevertheless, all C_i can be valued by a shadow price ξ_i, see Becker (1991). These shadow prices can be expressed by the average cost of the inputs to produce each unit of the commodity C_i:

$$\xi_i = \frac{p_i g_i + \omega t_i^p}{C_i}, \; \forall\, i = 1, ..., N \tag{1.3}$$

with ω being the earnings per hour of labor work outside the household and p_i being the market-price for the corresponding input g_i. Note, that shadow prices in the equilibrium are usually found deriving the *marginal* cost of production. Due to assumed homogeneous production functions $f_i \, \forall i = 1, ..., N$ and the exclusive usage for each unit of input however, the marginal cost equals the average cost in (1.3). As a result, the entire household budget constraint B can be expressed by using ξ_i:

$$B = \omega \sum_{i=1}^{N} t_i^p + \sum_{i=1}^{N} p_i g_i = \sum_{i=1}^{N} \xi_i C_i \tag{1.4}$$

The maximization of household utility according to (1.1), with respect to the binding constraint in (1.4) leads to

$$\frac{\frac{\delta U}{\delta C_i}}{\frac{\delta U}{\delta C_j}} = \frac{\xi_i}{\xi_j},\ \forall\, i, j = 1, ..., N \tag{1.5}$$

Changes in the (relative) shadow prices are likely to affect the demand for the inputs of the corresponding commodities. The relation of input goods g_i to the time t_i^p used for production of C_i can finally be analyzed by separating (1.1) with respect to both inputs:

$$\begin{aligned}
\frac{\frac{\delta U}{\delta g_i}}{\frac{\delta U}{\delta t_i^p}} &\equiv \frac{\frac{\delta C_i}{\delta g_i}}{\frac{\delta C_i}{\delta t_i^p}} \cdot \frac{\frac{\delta U}{\delta C_i}}{\frac{\delta U}{\delta C_i}} \\
&= \frac{\frac{\delta C_i}{\delta g_i}}{\frac{\delta C_i}{\delta t_i^p}} \\
&= \Phi\left(g_i, t_i^p\right),\ \forall\, i = 1, ..., N, \qquad (1.6)
\end{aligned}$$

being the ratio of the marginal products of both inputs. This "separability" into goods g_k being purchased with the price p_k and time t_k^p spent in the household, valued by ω, can be applied to the questions motivated in this work: an increasing wage ω (or earnings-potential) in the labor market can lead to a reduced amount of time allocated to the time-intensive childcare in the household. A quick return into employment after child-birth with a rising demand for g_k will be the rational result for the utility-maximizing household with the common assumptions concerning rational behavior. The main contribution of human capital theory to the above motivated household production theory is made by explaining the central variable: the wage and indirectly the shadow prices ξ_i. Wages, which can be earned at the labor market can be alternatively interpreted as *shadow prices of children* (see Boll (2011)).

Human capital theory provides a consistent framework for analyzing changes in earnings, especially wages with respect to employment interruptions. The length of withdrawal from the labor market, either voluntarily or involuntarily, is likely to play a vital role in this discussion. In contrast to traditional micro-economic theory, human capital theory allows for heterogeneity of the input-factor *work*. The heterogeneity of work, expressed by different wages for different employees, is assumed to originate in different sizes of the accumulated stock of human capital: higher stocks of human capital lead to higher wages. However, the human capital theory is also built upon the traditional assumptions of complete information and foresight as well as on perfect competition among the individuals providing their work-force at the labor market. These assumptions have been criticized since different reasons for withdrawal, e.g. an involuntarily period of unemployment in contrast to an voluntarily period of childcare, are not taken into account when analyzing (a decreasing) ω with respect to an eroding stock of human capital . Empirical studies however, like Kunze (2002) focus on different reasons for a *net-depreciation* of the human capital stock and finds evidence about different rates of net-depreciation in Germany, depending on the reasons for withdrawal. The empirically related characteristics of human capital stocks with explanation concerning the accumulation of it over the lifetime is given in chapter 3.

The relation between the stock of individual human capital and the wage ω traces back to Ben-Porath (1967), who postulates the human capital of being the central argument for employees to influence their earnings. Abstracting from non-monetary effects of a large stock of human capital in this work, e.g. a possible higher status in society due to higher educational degrees, human capital is assumed to be accumulated exclusively for maximizing the income over the lifetime. Earnings gained at the labor market are assumed to be a function of the human capital stock accumulated by the employee up to the focused time point. In this work, the definition of "human capital" is therefore restricted to individual knowledge and skills, which can be used at the labor market to earn money.

The maximization of lifetime income however has to account for further investments in the individuals' stock of human capital as well. In the economic literature the *human capital earnings function* has been established with possible negative net investments due to eroding human capital, e.g. while being in maternity leave. A consistent and useful explanation of the human capital earnings function is given by Mincer and Polachek (1974):

The gross earnings of an employee in period t,

$$E_t \;=\; E_{t-1} \;+\; rI_{t-1}, \tag{1.7}$$

equals the earnings of the previous time period E_{t-1} plus the net investments in the human capital stock I_{t-1} in the previous period, valued by the average rate of return on this investment r. Defining $\kappa_t = \frac{I_t}{E_t}$ as the investment ratio of the earnings, (1.7) can be changed to

$$E_t \;=\; E_{t-1}\left(1 + r\kappa_{t-1}\right). \tag{1.8}$$

Due to the time-sequence

$$\begin{aligned} E_t &= E_0\left(1 + r\kappa_0\right)\left(1 + r\kappa_1\right)\cdots\left(1 + r\kappa_{t-1}\right) \\ &= E_0 \prod_{i=0}^{t-1}\left(1 + r\kappa_i\right) \end{aligned} \tag{1.9}$$

and the approximation

$$r\kappa \approx \ln\left(1 + r\kappa\right), \tag{1.10}$$

since $r\kappa$ is assumed to be small, the (logarithmical) earnings in period t can be expressed by

$$\ln E_t \;=\; \ln E_0 + r\sum_{i=1}^{t-1}\kappa_i. \tag{1.11}$$

Since time and effort for current human capital investments in period t have to be taken into account, the complete income of the employee is defined as

$$Y_t \;=\; E_t\left(1 - \kappa_t\right) \tag{1.12}$$

or

$$\ln Y_t \;=\; \ln E_0 + r\sum_{i=0}^{t-1}\kappa_i + \ln\left(1 - \kappa_t\right). \tag{1.13}$$

In this work, I focus on females giving birth to a child after having completed school as one central aspect of human capital accumulation. The resulting employment interruption therefore takes place while being in formal and informal training in the job after exclusive usage of time for human capital investments in school. The earnings therefore can be changed to

$$\ln E_t \;=\; \ln E_0 + r\sum_{i=1}^{s-1}\kappa_i + r\sum_{j=s}^{t-1}\kappa_j, \tag{1.14}$$

with κ_i being the exclusive investments during school time and κ_j being the investments made at the labor market afterwards. Note that this definition abstracts from a variety of other ways for human capital accumulation. It is reasonable to assume $\kappa_i = 1\,\forall\, i = \{1,\ldots,s-1\}$ with resulting

$$\ln E_t \;=\; \ln E_0 + rs + r\sum_{j=s}^{t-1}\kappa_j. \tag{1.15}$$

For models describing the optimal distribution of I_t and κ_j among the lifetime, see Ben-Porath (1967) and Becker (1991).

Focusing on females and their employment-pattern around childbirth, a decomposition of net investments into gross investments and the depreciation of the accumulated stock of human capital is useful. With I^g_{t-1} being the gross investments and ϱ_{t-1} being the depreciation rate in $t-1$,

$$E_t \;=\; E_{t-1} + rI^g_{t-1} - \varrho_{t-1}E_{t-1} \tag{1.16}$$

and according to (1.8)

$$\frac{E_t}{E_{t-1}} \;=\; 1 + r\kappa^g_{t-1} - \varrho_{t-1} \;=\; 1 + r\kappa_{t-1}, \tag{1.17}$$

with $\kappa^g_t = \frac{I^g_t}{E_t}$ and $r\kappa_t = r\kappa^g_t - \varrho_t$.

(1.11) can be changed to

$$\ln E_t \;=\; \ln E_0 + \sum_{i=0}^{t-1}\left(r\kappa^g_i - \varrho_i\right). \tag{1.18}$$

A net depreciation of human capital in period t, expressed by $\varrho_t > r\kappa_t^g$, is therefore likely to reduce earnings in $t+1$. Such losses, tracing back to low (or even non) κ_t^g can be observed by analyzing intermittent work experience, e.g. if mothers withdraw from the labor market for childbirth and -care and return afterwards into employment. Post-school investments according to (1.15) can be split into periods or segments with participation in the labor market and periods of withdrawal.

Following Mincer and Polachek (1974), the investment ratio can be expressed as

$$\kappa_i \; = \; a_i + b_i t, \;\; i = 1, \dots, n \tag{1.19}$$

with a_i being the starting investment ratio and b_i being the corresponding rate of change in the i-th period or segment in the biography of the employee: $(t_{i+1} - t_i) = \psi_i$. Therefore:

$$\ln E_t \; = \; \ln E_0 + rs + r \sum_{i=1}^{n} \int_{t_i}^{t_{i+1}} (a_i + b_i t) \, dt, \tag{1.20}$$

with the initial investment ratio referring to the beginning of the working career of the employee. Since the amount of investment in individual human capital is likely to differ, (1.20) can be changed to

$$\ln E_t \; = \; \ln E_0 + rs + \sum_{i=1}^{n} \int_{0}^{\psi_i} (a_i + b_i t) \, dt, \tag{1.21}$$

with a_i being defined as the ratio of investment at the beginning of the i-th period. By assuming a constant rate of net investments *within* a given $\psi_c, c \in \{1, \dots, n\}$ and a differing ratio between the ψ_i, the (female) earnings function can finally be simplified to

$$\ln E_t \; = \; \ln E_0 + rs + r \sum_{i} a_i \psi_i. \tag{1.22}$$

Periods of maternity leave are assumed to reveal $(ra_i < 0)$, which is a net depreciation of human capital with empirically found consequences for the earnings in chapter 3. The relevant periods of withdrawal are under investigation in chapter 2.

1.2 The Idea of P-splines as the Employed Statistical Technique

Modeling wages, wage losses and the duration of withdrawal from the labor market can be carried out by employing statistical regression techniques with the wage, the wage loss and the hazard rate of return into employment as response variables, respectively. Due to their numerically stable estimation routines and the easy way of interpreting the results, linear regression models are still the core of statistical and econometrical applications. The "price" of simple and fast results however is the lack of flexibility if the a priori specified parametric structure of the model does not match with the underlying data. Although an a-priori parametric linear model could easily be derived from the established economic literature, Becker (1991) and Becker (1993) addresses possible non-linearities and other complex structures when investigating employment behavior in the context of human capital theory empirically. I follow his idea by allowing for non-linearities in the empirical models. Advances concerning the employed statistical methods however are not the aim of this work. In the following of this section, I give a short and rather technical introduction for the use of modern P-spline methods.

The aspired flexibility can be achieved by easing the restrictions on the model, e.g. the a-priori assumed structure of the model. Many techniques have been developed to achieve this goal and are still the topic of many research publications. In the last years however, *smoothing* techniques have been established in statistical literature. Spline regression, as one smoothing technique, is employed in this work and is introduced in the following section. Although these techniques are coined "non-parametric", they are built upon the classical regression techniques known from the linear (parametric) model. The main difference is found by looking at the parameters that have to be estimated when specifying a model: linear models can easily be specified by a few parameters, like the well-known coefficients $\hat{\beta}$ and $\hat{\sigma^2}$. In contrast, non-parametric models are characterized by many parameters which gain their importance in combination with others characterizing the model and could isolated hardly be interpreted. See Sprent and Smeeton (2007) for details about the parameters of statistical models.

While many promising techniques have been developed to overcome the disadvantages of (full) parametric models, P-splines have become an applicable and stable smoothing technique, which meets the demand for flexible but also reliable methods. In the following, P-splines techniques are introduced as a background for both, (generalized) additive modeling and non-proportional hazard models, which can be transformed to the first. The simple but well known model

$$y \;=\; \beta_0 + \sum_{j=1}^{q} \beta_j x_j \;+\; \epsilon \tag{1.23}$$

with $\epsilon \sim N(0, \sigma^2)$ traces back to

$$y \;=\; f(x_0, \ldots, x_q) \;+\; \epsilon \tag{1.24}$$

with the same assumption concerning the error-term. For simplicity however, an additive model structure is often assumed, leading to

$$y \;=\; f(x_0) \;+\; f(x_1) \;+ \ldots +\; f(x_q) \;+ \epsilon. \tag{1.25}$$

P-splines provide one technique for specifying $f(\cdot)$, which is a flexible and smooth but otherwise unspecified function. The following introduction to (P-)spline smoothing follows DeBoor (1978), Ruppert et al. (2003), Fahrmeir et al. (2007) and Krivobokova (2006). The techniques applied in chapters 2 and 3 are built upon these statistical background and are specified in the corresponding chapters.
The underlying idea can be easily shown by starting with a single covariate x, a response y and the corresponding function $f(x)$. The latter can be estimated by dividing the domain of x into sections marked by K knots: $k_1, \ldots, k_K$. Within these segments, parametric regression is carried out in the form of a polynomial model of degree d:

$$y_i \;=\; f(x) \;+\; \epsilon_i \;=\; \sum_{i=0}^{d} \beta_i x^i \;+ \sum_{j=1}^{K} u_j \,(x - k_j)_+^d \;+\; \epsilon_i \tag{1.26}$$

with

$$(x - k_j)_+ \; = \; \max\{(x - k_j; 0)\}, \tag{1.27}$$

employing a truncated polynomial basis function, which includes deviations from the polynomial functions by truncated terms within the segments marked by the knots. (1.26) can be restated to

$$y \; = \; X\beta \, + \, Zu \, + \, \epsilon, \tag{1.28}$$

with

$$X \; = \; [1, x_i^1, \ldots, x_i^d]_{1 \leq i \leq n} \; , \; Z \; = \; [(x_i - k_1)_+^d, \ldots, (x_i - k_K)_+^d]_{1 \leq i \leq n}$$

and the coefficients

$$\beta \; = \; (\beta_0, \ldots, \beta_d)' \; , \; u \; = \; (u_1, \ldots, u_K)'.$$

According to classical well-known least-squares techniques with $M = [X, Z]$, the fit can easily be obtained by

$$\hat{y} \; = \; M(M'M)^{-1}M'y. \tag{1.29}$$

Although this technique is linked directly to the known inference method arising from the least-squares criterion used for linear models with numerically stable results, the choice of K and the position of $k_i, i \in \{1, \ldots, K\}$ in the domain of x is arbitrarily. The amount of K is a central issue when dealing with P-splines: choosing a large K, the resulting fit will be too flexible resulting in a wiggly, non-smooth effect with low bias but a very high variance. On the other hand, choosing a small K, the data will probably be 'underfitted', leading to a small variance but a large bias of the estimation. To prevent a wiggly estimate, overfitting the data, a penalty is placed on the spline coefficients, resulting in

$$\hat{y} \; = \; M(M'M + \lambda D)^{-1}M'y \tag{1.30}$$

with $\lambda \geq 0$ being the penalty parameter and D being a block diagonal matrix of $[\mathbf{0}_{(d+1)\times(d+1)}, \mathbf{1}_K]$.

Due to numerical problems arising from a large K and a small λ (close to zero), a necessary inversion of $(M'M + \lambda D)$ for estimating y can often hardly been carried out. As one solution, B-splines have been introduced. Following DeBoor (1978) and Eilers and Marx (1996), B-spline basis are determined recursively for K knots by

$$B_j^d(x) = \frac{x - k_j}{k_{j+d} - k_j} B_j^{d-1}(x) + \frac{k_{j+1} - x}{k_{j+d+1} - k_{j+1}} B_{j+1}^{d-1} \tag{1.31}$$

with $B_j^d(x)$ being the j-th B-spline of degree d and initially

$$B_j^0 = \mathbf{1}_{(k_j, k_{j+1})(x)}. \tag{1.32}$$

Numerical properties of B-splines and advantages compared to truncated polynomials are discussed by Eilers and Marx (1996).

