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Quantity Rationing vs. IS-LM  
- A Synthesis

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Discussion Paper No. 252/83

**BEITRÄGE  
ZUR  
ANGEWANDTEN WIRTSCHAFTSFORSCHUNG**

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- A Synthesis

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\*The results presented in this paper are the outgrowth of some continuous research over the past years. I am indebted to many colleagues, in particular to Costas Azariadis, for critical comments and remarks. Financial support from the Deutsche Forschungsgemeinschaft for part of this research is gratefully acknowledged.

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## 1. Introduction

The attempts in the recent past to describe a microeconomic model which not only allows to describe Walrasian states but also situations where not all markets clear lead in a natural way to the problem of finding a microeconomic model which supports and/or incorporates elements of most of the traditional macroeconomic models. While most of the criticism from macroeconomists concerning the general equilibrium models with quantity rationing centers on the missing explanation for wage and price rigidities, some important aspects of non-market clearing are frequently overlooked. It should be clear by now that, apart from monopolistic or strategic equilibrium concepts, both branches of the theory, i.e. macroeconomics and the theory of quantity rationing, use the same basic notions to describe non-market clearing situations. As a consequence it should be possible to construct a rigorous microeconomic model to address most macroeconomic questions which are usually dealt with in more or less ad hoc models.

Rather than presenting a very general model the present paper addresses the particular problem of designing a simple structure which provides a microeconomic foundation for the traditional Keynesian model. In order to do this, the essential markets for commodities, labor, money, and bonds have to be described. If this is to be done in a meaningful way the model has to be genuinely dynamic. Rigidities of nominal wages will be the essential source for disequilibrium situations. However,

no theory or explanation for these rigidities are provided in the paper. All other prices including the interest rate are assumed to be sufficiently flexible, although the techniques used can handle other rigidities as well. The essential goal is to investigate the effects of standard policy measures and to compare their effectiveness in the short run and in the long run.

Several authors have tried to identify the prototype model of quantity rationing presented by Malinvaud with the standard IS-LM-model. (See e.g. Danthine and Peytrignet (1980), Sneesens (1981)). The missing link, however, between the two types of models has been a bond market or credit market and/or the description of investment demand which is derived from some choice theoretic microeconomic basis, although some work has been directed towards these problems (e.g. Malinvaud (1980), Fourgeaud, Lenclud and Michel (1981), Hool (1980), Benassy (1982), and Grandmont (1982)).

The present paper represents an attempt to incorporate a bond market into the now wellknown simple model with quantity rationing and to demonstrate its relationship with the traditional IS-LM-model under wage rigidity alone. Since the aim is to establish the fact that the models with rationing provide a microeconomic basis for such models, emphasis will be laid on pointing out similarities between the two models rather than on the most general framework one could construct. To

capture the essential dynamic elements an overlapping generations model will be used which avoids some of the conceptual problems of the planning of infinitely long lived agents. The basic innovation will be the introduction of a retradeable bond which is issued by the government. This yields in a natural way effective supply and demand functions of consumers which depend on the price of bonds, i.e. the interest rate. It can be argued that the qualitative properties of the model will not be changed substantially if investment demand were introduced which depends on the interest rate as well, as long as such investment has no capacity effect. The government and the banking sector are kept in the simple traditional format. For a model with a disaggregated public sector see Eichberger (1983).

Given a model with four markets and its associated prices, the interdependence in general will not allow specific comparative statics results. Therefore, at different stages a number of assumptions are required which may appear more or less appealing to the reader. None of these, however, claim to reflect reality. They are introduced and justified on the grounds of economic common sense. How sensitive some of the well established qualitative results can be to some of the assumptions is exemplified in Eichberger (1983).

## 2. Government

The model describes an ongoing economy at a particular time period, called period 1, with the previous period zero and the next future period 2. One homogeneous output is produced from one input factor labor. The government purchases a fixed quantity  $g > 0$  of the commodity at the market price  $p$ , and raises taxes from profits at a proportional tax rate  $\tau$ ,  $0 < \tau < 1$ . Furthermore, the government issues bonds which are chosen to be consols i.e. each bond which is sold at the price  $s_1$  in the current period earns one unit of money as interest paid at the beginning of the next period. Each bond can be retraded in the bond market in the next period. There exists a non-interest-bearing asset, called money, which serves as a unit of account with a price equal to one in all periods.

Let  $B_0$  and  $B_1$  denote the amounts of outstanding bonds at the beginning and at the end of the current period respectively,  $\Pi$  current period's profits and  $\Delta M$  the net change in the money stock. Then, the government budget equation is

$$p_1 g + B_0 = \Delta M + \tau \cdot \Pi + s_1 (B_1 - B_0).$$

A triple  $(g, \tau, B_1)$  is called a government policy. A possible deficit or surplus of the government is covered by creating or destroying money. A stationary bond policy is a policy with  $B_1 = B_0$ . A stationary balanced budget policy is one with  $\Delta M = B_1 - B_0 = 0$ . It should be noted that the balanced budget condition for a stationary bond policy primarily defines a relation between commodity prices and profits which is independent of the bond price.

### 3. Consumers and production

The model contains two groups of consumers, namely worker-consumers who supply labor and consume and capitalist-consumers who control production facilities, receive profits and consume. For each group the overlapping generations structure will be chosen which seems to be the natural framework for an ongoing stationary economy. Generations of each group of consumers are identical and of equal size. Each generation lives two periods.

Consider first the behavior of worker consumers who consume in both periods of their life but work only in the first period. Consumption in the second period is financed by savings, i.e. accumulation of wealth in the form of money and bonds. Old workers hold money  $m_{ow} \geq 0$  and bonds  $b_{ow} \geq 0$ , all of which they will spend on consumption in the current period, i.e. their consumption demand is

$$x_{ow} = \frac{m_{ow} + (1 + s_1)b_{ow}}{p_1}.$$

Let  $v_w(x_{1w}, x_{2w}, \ell_1)$  denote the concave von Neumann-Morgenstern utility function of a typical worker, where  $\ell_1$  is labor supply and  $x_{iw}$ ,  $i = 1, 2$ , consumption in the  $i$ -th period of his life. Given current commodity prices  $p_1$ , the wage rate  $w_1$  and the price  $s_1$  for bonds, the worker makes a point forecast  $p_2$  for next period's commodity prices. The bond price in the next period, however, is considered as a random variable  $\tilde{s}_2$  whose distribution depends on  $s_1$ . The behavior of the young worker is therefore determined by the solution of the problem



$$\text{Max } E v_w(x_{1w}, x_{2w}, \ell_1)$$

subject to

$$p_1 x_{1w} + m_{1w} + s_1 b_{1w} = w \ell_1$$

$$p_2 x_{2w} = m_{1w} + (\tilde{s}_2 + 1) b_{1w}.$$

As the solution one obtains the notional (unconstrained) demand and supply functions

$$x_{1w}^* = c_w(p_1, w_1, s_1)$$

$$\ell^* = a(p_1, w_1, s_1)$$

$$m_{1w}^* = m_w(p_1, w_1, s_1)$$

$$b_{1w}^* = b_w(p_1, w_1, s_1)$$

where the price  $p_2$  of the second period has been suppressed as an argument since  $p_2$  is a function of  $p_1$ . In the context chosen here it seems to be appropriate to work with the price of bonds  $s_1$  rather than with the interest rate  $r$ . Since  $\tilde{s}_2$  is a random variable the effective interest rate  $r$  is also random due to capital gains or losses with the expectation

$$E r = \frac{E \tilde{s}_2 + 1 - s_1}{s_1}$$

Clearly, if  $E \tilde{s}_2 = s_1$  then one obtains the usual formula  $r = 1/s_1$ .

In case of unemployment a worker faces a binding constraint  $z < \ell^*$ . Maximization of expected utility under the additional constraint  $\ell \leq z$  yields the effective demand functions of the worker

$$\begin{aligned} x_{1w} &= c_{uw}(p_1, w_1, s_1, z) \\ m_w &= m_{uw}(p_1, w_1, s_1, z) \quad z < \ell^* \\ b_w &= b_{uw}(p_1, w_1, s_1, z) . \end{aligned}$$

Capitalist-consumers (or producers) consume in both periods of their life, produce and save during the first period and consume all of their wealth during the second period. Old producers, therefore, hold money  $m_{op} \geq 0$  and bonds  $b_{op} \geq 0$  which they spend on consumption, i.e. their consumption demand is

$$x_{op} = \frac{m_{op} + (s_1 + 1)b_{op}}{p_1} .$$

Let  $v_p(x_{1p}, x_{2p})$  denote the concave von Neumann-Morgenstern utility function of a typical producer and  $F$  his strictly concave, twice differentiable production function. Expectations for the future take the form of a point forecast for  $p_2$  and a probability distribution for the price of bonds  $\tilde{s}_2$  in the second period. A young producer determines his optimal consumption, asset and production plan by solving the maximization problem

$$\text{Max } E v_p(x_{1p}, x_{2p})$$

subject to

$$p_1 x_{1p} + m_{1p} + s_1 b_{1p} = (1 - \tau)\Pi$$

$$\Pi = p_1 F(z) - wz$$

$$p_2 x_{2p} = m_{1p} + (\tilde{s}_2 + 1)b_{1p},$$

where  $\Pi$  denotes profits and  $z$  labor demand. The solution deter-

mines the notional supply and demand functions

$$x_{1p}^* = c_p(p_1, w_1, s_1)$$

$$m_p^* = m_p(p_1, w_1, s_1)$$

$$b_p^* = b_p(p_1, w_1, s_1)$$

$$z^* = h\left(\frac{w_1}{p_1}\right)$$

$$y^* = F\left(h\left(\frac{w_1}{p_1}\right)\right) .$$

Writing labor demand as a function of the real wage above makes explicit that the optimal program of the producer implies the maximization of current net profits and that the bond price  $s_1$  is only relevant in the producer's portfolio and consumption decision. The tax rate  $\tau$  has been suppressed for the moment to reduce notation although it should be listed as an argument of all functions. This will be done at a later stage, when government policies are discussed.

Without loss of generality it will be assumed for the remainder of this paper that each generation of consumers consists of one of each type. Aggregate notional demand and supply functions (which will be described by capital letters) are therefore defined in a straightforward way. Since all prices refer to the current period the subscript 1 will be suppressed.

Consumption demand:

$$C(p, w, s, B_o, M_o) = c_w(p, w, s) + c_p(p, w, s) \\ + \frac{1}{p} [M_o + B_o(s + 1)],$$

where  $M_o = m_{oc} + m_{op}$  and  $B_o = b_{oc} + b_{op}$ ,

demand for money:

$$M(p, w, s) = m_w(p, w, s) + m_p(p, w, s),$$

demand for bonds:

$$B(p, w, s) = b_w(p, w, s) + b_p(p, w, s),$$

labor supply:

$$A(p, w, s) = a(p, w, s).$$

labor demand:

$$H\left(\frac{w}{p}\right) = h\left(\frac{w}{p}\right),$$

and commodity supply:

$$G\left(\frac{w}{p}\right) = F\left(H\left(\frac{w}{p}\right)\right).$$

The following assumptions will be made for the aggregate functions which are taken to be continuously differentiable.

- (C1) (i)  $\frac{\partial C}{\partial p} < 0$ ;  $\frac{\partial C}{\partial s} > 0$ ;  $\frac{\partial C}{\partial w} > 0$
- (ii)  $\frac{\partial M}{\partial p} < 0$ ;  $\frac{\partial M}{\partial s} > 0$ ;  $\frac{\partial M}{\partial w} > 0$
- (iii)  $\frac{\partial B}{\partial p} < 0$ ;  $\frac{\partial B}{\partial s} < 0$ ;  $\frac{\partial B}{\partial w} > 0$
- (iv)  $\frac{\partial A}{\partial p} \leq 0$ ;  $\frac{\partial A}{\partial s} \leq 0$ ;  $\frac{\partial A}{\partial w} \geq 0$
- (v)  $\frac{\partial H}{\partial p} > 0$ ;  $\frac{\partial G}{\partial p} > 0$ ;  $\frac{\partial H}{\partial w} < 0$
- (vi) for each  $(p, w)$
- a) if  $s^n \rightarrow \infty$ , then  $B(p, w, s^n) \rightarrow 0$
- b) if  $s^n \rightarrow 0$ , then  $B(p, w, s^n) \rightarrow \infty$  .

Some justification is needed for these assumptions, since the changes of current market prices affect young and old consumers in different ways. Wage changes do not affect old consumers so that the aggregate effect is the sum of the two effects of workers and producers. Since their income changes in opposite directions when the wage rate changes, their corresponding demand will, under similar preferences, change in opposite directions. The fact that producers pay taxes may lower the net demand effect. Here it is assumed that the demand of workers responds more strongly to wage changes than the demand of producers. Moreover, if current and future consumption and current consumption and leisure are substitutes then the wage effects in (i) - (iv) follow. A similar argument can be made to sign the aggregate price effect of young consumers on money and bonds. In the commodity market the demand of old consumers has a price elasticity of minus one which strengthens the negative effect.

Increases of the bond price have two effects on consumption. Old consumers receive a real income gain which yields a positive consumption effect with a marginal propensity to spend equal to one. Young consumers will substitute bonds for money. Their consumption demand, however, may increase or decrease, depending on expectations. Here it is assumed that a possible negative substitution effect by the young consumers is dominated by the positive consumption effect of wealth holders. Assumption (v) follows directly from the concavity of the production function. (vi) assumes that expectations on bond prices are sufficiently inelastic to make bond demand sufficiently elastic.

#### 4. Unemployment equilibria with fixed wage rates

Taking the government policy  $(g, \tau, B)$  as given, a full temporary equilibrium would be associated with a system of commodity and bond prices and a wage rate such that all four markets are in equilibrium. Since Walras Law holds for the economy as a whole, it suffices to have zero excess demand on three markets. In contrast to the traditional Keynesian analysis it is more appropriate and more informative here to consider the bond market explicitly and eliminate the money market. Therefore, given a government policy  $(g, \tau, B)$  a triple  $(\hat{p}, \hat{w}, \hat{s})$  is called a temporary equilibrium if excess demands on the three associated markets equal zero, i.e.:

$$(4.1) \quad C(\hat{p}, \hat{w}, \hat{s}, M_0, B_0) + g - F\left(H\left(\frac{\hat{w}}{\hat{p}}\right)\right) = 0$$

$$(4.2) \quad H\left(\frac{\hat{w}}{\hat{p}}\right) - A(\hat{p}, \hat{w}, \hat{s}) = 0$$

$$(4.3) \quad B(\hat{p}, \hat{w}, \hat{s}) - B = 0 .$$

It will be assumed that there exists a unique temporary equilibrium for each government policy. If  $B = B_0$  then the government bond policy is stationary.

If commodity prices and bond prices are sufficiently flexible to clear the two associated markets, but wages are rigid, then two distinct equilibria under-rationing may occur. One consists of states with unemployment. Since one-sidedness of the rationing mechanism is assumed, young worker-consumers will be rationed on the labor market whereas all old consumers as well as producers realize their unconstrained supplies and demands. This implies that the price which clears the commodity market has to be such that the real wage is equal to the marginal product of labor. At the given prices of commodities and of bonds young workers realize their effective demands on the two markets, given their level of employment. This seems to be the unemployment situation which is at the center of the Keynesian analysis.