Eilers and Marx (2000) link B-splines to truncated polynomials by showing that B-splines like (1.31) can be computed by differencing the latter. Assuming equidistant knots $k_m, m \in \{1, \ldots, K\}$, (1.31) can be expressed as

$$B_j^d = \frac{(-1)^{d+1} \Delta^d Z_j^d(x)}{(k_{j-1} - k_j)p!} \tag{1.33}$$

with $Z_j^d = (x - k_j)_+^d$ and Δ^{d+1} defined by

$$\begin{aligned}
\Delta^1 a_j &= a_j - a_{j-1}, \\
\Delta^2 a_j &= \Delta^1(\Delta^1 a_j) \\
\vdots \quad & \vdots \quad \vdots \\
\Delta^w a_j &= \Delta^1(\Delta^{w-1} a_j)
\end{aligned}$$

The resulting B-spline basis matrix of degree d, constructed over K knots and corresponding to n observations will therefore have the dimension $n(K + 1 + d)$. As Ruppert et al. (2003) show, (1.33) leads to to

$$\hat{y} \;=\; B(B'B + \lambda\tilde{D})^{-1}B'y, \tag{1.34}$$

including the B-spline basis matrices and $\tilde{D} = \Delta_q'\Delta_q$ with Δ_q as a $(K+p-q+1)$x$(K+p+1)$ matrix of the difference term Δ of order $d+1=q$. DeBoor (1978) gives the formula for the q-th derivative of the B-spline of degree d:

$$h^q \sum_j \theta_j (B_j^p)(x) \;=\; \sum_j \Delta_q \theta_j B_{d-q}^j(x), \tag{1.35}$$

with θ_j as spline coefficients. In many software packages, B-splines of degree $d=3$ and second order difference penalty are a common choice but can be changed if necessary.
The selection of the penalty or smoothing parameter $\lambda > 0$ is motivated by the *bias-variance trade off*: A penalized smoother can alternatively be expressed by

$$\|y - M\theta\|^2 \;+\; \lambda\theta' D\theta \tag{1.36}$$

with $M = [X, Z]$ and $\hat{\theta}$ as the resulting minimizer of (1.36). D has to be positive definite and M contains the basis functions defined above. Using P-splines for fitting, K and the location of the knots in the domain of x have to be chosen. In addition, the employed basis functions, the degree of the spline and the penalty matrix D have to be determined. The fitting routines however make use of the definition of the hat matrix H to obtain $\hat{y} = X(X'X)^{-1}X'y = Hy$ in the parametric (linear) model. According to (1.30), the corresponding matrix for P-spline fitting can be defined as the smoothing matrix

$$S_\lambda \;=\; M(M'M + \lambda D)^{-1}M' \tag{1.37}$$

with

$$df \;=\; \mathrm{tr}(S_\lambda) \tag{1.38}$$

being the degrees of freedom for the model, which are linked to the penalty parameter λ as the degree of the smoother.
The residuals degrees of freedom can be obtained from the linear model by

$$E(RSS) \;=\; E(y'(\mathbf{1}-S_\lambda)'(\mathbf{1}-S_\lambda)y) \;=\; \sigma^2 \mathrm{tr}(\mathbf{1}-2S_\lambda+S_\lambda S_\lambda) \;+\; \|f(x)(1-S_\lambda)\|^2 \quad (1.39)$$

An unbiased estimate for σ^2 can be derived with assumed $\|f(x)(1-S_\lambda)\|^2$ being close to zero, by

$$\hat{\sigma^2} \;=\; \frac{y'(\mathbf{1}-S_\lambda)'(\mathbf{1}-S_\lambda)y}{n-\mathrm{tr}(2S_\lambda - S_\lambda S_\lambda)} \;=\; \frac{RSS}{df_{res}} \quad (1.40)$$

To measure the error of the smoother, the *mean squared error (MSE)*

$$\mathrm{MSE}(\hat{f}(x)) \;=\; \mathrm{Var}(\hat{f}(x)) \;+\; \Big(E(\hat{f}(x)) - f(x)\Big)^2 \quad (1.41)$$

for a singe data point x can be transformed to a *mean average square error (MASE)*

$$\mathrm{MASE}(\lambda) \;=\; \frac{1}{n}\sum_{i=1}^{n}\mathrm{MSE}(\hat{f}(x_i)) \;=\; \frac{1}{n}\left(\sigma^2\mathrm{tr}(S_\lambda S_\lambda) + \|f(x)(1-S_\lambda)\|^2\right) \quad (1.42)$$

for the fit corresponding to all points in the dataset. The definition of MASE reflects the bias-variance trade-off in the resulting fit when dealing with P-spline approaches to estimate $\hat{f}(x)$. The optimal amount of smoothing, and therefore the value of $\lambda > 0$, will be a compromise between the goodness of fit and the complexity of $\hat{f}(x)$. In many statistical software packages and in this work, a (generalized) cross validation criterion is minimized to find optimal values for the penalty parameter. For details about the numerical challenges of finding the optimal λ, see Wood (2000) and Wood (2006).
The well-known definition of the residual sum of squares

$$\mathrm{RSS} \;=\; \sum_{i=1}^{n}(y-\hat{y}_i)^2 \quad (1.43)$$

is used for the estimation of an optimal λ by minimizing a cross validation criterion

$$\mathrm{CV} \;=\; \frac{1}{n}\sum_{i=1}^{n}(y_i-\hat{y}_i^{(-i)})^2. \quad (1.44)$$

Note, that $\hat{y}_i^{(-i)}$ is the fit of the model by leaving the i-th observation unconsidered. The computations of the resulting fits of n separate models however can be time consuming.

As Ruppert et al. (2003) show, the computational efforts can by minimized by the usage of the smoothing matrix S and the approximation

$$\hat{f}(x) \approx \frac{\sum_{i \neq j} S_{ij} y_i}{\sum_{i \neq j} S_{ij}}. \tag{1.45}$$

The resulting criterion

$$\text{CV} = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{y_i - \hat{f}(x)}{1 - S_{ii}} \right) \tag{1.46}$$

still depends on the smoothing matrix S, which can be time consuming to compute if n is large. Craven and Wahba (1979) replaced (1.46) by a generalized cross validation criterion:

$$\text{GCV} = \frac{1}{n} \sum_{i=1}^{n} \left(\frac{y_i - \hat{f}(x)}{1 - \frac{\text{tr}(S)}{n}} \right), \tag{1.47}$$

which uses the above introduced definition of the degrees of freedom in (1.38). See Wood (2006) for numerical details about the grid search carried out to find the optimal value of λ by minimizing (1.47) when considering only a single covariate.

In this work, longitudinal data is used for the empirical analyses. Classical statistical regression models taking the characteristics of longitudinal data into account by adding random effects are coined "mixed models": due to additional random effects in the model structure, mixed models can capture possible correlation structures arising from repeated measurements of identical statistical units, e.g. participants in the survey. Let

$$y = X\beta + Zu + \epsilon \tag{1.48}$$

be a classical linear mixed model with β being the coefficient vector of dimension $q + 1$ for the assumed fixed effects, which are coined "population effects" in the context of mixed models. While X and Z can be assumed to be the well-known model matrices, u captures the *individual* random effects in the model. Since ϵ is also assumed to be a random variable:

$$\begin{pmatrix} u \\ \epsilon \end{pmatrix} \sim N\left(\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} G & 0 \\ 0 & R \end{pmatrix}\right) \tag{1.49}$$

with G and R being the positive definite covariance matrices for the random terms. Applying the linear mixed model, the estimation of the fixed effect vector $\hat{\beta}$ is straightforward with

$$y = X\beta + \varepsilon, \tag{1.50}$$

with $\varepsilon = Zu + \epsilon$ and $Cov(\varepsilon) = ZGZ' + R = A$. The best linear (unbiased) estimator for β is obtained by a generalized least squares approach and follows well-known classical regression techniques.

$$\tilde{\beta} = \left(X'A^{-1}X\right)^{-1} X'A^{-1}y. \tag{1.51}$$

The resulting best linear unbiased predictor for the random vector is given by

$$\tilde{u} = GZ'A^{-1}\left(y - X\tilde{\beta}\right) \tag{1.52}$$

See McCulloch and Searle (2001) for details about $\tilde{u}$. Due to the assumption expressed in (1.49), $\tilde{u}$ can also be obtained by maximizing the joint density of y and u:

$$(2\pi)^{\frac{-n+l}{2}} \left(\det\begin{pmatrix} G & 0 \\ 0 & R \end{pmatrix}\right)^{-1/2} \exp\left\{\frac{1}{2}\begin{pmatrix} u \\ y - X\beta - Zu \end{pmatrix}' \begin{pmatrix} G & 0 \\ 0 & R \end{pmatrix}^{-1} \begin{pmatrix} u \\ y - X\beta - Zu \end{pmatrix}\right\} \tag{1.53}$$

with l being the dimension of the random effects vector u. (1.53) leads to

$$(y - X\beta - Zu)' R^{-1} (y - X\beta - Zu) + u'G^{-1}u \tag{1.54}$$

(1.54) is minimized by $\tilde{\beta}$ and $\tilde{u}$. The resulting best linear predictor for the response y is gained by

$$\tilde{y} \;=\; M\tilde{\theta} \;=\; M(M'R^{-1}M + P)^{-1}M'R^{-1}y, \tag{1.55}$$

with $\tilde{\theta} = (\tilde{\beta}, \tilde{u})'$ and P being a block diagonal matrix of $(0, G^{-1})$.

The covariance matrix A can be estimated by using the so-called *profile log-likelihood*

$$-2l^p(A) \;=\; (y - X\tilde{\beta})'A^{-1}(y - X\tilde{\beta}) + \log|A| \tag{1.56}$$

and via Fisher scoring algorithms known from linear models with generalized response, e.g. being described in Searle et al. (1992).

The resulting estimate $\hat{A}$ finally yields

$$\hat{\beta} \;=\; \left(X'\hat{A}^{-1}X\right)^{-1} X'\hat{A}^{-1}y \tag{1.57}$$

and

$$\hat{u} \;=\; \hat{G}Z'\hat{A}^{-1}(y - x\hat{\beta}). \tag{1.58}$$

Interpreting the estimation and prediction of $\hat{\beta}$ and $\hat{u}$ has to take the variance into account. According to (1.55), it can be found that

$$Cov(\tilde{\theta}|u) \;=\; \left(M'R^{-1}M + P\right)^{-1} M'R^{-1}M \left(M'R^{-1}M + P\right) \tag{1.59}$$

and

$$Cov(M\tilde{\theta}|u) \;=\; M\,Cov(\tilde{\theta}|u)M'. \tag{1.60}$$

When analyzing the prediction,

$$Cov(\tilde{\theta} - \theta) \;=\; Cov\begin{pmatrix}\tilde{\beta} - \beta \\ \tilde{u} - u\end{pmatrix} \;=\; Cov\begin{pmatrix}\tilde{\beta} \\ \tilde{u} - u\end{pmatrix} \tag{1.61}$$

with (1.55), it can be stated that

$$
\begin{aligned}
Cov \begin{pmatrix} \tilde{\beta} \\ \tilde{u} - u \end{pmatrix} &= \left(M'R^{-1}M + P\right)^{-1} M'AM \left(M'R^{-1}M + P\right)^{-1} \begin{pmatrix} 0 & 0 \\ 0 & G \end{pmatrix} \\
&= \left(M'R^{-1}M + P\right)^{-1}. \qquad (1.62)
\end{aligned}
$$

The final variance for the estimates can be found by using

$$
Cov\left(M\tilde{\theta} - M\theta\right) = MCov(\tilde{\theta} - \theta)M' = M(M'R^{-1}M + P)^{-1}M'. \qquad (1.63)
$$

The notation used in (1.28) to introduce P-plines is not *at random* similar to the notation for mixed models. Indeed, assuming u in (1.28) to be random leads to the model

$$
y = X\beta + Zu + \epsilon \qquad (1.64)
$$

with $\epsilon \sim N(0, \sigma_\epsilon^2 \mathbf{1}_n)$, $u \sim N(0, \sigma_u^2 \mathbf{1}_K)$ and X and Z containing the polynomial base and the base constructed with truncated polynomials, respectively.
According to (1.49) with $R = \sigma_\epsilon^2 \mathbf{1}_n$ and $G = \sigma_u^2 \mathbf{1}_K$, (1.55) can be changed to

$$
\tilde{y} = M\left(M'M + \frac{\sigma_\epsilon^2}{\sigma_u^2}\right)^{-1} M'y \qquad (1.65)
$$

with D being a block diagonal matrix of $\left(\mathbf{0}_{(d+1)x(d+1)}\mathbf{1}_K^{-1}\right)$. The estimation of linear mixed models is directly linked to P-spline smoothing due to $\frac{\sigma_\epsilon^2}{\sigma_u^2}$ in (1.65) being the smoothing parameter λ in the context of P-splines. For a discussion concerning adjustments, which are necessary if using B-splines bases, see Ruppert et al. (2003). However, the link from mixed models to P-splines smoothing is also useful by obtaining confidence intervals for the (often graphically shown) estimated function of $f(\cdot)$. It can be shown, that $\tilde{f}(x) = X\tilde{\beta} + Z\tilde{u}$ is unbiased for $f(x)$ with the assumptions belonging to (1.64), leading to

$$
\tilde{f}(x) - f(x) \sim N(0, \sigma_\epsilon^2 S_\lambda), \qquad (1.66)
$$

with $Cov(\tilde{f}(x) - f(x)) = \sigma_\epsilon^2 S_\lambda$ according to (1.63). The resulting (bias adjusted) 95% confidence band can be displayed by

$$\tilde{f}(x_i) \pm 2\hat{\sigma}_\epsilon \sqrt{S_\lambda^{ii}} \quad \forall\, i = 1, \ldots, n, \tag{1.67}$$

employing only the diagonal entries of the smoothing matrix S_λ.

The above introduced P-spline technique is not limited for specifying $f(\cdot)$ with respect to only a single covariate. With the aim of enhancing empirical research techniques, multiple covariates have to be taken into account, resulting in specifying $v = 1, \ldots, w$ different functions $f_v(x_v)$. For

$$y = \beta_0 + \sum_{v=1}^{w} f_v(x_v) + \epsilon \tag{1.68}$$

with $\epsilon \sim N(0, \sigma_\epsilon^2)$ and n observation-pairs, P-spline techniques allow to estimate $f_v(\cdot)$ simultaneously by *assuming an additive structure*. Employing truncated polynomials, each function can be represented as

$$f_v(x_v) = X_v \beta_v + Z_v u_v \quad \forall\, v = 1, \ldots, w \tag{1.69}$$

with u_v being a vector of dimension K_v. The parameters can be estimated in two ways: either from minimizing

$$\|y - X\beta - Zu\|^2 + \sum_{v=1}^{w} \lambda_v u_v' u_v \tag{1.70}$$

or with the introduced mixed model notation according to (1.64) with $\beta = (\beta_{0,\ldots,\beta_w})$, $u = (u_1', \ldots, u_w')$, $X = [1, x_{i1}, \ldots, x_{iw}]_{1 \le i \le n}$ and $Z = [Z_1, \ldots, Z_w]$. u however is assumed to be normally distributed with the variance expressed by a block diagonal matrix of $(\sigma_{u_1}^2 \mathbf{1}_{K_1}, \ldots, \sigma_{u_w}^2 \mathbf{1}_{K_w})$ and Z_v being the basis matrix of dimension (nxK_v). The resulting fit is obtained by

$$\hat{y} = M \left(M'M + \tilde{D} \right)^{-1} M'y \tag{1.71}$$

with $M = [X, Z]$ and $\tilde{D}$ being a block diagonal matrix of $(\mathbf{0}_{(w+1)x(w+1)}, \lambda_1 \mathbf{1}_{K_1}, \ldots, \lambda_w \mathbf{1}_{K_w})$. Again, the smoothing parameters $\lambda_v, v = 1, \ldots, w$ can be found with $\lambda_v = \frac{\sigma_\epsilon^2}{\sigma_{u_v}^2}$. Fitting an additive structure with B-spline bases is shown by Durban and Currie (2003) and can also be carried out in the context of mixed models. The estimation of the smoothing parameter by employing a GCV criterion can be found in Wood (2004) and Wood (2006). This approach is used for estimating the empirical models in chapters 2 and 3. A readily introduction for extending P-splines techniques to models with non-gaussian responses can be found by Zuur et al. (2008). Wood (2006) gives an introduction with close relation to software-application, as it is used in the following chapters. Fahrmeir et al. (2007) give details and applications about semi-parametric models, which include fixed (parametric) and smooth effects.

1.3 The German Socio Economic Panel

Empirical research with the aim of drawing valid and reliable conclusions for a variety of questions concerning socio-economic topics is heavily dependent on the database. In the current work the German Socio Economic Panel (GSOEP) is employed to answer the above motivated questions. Due to a broad variety of available alternative datasets for research focusing on German families, females and their employment, this section will justify the use of longitudinal panel data in general and the GSOEP in specific by a brief discussion of the dis- and advantages of possible alternative sources of data. I follow thereby Boll (2011).

The method of survey employed to gain data for empirical research is one crucial characteristic for valid and reliable data. Cross section data as one survey result on the one hand is often provided in aggregated form to capture characteristics of many people (or other statistical units) at one single time point. These 'snapshots' are therefore not capable of measuring changes in individual characteristics over time. Since the latter is one aspect of this work, longitudinal data in the style of panel data is used here instead. Panel data can consist of retrospective and/or repeatedly collected data, which are often gained by personal interviews or via questionnaires. The first is likely to cause problems due to lacking knowledge of the respondent concerning events being in the far past. Although these information, like the years of schooling an adult has completed when entering a panel study might inherent inconsistencies or even lacking data, they are necessary in every panel study as a starting point in the survey and to give basic information of the respondent's biography. Especially quantitative panel data is measured more frequently, e.q. monthly or annually. As a combination of both, retrospective data covering the year anteceding an interview can display individual characteristics more reliable and are used in most surveys. The amendment of retrospective data with repeatedly measured data is therefore likely to provide a suitable database for this work. As one example, the educational level of participants is measured in detail retrospectively at the time of entering the survey and is amended only if changes since the last interview have occurred. The GSOEP, as one micro-level panel provides in addition a variety of socio-economic individual and family-related variables on a longitudinal scale, combining the advantages and minimizing the disadvantages of longitudinal data. However, other datasets and surveys provide such data as well:

The German Federal Bureau of Statistics provides data for demographic and labor market analysis tracing back to 1957 for West- and to 1991 to East-Germany. The *Mikrozensus*, as a one-percent-sample of the German population is a large database compared to many other datasets. With both, 830000 participants living in 390000 households and an obligation to participate, the lack of responses is a rather minor problem. However, households and their members have to leave the survey after participating four years, impeding to draw conclusions based on long-term observations. Due to lacking information on taxes and other income-relevant factors, the *Mikrozensus* can not provide a suitable database to analyze females' individual employment characteristics and their changes over a time period exceeding four years.

Two major alternatives to the GSOEP and the *Mikrozensus* are provided by the German Federal Labor Agency: One the one hand, the *IABS* contains data tracing from the public German health insurance, the pension entitlements and the public unemployment insurance. The two-percent-sample of employees paying for the public social security system consists of 200000 people in West-Germany, measured annually since 1973. Although the *IABS* provides data gained from measurement instead of personal questionnaires or interviews and therefore yielding high consistent data, the *IABS* has three main disadvantages for the aspired analysis in this work: first, it is restricted to

employees paying for the public social security system and is therefore lacking data on minor employment contracts. As second aspect, non-employment details are not provided consistently, impeding to analyze maternity leave or other time-usage while being off the labor market. Third, the *IABS* lacks collateral socio-economic variables necessary to explain possible reasons for the length of maternity leave and changes in wages. On the other hand, an employment sample of the German Federal labor Agency provides only aggregated cross-section data based on quarterly selected target dates. The rather short survey starting in 1998 in combination with lacking additional socio-economic variables is therefore not a valid database for this study. An overview to additional minor German datasets and further references for the above introduced surveys is provided by Centre for European Economic Research / German Research Foundation (DFG) (2010).
The GSOEP however is a representative panel survey on a micro level for Germany, starting in 1984 for West-Germany and expanded to East-Germany in 1990. The aim of the annually performed survey is to provide suitable data for individual characteristics of the German population in the context of employment and (family) living. Although the GSOEP can be characterized as an *representative* panel, it is heavily faced with problems resulting from low sample sizes and high panel mortality, compared to the above discussed alternatives. Having started in 1984, interviewing 12290 individuals in 5921 households successfully, only 3337 households with 5963 participants remained before starting with the 25th wave in 2008. Due to constructing new subsamples and adding them to the original wave-structure (i.e. a sample focusing on people with migrational background) the GSOEP provides data from more than 20000 people living in about 11000 households in Germany by the year 2008.
The disadvantages concerning the sample size and its structure are compensated by the construction of the questionnaires and the following collection of the data, resulting in valid and reliable empirical material for socio-economic research. Especially the consistently provided data for the employment status and the individual and family related income situation are major advantages for this work. The carefully collected and in-depth provided individual employment history, which traces back in the respondent's biography to the age of 16, is crucial for the aspired analyses: the labor force experience

as retrospective data in combination with annual updates results in monthly spell data, which are especially useful for the analysis in chapter 2. The construction of variables capturing the labor force experience working full- or part-time, amended by several other *human capital variables* yields a suitable database for modeling wage profiles in the short- and long-run in chapter 3. For further details concerning the sample construction and the methods of survey employed in the GSOEP, Hanefeld (1987), Haisken-DeNew and Frick (2005b) and Wagner et al. (2007) give details. Restriction on the dataset resulting in the actually used subsets are discussed in the corresponding chapters 2 and 3 in detail and have to be interpreted in the context of the chapters, respectively.

2 Duration of Maternity Leave in Germany: A Case Study of Nonparametric Hazard Models and Penalized Splines

2.1 Introduction

In the second half of the 20th century the female labor force participation rates have risen constantly in all western European countries. Possible reasons are discussed by Fitzenberger et al. (2004) and Rubery et al. (1999). Besides this, the educational attainment has increased for both, males and females. This goes hand in hand with a longer duration of schooling and/or vocational training resulting in a shifted labor market entry to a higher age. Especially mothers of small children have expanded their labor supply disproportionately in Germany and other western countries. As discussed by Rubery et al. (1999) especially mothers of small children are dependent on social and intra-family norms. In this context Dingeldey (2000) considers Germany as a "conservative welfare state" with the consequence of disincentives for mothers to return to work after being in maternity leave, see also Kreyenfeld and Geisler (2006). With maternity leave here and in the following we understand the period where the mother is not working, that is paid or unpaid maternity leave in the classical sense but also parental leave and voluntarily non-employment due to child care. Besides a social and labor market dimension, the employment break and the reentry into employment after giving birth is traditionally a crucial point in mothers' biographies and has individual and family related aspects which motivates to look at data on both, individual and intra-family level.

This chapter focuses on two questions: First, how do individual and intra-family effects like the educational attainment, the personal as well as the household income before entering maternity leave and the presence of a working spouse or life partner influence the time point of returning to work after bearing a child. As second aspect of this part, we analyze the dynamic behavior of these effects and how they vary with the duration of voluntarily staying at home and possibly loose their importance on mothers' decisions when to reenter the labor market as the time off the job continues. The aspired analysis is carried out with data from GSOEP.

The analysis of female labor market participation has been pursued in numerous research publications before. Fitzenberger et al. (2004) and Fitzenberger and Wunderlich (2004) analyze this issue from a more macroeconomic and aggregate focus, while other publications like Beblo et al. (2006), El Lahga and Moreau (2007), Hank and Kreyenfeld (2000) or Kreyenfeld et al. (2007) focus on single effects concerning motherhood. The withdrawal of mothers from the labor market and the transition from maternity leave to employment has been under investigation before in a number of countries. We refer exemplary to Shirley et al. (1998) for the UK and to Desai and Waite (1991) for the US, respectively. Several recent articles focus on the loss of human capital, especially for highly educated mothers while being out of the labor market, see for instance Baum (2002) or Gutierrez-Domenech (2005). After giving birth, mothers face a trade-off between the costs of institutional child care and a proposed continuing loss of their personal human capital while staying at home. The personal income a woman was able to earn at the labor market prior to childbirth can be considered as the labor market value and mirrors as the opportunity cost of staying at home for childcare. As a result, mothers with high income should be more likely to return to the labor market. This goes hand in hand with the standard model of labor supply, which predicts an increasing probability of working with the wage or realized income. In contrast to recent articles, we allow the effect of personal income and the effects of all other covariables to vary over the duration of maternity leave. That is to say we capture how the effects influence the probability to return to professional life and how such effects change in time while controlling for unobserved heterogeneity.

The use of panel data ensures a reliable analysis of individual effects on the probability of re-entering the labor market after bearing a child. The analysis is based on data form the GSOEP, see Wagner et al. (2007) and Haisken-DeNew and Frick (2005a) for a detailed introduction. The GSOEP provides suitable data and allows to empirically explore the returning-to-work-decision on a microlevel. We analyze the duration of maternity leave of 689 and 517 mothers for first and second maternity leave, respectively.

The statistical model being used in this chapter is built upon the classical Cox model, see Cox (1972), but we allow for non-proportional hazards in the style of varying coefficients as suggested in Hastie and Tibshirani (1993), see also Gray (1994) or Therneau and Grambsch (2000). For fitting we make use of penalized splines to estimate smooth dynamic covariate effects as proposed in Kauermann (2005). See also chapter 1 for a short introduction. The modeling exercise extends this work by allowing for unobserved heterogeneity. To do so we include an individual latent factor which is modeled by a finite mixture of mass points and weights, as described by Bover et al. (2002) or Heckman and Singer (1984). Alternatives are e.g. modeling the latent factor as Gamma distributed to obtain a coherent estimation framework following Klein (1992).

Applying the estimation routine to the data at hand we can graphically investigate the dynamics of the overall probability of returning into paid employment after maternity leave. Looking at the covariate effects it is shown that the effects of realized personal income as well as educational attainment of the mothers significantly change over the duration of maternity leave. This allows for an advanced interpretation compared to the classical but in our case misspecified coefficients of a proportional hazard-model. Overall the decision for returning into paid employment underlies different effects also depending on the mothers' attachment to the labor market expressed here by a possible reentering in the labor market between the first and second leave period.

The chapter is organized as follows. In section 2.2 we give details about the data and show some exploratory analysis based on Kaplan Meier curves. Section 2.3 introduces penalized spline smoothing and suggests some ideas of model selection. Section 2.4 gives the data analysis before we conclude in section 2.5.

2.2 Panel Data and the Duration of Maternity Leave

The analysis of two different periods of maternity leave is based on data from January 1995 to December 2006. As maternity leave we define the period off the job due to pregnancy in the last weeks prior to the birth and subsequently the time after childbirth staying at home. In this definition maternity leave is not restricted to a job-protection-period granted by law and also includes unpaid maternity leave due to child care, where the mother stays (voluntarily) at home and is not available for the labor-market. As event we consider the return into any kind of paid employment (including full and part time employment as well as self-employment). The maximum duration times of maternity leave observed in the data are 62 and 72 months after first and second childbirth respectively and correspond to the latest events observed. In addition to the maximum duration times an observation can be censored due to panel mortality, a further childbirth within maternity leave or a transition into a involuntarily non-employed status, i.e. being registered as unemployed and seeking for a job. According to the GSOEP-questionnaires we distinguish therefore between being a housewife and officially being registered as unemployed, which corresponds to voluntarily and involuntarily being out of the official labor-market, respectively. The analysis is restricted to mothers who were employed (full or part time) before having their child, i.e. before entering maternity leave. This restriction ensures the aspired consistent analysis of employment interruption due to childbirth. The data consists of individual spells from West Germany starting 1995. As covariates we focus on information about inflation-adjusted income, education and other personal variables. The realized personal income of mothers prior to their first and second maternity-leave period is defined as the maximum amount of labor income the mother earned in the five years anteceding birth, measured in euros (see Projectgroup SOEP - DIW (2009) for detailed outline and information about imputations.). The personal income in our discussion can be considered as the personal labor market value, which the mother has been able to realize at the labor market in advance of bearing a child. Apparently, this also mirrors as the opportunity cost for the female for staying at home and is linked to the shadow prices introduced in chapter 1. For the analysis the personal income is categorized into three levels: less than 1300 euros monthly income, between 1300 and 2400 euros monthly income (taken as reference category) and more than 2400 euros monthly income before entering maternity leave. Mothers, who withdrew voluntarily from the labor market between their childbirths for more than five years are excluded from the analysis of the second maternity leave. The thresholds correspond roughly to the 25% and 75% quartiles based on the data. Additionally to the personal income we look at the household income of the household the mother lives in while being off the job. We define household income as the maximum value of the provided generated net household income in the five years anteceding birth, see Projectgroup SOEP - DIW (2007) for details. The household income is categorized into three groups: less than 2100 euros monthly household-income, between 2100 and 3600 euros monthly household income (taken as reference category) and more than 3600 euros monthly household income. The thresholds again correspond to the first and third quartiles of both data sets.

The educational attainment of a mother is measured with the "International Standard Classification of Education (ISCED)" which is available in the version of 1997 for the GSOEP data and used for our analysis (see UNESCO Institute for Statistics (2006) for details). Our analysis is carried out on three different groups of ISCED-levels: a lower group consisting of levels 1 and 2, a medium group consisting of levels 3 and 4 (taken as reference category) and finally a higher group with levels 5 and 6. The age of the mother at time of her first or second childbirth is also categorized into three groups: younger than 26 years, between 26 and 32 years (taken as reference category) and older than 32 years with the thresholds corresponding to first and third quartiles. Besides a binary factor indicating whether the mother has a migration background, we construct a variable focusing on the spouse or life partner of the mother at time of the birth. We differentiate between mothers having a partner that is working at time of childbirth and mothers who do not have a working partner (including mothers living without a spouse or life partner). Two additional covariate effects are constructed for mothers being in their second maternity leave: First, an effect indicating whether the first child is older or younger than 3 years of age at the time point of the second delivery is added. Secondly, we observe whether the mother has been available for the labor market since the first maternity leave, i.e. being employed or seeking for a job and registered unemployed. Simple Kaplan-Meier estimators are shown in Figure 2.1. The structure of this plot and subsequent one is as follows: The first and third column display the effect for the first maternity leave, the second and fourth column give results for the second maternity leave period. The overall survivor curves (top left row) show a strong decrease and a jump like decrease at 4 and 36 months, which corresponds to the length of the mother-protection-period and the job-protection-period granted by law in Germany respectively. Overall, the decrease for the second child is weaker, i.e. more females remain in maternity leave, especially after 36 months of duration. The estimated probability of extending the maternity leave after 36 months is approximately 50% after first childbirth and about 60% after second bearing. Looking at the effect of education, higher educated women tend to return to work earlier than lower educated ones. Concerning income, the Kaplan-Meier curves of mothers in the high wage group are nearly always below the curves of individuals with lower personal income, concluding that a higher labor market income prior to maternity leave lets mothers return to work earlier. This conclusion can also to drawn from looking at the household income concerning at least for the second child. Interestingly enough the effect is not seen for the first maternity leave. While no clear difference can be found by looking at the effect of the migration background of the mother, the age at the time of giving birth seems to have a (small) effect after second childbirth indicating that older mothers return to the labor market earlier than younger mothers. Looking at mothers who do not have a working partner, the corresponding effect indicates a higher chance of returning to the labor market soon after childbirth. Mothers who worked between the end of their first and the beginning of their second leave period, or at least have been seeking for a job, reveal a strong attachment to the labor market with a higher probability for returning to a job after second delivery soon. Finally the presence of a first child younger than 3 years seems to have an effect on the duration in maternity leave, since mothers tend to stay at home as the second leave-period continues and both children are at home. In the remaining of the chapter these data are modeled using non-proportional as well as proportional hazard effects.