The other possible state neglected by Keynes is the one with demand rationing on the labor market. In such a case prices adjust so that young worker-consumers realize their notional supply and

demands and old consumers realize their notional commodity demand. However, prices are too high to equate the marginal product of labor with the real wage, i.e. the marginal product at the resulting level of employment is higher than the real wage.

*Inelastic labor supply*

Before treating the general case, consider the special situation where labor supply is completely inelastic, i.e.  $\partial A/\partial s = \partial A/\partial p = \partial A/\partial w = 0$ . In order to analyse alternative levels of fixed wage rates, it is useful to graph the notional market clearing loci given by equations (4.1)-(4.3) in a diagram with the two flexible prices  $(s,p)$ . Starting with the labor market at  $\hat{w}$ , it is clear that (4.2) can be represented by a horizontal line  $L(\hat{w})$  in  $(s,p)$ -space, which shifts upward (downward) for higher (lower) wages. For the commodity market, one obtains as a functional dependence  $p = X(w,s)$  an upward sloping graph  $X(w)$ , which shifts to the left (right) for higher (lower) wages, i.e.

$$(4.4) \quad \left. \frac{dp}{ds} \right|_w = - \frac{\frac{\partial C}{\partial s}}{\frac{\partial C}{\partial p} + F'H' \frac{w}{p^2}} > 0$$

$$(4.5) \quad \left. \frac{ds}{dw} \right|_p = - \frac{\frac{\partial C}{\partial w} - F'H' \frac{1}{p}}{\frac{\partial C}{\partial s}} < 0.$$

Similarly, analysing (4.3) yields a function  $p = B(w,s)$  with the properties



$$(4.6) \quad \left. \frac{dp}{ds} \right|_w = - \frac{\frac{\partial B}{\partial s}}{\frac{\partial B}{\partial p}} < 0$$

and

$$(4.7) \quad \left. \frac{ds}{dw} \right|_p = - \frac{\frac{\partial B}{\partial w}}{\frac{\partial B}{\partial s}} > 0.$$

Figure 4.1 contains the graphs of all three market clearing loci for  $\hat{w}$ . Therefore, all three have a common intersection at  $(\hat{s}, \hat{p})$ . A plus sign (a minus sign) on the side of a curve indicates that excess demand is positive (negative) in that region.

Consider a situation  $w > \hat{w}$ . Then commodity prices have to be increased by the same factor to clear the labor market. At  $\hat{s}$  this causes excess supply both on the commodity market and on the bond market, which will be partly offset by the higher wage rate  $w$ . If one assumes, which seems natural, that the price effect on the bond market dominates the income effect from the higher wages, there remains an excess supply on the bond market, which can be eliminated through a lower bond price  $s < \hat{s}$ . Algebraically, differentiating (4.2) and (4.3) with respect to  $w$ , one obtains

$$(4.8) \quad \left. \frac{ds}{dw} \right|_{\substack{L \\ B}} = - \frac{p \frac{\partial B}{\partial p} + w \frac{\partial B}{\partial w}}{w \frac{\partial B}{\partial s}}$$

If the elasticity of the demand for bonds with respect to the commodity price  $p$  is larger than the wage elasticity, then (4.8) is negativ. This means that the commodity price compensated bond price which clears the bond market is lower than  $\hat{s}$ . Analys-

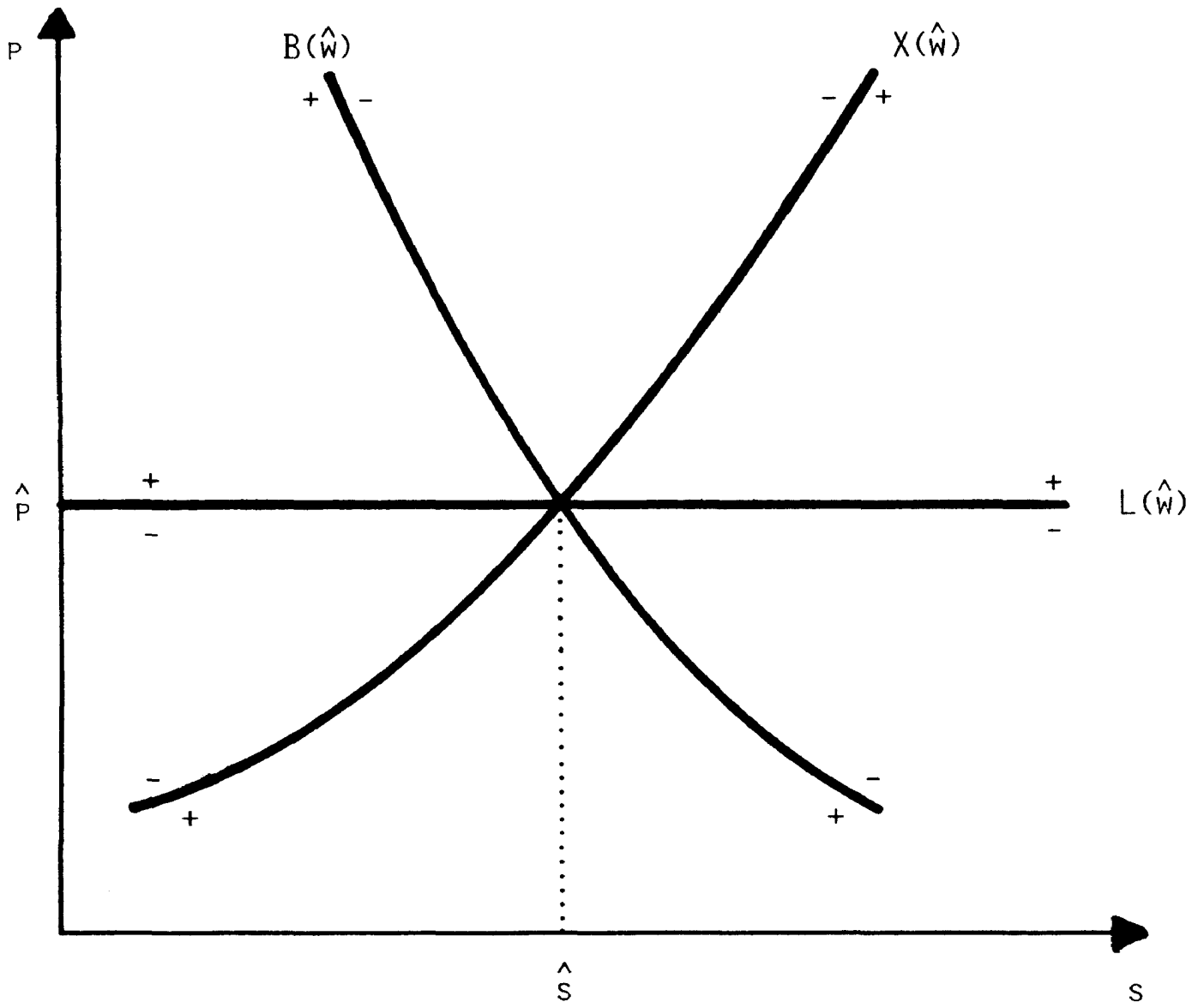


Figure 4.1

ing the labor market and the commodity market simultaneously in equilibrium yields a symmetric expression to (4.8)

$$(4.9) \quad \left. \frac{ds}{dw} \right|_{\substack{L \\ X}} = - \frac{p \frac{\partial C}{\partial p} + w \frac{\partial C}{\partial w}}{w \frac{\partial C}{\partial s}}.$$

The denominator of (4.9) is positive. The numerator, however, is the sum of a positive wage (i.e. income) effect and a negative price effect. The wage effect has to be small since it affects only young worker-consumers in a positive way. On the other hand the price elasticity for old consumers is minus one. Hence, it is reasonable to assume for (4.9) that the overall price elasticity of consumption is larger in absolute terms than the wage elasticity of consumption. Therefore, (4.9) is positive. The price compensated wage increase leaves an excess supply on the commodity market at  $\hat{s}$ , which requires an increase of the bond price  $s$  and thus a pure wealth effect for old consumers to equilibrate the commodity market. Since total supply remains constant with the compensated price increase, the real wealth loss of old consumers is compensated through the increase of the bond price. Summarizing these assumptions and their consequences one finds:

*If the price elasticities of the demand for bonds and for commodities are larger than the respective wage elasticities, i.e. if the assumption*

$$(C2) \quad (i) \quad p \frac{\partial B}{\partial p} + w \frac{\partial B}{\partial w} < 0$$

$$(ii) \quad p \frac{\partial C}{\partial p} + w \frac{\partial C}{\partial w} < 0$$

holds, then market clearing of the labor market and of the commodity market simultaneously requires a bond price  $s_x > \hat{s}$  if  $w > \hat{w}$ . Clearing of the labor market and of the bond market requires a bond price  $s_B < \hat{s}$  if  $w > \hat{w}$ .

The overall result on the market clearing loci is given in Figure 4.2 where the graphs for  $\hat{w}$  have been added as dotted lines. It is clear from the analysis that at  $w > \hat{w}$  clearing of all three markets is impossible since at the new wage rate the bond market requires a low bond price whereas the commodity market requires a high bond price. It will be shown that this leads to a situation of unemployment.

Consider now aggregate demands on the commodity market and on the bond market with unemployment. Since prices on both markets are flexible producers will not be rationed. Hence, aggregate effective consumption demand  $C_u$  and aggregate effective bond demand  $B_u$  are defined by

$$(4.10) \quad C_u(p, w, s, B_o, M_o, L) = c_{uw}(p, w, s, L) + c_p(p, w, s) + \frac{1}{p} [M_o + (1 + s)B_o]$$

and

$$(4.11) \quad B_u(p, w, s, L) = b_{uw}(p, w, s, L) + b_p(p, w, s).$$

One observes that the consumption function depends on wealth of old consumers whereas the bond demand does not. A consequence of this is that the marginal propensity to consume  $\partial C_u / \partial L$  is independent

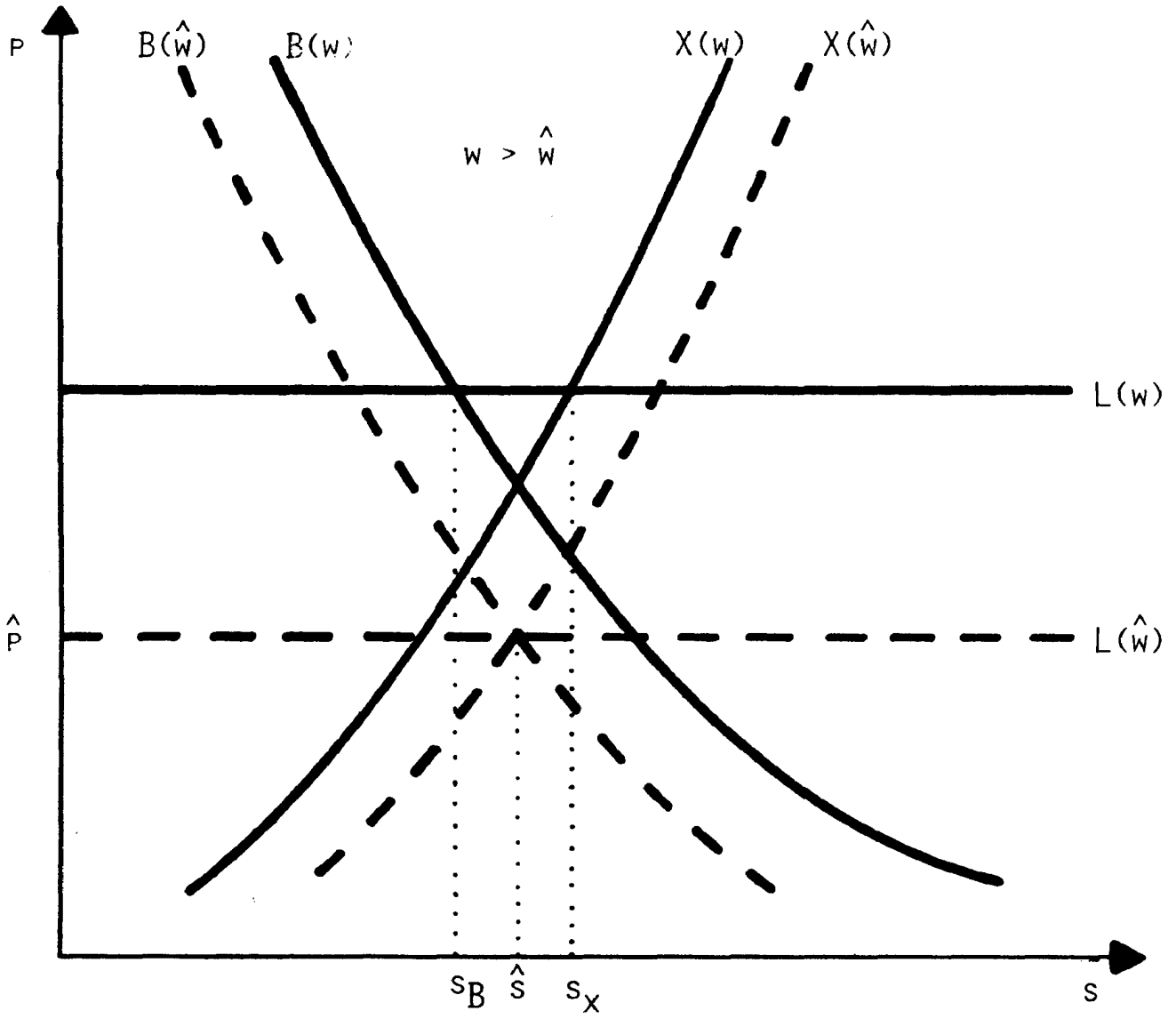


Figure 4.2

of current wealth held by the private sector. This feature seems to be a standard assumption in traditional Keynesian models. In the present model it arises out of the overlapping generations structure, since bequests are not allowed. The effective demand for money displays the same properties here as well as in Keynesian models. It will be assumed that all price effects of the effective demand functions have the same signs as the corresponding notional functions as stated in (C1). Moreover, the marginal propensity of young workers are between zero and one. Stated as assumption (C3) one has

$$(C3) \quad (i) \quad 0 < \frac{\partial C_u}{\partial L} < \frac{w}{p} ; \quad 0 < \frac{\partial B_u}{\partial L} < \frac{w}{s}$$

(ii) *all partial derivatives of  $C_u$  and  $B_u$  have the same signs as those of  $C$  and  $B$  listed in (C1).*

In order to describe the resulting disequilibrium situation under flexible commodity prices, any price change induces an employment change and vice versa according to the formula  $p = w/F'(L)$ . Substituting this relationship into the effective consumption demand function defines the price compensated effective consumption demand function  $\tilde{C}_u$  as

$$(4.12) \quad \begin{aligned} \tilde{C}_u(w, s, M_0, B_0, L) &= C_u\left(\frac{w}{F'(L)}, w, s, M_0, B_0, L\right) \\ &= c_{uw}\left(\frac{w}{F'(L)}, w, s, L\right) + c_p\left(\frac{w}{F'(L)}, w, s\right) \\ &\quad + \frac{M_0 + (s+1)B_0}{\frac{w}{F'(L)}} \end{aligned}$$

for  $L \leq L^*$  where  $L^*$  is the unconstrained fixed labor supply. Equilibrium on the commodity market for alternative levels of employment is then defined by

$$(4.13) \quad \tilde{C}_u(w, s, M_0, B_0, L) + g - F(L) = 0$$

For fixed  $w$  this defines the price compensated effective market clearing locus as a relation between the bond price  $s$  and the level of employment. Clearly, for  $w = \hat{w}$ ,  $(\hat{s}, L^*)$  solves (4.13) with  $\hat{w}/F'(L^*) = \hat{p}$ . Let  $L_g$  denote the amount of labor required to produce government demand  $g$ , i.e.  $F(L_g) = g$ . All feasible solutions  $L$  of (4.13) require that  $L_g < L \leq L^*$ . Define  $\underline{L}_u$  as the lowest employment level consistent with (4.13).  $\underline{L}_u$  depends primarily on initial total real wealth, i.e. on  $M_0$  and  $B_0$ , but also on  $g$  through the price  $p = w/F'(L_g)$ . Moreover,  $s$  has a positive effect on consumption. Hence, decreasing  $s$  to its lowest possible level will decrease effective consumption demand to its lowest level. Since  $s$  has no positive lower bound for the consumption decision it follows that  $s = 0$  and  $\underline{L}_u$  solve (4.13). Finally, differentiating (4.13) for  $w = \hat{w}$  yields

$$\left. \frac{ds}{dL} \right|_{\tilde{C}_u} = - \frac{\frac{\partial \tilde{C}_u}{\partial L} - F'}{\frac{\partial \tilde{C}_u}{\partial s}}$$

or

$$(4.14) \quad \left. \frac{ds}{dL} \right|_{\tilde{C}_u} = \frac{F' - \frac{\partial C_u}{\partial L} + \frac{\partial C_u}{\partial p} \frac{w}{(F')^2} F''}{\frac{\partial C_u}{\partial s}} .$$

Since  $F'' < 0$ , it follows from (C1) - (C3), that the expression (4.14) is positive. Plotting the graph of the price compensated effective market clearing locus for the commodity and labor market in  $(L,s)$ -space for  $\hat{w}$  gives the curve marked  $\tilde{C}_u$  in Figure 4.3.