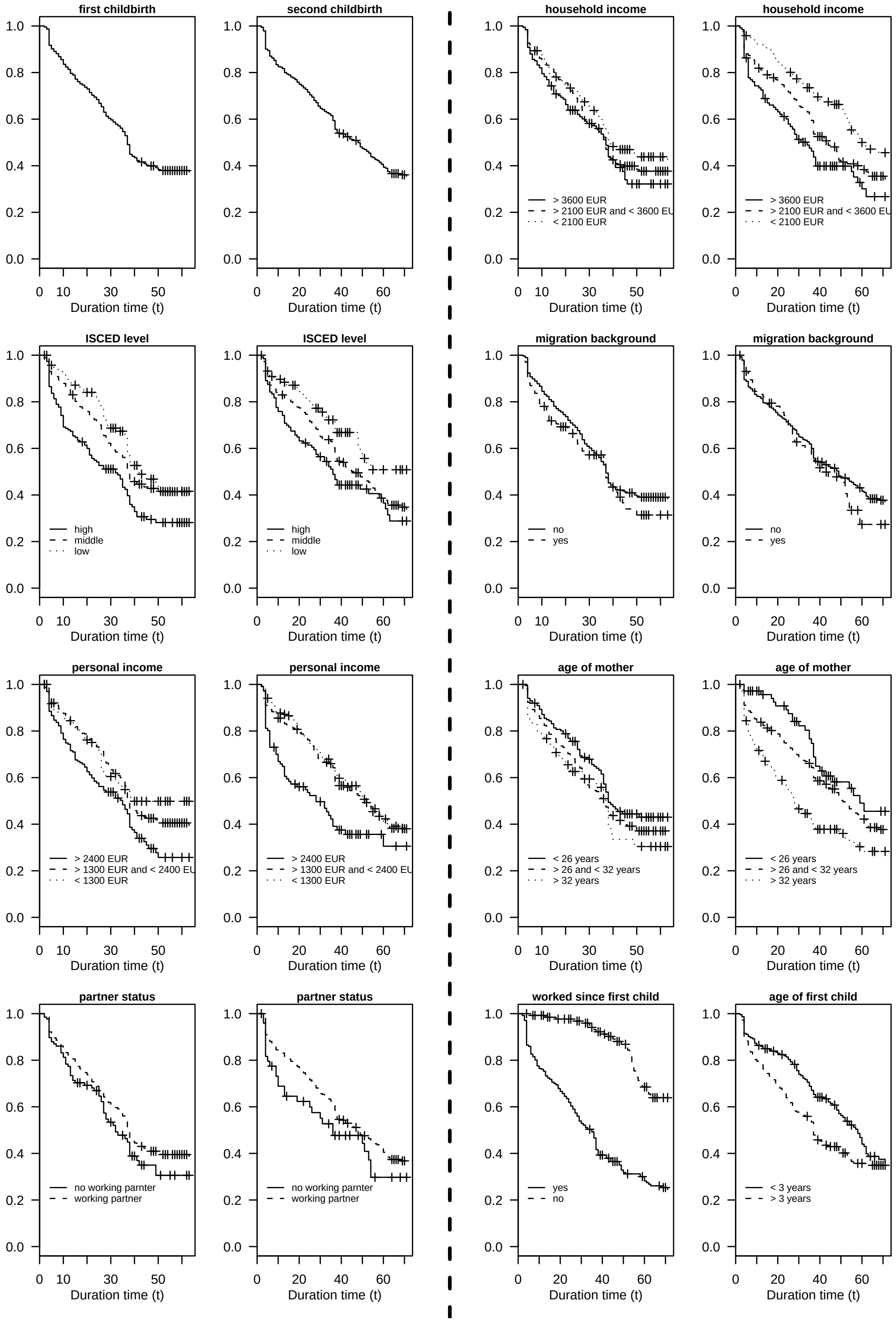

Figure 2.1: Kaplan-Meier curves for first and second maternity leave (left-hand and right-hand columns respectively)

2.3 Dynamic Hazard Model and Penalized Spline Smoothing

We denote with $h(t, x)$ the hazard function which mirrors the probability of returning to professional life after t months in maternity leave. The hazard depends on the covariates discussed in the previous section, notated here with x. The typical Cox type model takes the form

$$h(t, x) = \exp\{\beta_0(t)\} \exp\{x\beta_x\}, \tag{2.1}$$

where $h_0(t) = \exp\{\beta_0(t)\}$ is the baseline hazard and β_x give the covariate effects, see Cox (1972). The effects expressed in β_x are constant over time, so that model (2.1) implies proportional hazards. Looking at Figure 2.1 the proportionality assumption seems questionable since the Kaplan-Meier curves do not mirror proportionality. We therefore allow covariate effects to change with the duration of maternity leave. This interaction of effects is incorporated in the model in a functional form by setting

$$h(t, x) = \exp\{\beta_0(t)\} \exp\{x\beta_x(t)\}, \tag{2.2}$$

where $\beta_x(t)$ is a functional effect, which is assumed to change smoothly, that is not rapidly, with duration time t. Estimation is carried out using penalized splines. We follow thereby closely Kauermann and Khomski (2006): For simplicity of notation and presentation of the penalized spline idea we ignore covariates x in model (2.2) for the moment. The underlying idea for estimation is to replace the unknown smooth function by some high dimensional parametric function. This means, for instance, we model $\beta_0(t)$ as $B_0(t)u_0$ with $B_0(\cdot)$ as high dimensional basis. For fitting we impose a penalty on coefficient vector u_0 which guarantees that the resulting fitted curve $\hat{\beta}_0(t) = B_0(t)\hat{u}_0$ is smooth. This is achieved by adding the penalty component $\lambda_{0u} u_0^T D_{0u} u_0$ to the log likelihood, with D_{0u} as penalty matrix and λ_0 as penalty parameter steering the amount of smoothness.

Denote now with (t_i, δ_i) the observations (again omitting covariates for simplicity of presentation), where t_i is the length of maternity leave and δ_i the censoring indicator. The penalized likelihood for coefficients u_0 results now with classical theory, see Cox and

Oakes (1984), to

$$\ell(u_0, \lambda_{0u}) = \sum_{i=1}^{n} \left\{ \delta_i B_0(t_i) u_0 - \int_0^{t_i} \exp(B_0(z) u_0)\, dz \right\} - \lambda_{0u} u_0^T D_{0u} u_0. \tag{2.3}$$

For estimation two further aspects have to be considered. First, one has to numerically solve the integral in (2.3) resulting from the integrated Hazard function. A simple and numerically feasible way to do so is to use a trapezoid approximation. In formula this boils down to discretizing the continuous time scale. In our example we use trapezoids of width corresponding to one month which is also the finest resolution of the time scale. The second aspect is to select the smoothing parameter λ_{0u} appropriately, that is data driven. This can be done by comprehending the penalty as a priori normality imposed on the coefficient. In this case λ_0 becomes a parameter which can be estimated by maximizing the corresponding likelihood. In particular, trapezoid approximation and writing the penalty as a priori normal distribution lead to a generalized linear mixed model (GLMM) and the model can be easily fitted with available software.

To be more specific, let $0 = \tau_0 < \tau_1 < \ldots < \tau_K$ denote integration point at which we anchor our trapezoid approximation. In principle these can be the observed time points, even recommendable if duration times are observed on a discrete, rounded scale, like months. The integral component in (2.3) becomes now with trapezoid approximation and some simple calculus

$$\int_0^{t_i} \exp\left(B_0(z) u_0\right) dz \approx \sum_{k=0}^{K_i} \exp\left(B_0(\tau_k) u_0 + o_{ik}\right)$$

where $o_{i0} = \log\{\tilde{\tau}_{i0}\}$ and $o_{ik} = \log\{1/2[\tau_{i(k+1)} - \tilde{\tau}_{ik}]\}$ is a known offset with $\tilde{\tau}_{ik} = \min(\tau_k, t_i)$ and $K_i = \text{argmax}\{t_i \leq \tau_k\}$. Inserting this sum into (2.3) yields a penalized likelihood for artificial random variables d_{ik} taking values $d_{ik} = 0$ for $k < K_i$ and $d_{ik} = \delta_i$ for $k = K_i$ and having the Poisson distribution

$$d_{ik} | u_0 \sim Poisson\left(\lambda_{ik} = \exp\left\{B_0(\tau_k) u_0 + o_{ik}\right\}\right) \tag{2.4}$$

The next step is to formulate the penalty as normal prior leading to

$$u_0 \sim N(0, \lambda_{0u}^{-1} D'_{0u}) \tag{2.5}$$

with D' as (generalized) inverse. With (2.4) and (2.5) we obtain a Generalized Linear Mixed Model (GLMM) and the smoothing or penalty parameter becomes an a priori variance component which could be estimated following the likelihood principle. This idea has proved to be quite powerful, both in theory as well as in its numerical performance. For further details we refer to Wand (2003) and Kauermann (2005). The model can now be fitted using software for GLMMs in the style of Breslow and Clayton (1993). The idea is to treat spline coefficient u_{0t} as random so that the likelihood to be maximized results by integrating out the random terms. The latter is done by Laplace approximation. Clearly, the idea of penalized splines and its connection to GLMMs extends to model (2.2), that is for fitting the smooth covariates effect $\beta_x(t)$. A user-friendly implementation to fit the model is provided in R, see R Development Core Team (2008), with the R-package `TwoWaySurvival`, which can be downloaded from the CRAN server at `www.r-project.org`, see Khomski (2008) for details. The package is an enhancement of the routines provided with Ruppert et al. (2003) and allows to fit the model easily and relatively quickly. Moreover, using standard asymptotic arguments, one can derive variance formulas from the estimates, making use of asymptotic normality statements. This allows not only to fit functional shapes but also to provide confidence bands for the functional effects.

The model (2.2) is on a population basis and does not incorporate individual latent effects, that is unobserved heterogeneity among the females. We therefore extend (2.2) so that the i-th female has the individual hazard

$$h_i(t, x_i) = h(t, x_i)exp(v_i) \tag{2.6}$$

with v_i as unobserved latent effect with $E(v_i) = 0$ to maintain identifiability. There are two common distributional scenarios for v_i. The first is to assume a Gamma distribution, which is the conjugate distribution to Poisson and therefore allows for numerically simple estimation, see e.g. Klein (1992). Alternatively, and less restrictive in terms of the distributional shape is to approximate the continuous distribution of v_i by a finite mixture of mass points and weights, i.e. probabilities, see e.g. Bover et al. (2002) or Heckman and Singer (1984).

τ_k	-1.19 (0.18)	-0.04 (0.11)	0.12 (0.06)	0.37 (0.07)
$P(v_i = \tau_k)$	0.19 (0.02)	0.17 (0.02)	0.10 (0.03)	0.16 (0.02)

τ_k	0.39 (0.07)	0.43 (0.09)
$P(v_i = \tau_k)$	0.22 (0.03)	0.17 (0.03)

Table 2.1: Mixture Distribution of unobserved heterogeneity with bootstrap standard deviation in brackets, first childbirth

We now extend model (2.4) by assuming λ_{ik} to depend on some unobservable heterogeneity as well. We replace (2.4) by

$$d_{ik}|u_0, v_i \sim Poisson\left(\lambda_{ik} exp(v_i)\right) \tag{2.7}$$

with v_i being random coming from an unspecified distribution with $E(v_i) = 0$ for identifiability reasons. We assume that v_i is discrete valued with $P(v_i = \tau_k) = \pi_k$ for $k = 1, ..., K$ with the additional constraint

$$\sum_{k=1}^{K} \tau_k \pi_k = 0. \tag{2.8}$$

Fitting can be easily carried out with an EM algorithm. The choice of K is discussed for instance in McLachlan and Peel (2000). This approach is also known as Nonparametric Maximum Likelihood Estimation (see e.g. Laird (1978) for details) with variance estimation treated for instance in Friedl and Kauermann (2000). We pursue the latter approach which results in the discrete mixture distribution given in Table 1 and 2, with standard deviations derived by bootstrapping (100 bootstraps).

2.4 Data Analysis

Maternity leave in Germany is mostly regulated in two federal laws. An important role plays the law on the protection of expectant and nursing mothers (*Mutterschutzgesetz*

τ_k	-0.93 (0.14)	0.06 (0.13)	0.26 (0.07)	0.28 (0.08)
$P(v_i = \tau_k)$	0.24 (0.02)	0.12 (0.03)	0.17 (0.04)	0.15 (0.03)

τ_k	0.32 (0.07)	0.49 (0.10)
$P(v_i = \tau_k)$	0.14 (0.02)	0.18 (0.02)

Table 2.2: Mixture Distribution of unobserved heterogeneity with bootstrap standard deviation in brackets, second childbirth

MuSchG), which originally was introduced in 1952. This law regulates the rights for pregnant women and mothers after delivery. It has been modified several times in the last decades, mostly concerning the type of work a pregnant female is allowed to do on the job. In combination with the federal law on child support (*Bundeserziehungsgeldgesetz BErzGG*), which is replaced by the federal law of parental leave and financial support (*Bundeselterngeld- und Elternzeitgesetz BEEG*) coming into effect January 2007, mothers and fathers have the right to leave their paid employment, partly with ongoing payment, see John and Stutzer (2002) and Gottschall and Bird (2003) for a discussion on the legal framework. In 1993 the job protection period was expanded to 36 months. Other minor changes concern the amount of parental leave benefits, see Buchner and Becker (2008) for details. It is reasonable to assume that changes in the federal regulation of maternity and parental leave change the individual behavior of mothers, but the period we consider (1995 - 2006) did not see drastically amendments to the law so that we can assume a time constant legal framework for employed mothers. The aim of our analysis is now to analyze the effects of individual and family-related covariates on the decision to go back to the job within the legal framework.

In Figure 2.2 we present the resulting fit of model (2.6) including 95% (pointwise) confidence bands for first and second maternity leave in comparison. The distributions of the covariates are listed tables 2.4 and 2.4, while the estimated significant proportional effects from a Cox-model are added as dotted lines in figure 2.2. These estimates may be seen as benchmark and our smooth estimation clearly indicates that the proportional

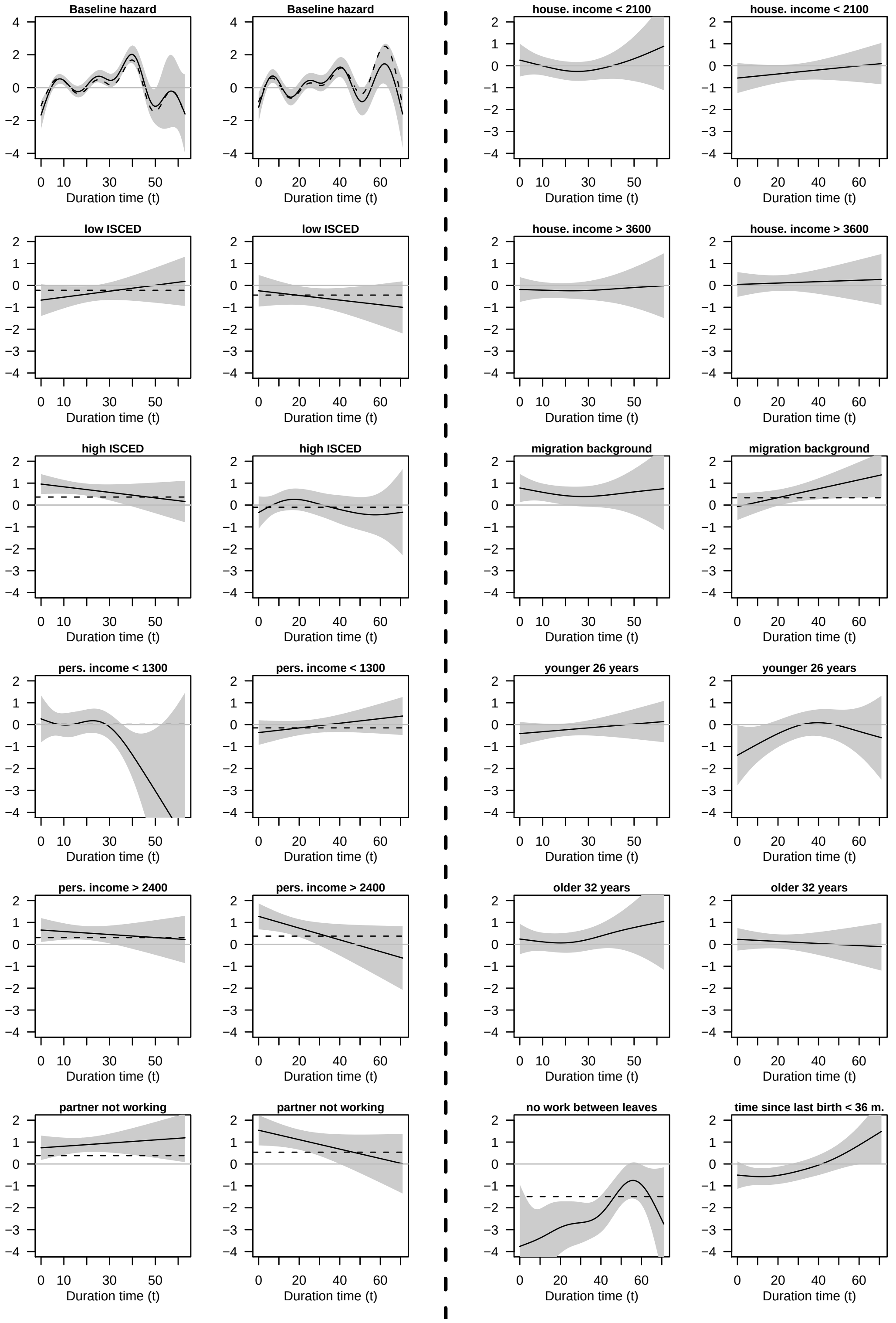

Figure 2.2: Fitted dynamic effects for duration time in maternity leave (in months) after first and second childbirth (first and second columns respectively)

hazard assumption is void. The first and third column show the fitted effects for first maternity leave, the second and fourth column give the corresponding fitted effects for the second child related maternity leave. The first plots in the first and second column show the baseline effects $\beta_0(t)$. These effects represent German-born mothers with a personal monthly income between 1300 and 2400 euros with an average achieved ISCED-level, who gave birth in the age between 26 and 32 and live with a working partner in a household with a monthly (net) income between 2100 and 3600 euros. Additionally, the baseline effect for mothers analyzed for second childbirth represents mothers who have been available for the labor market between their two leave periods and have a first child older than 3 years. We see a steep increase in the probability of returning into paid employment until about 4 months. Note that a fixed mother-protection period starts 6 weeks in advance of the scheduled birth date and ends 8 weeks after childbirth. During that time mothers are not allowed to work officially and they can return to their previous job at the earliest after being 4 months in maternity leave. Both baselines (first and second column on child respectively) also show a strong peak at 36 months of duration time. This time point indicates the end of the law-regulated job-protection period.

The effect of the educational attainment are different for low- and high educated mothers. The effect of females being in the low ISCED group shows a linear dynamic behavior postulating a weak negative effect on the chance to return into a job. On the other hand, highly educated mothers seem to be affected by their educational attainment , resulting in a higher chance to return into professional life. The effect of high ISCED level is less significant after bearing a second child.

Significant effects can be observed by looking at mothers with a high personal income realized at the labor market anteceding birth: while observing an almost constant positive effect after first childbirth, the effect is only strong positive in the first two years of second maternity leave. Afterwards the effect weakens and even becomes insignificant. In contrast, mothers with low personal income are not affected by their realized income significantly.

An almost constant positive effect can be observed by looking at the presence of a partner who is working at time of first birth. The positive effect of a working partner weakens

			job status		job returns in year			
		all	no return	return	1st	2nd	3rd	4th or later
ISCED-level	low	121	75	46	12	8	12	14
	middle	382	220	162	52	42	43	25
	high	186	82	104	56	20	14	14
personal	< 1300	125	79	46	18	10	15	3
income	1300 - 2400	335	180	155	48	33	41	33
[EUR]	> 2400	229	118	111	54	27	13	17
working	yes	557	306	251	94	60	53	44
life-partner	no	132	71	61	26	10	16	9
household	< 2100	155	88	67	25	13	17	12
income	2100 - 3600	346	184	162	55	39	40	28
[EUR]	> 3600	189	108	81	37	18	12	14
migration	yes	104	56	48	23	8	7	10
background	no	585	321	264	97	62	62	43
age of mother	< 26	210	127	83	29	15	24	15
[years]	26 - 32	370	192	178	67	44	39	28
	> 32	109	58	51	24	11	6	10
	Total	689	377	312	120	70	69	53

Table 2.3: Distribution of covariates for first maternity leave

		all	job status: no return	return	job returns in year: 1st	2nd	3rd	4th or later
ISCED-level	low	90	59	31	10	5	6	10
	middle	298	144	154	50	26	31	47
	high	129	59	70	32	15	12	11
personal income [EUR]	< 1300	186	99	87	23	18	17	29
	1300 - 2400	223	114	109	33	18	22	36
	> 2400	108	49	59	36	10	10	3
working life-partner	yes	467	241	226	77	43	43	63
	no	50	21	29	15	3	6	5
household income [EUR]	< 2100	121	68	53	9	13	10	21
	2100 - 3600	271	137	134	50	20	27	37
	> 3600	125	57	68	33	13	12	10
migration background	yes	87	41	46	14	8	11	13
	no	430	221	209	78	38	38	55
age of mother [years]	< 26	72	45	27	3	4	8	12
	26 - 32	304	157	147	48	25	25	49
	> 32	141	60	81	41	17	16	7
time since last birth [months]	< 36	226	123	103	31	9	26	37
	> 36	291	139	152	61	37	23	31
worked between the leave periods	yes	381	157	224	91	44	37	45
	no	136	105	31	1	2	5	23
	Total	517	262	255	92	46	49	68

Table 2.4: Distribution of covariates for second maternity leave

as the second maternity leave of the mother exceeds 3 years. In contrast, the income of the entire household where the mothers lives in does not effect the decision when to reenter the labor market.

A weak positive effect can be seen by looking at mothers with a migration background being in maternity leave for first and second leave period respectively. No clear effect can be found by looking at the age of the mother. Mothers being in their second maternity leave while having a child younger than 3 years are affected by the age of their first child until about 36 months after second delivery with a negative effect. Finally, the most dominant factor effecting the length of the second leave period is the labor market attachment of the mother since her first childbirth: Mothers who have not been available for the labor market in this time window are significantly negatively affected with an increase of the effect as the duration time continues.

2.5 Conclusion

The fitted smooth baseline and covariate effects reveal a two-dimensional framework for reentering the labor market after maternity leave: First, the legal framework of maternity leave in Germany drives some mothers back to their job after the maternity-protection-period which ends at 4 months and after the job-protection-period-ends which lasts 36 months. This is mirrored by the baseline effects. Secondly, the personal labor market income before entering maternity leave and the educational attainment of the mother are the most dominant factors on the decision when to return into a paid employment after first birth. Mothers with high personal income earned prior to the childbirth are more likely to return to a job after first childbirth. Assuming the income as opportunity costs of not working due to child care, high opportunity costs (and therefore high shadow prices for childcare) force mothers back to their jobs after giving birth to their first child. For the second child the effect of personal income is only strong in the first two years being in maternity leave and fading away thereafter, indicating decreasing importance of high income earned prior to childbirth and decreasing opportunity costs as duration time continues. The effect of high realized personal income show a dynamic behavior with vanishing importance as duration time continues. Low-paid mothers show weaker incentives to get reemployed after first and second childbirth. For these mothers, the opportunity costs of not-working are lower and the effects of low income are weaker than for mothers expecting high salary after maternity leave. It is not surprising that highly educated mothers are more likely to return to a job shortly after their childbirth (personal income and educational level are highly correlated in the data), establishing a dual-earner-model in contrast to the male-breadwinner-model after the first child. Looking at our fitted effects however, our analysis reveals a dynamic and vanishing effect of high education as maternity leave continues. A classical dual-earner-model, as discussed by Kreyenfeld et al. (2007) can only be obtained for families with mothers returning to work shortly after delivery. However, our analysis underlines Dingeldey (2000), considering Germany as a "conservative welfare state" with disincentives for mothers to work when the spouse or life partner contributes to the family income. Mothers without a working partner, including single mothers, strive back to the labor market sooner than mothers living with a partner being employed. Therefore, not only individual factors effect the length of the maternity leave. This underlines the proposed dependence on intra-family norms. In contrast however, the intra-family effect of household income only has a minor impact on the decision to reenter paid employment. The individual human capital of the mother, built up before entering motherhood and realized at the labor market in addition with the status of her spouse or life-partner is crucial for reentering the labor force in Germany after first childbirth. As Gustafsson et al. (1996) conclude, this holds for the UK as well, but not necessarily for all western countries. However, this does not hold as the family planing continues and the mothers enter their second maternity leave period. The attachment to the labor market, expressed by a readoptment or seeking of work between the two leave periods is more crucial to the decision when to reenter the labor market after second childbirth. A working partner however allows for an elongation of maternity leave during the entire leave-period after both, first and second childbirth, independently of income and educational attainment. A complete withdrawal from the labor market between the leave periods keeps mothers off the job as the child care of the second child continues.

We are reluctant to explain the different performances solely by different personal income, educational achievements and a readoptment of labor between two employment-breaks even though it seems plausible that this contributes to it. It is worth noting that the discussed socio-economic effects loose their impact as the mother continues to withdraw from the labor market voluntarily. The analysis therefore ends with the explanatory message based on our data analysis but does not go deeper into political and economical explanation. An investigation of major changes due to changed role allocation within German families and changed federal laws as well as a comparison of mothers living in West and East Germany is left for further research. The analysis however demonstrates the flexibility and capacity of penalized spline smoothing as estimation routine for functional data. This ensures the detection of time changing effects that even turn from positive to negative and vice versa during the analyzed periods of maternity leave. Especially the most crucial effects that influence mothers' decision to reenter the labor market (personal income, educational achievement, working between the leave periods) show a dynamical behavior. This cannot be observed specifying a classical proportional hazard model. A Cox-model averts a detailed analysis of the behavior of females facing an employment break due to childcare. Given that the software is available and the analysis did not require additional implementation, it seems inviting to make use of the non-proportional hazard model in other settings as well.

3 Female Wage Profiles: An Additive Mixed Model Approach to Employment Breaks Due to Childcare

3.1 Introduction

The rising labor market participation of females in western countries in the last century as described by Fitzenberger and Wunderlich (2004) and Rubery et al. (1999) forces females to combine family-related responsibilities with employment and to allocate their time between the labor market and the household. As a result, this issue is still not only discussed in the economic literature but is also a main political topic in many countries with different legislation for maternity leave rights.

Besides the more country-specific legal frameworks of parental leave policies, the wages females earn are the most valid economic value research focuses on when analyzing female labor force participation. Females' wage profiles and their labor supply have been discussed thoroughly in the economic literature over the last years. Corcoran et al. (1983) and Bloemen and Kalwij (2001) focus on female labor supply taking family decisions into account. The gender wage gap has been under investigation by Lundberg and Rose (2000), Kunze (2002) and more recently by Munasinghe et al. (2008). As a result, females have to face both, a wage penalty when interrupting their career due to child care (or other family related responsibilities) as well as consequences for their working career after having returned to the labor market.

In this chapter we analyze the effect of motherhood on wages. More precisely we investigate how temporary withdrawal from the labor market due to childcare and family reasons influences both, (a) the wage of a mother in the short-run when returning into work and (b) the ongoing wage profile in the long-run. Although the legal framework for parental leave has changed over the last decades in most countries, mothers are still faced with severe economic consequences after an employment interruption due to childbirth. We therefore restrict our analysis to women.

In this part we focus on three questions: First, how does an employment break due to childcare effect the wage of mothers in the short- and the long-run, dependent on their achieved educational level. Secondly, we analyze the dynamic behavior of covariates, like the labor force experience and other exogenous variables which are likely to directly influence the wage when reentering the labor market. Finally we look at a possible catch up of mothers with their wages compared to females not having children. The aspired analysis is carried out with the longitudinal data from the GSOEP, as being motivated in section 1.3.

The analysis of female wage profiles which are affected by employment breaks has been carried out for instance by Lundberg and Rose (2000) and Munasinghe et al. (2008) who explain wage reductions by human capital theory, referring to Becker (1993). Beblo and Wolf (2000) investigate how periods of non-employment and part-time work effect the gross hourly wage rate of females based on German data. The common consensus when analyzing employment breaks is empirical evidence about net-depreciated firm-specific and transferable human capital resulting in wage penalties when mothers return to work. These findings are independent from the reason of withdrawal, which are usually motherhood or unemployment. Kunze (2002) differentiates between reasons for employment breaks and concludes that females can lose more than 10 percent of wage per year due to maternity leave compared to the wage earned before leaving the labor market temporarily.

Classical human capital theory, as described by Becker (1993), allows to interpret wage losses due to employment-breaks as both: depreciation of the human capital accumulated prior to the interruption as well as lost rates of return due to failed human capital investigation when being off the job. The latter is likely to effect the wages in the long-run while net-depreciation of human capital is assumed to result in short-run losses just when the female reenters the labor market. To capture the effect of accumulated human capital and interrupted employment on personal income or wages empirically, the majority of authors, like Kunze (2002), Francesconi (2002), Ondrich et al. (2003) and Kreyenfeld et al. (2007) follow Mincer (1974) and Mincer and Ofek (1982). Thereby, many authors differentiate between firm-specific and transferable human capital when analyzing wages. The firm-specific human capital focuses on the experience an employee has earned while working in the current job due to on-the-job-training and the adaptation of job-specific skills, which only fit for the current job. In contrast, general or transferable human capital intends to capture the skills of the employee, which can be used in other jobs as well. The latter definition of human capital refers to the years of schooling, the age and the full-time work-experience in the past. This type of accumulated human capital is assumed to be transferable between different jobs in the labor market. More years of schooling, higher achieved educational levels as well as long full-time work experience are considered to have positive effects on the wage. In contrast, periods of unemployment and other employment interruptions result in wage losses due to human capital net-depreciation and lost rates of return.

In this chapter we investigate empirically if and how these rather static and fixed effects of depreciation are appropriate to describe the economic consequences for females around labor market transitions related to childbirth. We therefore use longitudinal panel data to obtain a reliable database of individual female wage losses and covariates affecting the ongoing wage profiles after employment breaks. The GSOEP provides suitable data and allows us to empirically explore the wage on a microlevel. We analyze wages from 1984 to 2008 for 3998 females contributing to 23445 observations in the study. For details about the GSOEP we refer to Wagner, Frick, and Schupp (2007) and Haisken-DeNew and Frick (2005b) and to section 1.3 of this work.

The statistical models employed in this article are built upon the classical regression model for longitudinal data (see Diggle et al. (2002).) Instead of a restrictive linear structure we allow for smooth functional effects which can capture potential non-linearities in the data without the challenge of specifying the structure of the model a priori. Note that non-linearities are likely so that the functional approach pursued in this chapter seems appropriate. For example, the experience a woman has earned while working full-time is supposed to have an increasing, but concave-running effect on the wage as her working career continues. Models with smooth, functional covariate effects have been coined 'varying coefficient models' by Hastie and Tibshirani (1993). The models allow for flexible fitting even in the presence of complex interaction terms. We refer again to Ruppert et al. (2003) for a readily introduction. For fitting we make use of penalized splines to estimate the smooth functional covariate effects as described by Wood (2006). Besides the functional estimation approach we have to take the unbalanced panel structure of the data into account also yielding unobserved heterogeneity. We do this by including unobserved individual (random) effects to the models. This yields a mixed model described in the statistical literature for instance in Wood (2006), Jiang (2007) and Zuur et al. (2008).

As a result of our data analysis we can graphically investigate the dynamics of the main covariates affecting wages and wage losses. Looking at the estimation results we underline that the effect of full-time work experience in females' biographies follows the assumed non-linear, concave shape and matches therefore with economic theory given for instance in Becker (1993). In addition, the duration of the employment interruption due to childcare affects the wage differently, depending on the levels of educational achievement. Most effects of the considered covariates reveal different labor market characteristics for mothers and non-mothers, modifying results of previous studies. Our models allow for an advanced analysis and interpretation compared to traditional parametric models which are likely to mis-specify severals coefficients and therefore lead to questionable conclusions concerning the underlying economic theory.