In a similar fashion one derives the price compensated effective demand function for bonds  $\tilde{B}_u$  for employment levels  $L < L^*$ . By definition one has

$$(4.15) \quad \begin{aligned} \tilde{B}_u(w, s, L) &= B_u\left(\frac{w}{F'(L)}, w, s, L\right) \\ &= b_{uw}\left(\frac{w}{F'(L)}, w, s, L\right) + b_p\left(\frac{w}{F'(L)}, w, s\right). \end{aligned}$$

Since only young consumers buy bonds, the effective bond demand is independent of wealth. Given aggregate net supply of bonds  $B$ , equilibrium on the bond market is defined by

$$(4.16) \quad \tilde{B}_u(w, s, L) - B = 0$$

Again,  $(\hat{w}, \hat{s}, L^*)$  solves (4.16). Feasibility requires only that  $0 \leq L \leq L^*$  and  $s \geq 0$ . For  $L = L_g$ , clearly, there exists a positive bond price  $s_g$  such that  $\tilde{B}_u(w, s_g, L_g) - B = 0$ . Moreover, differentiating (4.16) and rearranging terms yields

$$(4.17) \quad \left. \frac{ds}{dL} \right|_{\tilde{B}_u} = - \frac{\frac{\partial \tilde{B}_u}{\partial L}}{\frac{\partial \tilde{B}_u}{\partial s}} = \frac{pF''}{F'} \cdot \frac{\left[ \frac{\partial B_u}{\partial p} - \frac{F'}{pF''} \frac{\partial B_u}{\partial L} \right]}{\frac{\partial B_u}{\partial s}}.$$

Since  $F'' < 0$  and  $\partial B_u / \partial s$  are both negative, the sign of (4.17) equals the sign of the term in the bracket. The first term is



the price effect on the demand for bonds whereas the second term is the spillover onto the bond market generated by the commodity price increase. Note that  $-F'/pF'' = -wH'/p^2$  is the change in labor demand due to a price increase. If the direct effect dominates the spillover, then (4.17) will be negative. One possible justification for a large price effect could be that commodity price expectations are sufficiently inflationary and that bond price expectations are relatively inelastic. In such a case price increases lower the expected real return on bonds and thus have a strong negative effect on current bond demand. For the analysis here it will be assumed that the direct effect dominates the spillover. Formally stated, the results about the two price compensated market clearing loci are as follows:

*If the assumptions (C1) - (C3) hold, then*

$$\left. \frac{ds}{dL} \right|_{\tilde{C}_u} > 0.$$

*If in addition assumption*

$$(C4) \quad \frac{\partial B_u}{\partial p} - \frac{F'}{pF''} \frac{\partial B_u}{\partial L} < 0$$

*is satisfied, then*

$$\left. \frac{ds}{dL} \right|_{\tilde{B}_u} < 0.$$

For  $w = \hat{w}$ , the graphs of  $\tilde{C}_u$  and  $\tilde{B}_u$  are given in Figure 4.3.

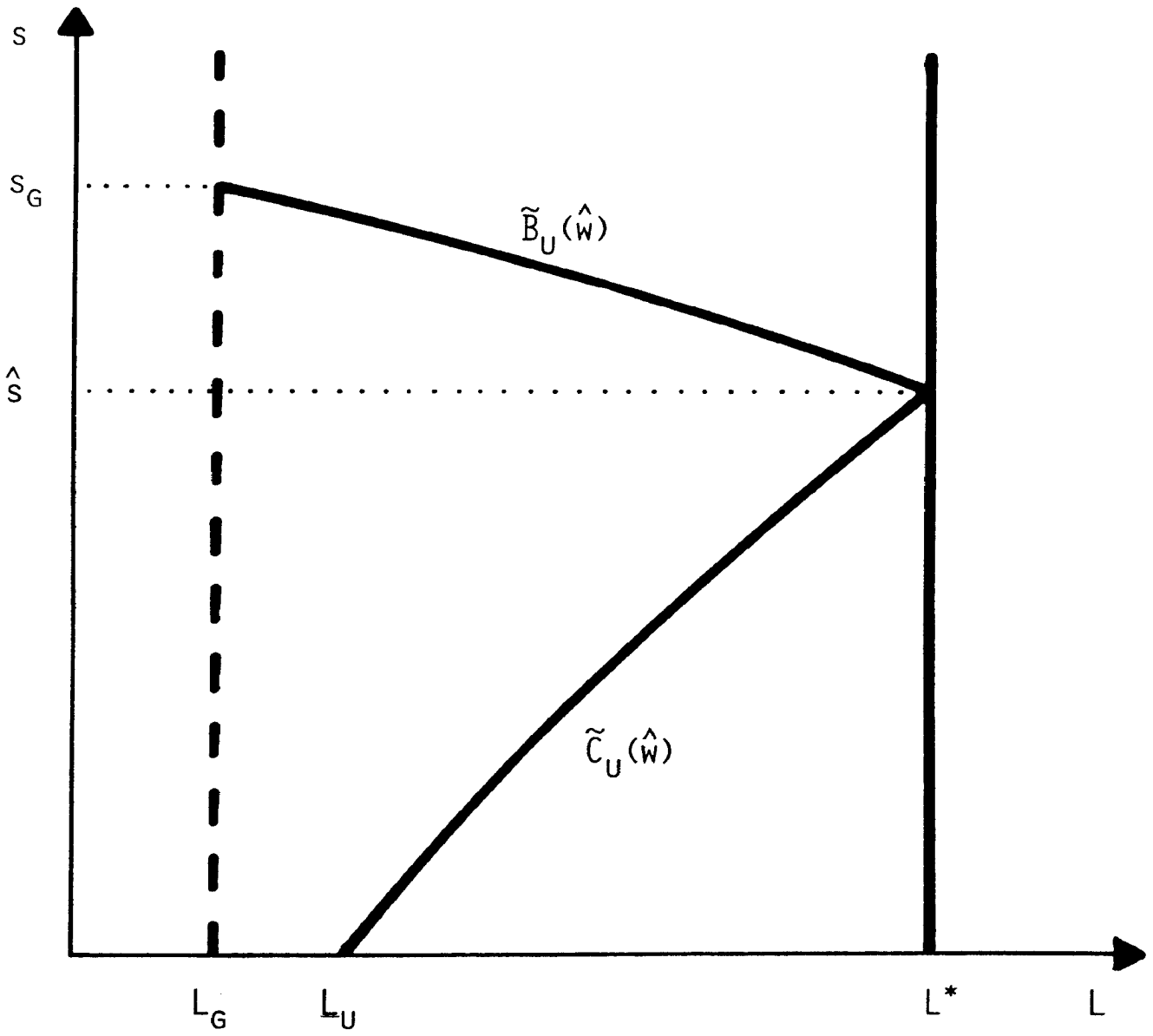


Figure 4.5

Finally, the original question of the consequences of a wage rate  $w > \hat{w}$  on the disequilibrium state can be approached using the price compensated market clearing loci. A state with unemployment, flexible commodity prices and bond prices is defined by a pair  $(L_u, s_u)$  which solves (4.13) and (4.16) at some  $L_u < L^*$ . Using the previous analysis of the notional market clearing conditions,  $w > \hat{w}$  implies that there exist  $s_x > \hat{s}$  such that  $(s_x, L^*)$  solves (4.13). Similarly, there exists  $s_B < \hat{s}$  such that  $(s_B, L^*)$  solves (4.16). More generally, it can be shown that higher wages imply that the price compensated locus  $\tilde{C}_u$  is moved upwards in Figure 4.3 and that  $\tilde{B}_u$  is moved downwards. Specifically,

$$(4.18) \quad \left. \frac{ds}{dw} \right|_{\substack{\tilde{C}_u \\ L=\text{const}}} = - \frac{p \frac{\partial C_u}{\partial p} + w \frac{\partial C_u}{\partial w}}{w \frac{\partial C_u}{\partial s}} > 0$$

and

$$(4.19) \quad \left. \frac{ds}{dw} \right|_{\substack{\tilde{B}_u \\ L=\text{const}}} = - \frac{p \frac{\partial B_u}{\partial p} + w \frac{\partial B_u}{\partial w}}{w \frac{\partial B_u}{\partial s}} < 0,$$

where the signs follow from an extension of the domination of the price effect over the wage effect (see (4.8), (4.9)) to the effective demand function. Hence, for  $w > \hat{w}$ , there exist unemployment states with  $L_u < L^*$  defined by the intersection of  $\tilde{B}_u$  and  $\tilde{C}_u$ , which are associated with lower bond prices. Figure 4.4 depicts such a typical situation. One observes that  $s_B < s_u < s_x$ . The wage increase above  $\hat{w}$  creates unemployment since the initial excess demand created by the wage increase is not sufficiently

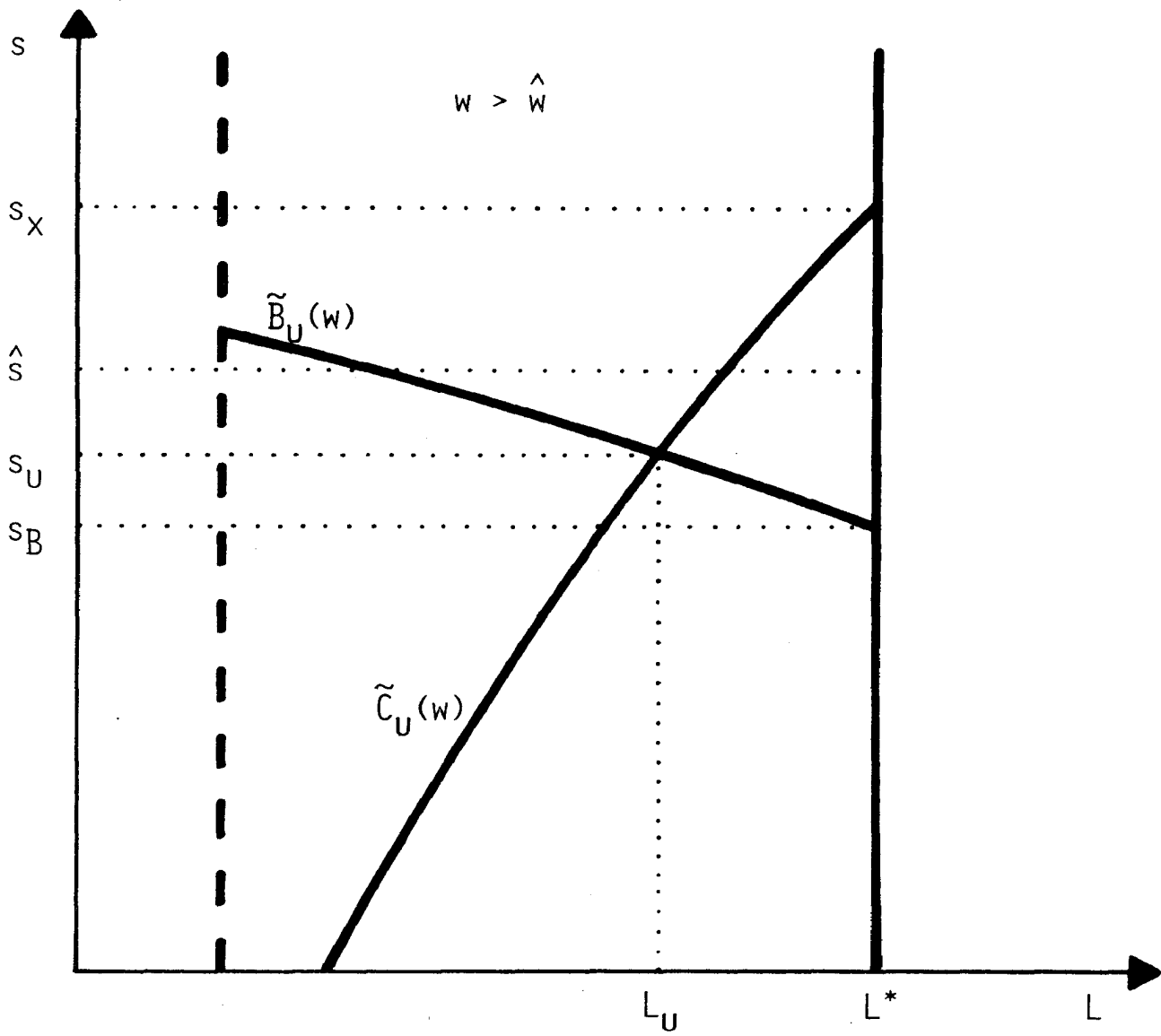


Figure 4.4

offset by an associated price increase. The real wage rises while the bond price may fall or rise.

*Flexible labor supply*

The general case with price and wage dependent labor supply is now a straightforward generalization which will not change the general structure substantially. Considering first the notional market clearing locus for the labor market, one obtains from (4.2)

$$(4.20) \quad \left. \frac{ds}{dp} \right|_w = - \frac{H' \frac{w}{p^2} + \frac{\partial A}{\partial p}}{\frac{\partial A}{\partial s}}$$

$$(4.21) \quad \left. \frac{ds}{dw} \right|_p = \frac{H' \frac{1}{p} - \frac{\partial A}{\partial w}}{\frac{\partial A}{\partial s}} .$$

Using the assumptions made in (C1), (4.20) is negative and (4.21) is positive. Thus, the notional market clearing locus for the labor market in  $(s,p)$ -space is downward sloping and shifts upwards for higher wages.