The chapter is organized in five sections: in section 3.2 we give details about the data used in this chapter and show some descriptive statistics. Section 3.3 introduces penalized spline smoothing within additive mixed models and explains the resulting statistical models. Section 3.4 gives the data analysis resulting from the models before we conclude in section 3.5.

3.2 Panel Data and Female Wages

3.2.1 Data Base and Variables

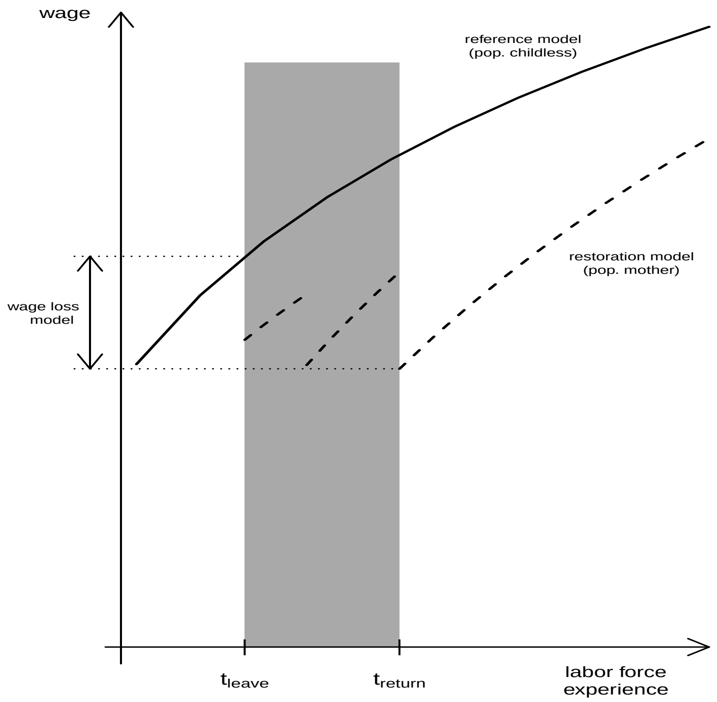

Figure 3.1: Female wage profiles with interruptions

In this part of the work we analyze female wages from three different aspects, as sketched in Figure 3.1: First, we model the wage a female receives in her working-biography if she has not given birth to a child (yet). This wage profile is drawn as solid line in Figure 3.1 and is defined as the reference model referring to the population of the childless women. Secondly, we consider females pausing their professional activity at time point t_{leave} due to maternity leave and possible subsequent child care. We model the wage loss a woman has often to accept when she stops working at t_{leave} and finally consequently starts working again at t_{return}. We define the wage loss as the (log) ratio of the wage earned when returning into a job in relation to the wage received before leaving the labor force temporarily at t_{leave}. We refer to this as the wage loss model. Finally, we capture the post-birth wage profile of mothers with restoration of human capital resulting in a possible catch-up to the wage level of the childless population. It is worth noting that t_{return} is defined as the time point in females work-experience of *final* return into the labor market with an assumed completed family planning. Due to this definition, t_{leave} can equal t_{return} if the mother stays out off the labor market for exclusive childcare and returns without additional gains in labor force experience. In contrast, taking multiple maternity leave periods off the job with multiple reentries into employment leads to $t_{return} > t_{leave}$, due to additional labor force experience gained since t_{leave}. We abstract from analyzing the working-behavior between t_{leave} and t_{return} as being sketched in the gray box in Figure 3.1 in detail. Instead, we make use of the additionally accumulated labor force experience by modeling the post-birth wage profile for the population of mothers if $t_{return} > t_{leave}$, named the restoration model throughout the chapter.

The analysis is based on data collected annually by the GSOEP, covering the years from 1984 to 2008. If available, we make use of the provided monthly spell data on individual labor force status, too. As response variable for modeling wage profiles and wage losses we calculate the hourly gross wages of females earned at the labor market. We use the contract-based weekly working hours to receive reliable values of the wages. This restriction excludes females being self-employed in the current year. As second aspect of the wage, we use the monthly gross labor income in euros. The income is inflation-adjusted by the German consumer price-index to the year 1995 and is provided by the GSOEP in euros.

As covariates we consider variables which are assumed to influence the individual wage in the context of human capital theory directly, as used by Mincer and Ofek (1982) and Becker (1993). We therefore abstract from variables which are assumed to influence the wage indirectly or rather focus on the reentering-decision, like the income of the spouse or the availability of childcare facilities. For a modeling of duration time of maternity leave and the point of time reentering the labor market we refer to chapter 2.

Human capital is generally accumulated by the educational achievements or the years of schooling and the labor force experience. See Ben-Porath (1967) and Mincer (1974).

As a proxy for the completed years of schooling and vocational training we divide the population into three different strata by looking at the achieved educational level of a female in the current year. The educational attainment of a woman is measured again with the „International Standard Classification of Education (ISCED)" which is consistently available in the version of 1997 for the GSOEP data and used for our analysis, see UNESCO Institute for Statistics (2006) and Projectgroup SOEP - DIW (2009) for details and definitions. Our analysis is carried out separating the females into three different groups of ISCED-levels: a lower group consisting of levels 1 and 2, a medium group consisting of levels 3 and 4 and finally a group with levels 5 and 6 representing high educated females. We exclude females with ISCED-level 0, which represent young females still being in school and therefore not available for the labor market full-time. The separation into three different strata is due to assumed major structural differences appearing in both, the population and the labor market depending on the different educational attainments. Taking this differentiation into account we will estimate the models for the three strata of the corresponding population separately. In addition to the education, the experience at the labor market and the experience within the current firm a female has collected are postulated to be major determinants when analyzing wages with respect to general and firm-specific human capital. The past labor market experience is separated into full- and part-time experience of the individual up to the current year. The variables *fulltime* and *parttime* give the entire time period of full- and part-time employment in the females' career up to the annual time point of the interview, respectively. To capture the stock of firm-specific human capital we include *firmtime* comprehending the years an employee has been working in the current firm. To contrast the withdrawal from the labor market due to childcare to unemployment spells of the female, we include the variable *unemployment* expressing the years a female has been registered unemployed and hence is off the labor market involuntarily. By taking individual unemployment into account, the model can also capture negative stigma-effects on wages related to unemployment, discussed in Belzil (1995). In all models being employed, we use the experience-variables *fulltime*, *parttime*, *unemployment* and *firmtime* for the entire recorded biography of females. Therefore we can capture possible temporarily reentries to the labor market between t_{leave} and t_{return} without going in detail for that time period. In addition we therefore allow for restoration of previously accumulated human capital with possible catch-up in wages.

While the above introduced exogenous variables are common in economic research to analyze wages, the inclusion of the weekly working hours as an explanatory variable is heavily discussed. Mincer and Polachek (1974) and Beblo and Wolf (2002) control for the working hours and find evidence about its importance when analyzing wages. However, the amount of working hours can either be interpreted as an endogenous or as an exogenous variable. According to classical economic theory, females allocate their time between labor work, work at home and leisure or to produce commodities within the family. See section 1.1 for details. As Polachek and Siebert (1993) point out, an increase of wages can lead to an increase in supply of labor work due to high opportunity costs of staying at home. When analyzing male working-supplies in contrast, substitution effects are revealed between labor work and leisure time. Females however often face the responsibility for the work at home as a third usage of time, which results in wage elasticities depending on how the work at home is substitutable to other time usage. In addition to the substitution effect of time-patterns, a causal relationship from the wage to the working hours can also be due to income effects: females earning low wages can be forced to work more hours to achieve a satisfactory monthly income. As a result, Mincer and Polachek (1974) estimated a negative relationship of wages on the working hours. While the income effects can be observed primarily for single-females, in households containing married couples, the substitution effect dominates. The progressive tax-system in Germany can be an additional reason of low incentives for married women to increase their amount of labor work. In Germany, as the Federal Bureau of Statistics found out, the time supply of labor work is heavily dependent on the availability and the opening-hours of childcare facilities. As a result, the proportion of mothers working full-time or at least with increased working hours starts rising again when the oldest child living in the household has reached the entering age of public schools. In Germany therefore, it is reasonable to treat the amount of working hours as an exogenous variable effecting the wage of females and not vice versa, see also Wolf (2002) and Statistisches

Bundesamt (2007). The resulting covariate *whours* can also be interpreted as the degree of labor-market attachment: the more hours a female works, the more attached to labor market is she assumed to be. A higher labor market attachment might result in more frequent offerings to participate in further professional trainings, since the employer can expect higher returns on his investment due to more working hours of the female within the firm. To link the change in working hours with the change in earned wages at t_{return}, we include the ratio of post-birth working hours to pre-birth working hours (*ratio.whours*) when modeling the wage loss. A low ratio indicates therefore a severe reduction in working hours when returning to a job with possible consequences for the wage earned. Due to the postulated time-dependency of absolute human-capital depreciation, we include the length of maternity leave as covariate (*duration.off*) when modeling the wage loss, too. In this chapter we interpret maternity leave as voluntarily and temporarily withdrawal form the labor market. As maternity leave we therefore define the period off the job due to pregnancy in the last weeks prior to the birth plus the time subsequently after childbirth staying at home. With this definition maternity leave is not restricted to a fixed job-protection-period granted by law in Germany and also includes unpaid maternity leave due to childcare where the mother stays voluntarily at home. For a discussion concerning the German legal framework we refer to Gutierrez-Domenech (2005) and Buchner and Becker (2008). According to the GSOEP-questionnaires and the provided monthly spell data we distinguish between *maternity leave*, *housewife* and *registered unemployment*. A time period being a housewife following immediately after being in maternity leave is interpreted as a voluntarily elongation of maternity leave while being registered unemployed is treated as an involuntarily change of the labor force status. Besides the individual covariates introduced so far, the personal economic situation is likely to be affected by general economic performance. We therefore include the year of observation as an exogenous variable to capture effects of business-cycles between 1984 and 2008. All covariates introduced above are metrical and will be included in each of the models in a functional form, allowing for smooth and dynamic effects if they are existing.

In addition to the above introduced metrical covariates, we create dummy-variables, leading to semi-parametric mixed models. Besides *firmtime* capturing firm-specific human capital, we estimate fixed effects for the size of the company the female is currently working in, with *small.firm* and *large.firm* indicate whether the female is either employed in a company with less than 20 employees or at least with 2000 employees, respectively. While controlling for business-cycle effects due to the year of the observation, we add the dummy-variable *south.germany* to the models, indicating if the female is living in either Bavaria, Baden-Württemberg or Hessen. These three southern states are assumed to perform economically stronger than others states in Germany and might therefore honor human capital and labor market attachment differently. For the population of the mothers we also created a dummy-variable indicating whether the mother has given birth to more than one child before finally returning into work.

Our analysis is restricted to females living in West-Germany without having a migrational background. Although we could include females living in East-Germany and/or having a migrational background in our analysis, it is not the intention of this chapter to estimate interesting but well known negative effects of the corresponding covariates indicating wage penalties. As already mentioned above, self-employed females lack reliable and valid values for the current contract-based weekly working hours and can not be included. We also exclude females who have been already mothers when joining the GSOEP and therefore lack data on their last maternity leave and the related covariates. Observations from females being younger than 18 years of age or older then 55 years are excluded, too. This ensures a reliable comparison between working mothers and working non-mothers. Observations are excluded from our analysis if a woman has been on her last maternity leave for more than 6 years. At this time children have reached the entering age of public schools in Germany and voluntarily intensive childcare has no longer to be provided by the mother. This restriction excludes especially several low educated females from the dataset. Nevertheless, the exclusion is inevitable since coming back to the labor market after more than six years reveals a completely changing preference towards work, impeding to draw valid conclusions for labor-attached females from the results in section 3.4.

3.2.2 Wage Losses and Female Work Behavior

Looking at the dataset we find descriptive evidence about a wage loss due to child related employment interruptions: while highly educated females have to face the highest absolute wage-losses due to a higher average wage-level, the percentage of losses is almost constant through the ISCED-strata: females having high, middle and low educational attainments loose on average 12%, 13% or 9% of their pre-birth wage, respectively. In contrast, the covariates assumed to affect these losses are different for the subsets as analyzed in section 4. Looking at the labor force status of the females, we find that childless women work with an average of 36.6 hours per week with only a standard deviation of 6 hours. With 35.7 hours, 36.7 hours and 37.3 hours for high, middle and low educated females, respectively, full-time employment is dominant for females who are not mothers. In contrast, the average working hours decrease after childbirth to an average of 23.9 with standard deviation of 9.2. With 23.4 hours, 24 hours and 23.7 hours, respectively, working hours do not change significantly with the ISCED-strata.

When analyzing female working behavior, the timing of births has to be considered as well. We refer to Wetzels (2001) and Bloemen and Kalwij (2001) for a discussion. Females in our dataset give birth to their first child with an average age of 28.2 years after having worked fulltime for on average 5.6 years. However, the timing is likely to differ between the employed strata: Highly educated women become mothers on average being 30.7 years old after having spent 5.8 years in fulltime-employment. Women belonging to the middle ISCED-strata give their first birth on average being 28 years old and having worked fulltime for 6.1 years. Finally, low educated females become mothers being only 24.8 years old after fulltime experience of 3.7 years on average. Differences can also be found by looking at the time point of final return into the labor market, corresponding to t_{return}. On average, mothers return finally into employment being 30.5 years old with 5.9 years of fulltime work experience. However, highly educated mothers return with 6.2 years of fulltime experience being 33 years old on average. Mothers belonging to the middle strata return on average being only 30.1 years old but having accumulated 6.3 years of fulltime labor force experience. Those low educated females returning to the labor market finally are on average 27.7 years old with only 4.1 years of experience gained when working fulltime.

In the remaining of the chapter we will model the wage losses and the wages using functional approaches to capture non-linear covariate effects.

3.3 Theoretical Background

3.3.1 Additive Mixed Models and Spline Smoothing

Classical regression models assume that a response or endogenous variable y, respectively, depends on some covariates $x_1, \ldots, x_q$ in a linear fashion

$$y = \beta_0 + x_1\beta_1 + \cdots + x_q\beta_q + \epsilon,$$

where ϵ is a random noise error usually assumed to be normally distributed. While the linear approach is simple it is certainly too simplistic for our covariates at hand. Instead, letting $x_1, \ldots, x_p$ with $p < q$ denote metrically scaled covariates (like full-time work experience) we replace the linear structure by a functional form

$$y = \beta_0 + f_1(x_1) + \ldots + f_q(x_p) + x_{p+1}\beta_{p+1} + \ldots + x_q\beta_q + \epsilon. \tag{3.1}$$

Here $f_j(x_j)$ are smooth but otherwise undetermined functions to be estimated from the data. Models of class (3.1) have been coined Generalized Additive Models by Hastie and Tibshirani (1990) and are extensively discussed in Wood (2006), see also Ruppert et.al. (2003, 2009). Apparently, model (3.1) itself is not identifiable since the offset can go in any function. One therefore needs the further constraint that $f_j(x_j)$ integrates out to zero with respect to the (empirical) distribution function of x_j. Fitting of model (3.1) will be carried out with penalized spline smoothing. The idea is thereby to replace function $f_j(x_j)$ by some high dimensional basis representation

$$f_j(x_j) = B_j(x_j)b_j,$$

where $B(.)$ can be taken as cubic smoothing spline, see Wahba (1978). To reduce the computational burden the use of so called “low rank smoothers” has proven to be reliable and stable, which explains the dominance of the routine in available software. Note that since basis $B_j(.)$ is high dimensional the resulting fit will be poor unless we impose a penalty in coefficient vector b_j. The common choice is to work with quadratic penalties of the form $\lambda_j b_j^T D_j b_j$ with D_j as penalty matrix (see Wood (2006) for more details) and λ_j as penalty parameter. Using cubic smoothing splines it can be shown that the quadratic form penalizes the integrated squared second order derivative of function $f_j(.)$. Following Wahba (1978), Wong and Kohn (1996) or Wood (2003) we can interpret the quadratic penalty as prior on the spline coefficients in the form $b_j \sim N(0, \lambda_j^{-1} D_j^{-1})$, which replaces the additive model (3.1) by

$$\begin{aligned} y|b_1, \ldots, b_j &\sim N\left(\beta_0 + \sum_{j=1}^{p} B_j(x_j)b_j + \sum_{j=p+1}^{q} x_j\beta_j, \sigma_\epsilon^2\right) \\ b_j &\sim N(0, \lambda_j^{-1} D_j^{-1}),\ j = 1, \ldots, p. \end{aligned} \tag{3.2}$$

The Bayesian formulation resulting from (3.2) is well established under the phrase Linear Mixed Model in statistics, see e.g. Searle et al. (1992) or McCulloch and Searle (2001) and estimation can be easily carried out with maximum likelihood theory. In fact, integrating out b_j in (3.2) gives the likelihood and we can comprehend σ_ϵ^2, λ_j, $j = 1, \ldots, 1$ as well as β_j, $j = p+1, \ldots, q$ as parameters. This is implemented in available software, where we make use of R, see Pinheiro and Bates (2000) and R Development Core Team (2010)

For our data analysis where we have multiple observations per individual we supplement model (3.2) by introducing an individual specific random effect. This takes on the one hand unobserved heterogeneity in the data into account and secondly controls for serial correlation. To be more specific we replace model (3.2) by

$$\begin{aligned} y_{it}|b_1, \ldots, b_j &\sim N\left(\beta_0 + \sum_{j=1}^{p} B_j(x_{jit})b_j + \sum_{j=p+1}^{q} x_{jit}\beta_j + \gamma_{i0}, \sigma_\epsilon^2\right) \\ b_j &\sim N(0, \lambda_j^{-1} D_j^{-1}), j = 1, \ldots, p \\ \gamma_{i0} &\sim N(0, \tau_0^2), \end{aligned} \tag{3.3}$$

where indices it refer to the t-th observation drawn from the i-th individual. Here γ_{i0}

is the latest individual effect. Though model (3.3) is a conceptually serious extension of model (3.2) it is again a Linear Mixed Model and hence fitting is done in the same fashion and with the same software.

3.3.2 Statistical Models

To model and to estimate female wages und wage losses we now employ the above introduced (Generalized) Additive Mixed Models. Economically the models trace back to Mincer (1974) and Mincer and Ofek (1982). Models (3.4) to (3.6) are each estimated based on data with respect to the achieved ISCED group with index i referring to the individual in the data set and index t referring to the time point of observation. Model 3.4 shows the assumed relationship for the reference model.

$$\begin{aligned}\log(wage_{it}) \;=\;& f_1(fulltime_{it}) + f_2(parttime_{it}) + f_3(unemployed_{it}) \\ & + f_4(firmtime_{it}) + f_5(whours_{it}) + f_6(Year_{it}) \\ & + \beta_0 + \beta_1\, small.firm_{it} + \beta_2\, large.firm_{it} \\ & + \beta_3\, south.germany_{it} + \gamma_{i0} + \epsilon_{it} \qquad (3.4)\end{aligned}$$

with $\gamma_{i0} \sim N(0, \tau_0^2)$ and $\epsilon_{it} \sim N(0, \sigma^2)$. Here $f_1(.)$ is the smooth but otherwise unspecified effect of full-time work experience and according definitions for the remaining functions.

In contrast, we model the wage loss due to childbirth related employment interruptions by

$$\begin{aligned}\log(ratio.wages_{it_r}) \;=\;& f_1(fulltime_{it_r}) + f_2(parttime_{it_r}) + f_3(unemployed_{it_r}) \\ & + f_4(firmtime_{it_r}) + f_5(ratio.whours_{it_r}) \\ & + f_6(duration.off_{it_r}) + f_7(Year_{it_r}) \\ & + \beta_0 + \beta_1\, small.firm_{it_r} + \beta_2\, large.firm_{it_r} \\ & + \beta_3\, south.germany_{it_r} \qquad (3.5) \\ & + \beta_4\, more.than.one.Kid_{it_r} + \epsilon_{it}\end{aligned}$$

with $ratio.wages_{it_r} = wage_{it_r}/wage_{it_l}$ and $ratio.whours_{it_r} = whours_{it_r}/whours_{it_l}$ and

$\epsilon_{it} \sim N(0, \sigma^2)$. Here, t_r and t_l define the year of returning to the labor market and the year of leaving the job due to childbirth, respectively, see Figure 3.1. In the wage loss model (3.5) we do not have to control for unobserved heterogeneity since females enter the model with only one observation each when they entered the labor market finally again.

Finally, modeling the wage profile for the population of the mothers (restoration model) differs in (3.6) by the amended dummy-variable indicating multiple births. To be specific

$$\begin{aligned}
\log(wage_{it}) \;=\;& f_1(fulltime_{it}) + f_2(parttime_{it}) + f_3(unemployed_{it}) \\
& + f_4(firmtime_{it}) + f_5(whours_{it}) + f_6(Year_{it}) \\
& + \beta_0 + \beta_1\, small.firm_{it} + \beta_2\, large.firm_{it} \\
& + \beta_3\, south.germany_{it} + \beta_4\, more.than.one.Kid_{it} \\
& + \gamma_{i0} + \epsilon_{it} \qquad (3.6)
\end{aligned}$$

with $\gamma_{i0} \sim N(0, \tau_0^2)$ and $\epsilon_{it} \sim N(0, \sigma^2)$ where again $f_j(.)$ are smooth functions.

The resulting smooth effects will be displayed on the scale of the linear predictor with point-wise 2-standard-error confidence lines for each estimation. The estimation of the models (3.4) to (3.6) is carried out in R using the package `gamm4`, which is based on the packages `mgcv` and `lme4`. See Wood (2010a), Bates and Maechler (2010) and Wood (2010b) for the packages, respectively.

3.4 Data Analysis

The aim of this section is to analyze the effects of the metrical and {0,1}-covariates on the wages for mothers and non-mothers and on the wage-loss for mothers just having reentered the labor market after child related employment interruptions. Figures 3.2 to 3.4 show the empirical results of the estimated functional effects for the employed models, while Tables 3.1 to 3.3 give the estimation results for the parameters β_j and τ_0^2. For all estimated models in Figures 3.2 to 3.4, we added the estimated intercept $\hat{\beta}_0$ to the major smooth effect of the experience gained working full-time ($\hat{f}_1(fulltime)$). This allows interpreting the smooth estimated effects below the first rows in the figures as deviation from this effect and additionally we can identify the different wage-levels for the three ISCED strata.

reference model:

While the effect of the full-time work experience indicates a clear increasing, but concave-running shape for all ISCED-levels, it is not surprising to see higher wages for higher educated women. Although females belonging to this strata earn higher average wages, the increase of the effect is less sharp in the first six years. In contrast, working part-time in the past seems to have weaker, but negative effects on the wage, strengthening with the length of experience gained in working with reduced weekly working hours. Time spent in unemployment starts to have negative effects after an accumulated duration of more than one year. Females having achieved middle and low educational attainments can profit from a short period of being unemployed, but they are penalized for longer durations, too. While almost every woman has to face negative effects on her wage when being employed for only up to five years in the current company, staying longer than 20 year in the same job is not rewarded to low-educated females, resulting in severe wage penalties as the duration exceeds 20 years of company affiliation. Surprisingly, working part-time currently with contract-based working hours less than 35 is rewarded by positive effects on the wage. This stays in contrast to the effects of the above analyzed duration of past part-time periods. Besides this, the general economic performance, captured by the effect of the current year in the last row, postulates positive and even rising effects on the wage up to the year 1999, followed by a sharp decline until 2004 with recovery on a low level thereafter. In Table 3.1 we find evidence about honoring individual human capital by large firms with more than 2000 employees and by working in one of the three southern German states. Wage penalties have therefore to be taken into account when working in companies with less than 20 employees and/or in northern West-Germany. These fixed estimated effects are almost identical for all three ISCED-subsets. The reference model is estimated for 964 females with high educational achievements contributing 4203 observations, for 2450 females belonging to the middle ISCED-subset with 10542 observations and for 1230 low educated females with 3532 observations.

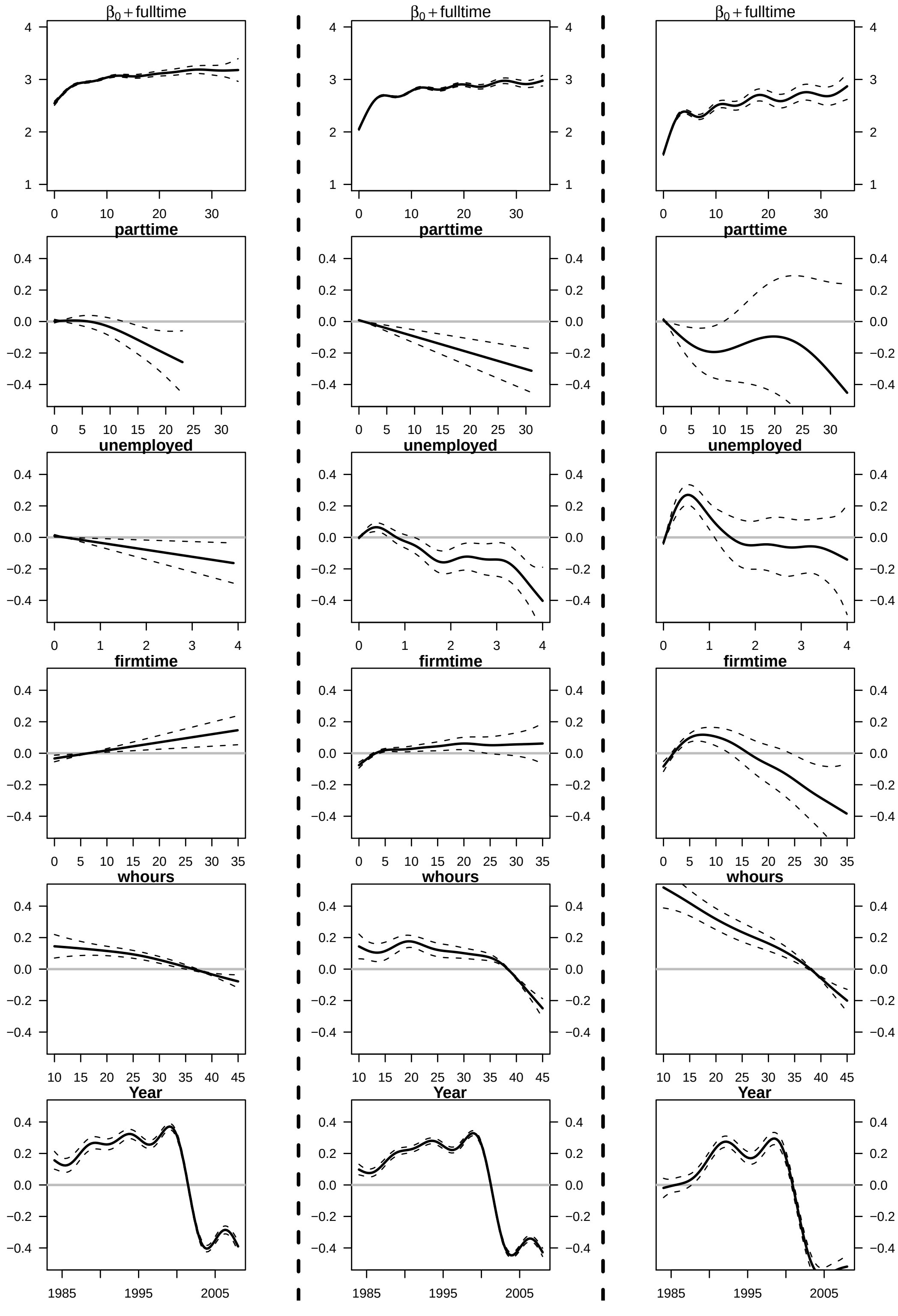

Figure 3.2: Fitted dynamic effects of the reference model for high, average and low educated females (first, second and third column respectively)

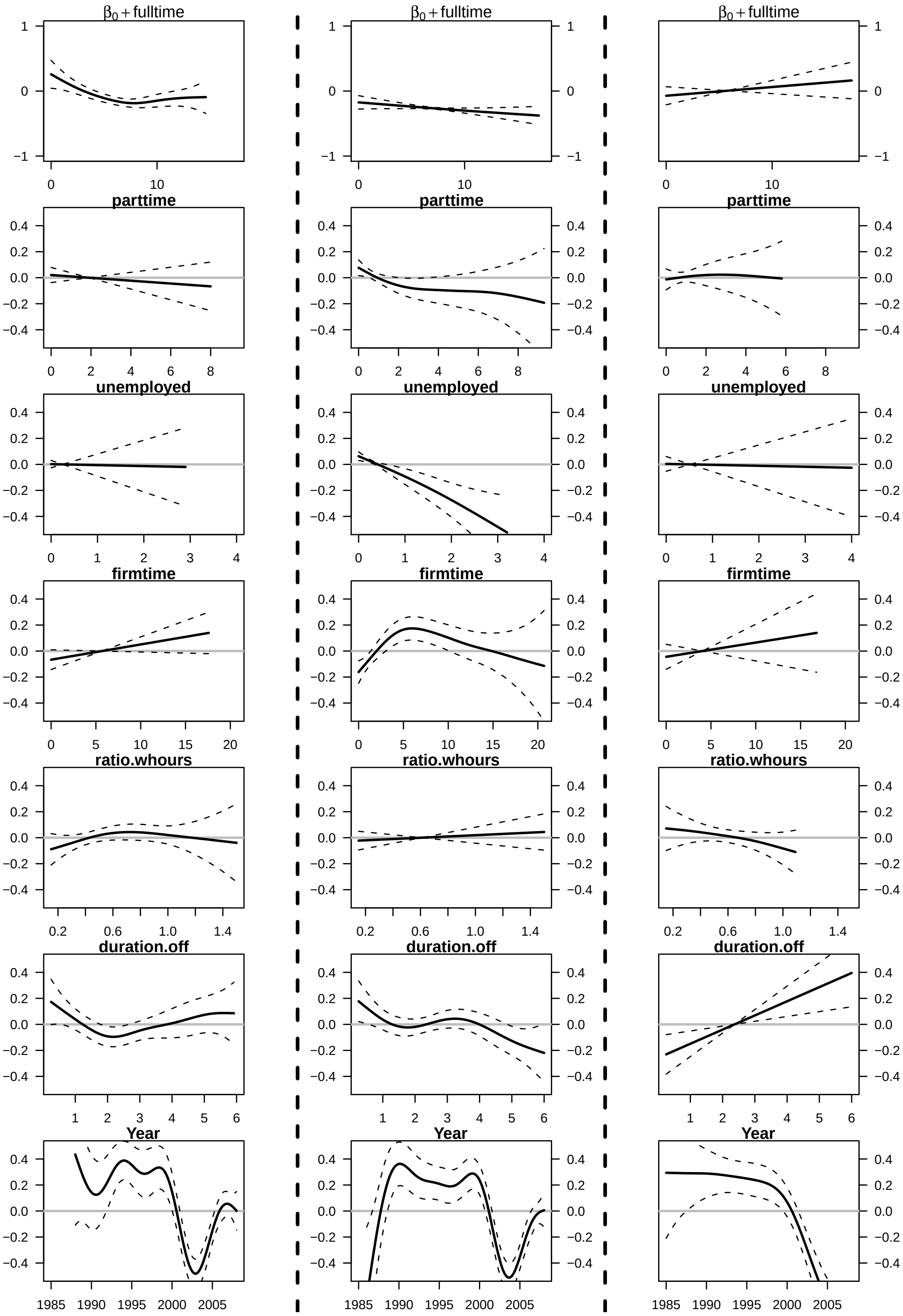

Figure 3.3: Fitted dynamic effects of the wage loss model for high, average and low educated mothers (first, second and third column respectively)