Considering next the price compensated labor supply schedule in  $(L,s)$ -space one obtains from

$$L - A\left(\frac{w}{F'(L)}, w, s\right) = 0$$

$$(4.22) \quad \left. \frac{ds}{dL} \right|_w = \frac{1 + \frac{\partial A}{\partial p} \frac{p}{L} \cdot \frac{F''}{F'} L}{\frac{\partial A}{\partial s}}$$

$$(4.23) \quad \left. \frac{dL}{dw} \right|_s = \frac{\frac{1}{w} \left( p \frac{\partial A}{\partial p} + w \frac{\partial A}{\partial w} \right)}{1 + \frac{\partial A}{\partial p} \frac{p}{L} \cdot \frac{F''}{F'} L} .$$

The second term of the numerator of (4.22) and of the denominator of (4.23) is a product of two elasticities, the price elasticity of labor supply and the marginal product elasticity, where the latter measures the elasticity of the output price required for an increase in labor demand. From (C1) it follows that (4.22) has a negative sign, i.e. the price compensated labor supply schedule is downward sloping. If the wage elasticity of labor supply is larger than the price elasticity, then (4.23) is positive, i.e. the price compensated labor supply schedule in  $(L, s)$  space shifts to the right with higher wages. It should be noted that this relationship determines the employment frontier for the economy as a whole. It indicates the maximum level of employment which is possible at any given bond price. The important fact to notice is that it is independent of total wealth and of any government policy parameters. It is determined completely by the technology, labor supply behavior and the wage rate.

Considering next the effective market clearing loci  $\tilde{B}_u$  and  $\tilde{C}_u$ , one observes that they are independent of the labor supply behavior of young workers. Hence,  $\tilde{B}_u$  and  $\tilde{C}_u$  remain unchanged and are still described by the properties derived earlier in (4.12)-(4.19). Therefore, it can be seen easily that for  $w > \hat{w}$

an unemployment situation will arise with an employment level  $L_u$  less than  $\hat{L}$ . Figure 4.5 depicts the typical situation for  $w < \hat{w}$ , where the corresponding graphs for  $\hat{w}$  have been added as dotted lines. The diagram also shows that a definite answer about the bond price cannot be given without further assumptions. Hence, the bond price may fall or rise for  $w > \hat{w}$ .

In order to determine the impact of  $w > \hat{w}$  on commodity prices, it is convenient to analyse the notional and effective market clearing conditions in the space of bond prices and of commodity prices. With flexible labor supply the notional market clearing locus of the labor market is downward sloping (see (4.20), (4.21)). Since all three functions in  $(s,p)$ -space shift upwards with a wage increase, it is necessary to make some further assumptions about the relative sizes of the different effects. These are listed as assumption (C5).

$$(C5) \quad \begin{aligned} (i) \quad & \frac{\partial B}{\partial p} \frac{\partial A}{\partial s} - \frac{\partial B}{\partial s} \frac{\partial A}{\partial p} < 0 \\ (ii) \quad & \frac{\partial B}{\partial w} \frac{\partial A}{\partial p} - \frac{\partial B}{\partial p} \frac{\partial A}{\partial w} > 0 \\ (iii) \quad & \frac{\partial C}{\partial w} \frac{\partial A}{\partial p} - \frac{\partial C}{\partial p} \frac{\partial A}{\partial w} > 0. \end{aligned}$$

(i) states that the market clearing locus of the bond market has a steeper slope than the one of the labor market, which is another way of saying that the bond price has a stronger impact on equilibrium conditions on the bond market than on the labor market. (ii) and (iii) postulate that wage changes imply higher price adjustments on the labor market than on the commodity and

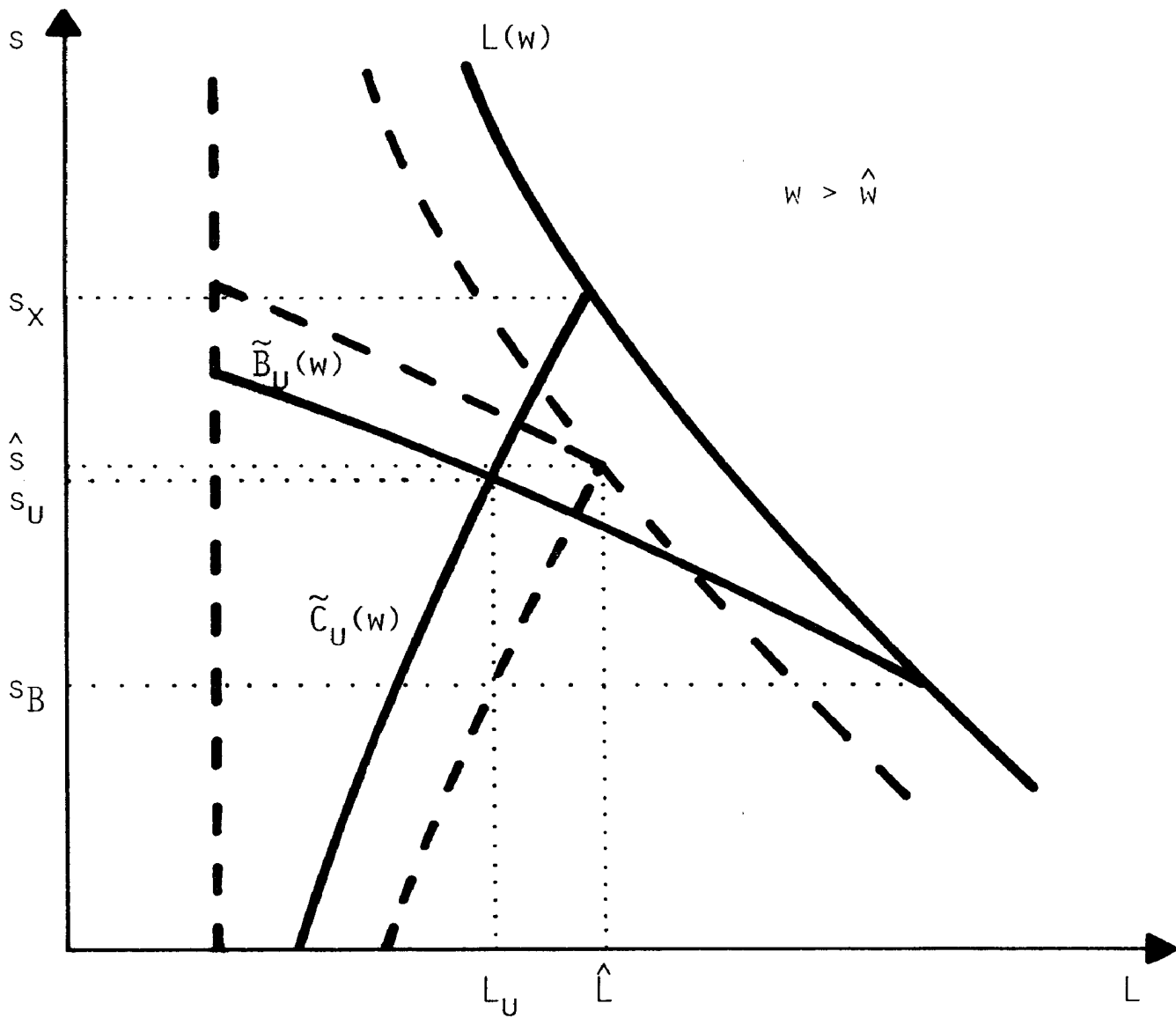


Figure 4.5



on the bond market, to reestablish equilibrium. The consequences of (C5) are shown in Figure 4.6, where the two sets of market clearing loci have been drawn for  $\hat{w}$  and for  $w > \hat{w}$ . One observes that  $s_B < \hat{s} < s_x$ . In fact, some straightforward calculations establish the following result.

Given assumptions (C1), (C2), (C5) then

$$\frac{ds_x}{dw} > 0 \quad \text{and} \quad \frac{ds_B}{dw} < 0,$$

*i.e.* at  $w > \hat{w}$ , simultaneous labor market clearing and commodity market clearing requires a higher bond price  $s_x$  and a lower commodity price  $p_x$  than the simultaneous clearing of the labor market and of the bond market. Formally stated: if  $(p_x, w, s_x)$  solves

$$C(p, w, s) + g - F\left(H\left(\frac{w}{p}\right)\right) = 0$$

$$H\left(\frac{w}{p}\right) - A(p, w, s) = 0$$

and if  $(p_B, w, s_B)$  solves

$$B(p, w, s) - B = 0$$

$$H\left(\frac{w}{p}\right) - A(p, w, s) = 0,$$

then

$$p_B > p_x$$

$$s_B < s_x.$$

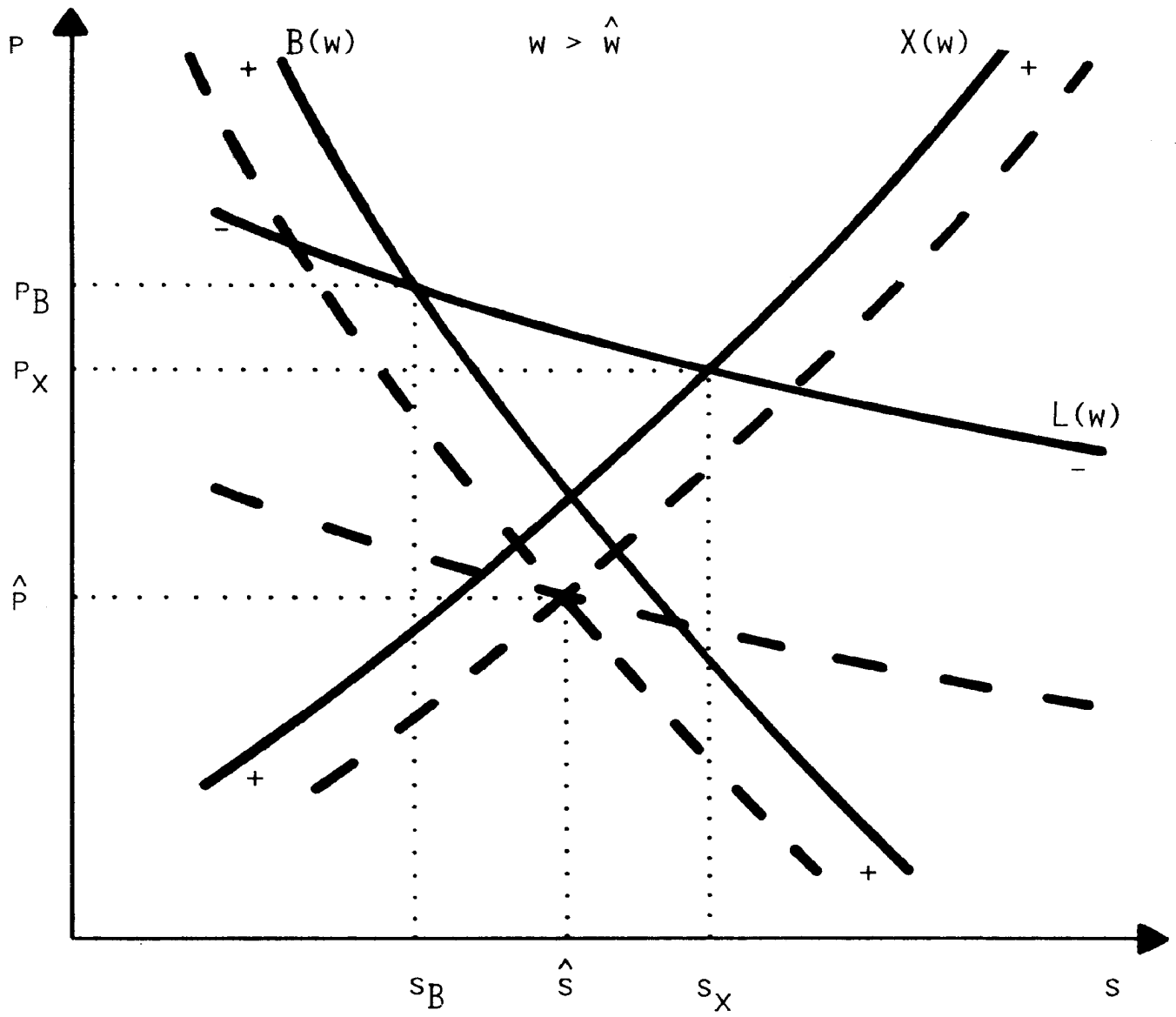


Figure 4.6

The next step is to analyse the effective market clearing conditions in  $(s,p)$ -space. It is easy to see that the price compensated market clearing conditions for the two markets  $\tilde{C}_u$  and  $\tilde{B}_u$  have as a mirror image a representation in  $(s,p)$ -space, i.e.  $\tilde{C}_u$  is defined by

$$(4.24) \quad C_u\left(p, w, s, H\left(\frac{w}{p}\right)\right) + g - F\left(H\left(\frac{w}{p}\right)\right) = 0$$

and  $\tilde{B}_u$  by

$$(4.25) \quad B_u\left(p, w, s, H\left(\frac{w}{p}\right)\right) - B = 0$$

with  $H\left(\frac{w}{p}\right) < A(p, w, s)$ . Given  $w > \hat{w}$ , one finds that for any pair  $(s, p)$  which solves (4.24) there must exist excess demand in notional terms on the commodity market since

$$C_u\left(p, w, s, A(p, w, s)\right) > C_u\left(p, w, s, H\left(\frac{w}{p}\right)\right).$$

Similarly, any pair  $(s, p)$  which solves (4.25) yields excess demand on the bond market in notional terms. Hence,  $\tilde{C}_u$  as a curve in  $(s, p)$ -space has to lie below  $L(w)$  and to the right of  $X(w)$ . Similarly  $\tilde{B}_u$  as a curve in  $(s, p)$ -space has to lie to the left of  $B(w)$ . The characterization of the corresponding graphs is given by the following results:

*Under the assumptions (C1), (C3) one has*

$$(i) \quad \left. \frac{dp}{ds} \right|_{\tilde{C}_u} > 0 \quad \text{and} \quad \left. \frac{dp}{dw} \right|_{\tilde{C}_u} > 0.$$

*If in addition (C4) and the assumption*

$$(C6) \quad \frac{\partial B_u}{\partial w} + \frac{1}{pF''} \frac{\partial B_u}{\partial L} > 0$$

holds, then

$$(ii) \quad \left. \frac{dp}{ds} \right|_{\tilde{B}_u} < 0 \quad \text{and} \quad \left. \frac{dp}{dw} \right|_{\tilde{B}_u} > 0.$$

These properties imply that prices rise for  $w > \hat{w}$ .

(C6) is an assumption parallel to (C4). Since  $1/pF'' = H'/p$ , the second term measures the spillover on the bond market due to a wage increase. (C6) postulates that the immediate impact on the bond market is larger than the spillover effect. Differentiation of (4.24) and substituting  $H' = 1/F''$  yields

$$(4.26) \quad \left. \frac{dp}{ds} \right|_{\tilde{C}_u} = \frac{-\frac{\partial C_u}{\partial s}}{\frac{\partial C_u}{\partial p} + \frac{F'}{pF''} \left( F' - \frac{\partial C_u}{\partial L} \right)}$$

and

$$(4.27) \quad \left. \frac{dp}{dw} \right|_{\tilde{C}_u} = \frac{-\frac{\partial C_u}{\partial w} + \left( F' - \frac{\partial C_u}{\partial L} \right) / pF''}{\frac{\partial C_u}{\partial p} + \frac{F'}{pF''} \left( F' - \frac{\partial C_u}{\partial L} \right)}.$$

Using (C1) and (C3) then implies that (4.26) and (4.27) are both positive. Hence, the effective market clearing locus  $\tilde{C}_u$  (employment compensated) in (s,p)-space is upward sloping, and it shifts upward for higher wages.

Differentiation of (4.25) yields

$$(4.28) \quad \left. \frac{dp}{ds} \right|_{\tilde{B}_u} = \frac{-\frac{\partial B_u}{\partial s}}{\frac{\partial B_u}{\partial p} - \frac{F'}{pF''} \frac{\partial B_u}{\partial L}}$$

and

$$(4.29) \quad \frac{dp}{dw} \Big|_{\tilde{B}_u, s} = - \frac{\frac{\partial B_u}{\partial w} + \frac{1}{pF''} \frac{\partial B_u}{\partial L}}{\frac{\partial B_u}{\partial p} - \frac{F'}{pF''} \frac{\partial B_u}{\partial L}} .$$

The denominator of both expressions is negative due to assumption (C4). Therefore, (4.28) is negative. The numerator of (4.29) is positive because of (C6). Hence (4.29) is positive. The effective market clearing locus  $\tilde{B}_u$  (employment compensated) in (s,p)-space has a negative slope, and it is shifted upwards for higher wages. Figure 4.7 gives the geometrical summary of these results in (s,p)-space. Since the two functions  $\tilde{B}_u$  and  $\tilde{C}_u$  shift upwards with  $w > \hat{w}$ , their intersection also shifts upwards, so that  $p_u > \hat{p}$ . In fact, differentiating (4.24) and (4.25) and solving for  $dp/dw$  yields

$$(4.30) \quad \frac{dp}{dw} = \frac{\frac{\partial B_u}{\partial s} \left[ \left( F' - \frac{\partial C_u}{\partial L} \right) - pF'' \frac{\partial C_u}{\partial w} \right] + \frac{\partial C_u}{\partial s} \left[ pF'' \frac{\partial B_u}{\partial w} + \frac{\partial B_u}{\partial L} \right]}{\frac{\partial B_u}{\partial s} \left[ F' \left( F' - \frac{\partial C_u}{\partial L} \right) + pF'' \frac{\partial C_u}{\partial p} \right] + \frac{\partial C_u}{\partial s} \left[ F' \frac{\partial B_u}{\partial L} - pF'' \frac{\partial B_u}{\partial L} \right]} .$$

Using (C1), (C2), (C4) and (C6) one establishes easily that (4.30) is positive. Hence prices rise as  $w > \hat{w}$ , but real wages rise as well. Therefore, the price increase is not sufficient to offset the wage increase, which in turn leads to an employment decrease.

In summary, this section established the following results:

*Under flexible commodity prices and bond prices wage rates  $w$  higher than the Walrasian one  $\hat{w}$  imply unemployment situations, higher commodity prices and higher real wages. The effect on bond prices and thus on the (expected) rate of interest is ambiguous.*

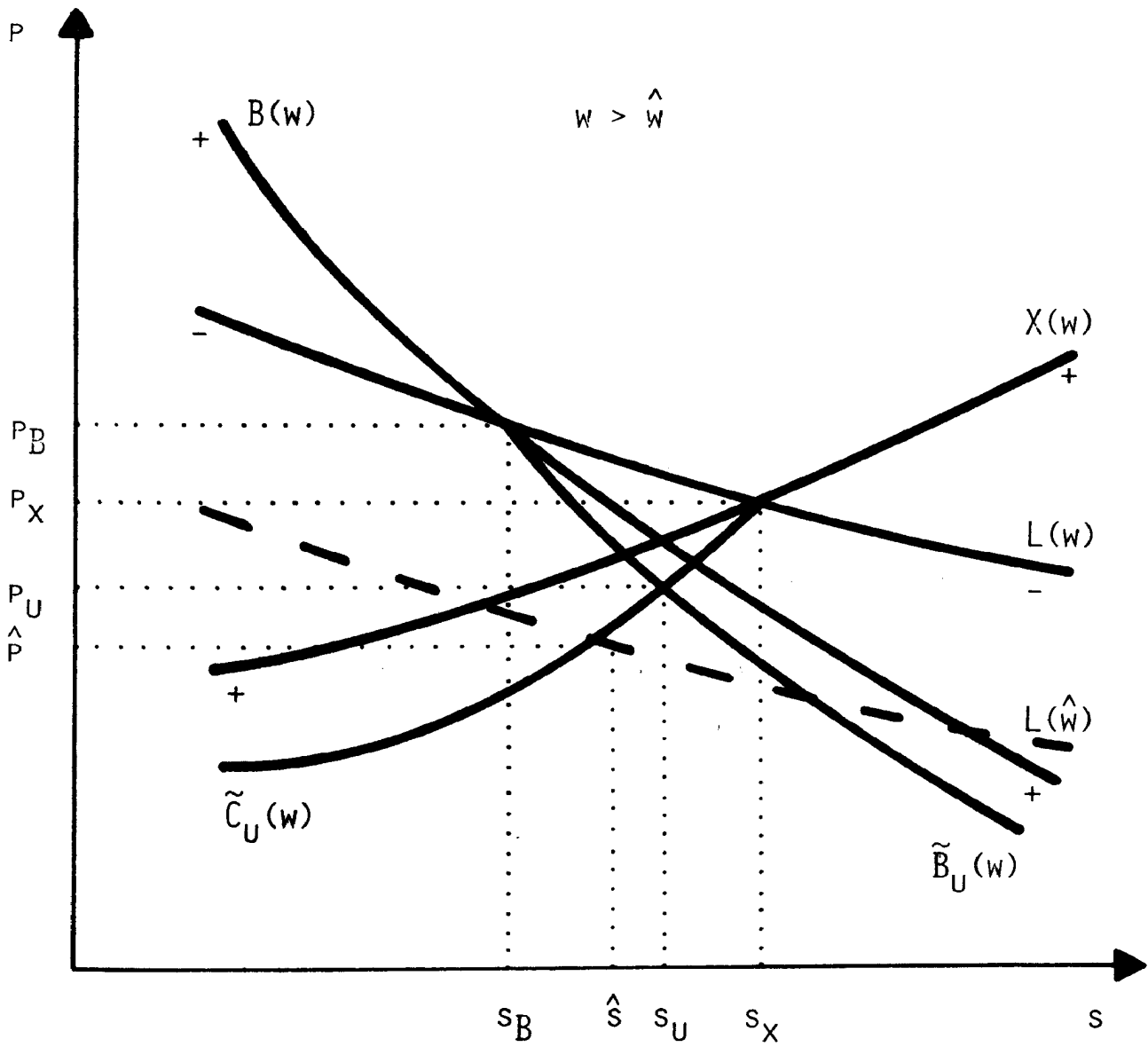


Figure 4.7

## 5. Comparative statics and IS-LM

The analysis of the model in situations of unemployment presented in the preceding section has shown a striking similarity with the traditional Keynesian IS-LM analysis while maintaining essentially the same simple structure. The reduced form of the model defines unemployment states as a solution  $(s, L)$  of a system of the two equations describing the effective market clearing loci for the commodity market and for the bond market. Given a government policy vector  $(g, \tau, B)$ , a fixed wage rate  $w$  and the wealth of old consumers  $(M_o, B_o)$  a pair  $(s, L)$  is an unemployment situation if it solves

$$(5.1) \quad \tilde{C}_u(w, s, \tau, M_o, B_o, L) + g - F(L) = 0$$

$$(5.2) \quad \tilde{B}_u(w, s, \tau, L) - B = 0$$

$$(5.3) \quad L < A \left( \frac{w}{F'(L)}, w, s \right).$$

The model has several distinct advantages over the traditional IS-LM characterization of unemployment states. It distinguishes between all price effects and all quantity effects. This will make it possible in a later section to analyse other types of disequilibria and to determine at which exogenous parameters unemployment states occur. The model is genuinely dynamic. Therefore, sequences of equilibria can be studied. Stationarity and the convergence of non-stationary paths can be examined. In particular, the consequences of exogenous shocks, of government policies as well as the dynamic paths of all endogenous variables under non-stationarity can be studied. Some of these problems will be approached in section 7.

One important difference, however, should be pointed out. The reduced form model treats the commodity market and the bond market explicitly eliminating the money market, while the traditional IS-LM-analysis considers the money market explicitly. By Walras Law, either procedure is possible. In the present context it seemed more natural to consider the bond market explicitly, since the concept of net supply of bonds is well defined. On the other hand, little structure of the money market has been incorporated into the model, a banking sector has not been modelled. Hence, changes in the money stock appear as slack variables in the government budget equation. With a more explicit modelling of a banking sector (see Eichberger (1983)), this approach might have to be changed.

In the model presented here, the demand for money and for bonds arises out of a pure portfolio decision. No transactions motive to hold money exists. As is well known from the literature (see e.g. Clower (1967), Hool (1980)), transactions restrictions can be introduced in a straightforward way which imply the holding of money over and above the store of value motive. For a macroeconomic model which uses such constraints see Eichberger (1983).

#### *Government purchases*

The comparative statics properties of the model can be derived in the usual way analyzing equations (5.1) and (5.2). Consider first an autonomous increase of government purchases  $g$ .



Differentiating (5.1) and (5.2) yields

$$(5.4) \quad \frac{dL}{dg} = \frac{\frac{\partial \tilde{B}_u}{\partial s}}{F' \frac{\partial \tilde{B}_u}{\partial s} + \left( \frac{\partial \tilde{C}_u}{\partial s} \frac{\partial \tilde{B}_u}{\partial L} - \frac{\partial \tilde{B}_u}{\partial s} \frac{\partial \tilde{C}_u}{\partial L} \right)}.$$

It follows from (4.14) and (4.17) that the second term in the denominator is negative. Hence, an increase in  $g$  implies an increase of employment. Rewriting (5.4) as

$$\frac{dL}{dg} = \frac{1}{F' - \frac{\partial \tilde{C}_u}{\partial L} + \frac{\partial \tilde{C}_u}{\partial s} \frac{\partial \tilde{B}_u}{\partial L} / \frac{\partial \tilde{B}_u}{\partial s}},$$

one observes that

$$\frac{dL}{dg} > \frac{1}{F' - \frac{\partial \tilde{C}_u}{\partial L}} = \frac{1}{F' \left( 1 - \frac{p}{w} \frac{\partial \tilde{C}_u}{\partial L} \right)}.$$

Since the marginal propensity to consume out of income is less than one, the output multiplier

$$\frac{dY}{dg} = F' \frac{dL}{dg} > \frac{1}{1 - \frac{p}{w} \frac{\partial \tilde{C}_u}{\partial L}}$$

is greater than one.

Similarly, one obtains for the change of the bond price

$$(5.5) \quad \frac{ds}{dg} = \frac{-\frac{\partial \tilde{B}_u}{\partial L}}{F' \frac{\partial \tilde{B}_u}{\partial s} + \left( \frac{\partial \tilde{C}_u}{\partial s} \frac{\partial \tilde{B}_u}{\partial L} - \frac{\partial \tilde{B}_u}{\partial s} \frac{\partial \tilde{C}_u}{\partial L} \right)},$$

which has a negative sign. It is worth considering these two results in separate steps. The increase in government demand creates excess demand on the commodity market which causes prices to rise. The real wage falls and therefore employment increases. Since the spillover on the bond market is dominated by the price effect the latter implies a decrease of the bond price, to clear the bond market. This in turn decreases demand of all old consumers. Hence, there is a direct demand stimulus from government purchases which is weakened through the wealth effect of rentiers.

The employment effect of government purchases clearly has a positive effect on tax revenue. The difference of the additional expenditures and additional tax revenue defines the necessary increment of the government's deficit, which has to be financed through additional money supply. It is assumed here that there is no change in the volume of bonds. From the government budget equation

$$pg + B_0 = \Delta M + \tau \Pi(p, w)$$

it follows that financing through additional money creation becomes necessary if and only if

$$\frac{d(p \cdot g)}{dg} > \frac{d(\tau \cdot \Pi)}{dg}.$$

Taking the appropriate derivatives one obtains as an equivalent condition

$$p > (\tau F(L) - g) \frac{dp}{dg}.$$

At a balanced budget, one always has  $\tau F(L) - g > 0$ . Hence, in this case the right hand side of the inequality is positive.

Therefore the size of the relative price effect determines primarily whether a deficit or a surplus will arise out of additional government purchases.

*Bond market policy*

An exogenous change in the supply of bonds can be analyzed in the same way. Differentiating (5.1) and (5.2) yields

$$(5.6) \quad \frac{dL}{dB} = \frac{\frac{\partial \tilde{C}_u}{\partial s}}{F' \frac{\partial \tilde{B}_u}{\partial s} + \left( \frac{\partial \tilde{C}_u}{\partial s} \frac{\partial \tilde{B}_u}{\partial L} - \frac{\partial \tilde{B}_u}{\partial s} \frac{\partial \tilde{C}_u}{\partial L} \right)}$$

and

$$(5.7) \quad \frac{ds}{dB} = \frac{F' - \frac{\partial \tilde{C}_u}{\partial L}}{F' \frac{\partial \tilde{B}_u}{\partial s} + \left( \frac{\partial \tilde{C}_u}{\partial s} \frac{\partial \tilde{B}_u}{\partial L} - \frac{\partial \tilde{B}_u}{\partial s} \frac{\partial \tilde{C}_u}{\partial L} \right)}.$$

Both expressions are negative, so that a decrease in the supply of bonds, i.e. an open market purchase, causes an increase of employment and of the bond price. In this case, the increase of the bond price creates an excess demand for commodities which causes prices and employment to rise.

An analysis of the government budget constraint shows the effect on the money stock. Differentiating the budget equation

$$pg + B_0 = \tau \Pi(p, w) + \Delta M + s(B - B_0)$$

with respect to B and evaluating the derivative at  $B = B_0$  yields

$$(5.8) \quad \frac{dM}{dB} = \left( g - \tau F(L) \right) \frac{dp}{dB} - s.$$

From (5.6) one deduces immediately that  $dp/dB$  is negative. As before, near a balanced budget situation  $\tau F(L) - g > 0$  holds. Therefore, the sign of (5.8) depends on the relative size of the two expressions. Hence,  $dM/dB$  will be negative if the price effect of the open market operation is small relative to the current bond price  $s$ . In such a case a decrease of the supply of bonds increases the amount of money held by the private sector.

The open market operation through the bond market in the model here is the counterpart to the so-called monetary policy in the traditional IS-LM model. The effectiveness of such a policy clearly depends on the interest elasticity of the demand for bonds. If  $\partial \tilde{B}_u / \partial s$  approaches minus infinity the expression in (5.6) tends to zero, i.e. open market operations become ineffective. This corresponds to the phenomenon of the liquidity trap. A decrease in the bond supply has almost no effect on the bond price, thus eliminating all effects on demand and on employment. This implies in particular that (5.8) is negative. Hence, an open market operation increases the stock of money. But the additional liquidity provided for the economy is held exclusively as a store of value by young consumers who substitute bonds for money.