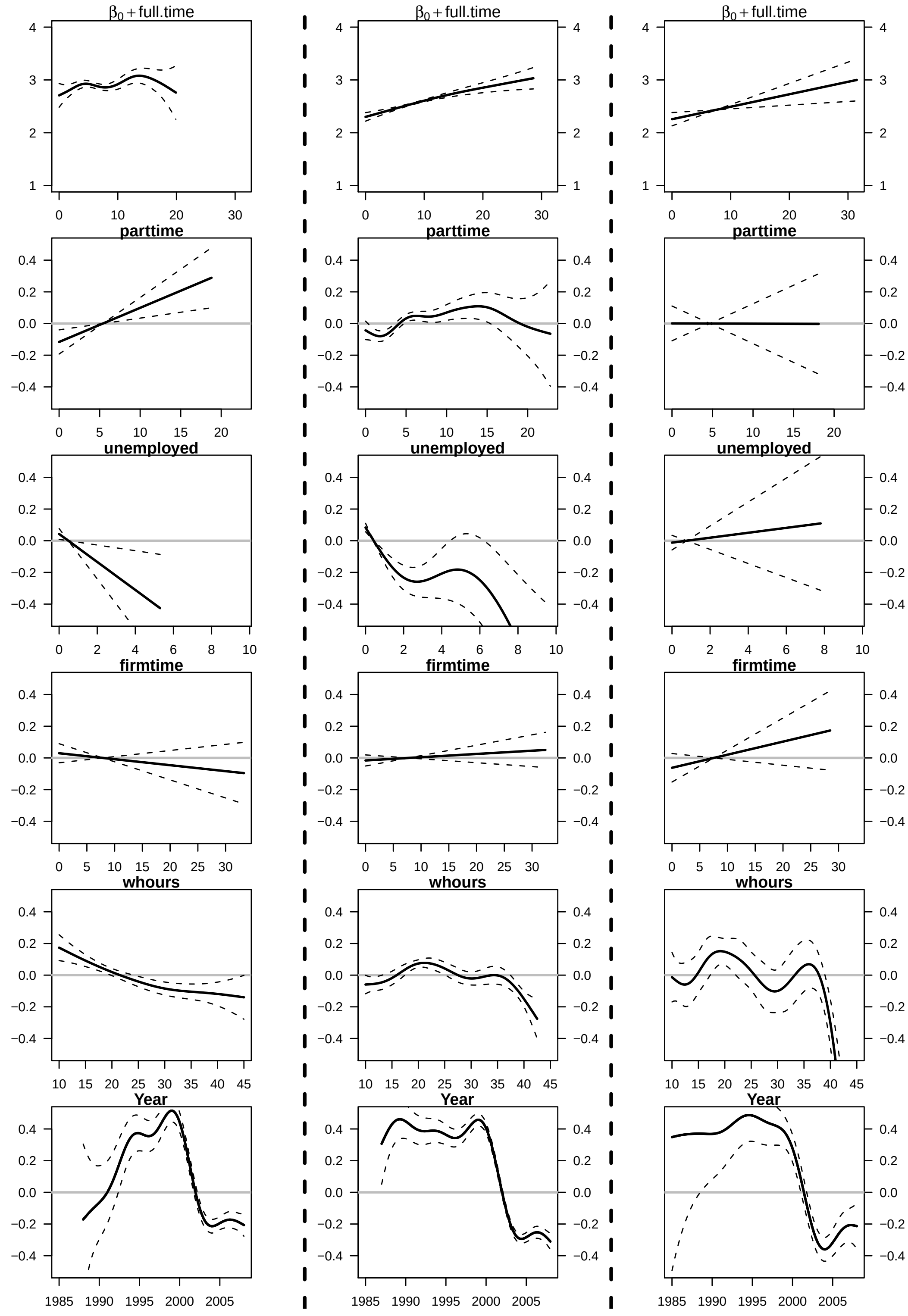

Figure 3.4: Fitted dynamic effects of the restoration model for high, average and low educated mothers (first, second and third column respectively)

effect	$\hat{\beta}_j$ (p-value) high ISCED	middle ISCED	low ISCED
(Intercept)	2.93 (< 0.01)	2.61 (< 0.01)	1.99 (< 0.01)
small.firm	-0.04 (< 0.01)	-0.07 (< 0.01)	-0.13 (< 0.01)
large.firm	0.04 (< 0.01)	0.07 (< 0.01)	0.13 (< 0.01)
south.germany	0.06 (< 0.01)	0.04 (< 0.01)	0.10 (< 0.01)
Var(γ_{i0})	0.12	0.10	0.18

Table 3.1: Parametric estimation results from the reference model

effect	$\hat{\beta}_j$ (p-value) high ISCED	middle ISCED	low ISCED
(Intercept)	-0.09 (0.09)	-0.27 (< 0.01)	0.01 (0.94)
small.firm	-0.01 (0.92)	-0.09 (0.1)	-0.27 (0.02)
large.firm	0.16 (0.03)	-0.03 (0.62)	-0.43 (< 0.01)
south.germany	-0.19 (0.02)	-0.1 (0.05)	-0.02 (0.84)
more.than.oneKid	-0.22 (< 0.01)	0.02 (0.74)	-0.19 (0.12)

Table 3.2: Parametric estimation results from the wage loss model

effect	$\hat{\beta}_j$ (p-value) high ISCED	middle ISCED	low ISCED
(Intercept)	2.89 (< 0.01)	2.56 (< 0.01)	2.44 (< 0.01)
small.firm	-0.11 (< 0.01)	-0.12 (< 0.01)	-0.33 (< 0.01)
large.firm	0.11 (< 0.01)	0.02 (0.17)	-0.03 (0.45)
south.germany	0.07 (< 0.01)	0.05 (< 0.01)	0.01 (0.75)
more.than.one.Kid	-0.22 (< 0.01)	-0.06 (< 0.01)	-0.03 (0.47)
Var(γ_{i0})	0.18	0.17	0.14

Table 3.3: Parametric estimation results from the restoration model

wage loss model:

Figure 3.3 and Table 3.2 give the results for modeling the wage loss. Therefore, high and average educated mothers can attenuate their (relative) wage loss due to child related employment-breaks by placing births at the beginning of their (full-time) work career. For low educated females, this conclusion can not be drawn. The part-time experience gained at the labor market seems to have no major effect on the relation of post-wages to pre-wages. Having not been unemployed is rewarded when reentering the labor market consistently over all three ISCED-subsets. In contrast to the past full- and part-time experience, giving birth to a child when only being employed in the current firm for a short period of time leads to wage penalties, while the ratio of the working hours does not seem to have significant effects on the wage loss. High and middle educated females can attenuate their losses by returning to the labor market within one year after leaving the job due to childcare. Note that this time period includes the weeks anteceding giving birth due to laws on maternity protection. Like the wage of non-mothers in the reference model, the year of return seems to influence the wage-ratio of mothers with decreasing and even negative effects from 1999 to 2004. In contrast, low educated mothers can not profit from a possible recovery after 2004. Highly educated females can attenuate their wage loss by returning into a large company while low educated mothers face heavy wage penalties when being employed in a large company. In contrast to model 0, giving birth to a child in southern Germany is likely to aggravate the loss for high and average educated females. However, bearing more than one child while being off the labor market temporarily has severe negative effects on the wage only for high and low educated mothers. The wage loss model is estimated for 507 females having high educational attainments, for 900 females belonging to the middle ISCED-strata and for 185 low educated mothers, contributing one observation each.

restoration model:

After having returned to the labor market, highly educated mothers do not catch up with their wages quickly, as the slow increasing effect of the newly gained full-time and part-time experiences in Figure 3.4 indicates. While the ongoing full-time experience influences post-birth wages positively for all three ISCED-groups, newly gained part-time experience is not rewarded to middle and low educated mothers. However, having been employed with reduced weekly working hours in the past can lead to wage increases for highly educated mothers if this experience exceeds five years. Past unemployment results in severe wage-penalties for high and middle educated females. The firm affiliation and therefore the firm-specific human capital seems to loose importance in post-birth employments as the almost non-significant effects of the covariate *firmtime* indicate. Interestingly, highly educated females face a weak, but decreasing effect as the time in the current firm continues. In contrast, wages from mothers coming from the middle and low ISCED-strata are affected about the same as the childless women by the time in the firm. The amount of contract-based working hours seems to influence the wages of highly educated females strongly: working more than 25 hours a week (abstracting from overtime) is penalized, while these negative effects are visible for middle and low-educated females only if they are employed for working more than 35 hours and therefore working almost fulltime. As a result, highly educated mothers can profit from working less than 25 hours according to their wages. This holds for past and current part-time employment. The business-indicator in the last row is almost identical to the findings in the reference model. In addition, Table 3.3 displays negative wage effects when working in small firms. Living in southern Germany leads to a wage plus only to highly and average educated females. Interestingly, giving birth to more than one child is only penalized when having achieved a high ISCED-level. $\hat{\beta}_4$ can also be interpreted as a proxy effect of multiple interruptions between t_{leave} and t_{return} since only a small minority of mothers give birth to more than one child within one maternity leave period. The restoration model is estimated with 881 observations coming from 201 highly educated mothers, with 2181 observations contributed by 429 average educated females and with 439 observations due to 106 mothers with low educational attainment.

3.5 Conclusion

The fitted smooth and parametric effects of the employed covariates reveal a multi-dimensional framework for analyzing females' wages and the consequences resulting from employment interruptions due to childcare. The different pattern of covariate effects as displayed in Figures 3.2 to 3.4 are likely to have their origin in structural differences in the labor market for more or less educated females. These findings support the use of strata referring to the educational achievements of the women. The theoretical assumptions postulating an increasing, but concave-running effect of the labor market experience is met when focusing on full-time labor experience for the population of childless females. However, this does not hold for mothers with average or low educational achievements who reveal an increasing but rather linear effect of full-time experience. Highly educated mothers can not thereby restorate their human capital accumulated prior to birth quickly, as indicated by a less sharp increase of the effect after giving birth to a child. The most interesting effects can be found by comparing past and current part-time employment. While non-mothers are likely to be penalized having worked part-time in their working biography, current part-time employment can lead to wage increase, even for low educated females who are assumed to work in jobs with a rather lousy image. In contrast, highly educated mothers can profit from working part-time currently and by having worked with reduced working hours in their work biography. The return of these "high potentials" might be an important goal of employers willing to avoid high costs due to searching for new females and make use of the firm-specific human capital the woman has built up before withdrawal. This demand of highly-educated mothers is valued by higher wages, even if they only want to work part-time. In contrast, mothers without high educational attainments seem to be more substitutable in their firm and perform poorer when reentering the labor market, indicated by less valued work-experience for both, full- and part-time work. Again, periods of unemployment after leaving the labor due to childcare result in wage penalties. A negative stigma-effect on unemployment is likely to force wages down for mothers and non-mothers. For mothers however, being unemployed in addition to voluntarily withdrawal from the labor market due to childcare might signal low labor market attachment to the employer. Surprisingly, the time working in a company as a proxy for specific human capital yields lower rates of return, compared to the rates of return gained from accumulated general human capital. The all over weak effect of time worked in the current company is almost vanishing for mothers trying to gain a foothold in the labor market again. However, the size of the firm seems to influence the wages since large firms pay higher wages. Highly educated females can loose this advantage of being employed in large companies if they decide to have more than one children. The personal human capital seems to determine the individual wages significantly but the time-period a woman works in seems to influence her wage, too. The sharp decline from 1999 to 2004 is prominent, no matter what educational attainment the woman has. Interestingly, no catch-up in wages can be found for low-educated females after 2004.

While the reference and the restoration models are likely to describe the wages of females quite well, underlining the findings of Mincer (1974) and Becker (1993), trying to model the wage loss is a more difficult task. This underlines the assumption of interpreting wage losses as a net-depreciation of the individual stock of human capital rather than being influenced by several covariates. However, the wage loss model reveals some factors which can influence the loss. The discussion of the timing of births in females' biographies, as carried out for example in Bloemen and Kalwij (2001), Bernardi et al. (2008) and Kunze (2002), often reveals a delay in giving birth to children for higher educated women. With our findings however, the average relative wage loss can be attenuated by timing the pregnancy in the first two years of full-time work. However, highly educated mothers can profit from early maternity leave periods in their career by working part-time when returning into the labor market. Note, that this conclusion on wage losses are based upon the relative rather than the absolute wage loss. When focusing on absolute earnings, fulltime employment is likely to be dominant by single mothers with the sole responsibility for their income. In the wage loss model, the attachment of the mother to the firm being employed in seems to play a more important role compared to the effects in the profile models. Bearing a child after having worked for at least five years in the current company can attenuate the relative wage loss in all three ISCED-groups. The above mentioned demand for highly educated females after maternity leave by their employers is underlined in the wage loss model. Low educated females can not make profit from quickly returning to the labor market. This might explain the rather long duration times of maternity leave for this subset and the low proportion of these women ever returning finally to the labor market. The common hypothesis of increasing wage penalties as the duration of leave holds on, can not be supported by our findings, since the effect vanishes as the duration exceeds two years.

The findings of Mincer (1974) and Becker (1993) are verified in general by our modeling exercise. Although many effects fit to the widely accepted coherence of determining wage profiles, our analysis reveals both, some deviations characterizing the German labor market for females and the flexibility and capacity of penalized spline smoothing as estimation routine for functional data. In addition, our results show that the wages of German females are not solely dependent on the individual human capital but are also influenced for example by the business cycle and even by the location of living in Germany. The functional approach pursued in this article lays open some non-linearities which are hard to anticipate a priori as well as some vanishing effects in the higher domain of some exogenous variables. The changing role allocation of males and females and their converging time patterns when taking care of a child is left to further research. We refer to Gutierrez-Domenech (2005) and Burgess et al. (2008) for a discussion. We recommend however using a functional approach to get interesting insights of families, not only in Germany.

4 Findings and Outlook

Although the award of the Nobel Prize to Gary S. Becker took place almost 20 years ago, his theoretical considerations are still the benchmark in many research publications. In this work, the human capital theory is dominant and has defeated its position in the field of labor economics: the theory provides the guideline for interpreting both: the duration of maternity leave and the consequences of childbirths to the individual wage. Many of the findings from Becker (1991), Becker (1993) and Mincer (1974) are still valid, even in Germany with its rather conservative labor market concerning maternity leave rights, job-protection-period and tax system. The criticism of the GfdS is therefore void since the phrase "human capital" does not necessarily degrade employees, but instead gives a solid fundament to analyze economic behavior of females. This behavior however can in turn provide the fundament for making the most of life, as George Barnard Shaw already noted at the beginning of the 20th century. The knowledge about the importance of human capital is therefore inevitable when politicians want to improve the compatibility of female employment and family responsibilities, especially if females have invested in a large stock of human capital and behave almost as Becker (1993) anticipates. However, the classical human capital theory is also limited with respect to two findings of this work: first, many effects that are assumed to influence wages and the withdrawal from the labor market follow a dynamic pattern, attenuating the static assumptions of the theory. This dynamic pattern, observable due to the application of modern statistical regression techniques, might reveal a slightly changing preference structure of mothers compared to the assumption of the classical theory of labor supply. As a result, a *New Human Capital Theory* might ease restrictions on the assumed behavior and therefore on the functional relationship between the accumulated stock of human capital on the one hand and the wage and the withdrawal from the labor market on the other hand. Easing restrictions on the statistical models employed to verify this modified theory is lucrative, as shown in this work. As second aspect of limitations concerning classical human capital theory, many effects that influence the responses used in the regressions models fade away and *loose significance*, either as the duration of the maternity leave or the domain of the covariates exceed some thresholds. This *fading away* of importance limits the application of human capital theory to real employment biographies. The latter are also influenced by the business cycle, slightly attenuating the impact of the human capital theory in general. The statistical techniques can therefore enrich the controversy debate in the field of labor economics by showing in detail when classical human capital theory is *verified*, *modified* or even *falsified.* This area of conflict has been analyzed in this work. To establish either a *New Human Capital Theory* or a *Modified Human Capital Theory*, which both should contain the dynamics and limitations found with respect to the classical theory, empirical findings for males and other social groups have to be made and put in comparison to international findings. I recommend using modern P-spline based regression techniques to support these findings. However, there is no a-priori assumed result, since the presence of heterogeneity has to be taken into account when employing empirical longitudinal data. Nevertheless, one goal, according the the Nobel Prize winner of 1938 in physics, has already been achieved and is the guideline for any empirical research:

"We're still confused, but on a higher level."
(Enrico Fermi)

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