The result of this section can be summarized as follows:

*In situations of unemployment active government policies on the commodity market and on the bond market predict the same employ-*

*ment and price effects as the traditional IS-LM-model whether the bond demand exhibits an infinite or a finite elasticity. However, the effects on the amount of money are only unambiguous in the case of a bond market policy when the demand for bonds is highly elastic.*

#### 6. Overemployment equilibria with fixed wage rates

In section 4 it was shown that wage rates  $w$  higher than the temporary Walrasian level  $\hat{w}$  led to situations with unemployment where the prices of commodities and bonds were sufficiently flexible to clear the two corresponding markets. It also can be seen immediately that an exogenous wage decrease below  $\hat{w}$  makes unemployment states impossible. There is no intersection of the two price compensated market clearing loci  $\tilde{B}_u$  and  $\tilde{C}_u$ . Redrawing the notional market clearing loci for all three markets in Figure 6.1 at a wage rate  $w < \hat{w}$ , indicates that simultaneous clearing of all three markets is impossible. Due to the assumption that the price response necessary to clear the labor market caused by a wage change is larger than the corresponding price changes in the other two markets, the pair  $(s, p)$  which clears the latter two lies above  $L(w)$ , i.e. in the region of excess demand for labor. Thus, labor demand rationing will occur. Denote, as in section 4, by  $(s_B, p_B)$  the pair of the bond price and of the commodity price which clears the labor and the bond market simultaneously and by  $(s_X, p_X)$  the pair which clears the labor and the commodity market simultaneously. It can be seen from Figure 6.1 that  $s_B > s_X$ .

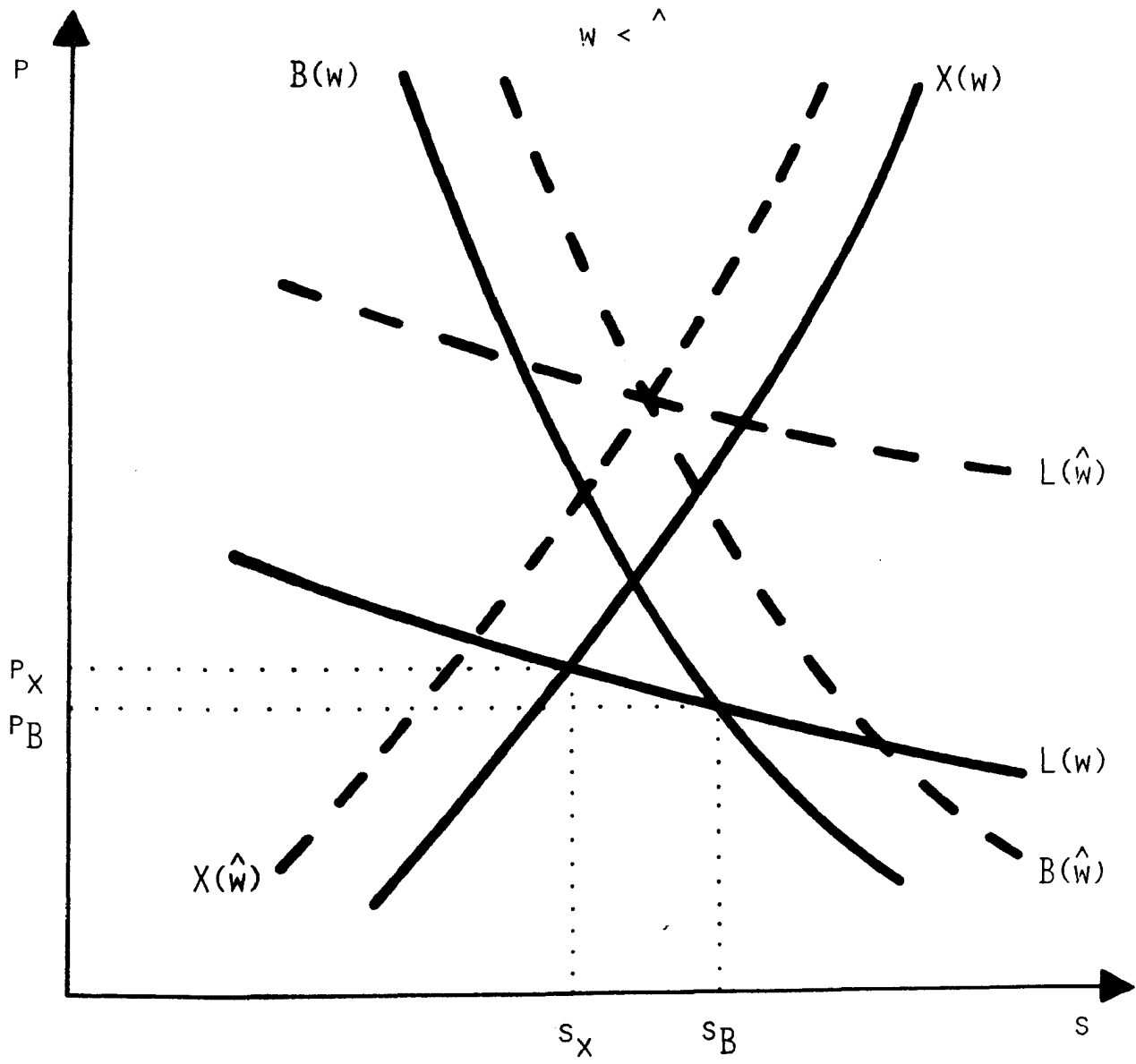


Figure 6.1

In case of demand rationing on the labor market young producer consumers face a binding constraint on the labor market given by the unconstrained labor supply decision of young workers. The solution of the producer's maximization problem

$$\text{Max } E v_p(x_{1p}, x_{2p})$$

subject to

$$p_1 x_{1p} + m_{1p} + s_1 b_{1p} = (1 - t)\pi$$

$$p_2 x_{2p} = m_{1p} + (\tilde{s}_2 + 1)b_{1p}$$

$$\pi = p_1 F(z) - wz$$

$$z \leq L < H\left(\frac{w}{p_1}\right)$$

yields effective demand functions

$$x_{1p} = c_{op}(p_1, w, s_1, L)$$

$$m_{1p} = m_{op}(p_1, w, s_1, L)$$

$$b_{1p} = b_{op}(p_1, w, s_1, L)$$

and a commodity supply function

$$y = F(L).$$

The properties governing these functions are listed in the following set of assumptions which are a natural extension of the previous ones. (The time subscript will be suppressed again since all prices refer to the current period).

$$(C7) \text{ (i) } F' > \frac{\partial c_{op}}{\partial L} > 0$$

$$\text{(ii) } \frac{\partial b_{op}}{\partial L} > 0$$

$$(iii) \quad \frac{\partial c_{op}}{\partial p} > 0; \quad \frac{\partial b_{op}}{\partial p} > 0$$

$$(iv) \quad \frac{\partial b_{op}}{\partial s} < 0$$

$$(v) \quad \frac{\partial c_{op}}{\partial w} < 0; \quad \frac{\partial b_{op}}{\partial w} < 0.$$

The two price effects in (iii) follow from normality and the fact that price increases induce higher profits. The same reasoning yields (v).

Define aggregate effective private consumption demand as

$$C_o(M_o, B_o, p, w, s, L) = \frac{M_o + (1+s)B_o}{p} + c_w(p, w, s) + c_{op}(p, w, s, L)$$

and aggregate effective bond demand as

$$B_o(p, w, s, L) = b_w(p, w, s) + b_{op}(p, w, s, L).$$

For a given wage rate  $w$ , a situation with demand rationing on the labor market is called a state of overemployment. It is defined by a triple  $(p, s, L)$  which solves the following set of equations.

$$(6.1) \quad C_o(M_o, B_o, p, w, s, L) + g - F(L) = 0$$

$$(6.2) \quad L - A(p, w, s) = 0$$

$$(6.3) \quad B_o(p, w, s, L) - B = 0$$

$$(6.4) \quad A(p, w, s) \leq H\left(\frac{w}{p}\right).$$

The following additional assumptions on the aggregate functions will be used later on.



$$\begin{aligned}
 \text{(C8) (i)} \quad & \frac{\partial C_o}{\partial s} > 0; \quad \frac{\partial B_o}{\partial s} < 0 \\
 & \frac{\partial C_o}{\partial p} < 0; \quad \frac{\partial B_o}{\partial p} < 0 \\
 & \frac{\partial C_o}{\partial w} > 0; \quad \frac{\partial B_o}{\partial w} > 0 \\
 \text{(ii)} \quad & \frac{\partial B_o}{\partial p} \frac{\partial A}{\partial s} - \frac{\partial B_o}{\partial s} \frac{\partial A}{\partial p} < 0 \\
 \text{(iii)} \quad & \frac{\partial C_o}{\partial p} \frac{\partial A}{\partial s} - \frac{\partial C_o}{\partial s} \frac{\partial A}{\partial p} > 0.
 \end{aligned}$$

(i) stipulates that the positive wealth effect of a bond price increase for old consumers dominates the possibly negative demand effect of young workers and of producers. Moreover, the aggregate commodity price effect and the wage effect are assumed to have the same signs as the ones in assumption (C1) with the same justification. Assuming that all terms in (ii) and (iii) are nonzero, these can be combined into the inequalities

$$0 > \frac{\frac{\partial B_o}{\partial p}}{\frac{\partial B_o}{\partial s}} > \frac{\frac{\partial A}{\partial p}}{\frac{\partial A}{\partial s}} > \frac{\frac{\partial C_o}{\partial p}}{\frac{\partial C_o}{\partial s}}.$$

Since these ratios are a measure of the responsiveness of the bond price due to a commodity price change given a constant transaction level on these markets, they indicate that the bond market is most price responsive, more than the labor market and the latter more than the commodity market. In other words, the price substitution effect between bonds and commodities is highest with respect to the own price concerned. The size of the corresponding

effect for the labor market is between the other two.

In order to utilize the technique developed in section 4, the bond market and the commodity market will be analyzed separately under the appropriate price compensation. Consider first the commodity and the labor market together described by equations (6.1) and (6.2) taking (6.4) into account. The set of solutions of (6.1) and (6.2) define a price compensated relation between employment and the bond price and, as a mirror image, a relation between the bond price and the commodity price. Both will be denoted by  $\tilde{C}_0$  in the appropriate space. The following result can be shown:

*If the assumptions (C1), (C7), (C8) hold and if in addition*

$$(C9) \quad \frac{\partial C_0}{\partial p} - \frac{\partial A}{\partial p} \left( F' - \frac{\partial C_0}{\partial L} \right) < 0$$

*is satisfied, then*

$$\frac{ds}{dL} \Big|_{\tilde{C}_0} < 0 \quad \text{and} \quad \frac{dp}{ds} \Big|_{\tilde{C}_0} > 0.$$

Assumption (C9) postulates that the price effect on commodity demand dominates the spillover generated through the induced change on employment. Differentiating (6.1) and (6.2) and solving for  $ds/dL$  yields

$$(6.5) \quad \frac{ds}{dL} \Big|_{\tilde{C}_0} = \frac{\frac{\partial C_0}{\partial p} - \frac{\partial A}{\partial p} \left( F' - \frac{\partial C_0}{\partial L} \right)}{\frac{\partial C_0}{\partial p} \frac{\partial A}{\partial s} - \frac{\partial C_0}{\partial s} \frac{\partial A}{\partial p}} .$$

The denominator of (6.5) is positive because of assumption (C8). The numerator is negative due to (C9). Hence (6.5) is negative.

Differentiation and solving for  $dp/ds$  yields

$$(6.6) \quad \left. \frac{dp}{ds} \right|_{\tilde{C}_0} = \frac{-\frac{\partial C_0}{\partial s} + \frac{\partial A}{\partial s} \left( F' - \frac{\partial C_0}{\partial L} \right)}{\frac{\partial C_0}{\partial p} - \frac{\partial A}{\partial p} \left( F' - \frac{\partial C_0}{\partial L} \right)}$$

The denominator and the numerator are negative. Hence (6.6) is positive. Therefore  $\tilde{C}_0$  is upward sloping in  $(s,p)$ -space, as was to be shown. Summarizing these two results one observes that simultaneous clearing of the labor market and the commodity market under overemployment requires that commodity and bond prices rise together while labor supply and therefore employment falls. Since higher prices imply higher notional labor demand, excess demand for labor increases as  $s$  rises above  $s_x$ .

Analyzing the bond market and the labor market simultaneously, i.e. equations (6.2) and (6.3), one obtains as a set of solutions the price compensated relation  $\tilde{B}_0$  between employment and the bond price in  $(L,s)$ -space and as its mirror image a relation between the bond price and the commodity price in  $(s,p)$ -space.  $\tilde{B}_0$  has the following properties:

*Under the assumptions (C1) and (C8)  $\tilde{B}_0$  is upward sloping in  $(L,s)$ -space and downward sloping in  $(s,p)$ -space, i.e.*

$$\left. \frac{ds}{dL} \right|_{\tilde{B}_0} > 0 \quad \text{and} \quad \left. \frac{dp}{ds} \right|_{\tilde{B}_0} < 0.$$

Differentiating (6.2) and (6.3) one obtains the two expressions

$$(6.7) \quad \left. \frac{ds}{dL} \right|_{\tilde{B}_0} = \frac{\frac{\partial A}{\partial p} \frac{\partial B_0}{\partial L} + \frac{\partial B_0}{\partial p}}{\frac{\partial B_0}{\partial p} \frac{\partial A}{\partial s} - \frac{\partial B_0}{\partial s} \frac{\partial A}{\partial p}}$$

and

$$(6.8) \quad \left. \frac{dp}{ds} \right|_{\tilde{B}_0} = - \frac{\frac{\partial B_0}{\partial s} + \frac{\partial B_0}{\partial L} \frac{\partial A}{\partial s}}{\frac{\partial B_0}{\partial p} + \frac{\partial B_0}{\partial L} \frac{\partial A}{\partial p}}.$$

Using (C1) and (C8) it is easy to establish that (6.8) is negative and that (6.7) is positive. Summarising these results, one can state that alternative situations of overemployment and clearing of the bond market and the labor market require higher commodity prices and lower bond prices, where the higher commodity prices imply excess demand on the labor market which is not sufficiently offset by lower bond prices. Figure 6.2 gives the characterization of  $\tilde{C}_0$  in (L,s)-space for  $w = \hat{w}$ .  $L_0$  denotes the smallest employment level compatible with (6.1) and (6.2).

In order to determine the impact of a wage rate  $w$  lower than  $\hat{w}$ , two further assumptions will be introduced. One is the counterpart of (C6) for the overemployment situation. It stipulates that the direct wage effect on consumption dominates the spillover effect generated through an increase of employment, i.e.

$$(C10) \quad \frac{\partial C_0}{\partial w} - \frac{\partial A}{\partial w} \left( F' - \frac{\partial C_0}{\partial L} \right) > 0.$$

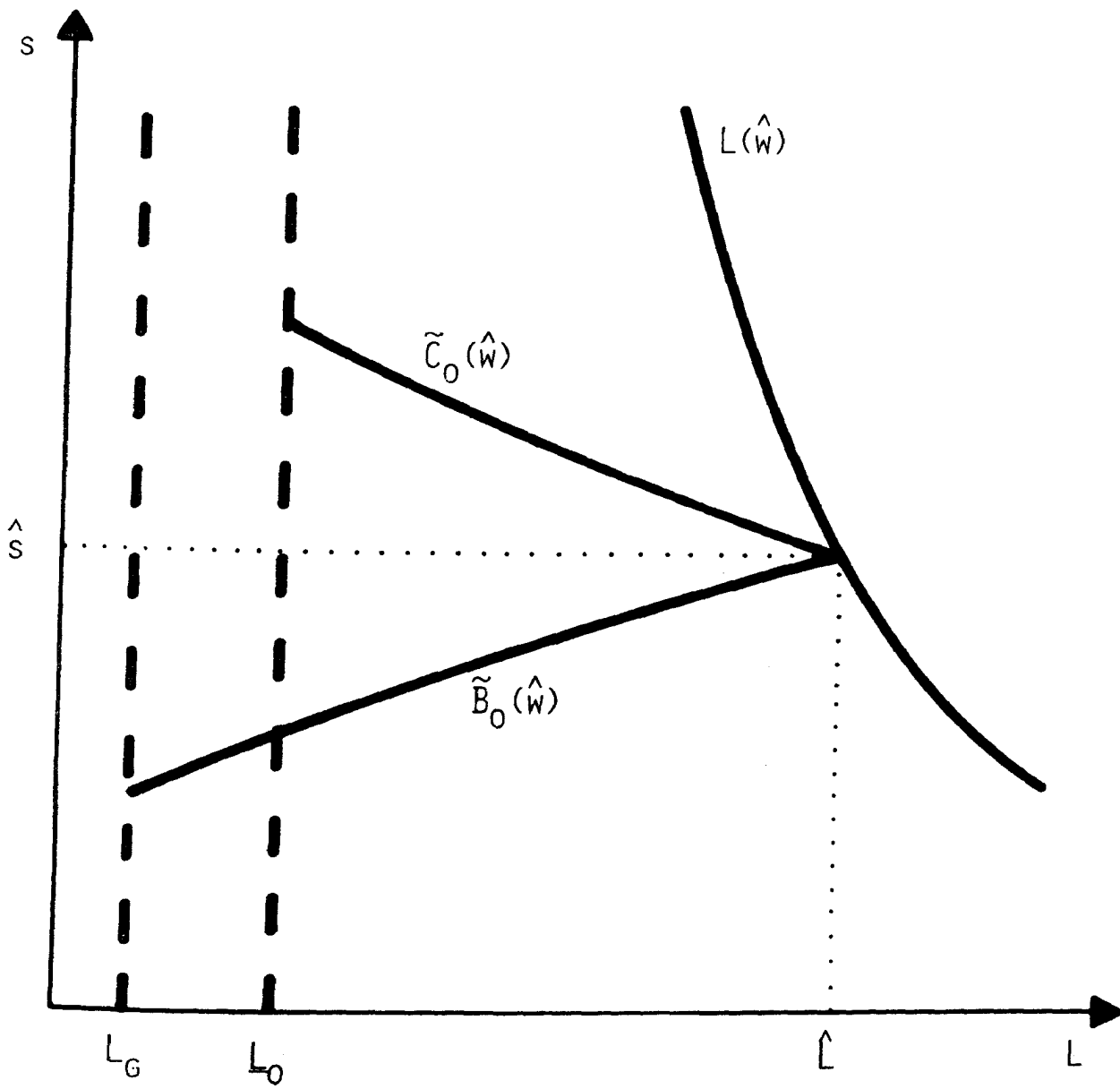


Figure 6.2

The other one is an extension of (C5) (ii) and (iii) to the overemployment case.

$$(C11) \quad (i) \quad \frac{\partial B_o}{\partial w} \frac{\partial A}{\partial p} - \frac{\partial B_o}{\partial p} \frac{\partial A}{\partial w} > 0$$

$$(ii) \quad \frac{\partial C_o}{\partial w} \frac{\partial A}{\partial p} - \frac{\partial C_o}{\partial p} \frac{\partial A}{\partial w} > 0.$$

Then the following results can be shown by straightforward calculations.

If the assumptions (C1), (C8), (C10) hold, then

$$(i) \quad \left. \frac{ds}{dw} \right|_{\substack{\tilde{C}_o \\ p}} < 0 \quad \text{and} \quad \left. \frac{ds}{dw} \right|_{\substack{\tilde{B}_o \\ p}} > 0.$$

If (C1), (C8) and (C11) hold, then

$$(ii) \quad \left. \frac{ds}{dw} \right|_{\substack{\tilde{C}_o \\ L}} > 0 \quad \text{and} \quad \left. \frac{ds}{dw} \right|_{\substack{\tilde{B}_o \\ L}} < 0.$$

As a consequence, for  $w < \hat{w}$ ,  $\tilde{C}_o$  is shifted downwards in (L,s)-space and to the right in (s,p)-space. Similarly,  $\tilde{B}_o$  moves upwards in (L,s)-space and to the left in (s,p)-space.

Figure 6.3 characterizes the situation in (L,s)-space when all three markets are taken into account. A state with overemployment, which is a solution of the equations (6.1)-(6.4), is given by the intersection of the two price compensated relations  $\tilde{C}_o(w)$  and  $\tilde{B}_o(w)$ . Since they have opposite slopes and since  $s_B > s_X$ , there exists a unique triple  $(s_o, p_o, L_o)$

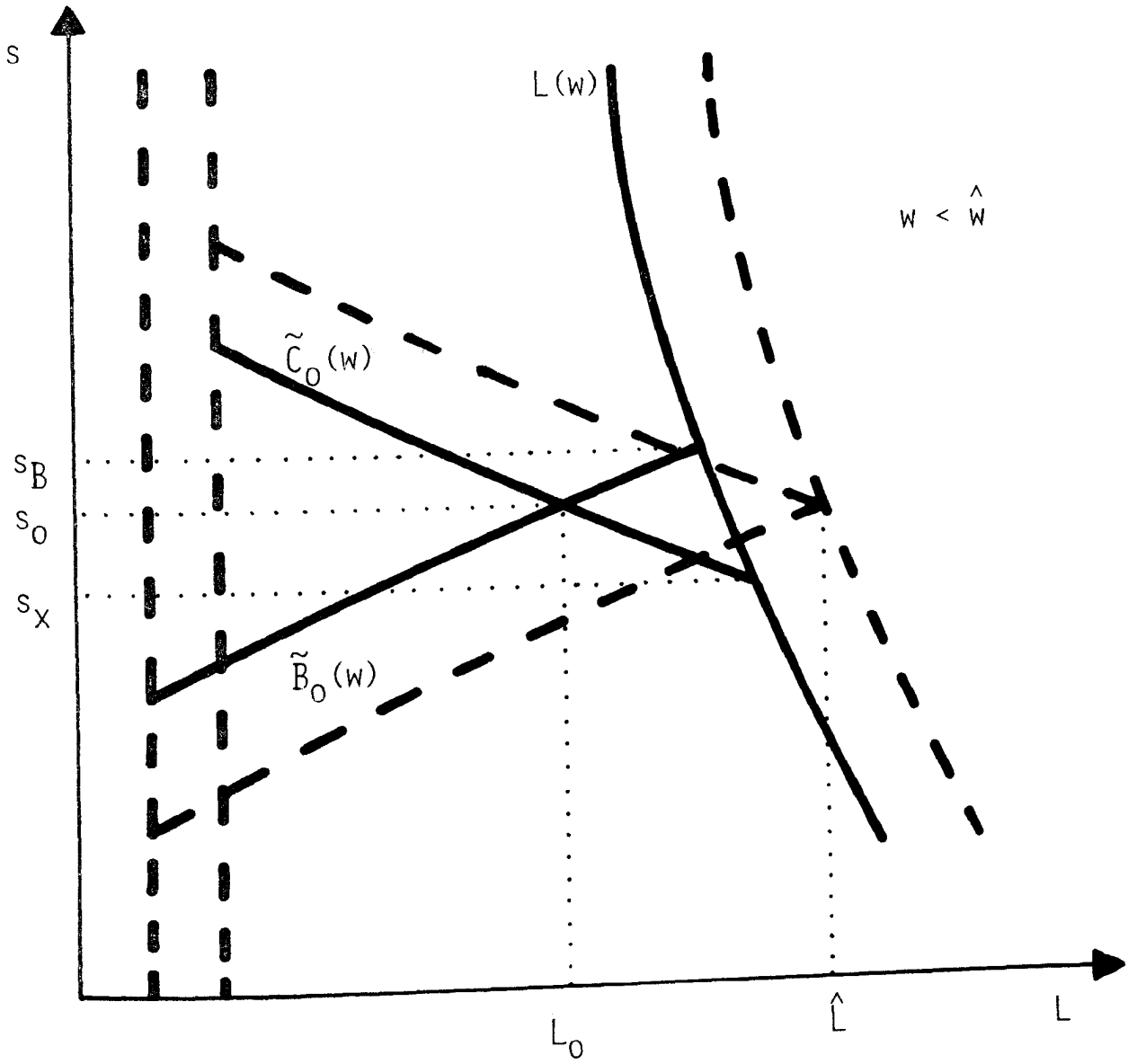


Figure 6-5

as an overemployment state. For  $w < \hat{w}$ , the notional labor market clearing locus  $L(w)$  lies to the left of  $L(\hat{w})$  and  $s_B > \hat{s}$ .  $w < \hat{w}$  implies that  $\tilde{C}_0$  shifts downward and  $\tilde{B}_0$  shifts upward. Therefore, employment falls, i.e.  $L_0 < \hat{L}$ . To see this algebraically, one observes that

$$\frac{dL}{dw} = \frac{\left. \frac{ds}{dw} \right|_{\tilde{C}_0} - \left. \frac{ds}{dw} \right|_{\tilde{B}_0}}{- \left. \frac{ds}{dL} \right|_{\tilde{C}_0} + \left. \frac{ds}{dL} \right|_{\tilde{B}_0}} \cdot$$

The previous results imply that  $dL/dw > 0$ . Hence,  $L_0 < \hat{L}$  for  $w < \hat{w}$ . Figure 6.3 also makes apparent that the sign of the bond price change is ambiguous.

On the other hand, the effect on commodity prices is unambiguous (see Figure 6.4).  $dp/dw$  is given by the expression

$$\frac{dp}{dw} = \frac{\left. \frac{ds}{dp} \right|_{\tilde{C}_0} - \left. \frac{ds}{dp} \right|_{\tilde{B}_0}}{- \frac{1}{\left. \frac{dp}{ds} \right|_{\tilde{C}_0}} + \frac{1}{\left. \frac{dp}{ds} \right|_{\tilde{B}_0}}},$$

which is positive. Therefore,  $p_0 < \hat{p}$  for  $w < \hat{w}$ . Finally, the impact on real wages can be deduced from the following observations. Let  $\tilde{p}$  denote the commodity price such that  $H(\frac{w}{\tilde{p}}) = A(\tilde{p}, w, \hat{s})$  at  $w < \hat{w}$ . It follows from (4.23) that

$$\hat{L} > H(\frac{w}{\tilde{p}}) = A(\tilde{p}, w, \hat{s}).$$

Therefore, the real wage  $w/\tilde{p}$  must be higher than  $\hat{w}/\hat{p}$ . Moreover,



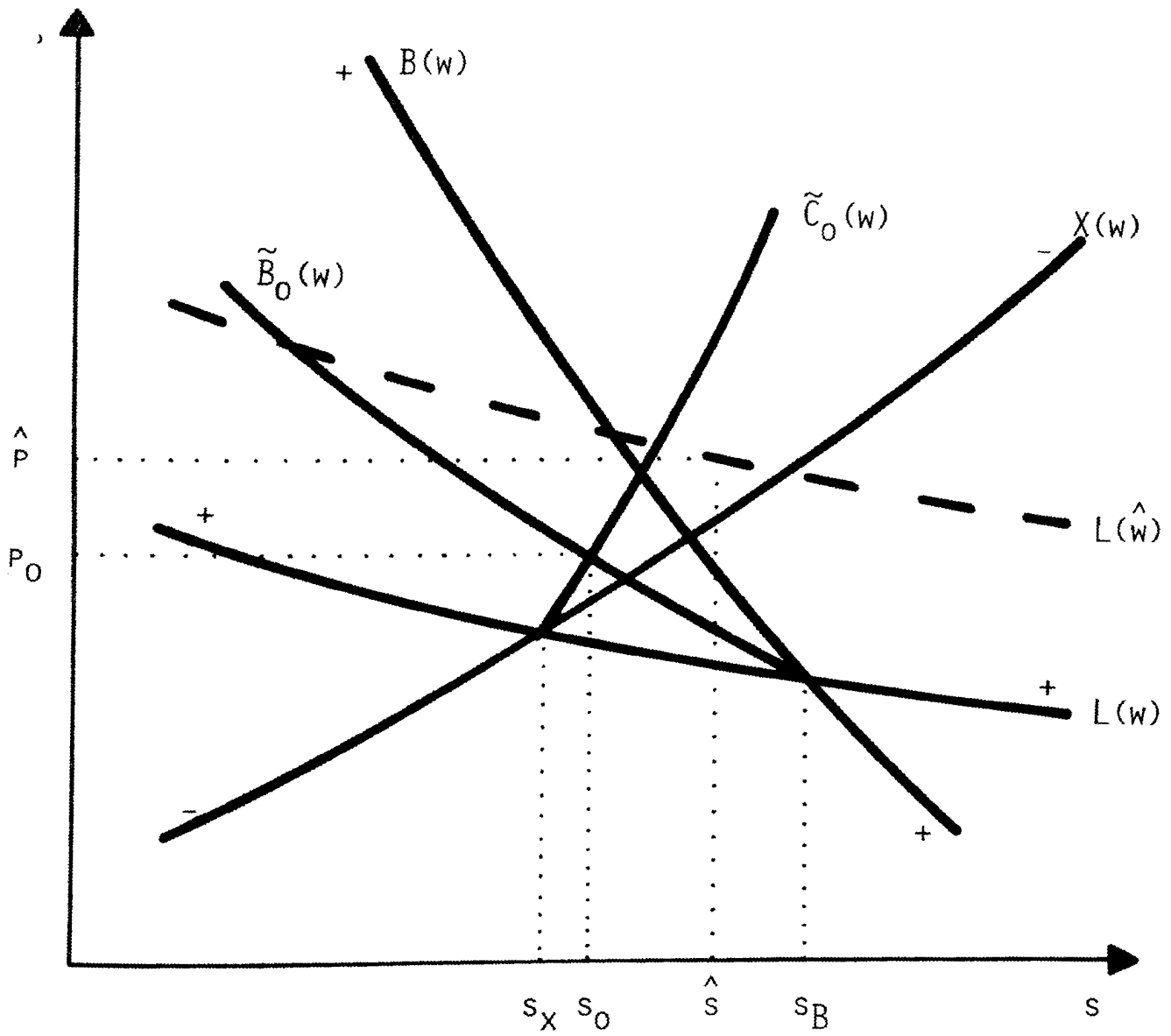


Figure 6.4

$$\begin{aligned} H\left(\frac{w}{p}\right) &= A(\tilde{p}, w, \hat{s}) \\ &> A(p_B, w, s_B) = H\left(\frac{w}{p_B}\right), \end{aligned}$$

since  $s_B > \hat{s}$ . Hence,  $w/p_B > w/\tilde{p}$ . Then,  $s_0 < s_B$  and the fact that  $\frac{dp}{ds} \Big|_{B_0} < 0$ , imply that  $w/p_0 > w/p_B$ . Thus,

$$\frac{\hat{w}}{\hat{p}} < \frac{w}{\tilde{p}} < \frac{w}{p_B} < \frac{w}{p_0}$$

shows that the real wage rises. The results of this section can be summarized as follows:

*For each wage rate  $w$  lower than  $\hat{w}$ , there exists a unique state of overemployment with an employment level  $L_0$  less than  $\hat{l}$ . Commodity prices fall as wages fall. However, real wages rise while excess demand on the labor market remains. This effect is primarily due to the assumption that wage changes affect labor supply behavior more than price changes. Therefore, labor supply falls faster than labor demand in spite of rising real wages.*

## 7. Short run vs. long run effects of government policies

The previous sections contained a detailed discussion of the possible states in any particular time period  $t$  where questions of the dynamic evolution of the economy were ignored. In particular section 5 analyzed the effects of changes of the government policy parameters, and the comparative statics properties of the model were regarded as the short run impacts of such changes. Since, in general, these changes induce a government deficit or surplus an adjustment of the quantity of money will follow which in turn implies changes of aggregate demand in subsequent periods. If the initial action of the government is taken at a stationary state, i.e. one with a balanced budget, the characteristics of the new stationary state as well as the dynamic path should be analyzed. This section largely ignores the adjustment problem but concentrates on the effects of a once and for all permanent change of the government policy variables, and it compares alternative stationary states. The interest in such an analysis stems from the fact that for some policy variables the short run and long run effects differ substantially. If they are of opposite sign the dynamic adjustment cannot be monotonically converging in general. In this section unemployment states will be considered only.

Given a time invariant, i.e. stationary government policy  $(g, \tau, B)$ , a stationary state of the economy is an equilibrium  $(p, w, s, L)$  such that the government deficit is equal to zero,

i.e.

$$pg + B = \tau\Pi$$

where  $\Pi = pF(L) - wL$ . If the equilibrium is one with unemployment, then the real wage is equal to the marginal product of labor and  $\Pi$  is the unrestricted maximal profit of young capitalists. Hence, the stationarity condition can be written as

$$(7.1) \quad pg + B = \tau\Pi(p,w)$$

where  $\Pi$  is the usual profit function with the well-known properties: i)  $\Pi$  is homogeneous of degree one and convex in  $(p,w)$ , ii)  $\partial\Pi/\partial p = F(H(\frac{w}{p}))$  and  $\partial\Pi/\partial w = -H(\frac{w}{p})$ . Figure 7.1 plots the left hand side of equation (7.1) against the right hand side. For a given wage rate  $w$  and a government policy  $(g,\tau,B)$  the stationary price level  $\bar{p}$  is unique as well as the associated employment level  $\bar{L}$  given by  $\bar{p}F'(\bar{L}) = w$ . The effects of changes of the wage rate  $w$  and of government policy parameters can be deduced in a straightforward way. Higher stationary levels of employment are associated with a lower tax rate, lower wages, higher government purchases, and a higher volume of government bonds, i.e.:

$$(7.2) \quad \frac{d\bar{L}}{d\tau} < 0; \quad \frac{d\bar{L}}{dw} < 0$$
$$\frac{d\bar{L}}{dg} > 0; \quad \frac{d\bar{L}}{dB} > 0.$$

Comparing these effects with the results of section 5 shows that the sign of the long run effect of a bond increase is opposite to the short run effect. Thus open

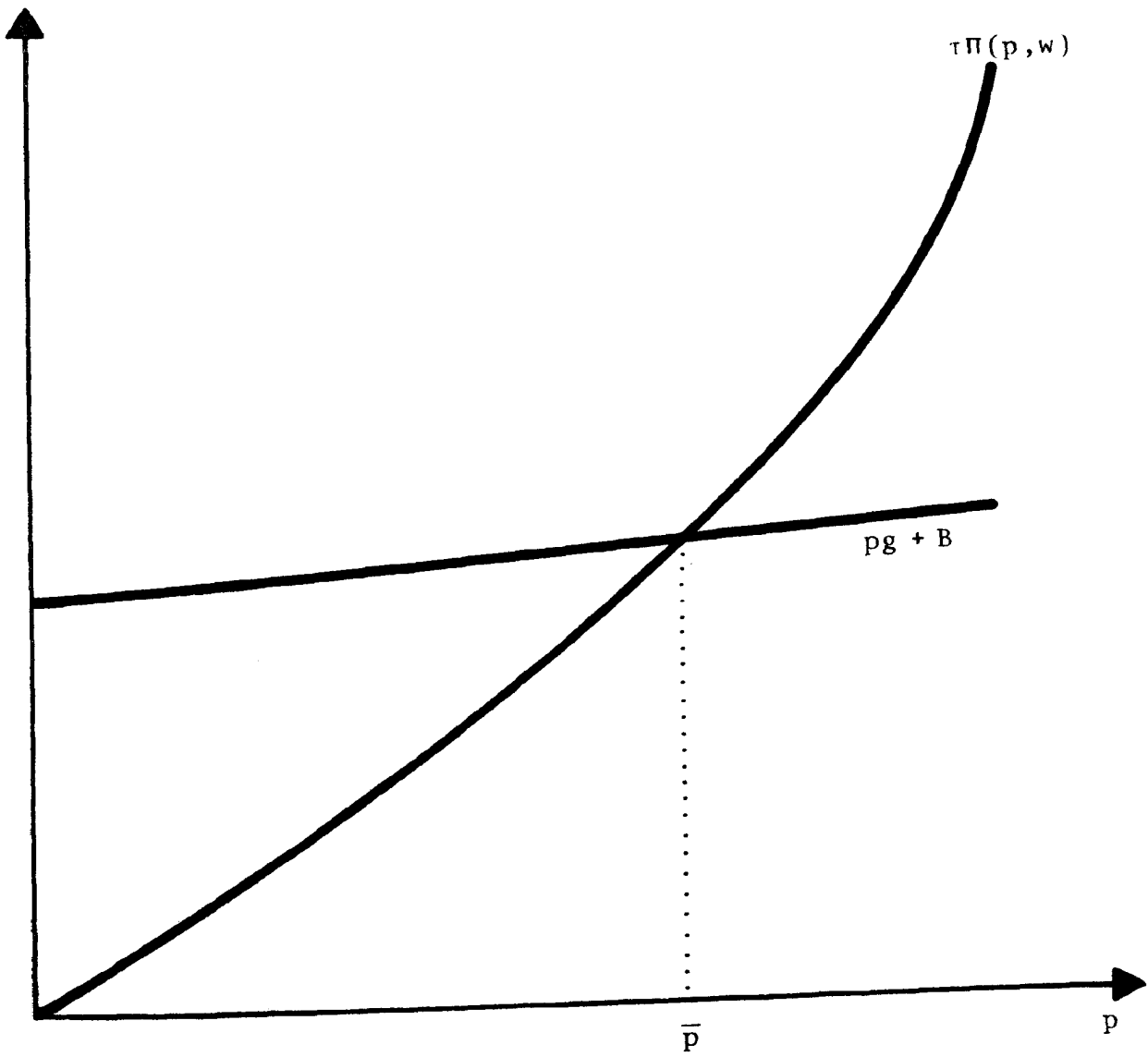


Figure 7.1

market purchases (decreases of public debt) yield an increase of employment in the short run, but they will have a negative effect in the long run. Short run effects and long run effects of changes in the tax rate, the wage rates and in government purchases have the same sign. However, the long run government multiplier will not be greater than one in general. Differentiating (7.1) and solving for  $d\bar{Y}/dg$  yields the condition

$$\frac{d\bar{Y}}{dg} \begin{matrix} > \\ < \end{matrix} 1 \quad \text{if and only if} \quad g \begin{matrix} \geq \\ < \end{matrix} F(L) \left[ \tau + \frac{F'(L) \cdot L}{F(L)} / \frac{F''(L) \cdot L}{F'(L)} \right].$$

The second term in brackets is the ratio of the elasticities of the production function and of the marginal product. Since the latter is negative, the multiplier will be greater than one if the tax rate is sufficiently small.

The effects of long run policy changes and the bond price can be calculated in a straightforward way. Since the demand for bonds is independent of wealth the long run price and employment effects determine primarily the change in the bond price. For all three policy variables one finds that increases have a negative effect on the bond price, i.e.:

$$(7.3) \quad \frac{d\bar{s}}{d\tau} < 0; \quad \frac{d\bar{s}}{dg} < 0; \quad \frac{d\bar{s}}{dB} < 0.$$

The following table summarises and contracts the results of the short run and long effects on the three state variables (p,L,s) at fixed wage rates w. It can be seen from the table that changes of government purchases and of the tax rate have the same quali-

(7.4)

	Short Run			Long Run		
	dg	dτ	dB	dg	dτ	dB
dp	+	-	-	+	-	+
dL	+	-	-	+	-	+
ds	-	-	-	-	-	-

tative effect on prices and employment in the short run as well as in the long run, whereas public debt policy (open market operations) induce opposite effects in the short run and in the long run. The reason for such a striking difference seems straightforward. In the short run an increase in public debt reduces aggregate private demand in two ways. Young consumers defer consumption to the later part of their life by increasing non-monetary wealth. On the other hand old consumer's wealth and hence their consumption demand decreases since the bond price falls. In the long run, however, higher public debt implies a higher steady flow of nominal interest payments which are spent on consumption at one hundred per cent, thus increasing aggregate demand. Analytically, what matters in the short run is the immediate impact on demand and the interdependence of all three markets. In the long run, however, the government budget constraint plays the essential role, a fact generally neglected in Keynesian models.

One further interesting result on the relationship between a long run wage policy and a public debt policy can be deduced directly from (7.1) and the fact that the tax function is homogeneous of degree one in prices and wages. If  $(\bar{p}, \bar{L})$  is the sta-

tionary price-employment pair at  $w$  and  $(g, \tau, B)$  then  $(\lambda \bar{p}, \bar{L})$  is also a stationary price-employment pair at  $\lambda w$  and  $(g, \tau, \lambda B)$  for all  $\lambda > 0$ . In other words, the long run employment level is homogeneous of degree zero in wages and public debt. Long run employment effects due to wage increases can be offset fully by equiproportionate increases in public debt and vice versa. This statement can be generalized directly to situations of more general tax functions which are homogeneous of degree one in prices and wages.

In summary the following results have been established:

- (i) *Government purchases and tax policies affect employment in the short run and in the long run in the same way.*
- (ii) *Public debt induces opposite effects in the short run and in the long run.*
- (iii) *The negative impact of higher wages on long run employment can be offset fully by equiproportionate increases of public debt if the tax function is homogeneous of degree one in prices and wages.*

#### *Policy trade offs*

Much of the debate on the effectiveness of government policy in macroeconomic theory centers around questions of how government purchases are financed. Different models produce quite opposing answers and it is not clear, in general, whether the different results arise from differences of explicitly modelled behavioral



and market clearing equations, or from some implicit assumptions, like rational expectations or some randomness, to name just two. The explicit treatment and description of short run states and of the government budget constraint here allows to draw some precise conclusions.

Some of the debate on policy trade offs centers around the role of deficits and of monetary policy. Apart from the simplest IS-LM model where open market operations are considered as the one-to-one counterpart of monetary policy, it should be clear that changes in the money stock are *not* just the negative of a change in public debt neither in the short run nor in the long run. Moreover, deficits in the short run always have two causes, namely changes of some policy variable and the induced price and tax revenue effects. The discussion in section 5 of this paper of the effects on the stock of money, i.e. on the deficit, indicated that the short run effects do not have an unambiguous sign. The same ambiguity arises already in the simplest income-expenditure model if, for example, taxes depend on income. In this respect, the result is not new. For a detailed analysis of these effects some more structure has to be imposed and it may be debatable whether the integrated public sector, i.e. central bank and government together with one budget constraint only, is the best way to do this. A separation of these two may give more insight into the issues which need clarification.

One aspect, however, seems clear from the above analysis. Once the government policy parameters ( $g, \tau, B$ ) are chosen the deficit,

i.e. money adjustments determine primarily the intermediate run evolution of the economy with the induced price and employment changes. In this sense the deficits as well as the stock of money become endogenous variables of the system. Therefore, in the present context, they are not an active policy variable which can be considered under the heading of policy trade offs. When the intermediate run of the evolution of the economy after an initial shock is at stake, the changes in the money stock will be the crucial variable to be analyzed.

Four situations of policy trade offs and their consequences on employment in the short run and in the long run will be looked at here: 1) debt financed government purchases, 2) tax financed government purchases, 3) substitution of taxes for bonds, and 4) substitution of government purchases for debt.

1) From table (7.4) one observes that increases of government purchases and of government debt have opposite effects on employment in the short run. Hence, the overall effect depends on the relative sizes of the changes of the government policy variables *and* on the induced price changes. However, in the long run purchases as well as debt exert a positive influence on employment, i.e. the employment effect of government purchases is reinforced by the long run demand effect generated through public debt. Since the short run effect and the long run effects differ in this way, it is possible that the intermediate run displays cyclical paths and large deficits.

- 2) Again, table (7.4) shows all the effects for the case of tax financed government purchases. In both runs the employment effects of taxes and of government purchases have opposite signs. Hence, the overall employment effect may be positive, zero, or negative.
- 3) The employment effect of a substitution of taxes for bonds in the short run can be read off table (7.4) as well. The policy variables have opposite employment effects, so that there exists a substitution policy with a lower tax rate and a higher public debt which maintains a constant level of employment. However, in the long run, a lower tax rate and a higher public debt both imply a higher level of employment. Thus, the two effects reinforce each other. Hence, for a balanced budget policy which attempts to maintain a given long run level of employment, a lower tax rate is associated with a lower level of public debt. Here as in case 1), large deficits as well as cyclical developments are possible in the intermediate run.
- 4) Substitution of government purchases for bonds yields opposite effects on employment (see Fig. 7.1) in the long run, so that a constant level of employment and prices can be maintained. In such a case the long run bond price must be lower which implies that consumers substitute consumption in youth for consumption in old age. Since government consumption decreases at a constant output level total private consumption increases. Moreover, total real wealth and therefore consumption in old

age increases by more than the decrease in government purchases. Since the higher total interest payments imply an increase of the two period budget set of each consumer the substitution of government purchases for bonds must be Pareto improving. In the short run, however, such a substitution leads to a strong negative effect on employment.

#### 8. Concluding remarks

This paper presented a model with an overlapping generations structure as its microeconomic base which generated an aggregate macroeconomic model which contains most of the elements of the typical Keynesian IS-LM model. It was shown that unemployment situations arise under flexible commodity prices and bond prices if nominal wages are fixed but too high. Although the model confirmed most of the standard short run comparative statics results, a systematic analysis of the government budget equation and comparisons of alternative stationary states showed that the short run policy implications may prove wrong in the long run. Policy measures which are advisable in the short run can be detrimental in the long run. Policy trade offs can differ substantially in the short run and in the long run.

One important issue which can be studied within this model, however, has only been mentioned in passing at one place or another. A complete understanding of the relationship between short run and long run effectiveness of government policies can arise

only out of a full dynamic analysis of the so-called intermediate run, i.e. the endogenous dynamic adjustment paths of the model after an initial policy shock. Some of the results of this paper seem to suggest that monotonic and stable adjustment paths cannot be expected in all situations. Hence, cyclical behavior seems possible. It is not clear at this stage whether reasonable conditions can be found to guarantee stability in spite of the fact that the dynamic mechanism is governed by one single variable alone, the stock of money. After the general dynamic structure has been analyzed it becomes justified to ask questions of economic stabilization policies. These and other related issues will be the subject of a subsequent paper.

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